Examiner’s Report
Pure Mathematics
Summary of Results

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>Abs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>46</td>
<td>99</td>
<td>116</td>
<td>44</td>
<td>75</td>
<td>198</td>
<td>37</td>
<td>615</td>
</tr>
<tr>
<td>%</td>
<td>7.5</td>
<td>16.1</td>
<td>18.9</td>
<td>7.1</td>
<td>12.2</td>
<td>32.2</td>
<td>6.0</td>
<td>100</td>
</tr>
</tbody>
</table>

Summary of Lower Marks

<table>
<thead>
<tr>
<th>≤</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>57</td>
<td>82</td>
<td>108</td>
</tr>
<tr>
<td>%</td>
<td>9.3</td>
<td>13.3</td>
<td>17.6</td>
</tr>
</tbody>
</table>

Comments on Candidates’ Performance

Paper 1

Question 1. Most of the candidates managed to do the first steps of part (a) correctly, either by direct substitution or using difference of two squares, but somehow some of them got lost in the final steps before getting the standard integral, or else they got the right integrand but used the incorrect standard integral. In part (b), practically all the candidates recognised the differential equation as “separable variables” type and also realised they have to use the answer in part (a). However a significant amount of candidates failed to integrate cosec y, despite being a standard integral.

Question 2. Most of the candidates obtained very high marks in this question. Practically all the candidates answered part (a) correctly and also in showing perpendicularity in part (b). Fewer candidates managed to show D is on the line and even fewer managed to get part (c) correct, either by dotting the incorrect vectors to find the angle or by incorrect use of the cross product to find the area.

Question 3. Almost all candidates, apart from very few exceptions, tackled this question in an improper way. It was like they had a recipe to follow and they put all the ingredients in it. Practically all the candidates started with the general equation of the circle, they substituted the given point, which is correct; then they substituted the point at the tangent, which is not correct since that is not a general point; and finally they substituted the point at the centre of the circle, which is also wrong. After all this they ended up with a system of three huge equations to solve, for some of them it took more than four pages of derivations, ending up nowhere. The most
interesting thing is that with a bit of thought, one could have done it without using the equation of the circle at all, obtaining a "rather" simple solution.

Question 4. Most candidates successfully proved the identity in (i) by starting from the LHS and using $\cos 3A = \cos (2A + A)$. The ones who started from the RHS were less successful. A minority applied De Moivre's theorem to prove the result. The majority attempted part (ii) but silly mistakes, such as wrong subtractions or substitutions, led to wrong answers. However, part (b) of the question was not as successfully accomplished. Those who did not recognize how to use parts (i) and (ii) to prove the given identity, were, in most cases, unsuccessful. Those who did, on the other hand, got to the answer or close to it. Some of the latter did not explain how $\cos x + \cos(x + 2\pi) + \cos(x + 4\pi)$ suddenly became $3\cos x$. Some candidates even tried to apply the result in question 9 to the solution of this part of the question.

Question 5. In part (a), the majority expanded the cubic equation and equated to zero. Some did not notice that $w$ was common so time was wasted in applying the remainder theorem to find a factor. Others wrote the answers without at least showing that $w$ was common, making it obvious that answers were obtained with the use of a calculator. In part (b), few realised that the required answer could have easily been obtained by adding 9 to both sides. Some used the quadratic formula, which still led to a correct answer. Others tried to substitute for $z$, and equated real and imaginary parts, but did not manage to find $z$. Some candidates even tried to find $z_0$, which was not required at all. Only a handful of candidates rearranged the brackets in (c) to compare to the given equation. The majority had no clue of where to start from and hardly attempted this part. Some assumed that the $z$’s in part (c) were the same as the $z$’s in the previous part of the question, and substituted for $z$, thus leaving them with no unknown to solve for.

Question 6. In part (a), a significant number of candidates got the first part correct although very few realised that the first integral could be used to evaluate the second very easily. As regards to the second part, few candidates managed to solve it with a significant number not even realising to use the first two answers. In part (b), a significant number of candidates managed to do the substitution correctly but there were still a lot of silly mistakes or interchanging simply $dx$ for $d\theta$. Even after the substitution many got the simplification of the trigonometric functions incorrect resulting in an incorrect integrand, not to mention all the mistakes with the limits, or how many candidates just left the limits unchanged as if they were integrating with the same variable.

Question 7. The vast majority of the candidates performed very badly in this question, despite the fact that the function given to sketch was pretty straightforward. The most common mistakes were either to draw the whole graph above the $x$-axis, maybe
the modulus sign induced this behaviour, or even worse where just a straight line was drawn. Surprisingly enough there were others who tried not to give a sketch at all. Obviously all this compromised the solution for the rest of the problem in most cases. All sort of manoeuvres were tried to show \( f(x) = 4 \) has no real roots and part (c), with most of them trying to obtain the discriminant for the quadratic in every possible way. Although this might have been used, many did it incorrectly, either treating the modulus as a square root etc. However there were far simpler ways to answer these parts of the question.

Question 8. Candidates did well in part (a), although some still added instead of multiplied the combinations. In part (b), the majority considered only 2 cases: when 3 higher level courses were chosen from \( H_1 \) to \( H_5 \) and when 3 higher courses were chosen from \( H_6 \) to \( H_{10} \). Few listed the other two possibilities (2 from \( H_1 \) to \( H_5 \) and 1 from \( H_6 \) to \( H_{10} \), and vice-versa). Very few got all the way to the final answer. Another common mistake was to take 1 higher level from \( H_1 \) to \( H_5 \) and choose the other 2 from any of the remaining 9, not taking into account that this would lead to repetitions and that if the other 2 were chosen from \( H_6 \) to \( H_{10} \), then \( L_2 \) and \( L_3 \) had to be selected. It is also suggested that students should, in such problems, explain (at least briefly) what they are finding, as different methods could lead to the same answer. In a considerable number of cases, candidates multiplied some values without explaining anything at all.

Question 9. In part (a), various methods were applied to prove the given equation. Candidates did well in part (b), however some ignored the 3 in the denominator, others multiplied the 3 by the three factors instead of by one. The three main correct approaches that were used were: removing the 3 in the denominator and then include it in the end, multiply the 3 by one of the factors, or taking the 3 as part of the numerator. A few candidates did not show any working at all, thus losing marks allocated to the finding of \( A \), \( B \) and \( C \).

Question 10. The majority managed to sketch \( f(x) \) correctly. On the other hand, they were not as successful in sketching \( g(x) \). It was noted that only a few were applying transformations to sketch \( g(x) \), even though this was not required, and most did not manage to find or mark the \( x \)-intercept on the negative side of the \( x \)-axis. In part (b), quite a number took matrix \( A \) to be an anticlockwise rotation instead of a clockwise. Some of those who sketched a diagram to find the images of \((1, 0)\) and \((0, 1)\) under \( A \) seemed not to know that \( \frac{\pi}{2} \) is equivalent to 90 degrees. However the majority still managed to prove that \( AB \) was equal to \( BA \). In part (c), only a few managed to reach the final answer. Some found the images of two points and tried to find the equation of the image assuming it was a straight line. Others tried to sketch the image and deduce the equation from the graph of the image.
Paper 2

Question 1. This was a very popular and well-answered question. About 20% of the candidates who attempted this question got full marks. In part (a), the $\int x^2 \cos^3 x \, dx$ caused numerous difficulties for many candidates. Instead of using the obvious substitution $u = x^3$, many tried integration by parts, with a predictable lack of success. The majority of candidates found part (b) very easy and straightforward. About 8% of the attempts indicated a very poor understanding of Ordinary Differential Equations and scored no marks.

Question 2. In part (a) the candidates wrongly tried to find the equation of the plane by working out the cross product of points $A$ and $B$, and many were unable to find the equation of the line where the two planes meet. In part (c) the candidates did not know the formula for the volume of a tetrahedron.

Question 3. This was by far the least popular question (attempted by only 28%) in the paper as is often the case with probability questions. It was also very poorly attempted. Only two candidates managed to obtain a complete correct solution. It is crucial at this point to reinforce that candidates must explain their reasoning in their answers. Simply writing expressions such as $3 \times 15$ is inadequate to gain all the available marks. In part (a) most candidates constructed the sample space for the roll of the two dice, but ignored completely the conditional probabilities when counting the possible case of success. Most candidates found part (b) difficult with only five correct answers. The key to this part of the question was the fact that John must win the second game if he is to win the tournament.

Question 4. Many candidates attempted this question and many knew how to apply the Newton-Raphson method. However, some were unable to differentiate $f(x) = \ln(1 + x^3) - 3$, and to solve $\ln(1 + x^3) = 3$. Candidates were familiar with Simpson’s rule and its applications.

Question 5. In this question, mostly on curve sketching, candidates tried to find the point of inflexion by using only the first derivative. Most candidates worked out correctly the equations for the asymptotes. Sketching $y = \frac{2}{f(x)}$ was beyond many students.

Question 6. This was quite a popular question, being attempted by 75% of candidates. There were only three perfect scores, while 31% scored less than quarter of the marks and 16% scored more than 9 marks. In part (a), the vast majority produced very neat sketches of the two polar curves, showing correctly the points of intersection, but very few bothered to find the tangents to the two curves and this made it very difficult to find the correct limits of integration when finding the required area.
In fact, only ten candidates managed to find the correct area. In part (b) a small number of candidates managed to simplify the determinant to obtain $2(\cosh x - \sinh x) = 4^x$, but for some reason the candidates found difficulties in solving the simple equation $e^{-x} = 4^x$.

Question 7. This was an unpopular and poorly answered question. Only 13% obtained more than seven marks and there were no full marks. In part (a), integration by parts was very poorly tackled. There was so much confusion in some of the handwriting that the candidates concerned were mistaking $\cos^2 x$ for $\cos 2x$. The volume of the solid generated was given correctly by $V = \pi I_4$, but many candidates tried to find $I_4$ from first principles instead of using the formula for $I_n$. Very few attempted part (b) with only one correct answer. This was a simple question on finding the minimum distance between the two points $A(1,0)$ and $P(x,y)$, where $x, y$ satisfy the equation of the given curve.

Question 8. A popular question attempted by 89% of the candidates. Seventeen candidates scored no marks, while twenty-four scored full marks. In part (a) many candidates made many silly mistakes, such as taking the unit matrix $I$ to be

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

or

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$  

Others made arithmetic mistakes obtaining the incorrect result $|A - 3I| \neq 0$ and concluded that the system of equations had only the trivial solution $x = y = z = 0$. Part (b) was generally well done. Only a few attempted part (c), but the majority of the attempts were very good, with 31 candidates obtaining full marks. Some of the candidates chose the hard way to find the image plane by using the transformation

$$A \begin{pmatrix} 4 \mu - \lambda \\ \mu \\ \lambda \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

and tried with difficulty to eliminate $\lambda$ and $\mu$. The easy way was to use the inverse matrix $A^{-1}$ found in part (b).

Question 9. Few candidates attempted this question. The candidates found it difficult to find the Cartesian equation from $\arg(z + i) - \arg(z - i) = \frac{\pi}{4}$ and were unable to describe the locus of $Q(x,y)$ using the two given relations.

Question 10. A very small number of candidates attempted this question and many of them obtained low marks. In both parts (a) and (b), the candidates were unable to show by mathematical induction that if a statement $P_n$ holds for $n = k$, then its also holds for $n = k + 1$. 