

**LIN1032 Formal Foundations for Linguistics**

Tuesday, 9<sup>th</sup> June 2009

13.00 - 14.30

**Surname:** \_\_\_\_\_ **Name:** \_\_\_\_\_ **I.D. No.** \_\_\_\_\_

**Complete ALL the tasks in sections A, B, C and D. Use the lined sheets provided for your answers. Write your name on every sheet of lines paper.**

**Section A. Set Theory**

1. Suppose we have a small world in which there are exactly 5 people. These people fall into three sets: *Heroes*, *Villains*, and *Tall people*.
  - Heroes (H) = {Hamlet, Desdemona}
  - Villains (V) = {Richard, Ophelia, Hamlet}
  - Tall people (T) = {Hamlet, Ophelia}

Draw a Venn Diagram representing the three sets and the things they have in common.

**(5 marks)**

2. Using the same sets as in question (a), indicate whether the following set-theoretic statements are true or false (circle the right answer in each case):
  - a.  $Hamlet \in (H \cap T)$
  - b.  $Ophelia \in (V \cup T \cup H)$
  - c.  $Hamlet \in (V - H)$
  - d.  $H \subseteq V$
  - e.  $\{Ophelia, Desdemona\} \subseteq (H \cup V)$

**(5 marks)**

3. For each of the following sets, give a definition **by description**.
  - a. the set of all pets
  - b. the set of all dogs with three legs
  - c. the set of all numbers greater than 0
  - d. the set of all lecturers who are Maltese

**(5 marks)**

4. Consider the following sets:
  - $A = \{1,6,7,9\}$
  - $B = \{4,8,10,6\}$

- a. Let us define a binary relation  $R: A \times B$ . List the elements of this relation.
- b. List the elements of the **inverse relation**  $R^{-1}$ .

**(5 marks)**

5. Suppose we have a small universe of discourse  $U$ , divided into two sets. The set  $F$  is the set of university faculties. The set  $D$  is the set of people who are deans of a faculty.
- $F = \{\text{Arts, Science, Theology}\}$
  - $D = \{\text{Martina, Pawlu, Thea}\}$

Assume that the following statements are true:

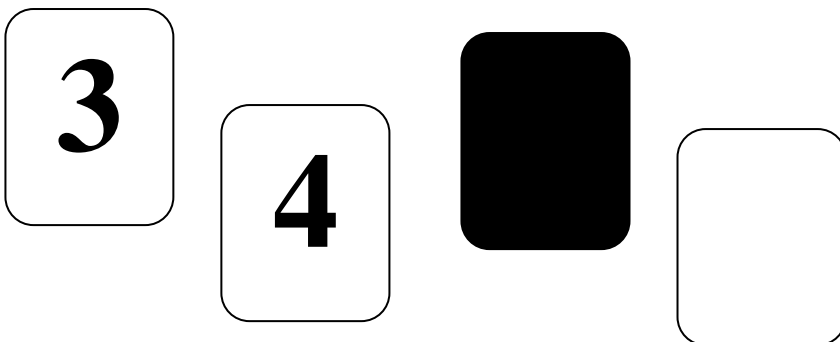
- Martina is the dean of the Arts Faculty.
  - Pawlu is the dean of the Theology Faculty.
  - Thea is the dean of the Science Faculty.
- a. We can think of the *dean-of-faculty* relation as a function. Draw a diagram to illustrate this function.
- b. What is the **range** of the function?
- c. What is the **domain** of the function?
- d. What is the term that best describes the *dean-of-faculty* function in our universe? Is it a **surjection**, an **injection**, or a **bijection**?

(5 marks)

## Section B. Propositional and Predicate Logic

6. Draw up the truth table for logical disjunction (*inclusive OR*).
- (3 marks)
7. Imagine that you're playing a game of cards. Each card has a number printed on one side. On the other side, it is either black or white. You are given the following statement, involving a logical implication (i.e. a proposition of the form  $p \rightarrow q$ ):
- *if a card has an even number on one side, **then** it is black on the other side.*

Your task is to check whether this statement is true or false. Now you are shown the following cards:



Which of the cards should you turn over, in order to check whether the statement is true? Use your knowledge of the truth table for implication to resolve the problem.

(7 marks)

8. For each of the following formulas, indicate whether it is an open sentence or a proposition:

- a.  $\forall x[\text{car}(x) \rightarrow \text{drive}(y, x)]$
- b.  $\exists z\exists y[\text{teacher}(z) \wedge \text{student}(y)]$
- c.  $\forall x[\text{teacher}(x)]$
- d.  $\forall z\exists y[\text{teacher}(z) \rightarrow \text{student}(y)]$
- e.  $\text{give}(x, y, z)$

**(5 marks)**

9. Translate the following sentences into formulas of predicate logic, using the following as your basic expressions:

- Individual variables:  $x, y, z, \dots$
- One-place predicates: *student*, *clever*
- Two-place predicates: *love*
- Individual constants:  $j$  (= John Smith),  $k$  (= King Canute)

- a. Every student is clever.
- b. There is a student who loves John Smith.
- c. King Canute is clever.
- d. Every student loves King Canute.
- e. Everyone loves someone.

**(6 marks)**

10. The following English sentence has at least two possible readings.

- *Everyone did not see Star Trek.*

Give the two readings as predicate logic formulas. Use the following as your basic expressions:

- Two-place predicates: *see*
- Constants:  $s$  (= Star Trek)

**(4 marks)**

## Section C: Graphs and Trees

11. Consider the following PS-rules and lexicon.

Phrase Structure Rules		Lexicon
S	→ NP VP ADV	Det: <i>a, the</i>
VP	→ V NP	N: <i>insect, fly, garden</i>
NP	→ Det (A) N (PP)	V: <i>swallowed</i>
PP	→ P NP	A: <i>huge</i>
		ADV: <i>yesterday</i>
		P: <i>in</i>

- Draw the parse-tree for the sentence *the huge insect swallowed a fly in the garden yesterday*
- Write down (a) the set of terminal (leaf) nodes, (b) the set of non-terminal nodes of the tree generated by the rules.

**(6 marks)**

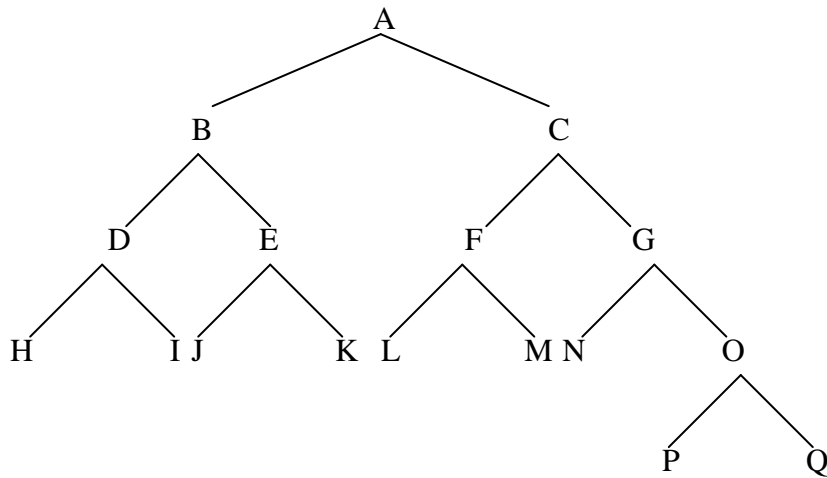
12. The following rules generate a number of *a/b* strings.

- $A \rightarrow a$
- $A \rightarrow B a$
- $B \rightarrow b$
- $B \rightarrow b A$

- Write down the tree for a one-element string.
- Write down the tree for an eight-element string generated by the rules.

**(5 marks)**

13. Look at the following tree and list the **sets** of objects listed below.  
NB. Make sure you use the usual conventions for brackets denoting unordered sets, pairs etc.



List the following:

- the set of all nodes
- the set of parent nodes
- the set of child nodes
- the set of leaf nodes
- the set of edges

**(5 marks)**

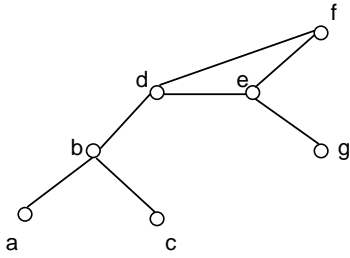
14. Look at the tree in (3) again and determine the height of:

- the tree
- the node B
- the node G

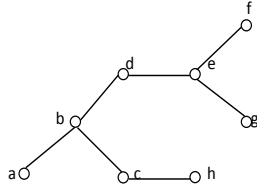
**(5 marks)**

15. Which of the following graphs are trees? For each non-tree graph, say why it is not a tree.

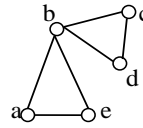
a.



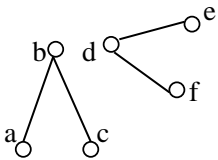
b.



c.

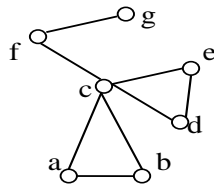


d.

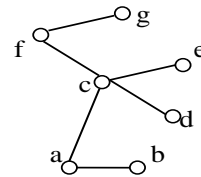


(6 marks)

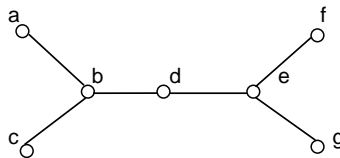
e.



f.



16. Change the following into a rooted tree, labelling the root node as R.



(3 marks)

### Section D: Feature Structures

17. Using the feature tableau below, write down the feature structures for the *noun*, *verb*, *adjective*, and *preposition* as feature matrices.

	+N	-N
+V	A	V
-V	N	P

(2 marks)

18. Draw the feature matrix and the DAG for the Maltese word *kitbet* ‘she wrote’, using the following attribute names: *majcat*, *N*, *V*, *agr*, *fm*, *pl*, *pst*, *per*.  
**(10 marks)**

19. In each of the following pairs, indicate whether the feature structure on the left (in A) subsumes that on the right (in B)?

A	B
a. $\left[ \begin{array}{l} \text{gender: feminine} \\ \text{number: singular} \end{array} \right]$	$\left[ \begin{array}{l} \text{number: singular} \\ \text{gender: feminine} \\ \text{person: second} \end{array} \right]$
b. $\left[ \begin{array}{l} \text{feminine: +} \\ \text{plural: -} \end{array} \right]$	$\left[ \begin{array}{l} \text{feminine: +} \\ \text{plural: -} \end{array} \right]$
c. $\left[ \begin{array}{l} \text{gender: masculine} \\ \text{person: third} \end{array} \right]$	$\left[ \begin{array}{l} \text{number: singular} \\ \text{gender: feminine} \\ \text{person: second} \end{array} \right]$
d. $\left[ \begin{array}{l} \text{gender: masculine} \\ \text{person: third} \\ \text{number: singular} \end{array} \right]$	$\left[ \begin{array}{l} \text{gender: masculine} \\ \text{person: third} \end{array} \right]$

**(4 marks)**

20. In each of the following cases, what is the result of unifying the feature structure on the left with that on the right?

A	B
a. $\left[ \begin{array}{l} \text{gender: feminine} \\ \text{number: singular} \end{array} \right]$	$\left[ \begin{array}{l} \text{number: singular} \\ \text{gender: feminine} \\ \text{person: second} \end{array} \right]$
b. $\left[ \begin{array}{l} \text{feminine: +} \\ \text{plural: -} \\ \text{per: 3} \end{array} \right]$	$\left[ \text{feminine: +} \right]$
c. $\left[ \begin{array}{l} \text{number: plural} \\ \text{person: third} \end{array} \right]$	$\left[ \begin{array}{l} \text{number: singular} \\ \text{gender: feminine} \\ \text{person: third} \end{array} \right]$
d. $\left[ \begin{array}{l} \text{gender: masculine} \\ \text{person: third} \\ \text{number: singular} \end{array} \right]$	$\left[ \begin{array}{l} \text{gender: masculine} \\ \text{person: third} \end{array} \right]$

(8 marks)