Examiner’s Report
Pure Mathematics
**Summary of Results**

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<th>C</th>
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**Summary of Lower Marks**

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**Comments on Candidates’ Performance**

**Paper 1**

Question 1. Most candidates recognised the differential equation as *separable variable* type and done the first step correctly. Also, most of them realised they need to use *partial fractions* on the LHS of the equation in order to integrate. However a significant number of candidates did not manage to get the correct value of the constant of integration and only few managed to get to the final step in order to find the values of \( p \) and \( q \).

Question 2. (a) Almost all candidates realised that the best way to tackle part (i) was by using the quotient rule, however a significant number of them did not manage to get the correct answer. As regards to part (ii), there were many candidates who failed to answer this correctly. In most cases they either just took the reciprocal of \( \frac{1}{2x^2-1} \), that is \( 2x^2 - 1 \), and multiplied it by its derivative, \( 4x \), or they tried to use the quotient rule, which does not work out in this case. (b) A significant number of candidates answered the first part correctly apart from some simplifying mistakes. Only few of those who answered the first part correctly were not able to use their answer to obtain the gradient of the tangent and hence, its equation.

Question 3. Almost all candidates answered questions (a), (b) and the first part of (c) correctly. However, only few managed to get the right answers for the distance from \( D \) to \( \ell_1 \) and the area of \( \triangle ABD \). A significant number of candidates who answered the last parts correctly used the vector product in their solution when they could have
given a much simpler solution. Most of those who failed to answer the last part correctly also used the vector product but they did not use it properly. It is worth noting that there are students who mix up the Cartesian form and the vector form of the equation of a straight line, in particular there were some who tried to use the equation of the distance of a point from a line in part (c) using the coefficients of the unit vectors i, j and k.

Question 4. Many students managed to answer part (a) correctly either by long division or by comparing coefficients. However there was a significant number of students who just showed that \( f(-k-2) = 0 \) concluding that \((x + k + 2)\) is a factor. In part (b) almost all students failed to write the expression \( 2P(x) \) as the sum of three perfect squares. However there were some students who still attempted to solve the equation \( f(x) = 0 \), doing some good steps on attempting to find the solutions. It is worth noticing that out of all the students that went directly to the last part, only one ended up with complex solutions and it was because of some mistakes and not by choice.

Question 5. Candidates did well in this question. Most answered part (a) correctly. Part (b) was well attempted, with the most common mistake being not interchanging the inequality sign when finding the reciprocal. A good attempt was made at part (c), however some candidates did not write the general solution and just found one value of \( x \). Others thought that \( e^x \) (and not \( x \)) had to be in the given range, hence did not find the second value for \( x \).

Question 6. Candidates did well in this question. The most common mistake in part (a) was that, after finding the correct values of \( A, B \) and \( C \) of the partial fractions, students forgot to write the \( x \) in the \( Bx \) term in the final answer, hence ending up with another integral. A lot of mistakes in logarithmic simplification led to a wrong value for \( p \). It was noted that a good number of students tried to avoid simplifying the logarithmic additions and subtractions with the use of the laws of logarithms, and gave \( p \) as a decimal number rather than in surd form. The majority of candidates had a very good idea how to tackle part (b). However, a lot of silly mistakes such as wrong multiplications and substitutions led to a wrong answer for the given integral. It was noted that most mistakes resulted when students skipped steps. Most also forgot the constant of integration in the final answer.

Question 7. Candidates did not do so well in this question. In part (a), quite a number assumed that the image was going to be a circle, thus the method they used was completely wrong. Few managed to find the area of the image, even though it could be worked out without finding the equation of the image. In part (b), quite a number were trying to find \( XZYZ \) and \( YZXZ \) instead of \( XYZ \)
and \( Y X Z \). It was noted that even though they ended up with a \( 2 \times 1 \) matrix being multiplied by another \( 2 \times 1 \) matrix, they kept on going and somehow multiplied these two matrices. Others interchanged the order of (i) and (ii). Although most guessed the answer to the last part, very few actually understood and explained why system (i) gave better results.

**Question 8.** (a) Many candidates managed to give the correct expression for \( h(x) \), a bit less for \( h^{-1}(x) \). However a significant number of students still failed to give the correct domain in each case.
(b) Many candidates failed to answer this question correctly, despite being an easy question. Most of them did not realise that what was being asked is simply the largest domain on which the function is real valued.

**Question 9.** Part (a) was answered correctly by most candidates, but very few tackled parts (b) and (c) correctly. Only a handful seemed to understand what was actually required to prove. Nobody obtained full marks in this question.

**Question 10.** Although this question was extremely easy to solve with the use of a Venn Diagram, which the majority drew, very few filled it in correctly. The vast majority did not realise that the 250, 350 and 150 included the 50 people who had the three measures. This reflects poor knowledge of intersection of sets and Venn Diagrams. As a result, when finding the number of tickets, students ended up with 1650 tickets for those with 2 or 3 measures, which was impossible. Others did not have the slightest idea of the topic, giving the impression they did not study the topic at all.

**Paper 2**

**Question 1.** Solutions to part (a) of the question were rather disappointing. Some of the attempted solutions were so poor that one wondered if some students had ever tackled Linear Differential Equations. The integration of the function \( (x + x^3)e^{x^2} \) proved to be difficult to many of the candidates. The most frequent errors were:
(i) the incorrect substitution \( u = x + x^3 \) instead of the obvious substitution \( u = x^2 \), and
(ii) Integration by parts, choosing \( dv = e^{x^2} \, dx \).
Part (b) was very well attempted. The majority of candidates were able to solve the given second order differential equation. A few number of them chose incorrectly \( y = ax^2 \) and \( y = ax^2 + \beta \) as trial solutions to find the particular integral.

**Question 2.** Some of the candidates who attempted this question failed to split \( \cos^n x \) into \( \cos^{n-1} x \cos x \). Only a few used the result of part (a)(i) to find the volume of the solid generated by the rotation through \( 2\pi \) radians of the region bounded by the curve \( y = \sin x \cos^3 x \) and the \( x \)-axis between \( x = 0 \) and \( x = \pi/2 \).
Question 3. This was a question which was attempted by nearly all the candidates. Candidates knew how to use the Newton-Raphson method and seemed to be also familiar with Simpson’s rule, however some did not manage to expand \( \ln \left( 1 + \frac{x^2}{9} \right) \) correctly.

Question 4. Part (a) was very poorly attempted. Almost all candidates were unable to show that \( \frac{dy}{dx} < 0 \) for negative \( a \). The sketching of the curve \( C_1 \) in part (b) was generally well done. Most candidates obtained correctly the two asymptotes, but in sketching the graph some forgot that \( a < 0 \) and plotted the vertical asymptote to the right of the \( y \)-axis instead of to the left. This changed the features of the curve. The main difficulty in part (c) was in differentiating the equation for \( C_2 \). Almost all candidates made many arithmetic mistakes in finding \( \frac{dy}{dx} \), although a few deduced the minimum point from the curve \( C_1 \). In part (d) the graph of \( C_2 \) was done well by only a few. Many assumed incorrectly that both curves had the same asymptotes. In fact, the vertical asymptote was the same for both curves, but not the horizontal asymptote. Part (e) proved to be too difficult for all the candidates and there were no correct solutions.

Question 5. Some candidates who attempted this question used laborious ways to prove that
\[
(sinh x + cosh x)^4 = cosh 4x + sinh 4x \quad \text{and} \quad (sinh x - cosh x)^4 = cosh 4x - sinh 4x.
\]
Only very few candidates used these results to solve part (b)(ii) of the question.

Question 6. Many candidates worked out correctly the point of intersection of the two lines and the angle between these lines. However few were able to find the equation of the plane \( \Pi_2 \) and the area of triangle \( \triangle PQR \).

Question 7. Only a few candidates tried to solve this question and most of them obtained low marks. Many did not manage to find the polar coordinates of the point of intersection of \( |z| = 1 \) and \( |z - w| = |z - 1| \). Finding the area of the isosceles triangle was beyond nearly all the students.

Question 8. In part (a)(i) many candidates were unable to find the first four derivatives correctly, mainly because they did not know the derivatives of \( \tan x \) and \( \sec x \). In (a)(ii), the solutions were rather disappointing. Some used the series for \( \ln(1 + x) \) to find \( \ln(\sqrt{2}) \) when specifically they were asked to use the series in (a)(i). A few number of students tried the erroneous substitution \( \cos x = 2 \). Very few substituted the correct values of \( x \).

In part (b), the induction method caused problems for many candidates. Although some candidates realized that they had to show that “true for \( n = k \) implies true for \( n = k + 1 \)” only a few were able to do it in this context. Common errors
were: \( f^{(k+1)}(x) = f^{(k)}(x) \times f^{(1)}(x) \) or \( f^{(k+1)}(x) = f^{(k)}(x) + f(x) \). It is worth mentioning that there were students who did not know how to locate the minimum point of a function.

Question 9. Part (a) was very well answered. The only mistakes were made in calculating the determinant of \( A \), but most candidates were awarded the seven marks which were allotted to this part of the question. In part (b), the explanations given as to why the equation \( Ax = 0 \) has at least one solution were very ambiguous. Very few stated that \( x = 0 \) is a solution independent of the value of \( k \). In the solution of \( Ax = 0 \) many candidates showed no evidence at all of understanding this part of this question. In finding the value \( k \) for which the equation has more than one solution, a large number of students thought that the system of equations in part (a) was related to the system of equations in part (b). Those who found the correct value of \( k = -1 \) by equating the determinant to zero also managed to solve the equation.

Question 10. Probability continues to be an area of difficulty. Even the simpler parts of problems are unapproachable to many candidates. In fact, only ten candidates managed to obtain more than eight marks. In part (a), counting principles still seem to continue to present problems. Those who attempted this question managed to do part (i), but made no progress in the remaining parts (ii) and (iii). Some candidates even tried to count the solutions. Others used incorrect arguments. Part (b)(i) was very well attempted except for some arithmetic mistakes. Students seem to be very happy when they can use Venn diagrams. Part (b)(ii) was disappointing. Most candidates thought that \( P(B \cap C) = P(B) \times P(C) \). Only two candidates realized that \( P(A \cup B) + P(C) - P(B \cap C) = 1 \).