UNIVERSITY OF MALTA

THE MATRICULATION CERTIFICATE EXAMINATION
ADVANCED LEVEL

PURE MATHEMATICS

May 2008

EXAMINERS' REPORT

MATRICULATION AND SECONDARY EDUCATION
CERTIFICATE EXAMINATIONS BOARD
Summary of Results

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Summary of Lower Marks

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Comments on Candidates' Performance

Paper 1

Question 1. (a) Generally this part was answered well. However some students had difficulty, or could not bother simplifying the algebraic expression, especially for part (ii). (b) To solve the two equations, the students ideally, had to use the answers in part (a). Many obtained that \( b = -c \), and \( b^2 = 4ac \), not realizing that this represents equal roots. Algebra gives \( c = 4a \) and \( b = -4a \). The resulting quadratic equation: \( x^2 - 4x + 4 = 0 \) was only achieved in half of the cases, to give the equal roots of \( \alpha = 2 \), \( \beta = 2 \). (c) Substituting \( x = y - 1 \) in the general algebraic equation of \( ax^2 + bx + c = 0 \) should have been a simple task, however about half the students did not manage the algebraic expansion. Once the equation \( ay^2 + (b - 2a)y + (a - b + c) = 0 \) was obtained not many students achieved the final result of new roots \( (\alpha + 1) \) and \( (\beta + 1) \). (d) When finding the equation of new roots \( (\alpha - \beta, \beta - \alpha) \), the sum of the new roots was answered well \( \alpha - \beta + \beta - \alpha = 0 \). However, for the product of the new roots \( (\alpha - \beta)(\beta - \alpha) \) in terms of \( a, b \) and \( c \) this was poorly answered to obtain \( \frac{-(b^2 - 4ac)}{a^2} \), again due to below standard algebraic manipulation.

Question 2. Most students recognized to solve the differential equation correctly as separation of variables. However a significant number of them did not manage to integrate correctly \( \frac{1 - x}{1 + x^2} \). Ideally the student had to separate the expression into two fractions and then integrate.
Question 3. (a) This was a poorly answered question, many had no idea regarding which series they were dealing with, the properties and general formulae. Some arrived to the first stage that \( u_1 + u_3 = 10 \) for the A.P., hence \( a + d = 5 \). However, moving on using this knowledge into the work of G.P.'s many made the mistake of using the symbol \( a \) for the first term of the new G.P. not realizing that the first term was in fact the 5 found above. Even so they could have followed the standard approach of generating the three terms, \( a + d, a + 3d, \) and \( a + 7d \) for the new G.P. and used the knowledge of the common ratio \( r = \frac{a + 7d}{a + 3d} = \frac{a + 3d}{a + d} \) to find \( a = d = \frac{5}{2} \). For the ones who reached this point they immediately obtained the correct answer of \( S_{15} = 300 \). (b) The first part of this question was well answered using the standard binomial expansion techniques. However even after getting both expansions individually correct some did not realize that their final answer was not as required to prove. The validity was answered poorly. The final part for taking \( x = \frac{1}{10} \) and substituting in the equation given in the question should have been well answered, however they had to show the correct substitution using both sides of the given equation.

Question 4. The first part of this question was answered correctly by most students. However, the second part illustrated the fact that the majority of the students have no idea how to work with vectors! Very few managed to arrive to the final answer. The majority did not tackle the problem in the right manner and so it became difficult for them to solve it. At least a significant number of students knew that the scalar product of two perpendicular vectors is zero and that the coordinates of the point of intersection of two lines satisfy the equation of both lines.

Question 5. This was a poorly answered question. Very few solved the first part correctly, which involved standard techniques and results about the product of two complex numbers. In the second part, many students did not know the exact meaning of \( \text{Re} \left( \frac{z + 2}{z - i} \right) \). In fact, even if some of them arrived at the equation of a circle, they did not manage to give the correct answers for the centre and radius.

Question 6. (a) A high percentage of students read \( e^{\frac{x}{2}} \) as \( e^{\frac{x}{8}} \). This did not really have an effect on the method of answering the first part of the question for \( \ln y \), still many did not give the most simplified form of the \( \ln \) equation. To obtain \( \frac{dy}{dx} \) using the \textit{hence} method, that is differentiating the resulting \( \ln y \) equation would have been the simplest approach and those how took this method generally got it correct. However, many who even got the first part correct took the \textit{otherwise} approach, that is, differentiating the original equation as a product and a quotient. About half managed to get the full derivation, but many got lost in the algebra. (b) This was generally a poorly answered question. The few
who managed to get \( \frac{dy}{dx} = \frac{2a}{y} \), failed to substitute \( y = 2at \), to obtain \( \frac{1}{t} \). Although the gradient of the normal should have been \(-t\), the expression was left in terms of \( y \) (and sometimes in terms of \( x \) when students put \( y = \sqrt{4ax} \)). The students who differentiated parametrically, \( \frac{dy}{dt} = 2a \), and \( \frac{dx}{dt} = 2at \), obtained the correct gradients of the tangent and normal. So long as the gradient of the normal was in terms of \( t \), the equation of the normal \( y = at(2 + t^2) - tx \) and the coordinates of \( G(a(a + t^2), 0) \) were found correctly. Otherwise it was impossible to continue. Finding the coordinates of \( G \) and \( N(at^2, 0) \) should have been simple even if they could not draw the resulting curve, line and intersections. The subtraction \( 2a + at^2 - at^2 = 2a \) or the distance between two points could have been found.

**Question 7.** It is difficult to understand how some students could not manage to find the constants \( A \) and \( B \) of part (a). In part (b) a significant number of students gave the incorrect domain or range, however most of them still got the correct sketch for \( f(x) \). The majority of the students gave a correct answer for \( f^{-1}(x) \), however they simply interchanged the answers of the domain and range of \( f(x) \) when giving the domain and range of \( f^{-1}(x) \) without even checking if their answers make sense. Very few students got the last part correct, they did not manage to obtain the correct expression for \( h(x) \). It is puzzling that a significant number of students interpreted \( (f^{-1} \circ g)(x) \) as \( (f^{-1} \circ g) \) multiplied by \( x \).

**Question 8.** On the whole those who attempted this question answered it correctly. (a) This was recognized as a combination \( \binom{6}{4} \) and \( \binom{10}{7} \) however, some added instead of multiplied. (b) The arrangement of the 8 letters TROTTING with the three repeats should have been \( \frac{8!}{3!} \). However, simply putting 8! would have a numerical consequence on parts (i) and (ii) but not on methodology. (b)(i) The arrangement of the letters \( N \) and \( G \) next to each other was \( \frac{7!}{3!} \) but many forgot to multiply by 2 because of \( NG \) or \( GN \). Some did not divide this answer with that of the first part of (b) to find the probability. (b)(ii) The arrangement of all the three \( T \)'s next to each other should have been \( 6! \), a common mistake was simply using \( 3! \) or \( \frac{6!}{3!} \). If the student divided (b)(i) by the answer of the first part of (b) then they similarly did the same thing here, otherwise it was completely omitted.

**Question 9.** On the whole there was a good attempt of the question. (a) This was answered quite well choosing \( u = \ln 2x \) and \( u' = x \), however the most common error was in the differentiation of \( \ln 2x \) which should have been \( \frac{2}{2x} \); many omitted the 2 in the numerator. (b) Many did not complete all the steps in integrating by substitution. The most common mistake was leaving the \( x \) and the \( dx \) as given in the question, therefore subsequently they could not perform the integration properly, some being completely oblivious of the problem they caused. Another common problem was that although the substitution was made correctly and the
integration process was also correct, the limits were not changed and therefore resulting in the wrong evaluation. (c) The first part on partial fractions was properly attempted by most students. However unnecessary algebraic mistakes resulted in the wrong values of the constants. The correct result should have been \( \frac{1}{x+1} + \frac{2x}{x^2+1} \). The next step provided a number of mistakes. The mistake for the first fraction was to express it as \((x + 1)^{-1}\) and integrate it by some new method. The second fraction when correct was to think that this was in the form of \(\tan^{-1} x\), (and a few put \(\sin^{-1} x\) or \(\sinh^{-1} x\)). Obviously when the partial fraction was incorrect many realized they were wrong and stopped. The students who reached the stage of complete correct integration normally got the evaluation correct but still some were making many basic log errors. The students who normally got part (b) correct usually sailed through part (c).

Question 10. Many of the students did not give the correct answer for part (a) of this question. They had simply to remember the relation between \(\sin \theta\) and \(\cos \theta\). The answers to part (b) were better although some errors were made in finding \(R\) and \(\theta\), and also in the use of the general solution of \(\sin \theta\).

Paper 2

Question 1. Students who attempted this question did not find any problems in proving that the two given planes were perpendicular to each other. There were still a few students who made mistakes when solving simultaneous equations, which were needed to find the vector equation of the line of intersection of the two planes. Also, some were unable to work out the cross product correctly. Determining the reflection of a point in a plane proved to be beyond the ability of most students.

Question 2. (a) Generally this part was poorly answered well. Many errors were made in using integration by parts. Very few students arranged the integral in the form

\[
I_n = \int_0^{\frac{\pi}{2}} \cos^n x \sin^2 x \, dx
\]

in order to integrate \(\cos^n x \sin^2 x\) by parts. \(I_4\), \(I_6\) and \(I_8\) were found in terms of \(I_0\), but \(I_0\) was incorrectly evaluated. (b) Most students sketched the polar curve correctly. Very surprisingly most students found many difficulties in proving the given expression for \(r^2\). Some even tried to use complex numbers. Only four candidates managed to obtain the correct value for the area. The expression for \(r^2\) was given as a hint to relate the area to part (a).
Question 3. This question was well attempted. (i) Most students gave the value of the determinant as

\[ |A| = \cos^2 \alpha \cos^2 \beta + \cos^2 \alpha \sin^2 \beta + \sin^2 \alpha \sin^2 \beta + \sin^2 \alpha \cos^2 \beta, \]

but seem to have forgotten the identity \( \cos^2 \theta + \sin^2 \theta = 1 \) which gives \( |A| = 1 \). (ii) Most students computed \( A^{-1} \) correctly, but again did not simplify the elements. (iii) Many students did not solve the system of equations by using \( X = A^{-1}H \). Instead they used elementary row operations without success.

Question 4. Almost all students attempted this question and many obtained good marks. However, some students tried to use unsuccessfully other methods instead of the integrating factor method to solve the differential equation in part (a). In working out part (b) of this question, errors were made in solving the auxiliary equation. The roots of the equation were given as 0 and -4 or 2 and -2 instead of 2i and -2i. The particular integral was correctly worked out in many cases.

Question 5. Very well attempted by most students. Many did not find the oblique asymptote.

Question 6. (a) The main error in this part of the question was that most students found the sum of \((n - 1)\) terms by summing from \( r = 2 \) to \( n \). (b) Very poorly answered. It is clear that most students were trained blindly on how to tackle induction problems. None of the students noted that an odd number is of the form \( 2r + 1 \). This would have made the solution straightforward.

Question 7. This question was unpopular and disappointing. (a) Deriving probability equations is something most students are not trained to do. Most students just drew a Venn diagram, but did not produce any mathematical proofs. Part (i) was straightforward, part (ii) is deduced from (i) and part (iii) is deduced from (ii) and (i). (b) A lot of confusion when dealing with conditional probability. Most candidates managed to do only parts (ii) and (iii). Many students still do not seem to know that \( 0 \leq P[E] \leq 1 \).

Question 8. Very few students tried to work out this problem and many of them obtained a low mark for it. There were a number of students who while calculating the surface area squared wrongly \( \frac{dy}{dx} \), writing 3x^4 instead of 9x^4. Nearly no one worked out part (b) of the question completely correct. The students were unable to determine \( \frac{dA}{dy} \) and \( \frac{dV}{dy} \).
Question 9. The students who attempted this question worked out correctly the proof by induction part. However, to prove that \( \cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1 \), some students preferred to use the trigonometric identities instead of De Moivre’s Theorem. When working out the last part of (c), the students did not realize that there were \( 2n + 1 \) terms and not \( n \) or \( 2n \) terms. Very few noticed that \( e^{-in\theta} \) was common to all the terms in the series and that the summation was the previous summation multiplied by \( e^{-in\theta} \).

Question 10. Many students attempted this question but some were unable to find the value of \( 1 - \frac{1 - \cos 1}{2 + \sin 1} \). Students were familiar with Simpson’s rule and integrating a series expansion and obtained good marks for these parts of the question.

Chairperson
Board of Examiners
July 2008