A new algorithm for characteristic extraction and matching in numerical relativity

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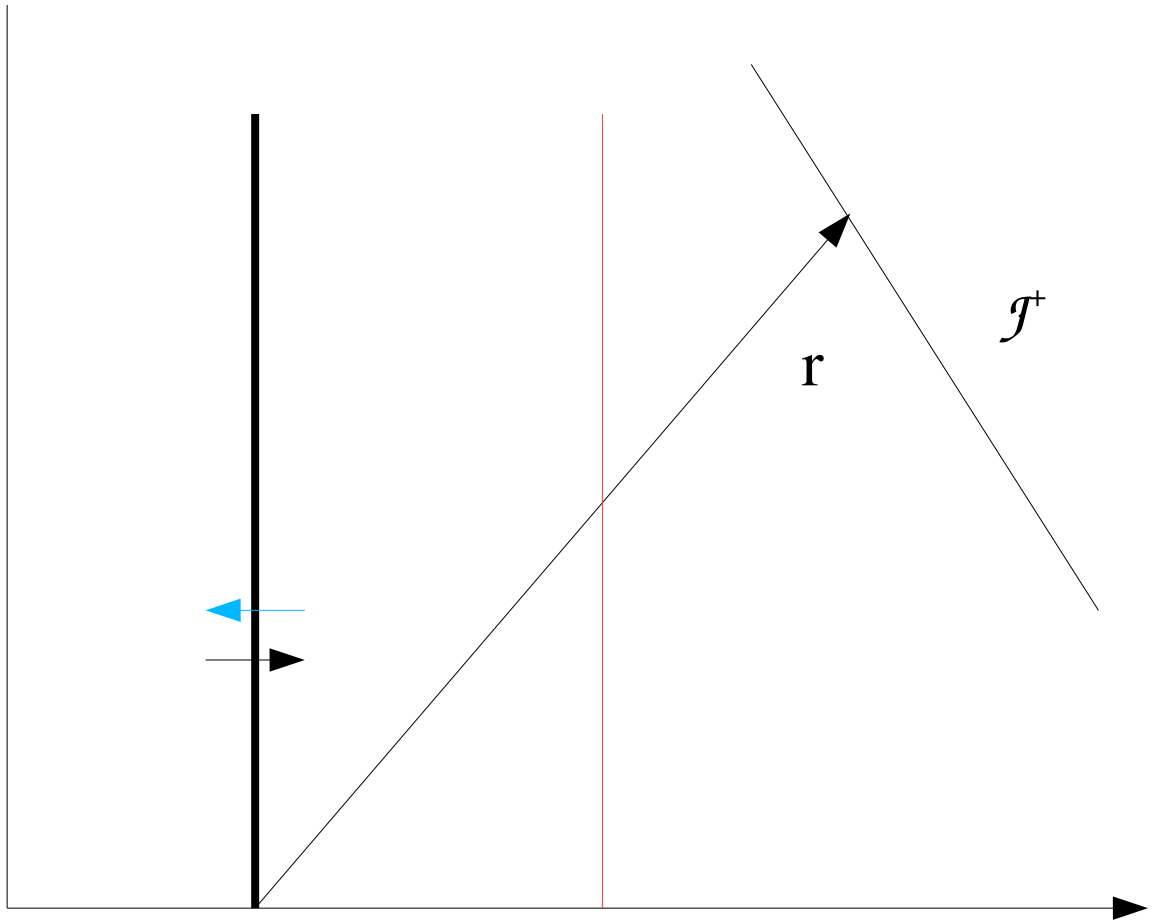
Outline

- Characteristic extraction: Original algorithm
- A new algorithm for characteristic extraction and matching
- Conclusion


r
$\mathcal{I}^+$
Worldtube
Outer
boundary
x, y, z

Characteristic formulation of numerical relativity

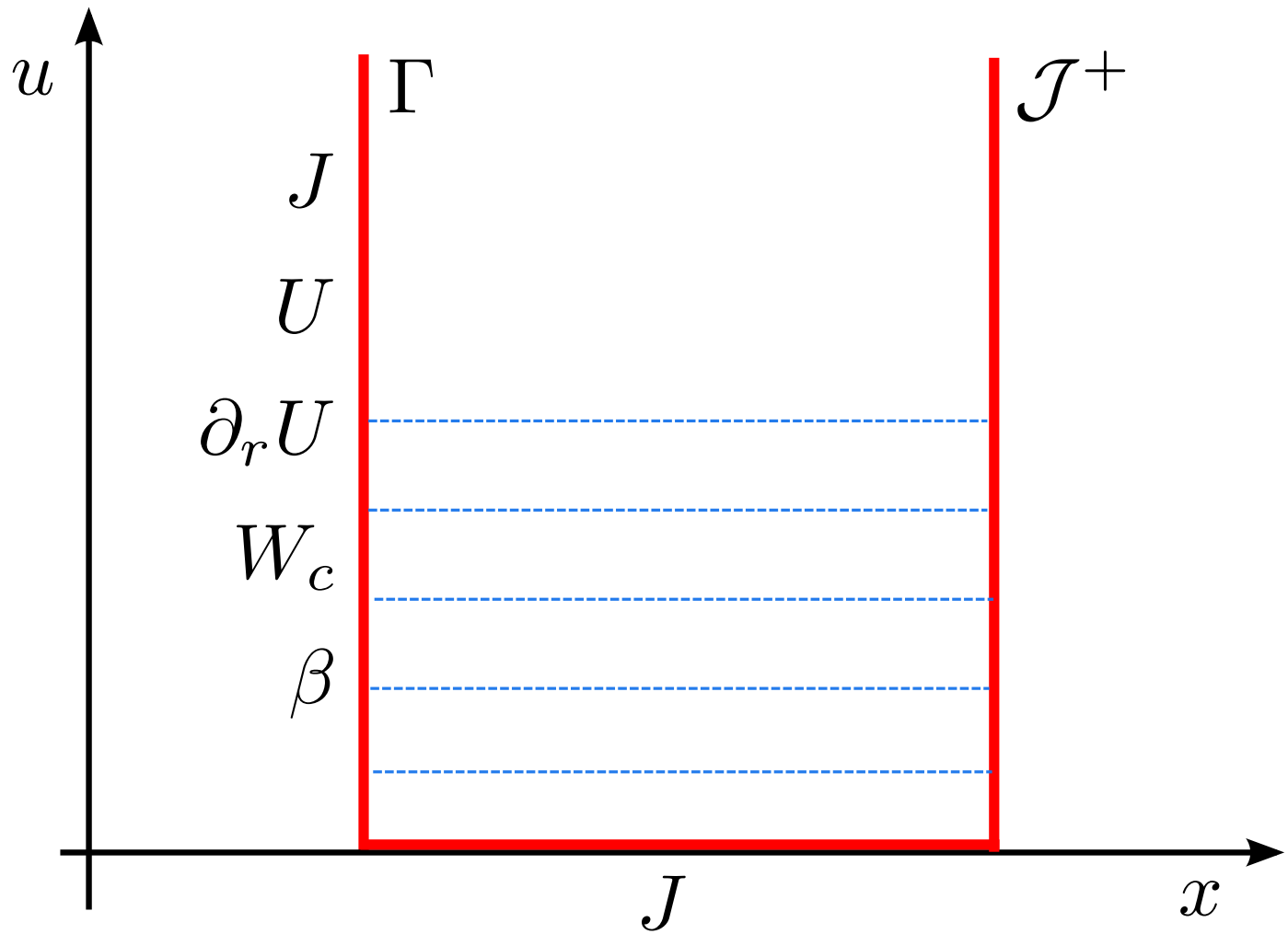
The Bondi-Sachs metric is

$$ds^2 = -\left(e^{2\beta}(1+W_c r) - r^2 h_{AB} U^A U^B\right) du^2$$
$$-2e^{2\beta} du dr - 2r^2 h_{AB} U^B du d\phi^A + r^2 h_{AB} d\phi^A d\phi^B,$$

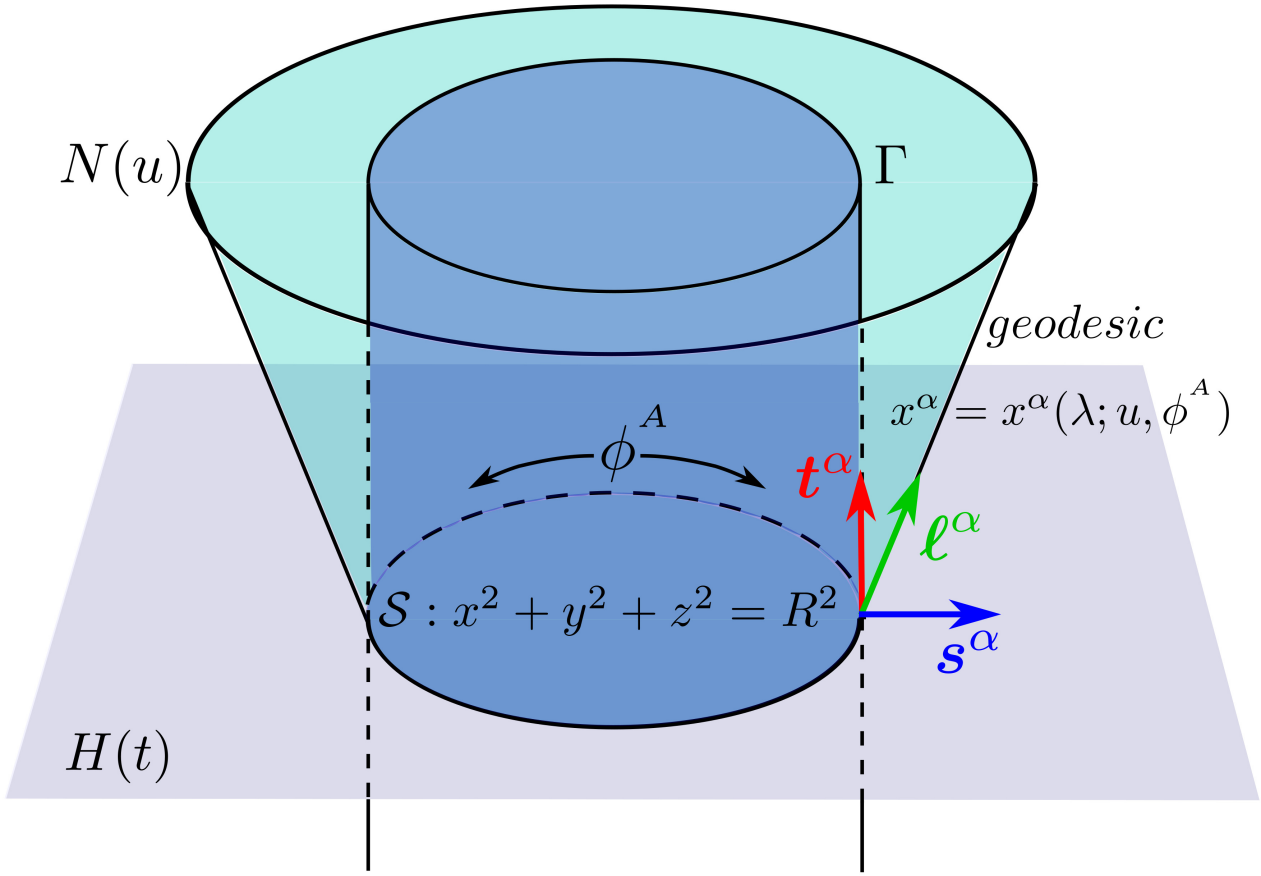
where r is a surface area coordinate. We introduce a complex dyad q_A (e.g., $q_A = (1, i\sin\theta)$ in spherical polars), then h_{AB}, U^A can be represented by complex numbers,

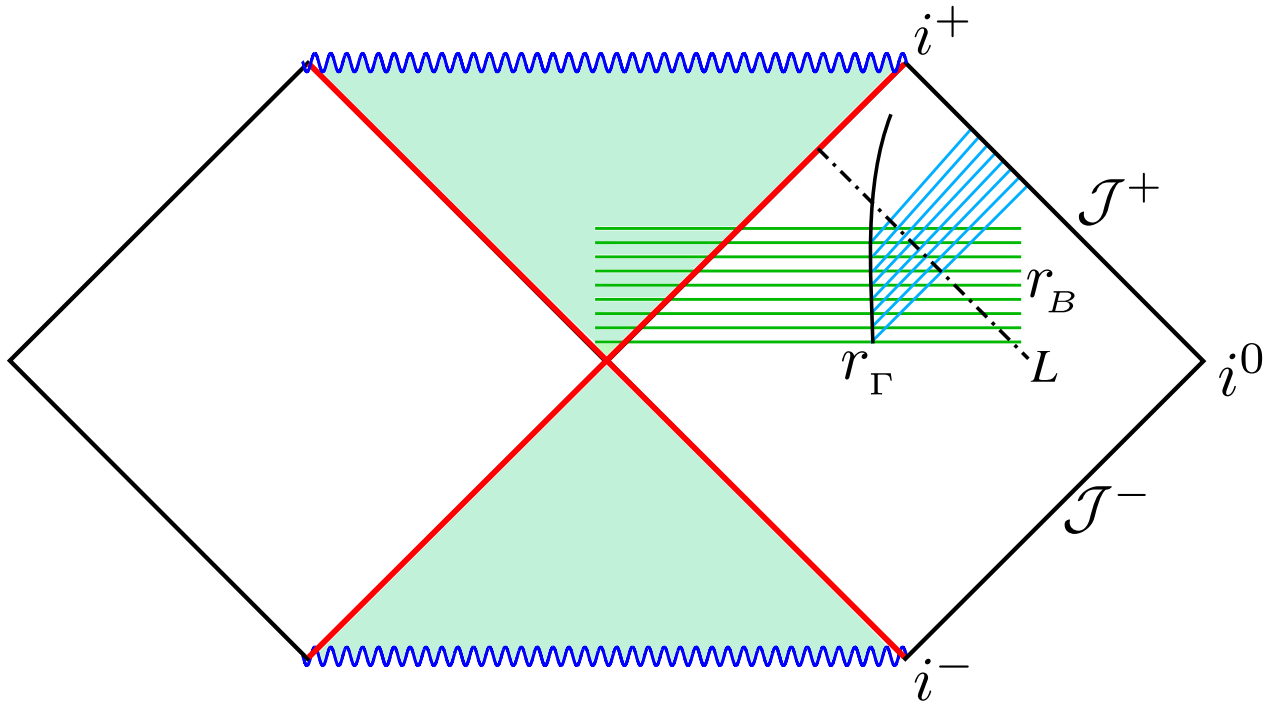
$$J = h_{AB} q^A q^B/2\,, \quad U = U^A q_A\,.$$

Einstein's equations $R_{\alpha\beta} = 8\pi(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T)$ **remain regular on compactification** $r \to x$ with $r = \infty \to x = 1$.


u
Γ
$\mathscr{J}^+$
J
U
$\partial_r U$
W_c
β
J
x

Construction of characteristic boundary data (C Reisswig *et al.* 2009, 2010; M Babiuc *et al.* 2011)





Error in GW extraction:

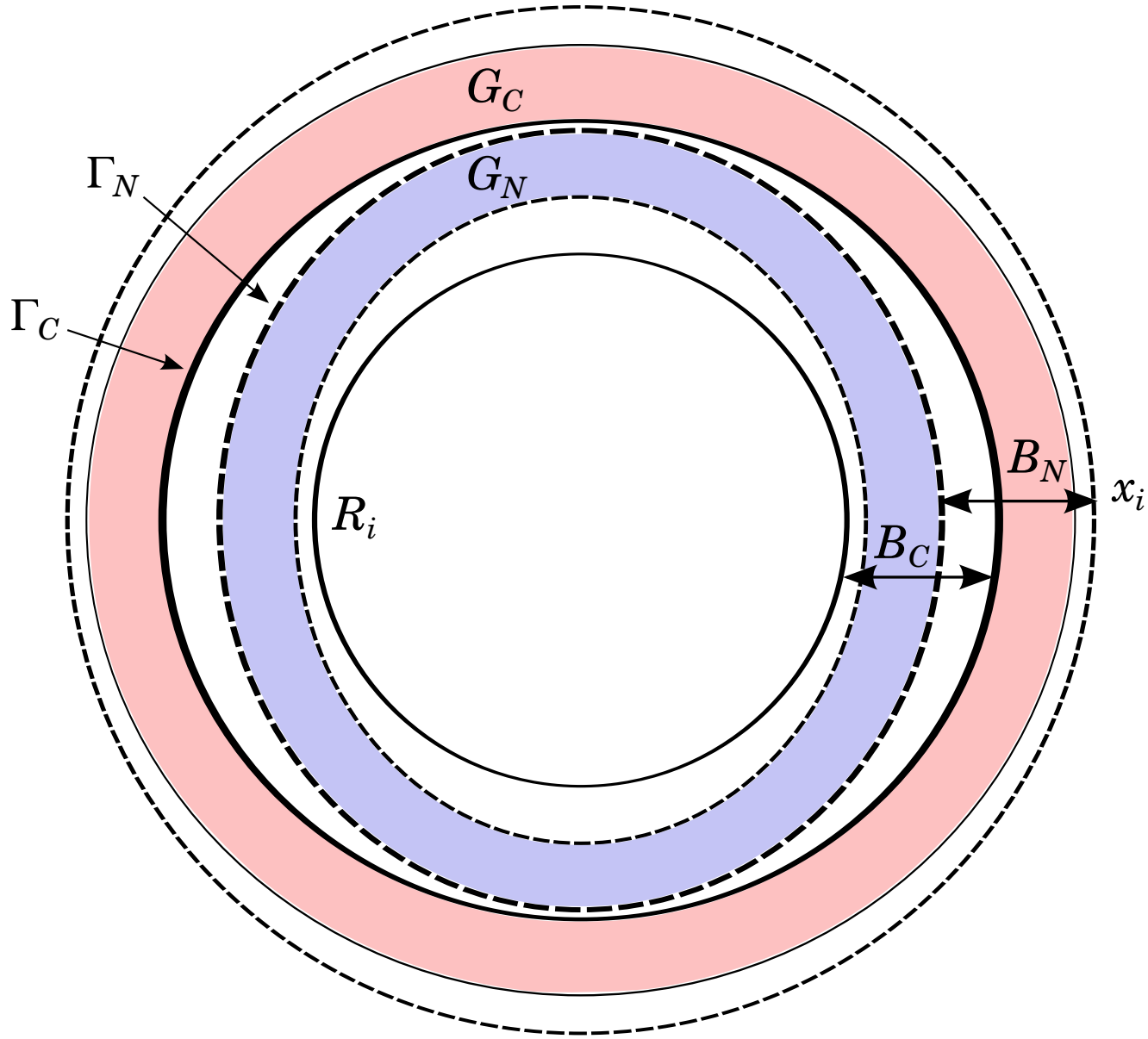
- Unphysical initial Cauchy data
- Unphysical outer boundary data if $r_b - r_\Gamma < t_{\text{final}} - t_{\text{initial}}$
- Unphysical initial characteristic data
- Truncation error, round-off error

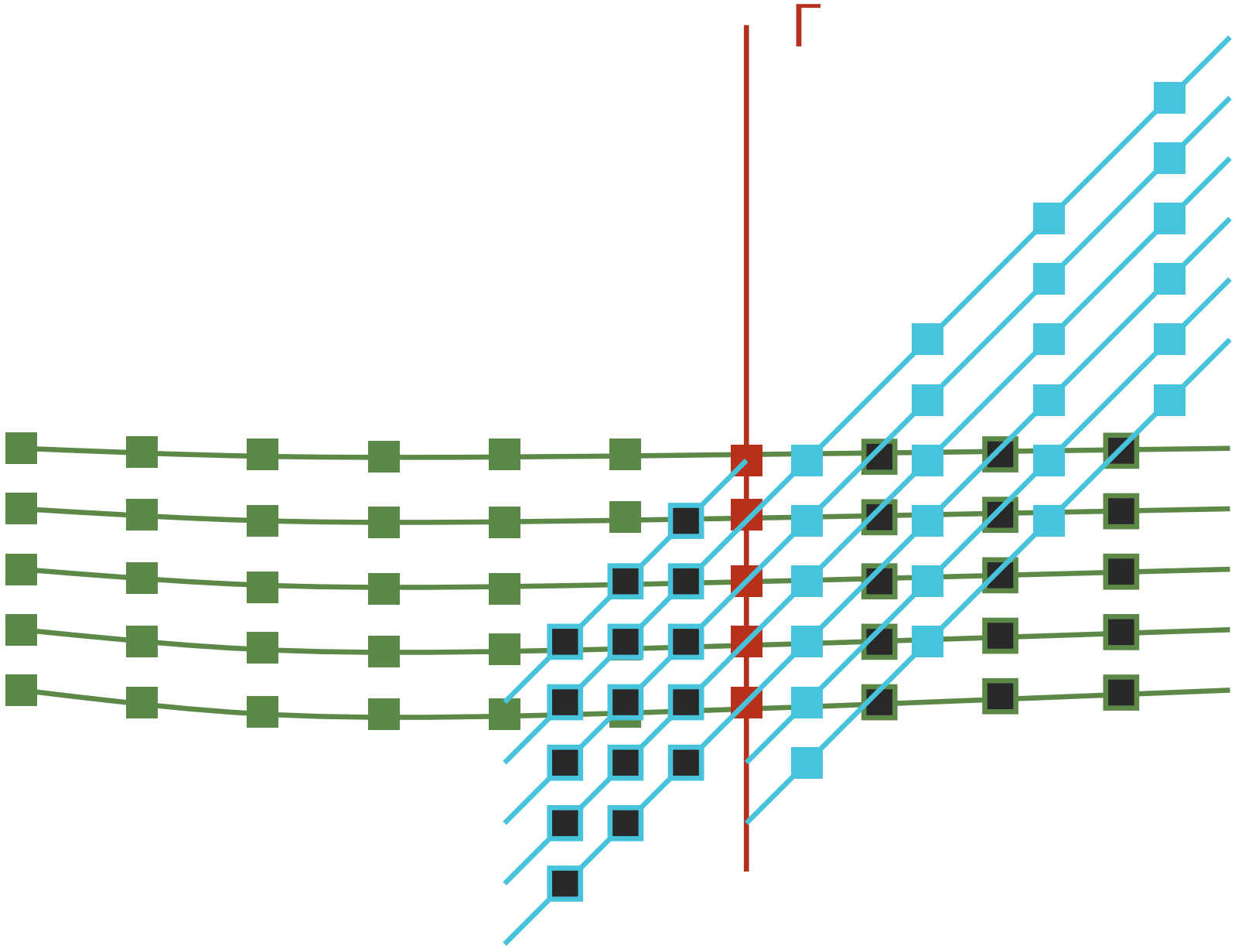
Implementation of characteristic extraction

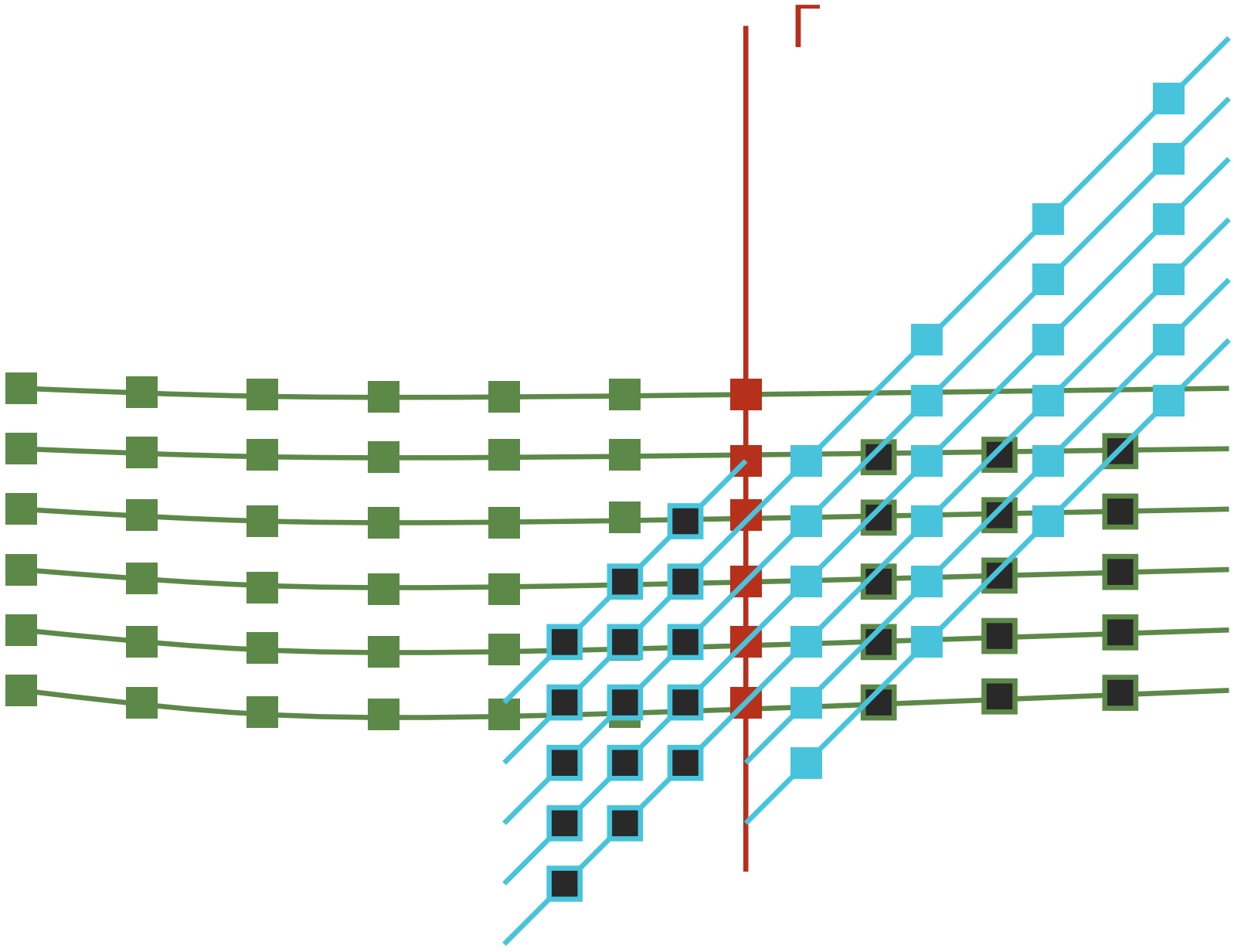
- Cauchy code must output to file metric data at worldtube in prescribed format.
- Characteristic extraction post-processes the data and typically requires 1% of Cauchy compute resources.
- Characteristic extraction is part of the Einstein toolkit.

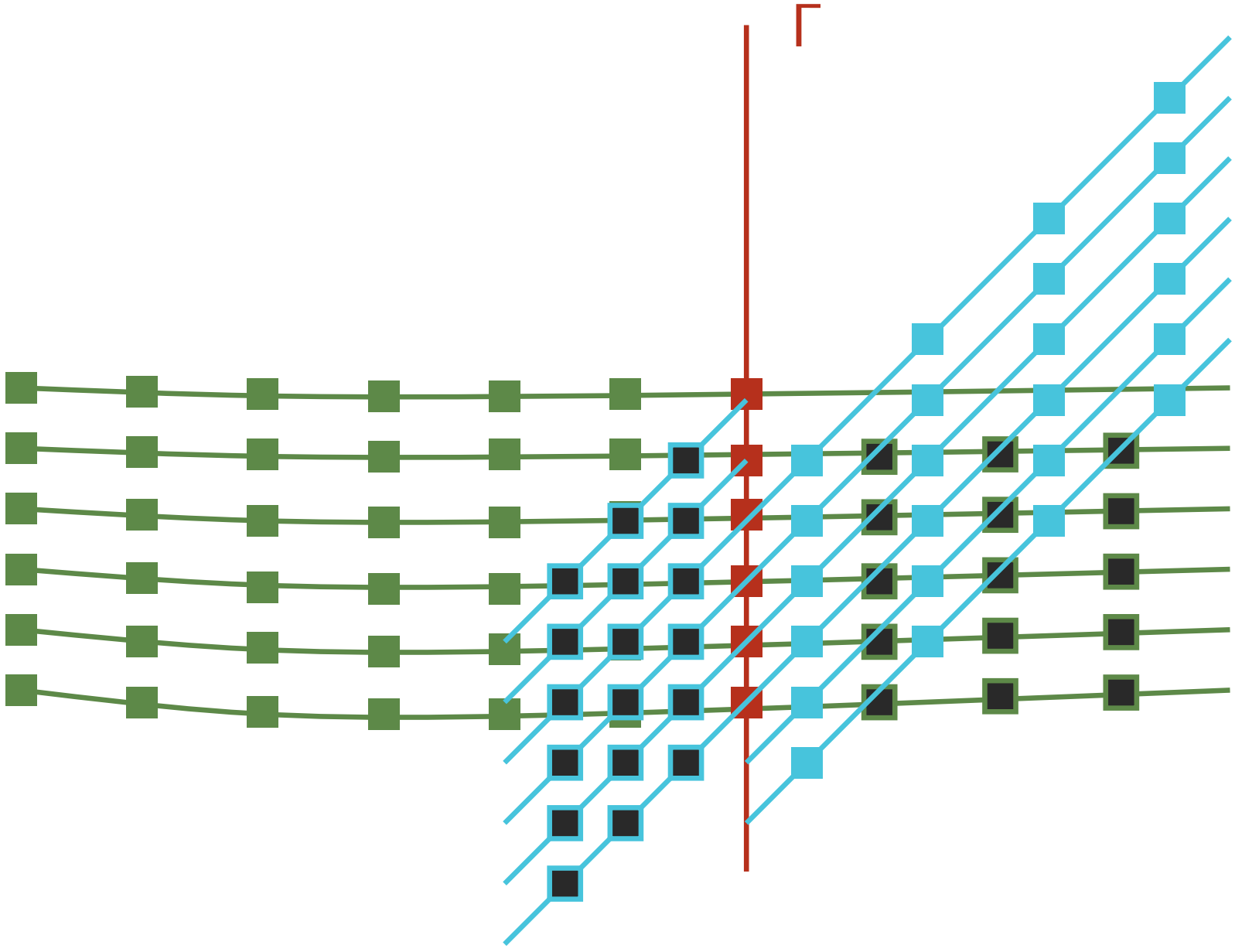
A new algorithm for extraction and matching

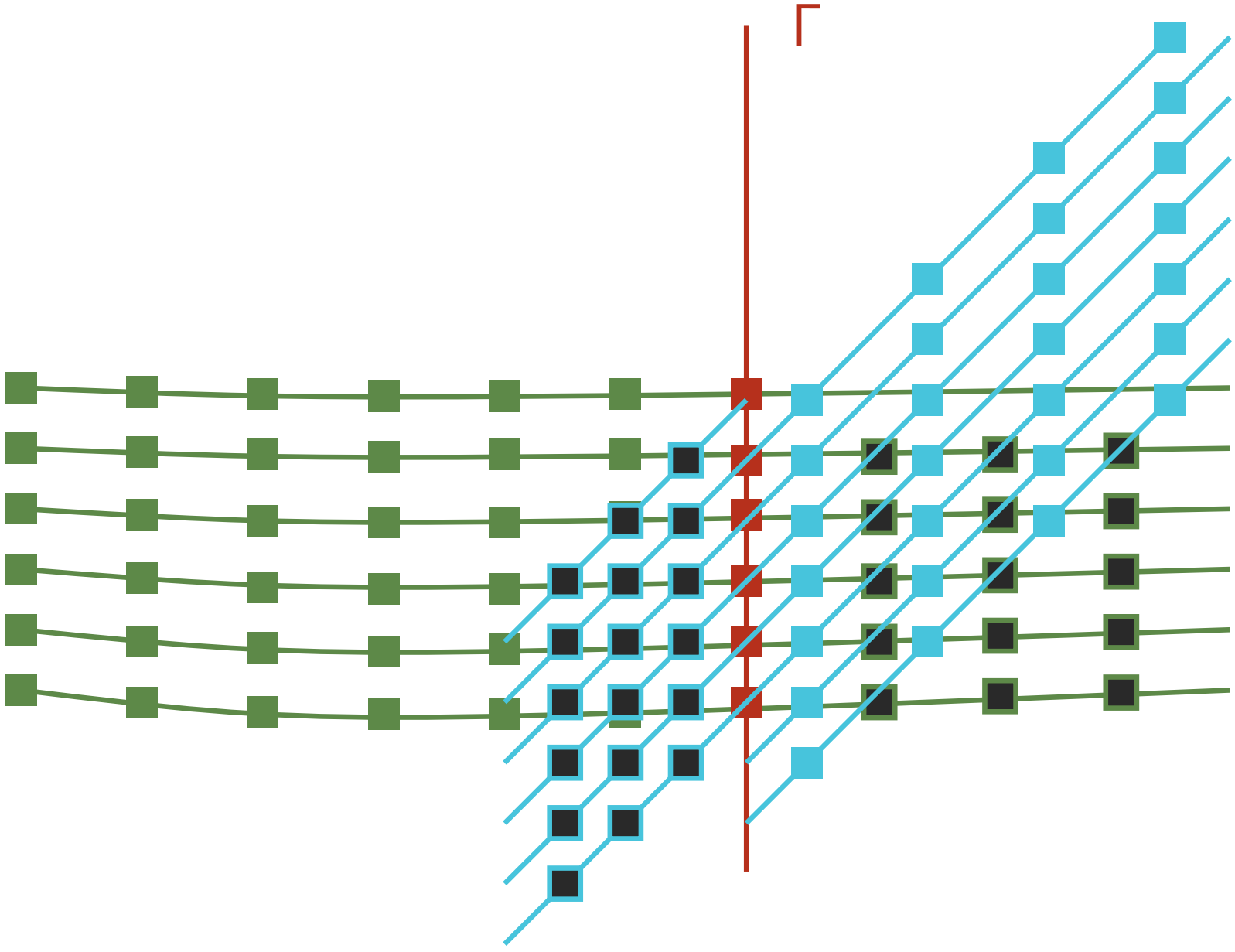
- Approach the problem as one of using different coordinate patches, i.e. Cauchy and characteristic, in the same way that other NR codes have patches with Cartesian coordinates and spherical coordinates.
- Regard grid points where one grid extends into the other coordinate patch as ghost points.

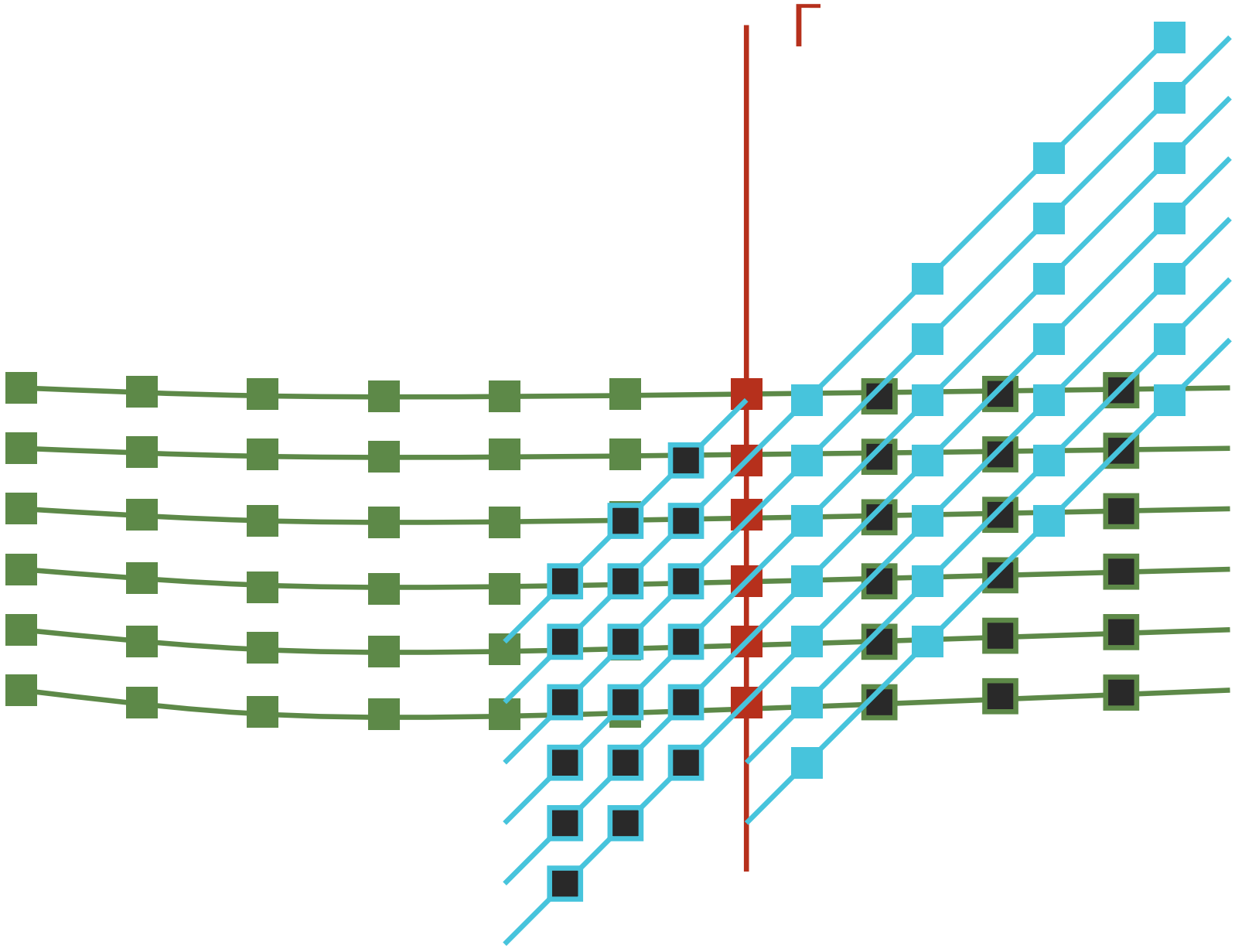

G_C
G_N
Γ_N
Γ_C
R_i
B_N
B_C
x_i





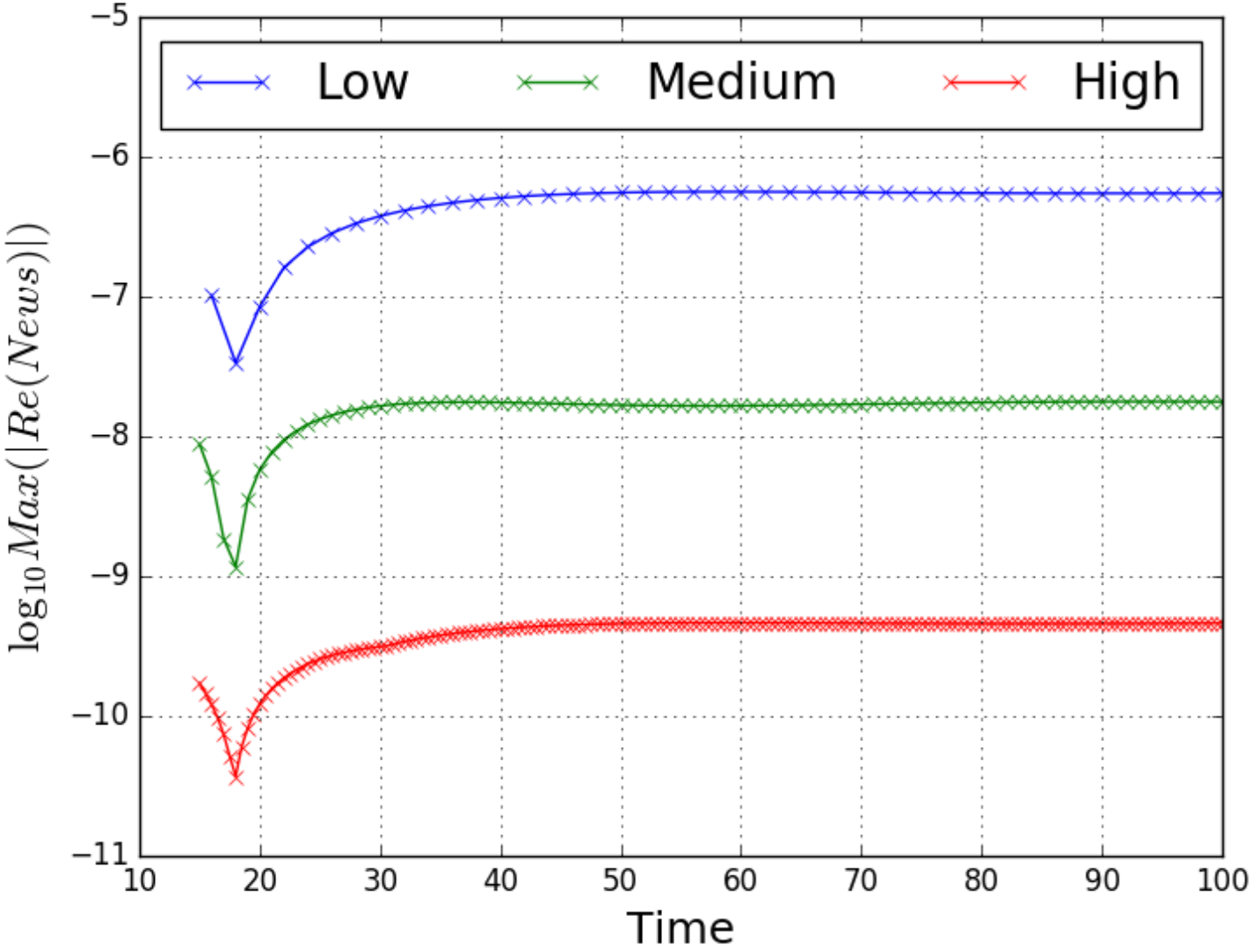


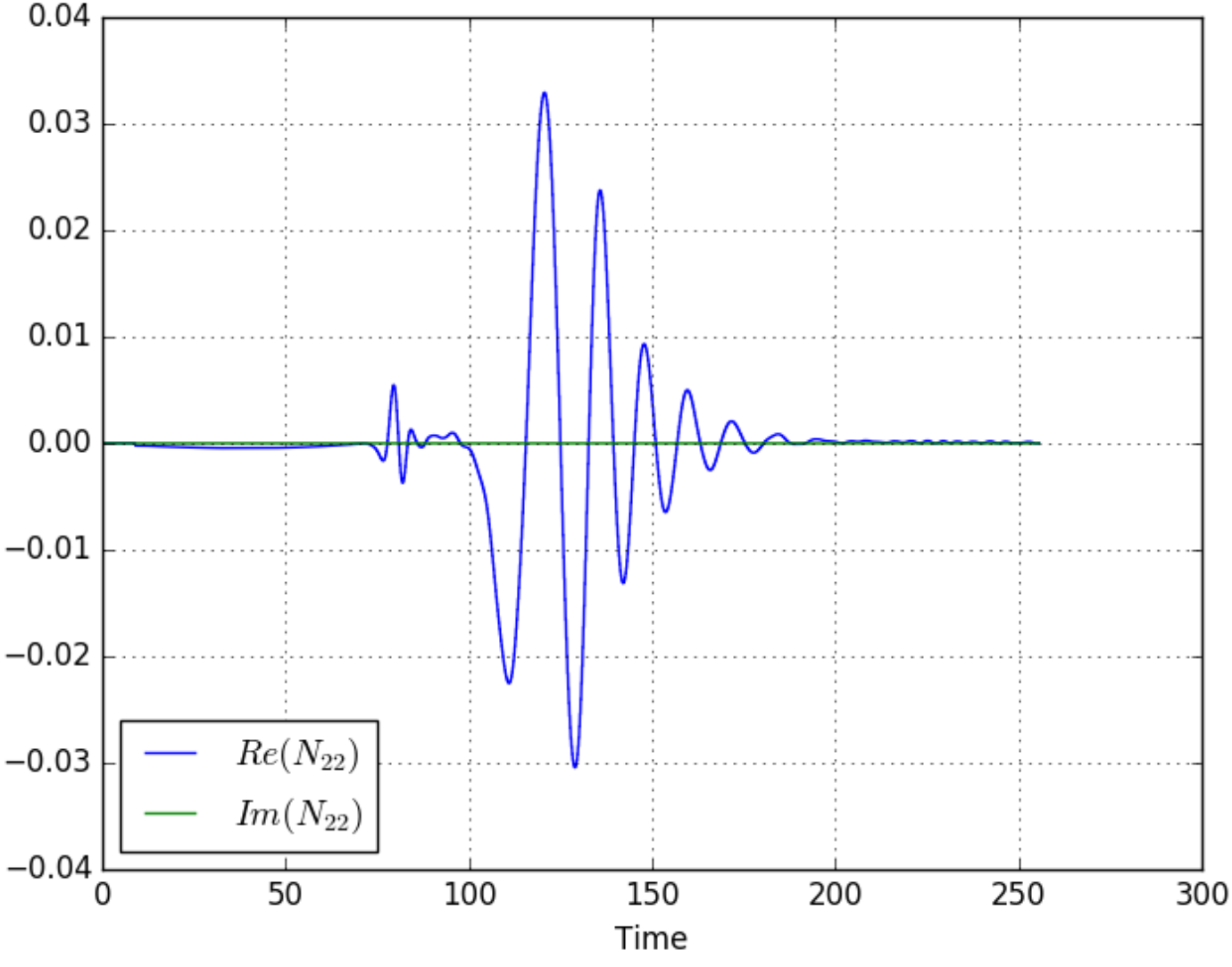




Current status of the matching code

- Code has been completed, and is being de-bugged.
- For the Schwarzschild solution, the emitted gravitational waves are 4^{th} order convergent to 0.
- Equal mass binary black hole grazing collision and merger has run until $t = 250M$. It produces a gravitational waveform that seems reasonable.


Low
Medium
High
$\log_{10} Max(|Re(News)|)$
Time
−5
−6
−7
−8
−9
−10
−11
10
20
30
40
50
60
70
80
90
100


0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04
0
50
100
150
200
250
300
Time
$Re(N_{22})$
$Im(N_{22})$

Conclusions

- The only gravitational wave extraction method method free of systematic error is characteristic extraction.
- A new algorithm has been developed for characteristic extraction and for Cauchy characteristic matching.
- The code has been written and is currently being de-bugged.
- Cauchy characteristic matching has the potential to achieve significant efficiencies in the computation of gravitational waveforms.

THANK YOU