EXAMINERS' REPORT

Pure Mathematics
Advanced Matriculation Level

First Session 2018

L-Università ta' Malta

MATSEC
Examinations Board

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Summary of Results

Grade	A	B	C	D	E	F	Abs	Total
Number	42	100	126	46	28	113	35	490
%	8.6	20.4	25.7	9.4	5.7	23.1	7.1	100

Summary of Lower Marks

≤	10	15	20
Number	23	39	54
%	4.7	8.0	11.0

Comments on Candidates' Performance

Paper 1

Question 1. Candidates did quite well in this question. Some candidates found difficulty in integrating $e^x \sin x$, particularly those who integrated the left and right hand side at one go. A minority of candidates introduced logarithms in the first step and lost most marks.

Question 2. Candidates did quite well in this question. In part (a), some candidates did not seem to understand the meaning of $^{\mathrm{d}}/_{\mathrm{d}x}$ and tried to integrate instead of differentiate the given function. Quite a number of candidates differentiated correctly but were unable to simplify to match the given answer. In the second part of the question, most found difficulty in finding or simplifying the second derivative as they did not use part (a) to differentiate $^{\mathrm{d}y}/_{\mathrm{d}x}$.

Question 3. The vast majority of candidates did very well in this question, 75% getting 5 marks or more, and even 47% getting more than 8 marks. The most common mistake was the incorrect choice of vectors when finding the required angle. However, most candidates showed that they know how to use the scalar product to find an angle.

Question 4. In part (a), the candidates who did well were those who realised that the equation could be changed into quadratic form. Most of the other candidates just wrote *log* in front of each term, losing all marks. In part (b), quite a large percentage of candidates did not follow the laws of logarithms, thus obtaining wrong values for the unknowns. Of those who managed to find the correct solutions, very few removed the negative values for x and y.

Question 5. (a) Most candidates were able to give the correct range and also found the inverse function, however a significant number of candidates gave the incorrect domain because they considered the expression obtained for the inverse instead of the range of the original function.
(b) Again most candidates managed to obtain the expression for $g \circ h$ but then only few were able to demonstrate how one obtains the domain of a composite function from the given functions.
However one must note that there was a significant improvement in this question when compared to the previous years with 69% getting 5 marks or more and 21% getting more than 8 marks.

Question 6. An unexpected number of candidates did not sketch a graph to solve the first inequality but treated it like an equation, thus obtaining wrong values for the range of u. Very few candidates managed the second part of (a). Most of them did not realise that $\sin^2\theta > 1$ has no solutions and very few candidates used a graph to solve the inequality $\sin^2\theta < 1/3$. Most of those who attempted this part just solved the equations $\sin\theta = 1$ and $\sin\theta = 1/\sqrt{3}$ and then placed a $>$ or $<$ sign in front of all of their answers. Most candidates managed the first part of (b), although some did forget to write $= 0$ in their equation. When solving for the last part, most candidates did not realise that they were better off changing all the terms in terms of $\sin\theta$ and only a handful of students managed to reach the correct answer.

Question 7. (a) A significant number of candidates had problems with dividing the numerator by the denominator, with some even trying to use partial fractions. However most of those who managed with the division ended up getting full marks for this part of the question.
(b) Most of the candidatess did the partial fractions part correctly but then by mistake they equated to zero the coefficient of x, instead of the coefficient of the constant, which gave rise to a different integral. However, most of the candidates were able to continue with the integration. In fact 67% got 5 marks or more in this question.

Question 8. Candidates performed very badly in this question as most of them did not understand what was being asked of them in part (a). It appears that candidates did not know what is meant by the equation of the locus. This effected the outcome of the rest of the question as it was based on the first part. In fact only about 20% managed to get 5 marks or more in this question.

Question 9. (a) Many candidatess got mixed up in this part of the question with most of them even getting the product in the denominator wrong.
(b) Most candidates managed to get the first part correctly, however a significant number of them forgot to divide before going for partial fractions and an even bigger number of them preferred to start from the RHS and take the common denominator instead of dividing and using partial fractions. Candidates performed much worse in the second part of (b) with a significant number of them not using the binomial theorem when approximating. Most of the candidates tried to get an approximation of the expression using $x = 1/2$.

Question 10. In part (a), around half of the candidates used a combination instead of a permutation, reflecting little knowledge of the topic.
In (b)(i), some candidates found the probability that Anna won a prize, or that Bernard won a prize, but very few found the probability that either one or both won a prize. When finding the probability that both won a prize, most multiplied $P(A)$ by $P(B)$. The majority of those candidates who answered this part correctly used the complement method.
In (b)(ii), most candidates listed all possible outcomes but very few reached the correct answer. Some were cunning enough to realise that the answer to (ii) was just half of (i). Unfortunately, a lot of candidates just multiplied fractions without explaining what they were finding, so it was very difficult to understand whether their working made any sense or not, particularly when the answer was wrong.

Paper 2

Question 1. Part (a) was a straightforward differential equation of the separation of variables type. However, most candidates preferred to treat the equation as a linear differential equation and a few number of them made errors in finding the integrating factor. One common error was $e^{\int \frac{x}{1+x^2}\,\mathrm{d}x} = (1+x^2)^2$. In part (b) most candidates solved the differential equation correctly although mistakes were made in finding the derivatives of y. Also, many arithmetic mistakes were made in finding the values of the constants A, B and C.

Question 2. Most candidates attempted this question and many of them obtained high marks. Candidates seem to be well acquainted with the Newton-Raphson method and Simpson's rule. However, a few candidates were unable to differentiate $e^{\sin x} = 2\cos x$.

Question 3. This question was attempted by many candidates but only a few obtained full marks. Some found difficulty in integrating by parts $\sec^{n-2} x \sec^2 x$ and others made the mistake of stating that $\tan^2\theta = 1 - \sec^2\theta$ or $(\sec^3\theta)^2 = \sec^5\theta$.

Question 4. Some of the candidates who answered this question did not find the oblique asymptote, thus sketching the graph wrongly. Also, many did not answer correctly part (e) of the question. In this part of the question candidates tried to find whether $f(x+a)$ had real or imaginary roots.

Question 5. Only a few candidates sketched the curve correctly. Finding the polar coordinates of the point P and the area bounded by OP, OQ and the curve was beyond nearly all the candidates who attempted this question.

Question 6. Most of the candidates managed to work out the vector equation of the plane passing through the three given points. However, some candidates tried unsuccessfully to find the position vector of point E, where the three planes intersect, by solving simultaneously the equations of the three planes.

Question 7. In part (a) most candidates failed to transform the quotient of two complex numbers into a complex number by multiplying the numerator and denominator by the complex conjugate of the denominator and concluded incorrectly that

$$\mathrm{Re}\left(\frac{z+ia}{z+b}\right) = \mathrm{Re}\left(\frac{x+i(a+y)}{(x+b)+iy}\right) = \sqrt{\frac{x^2+(a+y)^2}{(x+b)^2+y^2}}.$$

Also many candidates were apparently unaware of the difference between

$$\mathrm{Re}\left(\frac{z+ia}{z+b}\right) \le k \quad \text{and} \quad \mathrm{Re}\left(\frac{z+ia}{z+b}\right) = k.$$

This was evidently clear when describing the locus of $P(x, y)$ in part (b).

Question 8. In part (a) many candidates did not apply the Maclaurin expansion formula and simply wrote down

$$\ln(1+e^x) = e^x - \frac{e^{2x}}{2} + \frac{e^{3x}}{3} - \dots$$

There were also mistakes in finding the derivatives, for instance

$$\frac{\mathrm{d}}{\mathrm{d}x}[\ln(1+e^x)] = \frac{1}{1+e^x}.$$

Also some candidates wrote down the value of $\ln 2$ as 0.6931 and did not spot that $2\ln 2$ cancels with $\ln 4$ in the second part.

In part (b), when proving the proposition is true for $n = k+1$, few candidates made errors in finding $f^{(k+1)}$ such as $f^{(k+1)} = f^{(k)} \times f'$ or $f^{(k+1)} = f^{(k)} + f'$, but in general this part of the question was well tackled.

Question 9. Part (a) was very well attempted although arithmetic mistakes were made in finding $\mathbf{A}^{-1}$ which gave the incorrect vectors $\mathbf{u}$ and $\mathbf{v}$. Many candidates did not seem to have understood part (b) of the question and found the equation of line ℓ instead of the line that is transformed by $\mathbf{A}$ into the line ℓ. Part (c) defeated almost all candidates. From the solutions presented, it was clear that most of the candidates are unaware of the difference between the equation of a line and the equation of a plane.

Question 10. Although the question on probability is never popular, more than half of the candidates who attempted this question made very good attempts. In part (a) many candidates argued that the probability of a

$$\textbf{yes}\text{ answer} = P(\textbf{head}) \times P(\textbf{yes}) = 1/2 \times 1/2 = 25\%$$

and concluded that the percentage of students that used illegal drugs is therefore 10%, but did not spot the other possibility of a **yes** answer, i.e. $P(\textbf{tail}) \times P(\textbf{yes})$. From solutions of part (b) it was clear that there are candidates who do not know the meaning of the statement "*at least*". Most candidates did not realize that $P(\textbf{of at least one}) = 1 - P(\textbf{none})$ and that there are three ways of choosing exactly two students from three.