Gravitational waves from eccentric binary black hole coalescence

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January 23, 2018

Overview

- aim: include eccentricity in the modeling strategy of phenomenological frequency-domain inspiral-merger-ringdown (IMR) waveforms
- work in progress
- in collaboration with: Sascha Husa & Antoni Ramos (Universitat de les Illes Balears)
  A. Gopakumar (Tata Institute of Fundamental Research)
- supported by STSM CA16104 - 38108

GW data analysis and eccentricity

- stellar-mass BBH formed in isolation:
  $\rightarrow$ expected to be in quasi-circular orbits in the LIGO-Virgo band
  $\rightarrow$ shed formation eccentricities due to gravitational radiation reaction
- Waveform models used for construction of CBC template banks assume circularity!
- BBH with non-negligible eccentricity in the LIGO-Virgo band require formation with high eccentricity at small separations
  $\rightarrow$ N-body interactions in dense stellar environments (globular clusters, galactic nuclei, vicinity of supermassive BHs)
- plausible astrophysical mechanisms for formation/merger of BBH that retain non-negligible eccentricities: dynamical capture or Lidov-Kozai oscillations in hierarchical triples
- LIGO should be able detect of the order $\geq 1/yr$ events with non-negligible eccentricity

Goals of eccentric waveform modeling

- quantify sensitivity of quasi-circular template searches and unmodeled search pipelines to eccentricity...
- estimate eccentricity of detected BBH signals
- work towards dedicated template-based search for eccentric GW signals?
- unambiguous detection of GWs with non-negligible eccentricity could shed light on formation channel
- LISA!

Waveform development overview

1. NCSA, Cambridge, CITA & AEI:
   NCSA-CAM waveform model
   $\rightarrow$ time-domain eIMR model with hybrid inspiral waveform and GPR based merger waveform)

2. (formerly) AEI Potsdam:
   foundational work for an eccentric EOB model $\rightarrow$ reparameterized EOB-based inspiral + SEOBNRv2 merger-ringdown

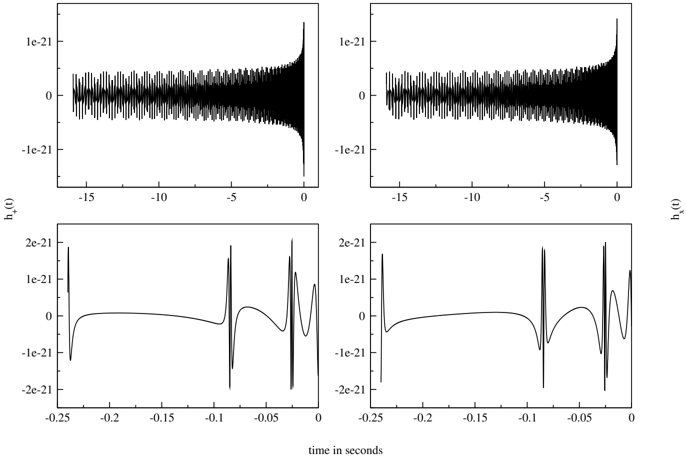
3. CMI, Montclair State University, KGWG (Seoul National University & Inje University):
   Extension of TaylorF2 to include leading-order eccentricity effects

4. TIFR Mumbai & University of Zürich:
   - Time- and frequency-domain eccentric inspiral waveforms
   - An effective frequency-domain eIMR model for low eccentricities (with UIB Palma)

Eccentric waveform modeling in the time domain

- to solve 2-body problem for binaries on eccentric orbits and to generate $h_{\times,+}(t)$: formalism adapts widely used PN-accurate Damour-Deruelle timing formula for binary pulsars
- for non-spinning compact binaries: approach consistently combines three times scales associated with the orbital motion, precession of periastron & radiation reaction
  Damour, Gopakumar, Iyer [2004] & Gopakumar, Schäfer [2012]
- PN-accurate 'Keplerian type' parametric solution is employed to describe temporal evolution of PN-accurate eccentric orbits
- currently, description of eccentric binary inspiral without spin is avalable to 3PN order in EOM and GW fluxes
  Hinder et al. [2011] , Tanay/Haney/Gopakumar [2016], Huerta et al. [2017]

Eccentric time-domain inspiral waveforms



EccentricTD: $m_1 = m_2 = 10M_{\odot}$, $f_0 = 10\mathrm{Hz}$, $e_0 = \{0.45, 0.85\}$

Eccentric waveform modeling in the frequency domain

- starting point: *post-circular formalism* [Yunes et al. (2009)]
  → provides analytic expression for Fourier transform $\tilde{h}(f)$ of the detector strain $h(t)$ in the small-eccentricity limit, invoking *Stationary Phase Approximation)*
  → $\tilde{h}(f)$ for eccentric compact binaries inspiraling under the influence of quadrupolar-order radiation reaction (Newtonian in both amplitude and Fourier phase evolution)

- decomposition of time-domain detector strain into harmonics in terms of mean anomaly l:

$$h(t) = -\left(\frac{G\ m\ \eta}{c^2\ D_L}\right)\ x \sum_{j=1}^{8} \alpha_j \left(\cos\phi_j \cos(jl) - \sin\phi_j \sin(jl)\right)\ . \qquad (1)$$

where α_j and ϕ_j are functions of $\{e_t, F_{+,\times}, \phi, \iota, \beta\}$, and where $x = (Gm\omega/c^3)^{2/3}$ is the gauge-invariant PN expansion parameter

Stationary phase approximation

- Fourier integral in SPA as

$$I(f) = \int_{-\infty}^{+\infty} a(t) e^{i\Psi(t)} dt \backsimeq a(t_0) \sqrt{\frac{2\pi}{\ddot{\Psi}(t_0)}} e^{i(\Psi(t_0) - \pi/4)} \tag{2}$$

where t_0 is the stationary point of the Fourier phase Ψ, i.e., $\dot{\Psi}(t_0) = 0$

- for eccentric orbits, an analytic expression for the Fourier phase

$$\Psi[F(t_0)] = 2\pi \int^{F(t_0)} \tau' \left(j - \frac{f}{F'} \right) dF' \tag{3}$$

at the stationary point defined by $j \times \dot{l}(t_0) = 2\pi f$ is only possible in the small-eccentricity limit

- evolution equations for frequency ω and eccentricity e_t are coupled!
$\rightarrow \tau = F/\dot{F}$ requires an appropriate analytic expression for $e_t = e_t(f, f_0, e_0)$
- asymptotic eccentricity invariant $e_t = e_0 \left(f/f_0\right)^{-19/18}$ at leading order introduced by [Krolak et al. (1995)]; PC formalism extends $e_t = e_t(f, f_0, e_0)$ to $\mathcal{O}(e_0^8)$

PN-consistent extension of the PC formalism

- independent work by different groups to adapt and extend the PC formalism (i.e., to incorporate higher-order PN corrections and include effects of non-negligible orbital eccentricities in the `TaylorF2` approximant)
- more recent work ([Moore et al. (2016), Tanay/Haney/Gopakumar (2016)]): $\rightarrow$ fully analytic Fourier domain inspiral approximant that incorporates all the effects of PN-accurate orbital eccentricity evolution in a PN consistent manner.
- crucial ingredient: the orbital eccentricity e_t as bivariate PN-accurate expansion in e_0 and x_0, assuming $e_0 \ll 1$ and $x_0 \ll 1$ (through hierarchical computations at each PN order)
- further improvements include: explicitly the effects of periastron advance on $h(f)$, leading-order spin-orbit and spin-spin effects in the phasing

Explicit results at 2PN order

- The expression for $\tilde{h}(f)$ with 2PN level Fourier phase and Newtonian order amplitude

$$\tilde{h}(f) = \tilde{\mathcal{A}} \left(\frac{Gm\pi f}{c^3} \right)^{-7/6} \sum_{j=1}^{8} \xi_j \left(\frac{j}{2} \right)^{2/3} e^{-i(\pi/4 + \Psi_j)} \,, \quad \xi_j = \frac{\left(1 - e_t^2\right)^{7/4}}{\left(1 + \frac{73}{24} e_t^2 + \frac{37}{96} e_t^4\right)^{1/2}} \alpha_j e_t^{-i\Phi_j} \tag{4}$$

where α_j and ϕ_j are functions of $\{e_t, F_{+,\times}, \phi, \iota, \beta\}$, and where $x = (Gm\omega/c^3)^{2/3}$ is the gauge-invariant PN expansion parameter.

- The following 2PN-accurate analytic expression for e_t is required to specify the frequency dependence of the harmonic coefficients ξ_j and to apply the SPA:

$$\begin{aligned} e_t \sim e_0 \Bigg\{ & \chi^{-19/18} + x \left(\frac{2833}{2016} - \frac{197\eta}{72} \right) \left[-\chi^{-19/18} + \chi^{-31/18} \right] \\ & + x^{3/2} \left(\frac{377\pi}{144} \right) \left[-\chi^{-19/18} + \chi^{-37/18} \right] + x^2 \Bigg[\chi^{-19/18} \left(\frac{78276085}{24385536} - \frac{1015247\eta}{145152} + \frac{43807\eta^2}{10368} \right) \\ & + \chi^{-31/18} \left(-\frac{8025889}{4064256} + \frac{558101\eta}{72576} - \frac{38809\eta^2}{5184} \right) \\ & + \chi^{-43/18} \left(-\frac{30120751}{24385536} - \frac{100955\eta}{145152} + \frac{33811\eta^2}{10368} \right) \Bigg] \Bigg\} \,. \end{aligned} \tag{5}$$

Explicit results at 2PN order

- The explicit 2PN order expression for Ψ_j that incorporates $\mathcal{O}(e_0^2)$ at each PN order is given by

$$
\begin{aligned}
\Psi_j \sim\; & j\phi_c - 2\pi f t_c - \left(\frac{3j}{256\eta}\right) x^{-5/3} \Bigg\{ 1 - \frac{2355 e_0^2}{1462}\chi^{-19/9} + x \Bigg[\frac{3715}{756} + \frac{55\eta}{9} \\
& + \left(\left(-\frac{2223905}{491232} + \frac{154645\eta}{17544} \right) \chi^{-25/9} + \left(-\frac{2045665}{348096} - \frac{128365\eta}{12432} \right) \chi^{-19/9} \right) e_0^2 \Bigg] \\
& + x^{3/2} \left[-16\pi + \left(-\frac{295945\pi}{35088}\chi^{-28/9} + \frac{65561\pi}{4080}\chi^{-19/9} \right) e_0^2 \right] \\
& + x^2 \Bigg[\frac{15293365}{508032} + \frac{27145\eta}{504} + \frac{3085\eta^2}{72} + \Bigg[\left(\frac{1185955235}{1485485568} + \frac{1902055\eta}{130032} - \frac{14251675\eta^2}{631584} \right) \chi^{-31/9} \\
& + \left(-\frac{5795368945}{350880768} + \frac{4917245\eta}{1566432} + \frac{25287905\eta^2}{447552} \right) \chi^{-25/9} \\
& + \left(-\frac{116151665}{14141952} - \frac{6138415\eta}{133056} - \frac{10688155\eta^2}{294624} \right) \chi^{-19/9} \Bigg] e_0^2 \Bigg] \Bigg\} ,
\end{aligned}
\tag{6}
$$

where $\chi = f/f_0$ and $x \equiv \left(G\, m\, \omega(t_0)/c^3\right)^{2/3}$ to ensure the SPA condition.

- We display here **only the leading order contributions in e_0**!
- We have obtained 3PN-accurate Ψ_j expression that includes all the $\mathcal{O}(e_0^6)$ contributions at every PN order.
  This requires $e_t = e_t(\omega, \omega_0, e_0)$ accurate to $\mathcal{O}(e_0^5)$ at every PN order.

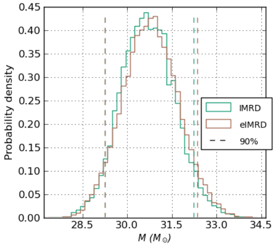
Eccentricity and IMRPhenom modeling

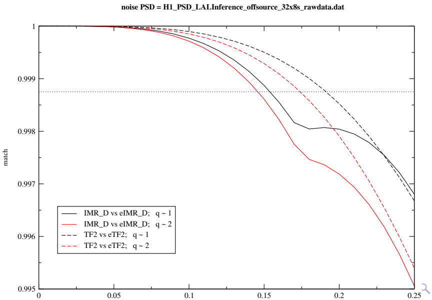
- current state-of-the-art of BBH waveform modeling:
  $\rightarrow$ phenomenological, frequency-domain waveforms (PN-based inspiral, phenomenological ansatz for merger-ringdown, tuned with 19 calbritation hybrids)
  $\rightarrow$ spin-aligned `IMRPhenomD`; `IMRPhenomPv2` for effective precession
- after BBH detections: we implemented a ready-to-use, eccentric `TaylorF2` approximant for small eccentricities
- Fourier phase includes leading-order eccentricity corrections up to 3PN by extending our results of Tanay et al. (2016)
- crucial input for the ready-to-use *effective eccentric variant* of the `IMRPhenomD` waveforms (restricted to non-spinning black holes) we developed subsequently, utilizing the modular structure of the IMRPhenomD waveform family (no calibration!)

Eccentric IMR waveforms: Preliminary data analysis

- Parameter estimation with such effective eccentric IMR waveforms has revealed no appreciable systematic errors from assuming circular orbits during the data analysis of GW150914.

- We have constrained the initial orbital eccentricity of the GW150914 black hole binary to $e_0 < 0.1$ at $10Hz$.





On-going work with UIB Palma

- improvement of our 'effective eccentric variant' of IMRPhenomD:
  $\rightarrow$ systematic study of GW data analysis implications for such a preliminary Phenom model of GWs from eccentric black-hole binaries without spin
- hybridize 3PN time-domain inspiral waveforms (EccentricTD) with long-evolution, non-spinning, eccentric NR simulations (generated with BAM code; $q = [1, 1.5, 2, 4]$, $e_0 = [0.1, 0.2, 0.5]$, convergence series with 4 resolutions)
- aim: use as eccentricity-exact injection waveforms to explore the accuracy of the eIMR model over the parameter space
  recalibrate model with eccentric NR simulations?
- long-term goal: phenomenological GW model for generic binaries with both eccentricity and spin precession (IMRPhenomE?)

Conclusions

- Most stellar-mass BBH are expected to have shed their formation eccentricity when they enter the LIGO band.
- Plausible astrophysical formation scenarios in dense stellar environments suggest $\geq 1/yr$ GW events with non-negligible eccentricity.
- LISA will observe stellar-mass BBH at earlier stages in their evolution when the orbits cannot generally assumed to be quasi-circular.
- The inspiral stage of eccentric binary coalescence without spin is well-modeled in time- and frequency domain.
- First models for full inspiral-merger-ringdown signal of non-spinning coalescence along eccentric orbits are available.
- Improved IMR models that include spin should be available for data analysis in O3.