Gravitational waves from eccentric binary black hole coalescence

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aim: include eccentricity in the modeling strategy of phenomenological frequency-domain inspiral-merger-ringdown (IMR) waveforms

work in progress

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stellar-mass BBH formed in isolation:
→ expected to be in quasi-circular orbits in the LIGO-Virgo band
→ shed formation eccentricities due to gravitational radiation reaction

Waveform models used for construction of CBC template banks assume circularity!

BBH with non-negligible eccentricity in the LIGO-Virgo band require formation with high eccentricity at small separations
→ N-body interactions in dense stellar environments (globular clusters, galactic nuclei, vicinity of supermassive BHs)

plausible astrophysical mechanisms for formation/merger of BBH that retain non-negligible eccentricities: dynamical capture or Lidov-Kozai oscillations in hierarchical triples

LIGO should be able detect of the order $\geq 1$/yr events with non-negligible eccentricity
Goals of eccentric waveform modeling

- quantify sensitivity of quasi-circular template searches and unmodeled search pipelines to eccentricity...
- estimate eccentricity of detected BBH signals
- work towards dedicated template-based search for eccentric GW signals?
- unambiguous detection of GWs with non-negligible eccentricity could shed light on formation channel
- LISA!
Waveform development overview

1. NCSA, Cambridge, CITA & AEI:
   NCSA-CAM waveform model
   → time-domain eIMR model with hybrid inspiral waveform and GPR based merger waveform

2. (formerly) AEI Potsdam:
   foundational work for an eccentric EOB model → reparameterized EOB-based inspiral + SEOBNRv2 merger-ringdown

3. CMI, Montclair State University, KGWG (Seoul National University & Inje University):
   Extension of TaylorF2 to include leading-order eccentricity effects

4. TIFR Mumbai & University of Zürich:
   - Time- and frequency-domain eccentric inspiral waveforms
   - An effective frequency-domain eIMR model for low eccentricities (with UIB Palma)
to solve 2-body problem for binaries on eccentric orbits and to generate $h_{x,+}(t)$: formalism adapts widely used PN-accurate Damour-Deruelle timing formula for binary pulsars

for non-spinning compact binaries: approach consistently combines three times scales associated with the orbital motion, precession of periastron & radiation reaction


PN-accurate ‘*Keplerian type*’ parametric solution is employed to describe temporal evolution of PN-accurate eccentric orbits

currently, description of eccentric binary inspiral without spin is available to 3PN order in EOM and GW fluxes

*Hinder et al. [2011], Tanay/Haney/Gopakumar [2016], Huerta et al. [2017]*
Eccentric time-domain inspiral waveforms

EccentricTD: $m_1 = m_2 = 10M_\odot$, $f_0 = 10\text{Hz}$, $e_0 = \{0.45, 0.85\}$
Eccentric waveform modeling in the frequency domain

- starting point: *post-circular formalism* [Yunes et al. (2009)]
  - provides analytic expression for Fourier transform $\tilde{h}(f)$ of the detector strain $h(t)$ in the small-eccentricity limit, invoking *Stationary Phase Approximation*
  - $\tilde{h}(f)$ for eccentric compact binaries inspiraling under the influence of quadrupolar-order radiation reaction (*Newtonian* in both amplitude and Fourier phase evolution)

- decomposition of time-domain detector strain into harmonics in terms of mean anomaly $l$:

  $$ h(t) = - \left( \frac{G m \eta}{c^2 D_L} \right) x \sum_{j=1}^{8} \alpha_j \left( \cos \phi_j \cos (jl) - \sin \phi_j \sin (jl) \right). $$

  where $\alpha_j$ and $\phi_j$ are functions of $\{e_t, F_+, x, \phi, \iota, \beta\}$, and where $x = (Gm\omega/c^3)^{2/3}$ is the gauge-invariant PN expansion parameter
Stationary phase approximation

- Fourier integral in SPA as

\[ I(f) = \int_{-\infty}^{+\infty} a(t)e^{i\Psi(t)} dt \cong a(t_0)\sqrt{\frac{2\pi}{\ddot{\Psi}(t_0)}} e^{i(\Psi(t_0) - \pi/4)} \]  \hspace{1cm} (2)

where \( t_0 \) is the stationary point of the Fourier phase \( \Psi \), i.e., \( \dot{\Psi}(t_0) = 0 \)

- for eccentric orbits, an analytic expression for the Fourier phase

\[ \Psi[F(t_0)] = 2\pi \int_{F'(t_0)}^{F(t_0)} \tau' \left( j - \frac{f}{F'} \right) dF' \]  \hspace{1cm} (3)

at the stationary point defined by \( j \times \dot{l}(t_0) = 2\pi f \) is only possible in the small-eccentricity limit

- evolution equations for frequency \( \omega \) and eccentricity \( e_t \) are coupled!

\[ \rightarrow \tau = \frac{F}{F'} \] requires an appropriate analytic expression for \( e_t = e_t(f, f_0, e_0) \)

- asymptotic eccentricity invariant \( e_t = e_0 (f/f_0)^{-19/18} \) at leading order introduced by [Krolak et al. (1995)]; PC formalism extends \( e_t = e_t(f, f_0, e_0) \) to \( O(e_0^8) \)
PN-consistent extension of the PC formalism

- independent work by different groups to adapt and extend the PC formalism (i.e., to incorporate higher-order PN corrections and include effects of non-negligible orbital eccentricities in the TaylorF2 approximant)

- more recent work ([Moore et al. (2016), Tanay/Haney/Gopakumar (2016)]): → fully analytic Fourier domain inspiral approximant that incorporates all the effects of PN-accurate orbital eccentricity evolution in a PN consistent manner.

- crucial ingredient: the orbital eccentricity $e_t$ as bivariate PN-accurate expansion in $e_0$ and $x_0$, assuming $e_0 \ll 1$ and $x_0 \ll 1$ (through hierarchical computations at each PN order)

- further improvements include: explicitly the effects of periastron advance on $h(f)$, leading-order spin-orbit and spin-spin effects in the phasing
Explicit results at 2PN order

- The expression for $\tilde{h}(f)$ with 2PN level Fourier phase and Newtonian order amplitude

$$\tilde{h}(f) = \tilde{A} \left( \frac{Gm\pi f}{c^3} \right)^{-7/6} \sum_{j=1}^{8} \xi_j \left( \frac{j}{2} \right)^{2/3} e^{-i(\pi/4+\Psi_j)}$$

$$\xi_j = \frac{(1 - e_t^2)^{7/4}}{(1 + \frac{73}{24} e_t^2 + \frac{37}{96} e_t^4)^{1/2}} \alpha_j e_t^{-i\phi_j}$$

where $\alpha_j$ and $\phi_j$ are functions of $\{e_t, F_+, x, \phi, \iota, \beta\}$, and where $x = (Gm\omega/c^3)^{2/3}$ is the gauge-invariant PN expansion parameter.

- The following 2PN-accurate analytic expression for $e_t$ is required to specify the frequency dependence of the harmonic coefficients $\xi_j$ and to apply the SPA:

$$e_t \sim e_0 \left\{ \chi^{-19/18} + x \left( \frac{2833}{2016} - \frac{197\eta}{72} \right) \left[ -\chi^{-19/18} + \chi^{-31/18} \right] 
+ x^{3/2} \left( \frac{377\pi}{144} \right) \left[ -\chi^{-19/18} + \chi^{-37/18} \right] + x^2 \left[ \chi^{-19/18} \left( \frac{78276085}{24385536} - \frac{1015247\eta}{145152} + \frac{43807\eta^2}{10368} \right) 
+ \chi^{-31/18} \left( -\frac{8025889}{4064256} + \frac{558101\eta}{72576} - \frac{38809\eta^2}{5184} \right) 
+ \chi^{-43/18} \left( -\frac{30120751}{24385536} - \frac{100955\eta}{145152} + \frac{33811\eta^2}{10368} \right) \right] \right\}. \quad (5)
The explicit 2PN order expression for $\Psi_j$ that incorporates $O(e_0^2)$ at each PN order is given by

$$\Psi_j \sim j\phi_c - 2\pi ft_c - \left( \frac{3j}{256\eta} \right) x^{-5/3} \left\{ 1 - \frac{2355e_0^2}{1462} \chi^{-19/9} + x \left[ \frac{3715}{756} + \frac{55\eta}{9} \right] \right.$$ 

$$+ \left[ \left(- \frac{2223905}{491232} + \frac{154645\eta}{17544} \right) \chi^{-25/9} + \left(- \frac{2045665}{348096} - \frac{128365\eta}{12432} \right) \chi^{-19/9} \right] e_0^2$$

$$+ x^{3/2} \left[ -16\pi + \left(- \frac{295945\pi}{35088} \chi^{-28/9} + \frac{65561\pi}{4080} \chi^{-19/9} \right) e_0^2 \right]$$

$$+ x^2 \left[ \frac{15293365}{508032} + \frac{27145\eta}{504} + \frac{3085\eta^2}{72} + \left( \frac{1185955235}{148548568} + \frac{1902055\eta}{130032} - \frac{14251675\eta^2}{631584} \right) \chi^{-31/9} \right.$$ 

$$+ \left(- \frac{5795368945}{350880768} + \frac{4917245\eta}{1566432} + \frac{25287905\eta^2}{447552} \right) \chi^{-25/9}$$

$$+ \left(- \frac{116151665}{14141952} - \frac{6138415\eta}{133056} - \frac{10688155\eta^2}{294624} \right) \chi^{-19/9} \right] e_0^2 \} \right\} ,$$

where $\chi = f/f_0$ and $x \equiv \left( \frac{Gm \omega(t_0)/c^3}{e_0^5} \right)^{2/3}$ to ensure the SPA condition.

We display here only the leading order contributions in $e_0$!

We have obtained 3PN-accurate $\Psi_j$ expression that includes all the $O(e_0^6)$ contributions at every PN order. This requires $e_t = e_t(\omega,\omega_0,e_0)$ accurate to $O(e_0^5)$ at every PN order.
current state-of-the-art of BBH waveform modeling:
→ phenomenological, frequency-domain waveforms (PN-based inspiral, phenomenological ansatz for merger-ringdown, tuned with 19 calibration hybrids)
→ spin-aligned IMRPhenomD; IMRPhenomPv2 for effective precession

after BBH detections: we implemented a ready-to-use, eccentric TaylorF2 approximant for small eccentricities

Fourier phase includes leading-order eccentricity corrections up to 3PN by extending our results of Tanay et al. (2016)

crucial input for the ready-to-use effective eccentric variant of the IMRPhenomD waveforms (restricted to non-spinning black holes) we developed subsequently, utilizing the modular structure of the IMRPhenomD waveform family (no calibration!)
Eccentric IMR waveforms: Preliminary data analysis

- Parameter estimation with such effective eccentric IMR waveforms has revealed no appreciable systematic errors from assuming circular orbits during the data analysis of GW150914.

- We have constrained the initial orbital eccentricity of the GW150914 black hole binary to $e_0 < 0.1$ at 10Hz.
On-going work with UIB Palma

- improvement of our ’effective eccentric variant’ of IMRPhenomD: systematic study of GW data analysis implications for such a preliminary Phenom model of GWs from eccentric black-hole binaries without spin

- hybridize 3PN time-domain inspiral waveforms (EccentricTD) with long-evolution, non-spinning, eccentric NR simulations (generated with BAM code; $q = [1, 1.5, 2, 4]$, $e_0 = [0.1, 0.2, 0.5]$, convergence series with 4 resolutions)

- aim: use as eccentricity-exact injection waveforms to explore the accuracy of the eIMR model over the parameter space recalibrate model with eccentric NR simulations?

- long-term goal: phenomenological GW model for generic binaries with both eccentricity and spin precession (IMRPhenomE?)
Conclusions

- Most stellar-mass BBH are expected to have shed their formation eccentricity when they enter the LIGO band.
- Plausible astrophysical formation scenarios in dense stellar environments suggest $\geq 1/yr$ GW events with non-negligible eccentricity.
- LISA will observe stellar-mass BBH at earlier stages in their evolution when the orbits cannot generally assumed to be quasi-circular.
- The inspiral stage of eccentric binary coalescence without spin is well-modeled in time- and frequency domain.
- First models for full inspiral-merger-ringdown signal of non-spinning coalescence along eccentric orbits are available.
- Improved IMR models that include spin should be available for data analysis in O3.