Compact binaries
in
alternative theories of gravity

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Is General Relativity correct?

- General Relativity passes all the Solar system tests, laboratory experiments and astrophysical observations at intermediate length/energy scales, but....

  1) GR is not a renormalizable theory $\rightarrow$ might be solved by adding quadratic curvature terms (i.e., high energy corrections) to the Lagrangian, guided by "Quantum gravity" theory (String theory or Loop QG)
  - these corrections might avoid black hole singularities

  2) Existence of dark energy/dark matter $\rightarrow$ modifications of GR with corrections at low energies/large length scales

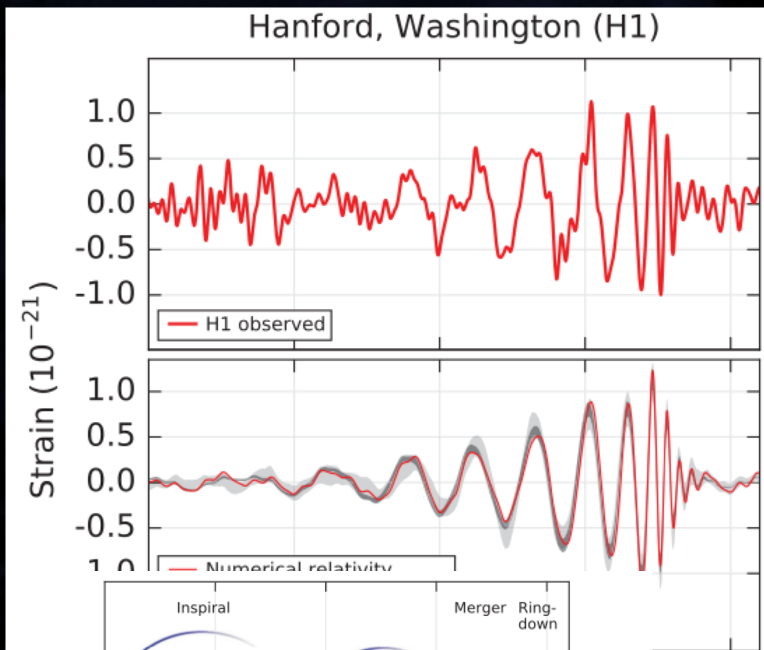
Viable alternatives theories of gravity

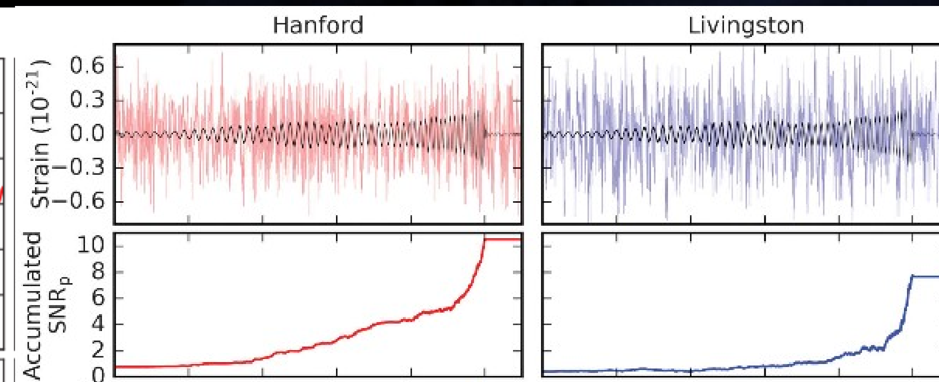
- Mathematical
  - field equations that follow from an action
  - well posed Initial-Boundary-Value-Problem

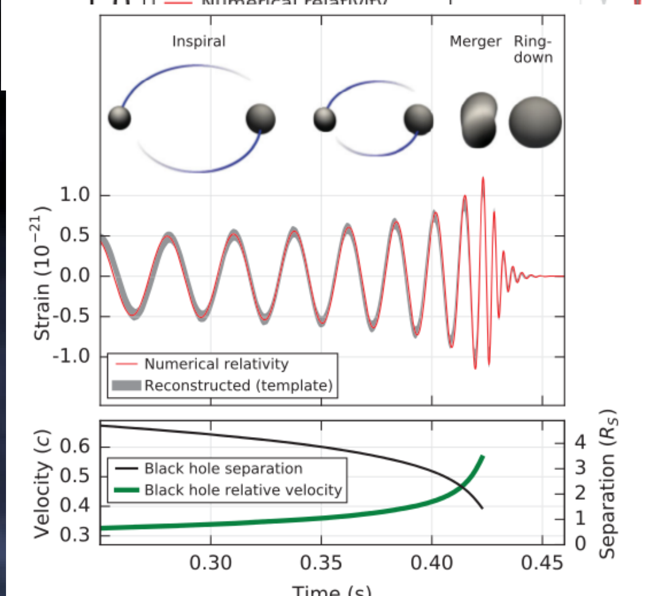
- Theoretical : solve the limitations of GR
  - high energy corrections to achieve a renormalizable theory
  - cosmological viable theory

- Observational : consistent with GR in the intermediate energy regime (and not too far in strong gravity?)
  - the weak gravity regime (Solar system and lab experiments)
  - the mildly relativistic regime (binary pulsars)
  - the weakly dynamic, strong regime (isolated BHs & NS)
  - the highly dynamic, strong gravity regime (mergers BH & NS)

From first GW detections in 2015...

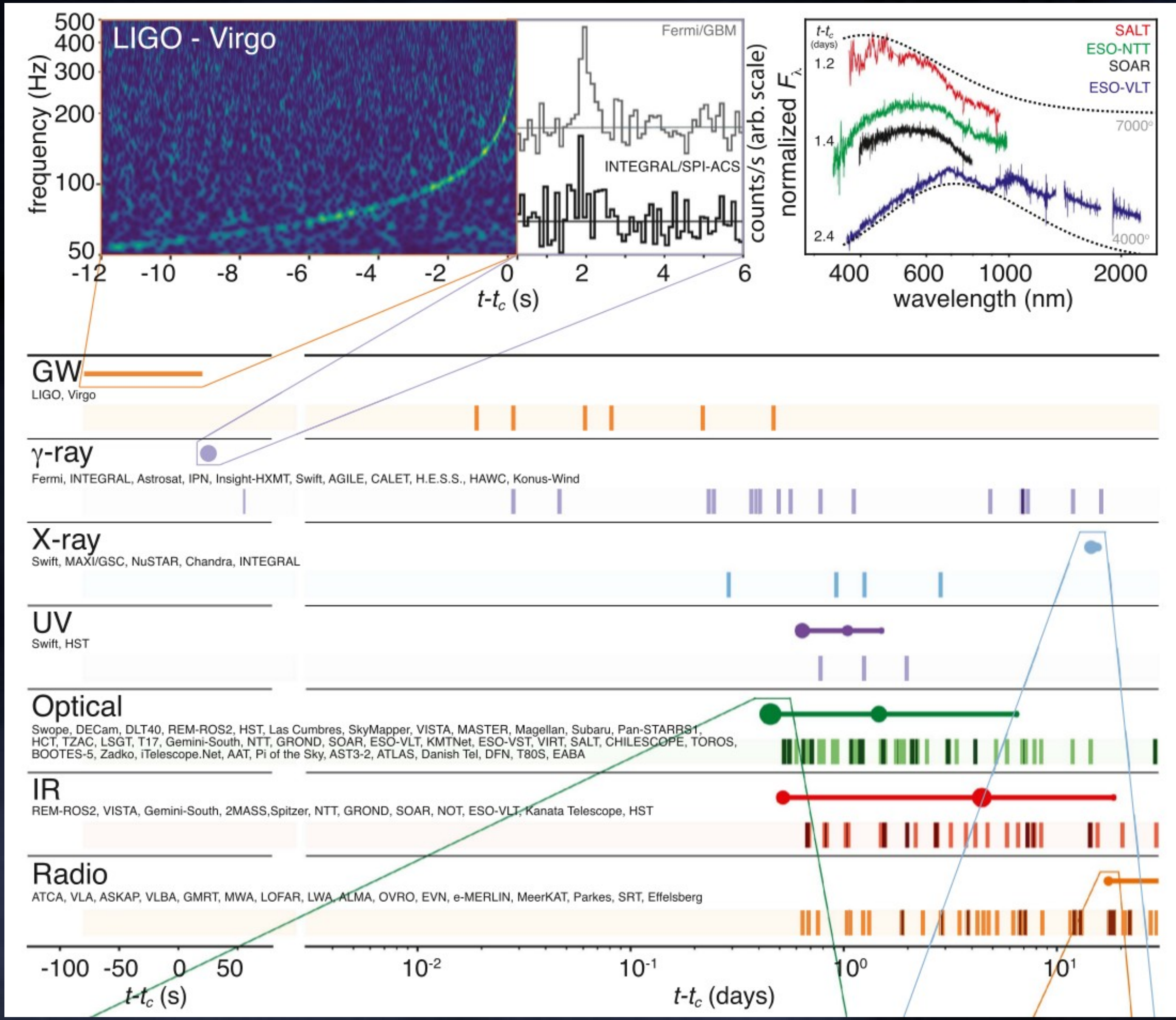






GW observations of 5 binary BH mergers with total mass O(10)- O(100)

….up to now



GW + EM observations of 1 binary NS merger

Viable alternatives to GR

- Unfortunately, most of the current alternative theories present pathologies and do not satisfy all (or any!) of these conditions

- GW observations could guide our quest, but...

  "the true potential of heavier BH mergers like GW150914 to test GR and exotic compact objects is limited by the lack of knowledge of how GWs behave during the merger phase in GR alternatives" [Yunes ++ 2016]

- On this talk I will review current numerical efforts to describe the coalescence (inspiral-merger-ringdown)

  - Scalar-Tensor (ST) theories + Extensions (EMD & f(R))
  - Quadratic theories : dynamical Chern-Simons (dChS)

Scalar-Tensor

theories of gravity

Scalar-Tensor theories of gravity

- Scalar fields appear naturally in phenomenological gravity and low-energy limits of String theory → non-minimally coupled scalar field to the scalar curvature (κ = 8πG, ω(ϕ) arbitrary function)

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}(\partial_\mu\phi\partial^\mu\phi) - V(\phi)\right] + S_M(g_{\mu\nu},\psi)$$

JORDAN or PHYSICAL FRAME

-Rewrite the previous action as the standard GR + a minimally coupled scalar field, by performing a conformal transformation

-

$$g^E_{\mu\nu} = \phi\, g_{\mu\nu}$$

$$\frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi} = \sqrt{\frac{2\kappa}{3+2\omega(\phi)}}$$

EINSTEIN FRAME

$$S = \int d^4x\sqrt{-g^E}\left(\frac{R^E}{2\kappa} - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \tilde{V}(\varphi)\right) + S_M(g^E_{\mu\nu}/\phi(\varphi),\psi)$$

Evolution equations

- By varying the action in the Einstein frame

$$G^E_{\mu\nu} = \kappa\left(T^\varphi_{\mu\nu} + T^E_{\mu\nu}\right),$$
$$\Box^E \varphi = \frac{1}{2}\frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi} T_E,$$
$$\nabla^E_\mu T^{\mu\nu}_E = -\frac{1}{2} T_E \frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi} g^{\mu\nu}_E \partial_\mu\varphi$$

-Effective coupling between matter and scalar field mediated by gravity, which can be expanded in Taylor series

$$\frac{1}{2}\frac{\mathrm{d}\log\phi}{\mathrm{d}\tilde{\varphi}} \approx \frac{1}{\sqrt{3+2\omega_o}} - \tilde{\beta}(\tilde{\varphi} - \tilde{\varphi}_0) + \ldots$$

- Brans-Dicke (BD) : ω = cte $\rightarrow$ $\Phi = \exp(\alpha_0\varphi)$
- Damour & Esposito-Farese (DEF) $\{\varphi_0, \beta\}$

$\omega = -3/2 + \kappa/(4\,\beta\,\log\phi)$ $\rightarrow$ $\Phi = \exp(\beta^2\varphi^2)$ with $\varphi \rightarrow \varphi_0$

No-hair black hole theorem

- There is a generalization of the no-hair BH theorem to Scalar-Tensor theories, so isolated BHs are equivalent to those in GR

- The dynamics of BH binaries in Scalar-Tensor theories can not be distinguish from that in GR

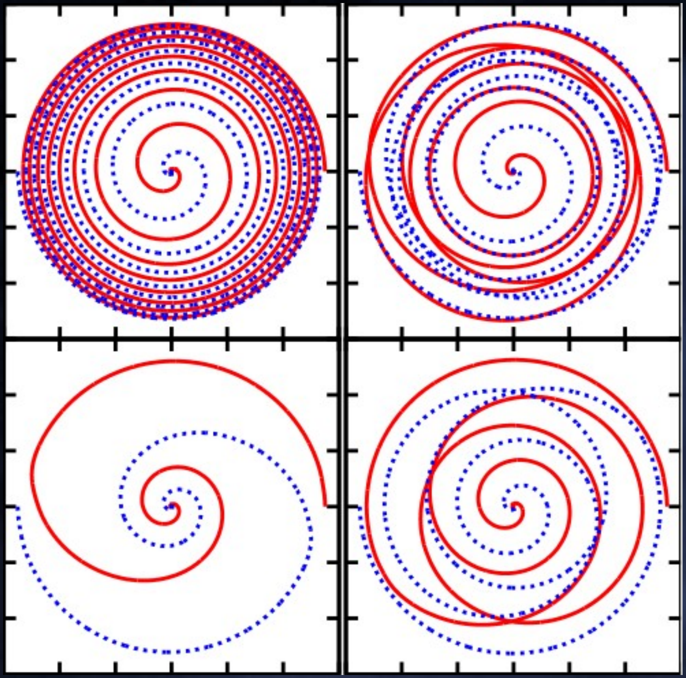
[Sotiriu++ PRL2012, Mirsheraki++ PRD2013 , Yunes++ PRD2012]

- Deviations from GR in the radiation from BH binaries can NOT occur if these assumptions are satisfied:

1) the spacetime contains no matter
2) the potential V (φ) vanishes
3) the metric is asymptotically flat and
   the scalar field is asymptotically constant

Beyond no-hair BH theorem I

- Binary BHs in bubbles of SF with non-vanishing $V(\varphi)$ [Haley++ CQG2012]
  - indistinguishable of GR unless there is a significant accretion



- Binary BHs with non-constant asymptotic scalar field due to cosmological considerations [Berti++ PRD2013]
  - dipolar emission of scalar-field
  - GWs (i.e., Newman-Penrose Ψ_4) indistinguishable from GR

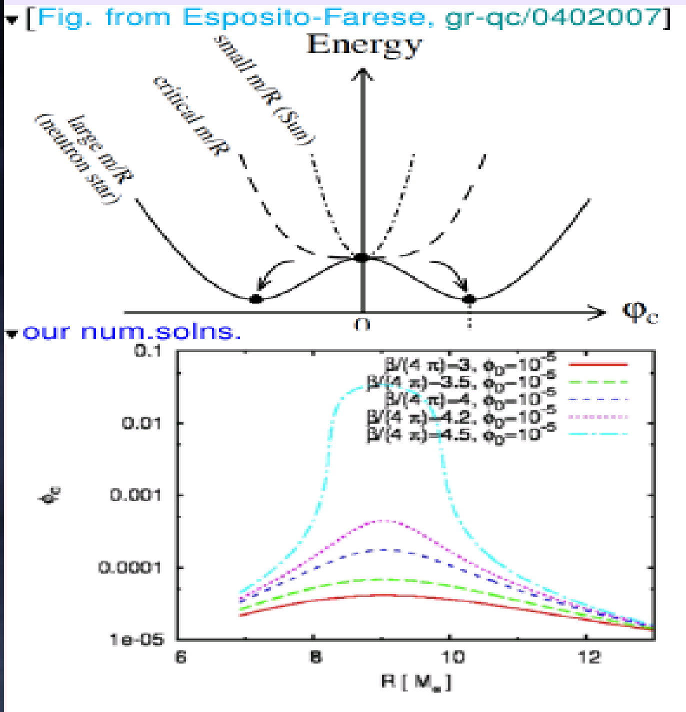
Beyond no-hair BH theorem II

- Spontaneous scalarization : non-trivial SF around compact matter solutions (i.e., neutron stars) due to the non-minimal coupling to the matter in Damour-Esposito-Farese families (which depends on the asymptotic value of the scalar field at infinity φ_0 and β) for $\beta/(4\pi G) \leq -4.5$ [Damour++ 1993]

$$\varphi = \varphi_0 + \varphi_1/r + O(1/r^2)$$

scalar charge $\alpha = (4\pi G)^{1/2}\, \varphi_1/m$

$$[\ g_{tt} = -1 + 2M/r + O(1/r^2)\ ]$$



Non-GR effects

1) Differences in the 1PN terms => (e.g., Solar system)

$$ds^2 = -\left[1 - \frac{2M_\odot}{r} + 2\beta^{PPN}\left(\frac{M_\odot}{r}\right)^2\right]dt^2 + \left[1 + 2\gamma^{PPN}\left(\frac{M_\odot}{r}\right)\right](dx^2 + dy^2 + dz^2)$$

2) Dipolar radiation at 1.5 PN (not only 2.5 PN) => binary pulsars

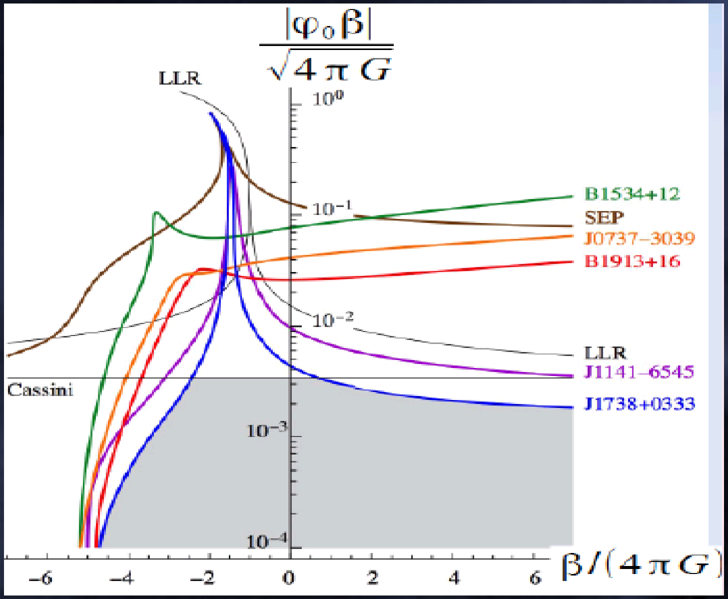
$$\dot{E}_{\rm dipole} = \frac{G}{3c^3}\left(\frac{G_{\rm eff}m_1m_2}{r^2}\right)^2(\alpha_1 - \alpha_2)^2$$

$$\dot{E}_{\rm quadrupole} = \frac{32G}{5c^3}\left(\frac{G_{\rm eff}m_1m_2}{r^2}\right)^2\left(\frac{v}{c}\right)^2$$

- Constraints in the free parameters of the theory [Antoniadis++ Science 2013]

$\varphi_0 < 10^{-4}\, G^{1/2}/|\beta|$

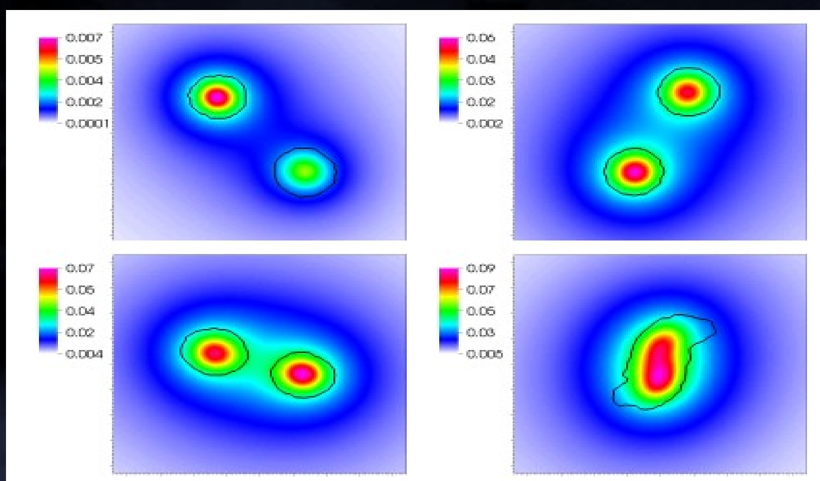
$\beta/(4\pi G) \leq -4.5$

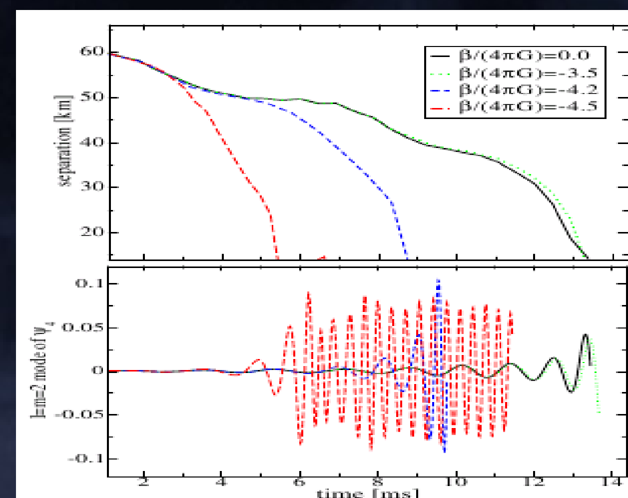


Simulations of neutron star coalescence

- Evolve numerically the ST + Hydro evolution equations

- Binary (non-scalarized) NS with $\beta/(4\pi G) \geq -4.5$ and $\varphi_0 = 10^{-5}$
  [Barausse++ PRD 2013, Shibata++ PRD 2014]



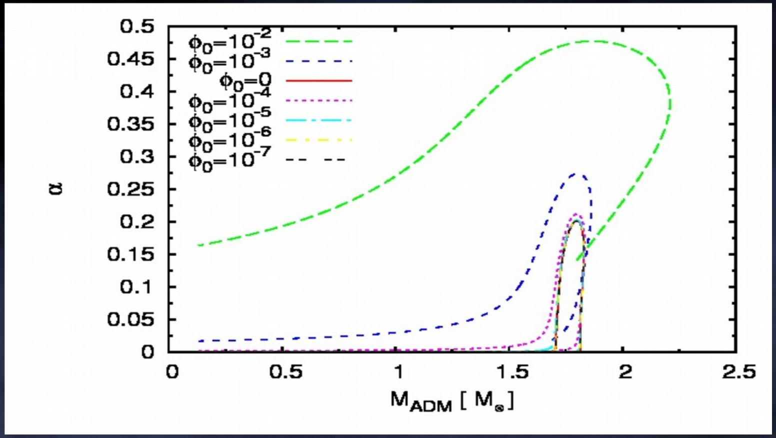


Dynamical Scalarization : the presence of the other star increases the scalar charge of both stars during the coalescence

Spontaneous, Induced and Dynamical Scalarization

- The system can undergo either spontaneous (SS), induced (IS) or dynamical (DS) scalarization, depending on the masses of the stars

- Massive (compact) stars : SS for $\beta/(4\pi G) \leq -4.2$
  * IS in a binary : increases the scalar charge of the other star
- Light (loose) stars : DS for $\beta/(4\pi G) \leq -4.2$ at a given separation



-additional force proportional to the scalar charges

$$F = \frac{G_{\rm eff} m_1 m_2}{r^2}$$

$$G_{\rm eff} = G(1 + \alpha_1 \alpha_2)$$

Post-Newtonian framework in ST

- In order to study the parameter space and produce full set of waveforms for quantitative analysis (i.e., GW templates and detection analysis) requires a Post-Newtonian approximation (at least 2.5 PN order) [Mirshekari & Will, PRD 2013]

$$
\begin{aligned}
\frac{d^2\mathbf{x}}{dt^2} &= -\frac{G_{\mathrm{eff}} M}{r^2}\mathbf{n} \\
&+ \frac{G_{\mathrm{eff}} M}{r^2}\left[\left(\frac{\mathcal{A}_{PN}}{c^2} + \frac{\mathcal{A}_{2PN}}{c^4}\right)\mathbf{n} + \left(\frac{\mathcal{B}_{PN}}{c^2} + \frac{\mathcal{B}_{2PN}}{c^4}\right)\dot{r}\mathbf{v}\right] \\
&+ \frac{8}{5}\eta\frac{(G_{\mathrm{eff}} M)^2}{r^3}\left[\left(\frac{\mathcal{A}_{1.5PN}}{c^3} + \frac{\mathcal{A}_{2.5PN}}{c^5}\right)\dot{r}\mathbf{n}\right. \\
&\left. - \left(\frac{\mathcal{B}_{1.5PN}}{c^3} + \frac{\mathcal{B}_{2.5PN}}{c^5}\right)\mathbf{v}\right]
\end{aligned}
$$

$A_{PN}, B_{PN} = f[\alpha_1, \alpha_2]$ $\rightarrow$ assume that α is "small and constant in time"

Post-Newtonian + full scalarization

- Perturbation expansion of the scalar charge can not capture spontaneous or dynamical scalarization

- Post-Newtonian framework to include non-linear scalarization
  [CP++ PRD 2014]

  (1) Consider the PN equations without assumptions on α
  (2) update the scalar charge
  at each time by solving

$$\varphi_B^{(1)} = \varphi_0 + \frac{\varphi_1^{(2)}(\varphi_B^{(2)})}{r(1-\dot{r})} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

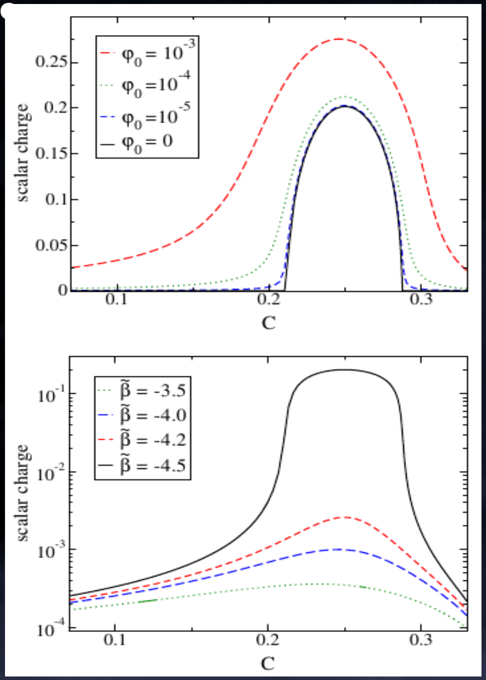
$$\varphi_B^{(2)} = \varphi_0 + \frac{\varphi_1^{(1)}(\varphi_B^{(1)})}{r(1-\dot{r})} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

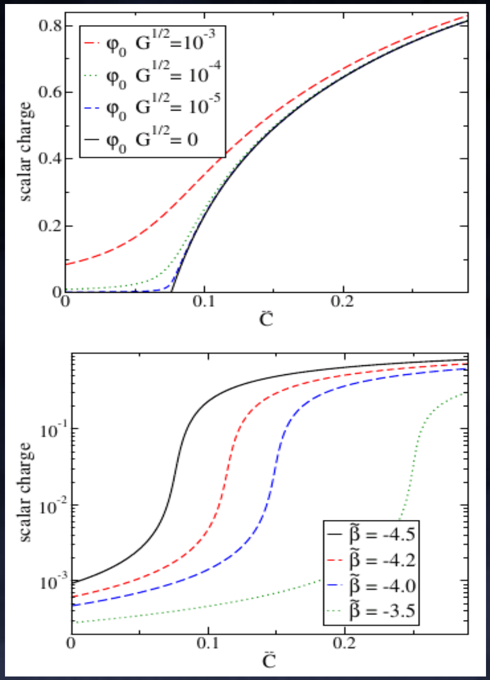
- For a given (φ_0, β), all the relevant quantities can be obtained from the numerical solution of isolated NS

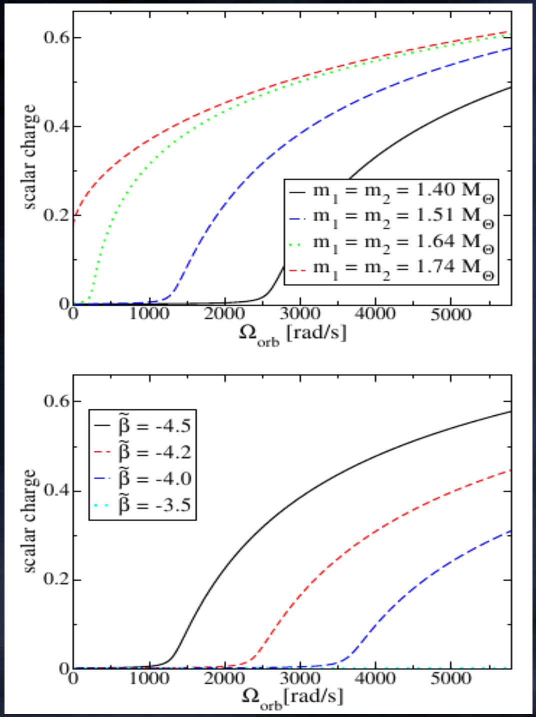
$\varphi_1 = \varphi_1(\varphi_B) \rightarrow \alpha$ $\qquad\qquad$ $\varphi_c = \varphi_c(\varphi_B)$

Spontaneous Scalarization vs Dynamical Binary Scalarization

The scalar charge in binary systems and isolated NS presents a similar dependence on the compactness of the system







Spontaneous

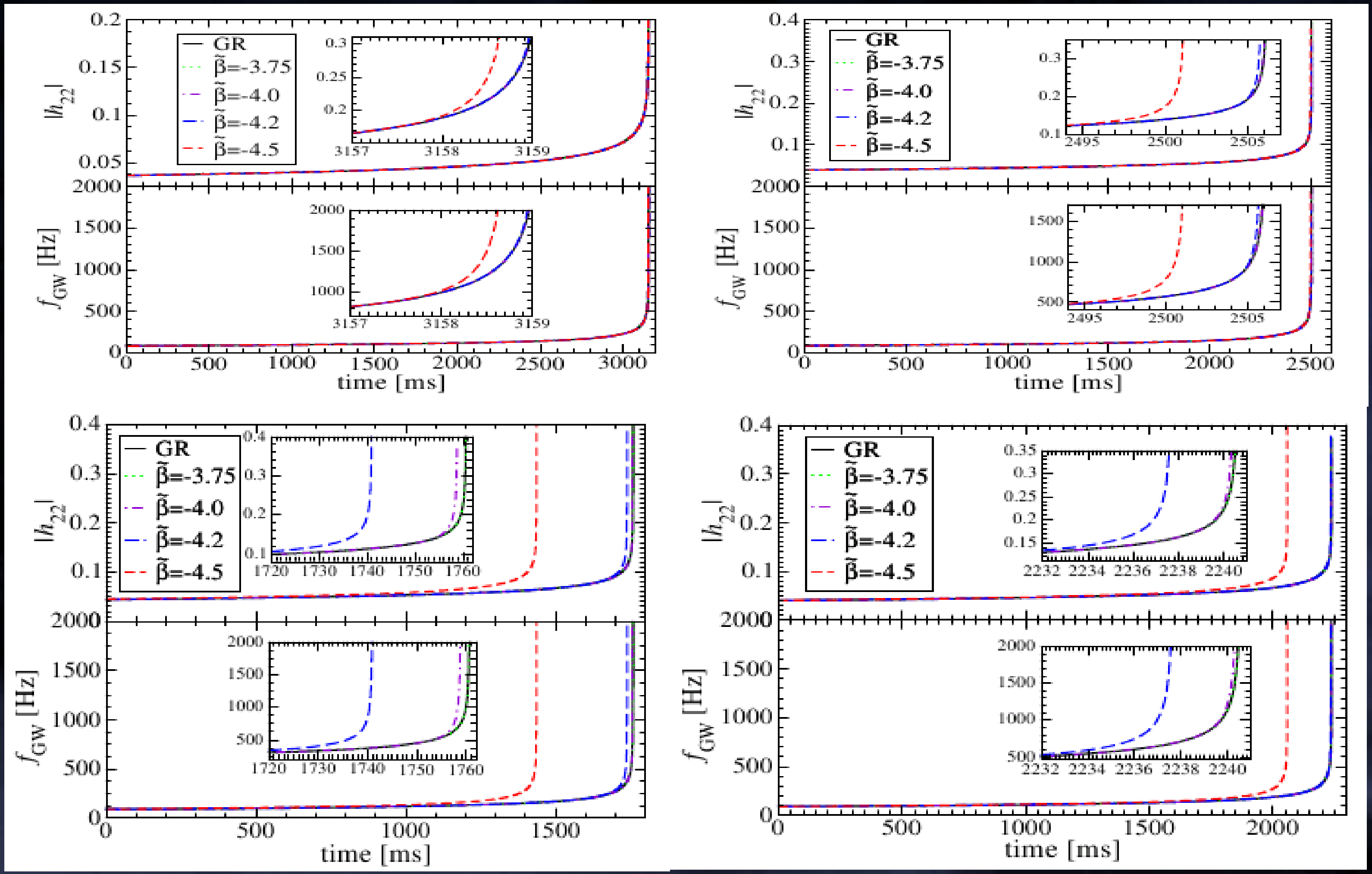
$C_{single} = M/R_{NS}$

Dynamical

$C_{binary} = E_{1PN}/a$

$$\Omega_*[rad/s] = A\left(\frac{m_1}{M_\odot} - m_*\right)\left(\frac{m_2}{M_\odot} - m_*\right)$$

Waveform templates for ST



Includes Spontaneous, Induced and Dynamical scalarization !!

[CP++ PRD2014, Sennett++ PRD2016, Sampson++ PRD2014, Ponce++ PRD2015]

Extensions of Scalar-Tensor

- Einstein-Maxwell-Dilaton
  (Scalar-Vector-Tensor)

Einstein-Maxwell-Dilaton gravity

- EMD is a well posed theory that appears as a low energy limit of string theory and includes a U(1) gauge field F_{ab} and a scalar field Φ

$$S = \int d^4x \sqrt{-\tilde{g}}\, e^{-2\phi} \left[R + \Lambda + 4(\nabla\phi)^2 - F^2 - \frac{H^2}{12} \right]$$

JORDAN or PHYSICAL FRAME

-Rewrite the previous action as the standard GR + a minimally coupled scalar field and an EM field by performing a conformal transformation $g_{ab} = e^{-2\phi}\tilde{g}_{ab}$

$$S = \int d^4x \sqrt{-g} \left[R - 2(\nabla\phi)^2 - 2V - e^{-2\alpha_0\phi} F^2 \right]$$

EINSTEIN FRAME

α_0 parametrizes a family of theories (α_0=0 Einstein-Maxwell, α_0=1 EMD, α_0=√3 Kaluza-Klein)

Evolution equations of EMD

-The evolution equations in the Einstein frame are reduced to the standard Einstein-Maxwell-Klein-Gordon equations with new source terms

$$
\begin{aligned}
R_{ab} &= 2\left(T_{ab} - \frac{1}{2} g_{ab} T\right) \\
\nabla^a \nabla_a \phi &= \frac{1}{2}\frac{\partial V}{\partial \phi} - \frac{\alpha_0}{2} e^{-2\alpha_0 \phi} F^2 \\
\nabla^a F_{ab} &= -I_b \quad .
\end{aligned}
$$

$$
\begin{aligned}
I_b &= -2\alpha_0 \nabla^a \phi F_{ab} \\
T_{ab} &= T^{\phi}_{ab} + e^{-2\alpha_0 \phi} T^{\mathrm{EM}}_{ab} \\
T^{\phi}_{ab} &= \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \left[\nabla_c \phi \nabla^c \phi + V(\phi)\right] \\
T^{\mathrm{EM}}_{ab} &= F_{ac} F_b{}^c - \frac{1}{4} g_{ab} F^2 .
\end{aligned}
$$

Single BH analytical solutions

- The gauge field F_{ab} might correspond either to:
  - hidden part of the gravity sector (i.e., like the scalar field): a priori there are no restrictions on its magnitude
  - EM sector: charge in BHs can not be large in general
    → but it could be important in NS with strong B fields!!

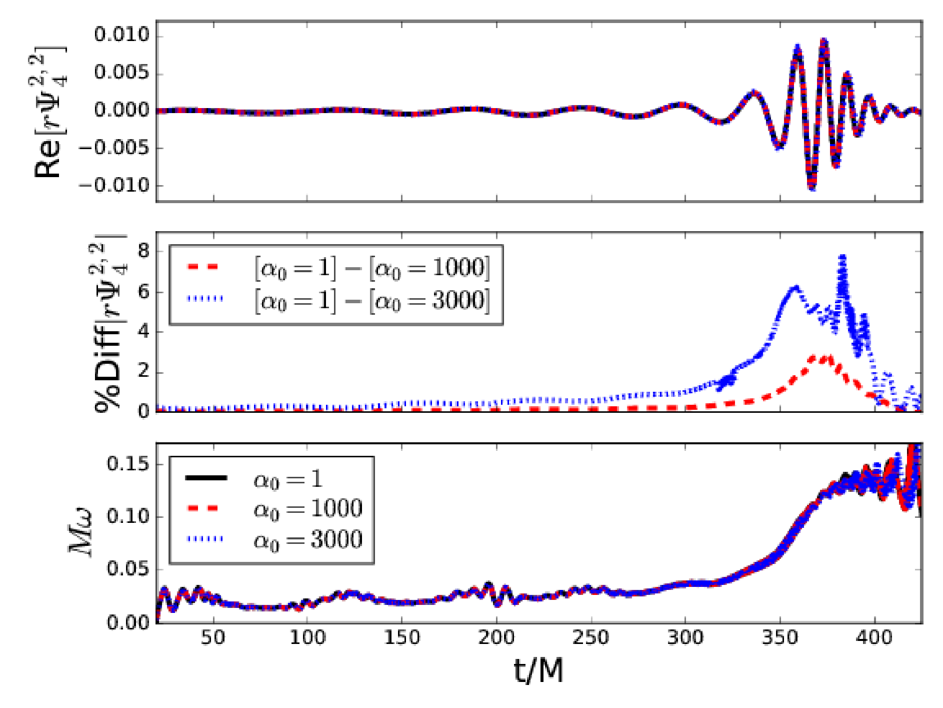
- Analytical solutions for single charged BHs show that there is a scalar charge associated to the EM charge

$$\Phi(r) \approx \Phi_0 + \Phi_1/r \qquad \Phi_1 \approx \alpha_0 Q^2/(2\,M)$$

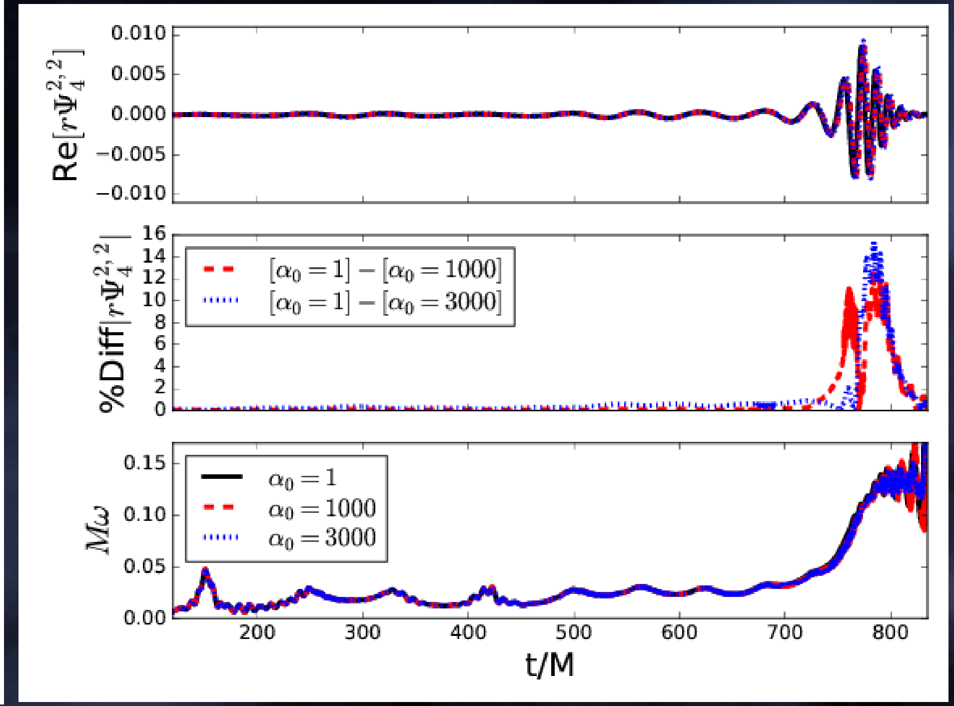
Similar to Scalar-Tensor theories with matter!!
Enhancement of gravitational force & dipolar radiation

Binary BH numerical solutions

- Equal and unequal binary BHs with a small charge Q/M=0.001 and different values of α_0 [Hirschmann++, PRD2018]



equal mass



unequal mass

Extensions of Scalar-Tensor

- f(R) gravity

Equivalence of f(R) and ST theories

- Consider an action with an arbitrary function f(R) of the scalar curvature

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,f(R) + S_{\rm M}[g_{\mu\nu},\psi]$$

- Perform the transformations

$$\phi \equiv \frac{df(R)}{dR}\,, \quad U(\phi) \equiv R\,\frac{df(R)}{dR} - f(R)$$

$$S^{\rm J} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\Big(\phi R - U(\phi)\Big) + S^{\rm J,\,M}[g_{\mu\nu},\psi]$$

JORDAN or PHYSICAL FRAME

$$g^{\rm E}_{\mu\nu} \equiv \phi(\varphi)\,g_{\mu\nu}\,, \quad \phi(\varphi) \equiv e^{2\beta\frac{\varphi}{M_{\rm Pl}}}$$

EINSTEIN FRAME

$$S^{\rm E} = \int d^4x\,\sqrt{-g^{\rm E}}\left(\frac{M_{\rm Pl}^2}{2}R^{\rm E} - \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - V(\varphi)\right) + S^{\rm E,\,M}[g^{\rm E}_{\mu\nu}\,\phi(\varphi)^{-1},\psi]$$

Binary NS simulations in f(R)

- First numerical simulations of NS binaries in f(R) [Sagunski++2017]

$$f(R) = R + a_2 R^2$$

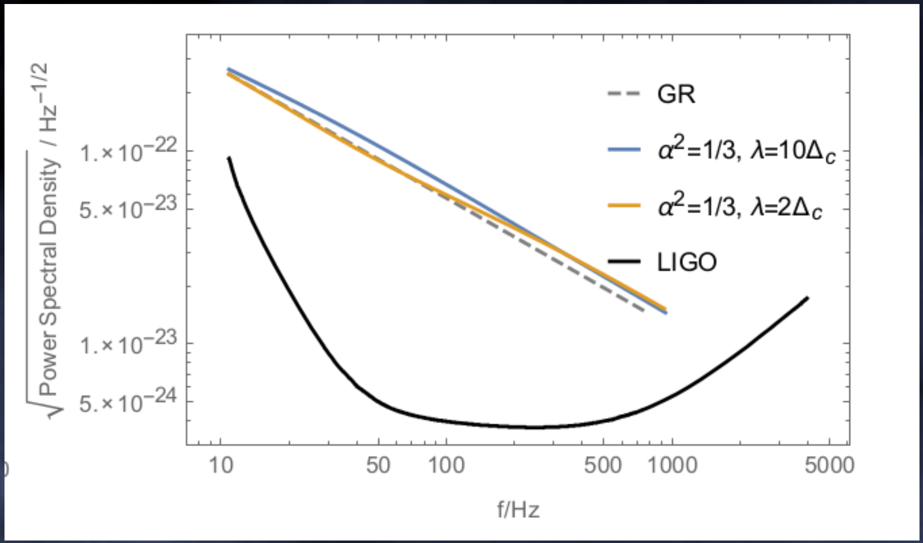
$$\phi = 1 + 2\,a_2 R\,, \quad U(\phi) = \frac{1}{4a_2}\,(\phi - 1)^2$$

Jordan frame

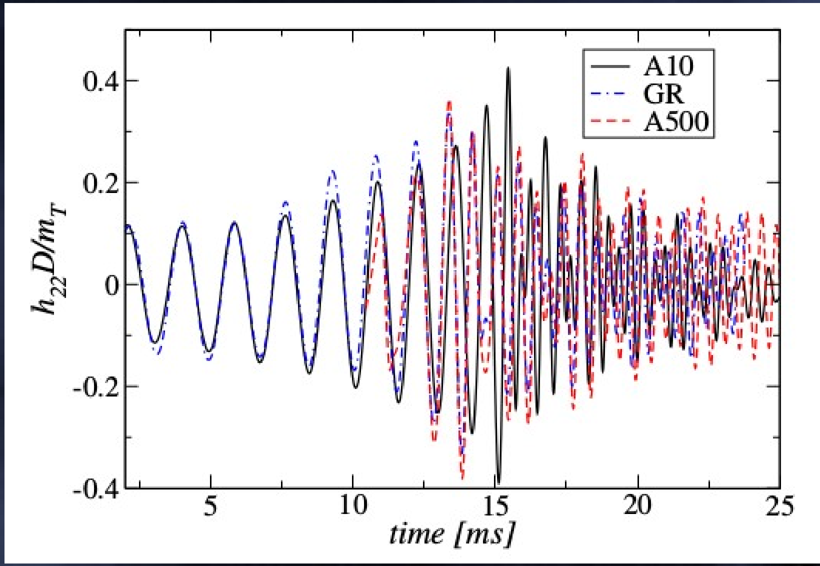
$$V(\varphi) = \frac{M_{\rm Pl}^2}{8a_2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm Pl}}}\right)^2$$

Einstein frame

PN theory



Full GR



Summary on ST

- ST allows for non-linear effects (i.e., scalarization) that produces differences wrt GR in the strong gravity regime

- Spontaneous and Dynamical scalarization are presents in ST and similar theories (f(R) and EMD) and might produce measurable differences on the waveforms → smoking gun!

- New results in EMD
  - spinning BH solutions, stability,…
  - the effects on the GWs produced during the merger of binary BHs are small for low values of the charge Q/M,
  even if α_0 >>1 (i.e., they scale as $\alpha_0(Q/M)^2$)

Dynamical Chern-Simons
as an
Efective Field Theory (EFT)

Effective Field Theory

- Most of the alternative/modified gravity theories seems to be ill posed (i.e. dynamical Chern-Simons [Delsate++ PRD2015] and Einstein-dilaton-Gauss-Bonnet [Papallo arxiv2017])

- For theories that are sufficiently close to GR it is possible to use an effective field-theory (EFT) approach, assuming that there is a high-energy theory whose low-energy limit leads to GR plus small corrections [Okounkova++ PRD2017]

- Perturb the solutions around GR in powers of the small coupling parameter $\rightarrow$ hierarchy of equations at each order, which inherit the same principal part and well-posedness properties of the background GR system.

DCS as an Effective Field Theory

- Write action and field equations as GR + $O(l^2 \sim \varepsilon)$

$$I_\varepsilon = \int d^4x \sqrt{-g}\,[L_{\rm EH} + L_\vartheta + \varepsilon L_{\rm int} + L_{\rm mat} + \ldots]$$

$$L_{\rm EH} = \frac{m_{\rm pl}^2}{2} R\,,$$
$$L_{\rm int} = -\frac{m_{\rm pl}}{8}\ell^2 \vartheta\, {}^*RR$$
$$L_\vartheta = -\tfrac{1}{2}(\partial\vartheta)^2$$

$$\Box\vartheta = \varepsilon \frac{m_{\rm pl}}{8}\ell^2\ {}^*RR$$

$${}^*RR = \tfrac{1}{2}\epsilon^{abef} R_{ef}{}^{cd} R_{abcd}$$

$$m_{\rm pl}^2 G_{ab} + m_{\rm pl}\varepsilon\ell^2 C_{ab} = T^{\vartheta}_{ab} + T^{\rm mat}_{ab}$$

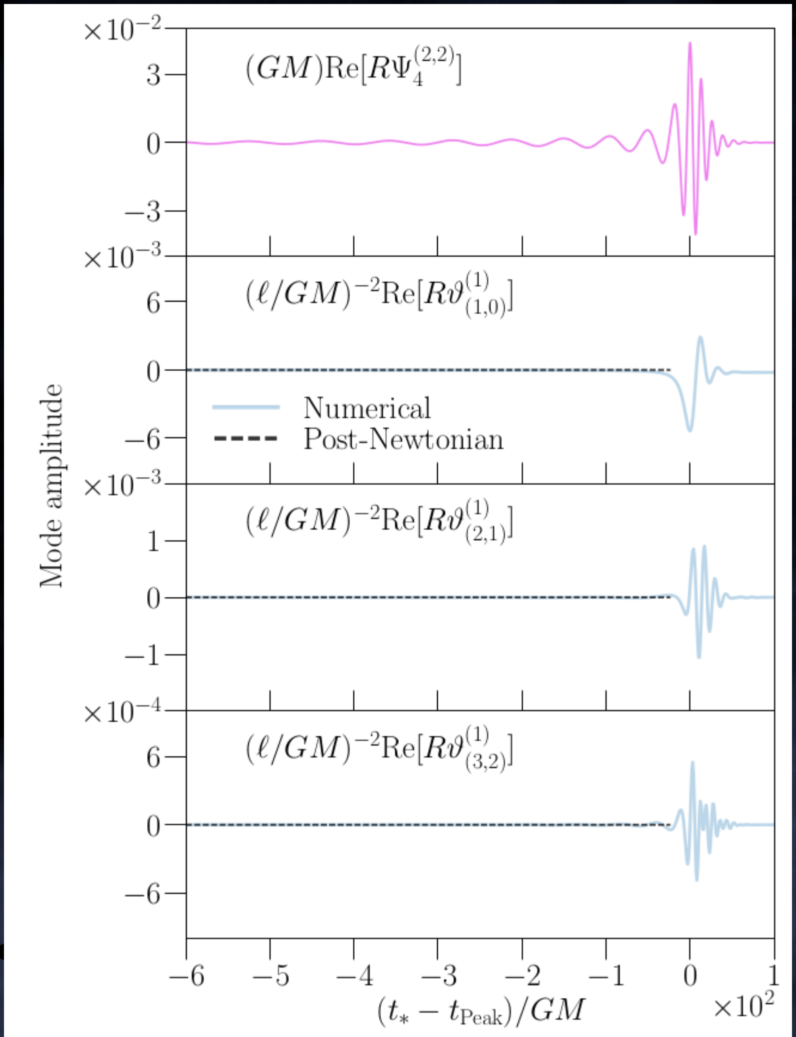
$$C_{ab} \equiv \epsilon_{cde(a}\nabla^d R_{b)}{}^c \nabla^e\vartheta + {}^*R^c{}_{(ab)}{}^d \nabla_c\nabla_d\vartheta$$

- Expand the solutions around the GR one [Okounkova++ PRD2017]

$$g_{ab} = g^{(0)}_{ab} + \sum_{k=1}^{\infty} \varepsilon^k h^{(k)}_{ab}$$
$$\vartheta = \sum_{k=0}^{\infty} \varepsilon^k \vartheta^{(k)}\,.$$

$$\mathcal{O}(\varepsilon^0):\quad G_{ab}[g^{(0)}] = 0\,,\qquad \vartheta^{(0)} = 0$$
$$\mathcal{O}(\varepsilon^1):\quad \Box^{(0)}\vartheta^{(1)} = \frac{m_{\rm pl}}{8}\ell^2[{}^*RR]^{(0)}\,,\qquad h^{(1)} = 0$$
$$\mathcal{O}(\varepsilon^2):\quad G^{(1)}_{ab}[h^{(2)}] = m_{\rm pl}^{-2} T^{\rm eff}_{ab}\,,\qquad \vartheta^{(2)} = 0$$

Black hole simulations in DCS at O(l²)



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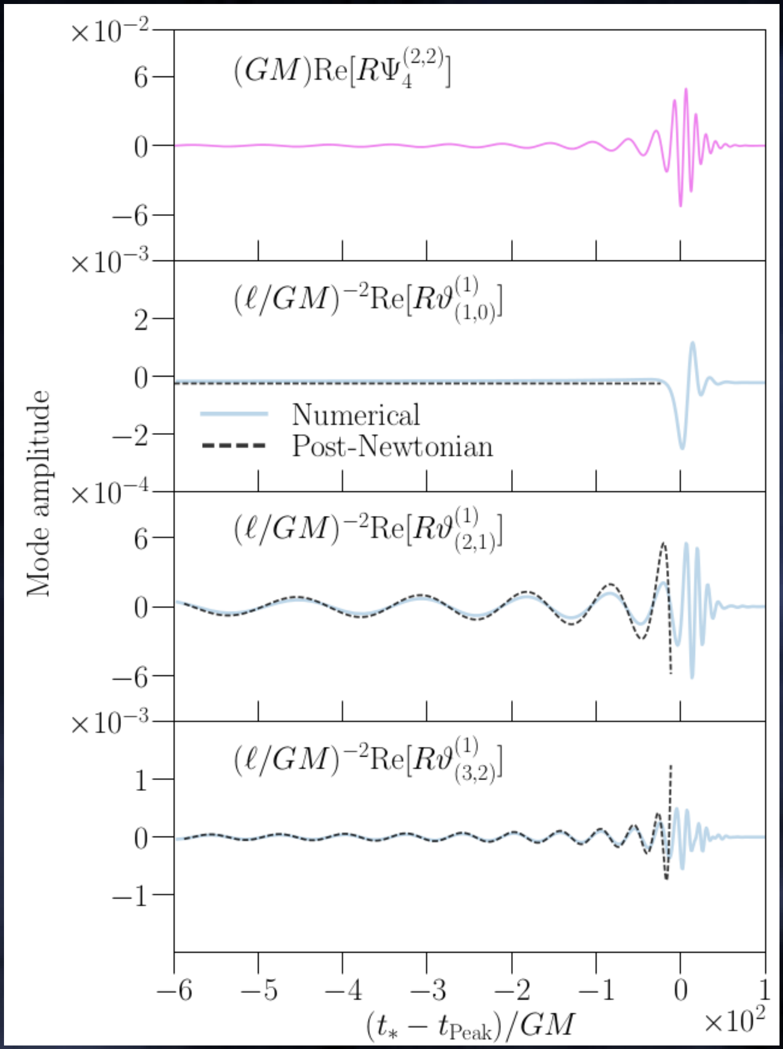

a=0.3

Well posed Full Theory

- A second approach to avoid pathologies, based on the solution for viscous relativistic hydrodynamics introduced by Israel and Stewart, allows to control undesirable effects of higher order derivatives in gravity theories [Cayuso++ PRD2017]

- Notice that a priori there is no guarantee that the solutions will converge at all, or that they will converge to the correct physical solution (i.e., will the higher order perturbations remain small?-→ energy cascade to long or short wavelengths?)

Summary and prospects

- ST theories (and extensions) are well posed, but
  - already quite constrained by observations
  - not clear if they can satisfy the theoretical requirements

- New approaches to construct well-posed formulations for generic alternative gravity theories
  - satisfy some of the theoretical requirements
  - do they converge to the solution of the full theory?

- Many theories, too many options and too much work..
  - is it possible to fill the requirements at low order of EFT?
  - can we have any guidance from GW during the inspiral?
    parameterized post-Einsteinian (ppE) [Yunes++,PRD2009]