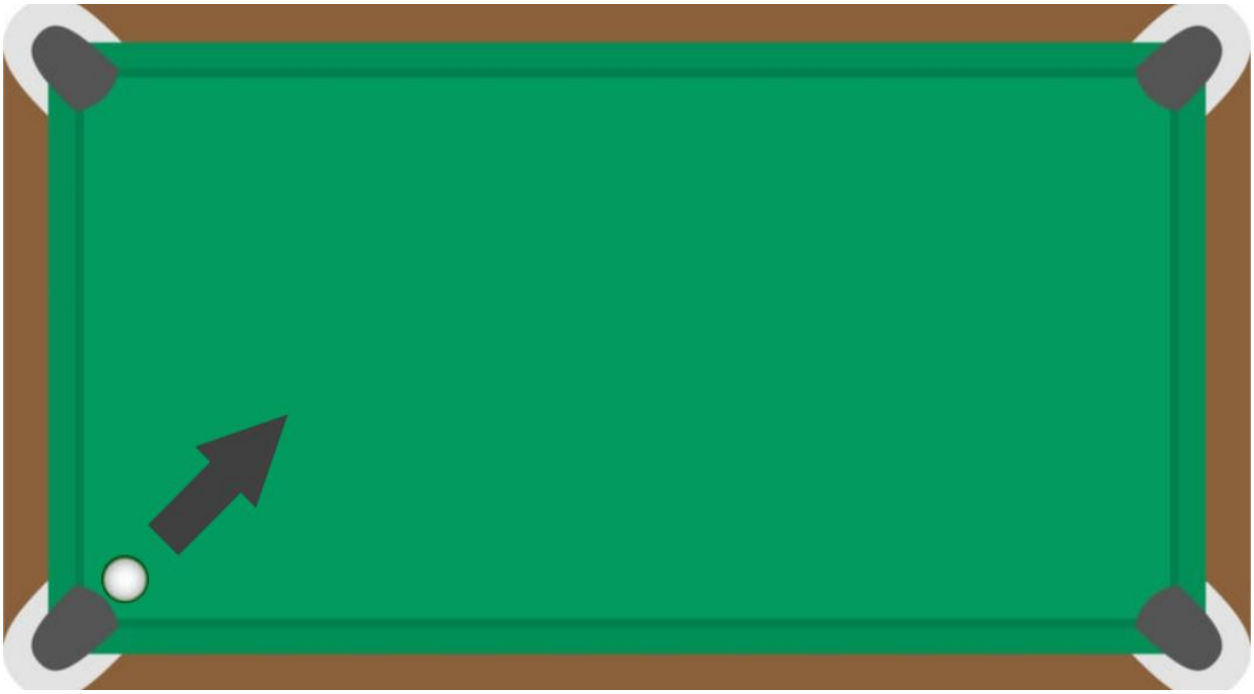
The Billiard Ball Problem

Place a cue ball at the bottom left corner of a billiard table and shoot it at 45° across the table. Into which pocket will it fall? Take this challenge, it is fun!



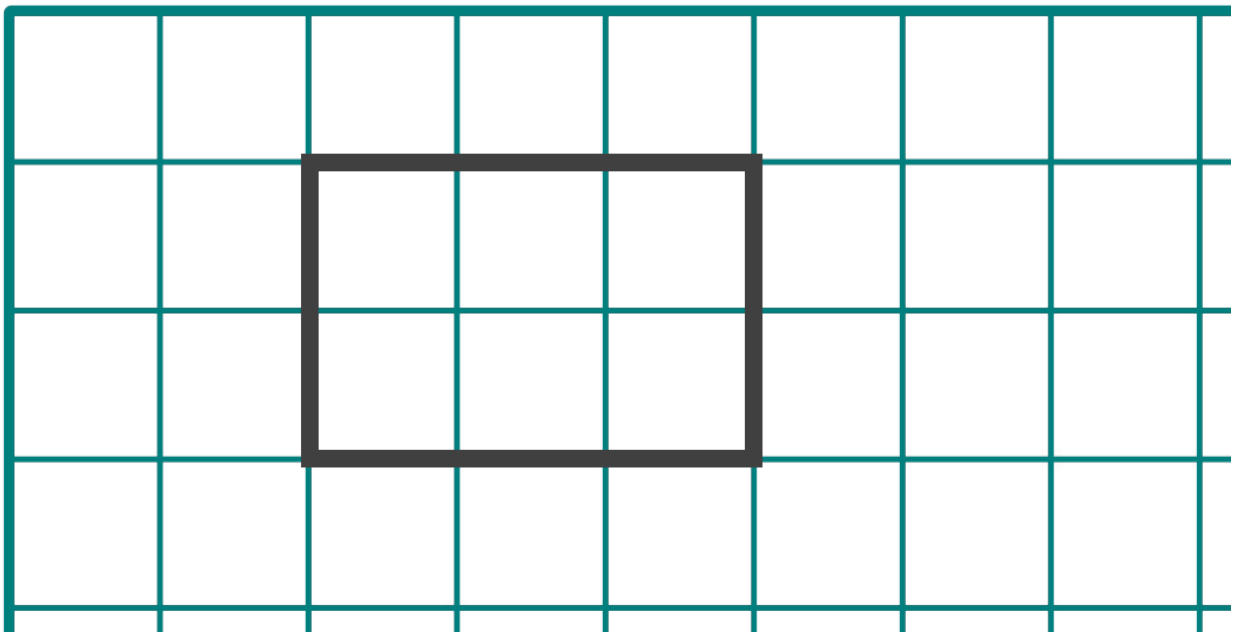
Our billiard table has four pockets, one at each corner. The pocket where the ball will fall into depends on the size of the table. We shall solve this problem with different table sizes. Then we will try to spot a pattern in the solutions!

What do you need?

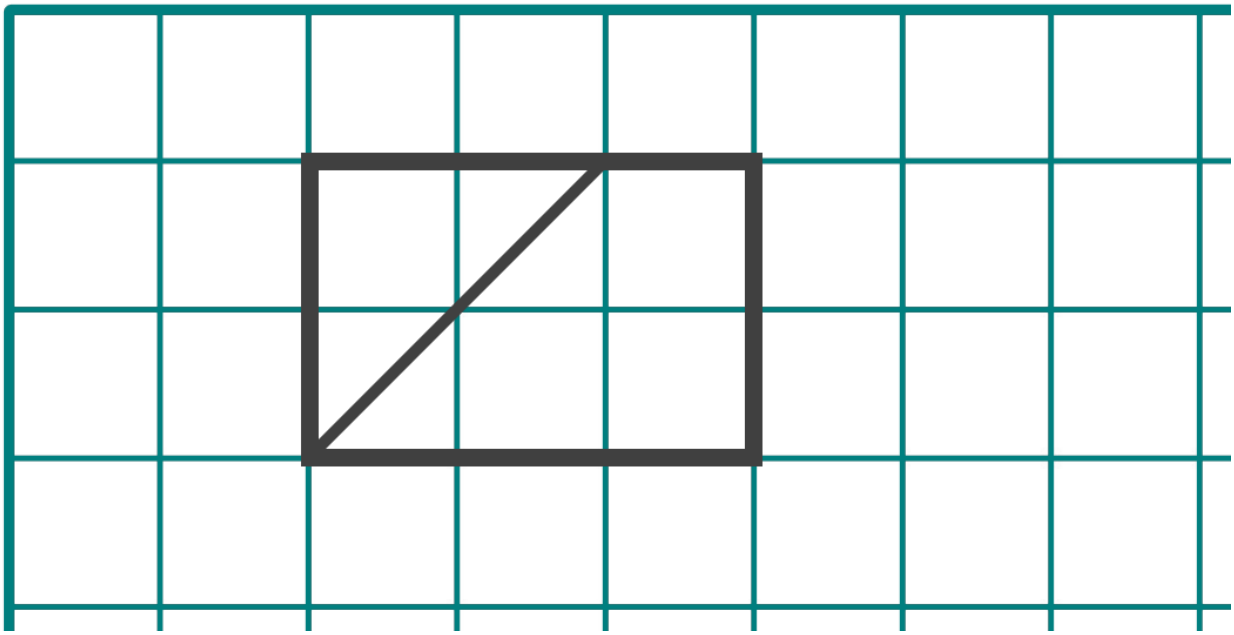
- Blank paper
- Squared paper. If you don't have squared paper, it is available here: http://www.greatlittleminds.com/pdfs/graph-paper-to-print/graph-paper-1cm-sq.pdf
- A ruler, a pencil and a rubber
- The '*I can do it*' attitude! 😉

What do you have to do?

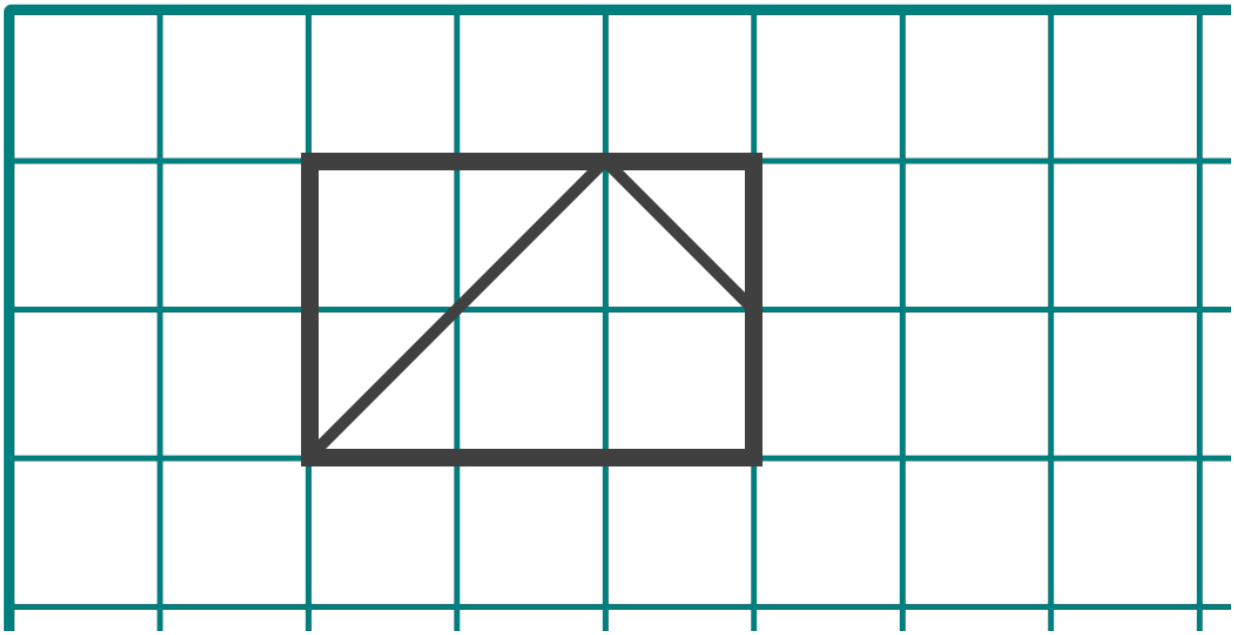
Draw a 3 units by 2 units rectangle on a squared paper, like so:



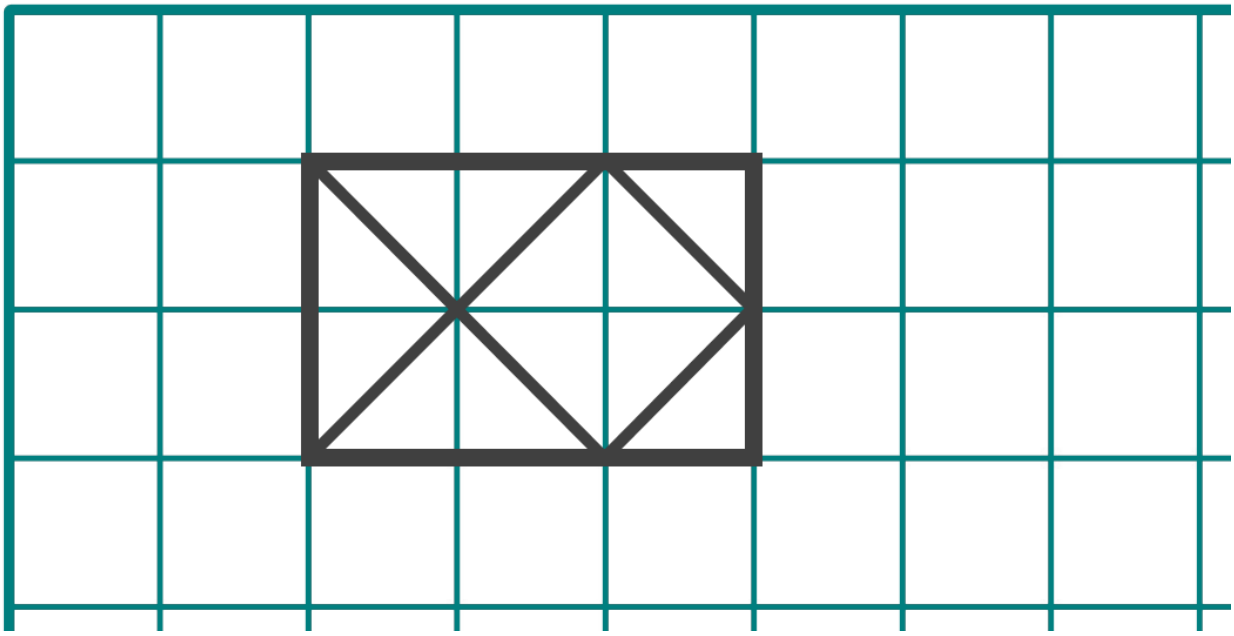
There are four corners, one in each corner of the rectangle. Since the ball starts from the bottom left corner, draw a straight line at 45° from the bottom left corner. This can be done by drawing a line from that corner that cuts through every square of your graph paper exactly along the diagonal; like this:



Now, what happens when the ball hits the side of the table? It bounces off at the same angle and continues along the diagonal of the square grids!



Continue drawing lines like this until your line reaches a corner.



Which corner did it reach? The upper left corner!

The following video should help you see why:

3x2 Billiard Table: https://youtu.be/BNgvIROyE1Y

Now repeat the above, but this time, start with a 5 by 3 rectangle. Where does the ball fall if the table is a 5 by 3 rectangle? The following video may help.

5x3 Billiard Table: https://youtu.be/zgBAEtpmerM

Note that a 2 by 3 rectangle, for example, is a rotated 3 by 2 rectangle, but is otherwise identical. Because of this, we shall only consider rectangles whose width is at least as big as their height.

Repeat the above procedure using some, or all, of the following rectangle sizes:

2 by 1	3 by 1	4 by 2	5 by 2	6 by 2	3 by 3
4 by 3	7 by 3	5 by 4	6 by 4	7 by 4	6 by 5

Questions

1. In which corner does the ball fall in each case?
2. Group the cases where the ball falls in the same corner. What do they have in common? Can you see a pattern?
3. Without drawing anything, can you guess where the ball will fall in each of the following table sizes?

10 by 9	15 by 12	33 by 17	142 by 105	52 by 30	280 by 240

4. Why does the pattern work?

Solution:

https://www.jc.um.edu.mt/__data/assets/pdf_file/0007/450754/TheBilliardBallSolution.pdf

Links Related to the Billiard Ball Problem

A small program in Scratch that helps you solve the Billiard Ball Problem: https://scratch.mit.edu/projects/152682559/

The Billiard Ball Problem for Kids: https://mathforlove.com/wp-content/uploads/2017/04/Billiard-Ball-Problem.pdf

The Billiard Ball Problem for Slightly Older Kids: https://www.thehindu.com/children/the-billiard-ball-problem/article20314985.ece

Billiards for Big Kids! https://mathworld.wolfram.com/Billiards.html