Introduction

Mathematicians and scientists come up with several ideas and with hypotheses. These can be regarded as seeds which develop in proven facts and theories that are then harvested when used in various sciences. Here we will be discussing the Fibonacci sequence, which is used and observed all around us. Various activities, crafts and problems for all ages and abilities were developed on this idea.

Next to each activity, the intended age of the child attempting it is marked. Please note that this is just a guideline; different students have different abilities. Furthermore, if students are being accompanied (and encouraged) by an older person, then they can also work on activities intended for older students.

What is a sequence?

A sequence is a list of numbers with a particular pattern.

Example: 1, 2, 4, 8, …..the numbers in this sequence are obtained by multiplying the previous terms by two.

Example: 1, 4, 7, 10, 13, 16…. The numbers in this sequence are obtained by adding 3 to the previous number.

Activity 1: Make up your own sequence (10 years)

Think of a list of numbers having a particular pattern. Tell your friend the first few terms of the sequence and invite him/her to guess the next few terms.

It's his/her turn now. He will think of a sequence and you will guess the next few terms. You can write your sequences in the worksheet entitled 'Activity 1'.

What is the Fibonacci sequence and who invented it?

Leonardo from Pisa (Leonardo da Pisa), better known as Fibonacci, meaning 'son of Bonaccio invented the sequence whereby each term is the sum of the previous two numbers. The sequence starts 0, 1, 1, 2, 3…..

Activity 2 (10 years)

Obtain the first 20 terms of the Fibonacci sequence by following the pattern. You may use the worksheet entitled 'Activity 2'.

Activity 3: Generalised Fibonacci Sequence Game (Part 1: 11 years, Part 2: 13 years)

Part 1:

Ask a friend to write two small positive integers (choosing small numbers will help you work mentally). These will be the first two terms of your sequence. The 3rd term will be the addition of the first 2 terms, the 4th term will be the addition of the 2 previous terms (2nd term and 3rd term and so on).

Write the first 10 numbers. Secretly look at the 7th number obtained and multiply it by 11, this is the sum of all the ten numbers. You may write your work on the worksheet entitled 'Activity 3'. Your friend will be impressed at how fast you 'added' all those numbers.

Example: 4, 7, 11, 18, 29, 47, 76, 123, 199, 322… the sum of these 10 terms is 836 which is 11 times the 7th term 76

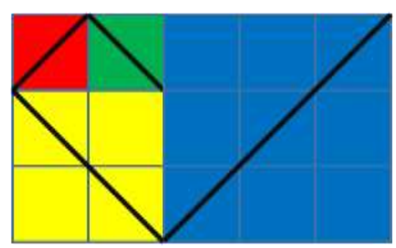
Part2:

Is it always the case that the sum of the first 10 numbers in a generalised Fibonacci sequence is 11 times the seventh number? Think…

(Hint: You can use 'Activity 3' worksheet using x and y to represent any two numbers)

Activity 4: The Fibonacci sequence, the Golden ratio, the Spiral (8 years)

In this activity we are going to obtain a spiral using squares of dimensions equal to the numbers in the Fibonacci sequence. Such spiral is present in nature (such as the snail shell, sunflower, sea horse, some horns of particular animals) and is very pleasing to the eye. Look around you in nature and try to spot these example, if not possible surf the internet for some examples.



For this activity we suggest using Worksheet entitled 'Activity 4' but you can use any squared paper.

Recall the Fibonacci sequence 1, 1, 2, 3, 5 ….. We will be drawing square with the length of the edge equal to the numbers in the Fibonacci sequence.

Draw a green square of edge 1unit (use the size of the square on your squared paper) and on the left hand side of it draw a red square of edge 1unit. The next number in the Fibonacci sequence is 2 so we need a square with edge length 2units.

(You may use different colours than those suggested here but use a colour for each number in the sequence.)

Place the 2 by 2 square below the red and green square. The next number is 3, hence we need a 3 by 3 square to the right of the squares just drawn…. the process will continue as described so far…but let us start marking the spiral first…

Draw the diagonal from the top right corner of the green square and continue marking a diagonal in each square each time choosing the diagonal touching the marked diagonal in the previous square. …yes you are right you are forming a spiral with straight lines. If you prefer a round spiral, change the lines of the diagonals by arcs of the size of the edge of the squares.

You can now continue with the diagram.

What is the size of the edge of the next square? Remember we are using the Fibonacci sequence.

Where should we place this square? Remember that the diagonals have to form a spiral?

Continue to draw the squares according to the Fibonacci sequence and marking the spiral until you use the optimum space on your paper.

The Golden ratio

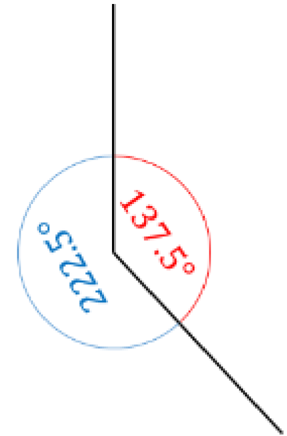
The Golden ratio, denoted by phi φ, is an irrational number (a number which cannot be expressed as a fraction of two integers) which is equivalent to $\frac{1+\sqrt{5}}{2}$, that is, 1.618033989…

The Golden ratio is a value between the ratios $\frac{F_{N+1}}{F_N}$ and $\frac{F_N}{F_{N-1}}$ for any Fibonacci number F_N. As N increases, the gap between these two ratios becomes smaller narrowing to the golden ratio φ.

Ratio of successive pair of Fibonacci numbers									
$^1/_1$	$^2/_1$	$^3/_2$	$^5/_3$	$^8/_5$	$^{13}/_8$	$^{21}/_{13}$	$^{34}/_{21}$	$^{55}/_{34}$	$^{89}/_{55}$
1	2	1.5	$1.\dot{6}$	1.6	1.625	1.615384625	1.619047619	1.617647059	1.6181818

The Golden angle

Do petals in a flower grow in the flower at random positions? Do leaves along the stem grow in any direction haphazardly? No, not really. Nature does not do things at random. The position of some petals and leaves is determined by the Golden angle. Petals which have a gap equal to the golden angle between them capture most light and catch the greatest amount of rain. This arrangement is also used in sunflower seeds and at the centres of other compound flowers. Wow, these plants know their mathematics really well! It would be interesting if you have the opportunity to spot the presence of the golden angle and the spiral directly in nature, if not, do surf the internet and see some pictures.



What is the golden angle? The golden angle , which is given by $\frac{1}{\varphi} \times 360°$, which is approximately $222.5°$ or $137.5°$ in the opposite direction (we usually refer to this obtuse angle rather than the reflex one when using the golden angle).

For an animation on the positioning of such leaves/ petals/ seeds see Animation 'The Golden Angle Animation'.

Activity 5: The Golden Ratio in Flowers (9 years)

In this activity you are going to build your own flower. Make sure that all the petals are exposed to the maximum amount of sunlight.

You need: 3 different cardboards, protractor and a pencil

Some preparation is needed: prepare some cardboard petals of the same shape and colour, prepare 2 circles of the same size to represent the centre of the flower.

Start by positioning the first petal. Here we chose the vertical position but any position will do. Using a protractor, place the 0° line with the position of the first petal and mark 137.5° as the position of the new petal. Continue this procedure depending on how many petals you want your flower to have. If you are not familiar with using the protractor, copy the diagram of the golden angle shown above on a tracing paper and use it so that from one petal you obtain the position of the next petal. Some pictures of this process are found in 'Activity 5' sheet.

Answers to the Activities presented here can be verified on:

<https://www.facebook.com/um.jcmaths>

Some references and links:

Martin Gardner, 1992, *Mathematical Circus*, edition 7, American Mathematical Society, United States of America

It's ok to be Smart, 10 March 2021, *The Golden Ratio: Is It Myth or Math?* [Video], PBS (The Public Broadcasting Service of America) < https://youtu.be/1Jj-sJ78O6M >

Akash Peshin, 2021, *What Is The Fibonacci Sequence? Why Is It So Special?*, Science ABC, accessed on 2nd August 2021, <https://www.scienceabc.com/eyeopeners/why-are-fibonacci-numbers-so-important.html >

Activity 1: Sequences

Start of sequence	Continuation of sequence
Example 800, 400, 200….	….100, 50, 25 …….

Activity 2: **Fibonacci Sequence**

$$\boxed{0} + \boxed{1} = \boxed{1}$$

$$\boxed{1} + \boxed{1} = \boxed{2}$$

$$\boxed{1} + \boxed{2} = \boxed{3}$$

$$\boxed{2} + \boxed{3} = \boxed{5}$$

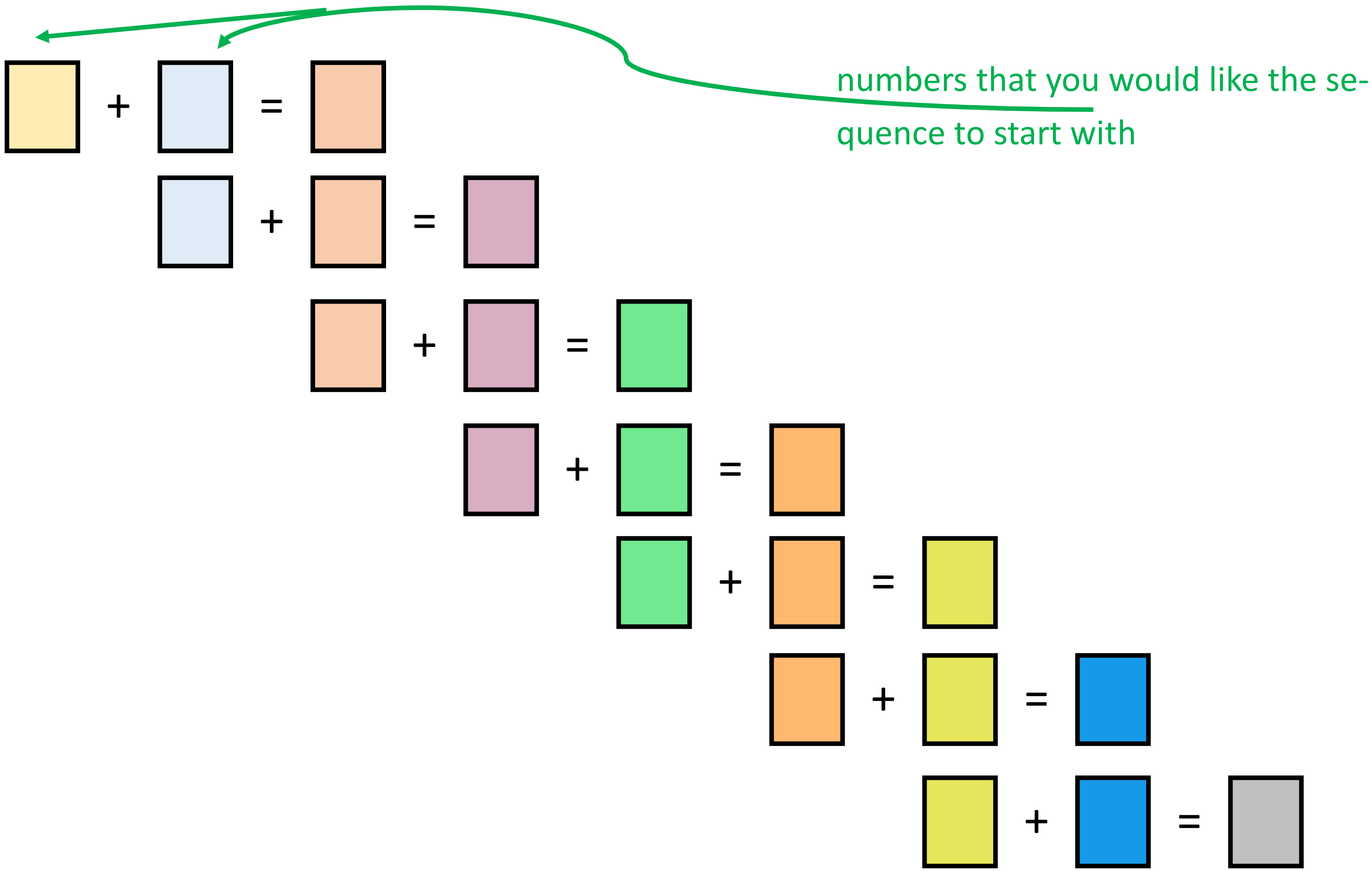
$$\boxed{} + \boxed{} = \boxed{}$$

$$\boxed{} + \boxed{} = \boxed{}$$

$$\boxed{} + \boxed{} = \boxed{}$$

The Fibonacci Sequence: 1,2, 3, 5, 8, ………….

Activity 3: **Generalised Fibonacci Sequence Game**

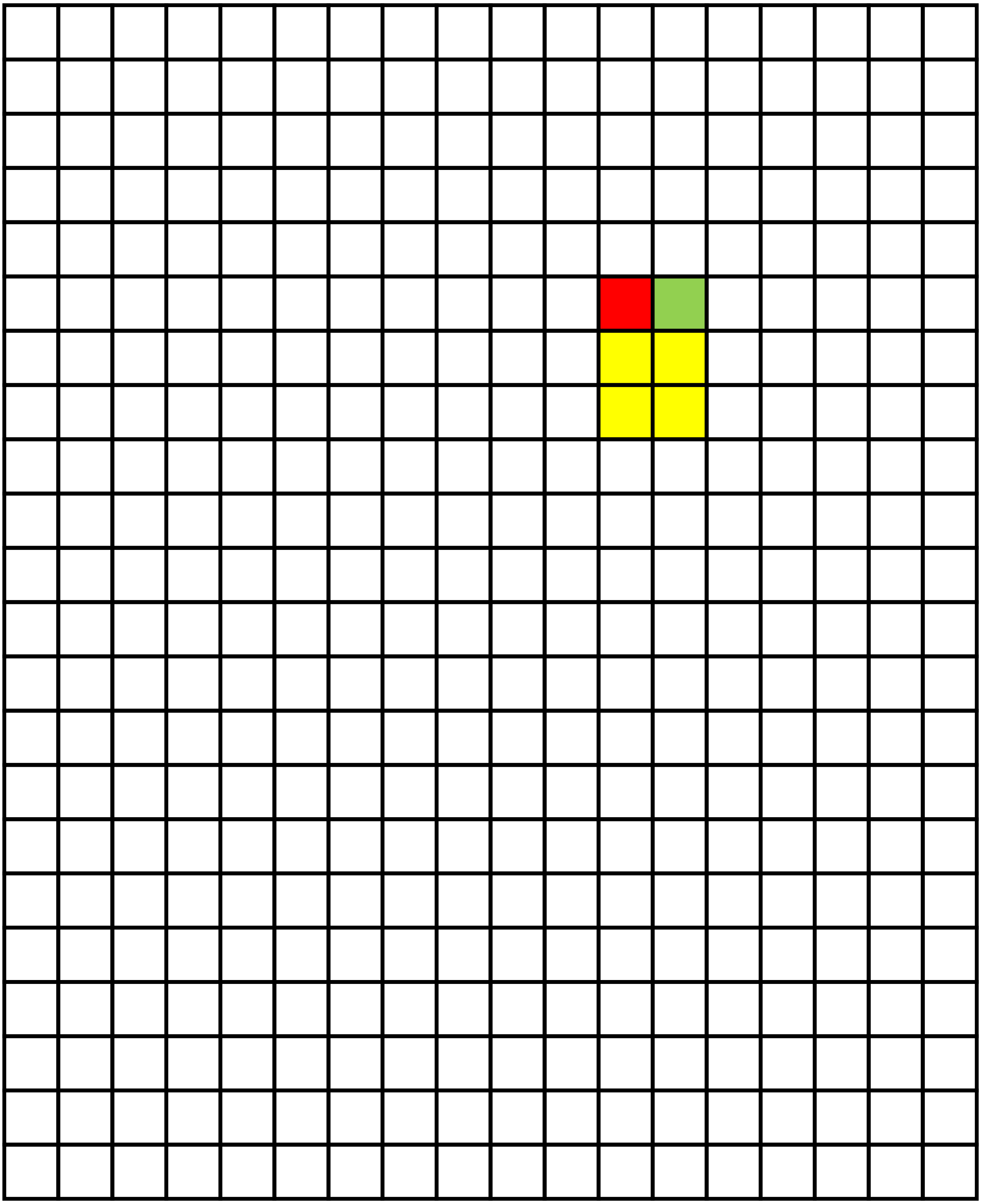


My first 10 terms : ____

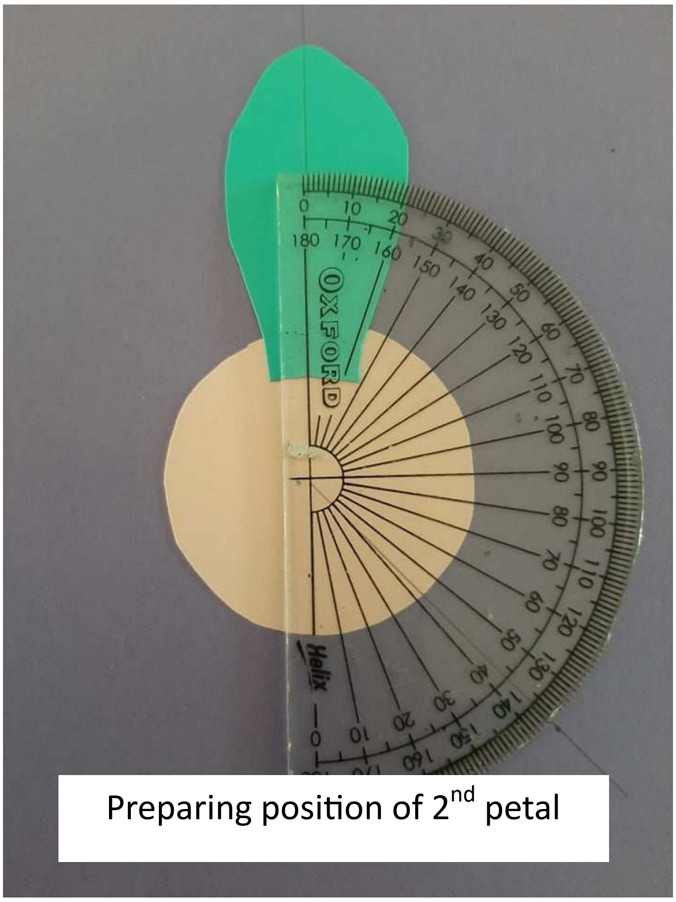
The sum of the first 10 terms: ______

Activity 4:

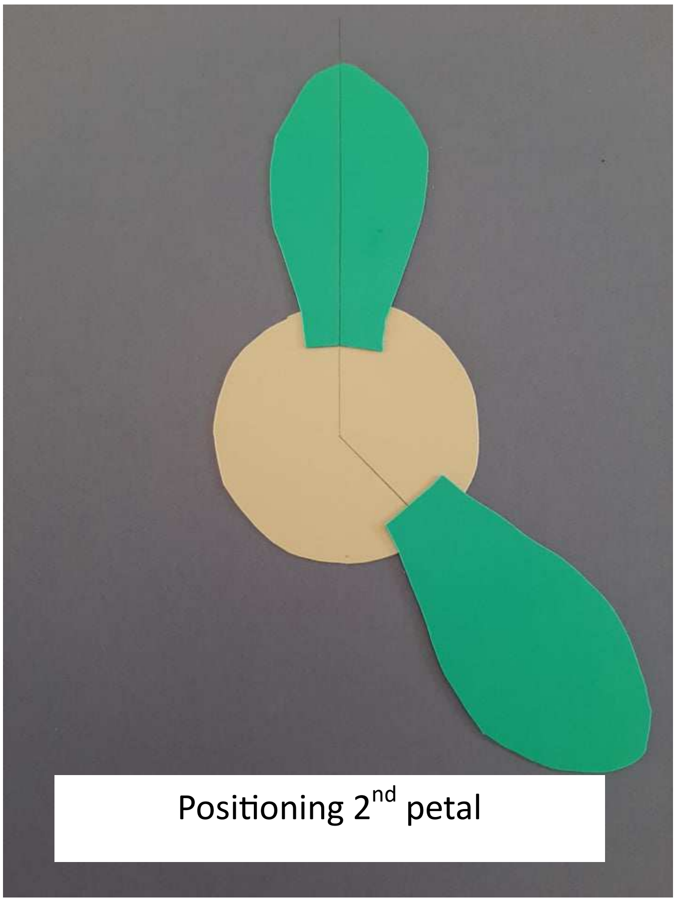
Fibonacci Sequence, the Golden ratio and spirals around us



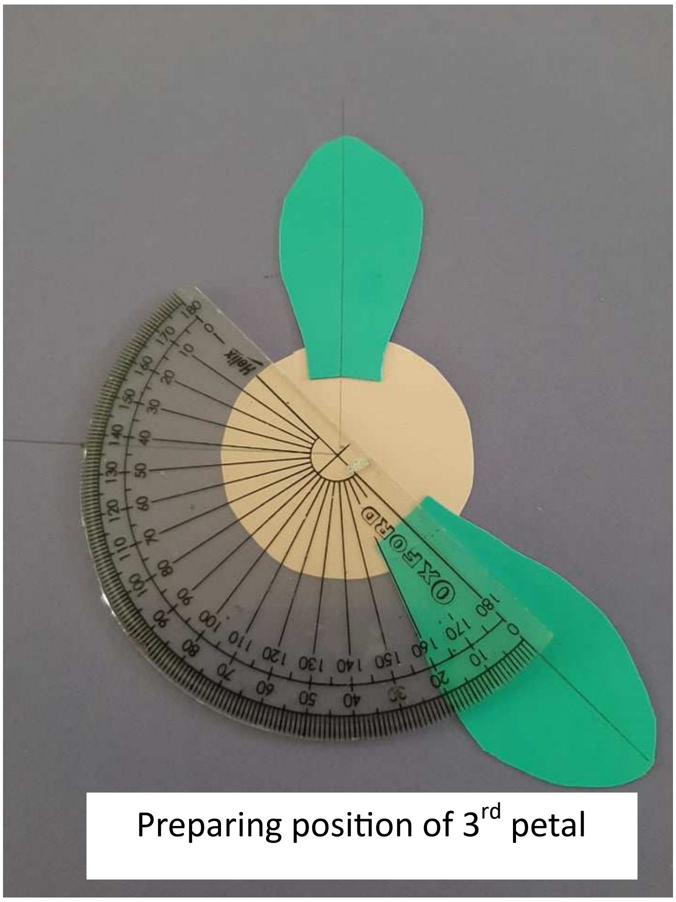
Activity 5: Building a flower model



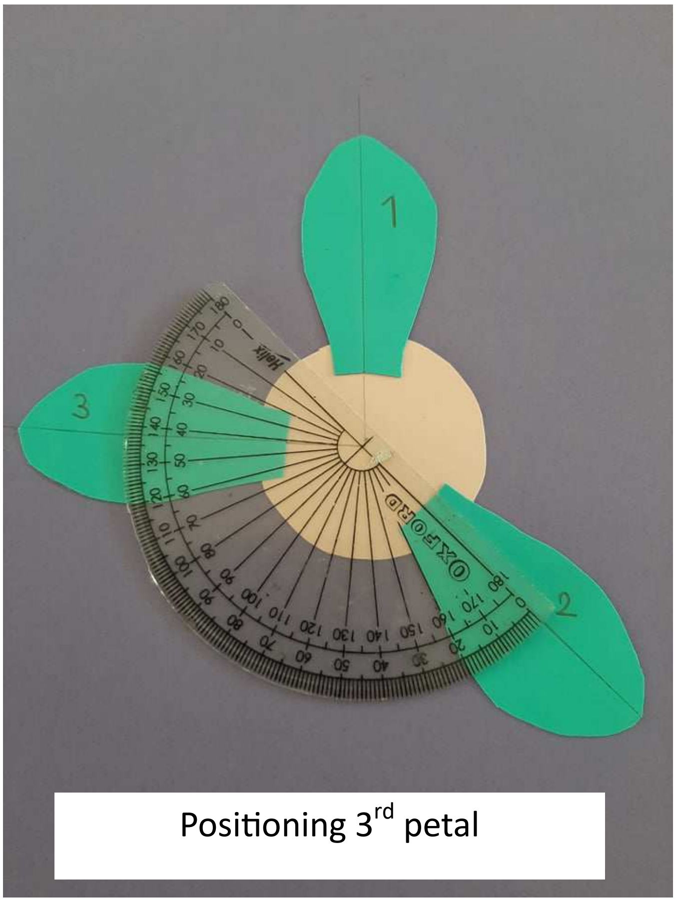
Preparing position of 2nd petal



Positioning 2nd petal

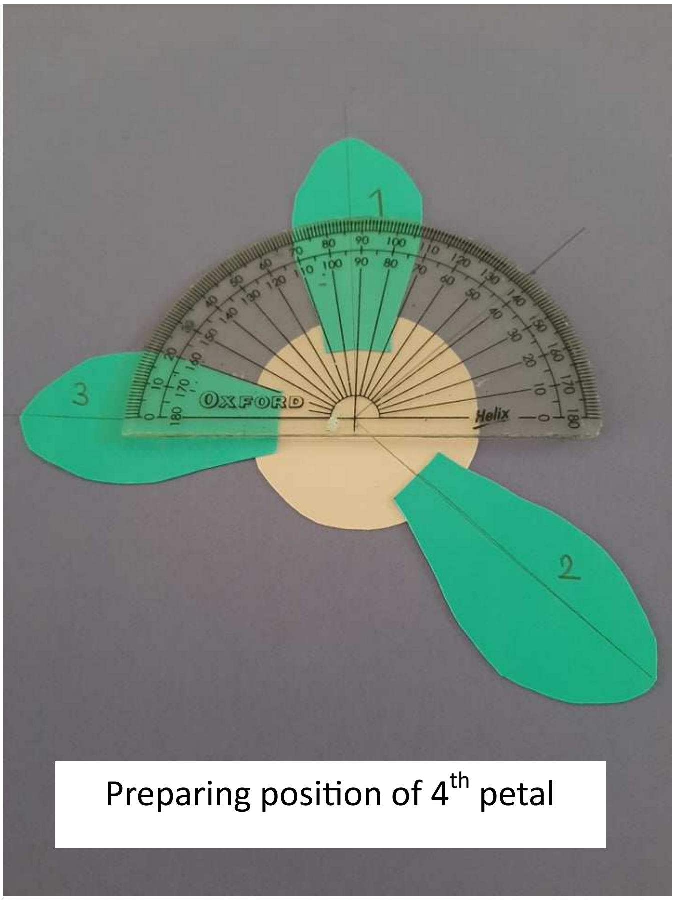


Preparing position of 3rd petal

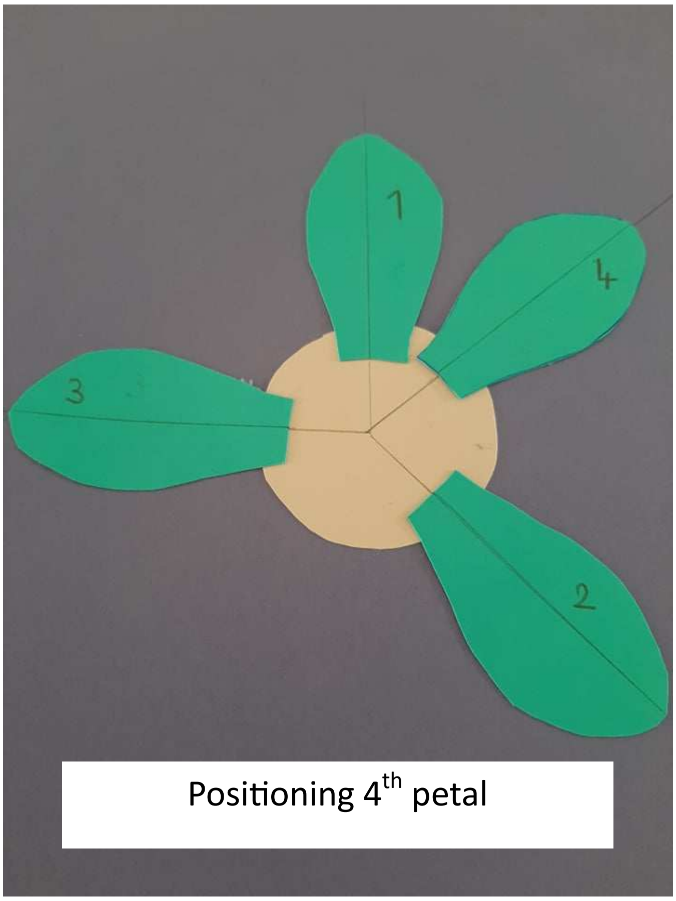


Positioning 3rd petal

Activity 5: Building a flower model

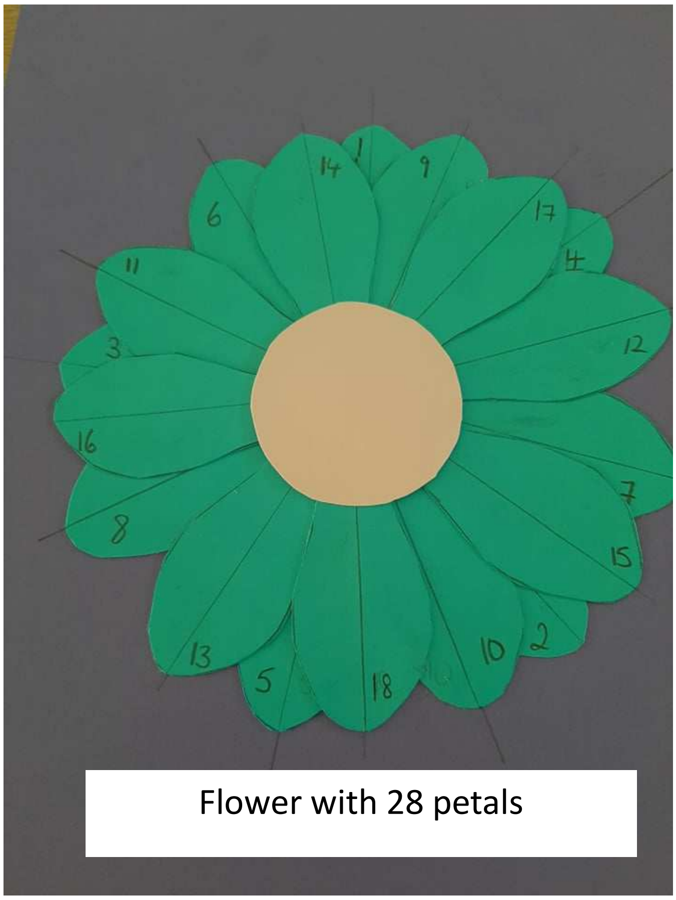


Preparing position of 4th petal



Positioning 4th petal

Repeating the process over and over again and placing the other circular piece of cardboard (to hide the ends on the petals) we obtain this lovely flower where irrespective of having a lot of petals each petal has a visible part, hence a part which is exposed to the rain and sun.



Flower with 28 petals