Fourier domain gravitational waveforms for precessing eccentric binaries

Yannick Boetzel
Physik-Institut der Universität Zürich

In collaboration with: A. Klein, A. Gopakumar, P. Jetzer, L. d. Vittori

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Introduction

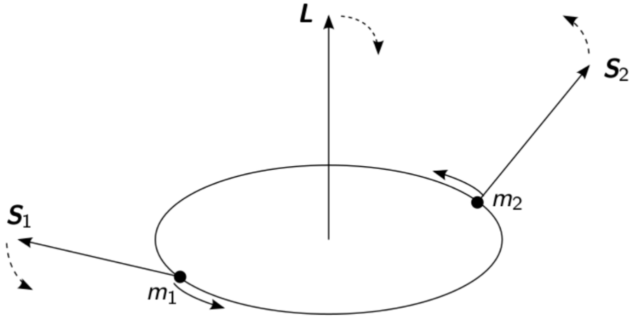
It is important to be able to simultaneously measure the eccentricity and spin distributions of binaries.

Moderate eccentricities, if present, might lead to a significant bias in parameter recovery

Current eccentric waveform models do not incorporate spin

Precession

In the construction of precessing waveforms there are four different timescales: orbital, spin precessional, periastron precessional and radiation reaction



Basic ingredients

We choose to express quantities as a function of the PN parameter $y = \omega^{1/3}/\sqrt{1-e^2}$ and the eccentricity parameter $e = e_t$.

We use

- A 2PN accurate quasi-Keplerian parametrization, including spin terms
- The 3PN accurate evolution equations for y & e, including spin terms at 2PN
- Orbital averaged spin evolution equations at 2PN order

We then

- Expand the amplitudes in small eccentricity to get 2PN accurate instantaneous amplitudes
- Solve the evolution equations numerically to get the phasing with all eccentricity effects

Eccentric waveform

The waveform emitted by an eccentric system has the following structure

$$h_{+,\times}(t) = \sum_{n\in\mathbb{Z}} H^{(n)}_{+,\times}(y, e, u) e^{in(\phi+\phi_T)} \,.$$

In order to use the SPA or SUA* we need to express this with a linearly growing phase with some corrections on the precession timescale.

*Shifted uniform asymptotics (https://arxiv.org/abs/1408.5158)

Eccentric waveform

Splitting $\phi = \lambda + W$ and defining $\delta\lambda = \lambda - l$ s.t. $\dot{\delta\lambda}/\dot{\lambda} = \mathcal{O}(y^2)$ allows us to write the waveform as

$$h_{+,\times}(t) = \sum_{m,n} H^{(m,n)}_{+,\times}(t) e^{-i(n\lambda + m\delta\lambda)}\,,$$

where all precession timescale effects are included in the amplitudes. We expanded the amplitudes in y and e, but the phasing is only PN expanded and includes all eccentricity effects

Fourier transform

This allows us to use a SUA transform to calculate the Fourier transform

$$\tilde{h}(f) = \sum_{n=\max(1,2-N)}^{2+N} \tilde{h}_n^{(0)}(f)\tilde{h}_n^{\text{PP}}(f)\,,$$

$$\tilde{h}_n^{(0)}(f) = \sqrt{2\pi}\; T_n \exp\left[i\left(2\pi f t_n - n\lambda(t_n) - \frac{\pi}{4}\right)\right],$$

$$2\pi f = n\dot{\lambda}(t_n)\,,$$

$$T_n = \left[n\ddot{\lambda}(t_n)\right]^{-1/2}\,,$$

$$\tilde{h}_n^{\text{PP}}(f) = \sum_{m=-M}^{M} e^{i\Delta\Psi_{n,m}} \sum_{k=-k_{\max}}^{k_{\max}} a_{k,k_{\max}} \mathcal{A}_{n,m}(t_n + \Delta t_{n,m} + kT_n)\,.$$

We get a family of eccentric waveforms characterized by the order M of the m expansion (accounting for periastron precession) and the order P of the eccentricity expansion of the amplitudes.

Simulations

We did two different sets of simulations:

- The late inspiral runs were made looking at systems in the last decade in frequency leading to ISCO, and therefore they are representative of what we might expect for BHBs in LIGO/Virgo
- The early inspiral runs were made for stellar origin BHs with LISA, and thus they span a greater frequency range

For each simulation we:

- Randomly sample eccentricity and spin (magnitude & direction) for a few thousand different systems
- Use a reference time domain waveform and compare with
  - our eccentric, precessing waveform
  - a quasicircular, precessing waveform[†]
- Average over all spin combinations

[†] https://arxiv.org/pdf/1408.5158.pdf

Late inspiral, highly spinning

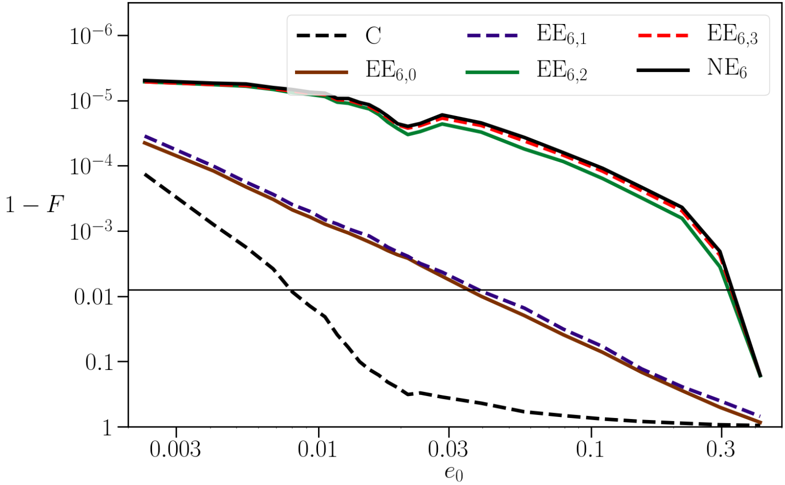


Figure: Late inspiral run, with starting eccentricity $10^{-5} < e_0 < 0.5$ and spin magnitudes $0 < \chi_i < 1$. F is median faithfulness.

Late inspiral, highly spinning, fully circularized

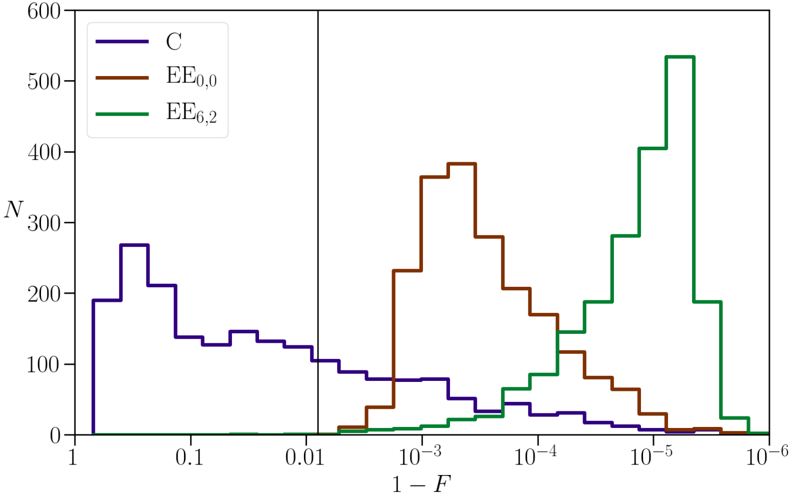


Figure: Late inspiral run, starting eccentricity $e_0 = e_{\min}$ and spin magnitudes $0 < \chi_i < 1$. N is the number of simulation runs in this bin.

Early inspiral, slowly spinning

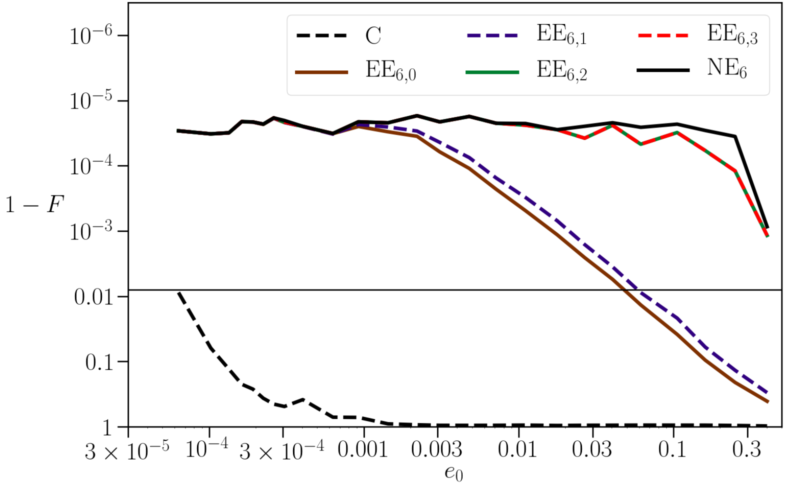


Figure: Early inspiral run, with starting eccentricity $10^{-5} < e_0 < 0.5$ and spin magnitudes $0 < \chi_i < 0.1$.

Early inspiral, slowly spinning, fully circularized

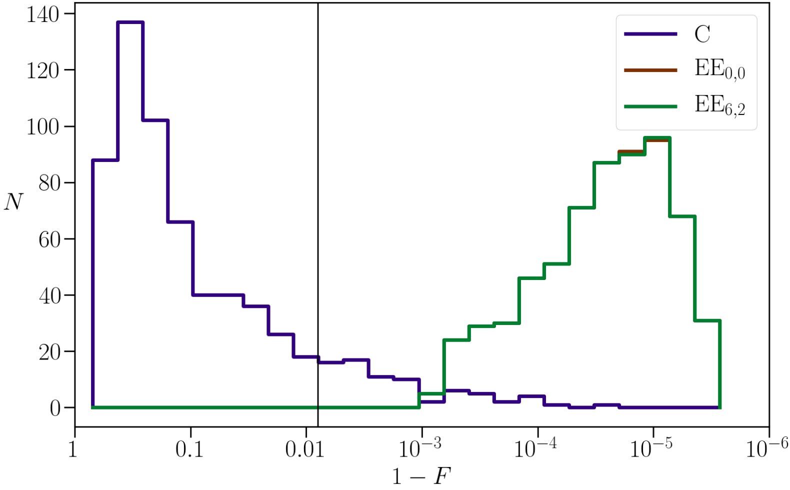


Figure: Early inspiral run, with starting eccentricity $e_0 = e_{\text{min}}$ and spin magnitudes $0 < \chi_i < 0.1$.

Conclusion

The spin effects in the evolution equations imply a minimum eccentricity which leads to important effects even for fully circularized binaries.

This is all the more important when more cycles are visible.