Bayesian hierarchical modelling of traffic flow — with application to Malta’s road network

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October 2013
Analysis of Traffic

- With 229,016 private vehicles in 2010 Malta is one of countries with the highest car ownership rates in the world.

![Figure: The number of cars/1000 persons in 2010](image-url)
Analysis of Traffic

- USA = 24 motor vehicles/km$^2$
- Malta = 804 motor vehicles/km$^2$
Traditionally, traffic assignment solutions have been derived from OD flows using two different methods: optimization based methods and statistical based methods.

Although widely used, the optimization based methods, ignore the uncertainty on the measured data.

Classical traffic assignment models make use of only the OD data to deliver the probabilities of the different route choices available to a traveller.

In many cases, traffic assignment models have been applied to small networks where the computational demand of the model is not a burden.
Traffic Assignment Modelling

Figure: Data Driven Traffic Assignment Model

- DAILY OD DATA
- ANNUAL AVERAGE DAILY TRAFFIC COUNTS ON LINKS
- TRAFFIC ASSIGNMENT MODEL
- TRAFFIC COUNTS ON LINKS
- TRAFFIC COUNTS AT EACH JUNCTION
- PROBABILITY OF CHOSEN ROUTE
Traffic Network for Malta

Figure: Traffic Network for Malta representing the location of the links under study.
Methodology

- Graphical Representation of the Traffic Network for Malta
  - Arterial Roads
  - Distributor Roads
  - Rural and Urban Roads with 'linking' functions [1]

- Connections of Origin Destinations - Search Algorithm

- Estimation of Route Choice Proportions - Traffic Assignment Problem

- Estimation of Traffic Counts on all links in the network

Bayesian Hierarchical Model

- **Bayesian Formulation of the hierarchical model**

  \[
  P(\text{process, parameters}|\text{data}) \propto P(\text{data}|\text{process, parameters}) \\
  \quad \cdot P(\text{process}|\text{parameters}) \cdot P(\text{parameters}) \quad (1)
  \]

- **Data Model**: \( P(\text{data}|\text{process, parameters}) = P(X, N_s, \alpha_s) \)
  where \( X \) represents the traffic counts on the links, \( N_s \) represents the OD trips for each OD pair \( s \), \( \theta_s \) represents the route choice preferences given in the process model and \( \alpha_s \) represents the hyperparameter for \( \theta_s \) given in the parameter model.

- **Process Model**: \( P(\text{process}|\text{parameters}) = P(\theta_s|\alpha_s) \)

- **Parameter Model**: \( P(\text{parameters}) = P(\alpha_s) \)
Bayesian Hierarchical Model

- Posterior Distribution:
  \[ P(\theta_s, \alpha_s | X, N_s) \propto P(X | N_s, \theta_s, \alpha_s)P(\theta_s | \alpha_s)P(\alpha_s)P(N_s) \]

**Table:** Hierarchical Bayesian Formulation of the Posterior Distribution

<table>
<thead>
<tr>
<th>Model Distributions</th>
<th>Chosen Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P(X</td>
<td>N_s, \theta_s, \alpha_s) ]</td>
</tr>
<tr>
<td>[ P(\theta_s</td>
<td>\alpha_s) ]</td>
</tr>
<tr>
<td>[ P(\alpha_s) ]</td>
<td>[ Multinomial(N_s, \theta_s) ]</td>
</tr>
<tr>
<td>[ P(N_s) ]</td>
<td>[ Poisson(N_{s_{data}}) ]</td>
</tr>
</tbody>
</table>

where \( \lambda \) represented the mean of the normal distribution and \( \psi \) represent the variance of the normal distribution.
Sampling from the posterior distribution

- Direct sampling of the model’s posterior distribution is not possible because the model’s posterior distribution does not represent any standard distribution.

- A Metropolis Hasting’s Algorithm (MH) is applied where new set of values for $\theta_s$ and $\alpha_s$ are proposed according to a Dirichlet distribution centered at current parameter values providing good mixing conditions.

- Metropolis Hasting’s ratio:

$$r = \frac{P(\theta_s^*, \alpha_s^* | X, N_s)}{P(\theta_s^{t-1}, \alpha_s^{t-1} | X, N_s)} \cdot \frac{J(\theta_s^{t-1}, \alpha_s^{t-1} | \theta_s^*, \alpha_s^*)}{J(\theta_s^*, \alpha_s^* | \theta_s^{t-1}, \alpha_s^{t-1})}$$
Estimated Traffic Flow in Malta

**Figure:** Boxplot representation of traffic counts on some of the roads in Malta with link 14 representing Triq Valletta (Mosta to Lija), link 11 representing Triq Dawret il-Gudja (Gudja to Luqa), and link 102 representing Kirkop Tunnels going into Luqa.
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Traffic Assignment Modelling

Percentage Error of Traffic Counts

Figure: Percentage Error of Traffic Counts on links
Estimated Route Choice Preference

**Figure:** Chosen route paths connecting Paola to Fgura using the BFS algorithm
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Traffic Assignment Modelling

Estimated Route Choice Preference

Traffic originating at Paola and ending at Fgura

Route Options

Traffic Counts

Legend
- Traffic Points
- Arterial, Distributor and Roads with Linking Function
- Local Councils
- Start Node
- End Node
Estimated Route Choice Preference

Traffic originating at Paola and ending at Fgura

Legend
- Traffic Points
- Arterial, Distributor and Roads with Linking Function
- Local Councils
- Start Node
- End Node

Traffic Counts

Route Options

0 100 200 300
0 50 100 150 200

Route 1
Route 2
Route 3
Route 4
Conclusions

Traffic Assignment Model

- Utilizing both OD information and traffic counts;
- Estimating route choice preference and traffic counts on the links;
- Uncertainties in traffic data;
- Dimensionality of the network size;
- Application - Traffic Network in Malta - resulting in a mean of 4% estimation traffic count error with a std dev of 10%.

Limitations and Future Work

- Improvement in computational efficiency - Variational Bayesian Estimation;
- Time variations in traffic flow - Dynamic Traffic Assignment (DTA) - Sequential Monte Carlo (SMC) estimation techniques.
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Conclusions

THANK YOU

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