Static Orbits in Rotating Spacetimes

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Introduction

Our understanding of the properties of spacetimes largely rests on the analysis of their geodesics. Therefore the study of geodesics has been in the focus of gravity research from its early beginnings. Indeed, describing the motion of particles and light, geodesics rapidly established General Relativity, predicting the observed orbital precession of Mercury and the deflection of light by the sun.

The orbits around Kerr black holes are well-known. In contrast, much less is known about the orbits in spacetimes of other compact rotating objects, such as boson stars, wormholes or hairy black holes. If such objects were to represent serious contenders for compact astrophysical objects, the presence of unique signatures - not present for Kerr black holes - would be highly valuable for their possible identification.

Here we would like to point out such a feature, which is found in a number of rotating non-Kerr spacetimes. We show that under certain conditions an axisymmetric rotating spacetime contains a ring of points in the equatorial plane, where a particle at rest with respect to an asymptotic static observer remains at rest in a static orbit. We illustrate the emergence of such orbits for boson stars. Further examples are wormholes, hairy black holes and Kerr-Newman solutions.

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Equatorial Geodesics

Line Element

$$ds^2 = -Adt^2 - 2Bdtd\varphi + Cd\varphi^2 + Ddr^2, \tag{1}$$

where A, B, C and D are evaluated at $\theta = pi/2$ and are then only functions of the radial coordinate r.

Equations of Motion

$$\dot r^2 = \frac{C}{\Delta D}(E - V_+)(E - V_-), \qquad \dot\varphi = \frac{B}{\Delta}\left(E - V_\varphi\right), \tag{2}$$

with the defined effective potentials

$$V_\pm = \frac{BL \pm \sqrt{\Delta(C + L^2)}}{C}, \qquad V_\varphi = -AL/B, \tag{3}$$

and $\Delta = AC + B^2$. E and L are the energy and angular momentum per unit mass of the test particle, respectively.

Orbits Starting from Rest

At $\tau = 0$, $r = r_0$, $\dot r = \dot\varphi = 0$, then

$$\sqrt{A}|_{r_0} = E, \quad B|_{r_0} = -EL. \tag{4}$$

Orbits Remaining at Rest

When the condition (4) is met together with the circular orbit condition, namely $\partial_r V_+ = 0$, the particle remains at rest at a distance r_{st} from the origin, as seen by an observer at infinity, with energy and angular momentum constrained by the above equations.

These conditions are combined simply by

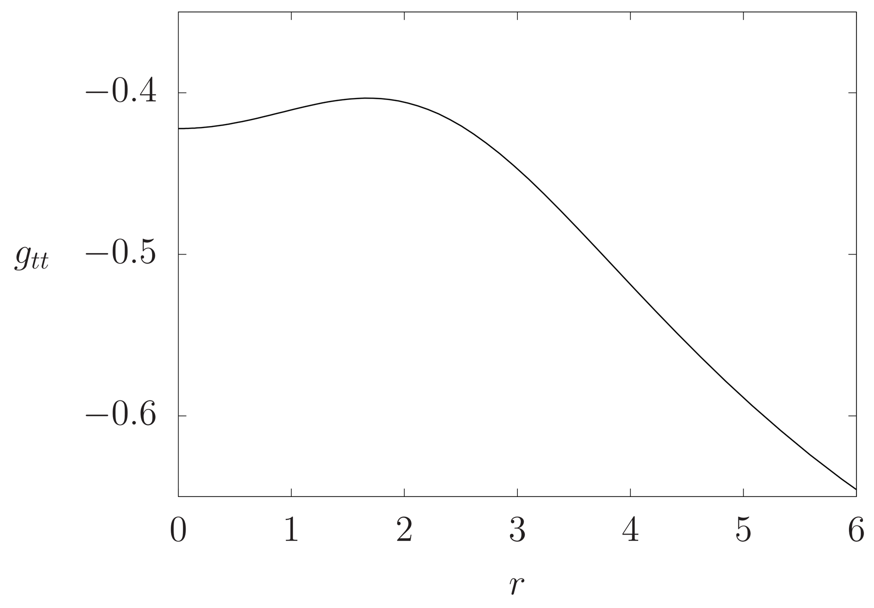
$$\boxed{\partial_\mathbf{r}\mathbf{A} = \mathbf{0}} \tag{5}$$

Therefore, a local maximum in g_{tt} where its value is negative (on the equatorial plane) is a necessary and sufficient condition for a spacetime to contain a ring of points where massive particles at rest remain at rest.

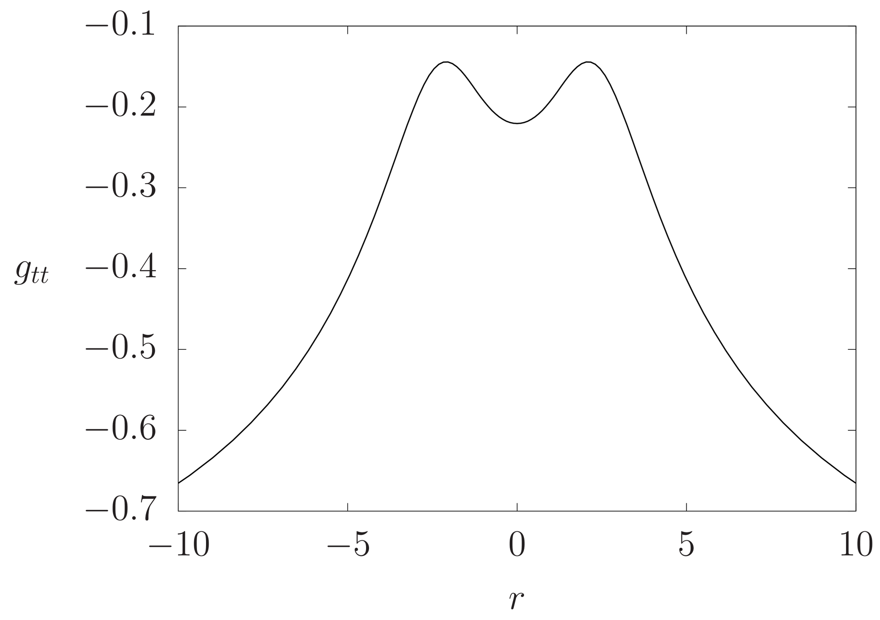
This feature is independent of the chosen parametrization, for $\delta\dot\varphi/\delta\dot r \propto \sqrt{r - r_{st}}$.

Spacetime Examples

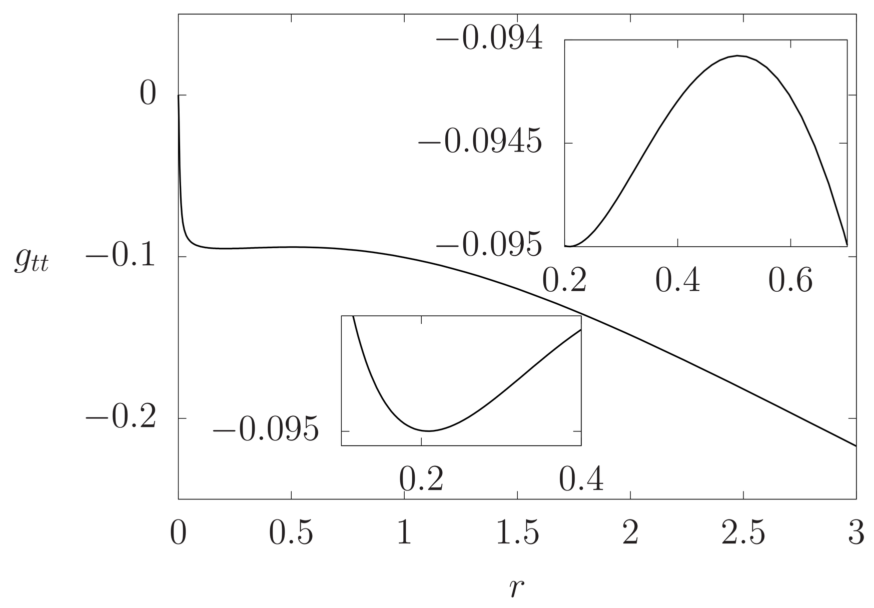
Rotating Boson Star [1]



Wormhole Inside Rotating Matter [2]



Rotating Hairy Black Hole [3]

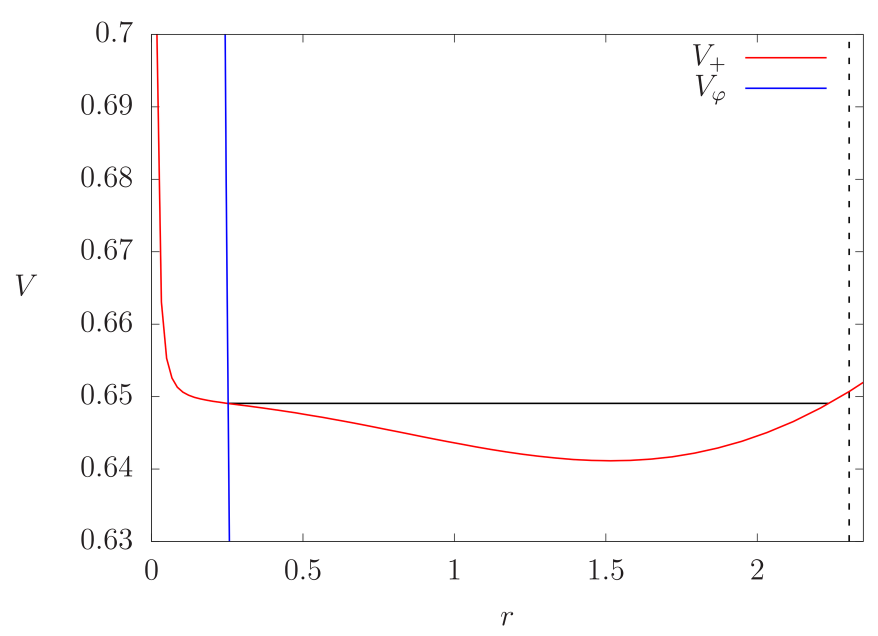


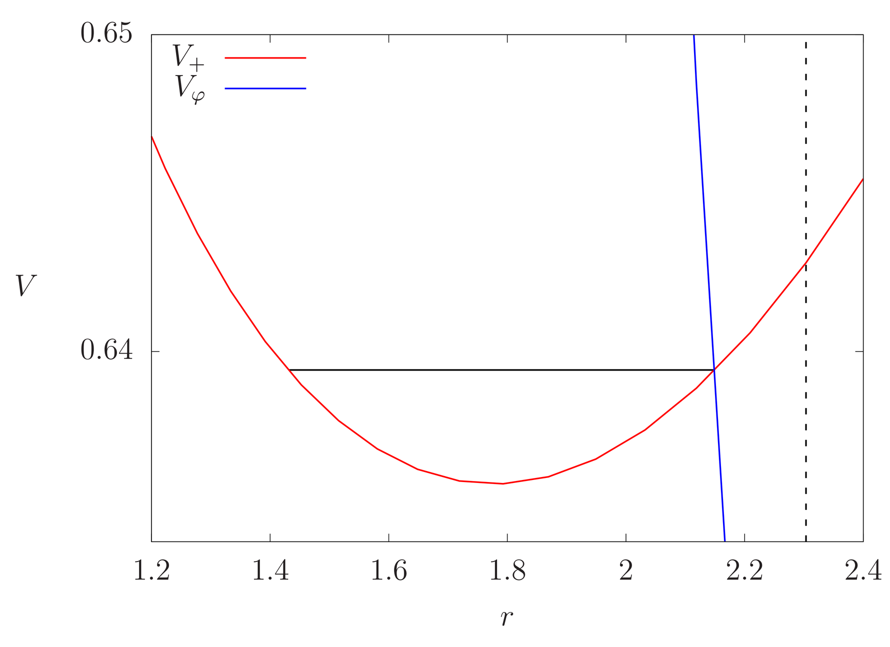
Kerr-Newman Solution

$$r_{st} = Q^2/M, \tag{6}$$

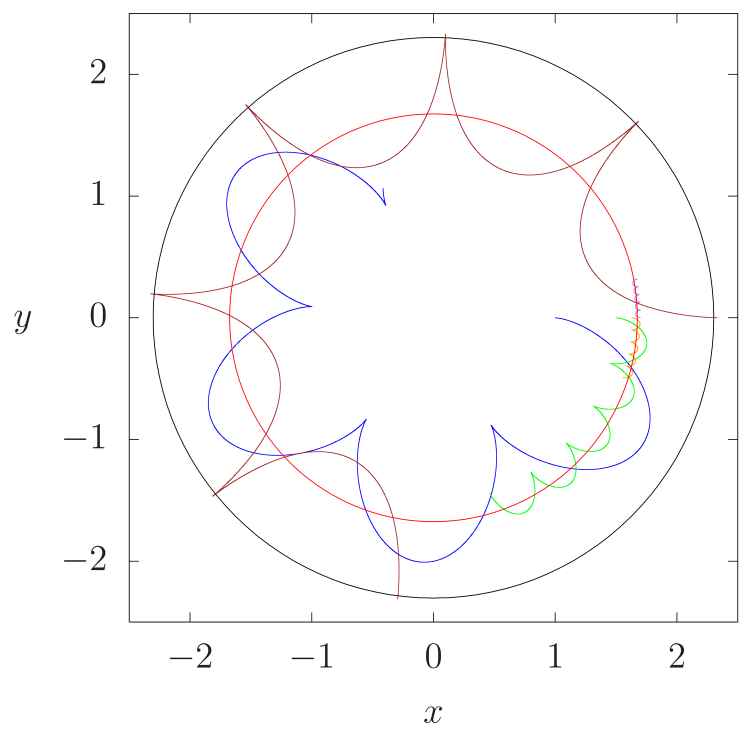
which is only observable for naked singularities. For black holes, the conditions are fulfiled inside both horizons iff $J^2 \geq \alpha^2 - \alpha^4$, with $\alpha = Q/M$.

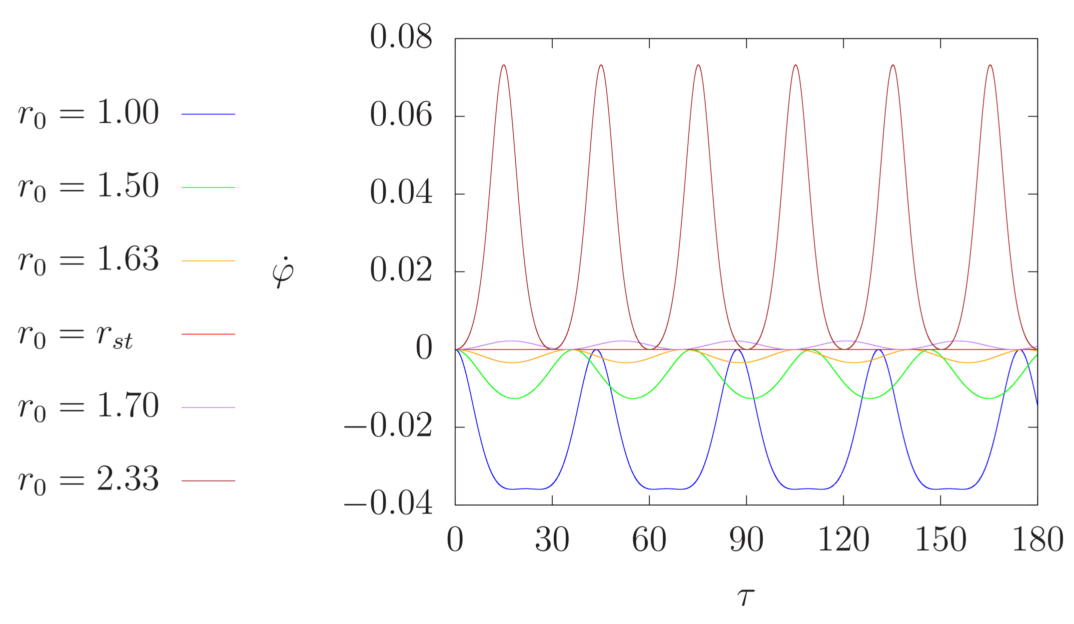
Boson Star Case and Discussion





Particles placed at rest at $r_0 \leq r_{st}$ get initially pushed away and engage in a *semi orbit* co-rotating with the star, while those for which the motion starts at $r_0 \geq r_{st}$ are pulled towards the center and retain a *pointy petal orbit* counter-rotating with respect to the star [4, 5]. The solid horizontal black line is the particle's energy per unit mass, while the vertical dashed line is the location of the maximum of the scalar field. The static radius is found well inside the star's interior, but since the scalar field interacts only gravitationally with ordinary matter, the boson star is the perfect playground to observe this feature. As r_0 approaches r_{st}, the amplitude in the radial motion and angular velocity decreases monotonically. All the trajectories exhibited below are performed during the same time span. Contrary to charged particles in static orbits around charged black holes, the static ring presented here arises from a *purely inertial phenomenom*.





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