Linear potentials in galaxy halos by Asymmetric Wormholes

Sebastian Bahamonde*
Department of Mathematics, University College London, Gower Street, London, WC1E 6BT, United Kingdom

David Benisty†
Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel Fachbereich Physik der Johann Wolfgang Goethe Universität, Campus Riedberg, Frankfurt am Main, Germany and Frankfurt Institute for Advanced Studies, Giersch Science Center, Campus Riedberg, Frankfurt am Main, Germany

Eduardo I. Guendelman‡
Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel Frankfurt Institute for Advanced Studies, Giersch Science Center, Campus Riedberg, Frankfurt am Main, Germany and Bahamas Advanced Study Institute and Conferences, 4A Ocean Heights, Hill View Circle, Stella Maris, Long Island, The Bahamas

(Dated: January 26, 2018)

In this letter we study the spherically symmetric space-time for a diffusive two measures theory. It is demonstrated that by imposing an asymmetric wormhole geometry, one gets a metric coefficient which for galactic distances has a linear term and from the analysis of Mannheim and collaborators, can be used to describe the galactic rotation curves and for cosmological distances one can obtain a de-Sitter space-time. Center of gravity coordinates for the wormhole are introduced and it is argued that these coordinates are the most suitable for the collective motion of a wormhole. The metric coefficients depend on the asymmetric wormhole parameters, The coefficient of the linear potential is proportional to both the mass of the wormhole and the cosmological constant. Similar results are also expected in other theories like k-essence theories, etc. that may support wormholes.

I. INTRODUCTION

One of the most challenging questions in astrophysics is the mismatch between the measuring velocity of stars in galaxies, and the predictions for galaxy rotation curves from the standard General theory of Relativity. This question led the astrophysicists arguing about the existence of dark matter. Other theorists instead tried to modify GR or Newton laws. The most prevailing belief is that for explaining the galaxy rotation curves, the galaxy has to be soaked in a dark matter halo [1–3]. Regardless of this question, a theoretical spherically symmetric solution for General Relativity (GR), called a wormhole, gives a concept with a non-trivial structure linking separate points in space-time [4–7]. This property of the space-time is different from black holes solutions. In this letter, we propose a modified theory of gravity, which for a spherically symmetric solutions produces asymmetric wormholes, and for a large distances produces gravitational potentials that can be suitable for the explanation of galaxy rotation curves. The parameters that defines the asymmetry of the wormhole hole determine linear gravitational potentials and therefore the rotation curves in galaxy haloes.

A. Two Measures Theory

Many modified theories of gravity have been formulated for explainig phenomena beyond GR. One example is the two measures theory [8]-[16] where in addition to the regular measure of integration in the action $\sqrt{-g}$, includes another measure of interation which is also a density and a total derivative. In this case, one can use for constructing this measure 4 scalar fields φ_a, where $a = 1, 2, 3, 4$. Then, we can define the density $\Phi = \varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{abcd}\partial_\alpha\varphi_a\partial_\beta\varphi_b\partial_\gamma\varphi_c\partial_\delta\varphi_d$,

* sebastian.beltran.14@ucl.ac.uk
† benidav@post.bgu.ac.il
‡ guendel@bgu.ac.il

and then we can write an action that uses both of these densities:

$$S = \int d^4x \Phi \mathcal{L}_1 + \int d^4x \sqrt{-g} \mathcal{L}_2 \,. \tag{1}$$

As a consequence of the variation with respect to the scalar fields φ_a, assuming that $\mathcal{L}_1$ and $\mathcal{L}_2$ are independent of the scalar fields φ_a, we obtain that

$$A^\alpha_a \partial_\alpha \mathcal{L}_1 = 0 \,, \tag{2}$$

where $A^\alpha_a = \varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{abcd}\partial_\beta\varphi_b\partial_\gamma\varphi_c\partial_\delta\varphi_d$. Since $\det[A^\alpha_a] \sim \Phi^3$, then for $\Phi \neq 0$, (2) implies that $\mathcal{L}_1 = M = const$. This result can be expressed as a covariant conservation of a stress energy momentum of the form $T^{\mu\nu}_{(\Phi)} = \mathcal{L}_1 g^{\mu\nu}$, and using the 2nd order formalism where the covariant derivative of $g_{\mu\nu}$ is zero, we obtain that $\nabla_\mu T^{\mu\nu}_{(\Phi)} = 0$ implying $\partial_\alpha \mathcal{L}_1 = 0$. This suggests the idea of generalizing the two measures theory by imposing the covariant conservation of a non-trivial kind of energy momentum tensor, which we denote as $T^{\mu\nu}_{(\chi)}$ [22]. Therefore, we consider an action of the form

$$S = S_{(\chi)} + S_{(R)} = \int d^4x \sqrt{-g}\chi_{\mu;\nu} T^{\mu\nu}_{(\chi)} + \frac{1}{2\kappa^2}\int d^4x \sqrt{-g} R \,, \tag{3}$$

where ; denotes covariant derivative, $\kappa^2 = 8\pi G$ and $\chi_{\mu;\nu} = \partial_\nu\chi_\mu - \Gamma^\lambda_{\mu\nu}\chi_\lambda$. If we assume $T^{\mu\nu}_{(\chi)}$ to be independent of χ_μ and having $\Gamma^\lambda_{\mu\nu}$ being defined as the Christoffel connection coefficients, then the variation with respect to χ_μ gives a covariant conservation: $\nabla_\mu T^{\mu\nu}_{(\chi)} = 0$. A full phenomenology for using these theories is described in [17, 18].

B. Diffusive Energy theory from Action principle

Calogero [21] proved that the diffusion equation in a curved space-time implies a non-conserved stress energy tensor $T^{\mu\nu}$, which has some current source f^μ:

$$\nabla_\nu T^{\mu\nu} = 3\sigma f^\mu \,, \tag{4}$$

where σ is the diffusion coefficient of the fluid. This generalization is Lorentz invariant and the current f^μ is a time-like covariantly conserved vector field and its conservation tells us that the number of particles in this fluid is constant. This non-conservative stress energy tensor can emerge from variations in the action (3), by replacing the dynamical time vector field to a gradient of a scalar field $\partial_\mu\chi$:

$$S_{(\chi)} = \int d^4x \sqrt{-g}\, (\partial_\mu\chi)_{;\nu} T^{\mu\nu}_{(\chi)} \,. \tag{5}$$

The variation with respect to χ gives a covariant conservation of a current f^μ

$$\nabla_\mu T^{\mu\nu}_{(\chi)} = f^\nu \,, \quad \nabla_\nu f^\nu = 0 \,, \tag{6}$$

which it is the source of the stress energy momentum tensor. Equation (6) has a close correspondence to (4). By taking variations with respect to χ_μ, we obtain 4 equations of motion which correspond to a covariant conservation of the energy momentum tensor $\nabla_\mu T^{\mu\nu}_{(\chi)} = 0$. By changing the 4 vector to a gradient of a scalar $\partial_\mu\chi$, we change the conservation of energy momentum tensor to an asymptotic conservation of energy momentum tensor (6) which corresponds to a conservation of a current $\nabla_\nu f^\nu = 0$. From a variation of the action with respect to the metric, we get a conserved stress energy tensor $T^{\mu\nu}_{(G)}$:

$$T^{\mu\nu}_{(G)} = \frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g^{\mu\nu}} \,, \quad \nabla_\mu T^{\mu\nu}_{(G)} = 0 \,. \tag{7}$$

By considering $T^{\mu\nu}_{(\chi)}$ being equal to $\mathcal{L}_1 g^{\mu\nu}$, the original measure Φ is modified to a Galileon measure $\Phi(\chi) = \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\chi)$, and the action (5) gets the following form

$$S_{(\chi)} = \int d^4x \Phi(\chi)\mathcal{L}_1 \tag{8}$$

Here if we take variation with respect to the scalar χ, the equation of motion gives $\Box\mathcal{L}_1 = 0$. This idea was also used in the context of String theory in [19, 20].

II. THE ACTION

Let us start with the following two-measure action

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left(\Lambda(\phi,X) + V_1(\phi)\right) + \int d^4x\Phi(\chi)\,\Lambda(\phi,X)\,, \tag{9}$$

where ; represents covariant derivative with respect to the Levi-Civita connection and ϕ is a scalar field. The first two terms in the above action represents standard k-essence theories whereas the last term has another contribution with a different Galilean measure.

Then the variation with respect to the scalar χ gives $\Box(\mathcal{L}_1 + V_1(\phi)) = 0$, which for a cosmological solution leads to an interactive unified DE/DM scenario [22–24]. The second term on this action depends on $\Lambda(\phi, X)$ which is a function of a scalar field ϕ and a kinetic term

$$X = -\frac{1}{2}\epsilon\,\partial_\mu\phi\partial^\mu\phi \tag{10}$$

that contains any k-essence theory. If $\epsilon = +1$, the scalar field ϕ represents a canonical scalar field, whereas when $\epsilon = -1$ represents a phantom scalar field. The third term in the action (9) also depends on an energy-momentum tensor $T^{\mu\nu}_{(\chi)}$ that couples to the vector field and it is assumed to be independent of it. In [22], the authors studied the specific case where the function $\Lambda(\phi, X)$ is defined as follows

$$\Lambda(\phi, X) = K - V_2(\phi) = -\frac{1}{2}\epsilon\,\partial_\mu\phi\partial^\mu\phi - V_2(\phi)\,, \tag{11}$$

where $V_2(\phi)$ is an energy potential, which in general is different than the potential $V_1(\phi)$. Note that the potentials are coupled with different measures. In [22], the special case where $V_1(\phi) = V_2(\phi) = 0$ was studied.

Variations of the action (9) with respect to the metric gives us the following field equations

$$G_{\mu\nu} = g_{\mu\nu}(\Lambda + \chi^\lambda\Lambda_{,\lambda}) - j_\mu\phi_{,\nu} + \chi_\mu\Lambda_{,\nu} + \chi_\nu\Lambda_{,\mu} - g_{\mu\nu}V_1(\phi)\,, \tag{12}$$

where we have assumed that commas denote differentiation, $\kappa^2 = 1$ and the vector field is equal to the gradient of the scalar field χ that appears in the Galileon measure, yielding

$$\chi_\mu = \partial_\mu\chi\,. \tag{13}$$

From a variaiton with respect to the scalar ϕ we obtain a non-conserved current, which is given by

$$j_\alpha = 2(\chi^\lambda_{;\lambda} + 1)\phi_{,\alpha}\,. \tag{14}$$

If we vary the action (9) with respect to the scalar field ϕ and the vector field χ_μ, we respectively get

$$\frac{\epsilon}{2}\nabla_\alpha j^\alpha = \frac{dV_1(\phi)}{d\phi} + \frac{dV_2(\phi)}{d\phi}\left(\chi^\lambda_{;\lambda} + 1\right), \tag{15}$$

$$\Box\Lambda = \Box(K - V_2(\phi)) = 0\,. \tag{16}$$

Here, $\Box = \nabla_\alpha\nabla^\alpha$ is the d'Alambertian. In the next section, a spherically symmetric space-time of this model will be introduced in order to then analyse the special case of asymmetric wormholes.

III. SPHERICALLY SYMMETRIC SPACE-TIME

A. General equations

Let us start with the most general spherically symmetric space-time metric given by

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2\,, \tag{17}$$

where $A(r)$, $B(r)$ and $C(r)$ are the metric coefficients which depends on the radial coordinate and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. In this space-time, the field equations (12) become

$$\frac{4BCC'' - 2CB'C' - BC'^2 - 4B^2C}{4B^2C^2} + \frac{\epsilon\phi'^2}{2B} + V_1(\phi) + V_2(\phi) + \frac{\phi'}{2B^3}\left(\epsilon B'\phi' - 2B^2V_2'(\phi) - 2\epsilon B\phi''\right)\chi_r = 0\,, \tag{18}$$

$$\frac{2CA'C' + AC'^2 - 4ABC}{4ABC^2} - \frac{\epsilon\phi'^2}{2B} + V_1(\phi) + V_2(\phi) - \frac{\epsilon\chi_r'\phi'^2}{B^2} + \frac{\phi'}{2AB^2C}\Big(2A\left(BCV_2'(\phi) - \epsilon(C\phi')'\right) - \epsilon CA'\phi'\Big)\chi_r = 0\,, \tag{19}$$

$$\frac{AC\left(BA'C' + C\left(2BA'' - A'B'\right)\right) - BC^2A'^2 - A^2CB'C' + 2A^2BCC'' - A^2BC'^2}{4A^2B^2C^2} + \frac{\epsilon\phi'^2}{2B} + V_1(\phi) + V_2(\phi)$$
$$+\frac{\phi'}{2B^3}\left(\epsilon B'\phi' - 2B^2V_2'(\phi) - 2\epsilon B\phi''\right)\chi_r = 0\,. \tag{20}$$

Here, prime denotes differentiation with respect to the radial coordinate r and $\chi_\mu = (0, \chi', 0, 0) := (0, \chi_r, 0, 0)$. Clearly, when $\chi_r = 0$ one recovers standard scalar-tensor theory. The modified Klein-Gordon equation (15) becomes

$$-\frac{\epsilon A'\phi'}{2AB} + \frac{\epsilon B'\phi'}{2B^2} + \frac{dV_1(\phi)}{d\phi} + \frac{dV_2(\phi)}{d\phi} - \frac{\epsilon\left(C'\phi' + C\phi''\right)}{BC} - \frac{1}{B^3}\left[\epsilon B\left(\frac{A'\phi'}{A} + \frac{2C'\phi'}{C} + \phi''\right) - 2\epsilon B'\phi' - \frac{dV_2(\phi)}{d\phi}B^2\right]\chi_r'$$
$$-\frac{\epsilon\chi_r''\phi'}{B^2} + \frac{\chi_r}{4A^2B^4C}\Big[\epsilon B^2CA'^2\phi' + 2AB\Big\{2\epsilon CA'B'\phi' + \frac{dV_2(\phi)}{d\phi}B^2CA' - \epsilon B\left(CA''\phi' + A'\left(2C'\phi' + C\phi''\right)\right)\Big\}$$
$$-A^2\Big\{2B^2\left(\frac{dV_2(\phi)}{d\phi}CB' + 2\epsilon\left(C''\phi' + C'\phi''\right)\right) + 5\epsilon CB'^2\phi' - 2\epsilon B\left(CB''\phi' + B'\left(4C'\phi' + C\phi''\right)\right) - 4\frac{dV_2(\phi)}{d\phi}B^3C'\Big\}\Big] = 0\,, \tag{21}$$

and the constraint (16) gives us

$$\frac{d}{dr}\Big[\frac{\sqrt{A/B}C\phi'\left(\epsilon B'\phi' - 2B^2V_2'(\phi) - 2\epsilon B\phi''\right)}{B}\Big] = 0\,, \tag{22}$$

which can be directly integrated yielding

$$\frac{\sqrt{A/B}C\phi'\left(\epsilon B'\phi' - 2B^2V_2'(\phi) - 2\epsilon B\phi''\right)}{B} = C_2\,, \tag{23}$$

where C_2 is an integration constant. There are four independent equations since the modified Klein-Gordon equation (21) can be also obtained by using (18)-(20) and (22). The constraint (22) is an additional equation that does not appear in standard k-essence theory. This equation comes directly by assuming that the vector field is a divergence of a scalar field. Note that if one subtracts (18) with (20), one gets

$$\frac{A''}{AB} - \frac{A'B'}{2AB^2} + \frac{A'C'}{2ABC} - \frac{A'^2}{2A^2B} + \frac{B'C'}{2B^2C} - \frac{C''}{BC} + \frac{2}{C} = 0\,, \tag{24}$$

which is an equation which does not depend on the scalar fields. Moreover, this equation is valid for any k-essence theory as it pointed out in [25]. The latter comes from the fact that in those theories $T_t^t = T_\theta^\theta$ and then all the contribution coming from the scalar field disappears.

B. Asymmetric Wormholes triggering linear potentials describing galaxy halos

In this section, we will assume a canonical scalar field ($\epsilon = +1$) and also we will choose that the metric coefficients are related as follows

$$B(r) = \frac{1}{A(r)}\,. \tag{25}$$

By replacing the above equation into (24), one gets

$$\frac{d}{dr}\Big(AC' - A'C\Big) = -\frac{d}{dr}\Big[C^2\frac{d}{dr}\Big(\frac{A}{C}\Big)\Big] = 2\,, \tag{26}$$

which is the same equation reported in [25]. Then, the global geometric structure would be the same as described in the latter mentioned paper. Then, one can easily integrate once to obtain

$$\frac{d}{dr}\Big(\frac{A}{C}\Big) = \frac{2(r_0 - r)}{C^2}\,, \tag{27}$$

where r_0 is an integration constant. This result is generic for any scalar field ϕ, χ_r and also for any energy potential $V_1(\phi)$ and $V_2(\phi)$.
Let us further assume an asymmetric wormhole geometry with the following metric coefficient function

$$C(r) = r^2 + b^2 + ar\,, \tag{28}$$

where b and a are the wormhole parameters. The parameter a measures the asymmetry of the wormhole throat under $r \to -r$ and therefore the asymmetry between the two sides of the wormhole, although as we will see, there is another radius in the wormhole with respect to which one can look at its asymmetry, which we will argue is more relevant, which is the center of gravity sphere (since it correspond to a certain radius, but the angles are arbitrary) . Then, one can directly solve (27) yielding

$$A(r) = A_0\left(ar + b^2 + r^2\right) + \frac{1}{c^2}\left[\frac{4(a+2r_0)\left(ar+b^2+r^2\right)\arctan\left(\frac{a+2r}{c}\right)}{c} + 2\left(a(r+r_0) + 2b^2 + 2rr_0\right)\right], \tag{29}$$

where A_0 is an integration constant and for simplicity we have defined $c = \sqrt{4b^2 - a^2}$. πc measures the circumferential radius of the wormhole at its neck. The parameters must satisfy $-2b < a < 2b$ which also ensures that the zeros of the equation $C(r) = 0$ will be imaginary making that the wormhole exists. Fig. 1 shows the metric coefficient $A(r)$ for some specific values of the parameters. We can notice that this function is describing an asymptotically de-Sitter space of one side of the wormhole (positive r) and anti de-Sitter space on the other side (negative r).

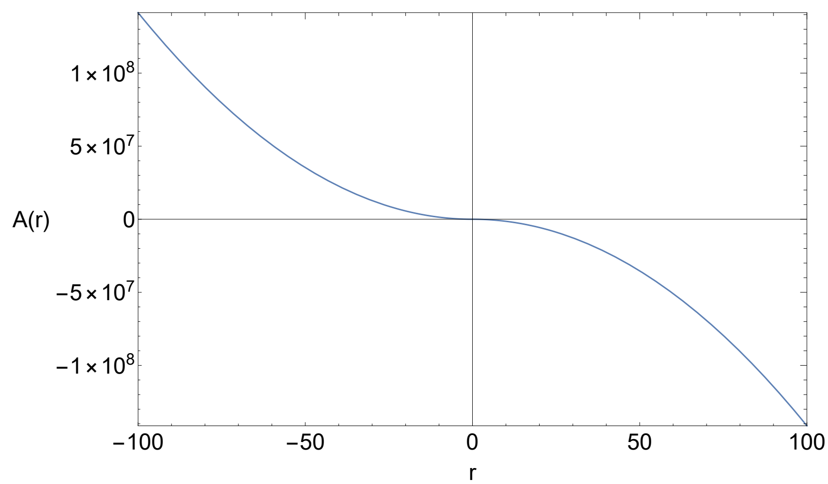

FIG. 1: Plot of the coefficient $A(r)$ versus r for $b = 2.5$, $a = 0.001$, $r_0 = -9.9$ and $A_0 = 1$. Negative values of r are also plotted which represents the other universe.

Let us study some special limit cases for our model. If one assumes that $2r + a \ll c$ and $r \ll a$, we can expand the metric coefficient up to second power-law orders in r, obtaining

$$A(r) \approx 1 + A_0 b^2 + \frac{ac(\lambda - A_0)}{\pi} + \frac{r(\pi a A_0 - 2A_0 c + 2c\lambda)}{\pi} + A_0 r^2 + \mathcal{O}(r^3)\,. \tag{30}$$

Here, we have used the expansion $\arctan(x) \approx x$ for $x \ll 1$ and for simplicity we have introduced the following constant

$$\lambda = A_0 + \frac{2\pi(a + 2r_0)}{c^3}\,. \tag{31}$$

Now, let us explore the limit case at very large scales. In this case, one can assume that $2r + a \gg c$ and $r \gg a$ and then one can expand the metric function (29) as follows

$$A(r) \approx 1 + b^2\lambda + \lambda r^2 + a\lambda r + \frac{c^3(A_0 - \lambda)}{6\pi r} + \mathcal{O}\Big(\frac{1}{r^2}\Big)\,, \tag{32}$$

where we have used the expansion $\arctan(x) \approx \pi/2 - 1/x$ for $x \gg 1$. Then, λ can be interpreted as a cosmological constant and the term λr^2 will be important at cosmological scales. At galactic scales, the leading terms will be proportional to linear potential and inverse potential, namely

$$a\lambda r - \frac{c^3(\lambda - A_0)}{6\pi r}\,. \tag{33}$$

As we will see in the next section, to see the physical implications one should express these results in terms of the center of gravity of the wormhole. Calculations in classical mechanics are often simplified when laws are formulated with respect to the center of mass. In this case, the center of mass is a hypothetical sphere where entire mass of an object may be assumed to be concentrated to visualise its motion. In other words, the center of mass is the single radius equivalent for the application of Newton's laws similar to the case of ordinary non relativistic mechanics. We now go on to define the analogous of this concept for the case of an asymmetric wormhole.

IV. CENTER OF GRAVITY COORDINATES

The wormhole has two special radiuses. The first one is the neck, where $C(r)$ is a minimum, and from (28) we obtain that the radial coordinate for this is $r_m = -a/2$. The second one is the center of gravity radius, or the equilibrium radius, where the Newtonian gravitational force vanishes. In general, when one considers an extenteded object, one defines a worldline on the basis of the center of mass, as discussed by Pound [28]. So here also, for analizing the behavior with respect to this special location, we have to use the coordinates of the center of gravity by considering a shift

$$r = r' + \Delta \,. \tag{34}$$

The coordinates of the center of mass are a type of collective coordinates used when one is dealing with extended objects, and therefore since a wormhole is an extended object, it is natural to use them. So if the wormhole interacts with another wormhole or with a point particle, the use of coordinates that vanish at the center of gravity is preferable since the center of gravity coordinate truly describes the collective motion of the extended object. Then, other coordinate choice will not be so physically correct.

Demanding that for small r', $A(r')$ does not contain linear terms in r' and inserting (34) into (29), we obtain that the linear term of $A(r')$ is cancelled for the following choice of Δ,

$$\Delta = \frac{c}{\pi} - \frac{a}{2} - \frac{c\lambda}{A_0\pi} \,. \tag{35}$$

Now, by expressing the small r' limit in terms of the center of gravity coordinates, we find

$$A(r') = 1 + A_0 b^2 + \frac{ac(\lambda - A_0)}{\pi} + A_0 r'^2 - A_0\Delta^2 \,. \tag{36}$$

where we see that the linear terms are now cancelled. We can see that for positive values of A_0 that the Newtonian potential produces attraction towards the center of gravity point $r' = 0$ for small r', so the radius $r' = 0$ is indeed the radius towards where test particles are attracted to. This is therefore the center of gravity radius. Notice that $r' = 0$, with whatever constant angles we choose, represent a geodesic motion. To ensure that the metric has the correct signature for small r', we require also that

$$1 + A_0 b^2 + \frac{ac(\lambda - A_0)}{\pi} - A_0\Delta^2 > 0.$$

In the case where $r' \gg c(\frac{1}{2} - \frac{c}{\pi} + \frac{\lambda}{A_0\pi})$, we obtain that the 00 component of the metric becomes,

$$A(r') = (1 + \lambda b^2 + a\lambda\Delta + \lambda\Delta^2) - \frac{2\lambda c(\lambda - A_0)}{A_0} r' + \lambda r'^2 - \frac{c^3(\lambda - A_0)}{6\pi r'} \,. \tag{37}$$

In the 00 component of the metric, the coefficient of the $1/r'$ term equals to $-2M$, where M is the mass of the wormhole. Therefore one has that

$$M = \frac{c^3(\lambda - A_0)}{12\pi} \tag{38}$$

and from Eq. (31) we can get the dependence of the mass in terms of a and r_0

$$M = \frac{a + 2r_0}{6} \,. \tag{39}$$

The linear term of $A(r')$ for large r' can be expressed in terms of M, obtaining,

$$A(r') = (1 + \lambda b^2 + a\lambda\Delta + \lambda\Delta^2) - \frac{24\lambda M}{A_0 c^2} r' + \lambda r'^2 - \frac{2M}{r'} \,. \tag{40}$$

The coefficient that multiplies r'^2 for large $|r'|$ identified the values of the cosmological constant at the two sides of the wormhole. Because of:

$$\lambda_{\pm} = A_0 \pm \frac{2\pi(a+r_0)}{c^3} = -\frac{\Lambda_{\pm}}{3} \,. \tag{41}$$

The discontinuity of the cosmological constant between the two asymptotic sides of the wormhole is:

$$\Lambda_{+} - \Lambda_{-} = -\frac{72M}{c^3} \,. \tag{42}$$

According to [26], a Newtonian potential with linear and inverse of r' can provide an explanation for flat rotation curves. An interesting point arises here. The wormhole parameters a, r_0 and b appear in the constants which are related to the flat rotation curves described in [26, 27]. Thus, the asymmetric wormhole is acting as a trigger of a dark matter behaviour. Notice that the linear term in the Newtonian potential $r'(4\Lambda_{+}M)/(A_0c^2)$ is proportional to the Mass of the wormhole M. For a positive values of M, A_0, Λ_{+} we obtain that the Newtonian potential produces an attractive force. Notice that using the coordinates of the center of gravity r', the linear term and the $1/r'$ term are both proportional to the mass M, as is the discontinuity in the cosmological constant across the wormhole. So the mass appears the source of gravitational attraction, at both lare and small distances. As well us being the source of the discontinuity of the cosmological cosntnant accross the whormhole.

This solution and also this interpretation would be also valid for any other k-essence theory. Then, one can say that if ones assumes an asymmetric wormhole geometry, the potential could describe dark matter and dark energy in a unified form.

V. THE BEHAVIOR FOR THE SCALAR POTENTIALS

In order to find solutions for our specific diffusive two measures theory, one needs to impose an additional ansantz since we have more variables than remaining equations. As an example and completeness, let us assume that the scalar field behaves as

$$\phi = \phi_0 \arctan\left(\frac{a+2r}{c}\right) . \tag{43}$$

Here, ϕ_0 is a constant. Then, by replacing this form into (22), one can easily find that the potential takes the following form

$$\begin{aligned} V_2(\phi) = \frac{1}{8\pi c^2\phi_0}\Big[& -2\phi_0^2\cos\left(\frac{2\phi}{\phi_0}\right)\left(2c^2\phi(\lambda-A_0)+\pi\phi_0\left(A_0c^2+4\right)\right)+c\Big(-4\phi\left(c\phi_0^2(\lambda-A_0)+4\pi C_2\right) \\ & +2c\phi_0^3(A_0-\lambda)\sin\left(\frac{2\phi}{\phi_0}\right)+c\phi_0^3(A_0-\lambda)\sin\left(\frac{4\phi}{\phi_0}\right)\Big)-2\pi\phi_0^3\cos\left(\frac{4\phi}{\phi_0}\right)\Big]+V_0 \,, \end{aligned} \tag{44}$$

where V_0 is an integration constant. Now, the scalar field χ_r can be directly solved by using (18), however, it depends implicitly on $V_1(\phi)$. In order to find $V_1(\phi)$ one needs to solve the remaining Eq. (19). This equation is an ordinary first order equation which in principle has solution but analytically, it can be easily solved. One can then solve that equation numerically to see the behaviour of the potential $V_1(\phi)$. As one can see form Fig. 2, the potential $V_1(\phi)$ behaves with well defined asymptotic properties. We present this to show the existence of solutions. A variety of other solutions, starting from an ansatz different than (43) could be explore also.

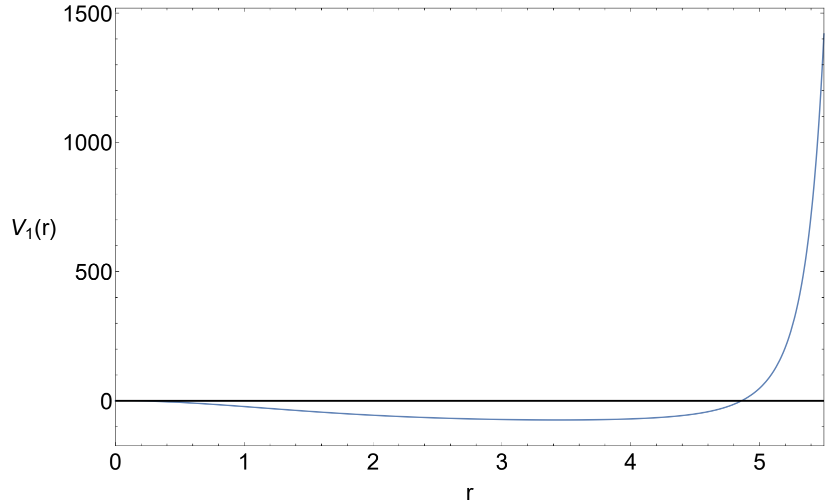

FIG. 2: Plot of the potential $V_1(r)$ versus r for $\phi_0 = 1$, $b = 2.5$, $a = 0.001$, $\lambda = 0.00001$, $C_2 = 0.1$, $V_0 = 100$ and $A_0 = -1$.

VI. CONCLUSIONS

In this paper we have explored wormholes solutions in a particular DE/DM unified model described in [22, 24]. In this case and for asymmetric wormholes, we have found that the asymmetry between the two universes connected through a wormhole induces a linear term in the gravitational potential, and have calculated the coefficient of these linear term in the coordinates of the center of gravity of the wormhole. These coordinates are expected to be the most suitable ones if we are interested in the collective motion of the wormhole as is the coordinates of the center of gravity in non-relativistic mechanics. As discussed in [27], these linear gravitational potentials can be used to explain the behaviour of galactic rotation curves.

ACKNOWLEDGMENTS

S.B. is supported by the Comisión Nacional de Investigación Científica y Tecnológica (Becas Chile Grant No. 72150066). The authors would also like to acknowledge networking support by the COST Action GWverse CA16104. E.G. is gratefull for many interesting discussions of the subject of this paper with Phillip Mannheim and Thom Curtright during the Miami 2017 conference and latter on during a visit to the Physics Department of the University of Miami. We also acknowlege Leor Barak for conversations concerning the need to refer to the center of mass during the first global event of the COST Action GWverse CA16104.

[1] V. C. Rubin, N. Thonnard and W. K. Ford, Jr., Astrophys. J. **238**, 471 (1980). doi:10.1086/158003
[2] V. C. Rubin and W. K. Ford, Jr., Astrophys. J. **159**, 379 (1970). doi:10.1086/150317
[3] E. Kulchoakrungsun, A. Lam and D. A. Lowe, arXiv:1712.06981 [gr-qc].
[4] F. S. N. Lobo, Phys. Rev. D **71**, 084011 (2005) doi:10.1103/PhysRevD.71.084011 [gr-qc/0502099].
[5] M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988). doi:10.1119/1.15620
[6] S. V. Sushkov, Phys. Rev. D **71**, 043520 (2005) doi:10.1103/PhysRevD.71.043520 [gr-qc/0502084].
[7] M. Visser, Phys. Rev. D **39**, 3182 (1989) doi:10.1103/PhysRevD.39.3182 [arXiv:0809.0907 [gr-qc]].
[8] E.I. Guendelman. Mod. Phys. Lett. A14 , 1043 (1999)e-Print: gr-qc/9901017
[9] E.I. Guendelman and A.B. Kaganovich, Phys. Rev. D53 , 7020 (1996)
[10] F. Gronwald, U. Muench, A. Macias, F. W. Hehl, Phys. Rev. D58 , 084021 (1998), e-Print: gr-qc/9712063.
[11] E.I. Guendelman, A.B. Kaganovich, Class. Quantum Grav. 25 , 235015 (2008), e-Print: arXiv:0804.1278 [gr-qc]
[12] H. Nishino, S. Rajpoot, Mod. Phys. Lett. A21 , 127 (2006), e-Print: hep-th/0404088
[13] E.I. Guendelman, A.B. Kaganovich, Ann. of Phys. 323 , 866 (2008), e-Print: arXiv:0704.1998 [gr-qc]
[14] E.I. Guendelman, A.B. Kaganovich, Phys. Rev. D75 , 083505 (2007), e-Print: gr-qc/0607111
[15] E.I. Guendelman, A.B. Kaganovich, Int. J. Mod. Phys. A21 , 4373 (2006), e-Print: gr-qc/0603070

[16] Eduardo Guendelman, Ramón Herrera, Pedro Labrana, Emil Nissimov, Svetlana Pacheva , Gen.Rel.Grav. 47 (2015) arXiv:1408.5344 [gr-qc]
[17] E. I. Guendelman, Int. J. Mod. Phys. A **25**, 4081 (2010) doi:10.1142/S0217751X10050317 [arXiv:0911.0178 [gr-qc]].
[18] D. Benisty and E. I. Guendelman, Mod. Phys. Lett. A **31**, no. 33, 1650188 (2016) doi:10.1142/S0217732316501881 [arXiv:1609.03189 [gr-qc]].
[19] T. O. Vulfs and E. I. Guendelman, doi:10.1142/S021773231750211X arXiv:1708.00458 [hep-th].
[20] T. O. Vulfs and E. I. Guendelman, arXiv:1709.01326 [hep-th].
[21] S. Calogero, JCAP **1111** (2011) 016 doi:10.1088/1475-7516/2011/11/016 [arXiv:1107.4973 [gr-qc]].
[22] D. Benisty and E. I. Guendelman, Int. J. Mod. Phys. D **26** (2017) no.12, 1743021. doi:10.1142/S0218271817430210
[23] D. Benisty and E. I. Guendelman, Eur. Phys. J. C **77**, no. 6, 396 (2017) doi:10.1140/epjc/s10052-017-4939-x [arXiv:1701.08667 [gr-qc]].
[24] D. Benisty and E. I. Guendelman, arXiv:1710.10588 [gr-qc].
[25] K. A. Bronnikov and J. C. Fabris, Phys. Rev. Lett. **96** (2006) 251101 doi:10.1103/PhysRevLett.96.251101 [gr-qc/0511109].
[26] J. G. O'Brien, T. L. Chiarelli and P. D. Mannheim, arXiv:1704.03921 [astro-ph.GA].
[27] P. D. Mannheim, Prog. Part. Nucl. Phys. **94**, 125 (2017) doi:10.1016/j.ppnp.2017.02.001 [arXiv:1610.08907 [hep-ph]].
[28] Pound, A, Fund.Theor.Phys. 179 (2015) 399-486 DOI : $10.1007/978-3-319-18335-0_13$ Conference: C13-02-17 Proceedings e-Print: arXiv:1506.06245 [gr-qc]