New Teleparallel Gravity Models

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Abstract

Motivated by success of $f(T)$ gravity we present two new approaches to construct novel modified teleparallel theories of gravity: $f(T_{\text{vec}}, T_{\text{ax}}, T_{\text{ten}})$ gravity, where the Lagrangian is taken to be a function of three quadratic torsion invariants, and a new approach motivated by axiomatic electrodynamics where various modified theories of gravity are realized through different gravitational constitutive relations.

1. Introduction

General relativity can be reformulated using teleparallel geometry instead of the usual Riemannian one, allowing us to obtain new insights and construct novel modified theories of gravity.

1.1 Teleparallel Geometry

is defined using tetrads, $h^a{}_\mu$, related to the metric tensor by

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu,$$

and the condition of vanishing curvature

$$R^a{}_{b\mu\nu}(\omega^a{}_{b\mu} \equiv 0$$

that leads to the teleparallel spin connection

$$\omega^a{}_{b\mu} = \Lambda^a{}_c \partial_\mu \Lambda_b{}^c.$$

The non-trivial geometry is attributed then to the torsion tensor

$$T^a{}_{\mu\nu}(h^a{}_\mu, \omega^a{}_{b\mu}) = \partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu + \omega^a{}_{b\mu} h^b{}_\nu - \omega^a{}_{b\nu} h^b{}_\mu.$$

1.2 Teleparallel Gravity

The torsion tensor can decomposed in irreducible pieces with respect to a local Lorentz symmetry

$$v_\mu = T^\lambda{}_{\lambda\mu} \qquad a_\mu = \frac{1}{6}\epsilon_{\mu\nu\sigma\rho} T^{\nu\sigma\rho},$$
$$t_{\lambda\mu\nu} = \frac{1}{2}(T_{\lambda\mu\nu} + T_{\mu\lambda\nu}) + \frac{1}{6}(g_{\nu\lambda} v_\mu + g_{\nu\mu} v_\lambda - 2 g_{\lambda\mu} v_\nu),$$

known as the vector, axial, and purely tensorial torsion. Taking their quadratic invariants

$$T_{\text{vec}} = v_\mu v^\mu, \qquad T_{\text{ax}} = a_\mu a^\mu, \qquad T_{\text{ten}} = t_{\lambda\mu\nu} t^{\lambda\mu\nu},$$

we can define the action of teleparallel gravity

$$\mathcal{L}_{\text{TEGR}} = \frac{h}{2\kappa} T = \frac{h}{2\kappa}\left(\frac{3}{2} T_{\text{ax}} + \frac{2}{3} T_{\text{ten}} - \frac{2}{3} T_{\text{vec}}\right),$$

which is (up to a surface term) equivalent to the Einstein-Hilbert action.

2. f(T) Gravity

The simplest and most popular modification

$$\mathcal{L}_f = \frac{h}{2} f(T),$$

where $f(T)$ is an arbitrary function of the torsion scalar.

Features

- Rich dynamics capable of explaining the two accelerated phases of expansion, at early and late times respectively
- Simplest toy model illustrating novel features/problems of modified teleparallel theories: issue of local Lorentz invariance, complicated non-linear dynamics, the problem of new degrees of freedom, etc

3. $f(T_{\text{vec}}, T_{\text{ax}}, T_{\text{ten}})$ Gravity

(Work with S. Bahamonde and Ch. Böhmer)

A natural generalization of $f(T)$ gravity is to consider a general function of three quadratic invariants appearing in the TEGR action, i.e.

$$\mathcal{L} = \frac{h}{2} f(T_{\text{vec}}, T_{\text{ax}}, T_{\text{ten}}),$$

3.1 Conformal transformations

Particularly interesting are conformal transformations in this theory

$$\hat{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}, \quad \hat{g}^{\mu\nu} = \Omega^{-2}(x) g^{\mu\nu},$$

under which the torsion invariants transform as

$$T_{\text{ax}} = \Omega^2 \hat{T}_{\text{ax}}, \qquad T_{\text{ten}} = \Omega^2 \hat{T}_{\text{ten}},$$
$$T_{\text{vec}} = \Omega^2 \hat{T}_{\text{vec}} + 6\Omega \hat{v}^\mu \hat{\partial}_\mu \Omega + 9 \hat{g}^{\mu\nu} (\hat{\partial}_\mu \Omega)(\hat{\partial}_\nu \Omega),$$

from where follows the non-existence of the Einstein frame in teleparallel theories with second order field equations.

3.2 Extension by the boundary term

The model can be extended by including higher derivatives, particularly interesting is

$$\mathcal{L} = \frac{h}{2} f(T_{\text{vec}}, T_{\text{ax}}, T_{\text{ten}}, B),$$

where B is the boundary term

$$B = \frac{1}{h} \partial_\mu (h v^\mu)$$

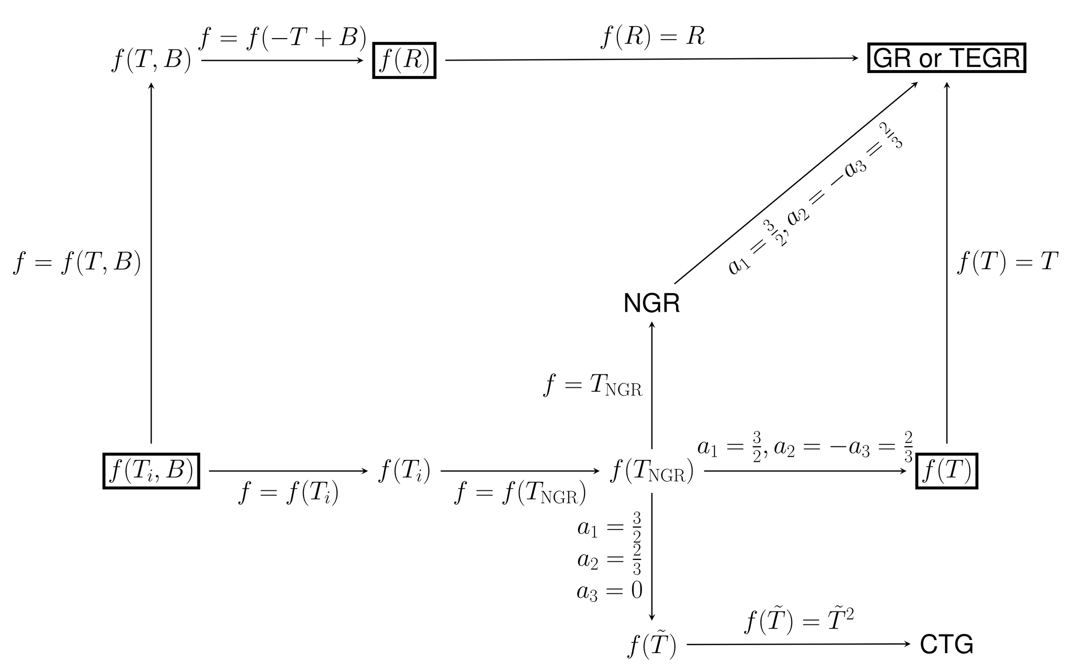
that under conformal transformations behaves as

$$B = \Omega^2 \hat{B} - 4\Omega \hat{v}^\mu \hat{\partial}_\mu \Omega - 18 \hat{\partial}^\mu \Omega \hat{\partial}_\mu \Omega + \frac{6}{\hat{h}} \Omega \hat{\partial}_\mu (\hat{h} \hat{g}^{\mu\nu} \hat{\partial}_\nu \Omega),$$

what allows us to show that the unique theory with the Einstein frame is $f(-T + B) \equiv f(R)$.

3.3 Unified scheme of modified theories

An unified picture of various modified teleparallel theories of gravity and their relation with $f(R)$ gravity:



Features

- Naturally includes all previously studied models: $f(T)$ gravity, New General Relativity, and teleparallel conformal gravity
- Suitable for analyzing behavior under conformal transformations and (non-)existence of the Einstein frame
- Extensions with the boundary term illustrate a possibly important role of higher derivatives
- Possibility of new modes of gravitational waves (ongoing project)

4. Modified Theories of Gravity Inspired by Electrodynamics

(Work with M.Hohmann, L. Järv, Ch. Pfeifer)

A novel approach to formulate modified teleparallel theories of gravity motivated by the axiomatic approach to electrodynamics.

4.1 Axiomatic Electrodynamics

allows us to write the field equations of any generalized electrodynamics (Born-Infeld, Plebanski,...) as Maxwell equations

$$\mathrm{d}F = 0,$$
$$\mathrm{d}H = J,$$

using *constitutive relation* $H = \kappa(F)$ relating the field strength F and the excitation form H.

4.2 Gravitational Models à la Electrodynamics

In a similar fashion, any gravity model with action

$$S_g = \frac{1}{2} \int_M T^a \wedge H_a,$$

can be described by the field equations

$$DT^a = 0,$$
$$D\Pi_a - \Upsilon_a = \Sigma_a,$$

where $\Pi_a = H_a + Q_a$ is the gravitational excitation form, Υ_a and Σ_a are gravitational and matter energy-momentum forms.

4.3 Constitutive Relations for Gravity Models

Various modified models can be realized through different *gravitational constitutive relations*:

- f(T) gravity

$$H_{a\mu\nu} = \frac{1}{2} \frac{f(T)}{T} |h| S_a.$$

- Plebanski-like gravity

$$H_a = U(T_{\text{vec}}, T_{\text{ax}}, T_{\text{ten}})\, T_b + V(T_{\text{vec}}, T_{\text{ax}}, T_{\text{ten}})\, S_b,$$

which includes $f(T_{\text{vec}}, T_{\text{ax}}, T_{\text{ten}})$ gravity as a special case.

Features

- Novel approach exploring the links between modified gravity theories and non-linear electrodynamics
- Field equations take the same simple form and allow to analyze their generic properties, e.g. local Lorentz invariance implies symmetry of the field equations, etc
- Large number of new gravity models realized through various constitutional laws, e.g. Plebanski gravity

References

[1] S. Bahamonde, C. G. Böhmer and M. Krššák, *New classes of modified teleparallel gravity models*, Phys. Lett. B **775** (2017) 37 [arXiv:1706.04920 [gr-qc]].

[2] M. Hohmann, L. Järv, M. Krššák and C. Pfeifer, *Teleparallel theories of gravity as analogue of non-linear electrodynamics*, arXiv:1711.09930 [gr-qc].