On some properties of purely axial torsion waves

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Introduction

A departure from Riemannian spacetime of GR consists of admitting torsion and possible nonmetricity – a 'metric-affine' spacetime. Characterisation of spacetime by an independent connection distinguishes MAG from GR. We give a characterisation of spacetimes whose (metric-compatible) connection is generated by purely axial torsion and an explicit formula for their torsion. We describe the properties of curvature and explicit formulae for all its irreducible pieces. We present possible applications to concrete cases of solutions of MAG with special attention to the (massless) Dirac equation and operator.

Connection

- Work in Minkowski space $\mathbb{M}^4$
- Connection $\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} A_\kappa \varepsilon^{\kappa\lambda}{}_{\mu\nu}$
- Metric compatibility ($\nabla g = 0$)
- Torsion $T = *A$ purely axial
- A vector potential of external electromagnetic field

Curvature

$$R^{(1)}{}_{\kappa\lambda\mu\nu} = \frac{1}{2} g_{\kappa[\nu} A_{\mu]} A_\lambda - \frac{1}{2} g_{\lambda[\nu} A_{\mu]} A_\kappa - \frac{1}{4} g_{\kappa[\nu} g_{\mu]\lambda} A_\eta A^\eta$$

$$R^{(2)}{}_{\kappa\lambda\mu\nu} = -\frac{1}{4} g_{\kappa[\nu} g_{\mu]\lambda} A_\eta A^\eta$$

$$R^{(3)} = 0$$

$$R^{(4)}{}_{\kappa\lambda\mu\nu} = -\frac{1}{4} \varepsilon_{\kappa\lambda\mu\nu} \partial_\eta A^\eta$$

$$R^{(5)}{}_{\kappa\lambda\mu\nu} = \frac{1}{2} \varepsilon^\eta{}_{\lambda\mu\nu} \partial_{(\kappa} A_{\eta)} - \frac{1}{2} \varepsilon^\eta{}_{\kappa\mu\nu} \partial_{(\lambda} A_{\eta)} - \frac{1}{4} \varepsilon_{\kappa\lambda\mu\nu} \partial_\eta A^\eta$$

$$R^{(6)}{}_{\kappa\lambda\mu\nu} = \frac{1}{2} g_{\kappa[\nu} (*dA)_{\mu]\lambda} - \frac{1}{2} g_{\lambda[\nu} (*dA)_{\mu]\kappa}$$

Discussion

Curvature also described using another formalism. Application to solutions QMAG in pp-spaces (generalised pp-waves with axial torsion). Yields a class of Weitzenböck spaces. Mathematical model? The spinor field which determines their complexified curvature satisfies the massless Dirac (or Weyl's) equation $\sigma^\mu{}_{a\dot{b}} \nabla_\mu \xi^a - \frac{1}{2} T^\eta{}_{\eta\mu} \sigma^\mu{}_{a\dot{b}} \xi^a = 0$.