Unimodular quantum gravity and the classical limit

Natascha Riahi
University of Vienna, Gravitational physics

CAN GRAVITY BE QUANTIZED? Time vanishes in quantum gravity! How to *predict then our classical world from a quantum theory?*

Unimodular Gravity

$$\delta g_{\mu\nu}\left(\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,R+S_{matter}\right)\Bigg|_{g=-1}=0$$

$$\Rightarrow \quad R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\kappa T_{\mu\nu}-\Lambda g_{\mu\nu}$$

- Varying the action with respect to the metric under the condition $det\, g_{\mu\nu}=g=-1$
- Getting Einsteins equations with an additional term
- Identifying Λ with the cosmological constant

$\Rightarrow$
Any solution of Unimodular Gravity is a solution of General Relativity with a certain cosmological constant!

Unimodular quantum cosmology:The flat Friedmann universe with a scalar field

spacetime:

$ds^2=-N^2(t)dt^2+a^2(t)d\Omega_3^2$
$a(t)\ldots$ scale factor
$N(t)\ldots$ lapse function

matter:

Lagrangian of the field Φ

$$L_{matter}=N\,a^3\left(\frac{\dot\Phi^2}{2N^2c^2}-V(\Phi)\right)$$

The unimodular Hamiltonian of the model reads

$$H=\frac{c^2}{2}\frac{p_\Phi^2}{a^6}-\frac{c^2}{4\epsilon}\frac{p_a^2}{a^4}+V(\Phi) \quad \text{where} \quad \epsilon\equiv 3/\kappa.$$

- **There is no Hamiltonian constraint!** (H is a constant of motion)

Quantization (with Laplace Beltrami operator ordering) yields a Schrödinger equation

$$\widehat{H}\,\Psi=\left(\frac{\hbar^2c^2}{4\epsilon}\frac{1}{a^5}\frac{\partial}{\partial a}a\frac{\partial}{\partial a}-\frac{\hbar^2c^2}{2}\frac{1}{a^6}\frac{\partial^2}{\partial\phi^2}+V(\Phi)\right)\Psi=i\hbar\frac{\partial\Psi}{\partial t}.$$

- The **problem of time** of standard canonical quantum gravity ($\widehat{H}\,\Psi=0$) **has disappeared !**
- It is possible to define a scalar product.

Introducing light-cone coordinates

$$u=\frac{a^3}{3}e^{-\frac{3}{\sqrt{2\epsilon}}\Phi} \qquad v=\frac{a^3}{3}e^{\frac{3}{\sqrt{2\epsilon}}\Phi},$$

we find

$$\widehat{H}=\frac{\hbar^2c^2}{\epsilon}\frac{\partial^2}{\partial u\partial v}+V\left(\frac{u}{v}\right), \qquad H=-\frac{c^2}{\epsilon}p_u\,p_v+V\left(\frac{u}{v}\right).$$

A superposition of eigensolutions Ψ_Λ

$$\frac{\partial^2}{\partial u\partial v}\Psi_\Lambda(u,v)=-\frac{\Lambda\epsilon}{3}\Psi_\Lambda(u,v),$$

yields wavepacket solutions of the form

$$\Psi(u,v,\tau)=\int_0^\infty e^{it\hbar c^2\frac{\Lambda}{3}}\psi_\Lambda(u,v)F(\Lambda)\,d\Lambda.$$

- It is possible to construct wavepackets with a **unitary time evolution** and to calculate expectation values.

The classical limit of unimodular quantum cosmology

We find for the variable A^2, related to the scalefactor by

$$A^2=uv=\frac{a^6}{9}$$

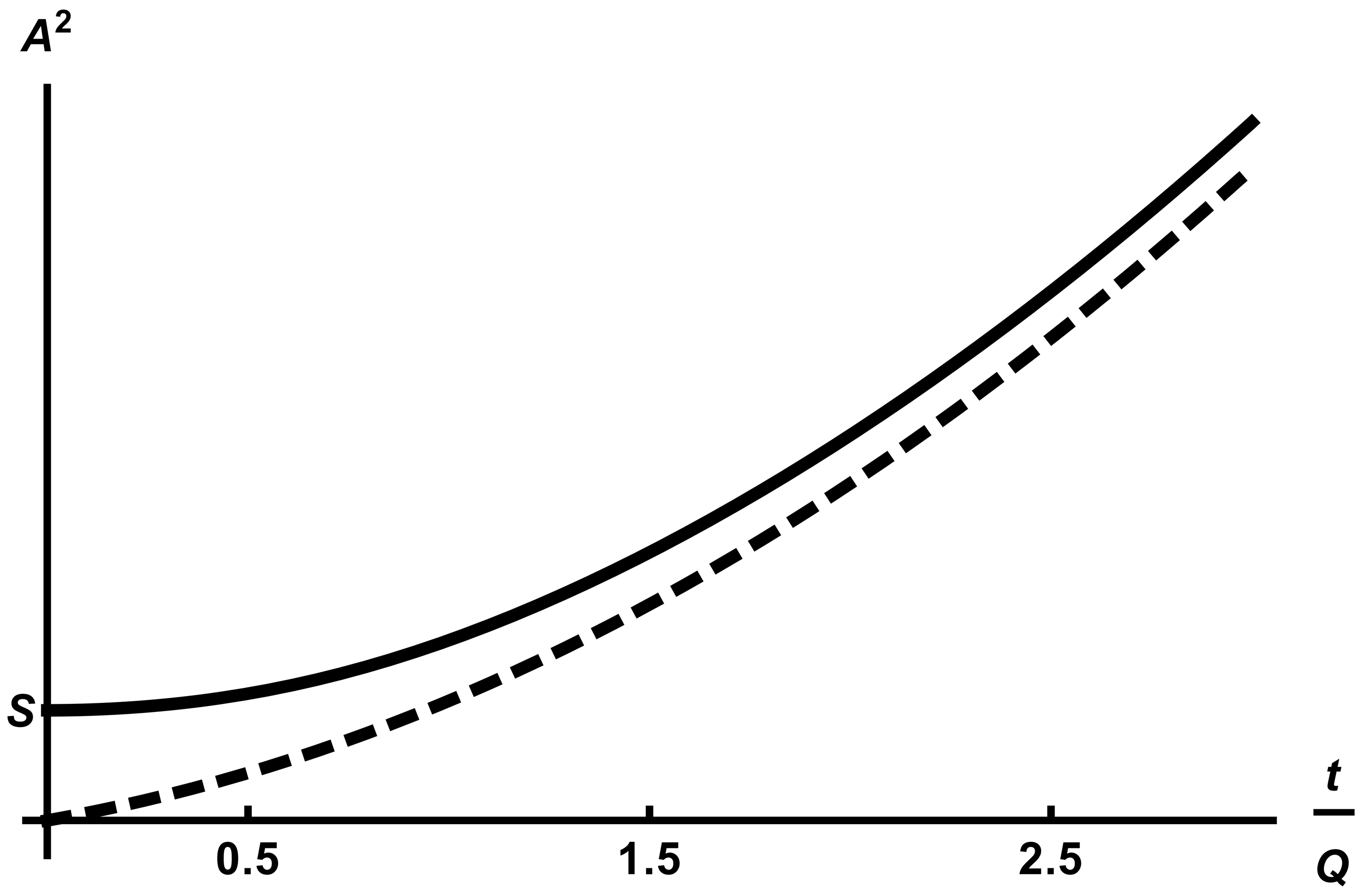
$$\lim_{t\to\infty}\frac{d^2}{dt^2}\langle A^2\rangle=\frac{c^2}{\epsilon}\left\langle -2\,\widehat{H}+V+u\frac{\partial V}{\partial u}+v\frac{\partial V}{\partial v}\right\rangle$$

$$\lim_{t\to\infty}\frac{d^2}{dt^2}A^2=\frac{c^2}{\epsilon}\left(-2\,H+V+u\frac{\partial V}{\partial u}+v\frac{\partial V}{\partial v}\right)$$

There is a **correspondence of classical and quantum behaviour for late times**!

Results for the massless scalar field

We found explicit solutions for the special case of a massless scalar field $V=0$(corresponds to stiff matter: $p=\rho c^2$) with a negative expectation value of the Hamiltonian [1].



The classical value A^2 and the expectation value $\langle A^2\rangle$ (time is scaled by $Q=\sqrt{\frac{2}{3}\cdot\frac{\epsilon}{\hbar c^2}}$)

- There is **no singularity** in the quantum case.
- The **classical and** the **expectation value of** A^2 have both the **same longterm behaviour**
  This is quite remarkable, since quantum effects use to enlarge with time!

Further questions

- In the case of the massless scalar field the uncertainty of the scaled factor increases: $\Delta A^2\sim t^2$ for late times.
  For which scalar field potentials a better quasiclassical behaviour can be expected?
- What are the results for **quantum black holes** in unimodular quantum gravity? Will the black hole solutions be stable?

[1] N.Riahi (2017): Wavepacket evolution in unimodular quantum cosmology, Galaxies 2018,6(1),8 (see also arXiv:1801.04833)