AM SYLLABUS (2019)

APPLIED MATHEMATICS AM 02

SYLLABUS

Applied Mathematics AM 02	(Available in September)
Syllabus	Paper I (3 hrs)+Paper II (3 hrs)

Applied Mathematics (Mechanics)

Aims

A course based on this syllabus should

i) provide a relevant, stimulating and motivating course of advanced study in mathematics, including the provision of a suitable foundation for further study in science and engineering;

ii) develop a variety of skills in modelling, logical reasoning and problem solving;

iii) encourage student interest and satisfaction through the development and use of mathematics in a variety of applications;

iv) promote an awareness of the relevance of mathematics to other fields of study and to other practical applications.

Assessment objectives

Candidates should be able to demonstrate that they can:

i) apply their knowledge of relevant mathematical techniques in a variety of contexts;

ii) construct rigorous mathematical arguments through an appropriate use of precise statements, logical deduction and by manipulation of mathematical expressions;

iii) evaluate mathematical models, including an appreciation of the assumptions made, and interpret, justify and present the results from a mathematical analysis in a form relevant to the original problem;

iv) communicate mathematical ideas and methods, including the use of appropriate mathematical notation, terminology, conventions and diagrams, in a clear, logical and well-structured presentation.

Scheme of Assessment

The examination consists of two papers of three hours each. Paper I will contain ten questions each carrying between eight and twelve marks. The total number of marks in Paper I is 100. Candidates will be required to answer all questions. Paper II will contain ten questions, of which candidates will be required to attempt seven. Each question will carry 15 marks.

Grade Description

Grades A, B, C, D, E will be awarded to candidates who pass in this examination, with Grade A being the highest grade, and Grade E the lowest passing Grade. Grade F signifies a failure in this examination.

Candidates who obtain Grade **A** are able to recall and select **almost all** concepts and techniques required in various contexts. Candidates who obtain Grade **C** are able to recall and select **most** concepts and techniques required in various contexts. Candidates who obtain Grade **E** are able to recall and select **some** concepts and techniques required in various contexts.

Syllabus

The syllabus is divided into two parts: Me1 and Me2 referring to Papers I and II respectively.

Applied Mathematics (Mechanics) Paper I – Me1.

Knowledge of the contents of the Intermediate Pure Mathematics syllabus is assumed. The level of difficulty involved will be consistent with the level attained in the Intermediate Pure Mathematics Syllabus.

The topics are not arranged in teaching order. Any examination question can test material from more than one topic. Questions may be set on topics which are not explicitly mentioned in the syllabus, but such questions will contain suitable guidance so that candidates would be able to tackle them with the mathematical knowledge they would have acquired during their studies of the material in the syllabus.

A problem on any part of the syllabus can be set in vector notation. Throughout the syllabus, it is assumed that students have a good understanding of the following:
Equality of vectors, the zero vector, addition of vectors and multiplication by a scalar.
The unit vectors $\mathbf{i}$, $\mathbf{j}$, the equation of a straight line in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.
Simple use of the scalar product of two vectors.
Simple differentiation and integration of vectors with respect to a scalar variable.

In this paper, typical applications of vectors could include:

i) finding the line of action, magnitude and direction of the resultant force of a simple coplanar system of forces,

ii) finding velocities and accelerations as the time derivatives of position vectors,

iii) deriving the acceleration of a particle moving in a circle with constant speed, and

iv) locating the centre of mass of a number of particles in two dimensions.

The use of the vector product will not be required in this paper.

Topics	*Notes*
Statics	
1. Coplanar forces acting at a point. General equilibrium. Friction, limiting equilibrium, coefficient and angle of friction. Elastic strings and springs: Hooke's law.	The concept of a force as a localised vector. Parallelogram and triangle rules for addition of forces. Resolution of forces. Resultant of a system of forces acting at a point. Forces in equilibrium. Lami's theorem. Reaction between bodies in contact. An understanding of the inequality $F \leq \mu R$. Tensions in inextensible strings and elastic strings satisfying Hooke's Law in the form $T=\lambda x/a$.
2. The principle of moments. Coplanar forces acting on a rigid body. Moment of a system of coplanar forces about an axis perpendicular to the plane containing the lines of action of the forces. Reduction of a coplanar system of forces to a force, to a couple, or to a force and a couple. Frameworks.	Forces may be given in vector form, but the moment of a force as a vector product is not required. Frameworks made up of light smoothly jointed rods.
3. Centre of mass of: a) a system of particles, b) a uniform body, including the use of integration, c) simple composite bodies.	Including problems of toppling.
Kinematics in one dimension	
4. Displacement, velocity and acceleration of a particle moving in a straight line.	Including the derivation and use of the formulae for constant acceleration, and the use of displacement–time and velocity–time graphs. Non-uniform acceleration problems involving the setting up and solution of first-order differential equations of the separable type. Appreciation of the identity $\mathrm{d}v/\mathrm{d}t=v\,\mathrm{d}v/\mathrm{d}x$.

5. Simple harmonic motion.	The solution of the differential equation $\ddot{x} = -\omega^2 x$ may be quoted without proof. Including angular simple harmonic motion.
Dynamics of Particles	
6. Newton's Laws of Motion. Mass, force.	Including motion of connected objects. Movable smooth light pulley systems.
7. Work, power, energy. The principle of conservation of mechanical energy.	The work energy principle. Kinetic energy, elastic and gravitational potential energy.
8. Impulse and momentum. Conservation of momentum in one dimension. Direct elastic impact. Newton's law of restitution.	Coefficient of restitution, *e*. Knowledge of the terms perfectly *elastic* ($e = 1$) and perfectly *inelastic* ($e = 0$).
Vectors	
9. Simple differentiation and integration of a vector with respect to a scalar variable. Velocity and acceleration as derivatives of a position vector. Momentum and force as vectors.	Cartesian components of velocity and acceleration. Use of unit vectors **i**, **j**. The resolved part of a vector in a given direction. The equation of motion of a particle in vector form: $\mathbf{F} = m\,\ddot{\mathbf{r}}$. Work, power and kinetic energy as scalar products: **F.s**, **F.v** and $^{1}/_{2}\, m\, \mathbf{v.v}$.
10. Relative velocity in two dimensions. Position and velocity vectors of a point relative to another point.	Problems could include motion in a uniformly moving medium, for example a boat moving in a stream. Collisions; closest approach.
Motion in a plane	
11. Angular speed, constant angular acceleration. Motion in a horizontal circle with uniform speed.	Including the conical pendulum and banked tracks.
12. Simple problems on projectiles.	Students will be expected to derive the equation of the path of a projectile, its horizontal range, its associated time of flight, and the maximum height.

Applied Mathematics (Mechanics) Paper II – Me2.

Knowledge of the topics in Me1and Pure Mathematics A-level is assumed, in particular second-order differential equations, further calculus, further trigonometry, vectors and polar coordinates.

The topics are not arranged in teaching order. Any examination question can test material from more than one topic. Questions may be set on topics that are not explicitly mentioned in the syllabus, but such questions will contain suitable guidance so that candidates would be able to tackle them with the mathematical knowledge they would have acquired during their studies of the material in the syllabus.

A problem on any part of the syllabus may be set in vector notation. In this paper, students are expected to have a good working knowledge of the unit vectors **i, j, k** , and the vector product. Typical applications could include:

i) problems of relative motion where the time of closest approach may be determined by a scalar product and finding the least distance between two particles moving at constant velocities in two or three dimensions,
ii) analysis of systems of forces in three dimensions,
iii) problems of oblique impact with velocities in three dimensions,
iv) locating the centroid of a number of particles in three dimensions,
v) the moment of a force about a point as a vector product,
vi) the angular momentum of a particle about a point.

Topics	*Notes*
Statics	
1. Heavy jointed rods.	Only problems in two dimensions will be set.
2. Shearing force and bending moment functions for horizontal beams in equilibrium under the action of concentrated and uniformly distributed loads. The relationship between shearing force and bending moment diagrams.	The beam may be simply supported, hinged or clamped. Problems on cantilevers are included. Sketching of the shearing force/bending moment diagrams; locating the maxima and minima.
3. Forces in three dimensions. Forces acting on a body in equilibrium. The moment of a force as a vector product. Reduction of a three-dimensional system of forces to a force and/or a couple.	
Relative Velocity	
4. Relative velocity in three dimensions. Position and velocity vectors of a point relative to another point.	
Further Dynamics	
5. Motion in a vertical circle under gravity and smooth constraints.	
6. Motion of a particle in two dimensions using polar coordinates.	Tangential and normal components of acceleration. Radial and transverse components of velocity and acceleration in polar coordinates. Derivation of relevant equations.
7. Further projectiles.	Problems may also be set involving an elastic collision. Projectiles on an inclined plane are included.
8. Motion in a resisting medium.	Motion under a force that could be a given function of time, displacement or velocity. Problems may involve setting up and solving simple first order differential equations with variables separable.

9. Systems of particles under the action of ordinary or impulsive forces. Conservation of Momentum.	Oblique impact of a smooth elastic sphere with a plane. Oblique impact of two smooth elastic spheres. Including problems on connected particles on a plane.
10. The work energy equation in two and three dimensions: $\int_0^{\tau} \mathbf{F}(t).\, \dot{\mathbf{r}}(t)\, \mathrm{d}t \;=\; {}^1/_2\,(m\mathbf{v}^2\text{-}m\mathbf{u}^2).$ Conservative forces, potential energy, conservation of mechanical energy.	
11. Forced and damped harmonic motion.	Setting up and solving the second order differential equation $a\ddot{x} + b\dot{x} + cx = f(t)$ where a, b, c are constants and $f(t)$ could be of the form $p+qt$, p cos qt, or p sin qt, where p, q are constants.
Rigid body dynamics	
12. Moments of inertia. The parallel and perpendicular axes theorem.	Including laminae, solid objects and simple hollow objects. Integration to find moments of inertia of standard bodies.
13. Motion of a rigid body about a fixed horizontal or vertical axis. Angular momentum. The principle of conservation of energy. The compound pendulum.	Equation of motion of the centroid and equation of rotational motion about the axis. Calculation of the force on the axis and the work done by a couple. Translational and rotational kinetic energies. Period of oscillation.
14. General two-dimensional motion of a rigid body. Simple cases of slipping and rolling.	
15. Impulse on rigid bodies, unconstrained or free to rotate about a fixed axis. Conservation of angular momentum.	Impulse perpendicular to the axis of rotation.