ICMI Study 22: Task Design in Mathematics Education Mathematical Investigations: The Impact of Students' Enacted Activity on Design, Development, Evaluation and Implementation

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This paper is based on a year-long action research study that I conducted in a Form Four mathematics class in a girls' state secondary school in Malta. The focus is on presenting the framework that was used to plan, develop, design, classify, implement and evaluate investigative tasks. This framework hence provides principles and guidelines on examining the differentiation between the tasks as designed and foreseen by the teacher and the actual instructional activities as undertaken by the students. For the teacher-researcher, students' perspectives become crucial within the developmental cyclic process of designing, modifying, implementing and evaluating tasks. The study reported in this paper shows that when students' contributions (students' classroom experiences, responses and understandings) are valued, the teacher-researcher gains more informed knowledge and improved understanding about the design and implementation of investigative tasks.

Investigations; active learning; reflective practice; students' perspectives; task resources

Introduction

This paper draws upon an action research study which I conducted with a Form Four (i.e., 14-year-olds) mathematics class during the scholastic year 2008-2009 that explored the integration of a set of 18 investigations within the mathematics syllabus (see Calleja, 2011). Since then, I have moved to a different state school and have also assumed the responsibilities of Head of Mathematics Department. The focus of this paper is to present a view of didactical goals, guidelines and frameworks that may be useful for collaborative design research projects that are specifically aimed at integrating tasks that promote active learning practices (see Anthony, 1996). The embedded understanding is enlightening my current collaborative project, which commenced in September 2012, on promoting the design and implementation of inquiry-based mathematical tasks with all the Form One students (i.e., 11-year-olds) in my new secondary school. As head of mathematics department, I am coordinating this year-long pilot project and acting as a teacher-mentor within a team consisting of the four other mathematics teachers. At present, we are busy selecting, planning, designing, implementing and evaluating a variety of inquiry tasks, including a range of resource materials (e.g., worksheets, measuring instruments, etc.). Working within our community of practice, we follow a teacher-researcher perspective of '*learning-to-develop-learning*' (Jaworski, 2006) while implementing our task-driven pedagogy (Walls, 2005).

The Research Area and Methodology

At the time of the action research study in my former school, I undertook the role of a reflective practitioner (see Schön, 1983) in the belief that "good research is not about good methods as much as it is about good thinking" (Stake, 1995, p. 19). This involved taking a reflective stance towards planning, designing, implementing, modifying and evaluating a set of mathematical investigations that I integrated within my scheme of work. Data collection included multiple sources. Apart from writing field notes of my classroom observations, I occasionally discussed classroom situations with a critical friend and kept a reflective journal to give my own interpretations as the events unfolded. However, I also valued students' reflections, thoughts and suggestions. Thus, I encouraged students to keep a learning log book through which they could write about their learning experiences, feelings and learning environment. Moreover, midway through the study, I conducted semi-structured qualitative interviews with the students. Data analysis was then an on-going process of searching for themes as these emerged and resurfaced. This sense-making journey was a search for 'some' truth through qualitative triangulation, namely, an account that presented the classroom situations from three different perspectives: those of the teacher-researcher, the students and a critical friend (McKernan, 1996). My qualitative account emerged as an informed story highlighting the salient features of the classroom community while engaged with a range of investigative tasks as an approach to learning mathematics. At the same time, this study offered me the possibility to better understand and eventually improve my practice.

Conceptual Framework

Reform-oriented approaches claim that doing mathematics should involve sense-making activities by using tasks that provide students with a variety of challenging experiences through which they can actively construct their mathematical meanings (Bishop, 1991). Within this 'active learning' approach – which is associated with experiential, collaborative and inquiry-based learning (see Anthony, 1996) – students gain autonomy and take control over the direction of their learning. Referring to the pedagogical implications, Ernest (1991, p. 288) proposes that the role of the teacher

> is understood in ways that support this pedagogy, as manager of the learning environment and learning resources, and facilitator of learning.

Thus, it is within the teacher's remit to include cognitively demanding tasks for their students, encouraging them to work together and to justify their solutions. Learners are thought of in turn as subjects who are responsible for learning, for making decisions and also for their behaviour (Teong, 2002). Put differently, whereas students are trusted and respected as responsible learners of mathematics and as constructors of their own meaningful knowledge, the teacher assumes the role of a facilitator who assists students' learning by observing, listening, questioning and challenging their inquiries.

Learning through active participation also carries an important social dimension. For as Fosnot (2005) claims, learners can negotiate meanings when they engage in cooperative social learning activities. This social constructivist dimension is based on the understanding that students should be active participants in their own learning by communicating and exchanging ideas with the teacher and other students. Learning is believed to occur by being part of and interacting within a social environment. Discussion with peers can assist learning as students develop conceptual understanding when they articulate their thoughts and points of view, when they learn to listen to others and when they ask questions (Orton & Frobisher, 1996). It follows that learning opportunities arise when student contributions are encouraged and when discussions invite students to challenge, argue and offer explanations.

Defining Tasks as Investigations

Mathematical tasks are an important vehicle through which classroom instruction can enhance students' learning. The tasks teachers present students with and the way in which students negotiate mathematical meaning by working on the set tasks largely determine students' classroom experiences and their learning of mathematics (Hiebert, et al., 1997; Shimizu et al., 2010). As Doyle (1983, p. 161) argues, tasks "influence learners by directing their attention to particular aspects of content and by specifying ways of processing information" and are "defined by the answers students are required to produce and the routes that can be used to obtain these answers". Hence, a key decision for teachers lies in their choice of tasks (Sullivan, 2011). In my action research study, I had chosen investigative tasks. The selected investigations required students to participate actively in their learning as a way of constructing mathematical knowledge within a social setting.

In this paper, an investigative task or investigation is defined as an inquiry into a mathematical situation presented by the teacher, but which can also initiate from a statement or a question posed by a student (Greenes, 1996). The topic of the investigation could arise from real-life or from a mathematically designed problem (e.g., *Investigate the sum of angles in polygons*). Again, an investigation can be a very open exploration or it can take the form of a more structured task that guides the learner into discovering mathematics (Yeo & Yeap, 2010).

Investigations offer opportunities for students to be more active in their learning. When students engage with such work they are involved in processes of exploration and explanation (Skovsmose, 2001). Students are expected to engage with finding ways to unravel the task assigned and be able to justify their work by presenting their method/s to the whole class. In the process, students engage in thinking critically, learning to ask, sharing ideas and communicating mathematically.

A Framework for Reflection on Design and Implementation

Integrating investigations takes into consideration how different mathematical processes and strategies can be embedded within the core topics which make up the content of a mathematics curriculum (Frobisher, 1994). Consequently, in planning and designing my 18 investigative tasks, I set out to achieve two aims. The tasks had to: (i) be related to the mathematics content prescribed in the Form Four syllabus; and (ii) engage students in mathematical inquiry, that is, thinking about, developing, using and making sense of mathematics (Breen & O'Shea, 2010). While a number of the selected investigative tasks integrated mathematical topics in order to allow students to experience different areas of mathematics, other tasks were intended mainly to help consolidate mathematical concepts and skills. Furthermore, in the belief that tasks should provoke curiosity in and be meaningful to students, the selection and creation of tasks and resource materials took into account the students' interests, mathematical ability and needs. However, when I came to setting future goals I took into consideration the responses/feedback that the students had provided during the study.

Within this conceptual framework, investigative tasks were classified according to the mathematics embedded within the activity, the degree of structure/guidance provided to students and the time devoted for students' activity. The main reason behind the decision to use investigations of varying levels was to smooth the transition for students from working on traditional exercises to engaging in more challenging tasks (Orton & Frobisher, 1996). Moreover, the spread of tasks along this classification helped to gradually introduce students to the cognitive processes of making and doing mathematics. In other words, the different levels offered graded entry points for students to familiarise themselves with the social experiences of mathematical inquiry, discussion and communication.

At the basic level, the investigations were *structured* tasks that lead students to mathematical discoveries. The given instructions guided students, who worked individually or in pairs, to use particular pre-determined mathematical concepts and apply them to arrive at a solution. At the next level, the investigations were *semi-structured*. This meant that they were either less structured or students were initially given some guidance in their work but were then free to explore and engage with the task using their own conceptual mathematical understanding and reasoning. Believing that learners benefit from discussing ideas and solutions when working on these more challenging tasks, the students were instructed to work in small groups of two or three. At the third and higher level, the students encountered *unstructured* investigations that were more process-oriented activities. These required students to investigate the problem posed or the situation presented in as many different ways as they wished and through different methods. These investigations placed greater demands on students to think through a solution, to make inferences and to test their own conjectures. As this type of investigation required students to challenge, argue about and justify their reasoning, the *unstructured* investigations were set as a group activity involving between three to four students.

Other than the level of structure, the investigations were also classified along the three 'reality levels' identified by Skovsmose (2001). Skovsmose sees mathematical investigations as a landscape that ranges across three levels of real-life contexts. These are: (i) *pure mathematics* which simply involves working with numbers or geometric figures; (ii) *semi-reality* which refers to an everyday-life problem that is rendered artificial as it is tackled in a classroom situation where variables can be controlled; and (iii) *real-life* situations where students are directly involved in carrying out the exercise in the actual setting.

Combining these two classifications, I came up with a rubric consisting of nine different types of investigations. Initially, the main purpose was to produce a template along which I could select and position the investigations (see Table 1). The matrix was eventually also useful in exploring how tasks were enacted in class and whether the students' actual engagement shifted the nature of tasks along the rubric.

INVESTIGATION	STRUCTURED	SEMI-STRUCTURED	UNSTRUCTURED
PURE MATHEMATICS	Type 1	Type 4	Type 7
SEMI-REALITY	Type 2	Type 5	Type 8
REAL-LIFE	Type 3	Type 6	Type 9

Table 1: The nine types of investigations

Investigations of types 1, 2 and 3 were *structured* tasks that varied according to the level of reality involved. While 'type 1' tasks resembled typical traditional exercises that are similar to those found in mathematics textbooks, 'type 2' and 'type 3' tasks were situated in a context of more practical mathematical experiences. Investigations of types 4, 5 and 6 were *semi-structured* tasks that again varied from a purely mathematical context to a real-life situation. The unstructured nature of investigations of types 7, 8 and 9, which again differed by context, placed the greatest cognitive demands on students as they were presented as more 'open-investigations'.

Selecting and Planning Investigations within Hypothetical Learning Trajectories

My quest to select tasks with goals in mind (Hiebert et al., 1997) involved thinking about how investigations would provoke inquiry and stimulate learning. As Simon and Tzur (2004, p. 93) argue:

> The tasks are selected based on hypotheses about the learning process; the hypothesis of the learning process is based on the tasks involved.

Along these lines, the day-to-day classroom experiences were vital in informing future planning that sought to integrate the learning goals with the trajectory of students' mathematical thinking and learning. This reflective process included the notion of 'hypothetical learning trajectory' (HLT) through which the teacher considers the learning goals, the instructional activities, and the thinking and learning in which the students might engage (Simon, 1995). A key aspect of this learning trajectory concerns a prediction of how students' thinking will evolve as they participate in the instructional activities. Actually, in classifying the tasks, I always started off with a HLT based on my expectations about students' explorations in learning – indicating the hypothetical learning trajectories from the syllabus (see Table 2). However, the actual learning trajectory cannot be known in advance as it depends on how the teacher and the students enact the tasks throughout the implementation process (see Stein et al., 2000). This eventuality of having students' task-inquiry leading to different learning trajectories than those anticipated by the teacher is indicated in Table 2, which shows a segment of my *task-delineated* scheme. More precisely, in my study, this occurrence rested upon how students actually went about working on the task and, in particular, their decisions regarding the use of the resources provided. This realization is crucial for the success of my current collaborative project. For I have come to appreciate that designing and implementing tasks requires teacher-decisions not only the number of resources, but also on the type and range of resources to be made available to students. The possibility of differentiating the resources that accompany tasks presents an opportunity for teachers to assign the same tasks to students of different abilities (discussed further in the following section).

WEEK	INVESTIGATION TITLE	TASK RESOURCES	HYPOTHETICAL LEARNING TRAJECTORY	STUDENTS' ENACTED LEARNING TRAJECTORY
6	The Netball Court	• Measuring tape • Squared paper • 1 cm grid	• Measuring lengths • Ratio notation • Scale drawing • Areas of rectangles, circles & semi-circles	• Measuring lengths • Mean, mode, median • Rounding numbers • Ratio notation • Scale drawing
7	Web Patterns	• Handout • Graph paper • Calculator	• Plotting coordinates • Finding areas of right-angled triangles • Sequences	• Plotting coordinates • Areas of triangles • Pythagoras' theorem • Sequences & n^{th} term

Table 2: A section showing how the scheme of work evolved

The scheme of work thus becomes a crucial reflective document for the teacher. By being responsive to students' inquiries, the teacher can occasionally fine-tune it as he or she re-plans and re-designs tasks and instructional practices. It is worth mentioning here that tasks might shift horizontally along the rubric presented in Table 1. In my case, a shift from structured to semi-structured to unstructured (or vice-versa) may results from the way students engage with the task.

Incorporating Students' Perspective in Design and Implementation

Reported and used extensively in research literature is the Mathematical Tasks Framework developed from the QUASAR (Quantitative Understandings: Amplifying Student Achievement and Reasoning) project team (Stein et al., 2000). This framework defines mathematical tasks as they unfold from design into implementation. In the framework outlined, mathematical tasks pass through three distinct yet related phases: as written by curriculum developers, as presented by the teacher in class, and as negotiated by students during classroom instruction. The framework is also useful in studying changes in task features and cognitive demands as instruction passes between any two successive phases.

As one might understand, depending on their beliefs, attitudes and experience, mathematics teachers are likely to implement the same task in different ways. For example, during the task presentation phase, different teachers may provide different kinds of instruction (information, guidelines and hints) to students about the task. I would argue that providing instruction towards the process rather than the product of students' inquiry is less likely to influence or modify the cognitive demands within the task. During this phase, teachers also tend to attribute titles to the tasks they assign. Reflecting on an incident from my research, I have come to understand that the task title might convey meaning to students about the kind of mathematical content involved in their investigation. Perhaps this focus indirectly provides unwarranted closure to the activity, thus directing students' attention to specific lines of inquiry and possibly shifting the intended 'open' nature of the task. I believe that presenting 'open' titles, such as, 'Investigate Right-Angled Triangles' rather than 'Investigate Pythagoras' Theorem' may offer more 'open' learning opportunities for students. In this case, a few students who attended private lessons already knew the theorem whereas others seemed puzzled by the title. These students, hence, offered resistance and lacked engagement. Apparently, the kind of title presented altered students' learning dispositions since they had 'good' reasons not to investigate.

During the presentation phase, teachers usually also provide students with resource materials to support task inquiry. Teachers might adapt tasks for students of lower ability by opting to provide more appropriate resource materials possibly without unduly reducing the cognitive demands within the task. As in my current collaborative project referred to earlier, teachers are investigating the possibility of designing and implementing inquiry-based learning tasks, on a weekly basis, along a whole scholastic year. These tasks are accompanied by a range of resource materials (including worksheets and instruments) with the intention to cater for students in our ability classes. For example, for the task 'Classifying Triangles' students in a high ability class are provided with a worksheet, scissors and glue, while students in a lower ability class are provided with cut-out triangles, glue, protractor and a ruler. Although students in the two classes are expected to classify different triangles, the ones in the lower ability class are provided with additional material to support their mathematical inquiry. Within this design principle, the 'process help' provided by a range of resources makes it possible for teachers to adapt tasks for all students. This prospect also offers students multiple entry points in engaging with the task. I therefore contend that, within a research task design project, incorporating a range of resources may be crucial in minimizing the gap between the intended and the enacted activity and in engaging all students in cognitively demanding mathematical inquiry.

Students may nevertheless interpret and negotiate tasks in ways that may be different from those intended by the teacher – the process of inquiry may either be undermined or sustained/improved during instruction. Occurrence of the latter trait is more likely to manifest itself in environments where students become truly responsible autonomous learners working within social norms of collaborative learning. When students become independent self-regulated learners, their activity might generate different mathematical trajectories to those intended by the teacher. The 'young mathematician' might be confident enough to choose what resources to use and to consider different lines of inquiry. Alternatively, as a number of studies show, teachers have a propensity in reducing the demand level of the task related to classroom norms, task conditions, and teachers' and students' dispositions (Henningsen & Stein, 1997). For example, Desforges and Cockburn (1987) report this tendency occurring when students are struggling or when they give up. Likewise, during my action research, I faced situations where a particular student continuously asked for help when she fell behind compared to the others. During our interview, this student reported that I usually made it easier for her to finish tasks. My task enactment evidently avoided the student's frustration and speeded up task completion. Bound by time constraints, I occasionally reverted to forms of telling to move on the class to the next phase, namely, the whole-class presentation and discussion. Such teacher interventions direct students' attention towards priority in task completion – at times at the expense of more meaningful and deeper understanding. With hindsight I argue that help directed towards the outcome of the activity hinders the process of learning by investigation and inhibits students' cognitive development.

Concluding Remarks

Implementing investigative tasks essentially involves four-phases: tasks as planned and designed by the teacher; tasks as presented to the students; tasks as negotiated by students; and tasks as concluded by the students and the teacher (Ponte, Segurado & Oliveira, 2003). The study of these inter-related phases has informed practitioners, researchers and academics about design issues originating from students' engagement with a range of tasks (see Stein et al., 2000). Of foremost importance here is the formulation of principles, guidelines and reflective frameworks for effectively exploring how students negotiate and engage with tasks. As I see it, this awareness is based on a reflective framework that includes: (i) a clear understanding of the purpose of tasks – one that resonates well with active learning; (ii) a classification within which different types of content-related tasks could be fitted – this alignment would be useful in examining how different tasks are designed and presented by the teacher and eventually enacted by students in class; (iii) a *task-delineated* scheme that defines the teacher's hypothetical learning trajectories and accounts for students' learning trajectories – this scheme hence also defines the didactical approach undertaken by teachers in class; and (iv) an account of the range and type of resources provided to students in order to render tasks more accessible to all students – this also provides a basis for investigating how students make use of the resource materials supplied and consequently on the ensuing activity.

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