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Robust ANOVA Tests for Exponentially Distributed Data

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Abstract

The presence of variance heterogeneity and non-normality many data may frequently invalidate the use of the analysis of variance (ANOVA) F test in one-way independent groups designs. In this study, we consider the violations of the assumptions in the one-way ANOVA. Robust Welch (RW), Brown-Forsythe (RBF) and also Modified Brown-Forsythe (MBF) test statistics for the one-way ANOVA under heteroscedasticity are developed for the non-normal data assumed to have exponential distribution with outliers. In order to get these robust test statistics, robust estimators of mean and variance of exponential distribution are obtained by using robust estimators of this distribution parameter. The influence function and breakdown point of these estimators are obtained to show that these estimators are robust. We consider balanced and unbalanced sample sizes with homogeneous and heterogeneous variances. The simulations results show up that the proposed robust tests have a good performance.

Keywords: ANOVA, Exponential Distribution, Welch, Brown-Forsythe, Modified Brown-Forsythe, Quantile, Quantile Least Square.

1. INTRODUCTION

The probability density function of exponential distribution has the form

$$f(x)=\frac{1}{\lambda}\exp\left(-\left(x/\lambda\right)\right),$$

where $x,\lambda>0$. Exponential cumulative distribution function is given by

$$F(x)=1\text{-}\exp\left(\text{-}\left(x/\lambda\right)\right)$$

The mean and the variance of an exponentially distributed random variable X with parameter λ is given by

$$E(X)=\lambda \qquad V(X)=\lambda^2.$$

Denote $\hat{q}_\alpha$, quantile of the observations $x_1,x_2,...,x_n$. Replacing the theoretical quantiles $q_\alpha=\lambda\left[-\log\left(1-\alpha\right)\right]^{1/\beta}$ with the corresponding empirical quantiles $\hat{q}_\alpha$, yields the following quantile-estimator of distribution parameter

$$\hat{\lambda}_Q=\frac{\hat{q}_\alpha}{\left(-\log\left(1-\alpha\right)\right)}$$

The Exponential distribution function in $q_\alpha=\lambda\left[-\log\left(1-\alpha\right)\right]$ can be transformed into a straight line by a double logarithmic transformation, with parameter that are non-linear functions of λ:

$$\log q_\alpha=\log\lambda+\frac{1}{\beta}\log\left(-\log\left(1-\alpha\right)\right)$$

a linear with intercept $b_0=\log\lambda$ and slope $b_1=1$. The quantile least squares estimator are:

$$\hat{b}_0 = \frac{1}{\tilde{n}} \sum_{i=\lfloor \alpha n \rfloor + 1}^{n-\lfloor \alpha n \rfloor} y_i - \frac{1}{\tilde{n}} \sum_{i=\lfloor \alpha n \rfloor + 1}^{n-\lfloor \alpha n \rfloor} z_i$$

where $y_i = \log \hat{q}_{i/(n+1)}$, $z_i = \log\left(-\log\left(1-\alpha_i\right)\right)$, $0 < \alpha < 0.5$, $\tilde{n} = n - 2\lfloor \alpha n \rfloor$.
Considering this estimator of the regression parameters, we can easily obtain the estimators of the Exponential distribution parameter $\hat{\lambda}_{QLS} = \exp\left(\hat{b}_{0QLS}\right)$.

Robust Welch (RW), Brown-Forsythe (RBF) and also Modified Brown-Forsythe (MBF) test statistics for the one-way ANOVA under heteroscedasticity are developed for the exponential distribution with outliers. In order to get these robust test statistics, robust estimators of mean and variance of exponential distribution are obtained by using the estimators based on quantile and quantile least square of this distribution parameter. The influence function and breakdown point of these estimators are obtained for robustness of estimator**s.**

2. CONCLUSIONS

By using the estimators based on quantile and quantile least square, the equations of the robust test statistics RW, RBF and RGBF are improved for the data are assumed to be exponential distribution. In the simulation study, using different experimental designs, type I error risks of the improved robust test statistics for the exponential distribution are obtained with respect to 2 robust methods for k=4,8 groups. We consider balanced and unbalanced sample sizes with homogeneous and heterogeneous variances. The simulations results show up that the proposed robust tests have a good performance.

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