

# **Parametric Study on the Seismic Vulnerability of Unreinforced Masonry Cross-Vaults**

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Dissertation submitted to the Faculty for the Built Environment, University of Malta  
in part fulfilment of the requirements for the attainment of the degree of  
Master of Engineering (Structural Engineering)

July 2024



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*To my family.*

*“All we have to decide is what to do with the time that is given to us.”*

J.R.R. Tolkien in *Lord of the Rings: The Fellowship of the Ring* (1954)



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## Acknowledgements

I would like to primarily express my sincere gratitude to my academic supervisor, Prof. Alex Torpiano B.E.&A. (Hons), M.Sc. (Lond), Ph.D. (Bath), D.I.C., M.I.Struct.E., C.Eng., Eur.Ing., Perit for his expert knowledge and advice on this topic and for his continuous guidance and unwavering support throughout this dissertation.

I would also like to extend my deepest appreciation to my academic co-supervisor Prof. Marc Bonello B.E.&A. (Hons)(Melit.), M.Sc. (Lond.), Ph.D. (Lond.), D.I.C., M.A.S.C.E., M.S.E.I., Eur.Ing. (FEANI), Perit for his invaluable insight, guidance and support throughout the course of this study.

Many thanks also go to the Faculty for the Built Environment for kindly providing a license of the HiStrA software for this dissertation. Additionally, I would also like to thank the technical team at Gruppo Sismica for their assistance and technical support during the construction of the numerical models.

I am deeply indebted to my family, whose encouragement, support and understanding provided the foundation for my academic pursuits. Special thanks also go to my friends for their support during my studies. Finally, words cannot express my gratitude to Julia, whose love and patience has been with me every step of the way.

I take full responsibility for any shortcomings in this dissertation.



## Abstract

This dissertation focuses on the seismic response of the cross-vault, a curved masonry structure typically roofing over impressive heritage buildings, and which extensively varies in form and geometry.

This study intends to investigate, within the local context, the seismic vulnerability of unreinforced masonry cross-vaults of varying geometry and constructional configurations. A numerical macro-modelling strategy is adopted, and through parametric variation, a set of fourteen numerical models were generated in the HiStrA software. The parameters primarily simulate cross-vaults of different aspect ratios on plan, the addition of diagonal ribs and cross-vaults in aggregate. The seismic performance of the models is assessed through a non-linear static pushover analysis, providing results in the form of capacity curves, observable deformation and lateral deflections.

On account of a comparative analysis, the findings established that a rectangular cross-vault characterised by an elliptical arch geometry, is stiffer and offers more resistance to sway as opposed to a square-plan vault. The results additionally shed light on the differences and similarities between local and global cross-vault behaviour. In all instances, the observed damage clearly indicates the four-hinge formation as the central collapse mechanism, and the results conclude that despite being complex three-dimensional curved structures, cross-vault behaviour is remarkably similar to that of a two-dimensional masonry arch. Apart from confirming that masonry compressive strength is irrelevant in lateral stability of cross-vaults, the findings suggest the notion of quasi-ductile behaviour, despite masonry being a brittle material.

Thus, this study concludes that geometric and constructional parameters evidently influence the response of unreinforced masonry cross-vaults when subjected to seismic loading. In this regard, the understanding of the effect of different forms and global configurations could translate into the identification of suitable retrofitting techniques as a means to safeguard heritage buildings in the event of an earthquake.

**Keywords** Cross-vault, unreinforced masonry, seismic vulnerability, pushover analysis, HiStrA.





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## List of Abbreviations

|                   |                                          |
|-------------------|------------------------------------------|
| <b>C1 – CV1</b>   | Configuration 1 – Cross-Vault 1          |
| <b>C1 – CV1-R</b> | Configuration 1 – Cross-Vault 1 – Ribbed |
| <b>C1 – CV2</b>   | Configuration 1 – Cross-Vault 2          |
| <b>C1 – CV2-R</b> | Configuration 1 – Cross-Vault 1 – Ribbed |
| <b>C1 – CV3</b>   | Configuration 1 – Cross-Vault 3          |
| <b>C1 – CV3-R</b> | Configuration 1 – Cross-Vault 1 – Ribbed |
| <b>C2 – CV1</b>   | Configuration 2 – Cross-Vault 1          |
| <b>C2 – CV1-R</b> | Configuration 2 – Cross-Vault 1 - Ribbed |
| <b>CV1</b>        | Cross-Vault 1                            |
| <b>CV1-R</b>      | Cross-Vault 1 – Ribbed                   |
| <b>CV2</b>        | Cross-Vault 2                            |
| <b>CV2-R</b>      | Cross-Vault 2 – Ribbed                   |
| <b>CV3</b>        | Cross-Vault 3                            |
| <b>CV3-R</b>      | Cross-Vault 3 – Ribbed                   |
| <b>DL</b>         | Damage Limitation                        |
| <b>DMEM</b>       | Discrete Macro-Element Method            |
| <b>DoF</b>        | Degree of Freedom                        |
| <b>EC8</b>        | Eurocode 8                               |
| <b>ELF</b>        | Equivalent Lateral Force                 |
| <b>EMS</b>        | European Macro-seismic Scale             |
| <b>EN1996-1</b>   | Eurocode 6 – Part 1                      |
| <b>EN1998-1</b>   | Eurocode 8 – Part 1                      |
| <b>EN1998-3</b>   | Eurocode 8 – Part 3                      |
| <b>FoS</b>        | Factor of Safety                         |
| <b>LF</b>         | Load Factor                              |
| <b>LS</b>         | Limit State                              |
| <b>MDoF</b>       | Multiple Degrees of Freedom              |
| <b>NC</b>         | Near Collapse                            |

|             |                                         |
|-------------|-----------------------------------------|
| <b>NTC</b>  | <i>Norme Technice delle Costruzione</i> |
| <b>PGA</b>  | Peak Ground Acceleration                |
| <b>PGD</b>  | Peak Ground Displacement                |
| <b>PGV</b>  | Peak Ground Velocity                    |
| <b>SD</b>   | Significant Damage                      |
| <b>SDoF</b> | Single Degree of Freedom                |
| <b>URM</b>  | Unreinforced Masonry                    |

## Glossary

|                                       |                                                                                           |
|---------------------------------------|-------------------------------------------------------------------------------------------|
| <b>Abutment</b>                       | Any structure which receives lateral thrust from an arch or vault.                        |
| <b>Aggregate</b>                      | A set of separate elements which together form a whole.                                   |
| <b>Aisle</b>                          | A passage located parallel to the nave, generally on both sides.                          |
| <b>Barrel Vault</b>                   | A type of vault; composed of a single arch extruded longitudinally to a specified length. |
| <b>Centring</b>                       | Falsework supporting the voussoirs during construction of arches or vaults.               |
| <b>Conoid</b>                         | A trapezoidal pocket between two adjacent bays of vaulting.                               |
| <b>Crown</b>                          | The highest point of the curvature of an arch or vault.                                   |
| <b>Damping ratio</b>                  | A dimensionless measure describing the decay of oscillations in a damped system.          |
| <b>Decentring</b>                     | The process of striking/removing the falsework after construction of an arch or vault.    |
| <b>Degree of freedom</b>              | A possible displacement of a structure.                                                   |
| <b>Epicentre</b>                      | The vertical projection of the hypocentre of an earthquake on the earth's surface.        |
| <b>Extrados</b>                       | The upper surface of an arch or vault.                                                    |
| <b>Focal depth</b>                    | The distance between the hypocentre and epicentre.                                        |
| <b>Hypocentre</b>                     | The point on the fault from which the earthquake originates.                              |
| <b>Intrados</b>                       | The inner surface of an arch or vault.                                                    |
| <b>Keystone</b>                       | The uppermost stone of an arch or vault.                                                  |
| <b>Natural frequency of vibration</b> | The number of cycles completed in one second during vibration.                            |
| <b>Natural period of vibration</b>    | The time taken for one whole cycle of free vibration.                                     |
| <b>Nave</b>                           | The middle portion of a church.                                                           |
| <b>Springing</b>                      | The point of departure of the curve of an arch or vault.                                  |
| <b>Thrust</b>                         | A lateral force in an arch or vault pushing downwards and outwards.                       |
| <b>Voussoir</b>                       | A wedge-shaped masonry unit used to construct arches or vaults.                           |
| <b>Web</b>                            | The masonry making up the vault.                                                          |



# 1 Introduction

## 1.1 Context

The cross-vault embodies a remarkable structural roofing typology; constructed in masonry and characterised by a true mastery of materials and craftsmanship. Universally considered as architectural heritage, cross-vaults can be found scattered all over Europe, most notably in France, England and Italy, in magnificent cathedrals, palaces and halls, and in many instances, intricately adorned with vibrant frescoes. Originating from the simpler Romanesque barrel-vault, the cross-vault reached its epitome in Gothic architecture, embellished with detailed sculptural pieces fashioned out of masonry.

Throughout its history, the Maltese Islands have had multiple occupants who undeniably left their architectural mark on the local built environment. With the arrival of the Order of the Knights of St. John of Malta in 1530, came construction techniques that had not yet been implemented locally, one of which was the masonry cross-vault. Locally one may find a few captivating examples of globigerina limestone (local stone) cross-vaults.

The following is a non-exhaustive list of local, built cultural heritage and religious buildings that feature masonry cross-vaults (refer to Appendix A for images):

- I. Cloister at the Convent of the Augustinian Friars, Rabat, Malta (c. 1530)
- II. St Anne's Church, in Fort St Angelo, Birgu (c. 1531)
- III. Santa Scholastica, Birgu (c. 1532)
- IV. Grandmaster's Staircase before the entrance of the Castellania, in Fort St Angelo, Birgu (c. 1533)
- V. Cloister at the Convent of the Franciscan Minor Observants, Rabat, Malta (c. 1533)
- VI. Inquisitor's Palace, Birgu (c. 1535)
- VII. The Lodge, Cospicua (c. 1580)
- VIII. Law Courts Annex, Valletta (c. 1600)
- IX. Old church of St Catherine, Żejtun (c. 1603)
- X. Spinola Palace (currently the Lombard Bank Annex building), Valletta (c. 1633)
- XI. Old church of St Nicholas, Siggiewi (only ruins remain and is now an archaeological attraction – c. 1637)
- XII. Magazine Ward, at the Mediterranean Conference Centre (also known as the *Sacra Infermeria*), Valletta (c. 1666)
- XIII. Carmelite Priory and Crypt, Mdina (c. 1670)
- XIV. Cloister at the Addolorata Cemetery (in the neo-Gothic style), Paola (1868)
- XV. Basilica of the National Shrine of the Blessed Virgin of Ta' Pinu, Għarb, Gozo (1932)



In structural design, it is an established fact that masonry does not perform well when subjected to lateral loading caused by an earthquake, and it has been the case where masonry heritage buildings have collapsed as a result. There are examples of this in the Maltese Islands, which although are located in a low-to-moderate seismic hazard zone, have been subject to earthquake tremors, and subsequent building damage in the past. On the basis of historic records, the 1693 Sicily earthquake emerges as the strongest seismic event the Maltese Islands have ever experienced, with severe damages documented, particularly the collapse of most of Mdina Cathedral, which had to be eventually completely rebuilt. Nonetheless, most of the local population discount the possibility of a damaging earthquake, despite it being evident that one may occur.

Seismic events are known to have catastrophic effects, contributing to loss of lives, infrastructural damage and destruction of the built environment. In present times, strict guidelines shape our designs to ensure stable and safe structures in an increased effort to minimise tragic incidents. This was not the case, however, for medieval builders who historically followed ancient rules of thumb and learned from past examples to construct buildings that have survived for centuries, and which we can still appreciate to this day. Although masonry structures have been studied extensively in the literature, the seismic vulnerability of cross-vault roofs within a building remains a largely unexplored subject.

## **1.2 Scope and Focus of the Study**

The scope of this study is to assess the influence of geometrical and constructional parameters on the performance of unreinforced masonry (URM) cross-vaults subjected to horizontal seismic loads. This study additionally aims to consider the divergence between global and local cross-vault behaviour.

The response of the different cross-vault geometries and configurations will be compared and analysed to investigate the behavioural changes of the masonry shells and the corresponding seismic vulnerability considering the Limit States (LS) as stipulated by Eurocode 8: Part 3 (EN1998-3). Thus, the focus of this study and the research questions to be addressed are:

- Q1**    *How does a masonry cross-vault perform when subjected to horizontal loading as a result of earthquake tremors?*
- Q2**    *How do different geometrical and constructional parameters influence the response of a masonry cross-vault subjected to seismic loading?*

### 1.3 Methodology and Outcomes

Since the nature of this research study is parametric, the methodology postulates a set of variable parameters to be identified and tested, such that comparisons could be extracted. In the context of this study, the parameters are related to the geometry and construction of masonry cross-vaults which were derived from the assessed literature.

The parametric variation resulted in the creation of a set of fourteen numerical models. To conduct the seismic analysis, the methodology adopts HiStrA, a macro-element numerical modelling software in which a non-linear static pushover analysis was performed for each numerical model. The output of the software is a series of pushover curves from which the behaviour of the various cross-vault models and their relative seismic vulnerability is evaluated. In light of the adopted methodology, the objectives of this dissertation are: (i) to numerically model typical masonry cross-vaults having different geometries and configurations; and (ii) to subject the test models to the design earthquake corresponding to the Maltese islands.

The following outcomes were expected to be fulfilled from the conducted seismic analyses:

- Assess the seismic vulnerability of unreinforced masonry cross-vaults through numerical modelling.
- Vary, investigate and compare different geometrical and constructional parameters of cross-vaults and their relative effect on seismic performance.
- Study the behaviour of cross-vaults in aggregate and its effect with respect to global and local behaviour.
- Outline general masonry shell behaviour, damage and collapse mechanisms and structural stability issues.

### 1.4 Dissertation Structure

With this being the introductory chapter, the remainder of this dissertation is structured into five chapters.

**Chapter 2** presents a brief overview of multiple aspects of unreinforced masonry cross-vaults. The reviewed literature, most of which is based on historical accounts, provides insight into the following sub-topics: (i) the historical development of the cross-vault typology; (ii) form and geometry; (iii) the construction of cross-vaults; and (iv) their static structural behaviour.

**Chapter 3** provides, through additional reviewed literature, further information in relation to: (i) seismicity and structural dynamics; (ii) masonry material properties and masonry shell behaviour; (iii) the factors affecting cross-vault performance subject to seismic load; and (iv) a brief description of numerical modelling techniques which are deemed suitable for masonry.

The information gathered in these two chapters provide the necessary tools to undertake the research methodology, and permits the results to be properly analysed, interpreted and hence discussed.

**Chapter 4** outlines the implementation of the research methodology with respect to the adopted numerical modelling strategy and the non-linear static pushover analysis using the HiStrA software. Apart from providing a brief explanation of the software's theoretical background and limitations, this chapter also describes: (i) the development of the test models and their geometry; (ii) the nature of the load application as per guidelines in Eurocode 8 (EC8); and (iii) the assumed analysis settings.

**Chapter 5** compiles the main results from the executed pushover analyses in the form of pushover curves, deformation images and lateral deflection values. This chapter discusses the findings by drawing out comparisons and observations of the behaviour of the various tested models with regards to: (i) the effect of the geometrical and constructional parameters; (ii) the response of the masonry shell in terms of hinge formation, cracking and collapse; and (iii) the seismic vulnerability criteria via a limit state analysis.

**Chapter 6** provides a synopsis of the principal outputs derived from this dissertation, suggests recommendations for future research work relevant to this field of study, and acknowledges other limitations of this research study.

## 2 Literature Review

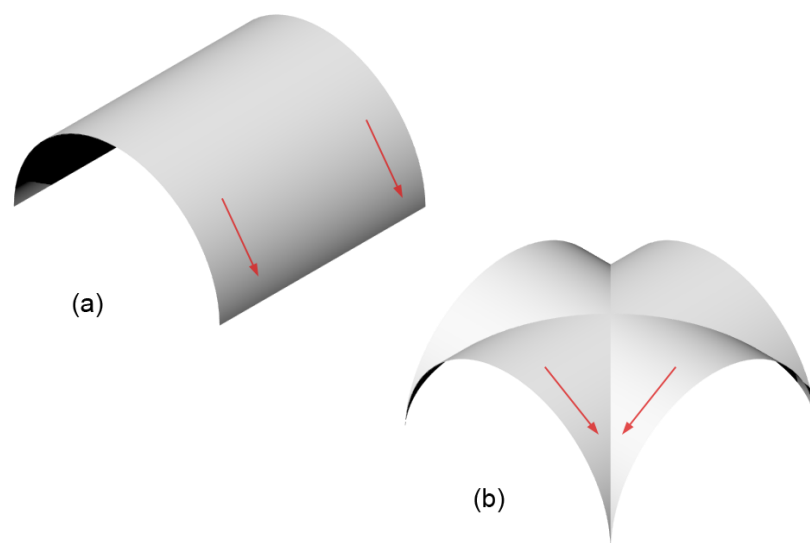
### 2.1 Introduction

The literature review provides a concise summary of the development of masonry cross-vaults. Without delving into detail, it explores the historical evolution of the cross-vault through its geometrical proportions, form, construction method, and then proceeds with the understanding of the static structural behaviour. This equips the author with the basic necessary context needed for the undertaken research study.

### 2.2 Historical Evolution – From the simple groin-vault to the Gothic ribbed vault

A dominant feature of many medieval cathedrals, the cross-vault was developed from the orthogonal intersection of two semi-cylindrical barrel-vaults<sup>1</sup> of equal span (Como, 2017). The diagonals formed by this intersection are referred to as groins, which form a cross when viewed from below (or above), hence the name (refer to Figure 2.1).

Also known as the groin-vault, its first appearance in Europe dates to the Roman Empire (Como, 2017). However, it is only at the beginning of the Middle Ages, that cross-vault construction resurfaced in Europe culminating in the elaborate design of the Gothic style (Gaetani et al., 2016).



**Figure 2.1** – Form and thrust: (a) Semi-cylindrical barrel vault; (b) Groin vault (Author, 2024).

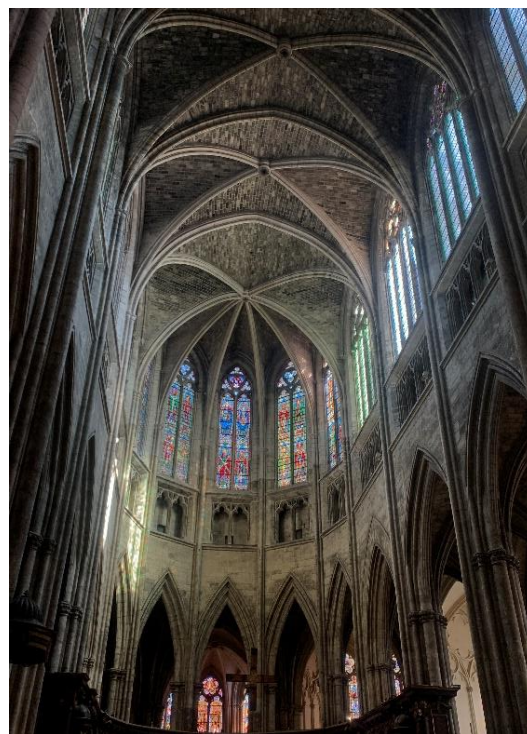
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<sup>1</sup> Barrel Vault – A type of vault; composed of a single arch extruded longitudinally to a specified length.

During the early Medieval era, the heavy Romanesque style dominated the design of cathedrals, with the typical barrel-vault used as the roofing over the nave<sup>2</sup> and the aisles<sup>3</sup>. The large lateral thrust<sup>4</sup> produced by these vaults meant that the opening of windows within the flanking walls was minimal (Fitchen, 1981). The subsequent re-emergence of the simple groin-vault, by nature of its geometry, meant that the Gothic notions of maximum height and maximum light could be achieved (Fitchen, 1981). The ability to support the self-weight and thrust of the cross-vault on pillars at the four corners implied that the lateral walls need no longer be structural, and therefore replaced with windows (Gaetani et al., 2016).

The configuration of the groin-vault as described above results in a square bay of vaulting, with semi-elliptical diagonals of intersection (Gaetani et al., 2016). However, the realisation of the ability to define the shape of the diagonal groins, with the vault's webs<sup>5</sup> following this pre-defined arch, sparked the development of the ribbed vault during the 11<sup>th</sup> Century (refer to Figure 2.2).

As a result of the possibility of having semi-circular diagonals, the crown<sup>6</sup> of the vault could rise higher than the lateral arches, forming double-curved webs which increased the stability of the vault during and after construction (Gaetani et al., 2016). The master builders realised



**Figure 2.2** – Ribbed vault roof of the St. Andre Cathedral, Bordeaux, France. Taken by Author (2023).

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<sup>2</sup> Nave – The middle portion of a church.

<sup>3</sup> Aisle – A passage located parallel to the nave, generally on both sides.

<sup>4</sup> Thrust – A lateral force in an arch or vault pushing downwards and outwards.

<sup>5</sup> Web – The masonry making up the vault.

<sup>6</sup> Crown (or key) – The highest point of the curvature of an arch or vault.

that a more economical design could be achieved as a benefit of this efficient load path, resulting in thin masonry shells of less self-weight, and hence less thrust (Como, 2017; Fitchen, 1981).

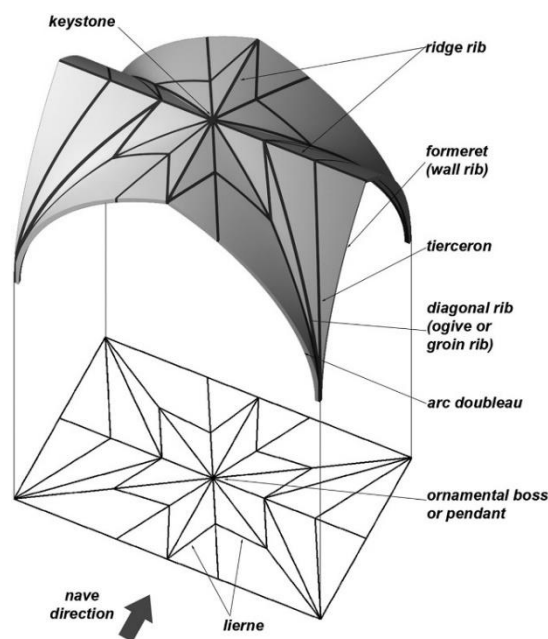
Up to this point, the cross-vault formed square bays of vaulting. The subsequent adoption and use of the pointed arch in the 12<sup>th</sup> century permitted the lateral arches to be as high as the crown of the vault, resulting in the Gothic system of rectangular quadripartite vaults. With this design approach mastered, more intricate vault designs emerged through the multiplication of diagonal ribs, resulting in tierceron, stellar, and fan vaulting systems (Gaetani et al., 2016).

## 2.3 Form and Geometry

### 2.3.1 Main Elements

Insofar as the main elements of a cross-vault are concerned, the fundamental difference between the groin and ribbed vault geometry is the addition of the rib at the intrados<sup>7</sup> of the vault. The groin-vault is composed of the transverse arches, and the masonry shell webbing, with the diagonal groins being formed by the masonry itself.

For the ribbed vault as seen in Figure 2.3, the configuration is essentially the same. The now-visible diagonal ribs meet at the crown forming the keystone<sup>8</sup>. The transverse arches (*arc doubleau* and *formeret*) meet the diagonal ribs and converge at the springing<sup>9</sup> to form the *tas-*



**Figure 2.3** – Main elements of a quadripartite bay of cross-vault. Reproduced from Gaetani et al. (2016).

<sup>7</sup> Intrados – The inner surface of an arch or vault.

<sup>8</sup> Keystone – The uppermost stone of an arch or vault.

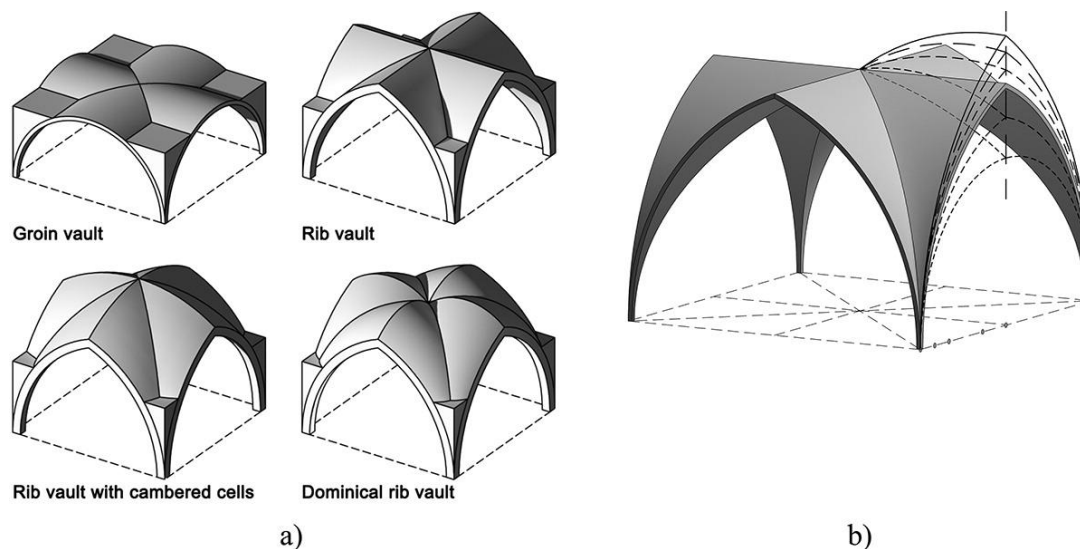
<sup>9</sup> Springing – The point of departure of the curve of an arch or vault.

*de-charge* (refer to Figure 2.5). This masonry element combines the ribs from each lateral arch and the diagonals into a unified base (Fitchen, 1981). Although not forming part of the cross-vault geometry, it is worth noting that buttressing, generally in the form of flying buttresses, was required to transfer the vault thrust at the springing to the ground.

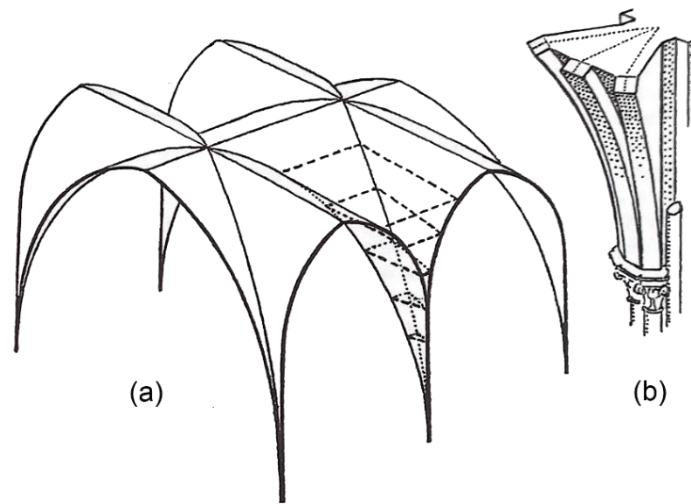
### 2.3.2 Geometrical Aspects

A large variety of quadripartite cross-vaults can be formed from the different geometrical bounds. As illustrated in Figure 2.4, it is evident that the height of the crown and of the lateral arches need not necessarily be at the same level. The height of the lateral arches in relation to the crown could be varied to create masonry webbing of varying degrees of curvature.

The arches forming the vault were semicircular or pointed. As explained previously, the employment of the pointed arch was constructional; whose unrealised benefit at the time was that it causes less thrust than a semicircular arch. Stiling was another common practice whereby the curve of the arch (whether semi-circular or pointed) began at a higher level than the arch of the diagonal and lateral ribs. Not only did this accentuate the effect of maximum height without the arch being highly pointed (as this would have been problematic), but this also allowed the vault thrusts to be directed vertically at the springing (Fitchen, 1981).



**Figure 2.4** – Cross-vault geometry: (a) Classification of type; (b) Varying geometry by lateral arch height. Reproduced from Gaetani et al. (2016).



**Figure 2.5** – (a) Stilted cross-vault bays showing conoid; (b) Point of convergence of all three ribs (*tas-de-charge*). Reproduced from Fitchen (1981).

### 2.3.3 Rules of Thumb

From the numerous surviving examples of medieval architecture, it is evident that the builders were able to design and construct safe buildings. They did so by following a '*scientia*', an empirical knowledge attained from the careful observation of earlier construction, allowing them to establish rules of thumb (Huerta, Santiago, 2006).

These geometrical and proportional rules were documented, such as Fr. Derand's Rule (also known as Blondel's Rule) which made it plausible to obtain through geometrical construction the width of the abutment<sup>10</sup> for an arch of a vault. This rule however did not include the thickness of the arch, which was later comprehended by Hernan Ruiz el Joven's rule, making it the first account to also consider the weight of the vault (Gaetani et al., 2016). Since these two rules considered solely the abutment width, other geometrical rules were formulated to obtain proportions of the principal members of the cross-vault. Without delving into the details of the various treatises which present these rules, to which the reader is referred to Gaetani et al. (2016) and Huerta, S. (2006), Table 1.1 provides the relevant rules in relation to a cross-vault incorporated in the design of a church.

<sup>10</sup> Abutment – Any structure which receives lateral thrust from an arch or vault.



| Elements                    | References                               | Dimensions                                            |
|-----------------------------|------------------------------------------|-------------------------------------------------------|
| <i>Arc doubleau</i>         | German late Gothic                       | $s_n/22.5$ (central nave)<br>$s_n/30$ (aisle)         |
|                             | Rodrigo Gil de Hontanon (1550)           | $s/20$                                                |
|                             | Hernan Ruiz el Joven (1560)              | Min. $s/10$                                           |
| Diagonal rib (central nave) | German late Gothic                       | Height: $s_n/30$<br>Thickness: $s_n/60$               |
|                             | Rodrigo Gil de Hontanon (1550)           | Height: $s/24$                                        |
| <i>Arc formeret</i>         | Rodrigo Gil de Hontanon (1550)           | Height: $s/30$                                        |
| Web thickness               | Como (2013)                              | $s/50 - s/75$ (Gothic vaults)                         |
| Infill                      | Friar Lorenzo de San Nicolas (1639-1664) | Up to one third of vault height (rounded cross vault) |
|                             | Hernan Ruiz el Joven (1560)              | Up to half of the height of <i>arc doubleau</i>       |

**Table 2.1** – Rules of thumb for the relevant elements of a cross-vault, where  $s_n$  is the span of the central nave and  $s$  is the span of the element considered. Adapted from Gaetani et al. (2016).

## 2.4 Cross-Vault Constructional Aspects

### 2.4.1 Stereotomy

Stereotomy is the technique whereby stones are cut and shaped into blockwork (Fitchen, 1981). Since cross vault geometry is defined by boundary arches, this implies that stones are predominantly shaped into voussoirs<sup>11</sup>. However, this was not always the case, where in early Gothic vaults the ribs were composed of cubic blocks whose arching effect was achieved through the mortar bedding (Wendland, D. et al., 2021). The webs of the vault were typically rubble or ashlar and coursed in either a radial or diagonal pattern (Theodossopoulos & Sinha, 2004). Heyman (1995) asserts that irrespective of the web pattern, the constructed shell bears the core structural action.

The dominant geometrical complication in the stereotomy of cross-vaults was the groin at the intersection of the webs. For a simple groin-vault, each groin voussoir would be of a different shape, and hence difficult to execute. In practice, the voussoirs were irregular, and the inconsistencies corrected by mortar joints (Fitchen, 1981). Out of the many functions of the rib as explained in the following subsection, these irregularities at the groin are covered in a

<sup>11</sup> Voussoir – A wedge-shaped masonry unit used to construct arches or vaults.

ribbed vault, whilst making the detail at the web joint easier to construct (Fitchen, 1981; Theodossopoulos & Sinha, 2004).

#### **2.4.2 The Role of the Rib**

The introduction of the rib at the intrados of a cross-vault unequivocally transpired in an easier and more economised construction sequence. The diagonal ribs at the groins function as a permanent formwork for the laying down of the intermediate masonry webs.

Whether the rib in a masonry cross-vault plays a structural role has been the subject of an extensive debate but is beyond the scope of this research and for further reading one may consult Huerta, S. (2009) and Lengyel & Bagi (2015). From his equilibrium analysis, Heyman (1977) concludes that large stresses are concentrated at the junction between two shells and hence that the diagonal ribs inadvertently reinforce and support the entirety of the vault. However, he recognises that a rib-less vault (groin-vault) of sufficient thickness possesses '*invisible ribs*' that carry the vault as part of the overall shell structure (Heyman, 1977). Fitchen (1981) also highlights that without the rib it would practically have been impossible to construct rectangular bays of vaulting due to the discrepancies in the coursing of the webs at the groin, which would have been arduous to fabricate.

#### **2.4.3 Construction Process**

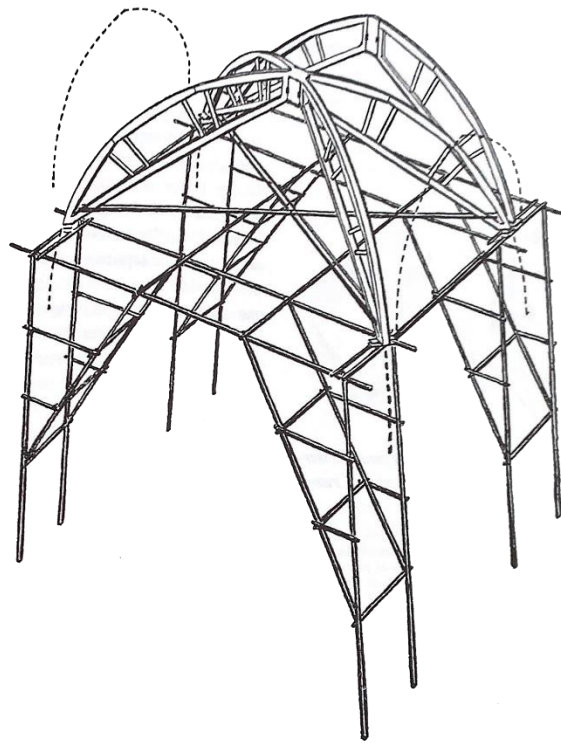
Contingent upon certain geometrical and constructional aspects of the cross-vault, a definite level of economy could be accomplished during the construction process. One possibility was to construct the cross-vault without formwork by laying self-supporting diagonal courses in an archwise fashion (Wendland, David, 2007). Adopting the radial direction, the vault would not be stable until completion and thus required intermediate points of support in the form of timber formwork (Alforno et al., 2022).

For the ribbed vault, a minimum amount of formwork was necessary (refer to Figure 2.6). A platform would first have to be built at the springing level from where the centring<sup>12</sup> of the arches could be erected. The centring would only be required at the position of the diagonal ribs. By applying mortar to the masonry blockwork, the webbing would then progress from the perimeter to the central part of the web towards the crown (Huerta, Santiago, 2006). The webs could be built entirely without formwork, presumably using a stone-weighted rope device (Fitchen, 1981). An alternative approach was to arrange the masonry in a diagonal as explained above with tilted bed-joint planes, achieving stability through the webs' double-curvature (Alforno et al., 2022; Wendland, David, 2007).

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<sup>12</sup> Centring – Falsework supporting the voussoirs during construction of arches or vaults.

The extrados<sup>13</sup> of the vault would then be protected by a coat of mortar, with the conoids<sup>14</sup> being customarily infilled with light rubble to prevent deformation (Fitchen, 1981; Theodossopoulos & Sinha, 2004). This infill within the vaulting conoid was necessary for equilibrium of the system, allowing the transfer of the thrust to the supporting flying buttresses (Heyman, 1977). In-depth accounts of the construction of cross-vaults can be found in Fitchen (1981) and Wendland, David (2007).



**Figure 2.6** – Sketch of lightest timber falsework required for the construction of a ribbed vault. Reproduced from Fitchen (1981).

## 2.5 Static Structural Behaviour of Masonry Cross-Vaults

### 2.5.1 Previous Methods of Analysis

Throughout history, scholars and engineers have tried to comprehend the complexity of the equilibrium of cross-vaults. From the 18<sup>th</sup> century onwards, concepts such as the slicing technique (graphical method), elastic theory, and the line of thrust dictated such attempts (Gaetani et al., 2016; Huerta, Santiago, 2009).

Viollet le Duc in his treatise discusses the robustness of Gothic structures in terms of elasticity, constantly referring to the ability of these structures to crack, deform and adapt in safe equilibrium (Huerta, S., 2021). However, le Duc's perception of elasticity was incompatible

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<sup>13</sup> Extrados – The upper surface of an arch or vault.

<sup>14</sup> Conoid – A trapezoidal pocket between two adjacent bays of vaulting.

and debated by many. Nonetheless, this was accepted up until the 20<sup>th</sup> century when modern limit analysis gained ground, transforming the realm of structural mechanics. For a comprehensive description of the development of modern limit analysis the reader is referred to Huerta, Santiago (2009).

### **2.5.2 Modern Limit Analysis**

Heyman structured the preceding observations into the modern theoretical framework of limit (or plastic) analysis (Huerta, Santiago, 2009). He realised that the analysis of all masonry structures, including masonry cross-vaults, falls within the remit of this theory, whereby safety is a condition of geometry provided that the material satisfies four assumptions (Heyman, 1995):

1. Masonry has infinite compressive strength.
2. Masonry has no tensile strength.
3. Sliding failure does not occur in masonry.
4. Masonry has infinite abutment strength.

Such assumptions were already accepted in the 19<sup>th</sup> century and used for analysis of masonry arches (Huerta, Santiago, 2009). Heyman (1995) implies that a structure is safe if a set of internal forces (or line of thrust) which is in equilibrium with the loads, lie within the geometry. It was also recognised that such set of forces is not unique, and through the equilibrium approach, any possible solution may be considered without knowing the compatibility conditions if it conforms with the yield condition (Heyman, 1995). This means that the thrust line could change due to changes in the boundary conditions (movement or settlement of abutments), but it would never exceed the masonry geometry. Accordingly, it is pertinent to recognise that safety in any masonry structure is an issue of stability rather than material strength, which is dependent on the geometry.

### **2.5.3 Cross-vault Cracking and Collapse**

For a masonry cross-vault carrying symmetrical dead load, cracking is a typical feature that may be observed. As in an arch, the masonry of the vault works in compression, and upon decentring<sup>15</sup> the vault thrust pushes outwards on the abutments. The potential movement of the abutments would have to be accommodated by an increase in span, resulting in cracks within the joints of the masonry.

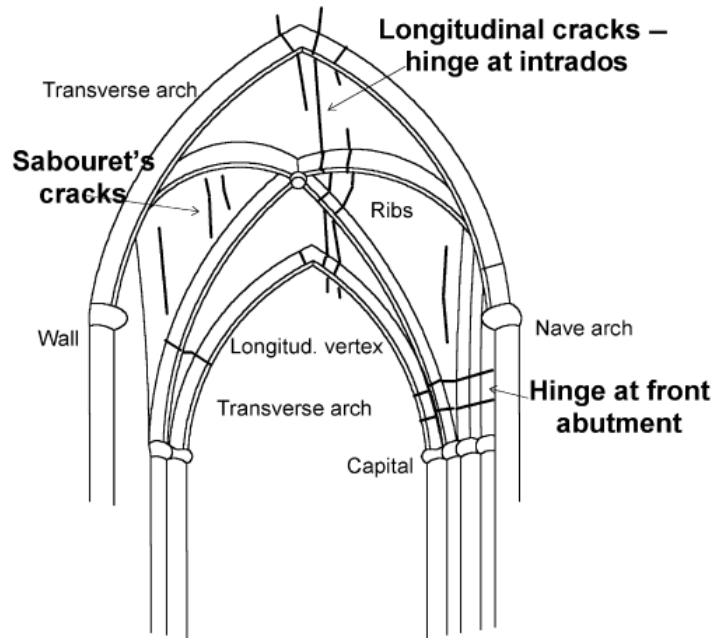
From arch theory, for which the reader is referred to Heyman (1995), such rotation at the joints is assimilated to a hinge. The arch would be stable and statically determinate with the

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<sup>15</sup> Decentring – The process of striking/removing the falsework after construction of an arch or vault.

formation of three hinges at the intrados and extrados of the arch. A four-hinge mechanism would render the arch or vault unstable and collapses (Heyman, 1995).

Within a cross-vault, these hinging cracks are typically visible at the crown. Additionally, other cracks referred to as *Sabouret's cracks* form parallel to the lateral arch ribs, resulting in a separation of the webs (refer to Figure 2.7). The latter may isolate the edges of the vault from the rest of the web (Heyman, 1995).



**Figure 2.7** – Cracking pattern of a masonry cross-vault. Reproduced from Theodossopoulos et al. (2004).

Hence, the main failure type for a cross-vault is due to lateral instability owing to the hinge formation. From research carried out by Theodossopoulos et al. (2002), it was also discovered that unsuitable design interventions can also result in damage and collapse. In relation to lateral instability, natural phenomena such as earthquakes impose unsymmetrical lateral loading which can lead to failure (Bertolesi et al., 2019; Heyman, 1995). This study focuses on such type of failure, which will be discussed in further detail in the next chapter.

## 2.6 Conclusion

In conclusion, the literature review briefly encapsulates the fundamental aspects of masonry cross-vaults. It provides the author with essential knowledge on the geometric and constructive parameters required to engage in this parametric study. Furthermore, a basic understanding of the static structural behaviour of cross-vaults is obtained, which is requisite in understanding the more complex dynamic behaviour.

## 3 Seismic Vulnerability of Unreinforced Masonry Shells

### 3.1 Introduction

Cross-vaults are a type of horizontal roofing structure and can be classified as masonry shells. Their structural response under dynamic action is substantially complex requiring careful analysis. Unreinforced masonry structures are considerably vulnerable to seismic action and this chapter thus investigates the general concepts of seismicity, masonry shell behaviour, and also the seismic vulnerability of cross-vaults and the possible methods to analyse this behaviour.

### 3.2 Seismicity & Structural Dynamics

#### 3.2.1 Seismic Action

An earthquake is a natural phenomenon generated from the sudden sliding between tectonic plate boundaries at faults within the earth's crust (Elghazouli, 2009). During this process, the high stresses created by the fault rupture exceed the strength of the material and releases the stored elastic strain energy in the form of seismic waves, hence causing an earthquake – generally defined by the elastic rebound theory (Kramer, 1996).

The location of the earthquake depends on the position of the hypocentre<sup>16</sup> below the earth's surface and specified geographically by the coordinates of the epicentre<sup>17</sup> and focal depth<sup>18</sup>. A greater focal depth means that seismic waves must travel larger distances, and, in the process, become attenuated (Kramer, 1996). Hence, the location affects seismic wave intensity and earthquake magnitude. The magnitude is related to the released energy and describes the size and intensity of an earthquake in terms of scale, such as the Richter Scale or the more recent European Macro-seismic Scale (EMS) (Tomazevic, 1999).

#### 3.2.2 Seismic Hazard and Risk

Seismic hazard and risk are two important and distinct terms in seismic structural design. Seismic hazards pertain to the possible detrimental effects of an earthquake such as ground shaking, while seismic risk concerns the potentiality of damage as a consequence of the interplay between seismic hazard, vulnerability and exposure (Wang, 2013).

Humans and the built environment are exposed to earthquake effects depending on location. Vulnerability refers directly to structures and their sensitivity to earthquake damage and is a factor which may be moderated through structural engineering and design (Elghazouli, 2009). The same cannot be said for historical existing buildings such as those containing cross-vaults,

---

<sup>16</sup> Hypocentre – The point on the fault from which the earthquake originates.

<sup>17</sup> Epicentre – The vertical projection of the hypocentre of an earthquake on the earth's surface.

<sup>18</sup> Focal depth – The distance between the hypocentre and epicentre.

since they were only designed for gravitational loads. These buildings are commonly vulnerable to seismic effects and in such cases, retrofitting and strengthening techniques may be employed to reduce vulnerability.

An earthquake event does not occur frequently and hence, seismic risk is articulated on what is known as a '*return period*' which refers to the '*average time interval between two events of a given (or greater) magnitude*'. The relationship between the probability of occurrence with return period can hence function as a tool for assessing and estimating occurrence probability for a longer return period (Tomazevic, 1999).

### **3.2.3 Ground Motion and Seismic Load Transfer**

When an earthquake occurs, seismic waves induce ground motion, which are translated into external forces on a structure. Ground motion is described in horizontal (x and y) and vertical (z) components, and accelerographs record relevant data related to seismic ground motion (Elghazouli, 2009). The external forces are contingent upon both the seismic wave intensity and the structural response to ground motion. Although the earthquake intensity proves useful in damage assessment for buildings subjected to similar intensities, it is not suitable as a design parameter (Tomazevic, 1999). For a singular earthquake there are multiple parameters which can be recorded, but for design load verification, the appropriate ground motion parameters to be used are the peak ground acceleration (PGA) and the elastic acceleration response spectrum. Through these parameters, earthquake actions can be expressed as equivalent static forces on a structure, providing the basis for seismic structural design (Elghazouli, 2009).

The PGA denotes the maximum ground acceleration, and hence the maximum force (as a result of the product of the mass and PGA) induced during a seismic event, and together with the peak ground velocity (PGV) and the peak ground displacement (PGD) define the amplitude of the seismic occurrence (refer to Figure 3.1). The elastic response spectra summarise the response of several structures with a Single Degree of Freedom<sup>19</sup> (SDoF) to a particular seismic event in the context of the natural period<sup>20</sup> and damping ratio<sup>21</sup> of the structure and hence describe the frequency content of an earthquake motion (Elghazouli, 2009).

In the design code Eurocode 8 (EC8), two response spectra are specified: Type 1 for high-seismicity areas and Type 2 for moderate-seismicity areas. The elastic spectra in EC8 assume a damping ratio of 5%, and for each type (1 or 2), five spectra are provided representing distinct soil classes (refer to Table 3.1 in EN1998-1) (Elghazouli, 2009). Ground type considerations in the response spectra are included to account for the effect of amplification of ground motion and duration of amplification due to local ground conditions on site. During

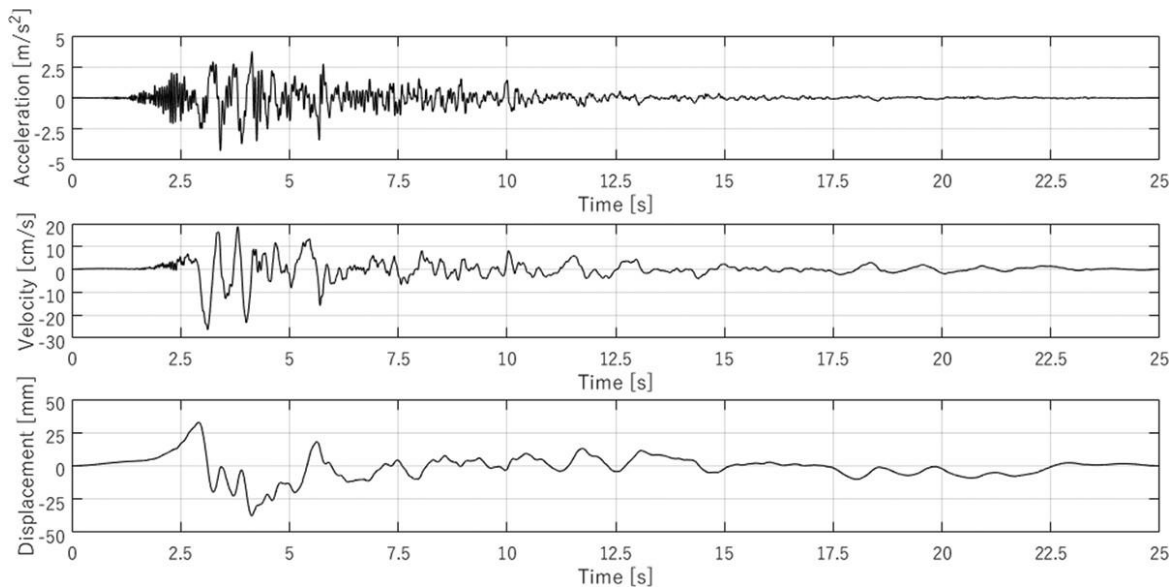
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<sup>19</sup> Degree of freedom – A possible displacement of a structure.

<sup>20</sup> Natural period of vibration – The time taken for one whole cycle of free vibration.

<sup>21</sup> Damping ratio – A dimensionless measure describing the decay of oscillations in a damped system.

a seismic event, the waves are dispersed within the ground material and are constantly reflected, refracted, and diffracted, with different ground materials (and their geo-mechanical properties) providing different levels of interference (Kramer, 1996).



**Figure 3.1** – Acceleration, Velocity and Displacement – time histories for the Aquila (2009) earthquake. Reproduced from Bianchini, Nicholetta (2024).

### 3.2.4 Structural Dynamics

The external forces (base shear,  $F_b$ ) induced in a structure due to ground motion provoke the structure to vibrate (forced vibration) and bring about displacements at the base which vary with time. Naturally this means that the structure possesses a natural frequency of vibration<sup>22</sup>, and as the structure oscillates, structural response can vary depending on stiffness, mass, and damping. For simplification purposes, sinusoidal oscillations against time are assumed to express complex ground motion. A devastating effect is when the ground motion frequency reaches the natural frequency of vibration of the structure, causing the structure to vibrate indefinitely – a phenomenon known as resonance. All structures possess some degree of damping and hence energy dissipation, and although this is typically minimal, it contributes to limit the amplitude of vibration (Tedesco et al., 1999).

A good understanding of SDoF systems is imperative in structural dynamics as generally, structures possess numerous degrees of freedom referred to as Multiple Degree of Freedom (MDoF) systems. This ultimately means that the motion of such a structure is described by the number of degrees of freedom it has. MDoF systems are complex to analyse, and it is common practice to describe their behaviour by discretising the system into connected nodes made of SDoF's, hence representing only one type of displacement (Tedesco et al., 1999).

In EC8, the total base shear (inertia force) is represented by the sum of equivalent static horizontal loads acting at each storey of the building in relation to storey heights, relative

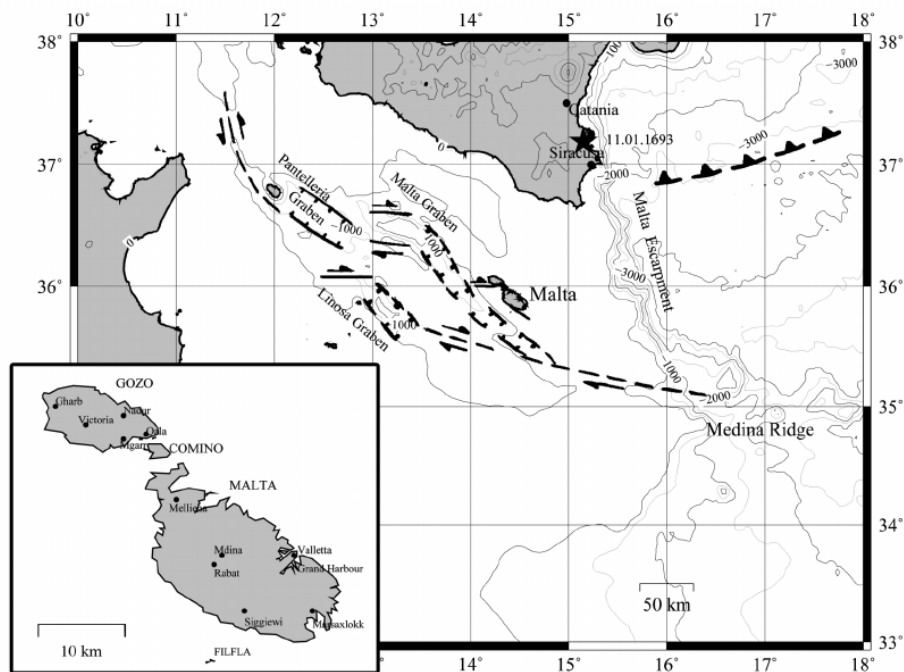
<sup>22</sup> Natural frequency of vibration – The number of cycles completed in one second during vibration.



stiffness, and the fundamental vibration mode shape of the structure. The computation of the base shear in this manner is the adopted approach of EC8 for structural design to earthquake resistance (Alexander, 2010), and this research study will follow EC8 procedures. Although EC8 guidelines are to be used for this study, mainly the respective limit states described in Chapter 4 are to be considered.

### 3.2.5 Local Seismicity

In the past and up until recently, the Maltese islands have been impacted by earthquake tremors. Galea (2007) provides a catalogue and description of past seismic occurrences which have been felt locally. Most notably, the 1693 earthquake which resulted in heavy damage and loss of buildings especially in Mdina and Rabat. From seismic hazard assessments (European Seismic Hazard Map), the Maltese islands are deemed to be an area of low-to-moderate seismic hazard, being positioned in the Sicily Channel on the moderately stable Pelagian Platform (refer to Figure 3.2). Effects have been felt locally with earthquake epicentres originating from the Sicily Channel and close to Sicily, and further away within the Hellenic Arc (D'Amico & Galea, 2013).



**Figure 3.2** – Map showing bathymetry and main tectonic features of Sicily Channel.  
Reproduced from Galea (2007).

The local building stock is predominantly in unreinforced masonry, and substantial damage to such buildings by even moderate ground accelerations should not be disregarded (D'Amico & Galea, 2013). From her research, Galea (2007) concludes that an earthquake of intensity VII-VIII (on the EMS), with a 475-year return period is sensible for the Maltese islands, and hence peak ground accelerations of 0.04 – 0.1g would be expected. This entails that EC8 seismic design rules are to be adopted for construction of buildings. However, local seismic awareness

is weak and up to this date, no local building regulation exists which demands the seismic design of buildings (Galea, 2007).

### **3.3 Unreinforced Masonry Shells**

#### **3.3.1 *Unreinforced Masonry***

Unreinforced masonry as a structural material has been used for thousands of years, and is comprised of the two main constituents, the masonry unit (rock, brick, etc.) and the mortar matrix, and whose interaction provides the structural behaviour (Heyman, 1995). Despite masonry being a heterogenous material, its mechanical properties are well-established in the literature, such as its compressive strength but limited tensile strength, non-linearity and anisotropy. Although the tensile strength of masonry is generally considered to be null, it still possesses marginal tensile capacity, and this fluctuates with orientation in relation to the joints, attributed to the anisotropic characteristic of masonry. Apart from that, when masonry is subjected to shearing, resistance is imparted by bond-shear strength and friction along the masonry-mortar joint (Torpiano, 1987). A concise review on masonry mechanical properties and behaviour can be found in Torpiano (1987), who also points out important considerations of masonry properties for masonry shell design.

#### **3.3.2 *Shell Theory and Masonry as a Material for Shells***

A shell refers to a structure characterised by a curved surface, whose behaviour differentiates if the shell has single or double curvature (Torpiano, 1987). Unlike other structures, shells are form-passive and do not adapt to changes in external loads, and hence the state of equilibrium could be permanently altered (Adriaenssens et al., 2014).

Structurally, apart from the structural action (internal forces) of normal beams or plates, a shell also utilises in-plane thrusts imparted by its curvature. This is referred to as shell action and these in-plane membrane stresses are the primary source of resistance to the applied loads on the shell (Heyman, 1977). However, deformations result in a diminished load-carrying ability and similarly to a column, the shell may buckle when compressive membrane stresses are present. Thus, apart from in-plane membrane action, a shell still requires bending stiffness to withstand buckling and deflections (Ugural, 1999). This description lightly touches on the mechanics of shells and for further reading, the reader should consult Adriaenssens et al. (2014), Calladine (2011), Heyman (1977) and Ugural (1999).

Stability in masonry is achieved through the compressive stress of the masonry units by gravity. Shells have long been built with masonry due to the compressive in-plane thrusts developed on their curved surface (Adriaenssens et al., 2014). However, for a masonry shell, its curved surface implies limited vertical compressive stress, derived by solely the self-weight of the masonry and if present, the weight of the infill on top. It is apparent that this lack of

compression causes vulnerability. Hence, masonry shells have reduced out-of-plane flexural resistance (Vintzileou et al., 2022).

As discussed in the previous chapter, the position of the thrust line within the geometry is a prerequisite for equilibrium in masonry arches. This is also true for vaults of single curvature, as the geometry of such structures is defined from the extrusion of a two-dimensional arch. Although having the possibility of being doubly curved, cross-vaults can be generally regarded as possessing webs of single curvature, and hence are stable if the line of pressure is contained within the geometry of the webs. When subjected to an earthquake, the thrust line within a particular cross-vault geometry is modified appreciably due to the induced seismic horizontal loads (Lagomarsino, 2006), further emphasizing the importance of comprehending seismic vulnerability of masonry shells.

### **3.4 Seismic Vulnerability of Cross-vaults**

Seismic ground motions induce an asymmetrical loading scenario on a vaulted structure, rendering the vaults vulnerable to lateral instability. The seismic vulnerability of cross-vaults and hence the type of damage incurred is affected by various factors as described below.

#### **3.4.1 Global vs Local Behaviour**

The cross-vault is but one element from a multitude of elements making up a structure and its seismic performance is hence relevant at both a local and global scale. Being a horizontal element, it acts as a diaphragm and when seismically loaded, considerable thrusts are potentially transferred onto the lateral supports (Alforno et al., 2022). Such thrusting action produces localised damage and failure mechanisms, possibly leading to loss of cultural heritage and lives. Consequentially, local failure has a major influence on the global seismic response of the whole building (Lagomarsino et al., 2022; Rossi et al., 2015).

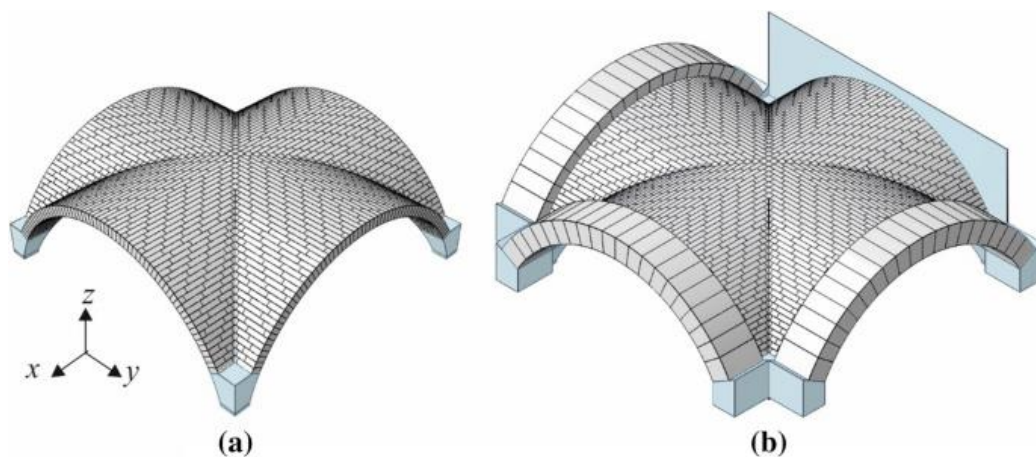
The global seismic vulnerability of a building can be obtained through an amalgamation of the separate response of various elements (Lagomarsino, 2006). The verification and integration of both global and local behaviour is central for seismic vulnerability studies (Lagomarsino et al., 2022). In this case, this study investigates mainly the local response of cross-vaults but also the partial global behaviour of cross-vaults in aggregate.

#### **3.4.2 Boundary Conditions**

The equilibrium of ‘incomplete’ shells of revolution (such as a cross-vault), is particularly sensitive to the edge conditions (Heyman, 1977). For a cross-vault, the boundary conditions represent the connection between the various structural elements. These generally take the form of lateral walls, arches, adjacent vaults, and pillars, serving as the supporting members. During a seismic event, the implication of different boundary conditions is various modes of behaviour and damage (Bianchini, Nicoletta et al., 2019; Gaetani et al., 2017).

By way of example, in a three-aisled church the boundary conditions of the nave cross-vaults differ from those of the aisles. Similarly, in an aggregate of cross-vaults an edge vault has different boundary conditions than an internal vault. Hence, different cross-vaults in their relative position within the global configuration may have symmetric or asymmetric boundary conditions, with the latter being more vulnerable (Bianchini, Nicoletta et al., 2019; Gaetani et al., 2017).

In a recently conducted study, Alforno et al. (2022) numerically tested under seismic load masonry groin vaults of different brick patterns and with two distinct boundary conditions (confinement and no confinement – refer to Figure 3.3). Despite not changing the collapse mechanism, the results show that the confinement condition substantially influences the resistance and stiffness of the groin-vault (Alforno et al., 2022).



**Figure 3.3** – Boundary conditions: (a) No confinement; (b) Rigid lateral arches and wall. Reproduced from Alforno et al. (2022).

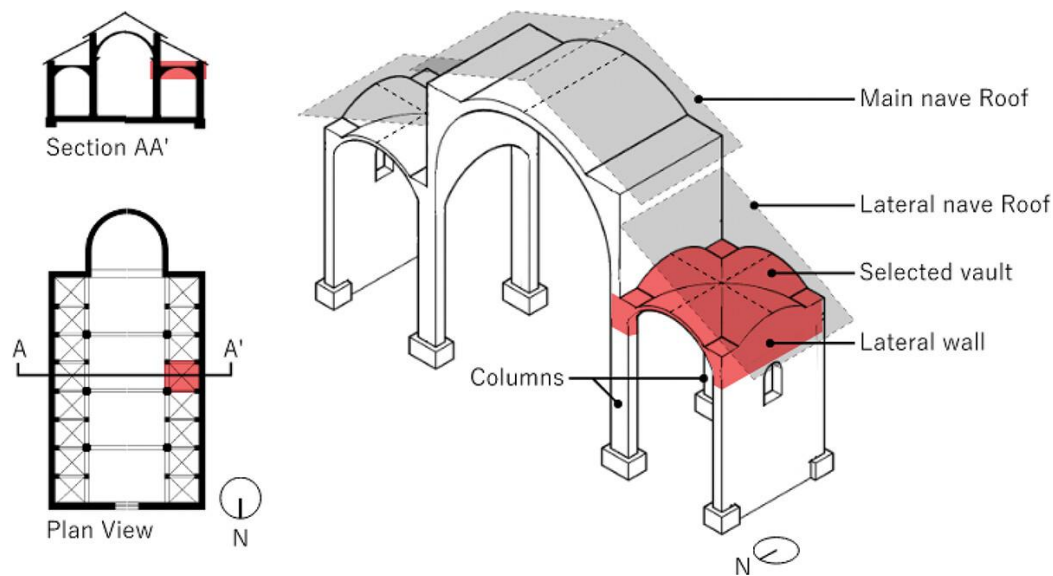
### 3.4.3 The Effect of Stiffness

The local stiffness of a cross-vault is clearly linked to its boundary conditions. For the three-aisled church example (refer to Figure 3.4), the asymmetric boundary condition accounts for a change in stiffness. In such a configuration, an aisle vault is on one side connected to the lateral wall and on the opposite side supported by the colonnade of the nave. Due to its structural and constructional nature, a column of the nave has a notably lower stiffness than the exterior wall. Consequently, with the occurrence of an earthquake, this change in stiffness develops a weak point within the system (Bianchini, Nicoletta et al., 2019; Carfagnini et al., 2018; Gaetani et al., 2017).

### 3.4.4 Seismic Damage and Collapse Mechanisms

Apart from the effect of edge conditions and stiffness, the type and extent of damage that a seismically loaded cross-vault endures is influenced by the construction, location (and hence ground conditions) and earthquake magnitude. Extensive research in the form of post-earthquake surveys commissioned by the Italian National Civil Protection Service documents

the reoccurring damages and cracking patterns (refer to Figure 3.5) in churches affected by recent earthquakes in Italy (Gaetani et al., 2017).

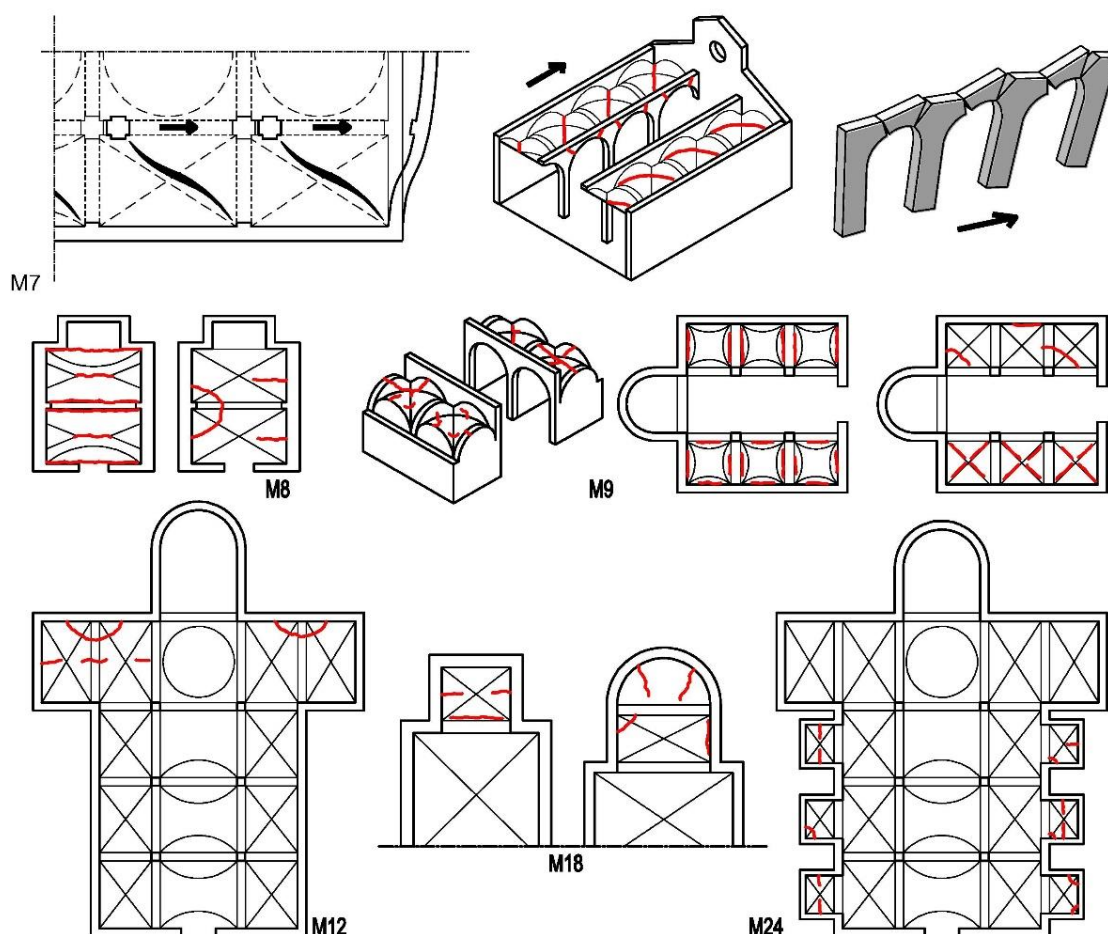


**Figure 3.4** – Sketch showing three-aisled church example and position of cross-vault in relation to boundary conditions and stiffness. Reproduced from Bianchini, Nicoletta (2024).

The two major causes of seismic damage in cross-vaults are displacements and shear-stress buildup due to dynamic and pseudo-static response of the vault. The former generally takes the form of rotations and translations, as a consequence of the relative movement of the supporting structural elements, whereas the latter is caused by the short uncontrolled vibration of the structure (Bianchini, Nicoletta et al., 2019; Carfagnini et al., 2018). Prior to cross-vault failure, it is generally the case that sizable vertical displacements rather than horizontal can be withstood (Bianchini, N. et al., 2022).

One common mechanism is an in-plane shear distortion resulting from the previously mentioned cross-vault having asymmetric boundary conditions and hence, elements of different stiffnesses triggering a longitudinal translation at the edges (Bianchini, Nicoletta et al., 2019). Independent from the cause of damage, the masonry of the cross-vault responds to the vibrational motion by cracking along its mortar joints. Typically observed patterns include diagonal cracking along the groins and cracks parallel to the lateral sides (Gaetani et al., 2017). Such cracking is inevitable, as during the movement the vault experiences a sudden and continuous change in its span causing the masonry to spread to accommodate this positional shift. It may be the case where the induced horizontal displacement exceeds the lateral capacity of the vault, rendering it laterally unstable and collapses partly or in whole. Regarding cross-vault failure, masonry compressive strength is rarely significant and fundamentally pertains to horizontal load and instability (Gaetani et al., 2017).

The brief discussion in the previous chapter on the formation of plastic hinges in masonry arches and vaults is relevant in comprehending the seismic collapse mechanism of cross-vaults. The possibility of the alternate four-hinge mechanism depends on the capacity of the



**Figure 3.5** – Sketches of observed cracking and damage mechanisms in cross-vaults. Reproduced from Gaetani et al. (2017).

vault (which in turn depends on the geometrical and constructive features) and the magnitude of the horizontal load and displacement (which depends on the earthquake magnitude, position, etc.). Upon the formation of the asymmetric four-hinge mechanism, the cracks would be propagated along the webs of the vault and eventually the masonry punches outwards, typically observed to commence from the crown. This phenomenon proceeds in this manner since at the crown the self-weight of the vault is minimal and hence the insufficient compression (mentioned in subsection 3.3.2) contributes to the instability (Rossi et al., 2015).

### 3.5 Methods of Analysis

The structural performance of masonry structures has been studied extensively in the literature. In spite of this, only a limited number of references have focused on the behaviour of masonry cross-vaults. The vast majority of these studies target the static behaviour, and research on the seismic vulnerability of such structures has gained ground only recently (Bertolesi et al., 2019).

Different seismic analysis procedures may be carried out for structures depending on the level of accuracy and are tabulated in Table 3.1. In the identified literature, various techniques for

assessing the seismic performance of masonry cross-vaults are implemented and can be summarised in two categories: physical laboratory testing and numerical modelling (using specialised software).

|                                                       |                                                                                                        |
|-------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| <b>Linear Static Analysis</b>                         | - Model building as equivalent SDOF system<br>- Translating seismic action as equivalent lateral force |
| <b>Linear Dynamic Analysis</b>                        | - Model building as equivalent MDOF system<br>- Seismic input modelled via a model spectral analysis   |
| <b>Non-linear Static Analysis (Pushover Analysis)</b> | - Application of monotonically increasing lateral force                                                |
| <b>Non-linear Dynamic Analysis</b>                    | - Seismic input through time-history analysis                                                          |

**Table 3.1** – Possible approaches for seismic analysis. Adapted from Brincat (2020).

A list of the relevant research in the current literature concerning strictly the seismic response of URM cross-vaults is presented in Table 3.2 below. Some of these studies will provide the basis for the comparison of results in Chapter 5 of this dissertation.

| Reference                       | Testing                        | Vault Type   | Geometric Parameters                           | Boundary Conditions   | Applied Loading                                |
|---------------------------------|--------------------------------|--------------|------------------------------------------------|-----------------------|------------------------------------------------|
| <i>Rossi et al. (2015)</i>      | Shake Table                    | Groin Vault  | Square plan (no variations)                    | Asymmetric condition  | Horizontal displacements                       |
| <i>Milani et al. (2016)</i>     | Tilting Plane Test & FEM model | Groin Vault  | Square plan (no variations)                    | Fixed/Rigid abutments | Ground acceleration & horizontal displacements |
| <i>Gaetani et al. (2017)</i>    | FEM model                      | Groin Vault  | Varying angle of embrace, diameter & t/d ratio | Asymmetric condition  | Self-weight & horizontal inertial forces       |
| <i>Carfagnini et al. (2018)</i> | Shake Table & FEM model        | Ribbed Vault | Square plan (no variations)                    | Asymmetric condition  | Shear displacements                            |

**Table 3.2** – List of current identified literature focused on the seismic performance of masonry cross-vaults, providing the testing methods and inputs used (Author, 2024).

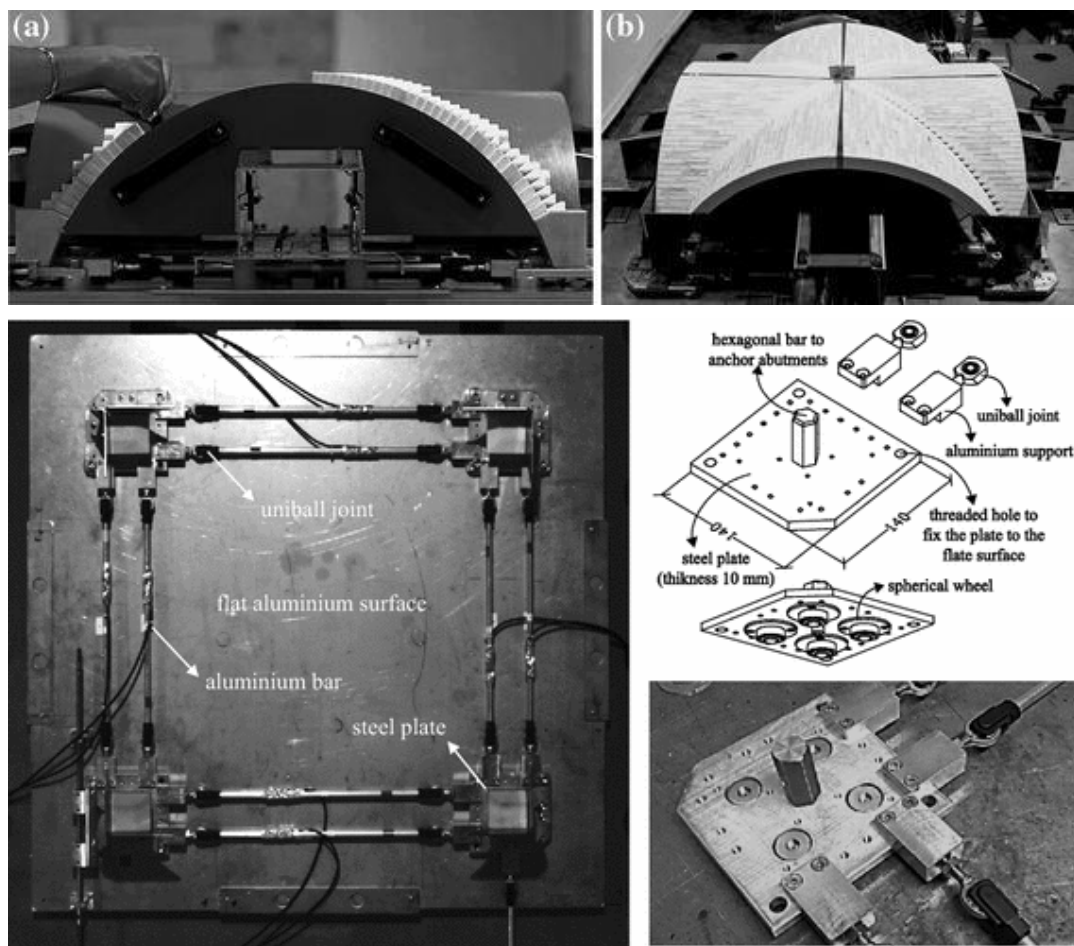
| Reference                       | Testing                       | Vault Type                 | Geometric Parameters                                   | Boundary Conditions                           | Applied Loading                             |
|---------------------------------|-------------------------------|----------------------------|--------------------------------------------------------|-----------------------------------------------|---------------------------------------------|
| <i>Lengyel et al. (2018)</i>    | DEM model                     | Groin Vault & Ribbed Vault | Same vault with addition of diagonal ribs              | Fixed/Rigid abutments                         | Self-weight & ground acceleration           |
| <i>Bianchini et al. (2019)</i>  | Shake Table & FEM model       | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Self-weight & ground acceleration           |
| <i>Alforno et al. (2022)</i>    | FEM model                     | Groin Vault                | (1) Radial brick pattern<br>(2) Diagonal brick pattern | (A) No confinement<br>(B) Rigid arches & wall | Ground acceleration at different directions |
| <i>Bianchini et al. (2022)</i>  | Shake Table & FEM model       | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Ground acceleration                         |
| <i>Vintzeleou et al. (2022)</i> | Shake Table                   | Groin Vault (UR & R)       | Square plan (no variations)                            | Asymmetric condition                          | Ground acceleration                         |
| <i>Chacara et al. (2023)</i>    | DMEM model                    | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Ground acc. (Aquila, 2009)                  |
| <i>Ferrante et al. (2023)</i>   | Hybrid FEM – DEM model        | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Ground acc. (Aquila, 2009)                  |
| <i>Gagliardo et al. (2023)</i>  | 3D Rigid block limit analysis | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Ground acc. (Aquila, 2009)                  |
| <i>Pellegrini, D. (2023)</i>    | FEM model                     | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Ground acc. (Aquila, 2009)                  |
| <i>Bianchini et al. (2024)</i>  | Shake Table                   | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Ground acc. (Aquila, 2009)                  |
| <i>Pintucchi et al. (2024)</i>  | FEM shell model               | Groin Vault                | Square plan (no variations)                            | Asymmetric condition                          | Ground acc. (Aquila, 2009)                  |

**Table 3.2 (continued)** – List of current identified literature focused on the seismic performance of masonry cross-vaults, providing the testing methods and inputs used (Author, 2024).



### 3.5.1 Shake-Table Testing

The physical testing generally comprises a scale model of a masonry cross-vault built on a shake-table apparatus and equipped with sensitive sensors (linear variable differential transformers – LVDT's) which measure the displacements of the model (refer to Figure 3.6). The shake-table not only provides the base for the model but also the application of the simulated ground motion or displacements. Such a test requires considerable time, effort, and cost with regards to the construction of the model. Since this would require loading the model to collapse, carrying out tests in succession may not be a viable option. Apart from this, the obtained results may not be wholly representative of the true behaviour due to potential human error during construction. In this research study, physical testing was excluded due to logistical and time constraints.



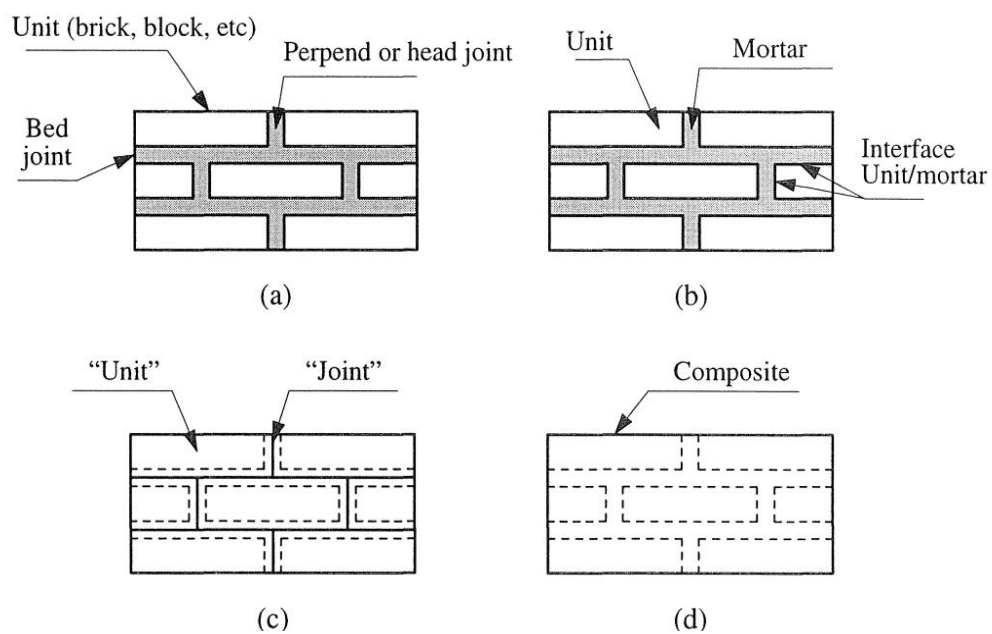
**Figure 3.6** – The physical (3D-printed) model testing setup on the shake table apparatus. Reproduced from Rossi et al. (2015).

### 3.5.2 Numerical Modelling

The advances made in numerical modelling strategies for the prediction of masonry structural behaviour has resulted in computational analysis attaining popularity amongst researchers. The development of such software is based on the capability of reproducing masonry cracking, and deformability, by considering the non-linear, anisotropic, and heterogeneous properties

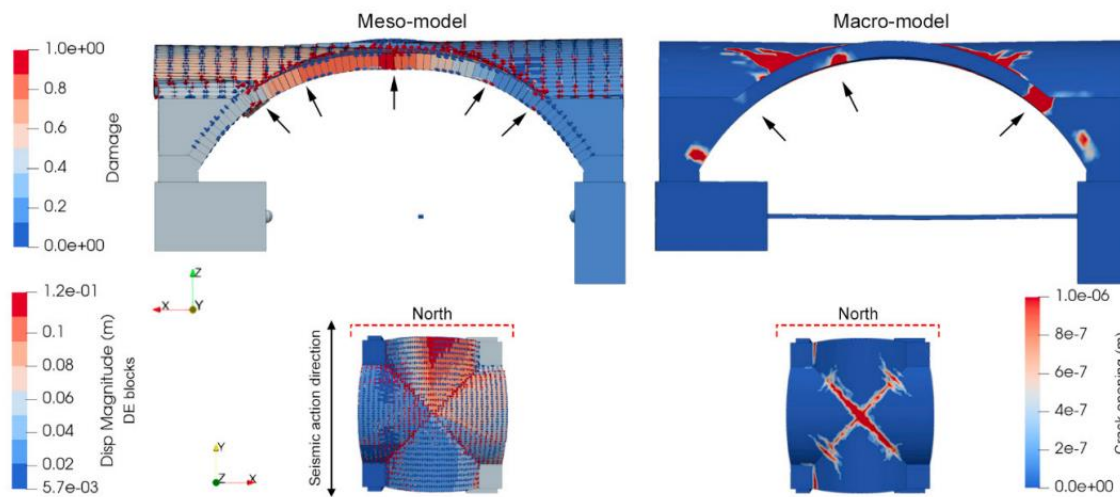
of this composite material (D'altri et al., 2019). In the realm of numerical simulations, there are three main approaches depending on the level of detail: micro, simplified micro (or meso-micro), and macro modelling (Lourenço et al., 1995).

Without delving into detail, the key difference between these methods is the representation of the masonry unit and the mortar joints (refer to Figure 3.7). Micro-modelling involves the separate modelling of both elements of masonry and hence requires more computational effort and produces more accurate results. On the other hand, macro-modelling involves the modelling of masonry as a composite and homogenised material without distinction between its elements, and hence this simplification reduces computational effort while still producing reliable results (Lourenço et al., 1995).



**Figure 3.7** – Modelling approaches for masonry: (a) general masonry; (b) micro-modelling; (c) simplified micro-modelling; (d) macro-modelling. Reproduced from Lourenço et al. (1995).

There are various types of models which can be used for the different levels of modelling such as Finite Element Method (FEM), Discrete Element Method (DEM), Applied Element Method (AEM), among others (Brincat, 2020). A concise review of such strategies can be found in D'altri et al. (2019) and Roca et al. (2010). This research study will involve a non-linear pushover analysis using the software HiStrA developed by Gruppo Sismica and will be discussed in further detail in the subsequent Chapter.



**Figure 3.8** – Numerical modelling of a cross-vault subject to seismic load: (Left) Meso-micro model of a cross-vault; (right) Macro model of a cross-vault. Reproduced from Ferrante et al. (2023).

### 3.6 Conclusion

In essence, this chapter provides the key considerations with respect to the understanding of the seismic vulnerability of masonry shells and in particular to URM cross-vaults. Despite the limited amount of literature, this chapter also underlines the various techniques that have been developed over time for the purpose of assessing the seismic performance of these structures.

## 4 Research Methodology

The parametric nature of this study implies a set of variable geometric parameters with regards to the construction of cross-vaults. The selection of the method of analysis hence depended on the appropriate depiction of the overall geometry, while balancing between user accessibility for the limited time available and adequately precise results.

The HiStrA – Arches & Vaults<sup>23</sup> software developed by Gruppo Sismica at the University of Catania (Gruppo Sismica, 2015) was chosen to conduct the seismic analysis of the several numerical models to be tested. This software possesses a great capability of rapidly varying the geometry of the cross-vaults and hence shortening the required modelling time, while providing an acceptable level of detail and suitable representation of masonry behaviour and performance.

### 4.1 Seismic Analysis using HiStrA

#### 4.1.1 *Discrete Macro-Element Method Theory*

The numerical code of the HiStrA package is based on the Discrete Macro-Element Method (DMEM), simulating masonry behaviour through discrete irregular macro-elements connected with non-linear interfacial links. Once the geometry is defined, the software automatically discretizes the geometric model into a mesh of discrete macro-elements which form the computational model that is to be used in the analysis (refer to Figure 4.1). The mesh discretisation is user-adjustable, with finer meshes increasing computational effort and hence, time (Gruppo Sismica, 2015).

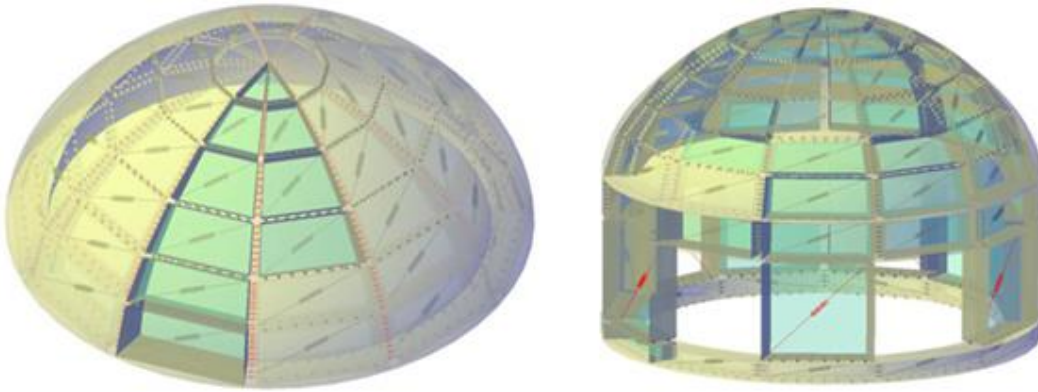
Since HiStrA enables the modelling of more complex curved geometry, it implements rigid, four and three-node macro-elements, referred to as Quad and Vertex respectively. Each macro-element is composed of the corresponding number of rigid sides between the nodes. The link between adjacent macro-elements (whether Quad or Vertex elements) is expressed as non-linear spring beds and these regulate the non-linear membrane, torsional, and sliding masonry behaviour (Gruppo Sismica, 2015).

For the Quad macro-element, the four rigid sides are stiffened by a singular diagonal spring which reproduces the in-plane shear deformability of the masonry panel (refer to Figure 4.2). This is not the case for the Vertex macro-element, and although this is an approximation, it does not extensively affect the veracity of results. This approximation hence implies a reduction in the number of degrees of freedom (DoF) of the macro-elements. The Quad elements possess seven degrees of freedom; with six corresponding to displacements in space with respect to the global axes (three rotations and three translations) and one to angular slip

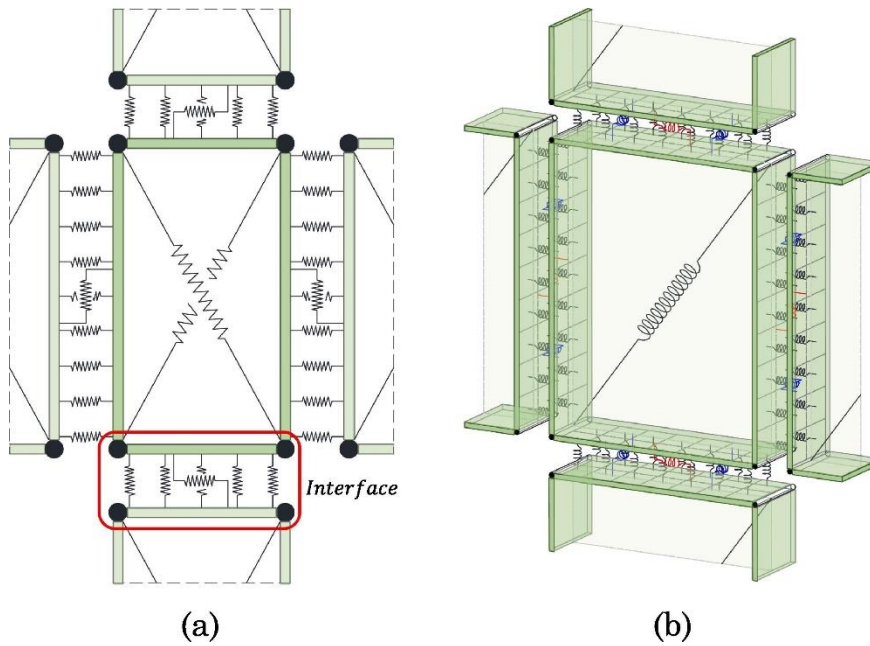
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<sup>23</sup> HiStrA is short for Historical Structures Analysis. There are two versions of HiStrA, the one being used in this study is HiStrA – Arches & Vaults and the other is HiStrA – Bridges.

(in-plane shear deformability), while the Vertex elements possess the six degrees of freedom pertaining to the displacements in space (Gruppo Sismica, 2015).



**Figure 4.1** – Subdivision of a masonry dome into Quad and Vertex macro-elements. Reproduced from Caddemi et al. (2017).



**Figure 4.2** – Visual representation of the DMEM approach: (a) 2D; (b) 3D. Reproduced from (Chácara et al., 2019).

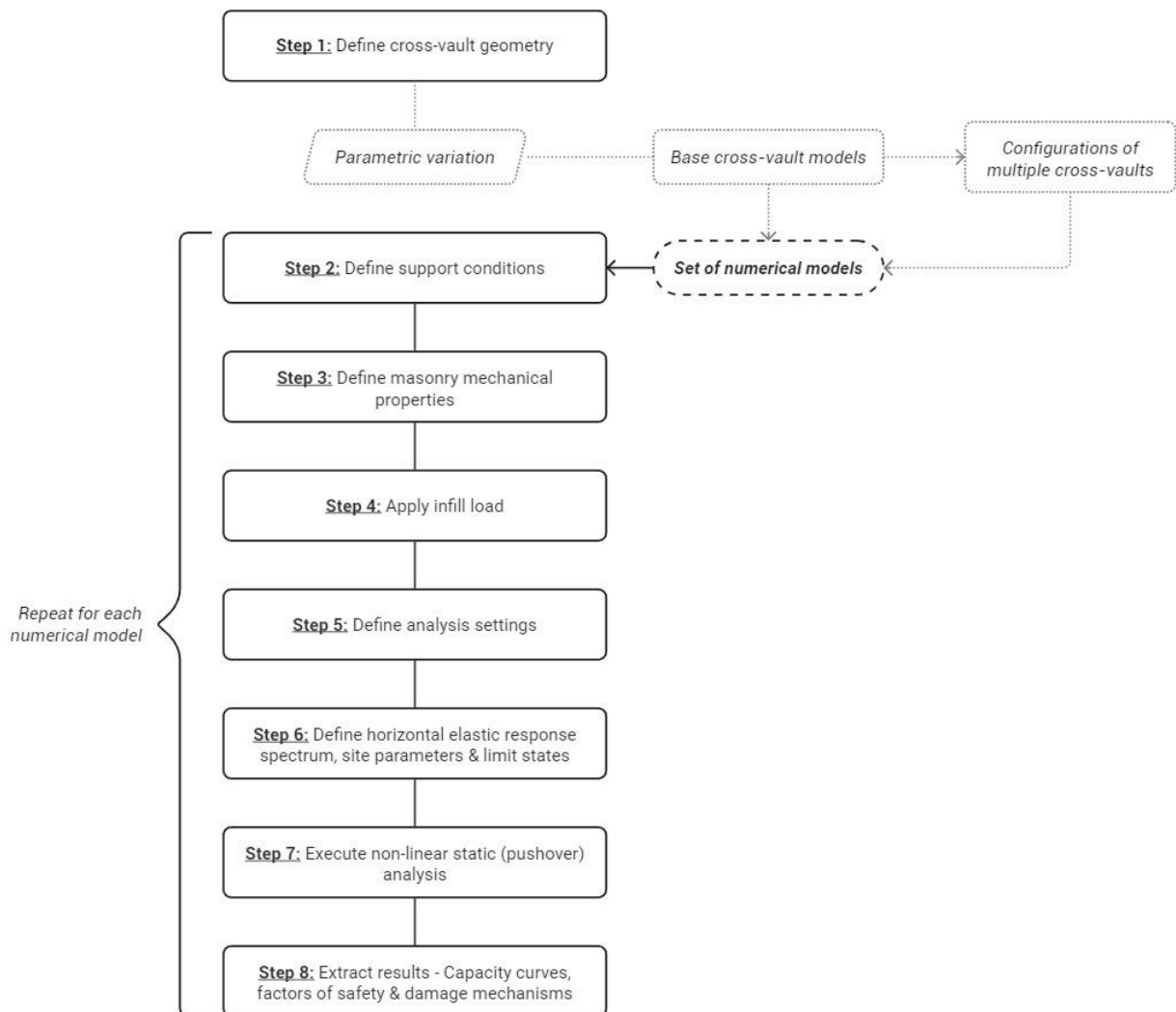
#### 4.1.2 Pushover Analysis

As previously stated, the seismic analysis for this research study involves a non-linear static pushover analysis – a commonly adopted approach for the evaluation of seismic performance of masonry structures. The nature of this procedure is incremental-iterative, in which the lateral load is increased gradually until the structure in contention reaches a state of collapse (D'altri et al., 2019). The default iterative procedure within HiStrA is the Newton-Raphson method (Gruppo Sismica, 2015).

The results are obtained in the form of plotted pushover curves (or capacity curves) which provide information on the structural load and displacement capacity and hence, on the non-linear behaviour of masonry for a given seismic input (Elghazouli, 2009). The capacity curves plot the horizontal displacement,  $U$  (in metres, m) of the structure against the load factor,  $\lambda$  (LF). The LF is a measure of the base shear divided by the weight of the model, and since both values are measured in kN, the LF is unitless.

#### 4.1.3 Method of Analysis

The testing regime for this parametric study adopts a set of numerical models with varying geometrical parameters and aggregate configurations. After application of the appropriate boundary conditions, material properties and loads, the computational models are subjected to a static monotonic load in a stepwise manner, until the subjects' failure criteria are reached. This procedure was reproduced for each numerical model and is summarised in Figure 4.3. The subsequent sections provide further detail on each step of the process.



**Figure 4.3** – Flowchart summarising numerical modelling strategy and methodology (Author, 2024).

## 4.2 Numerical Modelling

### 4.2.1 Geometric Parameters

The geometric definition of the cross-vault and its various forms has already been presented in Chapter 2. In this study, the chosen variable parameter was the ratio of the span of the cross-vault in the longitudinal and transverse directions (aspect ratio on plan). Through this variation, three different vaults of either a square or rectangular plan were generated, which provide a representation of different shapes of vaults as observed in historical buildings. This decision was facilitated by the fact that within the present literature, a square plan is generally adopted.

The creation of the geometrical models was carried out within HiStrA's built-in *Wizard* (refer to Figure 4.23). This data input window provides the editable dimensions and various options for the rapid creation of the model within the limits of the software. As illustrated in Table 4.1, the three cross-vault base models possess a common dimension in the X-direction, with the other reduced to vary the aspect ratio. The remaining numerical models to be tested in this study were developed from these three base models. Although the dimensions are not taken from real case studies, the dimensions are based on Table 2.1 and relevant data from the literature.

| Label <sup>24</sup> | Cross-Vault Parameters |       |     |       |       |           | Lateral Arches |          |          |
|---------------------|------------------------|-------|-----|-------|-------|-----------|----------------|----------|----------|
|                     | $L_1$                  | $L_2$ | $F$ | $t_0$ | $t_1$ | $L_{max}$ | $W_p$          | $T_{p0}$ | $T_{p1}$ |
|                     | In metres (m)          |       |     |       |       |           |                |          |          |
| <b>CV1</b>          | 4                      | 4     | 2   | 0.25  | 0.25  | 0.5       | 0.5            | 0.25     | 0.25     |
| <b>CV2</b>          | 4                      | 3     | 2   | 0.25  | 0.25  | 0.5       | 0.5            | 0.25     | 0.25     |
| <b>CV3</b>          | 4                      | 2     | 2   | 0.25  | 0.25  | 0.5       | 0.5            | 0.25     | 0.25     |

**Table 4.1** – Geometric parameters of the base numerical models

<sup>24</sup> The numerical models were labelled starting from Cross-Vault 1 and abbreviated as CV1. The ribbed versions of the base models are labelled as CV1-R, CV2-R and CV3-R respectively.

Keeping the dimensions constant, this set of base models was then modified to include a pair of diagonal ribs below the groins of the vaults, and hence creating a second set of models. The dimensions of the ribs were kept uniform for each model and are presented in Table 4.2.

| $W_d$                | $T_{d0}$ | $T_{d1}$ |
|----------------------|----------|----------|
| <i>In metres (m)</i> |          |          |
| 0.2                  | 0.35     | 0.35     |

**Table 4.2** – Dimensions of diagonal ribs

#### 4.2.2 Aggregate Configurations

Within historical buildings, it is predominantly the case that cross-vaults are found in aggregate<sup>25</sup>. A typical example is the nave or aisle of a cathedral, boasting a row of multiple cross-vaults, repeated along its length. As is observed in Gothic cathedrals, the vaults can also vary in shape within the same aggregate, which however, was not considered in this study.

In this case, two different aggregate configurations were simulated to study the behaviour of cross-vaults in aggregate (refer to Figure 4.4). The process to create these configurations involved a simple copy and paste operation of the base models, and subsequently applying the base restraints (as these cannot be copied and had to be manually defined).

##### 4.2.2.1 Configuration 1

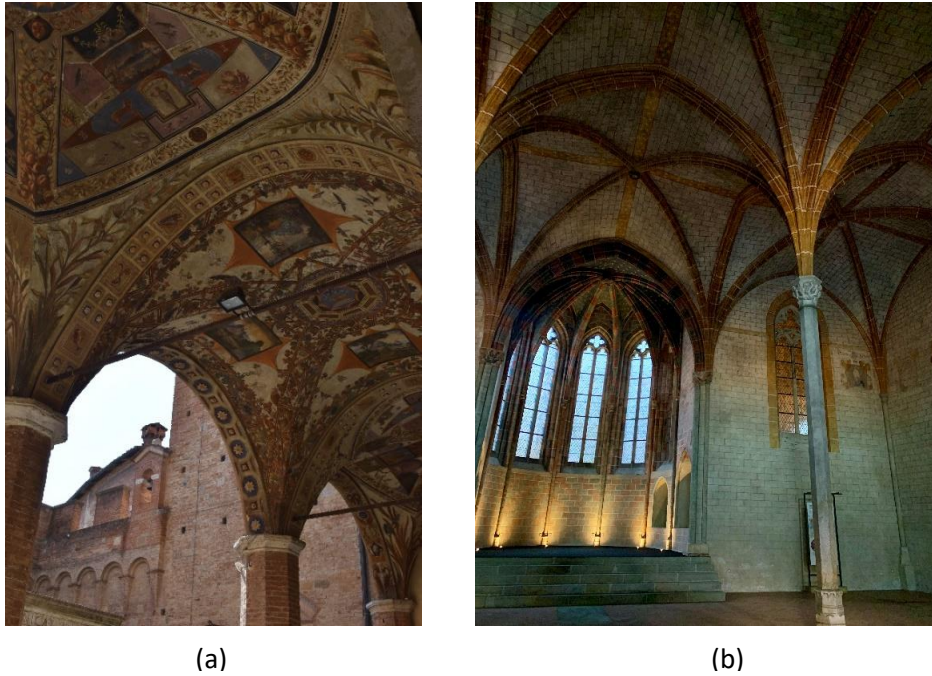
The first configuration is a row of three adjacent cross-vaults and was created with the six singular vault models. This configuration was chosen as it can be found in different roofing scenarios such as the nave or aisles in a church, or the portico of a palace or monumental building (frequently observed in the *centro storico* of Bologna, Italy).

##### 4.2.2.2 Configuration 2

The second configuration is a 3x3 grid of cross-vaults and was only created with the square-plan vaults. Although not as common as the previous configuration, this set of vaults can be observed in crypts, monasteries, and palaces.

<sup>25</sup> Aggregate – A set of separate elements which together form a whole.





**Figure 4.4** – Examples of aggregate configurations: (a) Configuration 1 – Portico in Bologna, Italy; (b) Configuration 2 – Couvet des Jacobins in Toulouse, France. Taken by Author (2023).

### 4.2.3 Support/Boundary Conditions

The restraints of the numerical model are defined in terms of the six degrees of freedom of displacement in the X-Y-Z space, three of which pertain to translations and the rest to rotations. Each DoF can be specified as either ‘Fixed’ or ‘Free’. Upon specifying a particular DoF as ‘Free’, the numerical model is updated and that DoF is now released.

For this study, the fully fixed support condition (FFFFFF) was adopted for all the numerical models (refer to Figure 4.5). Hence, it was assumed that the supports have equal stiffness. The asymmetric boundary condition mentioned in subsection 3.4.2 remains a relevant approach to model the change in stiffness of the supporting elements. However, since the central focus is the effect of geometrical and constructional parameters, this condition was excluded for this study.

| Restraint |       |   |
|-----------|-------|---|
| U1        | Fixed | ▼ |
| U2        | Fixed | ▼ |
| U3        | Fixed | ▼ |
| R1        | Fixed | ▼ |
| R2        | Fixed | ▼ |
| R3        | Fixed | ▼ |

**Figure 4.5** – Definition in HiStrA of restraints for translations and rotations of supports.

#### 4.2.4 Load Definition

The self-weight of the cross-vault models is computed automatically by the software and included within the default load combinations. Additionally, a uniformly distributed load of 20kN/m<sup>3</sup> was applied to the models to represent the self-weight of the infill within the conoids of the vaults (refer to Figure 4.15 for visual representation of the vault load). The default *SEISMIC* load combination of the software with preset factors of safety was assigned for the analyses.

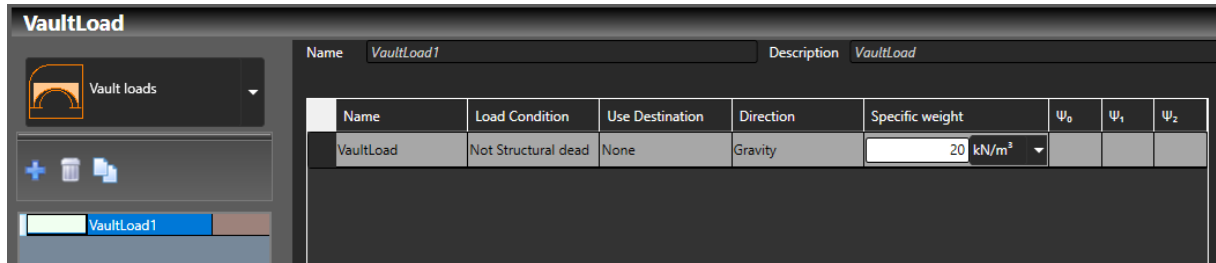


Figure 4.6 – Definition of vault infill load (above conoids).

#### 4.2.5 Material Properties

The mechanical properties of the masonry were kept identical for each numerical model. Within HiStrA, the properties of the masonry can be specified through either one of two modes: the simple or advanced mode (Gruppo Sismica, 2015). The former was chosen as the latter requires the input of a wide range of values that were not entirely available to the author, and which are probably not significant for the expected mode of failure. With the simple approach, the user chooses the masonry type, for which a set of preset values are automatically defined within the dialogue box as per the *Norme Tecniche delle Costruzione* (NTC). These values are corrected by a confidence factor depending on the knowledge level. In this case, ‘*muratura in mattoni pieni e malta di calce*’ was chosen, assuming average values with a knowledge level of the 2<sup>nd</sup> order and hence factored by 1.2.

Since HiStrA functions on a macro-element approach, these values represent the ‘*smeared*’ properties of masonry as a composite material. The software automatically provides the homogenised values which are computed using Equation 1.

$$X_t = \frac{A_b X_b + A_m X_m}{A_b + A_m}$$

Equation 1 – Computation of smeared material property. Adapted from Brincat (2020).

Where,

- $X_t$  = Smeared material property
- $X_b$  = Relative material property of the masonry block (units of property)
- $A_b$  = Reference wall area occupied by masonry blocks in  $m^2$
- $X_m$  = Relative material property of the mortar (units of property)
- $A_m$  = Reference wall area occupied by mortar in  $m^2$

Name: *Masonry1*      Description: *Muratura in mattoni pieni e malta di calce*

☒ Simple mode    ☐ Advanced mode      Specific weight,  $W = 18 \text{ kN/m}^3$

**Masonry typology**      *Muratura in mattoni pieni e malta di calce (NTC2008)*

**Description**      *Muratura in mattoni pieni e malta di calce*

**Knowledge level (Livello di conoscenza) LC**      *LC2*

**Elastic properties (E, G)**      ☐ Min values    ☒ Average values    ☐ Max values

**Strength properties**      ☐ Min values    ☒ Average values    ☐ Max values

**Corrective coefficients of mechanical properties (Circ. 2 / 2 / 2009 n. 617 C.S.I.L.P.P.)**      *Show/Hide*

| Parameter | Base value              | Corrective coefficient | FC  | Design value             |
|-----------|-------------------------|------------------------|-----|--------------------------|
| $f_m$     | 3200 $\text{kN/m}^2$    | 1                      | 1.2 | 2666.667 $\text{kN/m}^2$ |
| $\tau_0$  | 76 $\text{kN/m}^2$      | 1                      | 1.2 | 63.33333 $\text{kN/m}^2$ |
| E         | 1500000 $\text{kN/m}^2$ | 1                      | 1.2 | 1250000 $\text{kN/m}^2$  |
| G         | 500000 $\text{kN/m}^2$  | 1                      | 1.2 | 416666.7 $\text{kN/m}^2$ |

**Figure 4.7** – Chosen masonry material properties (in the simple mode).

#### 4.2.6 Computational Model

The generated geometric model is automatically discretised into the Quad and Vertex macro-elements to create the computational model. Each macro-element does not represent a singular unit of masonry but rather a collection of stone and mortar. Nonetheless, the size of the elements can be adjusted by the user through the  $L_{max}$  measurement as found in Table 4. A value of 0.5m was attributed to this dimension in order to be somewhat indicative of the general size of masonry units and in so doing achieving more accurate results through a finer mesh size.

### 4.3 Seismic action for analysis

The seismic input for the pushover analysis follows the guidelines provided by Eurocode 8.

#### 4.3.1 Site Parameters

The numerical models represent a set of fictional cross-vaults and hence are not physically tied to any specific location. The ground-structure interaction is not the focus of this study and in this case the site parameters were assumed and kept unaltered between the different tests for uniformity (refer to Figure 4.8). With regards to the ground conditions, soil category A was assumed, while for the building class, Type III was selected, corresponding to an importance factor of 1.2.

The image shows a software interface with two main sections: 'Structure parameters' and 'Site parameters'. The 'Structure parameters' section includes dropdown menus for 'New/Existing structure' (Existing structure), 'Structure typology' (Masonry structure), 'Knowledge level (LC)' (LC2), and 'Type of building' (Ordinary structure (building-)). It also has input fields for 'Nominal life age,  $V_n$ ' (50), 'Class of use' (II), 'Use coefficient,  $C_u$ ' (1), 'Reference period,  $V_R$ ' (50), and 'Equivalent damping,  $\xi$ ' (5 %). The 'Site parameters' section includes dropdown menus for 'Area Context' (detached building), 'Soil type' (A), and 'Topographic type' (T1). It also has input fields for 'Total height of the slope' (0 m) and 'Level of the structure (from the base of the slope)' (0 m).

**Figure 4.8** – Definition of site and structural parameters for seismic analysis. In this figure, Class II is defined due to limitations of the software, as explained in section 4.4.

| Ground type | Description of stratigraphic profile                                                                   | Parameters       |                        |             |
|-------------|--------------------------------------------------------------------------------------------------------|------------------|------------------------|-------------|
|             |                                                                                                        | $v_{s,30}$ (m/s) | $N_{SPT}$ (blows/30cm) | $c_u$ (kPa) |
| A           | Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface. | > 800            | –                      | –           |

**Figure 4.9** – Extract from Table 3.1 (in EC8) showing ground type A. Reproduced from EN1998-1.

| Importance class | Buildings                                                                                                                                                            |
|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I                | Buildings of minor importance for public safety, e.g. agricultural buildings, etc.                                                                                   |
| II               | Ordinary buildings, not belonging in the other categories.                                                                                                           |
| III              | Buildings whose seismic resistance is of importance in view of the consequences associated with a collapse, e.g. schools, assembly halls, cultural institutions etc. |
| IV               | Buildings whose integrity during earthquakes is of vital importance for civil protection, e.g. hospitals, fire stations, power plants, etc.                          |

**Figure 4.10** – The importance classes for the building type (Table 4.3 in EC8). Reproduced from EN1998-1.

#### 4.3.2 Limit States

Since cross-vaults are commonly found in existing buildings, the limit states (LS) prescribed in EN1998-3 were specified for the analysis. These limit states describe the expected performance requirements of existing structures subject to seismic load and are the following:

- Near collapse (NC)
- Significant damage (SD)
- Damage limitation (DL)

For the respective limit state, the code stipulates a reference return period which corresponds to a certain probability of exceedance, the values of which were inputted in the dialogue box as shown in Figure 4.11.

| Code Spectral Response Definition                                |       |         |       |     |         |     |   |        |        |        |        |        |
|------------------------------------------------------------------|-------|---------|-------|-----|---------|-----|---|--------|--------|--------|--------|--------|
| Add Delete <input checked="" type="checkbox"/> Custom definition |       |         |       |     |         |     |   |        |        |        |        |        |
| Limit state (LS)                                                 | Pvr % | Tr [ye] | Ag/g  | Fo  | Tc* [s] | Eta | S | Tb [s] | Tc [s] | Td [s] | Te [s] | Tf [s] |
| DL - Damage Limitation Limit State                               | 20    | 225     | 0.084 | 2.5 | 0       | 1   | 1 | 0.05   | 0.25   | 1.2    | 4.5    | 10     |
| SD - Significant Damage Limit State                              | 10    | 475     | 0.12  | 2.5 | 0       | 1   | 1 | 0.05   | 0.25   | 1.2    | 4.5    | 10     |
| NC - Near Collapse Limit State                                   | 2     | 2475    | 0.42  | 2.5 | 0       | 1   | 1 | 0.05   | 0.25   | 1.2    | 4.5    | 10     |

**Figure 4.11** – Spectral response values for each limit state, with pre-multiplied ground acceleration values.

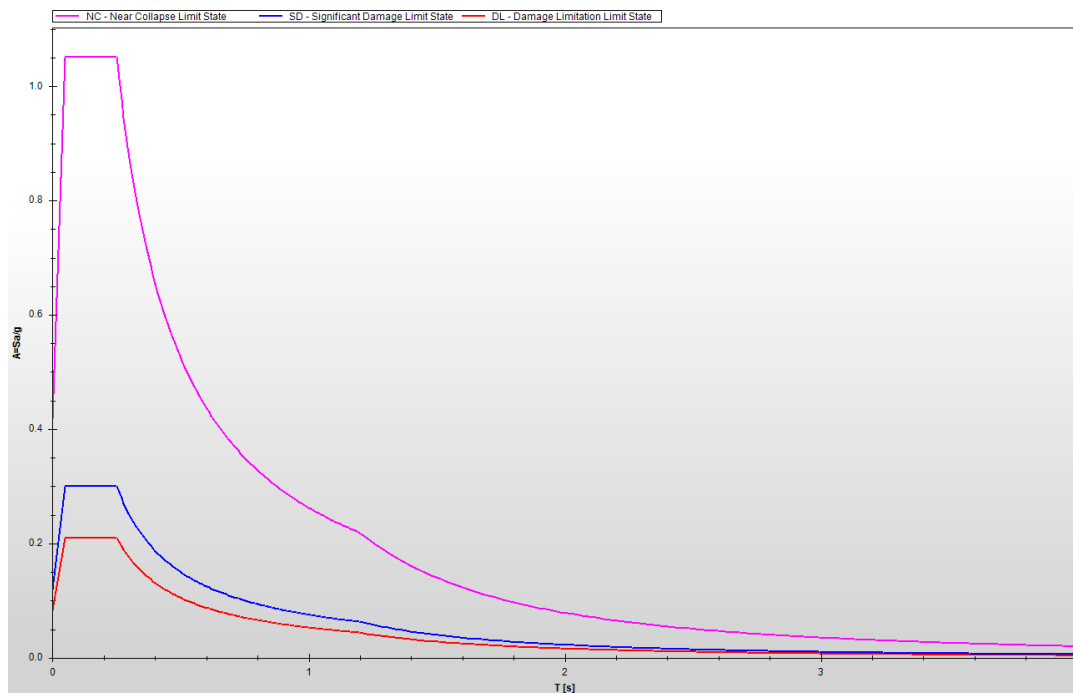
#### 4.3.3 Response Spectrum

The elastic response spectrum provided in EC8 is altered according to the values of the limit states and the site parameters. Since the Maltese islands are considered to be a low-to-moderate seismic hazard zone, the Type 2 horizontal elastic spectrum values were assigned as per Table 3.3 in EN1998-1 (refer to Figure 4.12). The designated PGA for the pushover analysis for each limit state is defined by the response spectra accordingly and whose values

were obtained from Brincat (2020). For each limit state, an elastic response spectrum is separately defined (refer to Figure 28).

| Ground type | $S$  | $T_B$ (s) | $T_C$ (s) | $T_D$ (s) |
|-------------|------|-----------|-----------|-----------|
| A           | 1,0  | 0,05      | 0,25      | 1,2       |
| B           | 1,35 | 0,05      | 0,25      | 1,2       |
| C           | 1,5  | 0,10      | 0,25      | 1,2       |
| D           | 1,8  | 0,10      | 0,30      | 1,2       |
| E           | 1,6  | 0,05      | 0,25      | 1,2       |

**Figure 4.12** – Values for Type 2 horizontal elastic response spectrum (Table 3.3 in EC8). Reproduced from EN1998-1.



**Figure 4.13** – Elastic response spectra for each limit state as was defined by author in HiStrA.

#### 4.3.4 Seismic Vulnerability

The seismic vulnerability criteria were obtained from Annex C in EN1998-3 and are represented in Figure 4.14. The assessment criteria are specific to non-linear analysis methods and are related to the displacement capacity of the structure. These requirements establish the failure criteria for the pushover analysis of the numerical models and hence provide information on the factors of safety,  $\alpha$  (FoS) for each limit state.

| Seismic Vulnerability Criteria    |                 |                             |                 |                |  |
|-----------------------------------|-----------------|-----------------------------|-----------------|----------------|--|
| Custom definition                 |                 |                             |                 |                |  |
| Limit state (LS)                  | Shear Reduction | Displacement of an other LS | Max shear force | Yielding point |  |
| DL - Damage Limitation Limit Sta  | 0 %             | 100 %                       | SD - Signi      |                |  |
| SD - Significant Damage Limit Sta | 0 %             | 75 %                        | NC - Near       |                |  |
| NC - Near Collapse Limit State    | 80 %            | 0 %                         | DL - Damu       |                |  |

| Seismic Vulnerability Checks        |                    |           |                  |                   |
|-------------------------------------|--------------------|-----------|------------------|-------------------|
| Custom definition                   |                    |           |                  |                   |
| Limit state (LS)                    | Displacement check | PGA check | Force (q*) check | q* <sub>max</sub> |
| DL - Damage Limitation Limit State  | ✓                  | ✓         |                  | 0                 |
| SD - Significant Damage Limit State | ✓                  | ✓         | ✓                | 3                 |
| NC - Near Collapse Limit State      | ✓                  | ✓         | ✓                | 4                 |

Figure 4.14 – Definition of seismic vulnerability criteria and checks.

#### 4.3.5 Seismic Analysis

Prior to carrying out the pushover analyses, a vertical load test was performed on the numerical models. This was necessary as the pushover analyses only commenced if the model did not fail when subjected to its self-weight. For each test, pushover analyses were carried out with respect to the X and Y global coordinates within the modelling space and in both the positive and negative directions (refer to Figure 4.15 for sign convention).

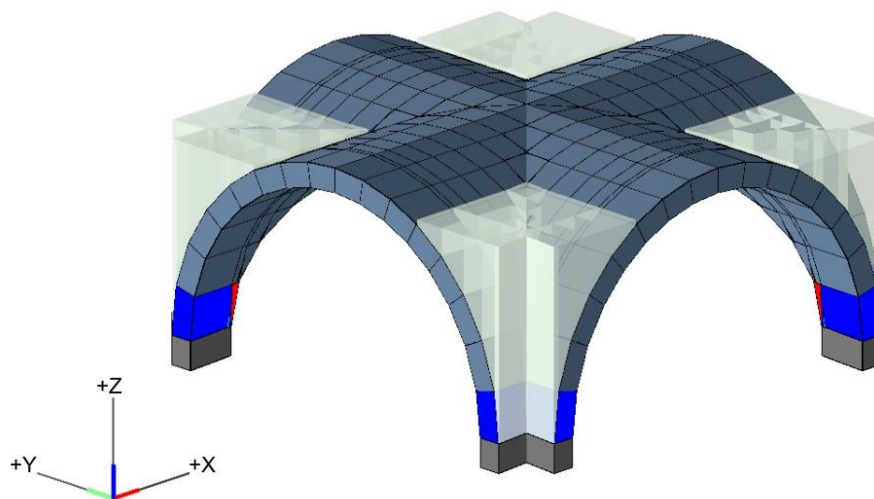


Figure 4.15 – A sample numerical model showing adopted sign convention for all tests.



Additionally, as required by Clause 4.4.4.2 in EN1998-3, two lateral load distributions were assumed, and hence pushover analyses were conducted for both. The first is ‘*Pushover Massa*’, which assumes a lateral force distribution proportional to the mass, and the second is ‘*Pushover Modal*’, in which the force distribution is proportional to a combination of the mode shapes, obtained through a modal analysis. The default number of modes in the modal analysis is four and since the first few mode shapes are the most significant, was retained at this value. Thus, for each numerical model, a total of eight capacity curves were obtained (refer to Figure 4.16).

| Run                                 | Stop                 | Continue              | Quick selector |                       |                                                                                                   |  | SEISMIC |
|-------------------------------------|----------------------|-----------------------|----------------|-----------------------|---------------------------------------------------------------------------------------------------|--|---------|
| Execute                             | Analysis             | Type                  | Start from     | State                 | Details                                                                                           |  |         |
| <input checked="" type="checkbox"/> | Vert                 | Static, Force control | Zero           | Analysis not executed | Base Analysis adopted for all analyzes derived from the seismic combination                       |  |         |
| <input type="checkbox"/>            | SLE_rara             | Static, Force control | Zero           | Analysis not executed | Analysis used to check SLE-rare combination                                                       |  |         |
| <input type="checkbox"/>            | SLE_frequente        | Static, Force control | Zero           | Analysis not executed | Analysis used to check SLE-frequent combination                                                   |  |         |
| <input type="checkbox"/>            | SLE_quasi_permanente | Static, Force control | Zero           | Analysis not executed | Analysis used to check SLE-quasi permanent combination                                            |  |         |
| <input type="checkbox"/>            | SLU_STR              | Static, Force control | Zero           | Analysis not executed | Base analysis used to check structural SLU combination (STR) of permanent loads                   |  |         |
| <input type="checkbox"/>            | SLU_GEO              | Static, Force control | Zero           | Analysis not executed | Base analysis used to check structural SLU combination (GEO) of permanent loads                   |  |         |
| <input checked="" type="checkbox"/> | Pushover_+X_Massa    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in +X direction with a force distribution proportional to the masses             |  |         |
| <input checked="" type="checkbox"/> | Pushover_-X_Massa    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in -X direction with a force distribution proportional to the masses             |  |         |
| <input checked="" type="checkbox"/> | Pushover_+Y_Massa    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in +Y direction with a force distribution proportional to the masses             |  |         |
| <input checked="" type="checkbox"/> | Pushover_-Y_Massa    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in -Y direction with a force distribution proportional to the masses             |  |         |
| <input checked="" type="checkbox"/> | Modal                | Modal                 | Zero           | Analysis not executed | Modal analysis                                                                                    |  |         |
| <input checked="" type="checkbox"/> | Pushover_+X_Modal    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in +X direction with a force distribution proportional to a combination of modes |  |         |
| <input checked="" type="checkbox"/> | Pushover_-X_Modal    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in -X direction with a force distribution proportional to a combination of modes |  |         |
| <input checked="" type="checkbox"/> | Pushover_+Y_Modal    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in +Y direction with a force distribution proportional to a combination of modes |  |         |
| <input checked="" type="checkbox"/> | Pushover_-Y_Modal    | Static, Displ.control | Vert           | Analysis not executed | Seismic analysis in -Y direction with a force distribution proportional to a combination of modes |  |         |

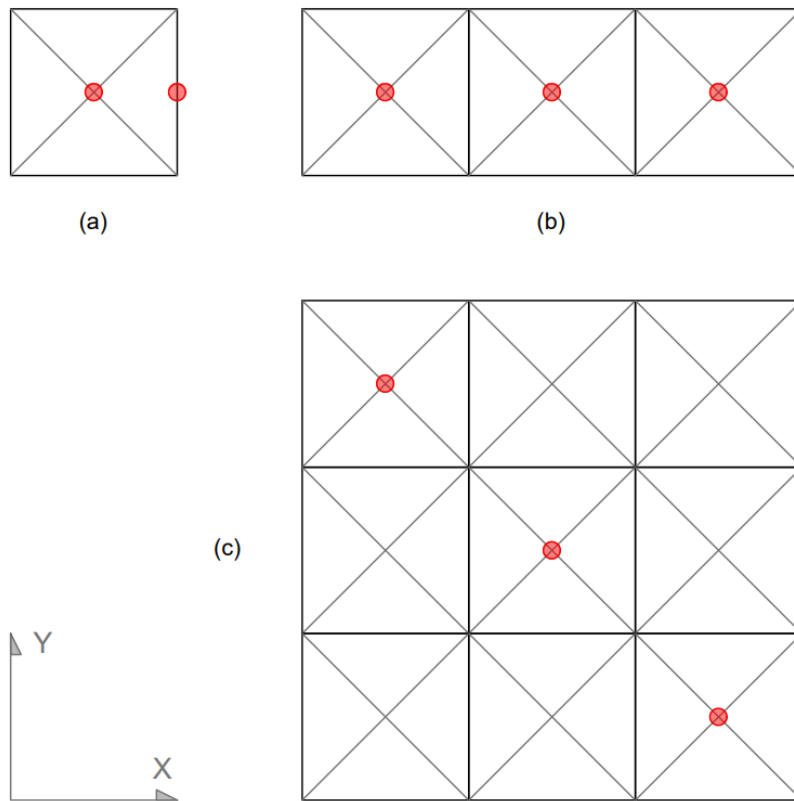
**Figure 4.16** – The selected analyses to be executed for each numerical model.

The pushover curves are plotted with reference to a control point within the numerical model. In this regard, the fundamental control point was the crown of the cross-vault represented by a singular node. However, for the different numerical models, multiple nodes were chosen as control points to gain insight on differential movement, and which are illustrated in Figure 4.17.

The length and hence the number of data points of the extracted capacity curves depend on the specified settings of the analyses. Despite a load reduction of 20% (default value) signifying collapse, the settings were configured to produce curves exceeding this point up to a default target displacement of 0.1m, allowing where possible to obtain post-peak information (refer to Figures 4.18 – 4.21). For such an operation, the number of iterations were increased to 10000 from the default 1000, which however, did not result in all executed analyses reaching the target displacement. In multiple cases, the convergence failed by reaching the maximum number of iterations and hence ‘*incomplete*’ curves were produced (refer to Figure 4.22). Since the numerical models of *Configuration 2* involve a total of nine cross-vaults each, the analysis settings were adjusted slightly for only those two tests, to mitigate the computational effort required (in a way which shortened the analysis).

Naturally, the assessment of seismic vulnerability and safety factors of the numerical models was based on the collected results.





**Figure 4.17** – Chosen model points (nodes): (a) Singular vaults; (b) Configuration 1; (c) Configuration 2 (Author, 2024).

Selected analysis: **Pushover +X Massa**

Control parameters | Iterative Strategy | Convergence Criteria | Model points | Loads applied | Load function

Type of load distribution: Force

Load control: rate of load to be applied: 100 %

Displacement control: target displacement: 0.1 m

Start also if previous analysis is not completed

Stop analysis if occurs a load reduction to: 0.8

Save results every N steps, N: 1

Control point: MaxDisplacement

Direction: Simple +x

Cart. x: 1 y: 0 z: 0

Polar  $\theta$ : 0 deg  $\phi$ : 0 deg

☒ Seismic: if this option is checked set the direction of all loads according to the direction above selected

**Figure 4.18** – Defined control parameters for all tests excluding of configuration 2.

Selected analysis: **Pushover +X Massa**

Control parameters | Iterative Strategy | Convergence Criteria | Model points | Loads applied | Load function

Integration method: Arc Length

Numerical method: Modified Newton Raphson

☒ Switch to modified strategy when stiffness is lower than its initial value

Percentage to switch Standard to Modified Newton-Raphson: 40 %

Max number of iterations: 10000

☒ Line search: Regula-Falsi Line Search

Tolerance: 0.8

Max number of iterations: 1000

Max Eta: 10

Min Eta: 0.1

Arc length method - advanced parameters

Compute  $d\lambda$  as function of: Control point selected (DISPLACEMENT CONTROL METHOD)

Arc length ray, dr: 0.001 m

Modify arc length ray during analysis

☒ Max load increment,  $d\lambda$ : 0.01

Number of iterations desired: 100

**Figure 4.19** – Specified iterative strategy for all tests excluding of configuration 2.

Selected analysis: **Pushover +X Massa**

Control parameters | Iterative Strategy | Convergence Criteria | Model points | Loads applied | Load function

Type of load distribution: Force

Load control: rate of load to be applied: 100 %

Displacement control: target displacement: 0.1 m

Start also if previous analysis is not completed

Stop analysis if occurs a load reduction to: 0.6

Save results every N steps, N: 1

Control point: MaxDisplacement

Direction: Simple +x

Cart. x: 1 y: 0 z: 0

Polar  $\Theta$ : 0 deg  $\Phi$ : 0 deg

Seismic: if this option is checked set the direction of all loads according to the direction above selected

Figure 4.20 – Defined control parameters for tests of configuration 2.

Selected analysis: **Pushover +X Massa**

Control parameters | Iterative Strategy | Convergence Criteria | Model points | Loads applied | Load function

Integration method: Arc Length

Numerical method: Modified Newton Raphson

Switch to modified strategy when stiffness is lower than its initial value

Percentage to switch Standard to Modified Newton-Raphson: 40 %

Max number of iterations: 5000

Line search: Regula-Falsi Line Search

Tolerance: 0.8

Max number of iterations: 1000

Max Eta: 10

Min Eta: 0.1

Arc length method - advanced parameters

Compute  $d\lambda$  as function of: Control point selected (DISPLACEMENT CONTROL METHOD)

Arc length ray,  $dr$ : 0.001 m

Modify arc length ray during analysis

Max load increment,  $d\lambda$ : 0.01

Number of iterations desired: 100

Figure 4.21 – Specified iterative strategy for tests of configuration 2.

```
#826: ##### CONVERGENCE FAILED #####
#827: Convergence failed, max number of iterations reached!
#828: Step=408, Iterations=10001, Error=2.99060505258685E-06, Toll=1E-06
#829: ##### CONVERGENCE FAILED #####
#830:
#831:
#832: ##### WARNING! #####
#833: Analysis interrupted at step 408 due to large iterations in Newton-Raphson algorithm
#834: > Iterations=10001 > 10000, error=0.0000, toll=0.0000
```

Figure 4.22 – Notification in output log file when analysis reaches maximum specified iterations.

#### 4.4 Limitations and Rationalisations of the Numerical Models

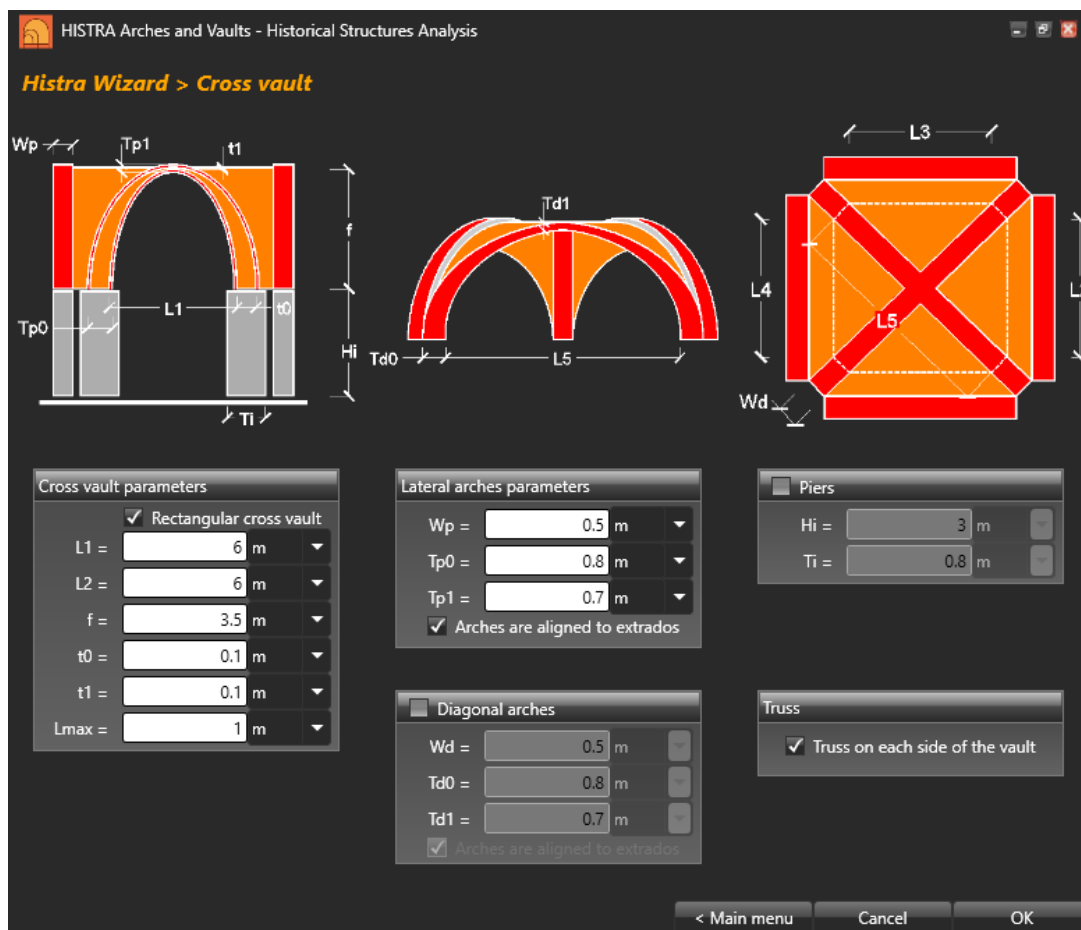
Like any other software package, HiStrA has its limitations, which had to be justified accordingly within the context of the methodology.

1. Specification of the appropriate design code: Since the software is developed by an Italian-based research group, it has by default the NTC as the specified design code. In this regard, the choice of other design codes, which in this case would be the EC8, was not possible. This shortcoming had a repercussion on two separate parts of the methodology:
  - a. Masonry mechanical properties – As explained in subsection 4.2.5 above, the values for the properties are according to NTC specification rather to the locally appropriate Eurocode 6: Design of Masonry Structures (EN1996-1). However, these values considerably approach the corresponding values that would have been obtained with EN1996-1, and hence were acceptable to use in this study.
  - b. Horizontal response spectrum – By default, the NTC limit states and their respective values were defined for the pushover analyses. Despite being quite similar in quantities, the NTC specifies four limit states rather than the three of EC8. Since there is no option to change the design code, the horizontal response spectrum and the limit states specified by EC8 had to be manually inputted for each numerical model. Likewise, the appropriate seismic vulnerability criteria for the non-linear analysis were also adjusted, and the importance factors corresponding to the building class were pre-multiplied to the PGA values. The latter was necessary as the importance factors specified by the NTC are different to those of the EC8.
2. Geometry definition: The built-in data input Wizard within HiStrA required the provision of dimensions for different elements of the cross-vault (refer to Figure 4.23). In this sense, there are five sub-elements which may be defined:
  - a. *Cross-vault parameters*
  - b. *Lateral arches parameters*
  - c. *Diagonal arches (ribs)*
  - d. *Piers (columns)*
  - e. *Truss (stiffening rod for retrofitting)*

Elements c-e can be simply excluded from the geometry, however, the software imposes on the user to define element b. In the context of this parametric study,

defining lateral arches with a different section thickness than the webs of the vault would invariably change its stiffness, and could be another geometrical variable.

This was tested through various trial runs on different dimensions of cross-vaults, and from the results it was understood that the lateral arches act as stiffening beam elements. When performing a dead load analysis on a 6m by 6m cross-vault, with and without defining a larger lateral arch section, the vault with the arch thickness equal to the web failed. Since the vertical analysis was a prerequisite for the software to execute the pushover analyses, the dimensions had to be scaled down to 4m by 4m with a uniform vault thickness. This model passed the vertical analysis and hence formed the basis of the other models. This rationalisation was sufficient as it did not change the analyses invariably (only change in mass) and thus, the deformation and cracking pattern obtained via the adopted model can be scaled to effectively any cross-vault dimension.



**Figure 4.23** – The HiStrA geometric Wizard with default values.



## 5 Seismic Analysis and Discussion of Results

### 5.1 General Overview and Results

A total of fourteen tests were performed, with the time of analysis for one test approximately ranging between 3 to 25 hours<sup>26</sup>, and referred to by the following notations:

- i. Singular cross-vault tests:
  - a. Cross-vault 1 (CV1)
  - b. Cross-vault 2 (CV2)
  - c. Cross-vault 3 (CV3)
  - d. Cross-vault 1 - Ribbed (CV1-R)
  - e. Cross-vault 2 - Ribbed (CV2-R)
  - f. Cross-vault 3 - Ribbed (CV3-R)
- ii. Aggregate Configuration 1 (C1) tests:
  - a. C1 – CV1
  - b. C1 – CV1-R
  - c. C1 – CV2
  - d. C1 – CV2-R
  - e. C1 – CV3
  - f. C1 – CV3-R
- iii. Aggregate Configuration 2 (C2) tests:
  - a. C2 – CV1
  - b. C2 – CV1-R

The output of the HiStrA software from the executed pushover analyses is in the form of capacity curves (8 for each test – plotted and formatted on Microsoft Excel), lateral deflection values and visual output with real-time model deformation. Hence, the discussion and comparison of the results is made with reference to the traced curve of the performed analysis and relevant values taken from said curve, limit state seismic vulnerability criteria (factor of safety –  $\alpha$ ) and the visually observed damage and collapse mechanisms of the cross-vault models.

The results derived from the tests were analysed and numerically compared. The peak load factor ( $\lambda$ ) values of each capacity curve were extracted, and these provide the maximum base

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<sup>26</sup> The time for the computation of the solutions depended on the model being tested. The aggregate configuration models had a significantly higher number of elements, degrees of freedom and nodes, resulting in an exponential increase in the time of analysis.

shear each numerical model sustained (in fact it is also referred to as the base shear coefficient). Furthermore, the gradient of the plotted pushover curves represents the relative sway stiffness of the model. The relevant pushover curves were overlain on the same set of axes, and the percentage increase between peak load factor values of certain tests was computed, forming the basis of comparison of results.

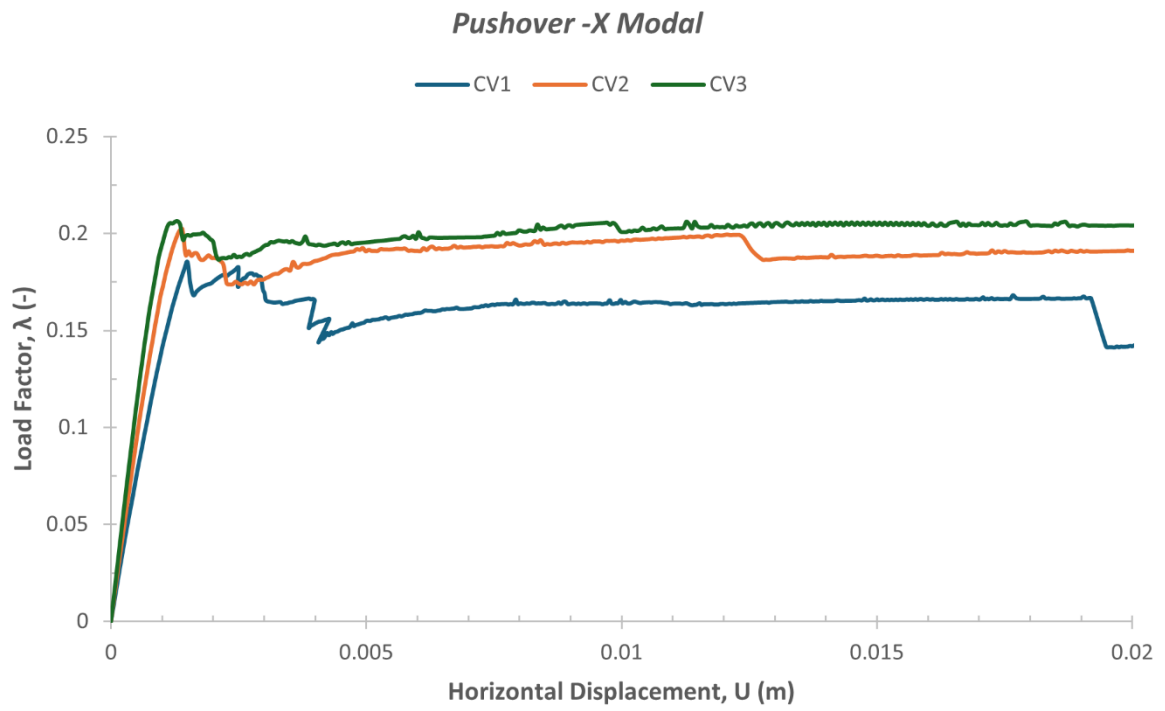
Additionally, for each pushover curve, a set of images were taken from various views showing the deformation of the numerical model. The software allows the deformation to be taken at any point along the pushover curve. The deformed model is displayed with a colormap representing the concentration of displacements in the Z global coordinate. Since not all curves were fully completed during the analyses, the images had to be extracted accordingly, and an appropriate scale factor was chosen to demonstrate clearly the movement. In the outputted images, the red signifies upward vertical displacement, while the blue represents downward vertical displacement.

From the analysis of results, it was realised that the '*Pushover Massa*' is analogous to the Equivalent Lateral Force (ELF) method, which is an approximate calculation method as prescribed in EN1998-1. Thus, in this discussion, importance is only given to the results obtained from the '*Pushover Modal*' analysis, as they yield a more accurate representation of the seismic performance of the vaults. Nonetheless, the complete set of results, including of '*Pushover Massa*', are presented distinctly in the respective Appendix (C-E), while the geometry of each numerical model is presented in Appendix B.

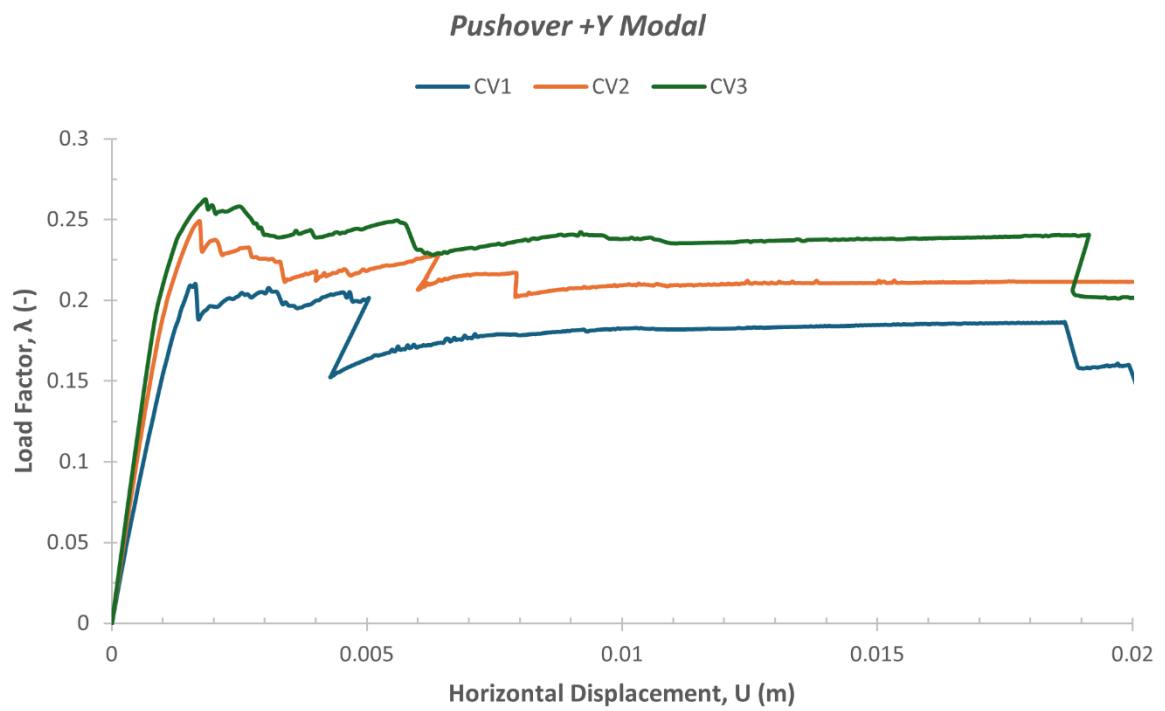
*Note: In Figures 54 and 58 and in Tables 11 and 16, it is evident that an error occurred during the analysis, and hence the results of C2 – CV1-R in the  $\pm Y$ -direction can be considered invalid. Since the model of Configuration 2 is symmetrical (made of square cross-vaults and square on plan), the results should have been equivalent to the results in the  $\pm X$ -direction. Refer to section 6.3 for justification.*

### **5.1.1 Pushover Curves**

Although for the pushover curves multiple model points were assumed (refer to subsection 4.3.5), the figures presented below only trace a single model point since regardless which point was chosen, the same peak  $\lambda$  values were obtained and only the lateral displacement varies. Additionally, only the pushover -X and +Y modal curves are presented, since the pushover +X and -Y modal curves are notably similar (refer to Appendix C). These directions were chosen as they generally had the lowest safety factor values.

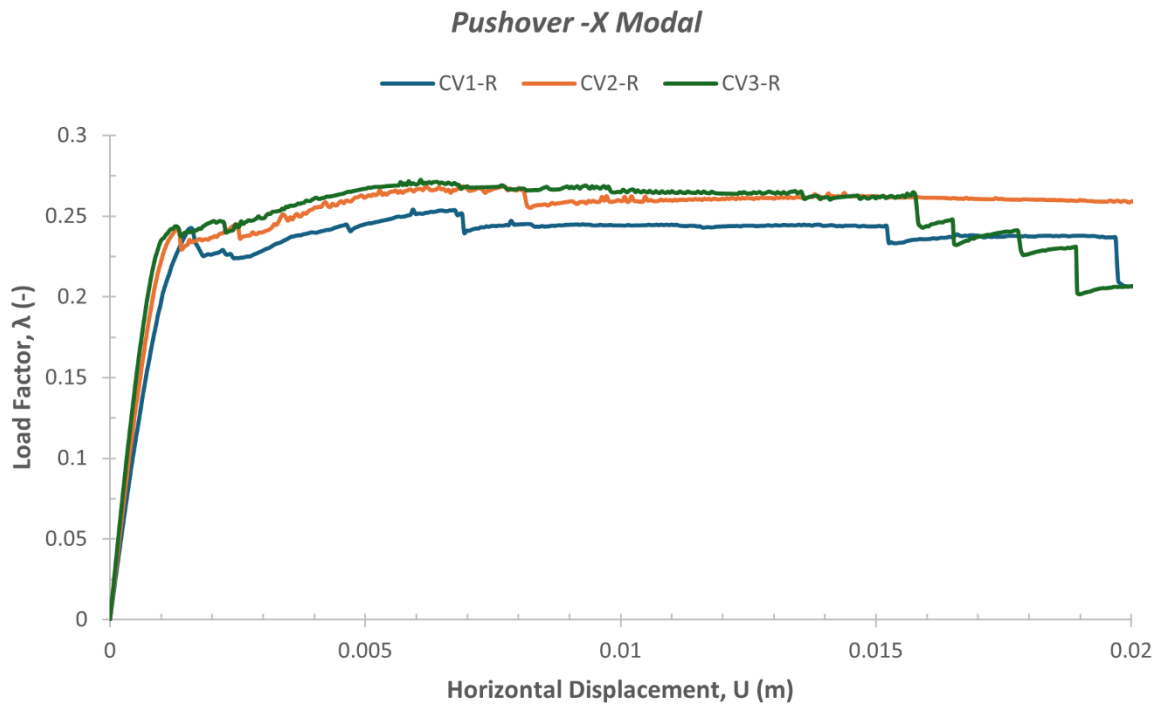


**Figure 5.1** – Comparison of -X pushover curves for CV1, CV2 & CV3.

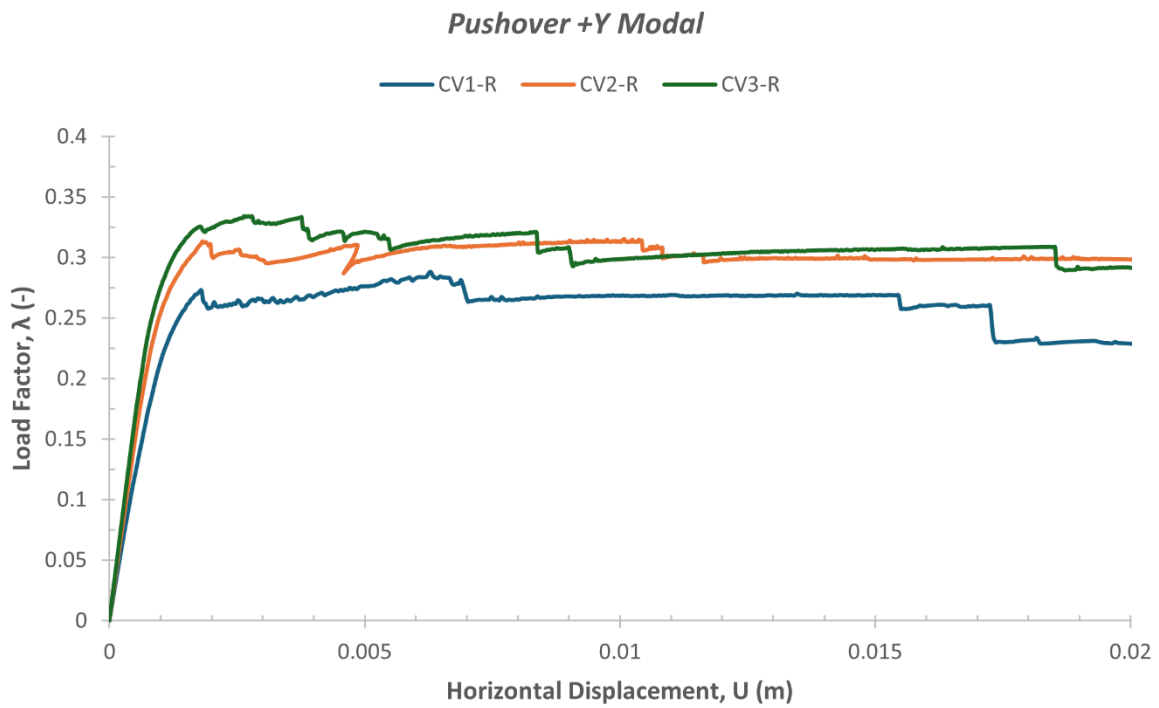


**Figure 5.2** – Comparison of +Y pushover curves for CV1, CV2 & CV3.

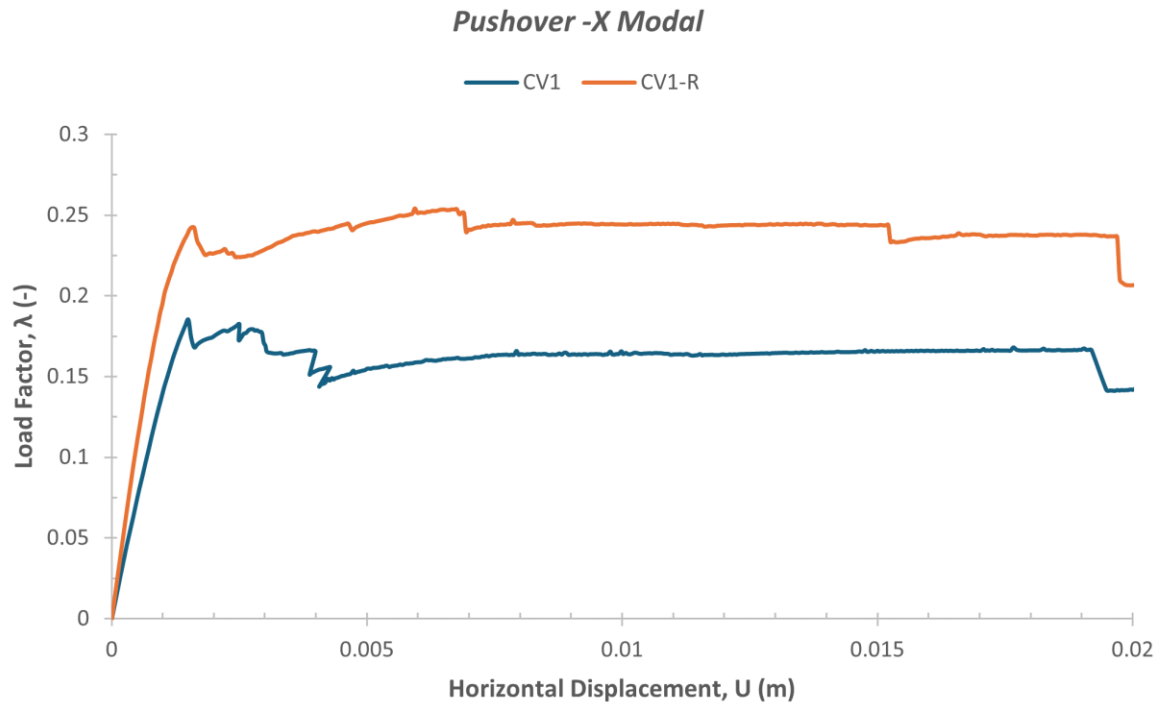




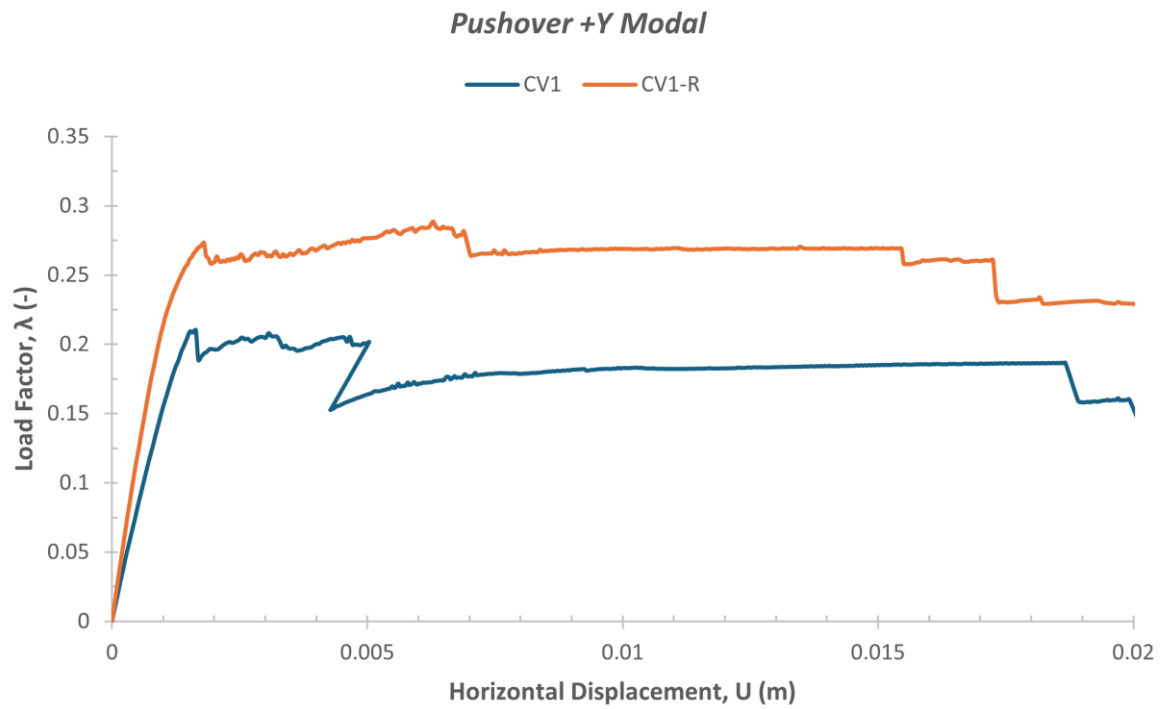
**Figure 5.3** – Comparison of -X pushover curves for CV1-R, CV2-R & CV3-R.



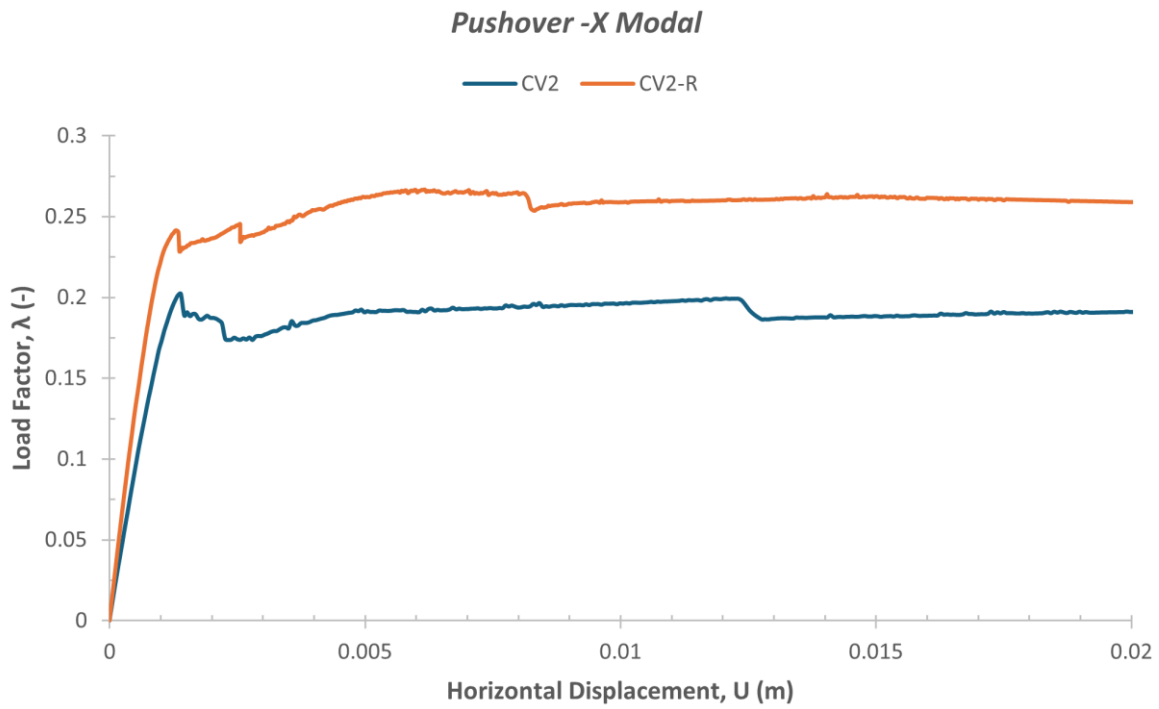
**Figure 5.4** – Comparison of +Y pushover curves for CV1-R, CV2-R & CV3-R.



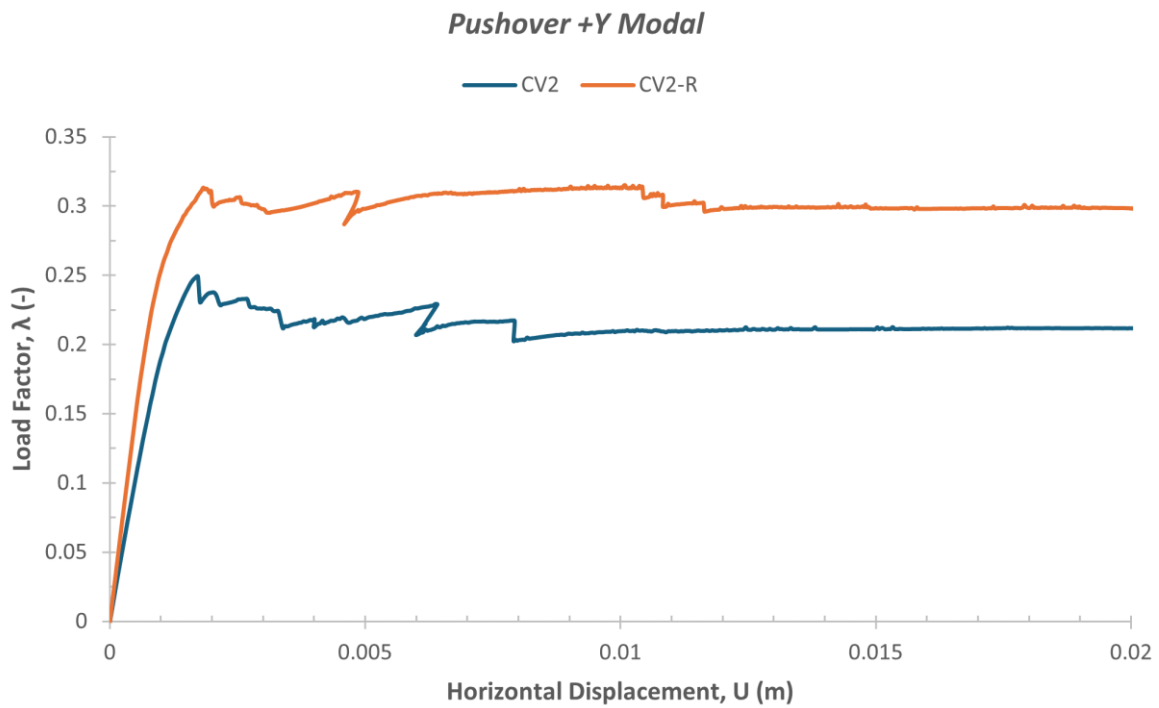
**Figure 5.5** – Comparison of -X pushover curves for CV1 & CV1-R.



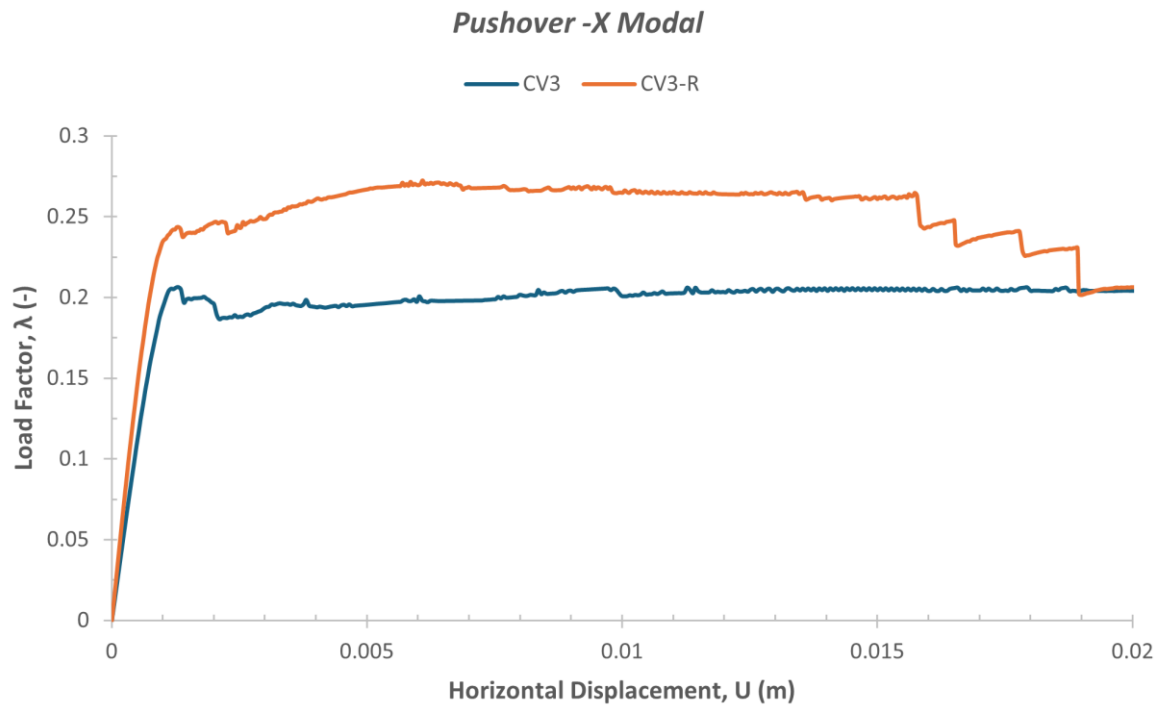
**Figure 5.6** – Comparison of +Y pushover curves for CV1 & CV1-R.



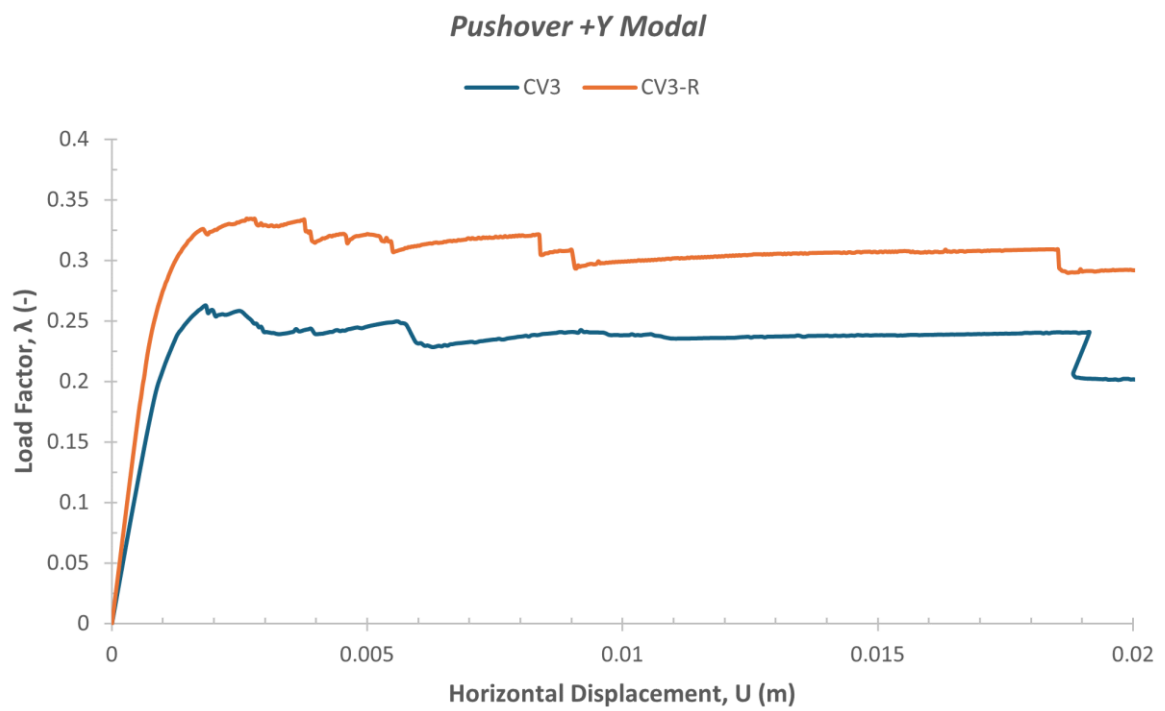
**Figure 5.7** – Comparison of -X pushover curves for CV2 & CV2-R.



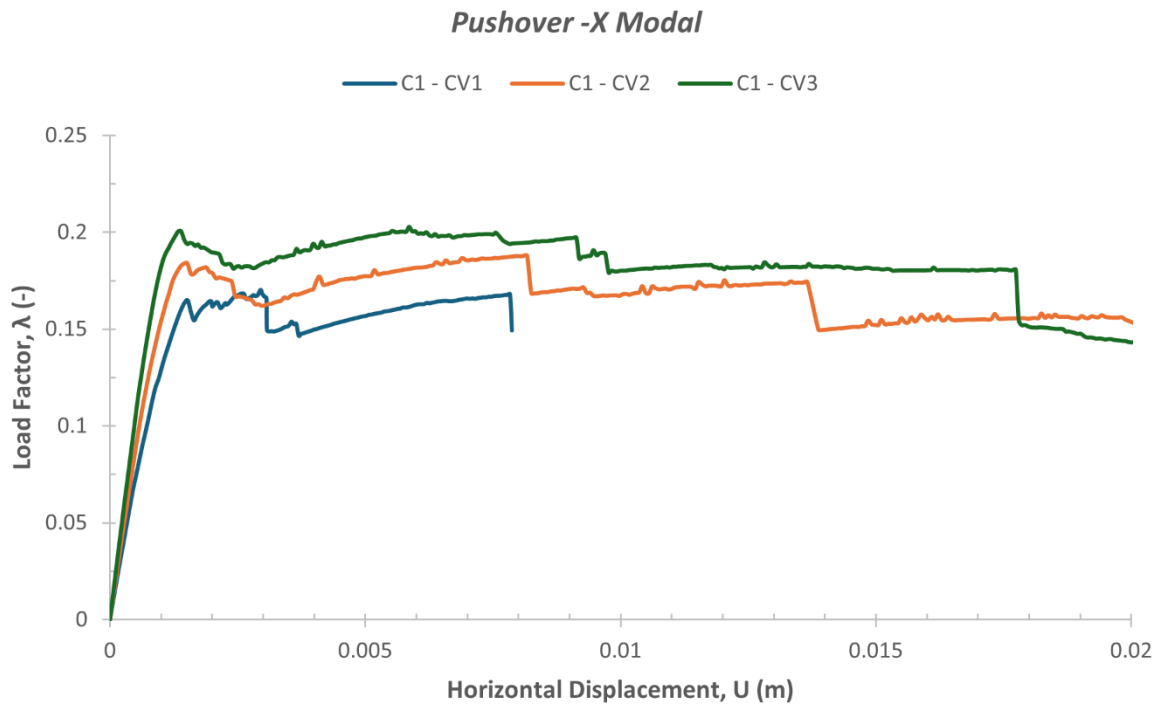
**Figure 5.8** – Comparison of +Y pushover curves for CV2 & CV2-R.



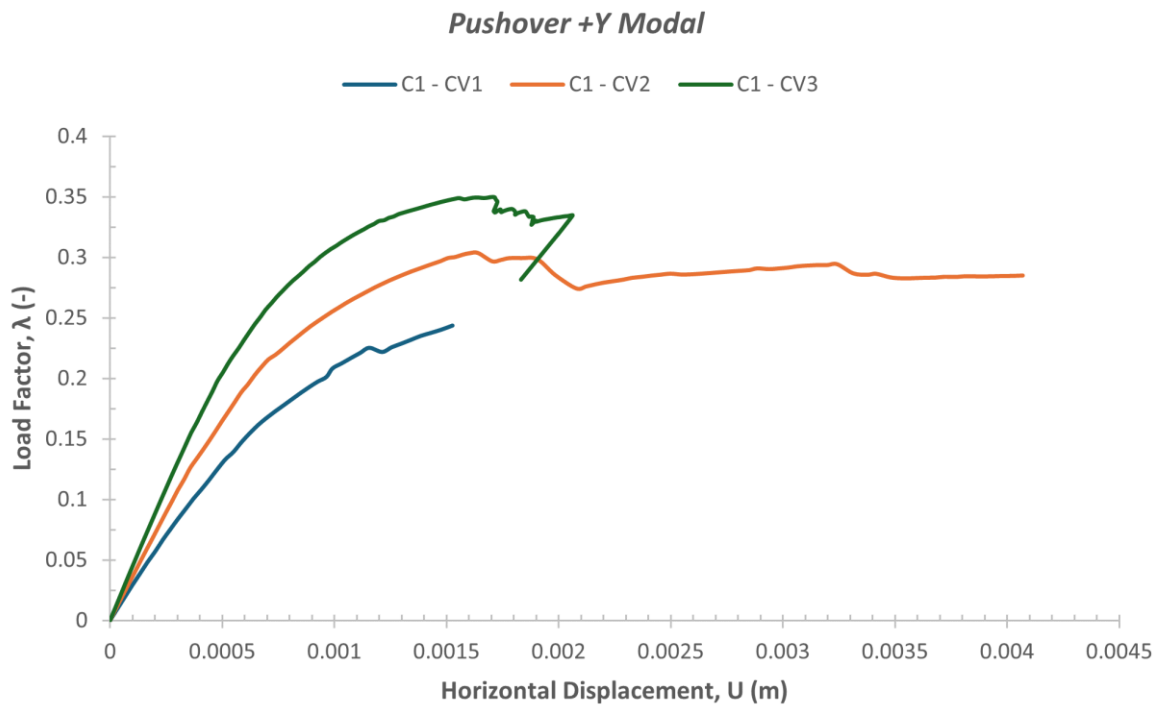
**Figure 5.9** – Comparison of -X pushover curves for CV3 & CV3-R.



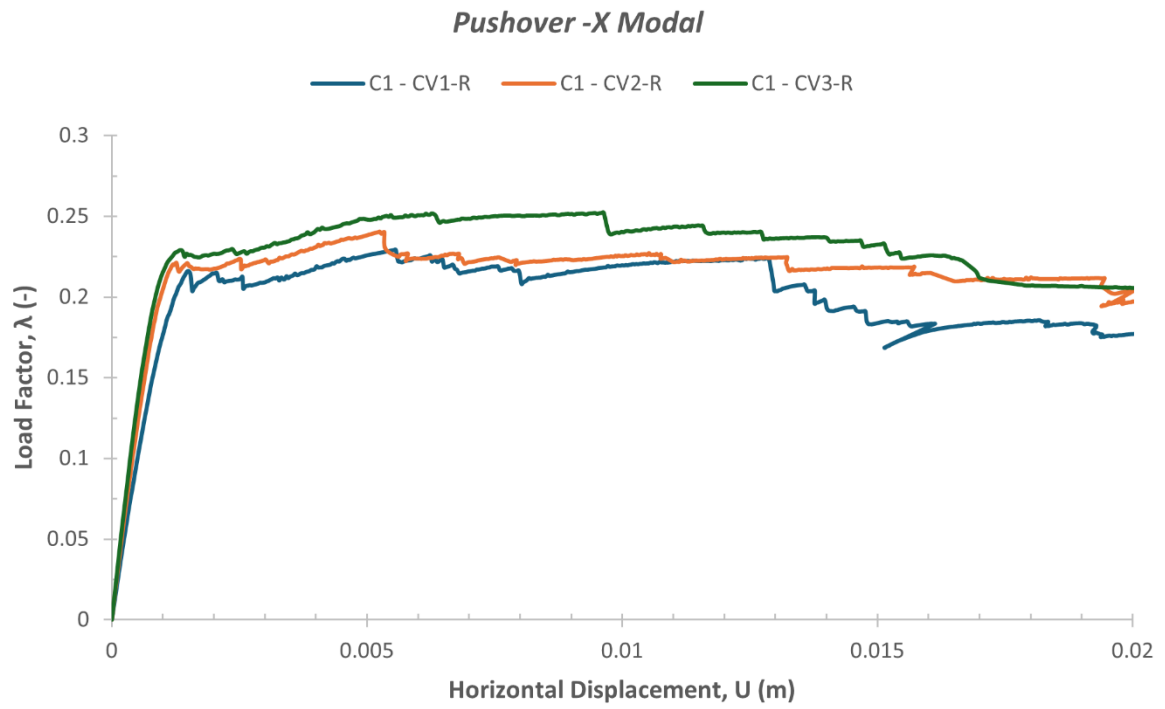
**Figure 5.10** – Comparison of +Y pushover curves for CV3 & CV3-R.



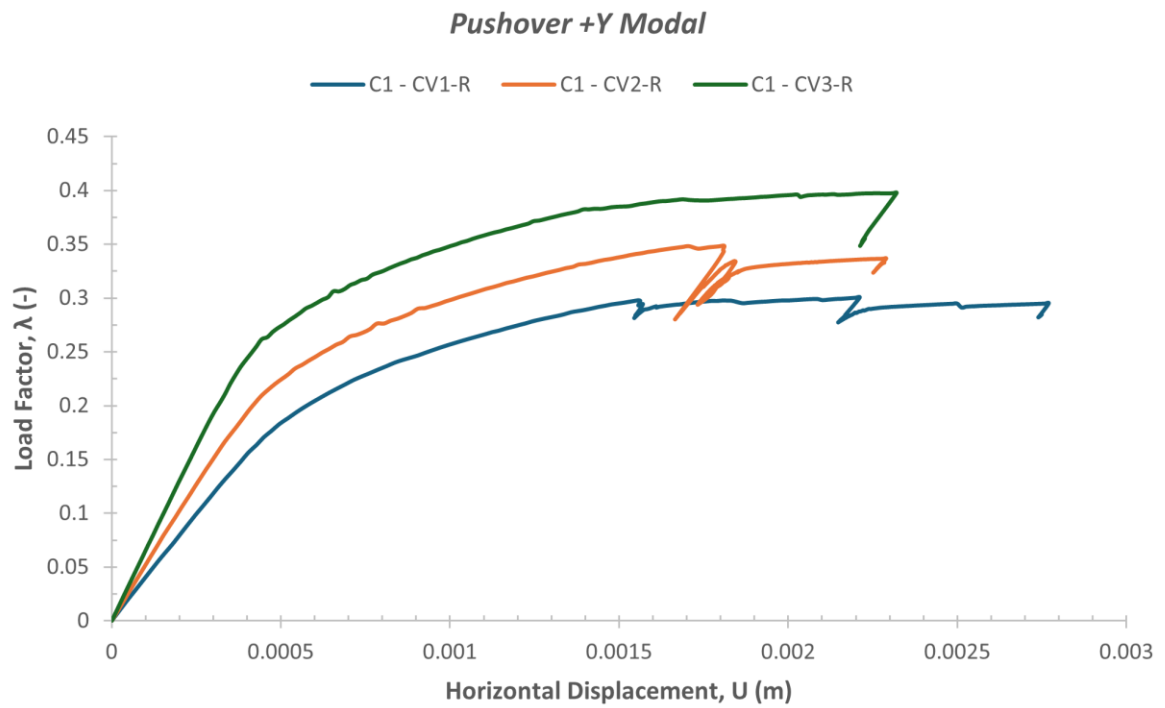
**Figure 5.11** – Comparison of -X pushover curves for Aggregate Configuration 1 of CV1, CV2 & CV3.



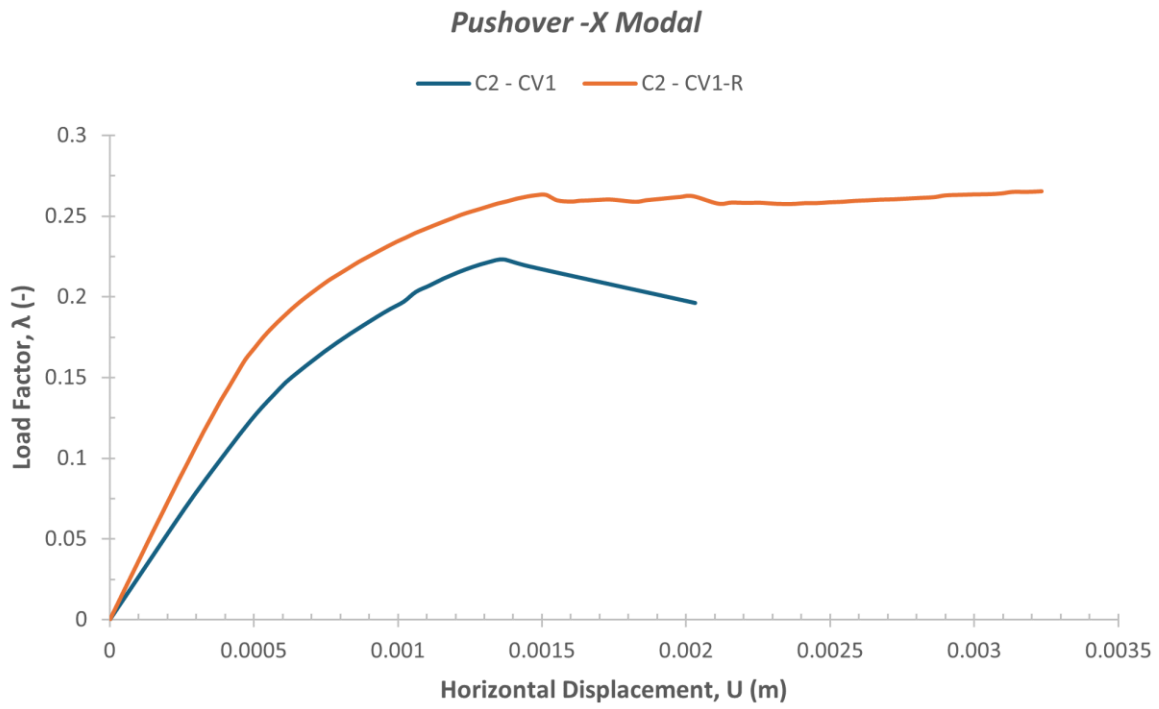
**Figure 5.12** – Comparison of +Y pushover curves for Aggregate Configuration 1 of CV1, CV2 & CV3.



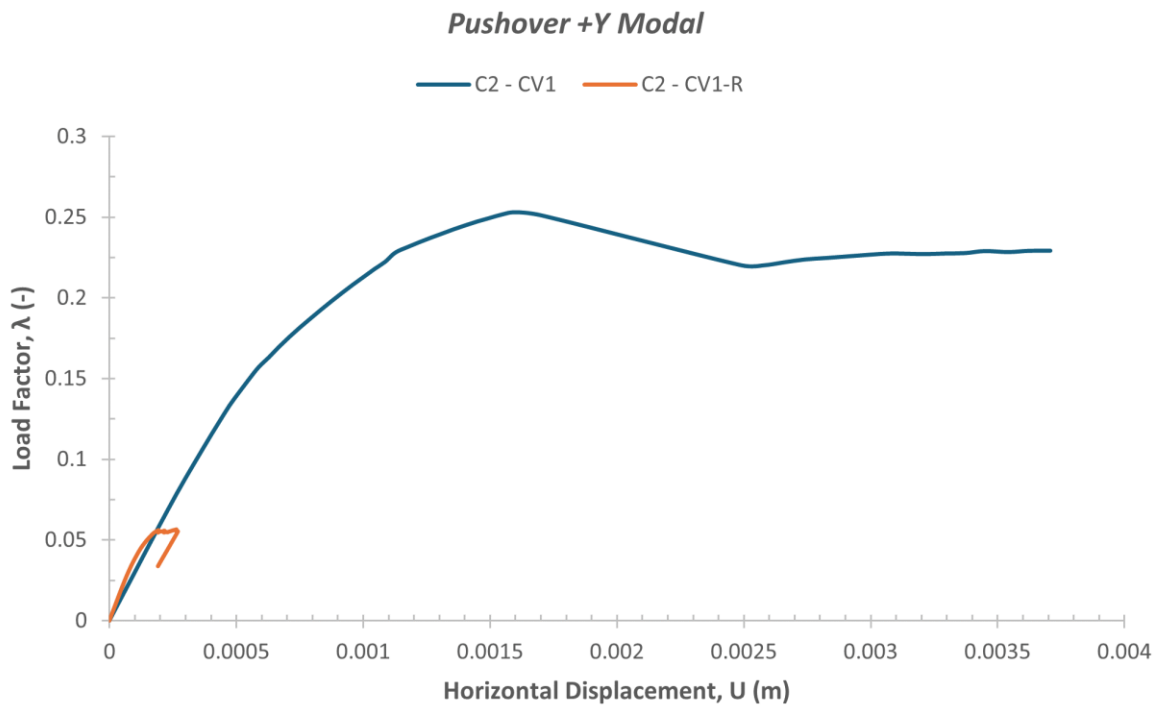
**Figure 5.13** – Comparison of -X pushover curves for Aggregate Configuration 1 of CV1-R, CV2-R & CV3-R.



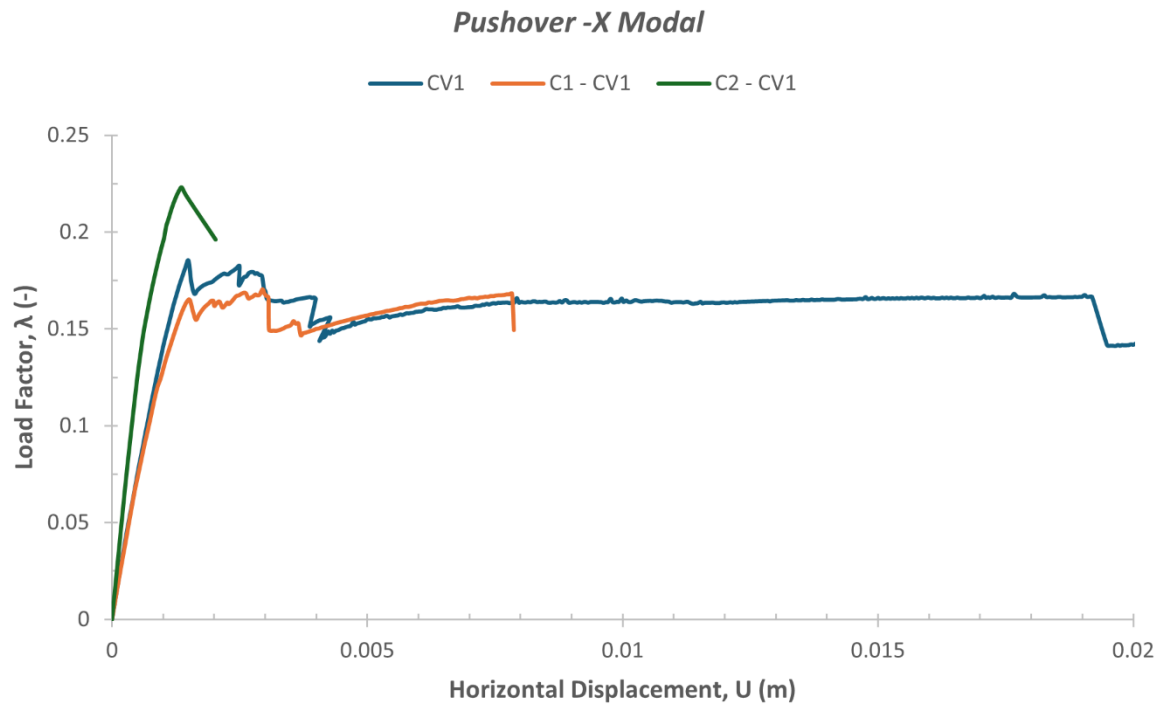
**Figure 5.14** – Comparison of +Y pushover curves for Aggregate Configuration 1 of CV1-R, CV2-R & CV3-R.



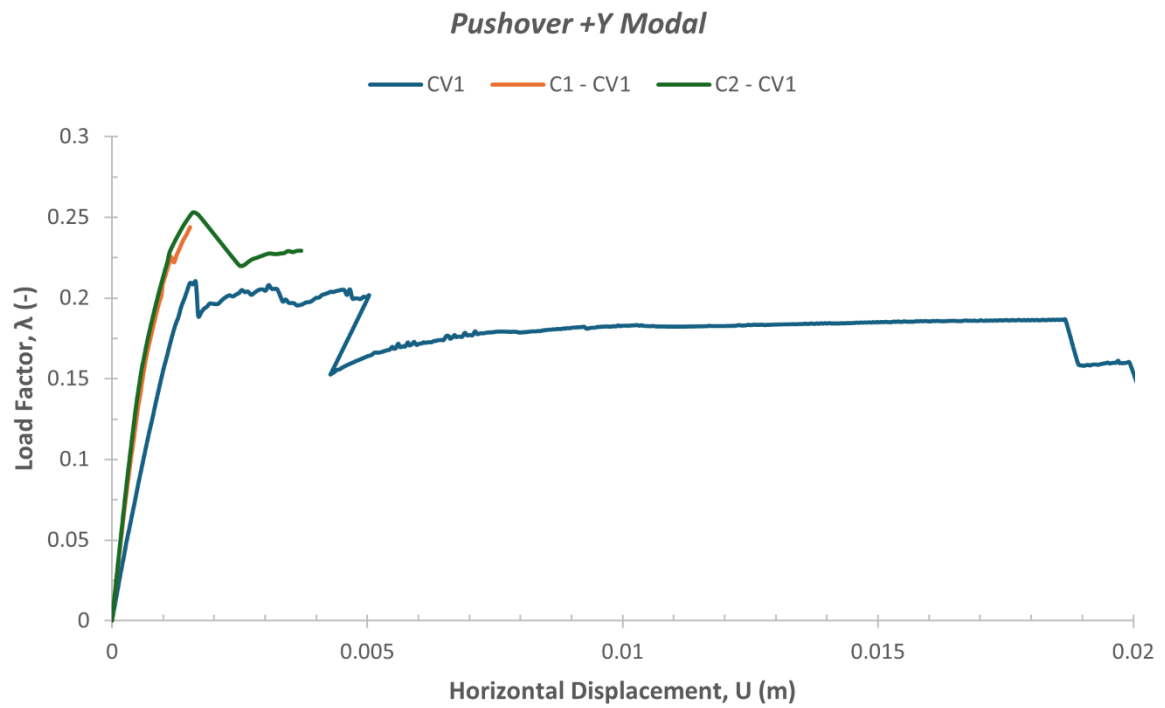
**Figure 5.15** – Comparison of -X pushover curves for Aggregate Configuration 2 of CV1 & CV1-R.



**Figure 5.16** – Comparison of +Y pushover curves for Aggregate Configuration 2 of CV1 & CV1-R (refer to note in section 5.1).

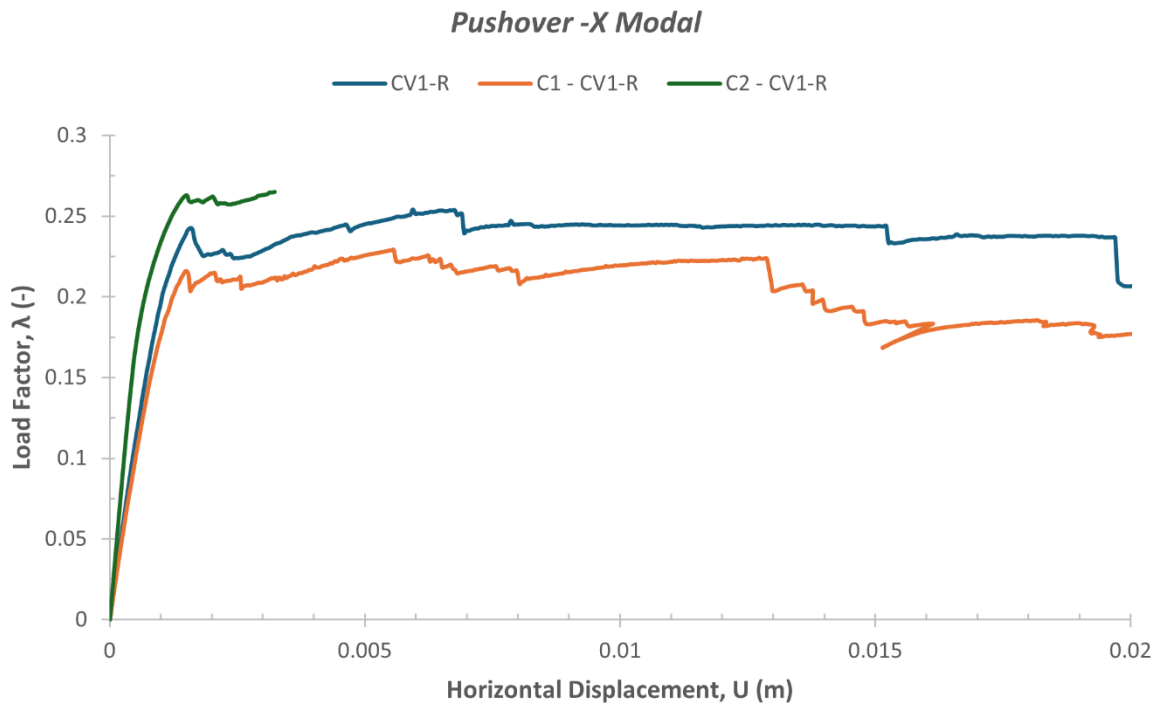


**Figure 5.17** – Comparison of -X pushover curves for CV1, C1 – CV1 & C2 – CV1.

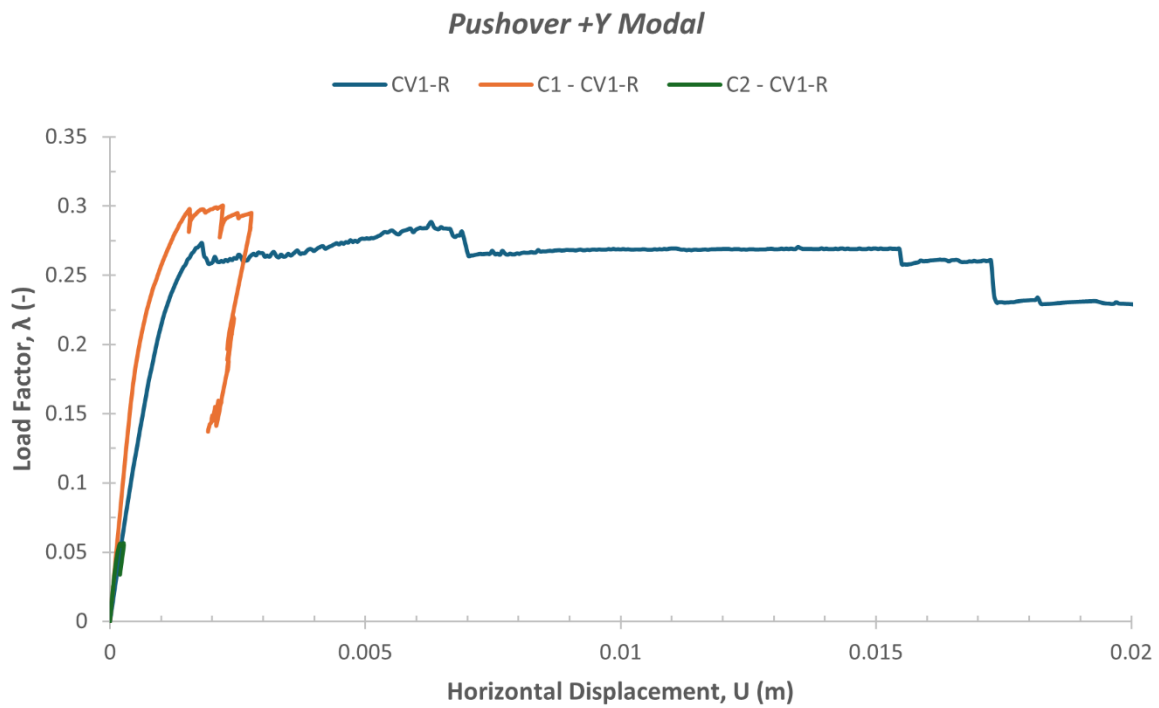


**Figure 5.18** – Comparison of +Y pushover curves for CV1, C1 – CV1 & C2 – CV1.





**Figure 5.19** – Comparison of -X pushover curves for CV1-R, C1 – CV1-R & C2 – CV1-R.



**Figure 5.20** – Comparison of +Y pushover curves for CV1-R, C1 – CV1-R & C2 – CV1-R (refer to note in section 5.1).

### 5.1.2 Peak Load Factor Values

| Plotted Graph            | CV1          |        | CV2          |        | CV3          |        |
|--------------------------|--------------|--------|--------------|--------|--------------|--------|
|                          | $\lambda$    | U (m)  | $\lambda$    | U (m)  | $\lambda$    | U (m)  |
| <i>Pushover +X Modal</i> | <b>0.185</b> | 0.0015 | <b>0.203</b> | 0.0014 | <b>0.207</b> | 0.0013 |
| <i>Pushover -X Modal</i> | <b>0.185</b> | 0.0015 | <b>0.202</b> | 0.0014 | <b>0.208</b> | 0.0241 |
| <i>Pushover +Y Modal</i> | <b>0.210</b> | 0.0016 | <b>0.249</b> | 0.0017 | <b>0.262</b> | 0.0018 |
| <i>Pushover -Y Modal</i> | <b>0.210</b> | 0.0016 | <b>0.249</b> | 0.0017 | <b>0.263</b> | 0.0019 |

**Table 5.1** – Peak load factor ( $\lambda$ ) values of CV1, CV2 & CV3.

| Plotted Graph            | CV1-R        |        | CV2-R        |        | CV3-R        |        |
|--------------------------|--------------|--------|--------------|--------|--------------|--------|
|                          | $\lambda$    | U (m)  | $\lambda$    | U (m)  | $\lambda$    | U (m)  |
| <i>Pushover +X Modal</i> | <b>0.254</b> | 0.0068 | <b>0.267</b> | 0.0062 | <b>0.271</b> | 0.0064 |
| <i>Pushover -X Modal</i> | <b>0.254</b> | 0.0059 | <b>0.268</b> | 0.0069 | <b>0.272</b> | 0.0061 |
| <i>Pushover +Y Modal</i> | <b>0.288</b> | 0.0063 | <b>0.315</b> | 0.0101 | <b>0.334</b> | 0.0026 |
| <i>Pushover -Y Modal</i> | <b>0.286</b> | 0.0066 | <b>0.315</b> | 0.0098 | <b>0.338</b> | 0.0026 |

**Table 5.2** – Peak load factor ( $\lambda$ ) values of CV1-R, CV2-R & CV3-R.

| Compared Tests           | Pushover $\pm$ X Modal | Pushover $\pm$ Y Modal |
|--------------------------|------------------------|------------------------|
| <b>CV1 &amp; CV2</b>     | 9.87%                  | 18.54%                 |
| <b>CV1 &amp; CV3</b>     | 11.93%                 | 24.90%                 |
| <b>CV1-R &amp; CV2-R</b> | 4.99%                  | 9.23%                  |
| <b>CV1-R &amp; CV3-R</b> | 6.66%                  | 15.94%                 |
| <b>CV1 &amp; CV1-R</b>   | 37.29%                 | 37.31%                 |
| <b>CV2 &amp; CV2-R</b>   | 31.19%                 | 26.53%                 |
| <b>CV3 &amp; CV3-R</b>   | 30.82%                 | 27.47%                 |

**Table 5.3** – Percentage increase in peak load factor value between tests.

| Plotted Graph            | C1 - CV1     |        | C1 - CV2     |        | C1 - CV3     |        |
|--------------------------|--------------|--------|--------------|--------|--------------|--------|
|                          | $\lambda$    | U (m)  | $\lambda$    | U (m)  | $\lambda$    | U (m)  |
| <i>Pushover +X Modal</i> | <b>0.170</b> | 0.0028 | <b>0.188</b> | 0.0082 | <b>0.201</b> | 0.0014 |
| <i>Pushover -X Modal</i> | <b>0.170</b> | 0.0029 | <b>0.188</b> | 0.0082 | <b>0.203</b> | 0.0059 |
| <i>Pushover +Y Modal</i> | <b>0.244</b> | 0.0015 | <b>0.304</b> | 0.0016 | <b>0.350</b> | 0.0017 |
| <i>Pushover -Y Modal</i> | <b>0.243</b> | 0.0015 | <b>0.304</b> | 0.0016 | <b>0.350</b> | 0.0017 |

**Table 5.4** - Peak load factor ( $\lambda$ ) values of Aggregate Configuration 1 of CV1, CV2 & CV3.

| Plotted Graph            | C1 - CV1-R   |        | C1 - CV2-R   |        | C1 - CV3-R   |        |
|--------------------------|--------------|--------|--------------|--------|--------------|--------|
|                          | $\lambda$    | U (m)  | $\lambda$    | U (m)  | $\lambda$    | U (m)  |
| <i>Pushover +X Modal</i> | <b>0.230</b> | 0.0054 | <b>0.240</b> | 0.0057 | <b>0.255</b> | 0.0094 |
| <i>Pushover -X Modal</i> | <b>0.229</b> | 0.0056 | <b>0.240</b> | 0.0053 | <b>0.252</b> | 0.0096 |
| <i>Pushover +Y Modal</i> | <b>0.300</b> | 0.0022 | <b>0.349</b> | 0.0018 | <b>0.397</b> | 0.0023 |
| <i>Pushover -Y Modal</i> | <b>0.300</b> | 0.0022 | <b>0.348</b> | 0.0017 | <b>0.398</b> | 0.0023 |

**Table 5.5** - Peak load factor ( $\lambda$ ) values of Aggregate Configuration 1 of CV1-R, CV2-R & CV3-R.

| Plotted Graph            | C2 - CV1     |        | C2 - CV1-R   |        |
|--------------------------|--------------|--------|--------------|--------|
|                          | $\lambda$    | U (m)  | $\lambda$    | U (m)  |
| <i>Pushover +X Modal</i> | <b>0.223</b> | 0.0014 | <b>0.304</b> | 0.0018 |
| <i>Pushover -X Modal</i> | <b>0.223</b> | 0.0014 | <b>0.265</b> | 0.0032 |
| <i>Pushover +Y Modal</i> | <b>0.253</b> | 0.0016 | <b>0.057</b> | 0.0000 |
| <i>Pushover -Y Modal</i> | <b>0.253</b> | 0.0016 | <b>0.057</b> | 0.0003 |

**Table 5.6** – Peak load factor ( $\lambda$ ) values of Configuration 2 of CV1 & CV1-R (refer to note in section 5.1).

## 5.2 The Effect of Geometry on Strength and Stiffness

One of the main aims of this research was to investigate through parametric variation whether a change in the aspect ratio of a cross-vault on plan has any effect on the seismic behaviour.

The aspect ratio was modified by decreasing the dimension in the Y-direction, with the X dimension kept constant; meaning that there was less masonry to resist the applied load in the Y-direction. It was initially assumed by the author that, as a consequence, the percentage increase in the peak  $\lambda$  would be greater in the X-direction rather than in the Y, since there would be more masonry material. However, the collected results show that the peak  $\lambda$  is greater when the load was applied in the Y-direction. Additionally, from Figures 5.1 – 5.4 and Tables 5.1 – 5.3, it is apparent that there is an increase in the  $\lambda$  of CV3 when compared to that of CV1.

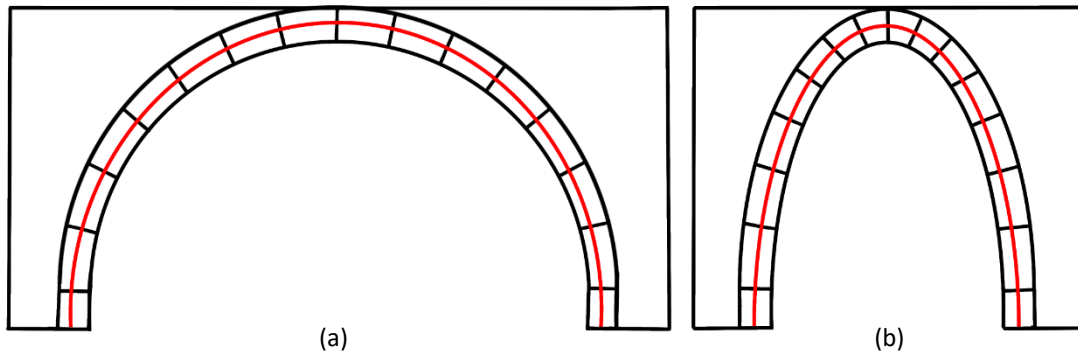
In this case, it can be noted from the results that CV3, which has an aspect ratio of 2:1, demonstrates higher  $\lambda$  values than both CV2 and CV1, which have an aspect ratio of 4:3 and 1:1 respectively. Naturally, CV2 also obtained higher  $\lambda$  values than CV1. In likewise manner as the base models, the peak  $\lambda$  values of CV3-R were also greater than those of CV2-R and CV1-R respectively. This percentage increase is however, of a lesser dimension than the un-ribbed version of the numerical models (CV1 – CV3), possibly due to the geometric definition within the software. Thus, in all singular vault models, the change in aspect ratio translates into an increase in stiffness in the shorter, transverse Y-direction.

Similarly, and as would be expected, the tests of *Configuration 1* also indicate the same trend (despite not having the same exact  $\lambda$  values) since the numerical model of this configuration was created from the base singular vaults. From the pushover curves, it is also clearly visible that the gradient of the curve of CV3 and its variants (CV3-R, C1 – CV3, C1 – CV3-R) is steeper than the gradient of the curves of the compared models. In this regard, the shallower the gradient, the less the sway resistance. This means that the CV3 models are more resistant to sway, and hence further confirms the above.

Such behaviour can be related to plate buckling theory as found in the field of structural analysis. When a rectangular plate is subjected to an increasing uniaxial compressive load in the longitudinal direction (longer dimension), the plate has the tendency to buckle outwards as the load approaches the critical buckling load (just as a column would). If the plate is idealised into a set of thin strips in the longitudinal and transverse directions, the strip in the shorter transverse direction offers more resistance to buckling than the strip in the longer direction, and hence, the shorter the width, the greater the resistance (Timoshenko & Woinowsky-Krieger, 1959).

Thus, it can be argued that a cross-vault with a rectangular plan is stiffer and stronger in the direction of the smaller transverse dimension (in this case the Y). This behaviour is a direct result of the degree of curvature of the vault section. As illustrated in Figure 5.21, CV1 has a semi-circular curve in both directions, while CV3 has a narrower, elliptical curve in the Y-

direction. Due to the length and the geometry of the two-dimensional (2D) arch, the former is easier to compress, while the latter offers more resistance. This also substantiates what was discussed in the literature regarding masonry compressive strength and its limited significance in masonry shell behaviour and stability.



**Figure 5.21** – The curvature of the cross-vaults in 2D: (a) semi-circular (CV1); (b) elliptical (CV3). (Author, 2024).

### 5.3 The Effect of Constructional Aspects on Strength and Stiffness

Apart from geometry, the effect of two constructional aspects was also considered – the addition of diagonal ribs and the aggregate configurations as specified in Chapter 4.

As anticipated, the ribbed vaults resulted in higher peak  $\lambda$  values and steeper gradients, representing an increase in strength, resistance to sway and stability. This outcome, however, does not end the debate on whether the diagonal rib is a structural element or not (refer to subsection 2.4.2). Since the rib is defined and positioned automatically by the software within the geometry of the model, it remains unsure how the rib is meant to perform in the context of the limitations of HiStrA.

With respect to the aggregate effect, it was observed in both configurations (C1 & C2) that the concentration of deformation was always at the edge vault on the opposite side of the point of application of the load (refer to Figures 5.30 & 5.32). For *Configuration 1* tests, this phenomenon was perceived only when the load was applied in the longitudinal direction of the aggregate of cross-vaults (even if the vault was rectangular in plan) and occurred in both negative and positive directions due to symmetry. For *Configuration 2* tests, it occurred in both directions. Such behaviour is deemed reasonable since the edge vault is absorbing the reactions from the adjacent vaults and is acceptable when taking into consideration the incremental nature of the static load application. A non-linear dynamic analysis would have potentially produced a different response due to the constant change in direction of the load, which however is not implementable in HiStrA, and a different software package would have had to be used.

In all instances the resulting peak  $\lambda$  values were substantially greater when tested in *Configuration 1* (C1 – CV2, etc.) in the longitudinal Y direction, in comparison to the singular

vault models. For cross-vault models in aggregate *Configuration 1*, the total seismic mass is tripled, meaning that the seismic force (base shear) is also higher. Similarly, the *Configuration 2* models computationally have nine times the seismic mass of the singular vault models. From the derived results, there is no evidence on the proportionality between the increase in stiffness in the Y-direction for C1 and C2 tests to the increase in seismic mass and force. Nonetheless, the gradient of the C2 – CV1 curve is steeper than the gradient of both C1 – CV1 and CV1 respectively, signifying that in aggregate, sway resistance is amplified despite the increase in base shear.

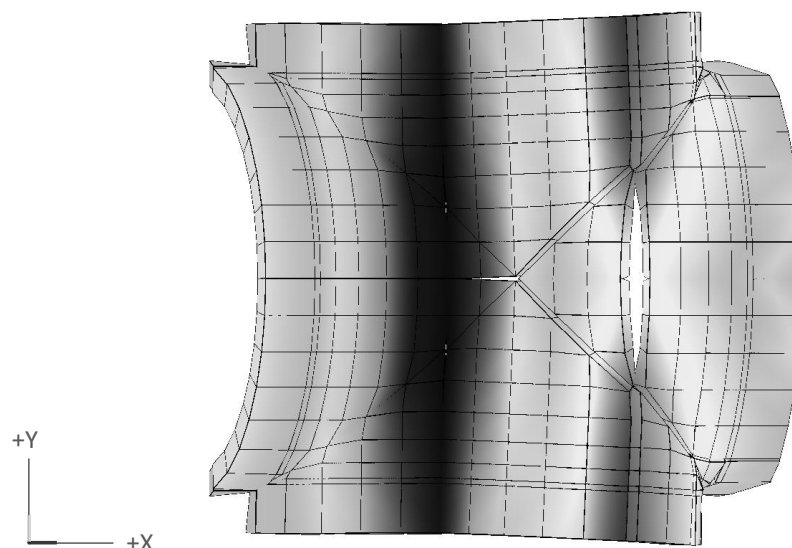
The significant increase exhibits the importance of the effect of diaphragm action in horizontal structural stability within a building. Additionally, the C1 – CV3 and C1 – CV3-R models sustained the largest base shear, further emphasizing the effect the geometry of the curvature has on the stiffness and strength of the cross-vault roof even in aggregate, when having a rectangular rather than a square-shaped cross-vault. These findings supplement further the importance in comprehending the difference between global and local seismic behaviour of the vaults.

## 5.4 Observed Masonry Shell Behaviour

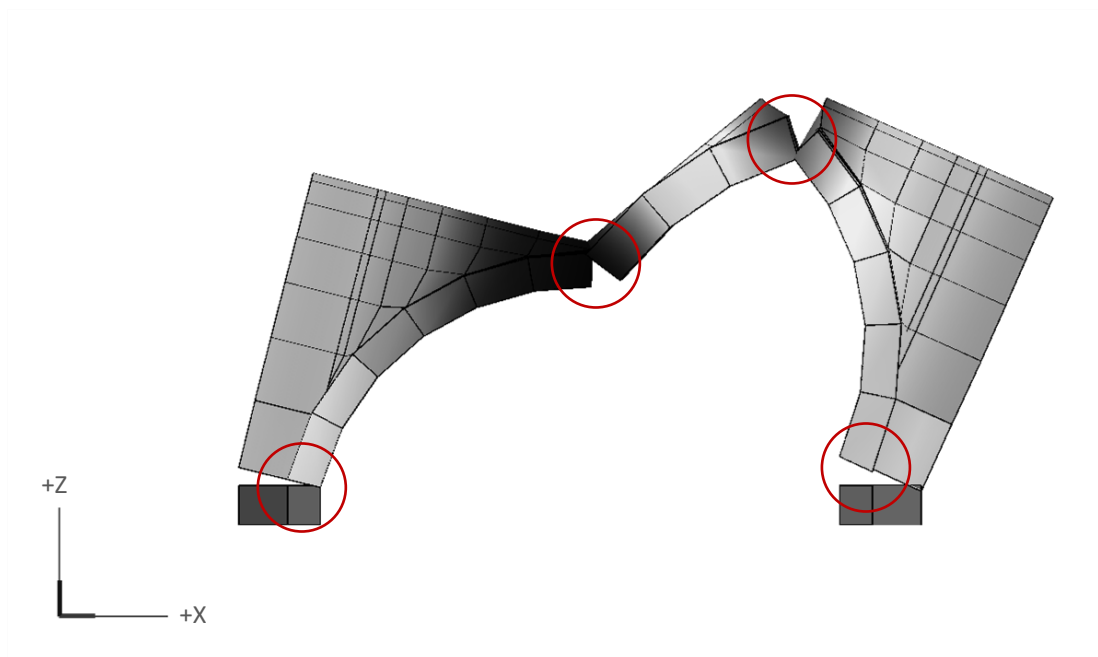
The output from the software allows for the behaviour of the masonry to be identified and described in the context of this study, with respect to the hinging and cracking of the masonry.

### 5.4.1 Deformation

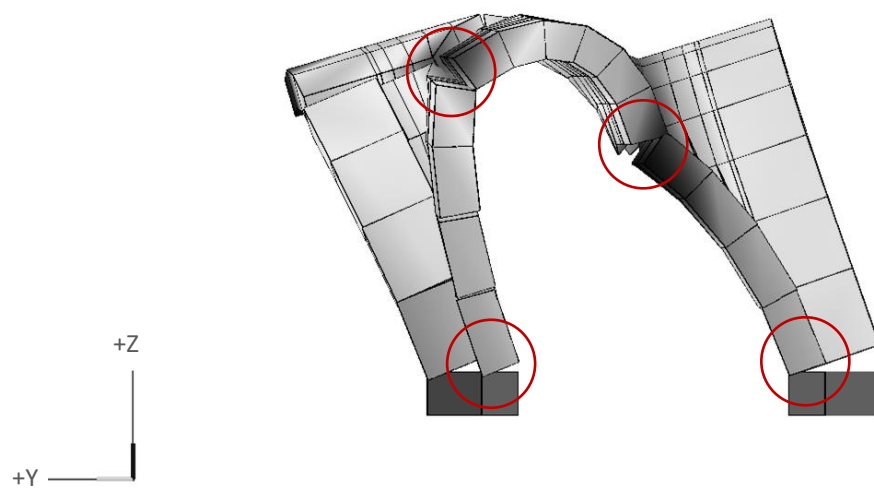
The deformation figures presented in this chapter are in grayscale for clarity. The original images for each performed test are shown in Appendix D.



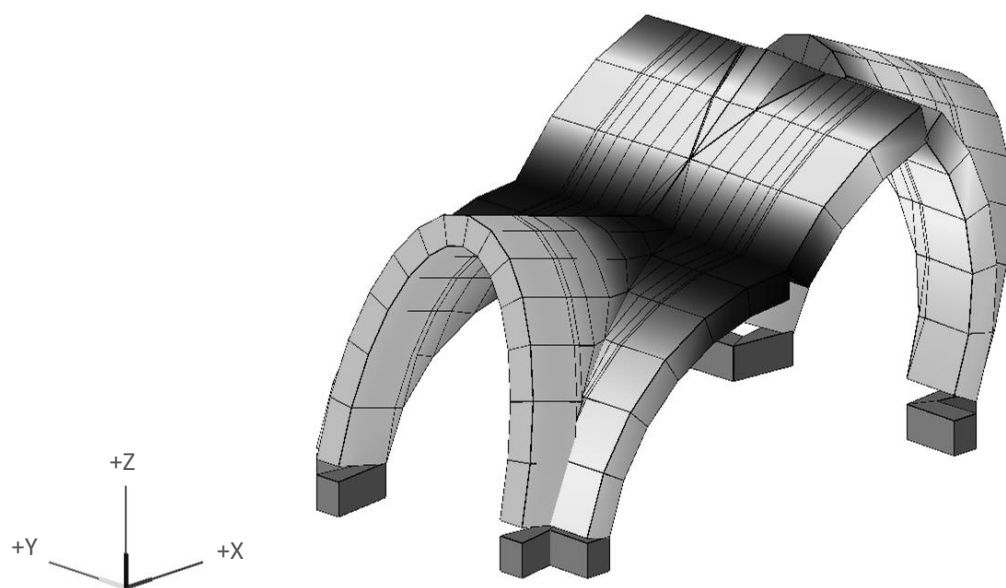
**Figure 5.22** – Plan view of CV1 with pushover in the +X direction.



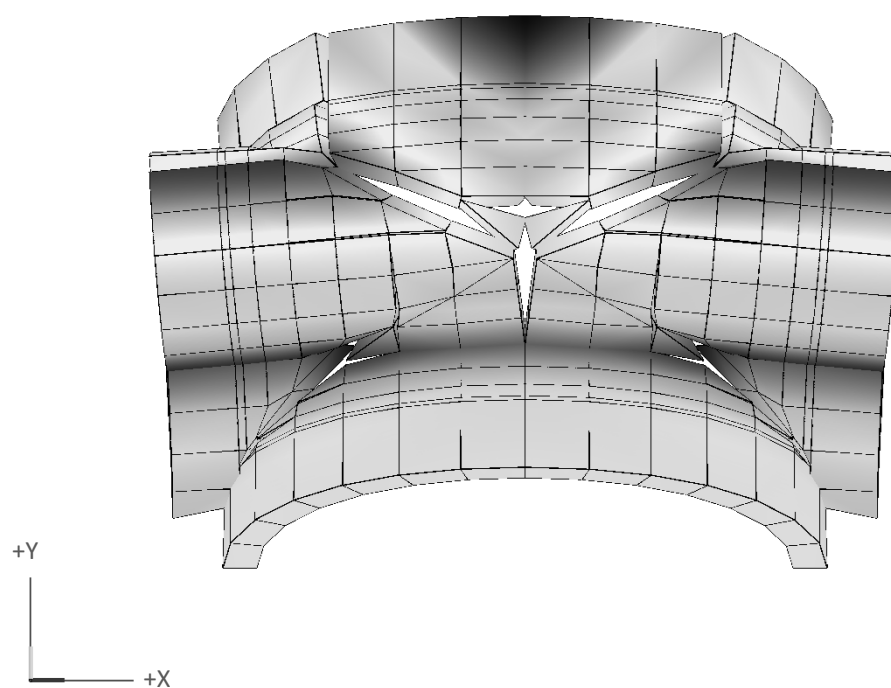
**Figure 5.23** – Four-hinge formation in the +X direction of CV3 (common dimension of all base models).



**Figure 5.24** – Four-hinge formation in the +Y direction of CV3.

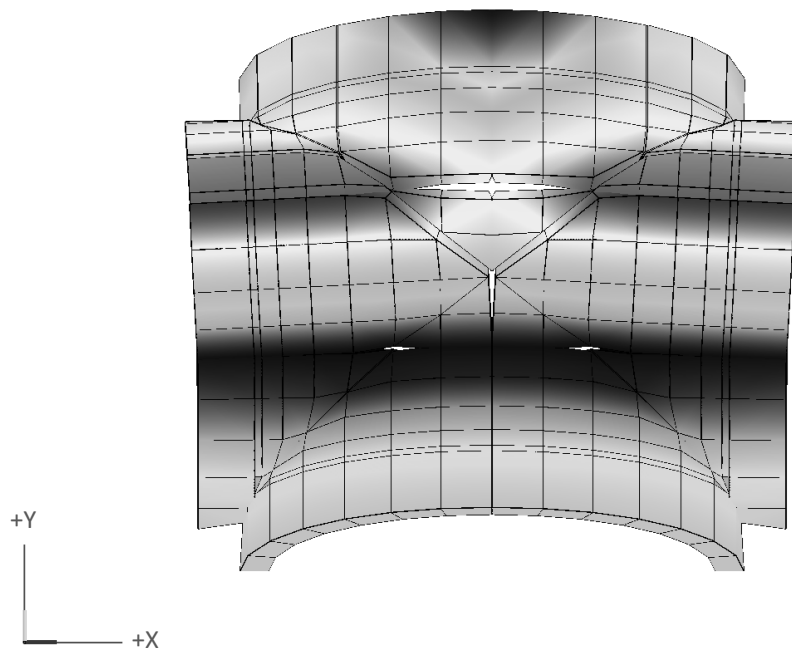


**Figure 5.25** – 3D view of CV3 with pushover in +X direction.

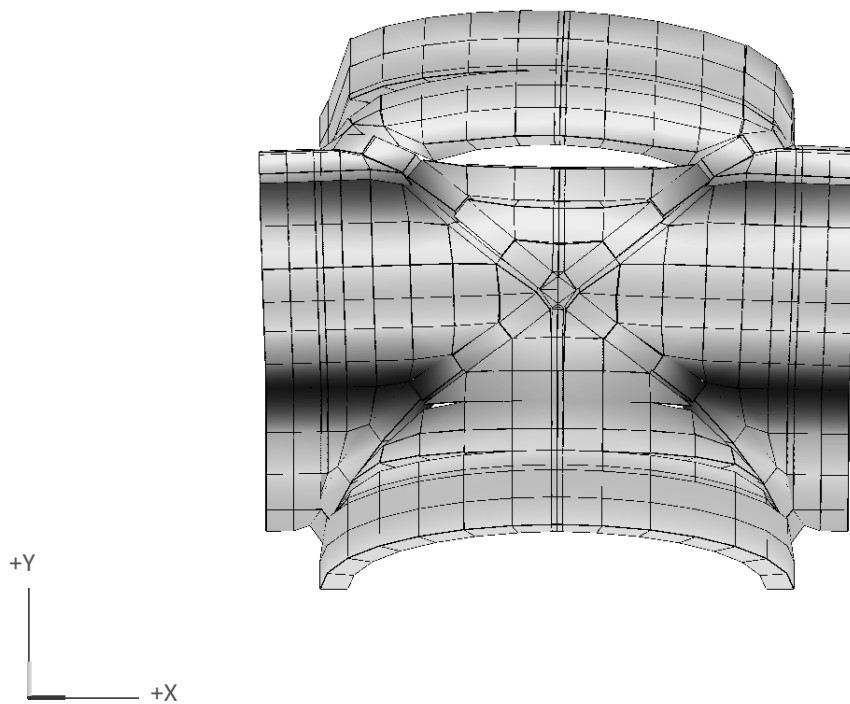


**Figure 5.26** – Plan view of CV3 with pushover in +Y direction.

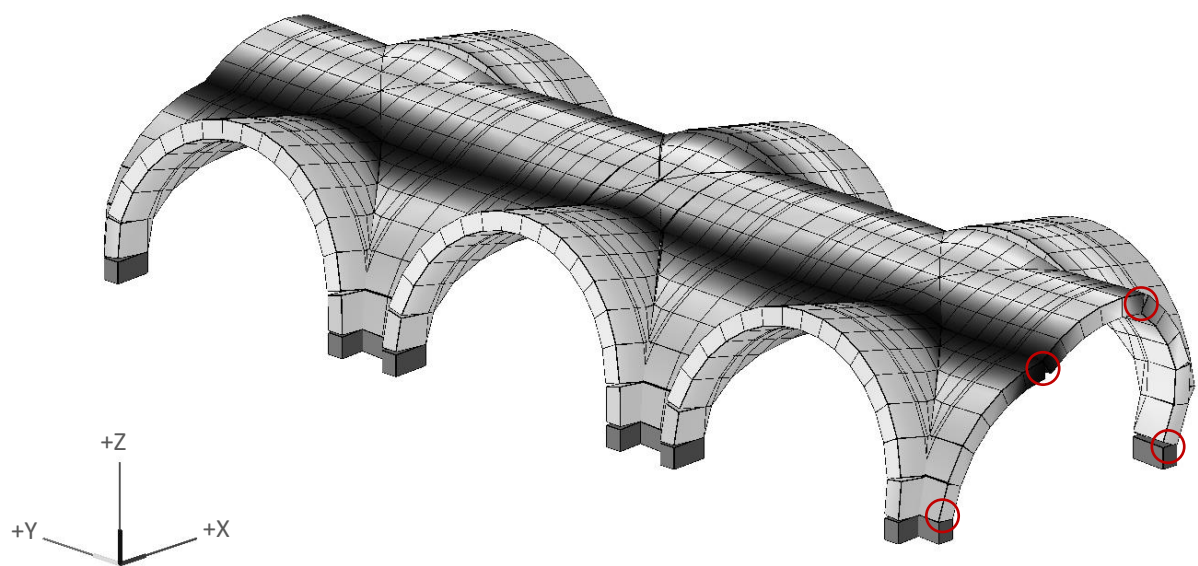




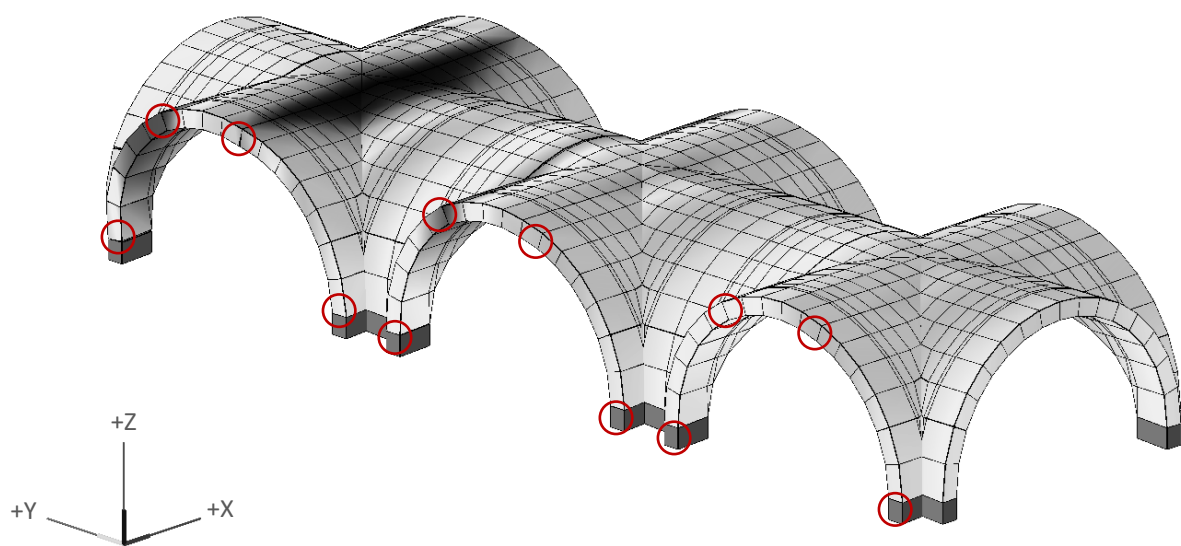
**Figure 5.27** – Plan view of CV2 with pushover in +Y direction.



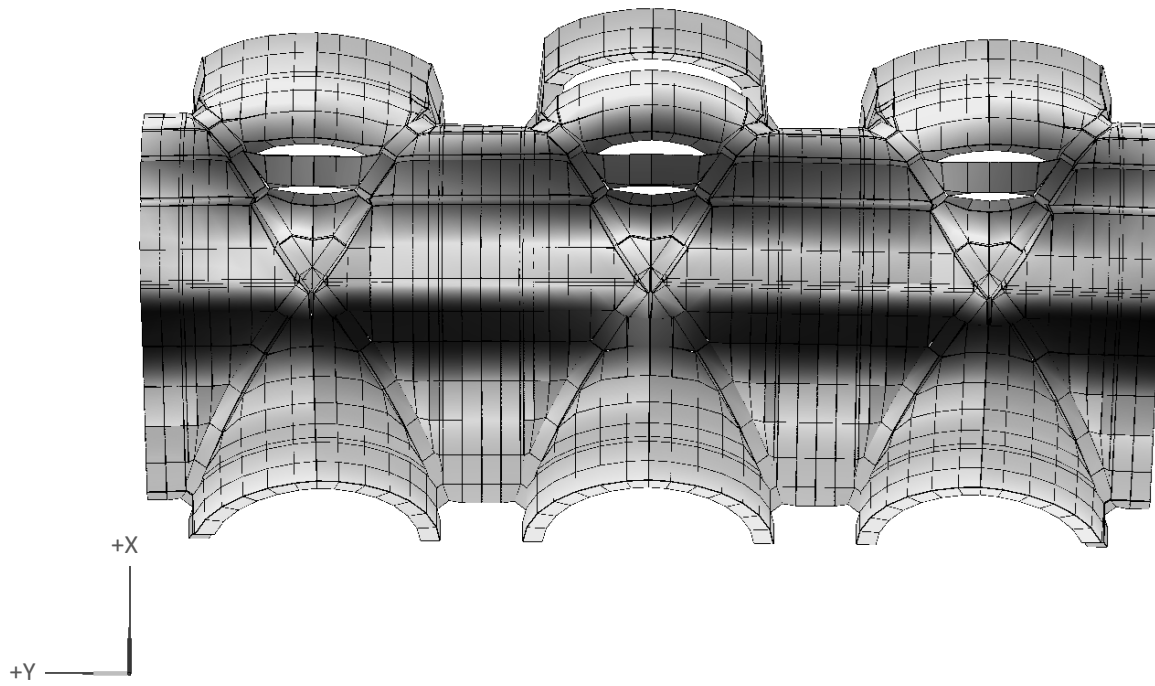
**Figure 5.28** – Plan view of CV2-R with pushover in +Y direction.



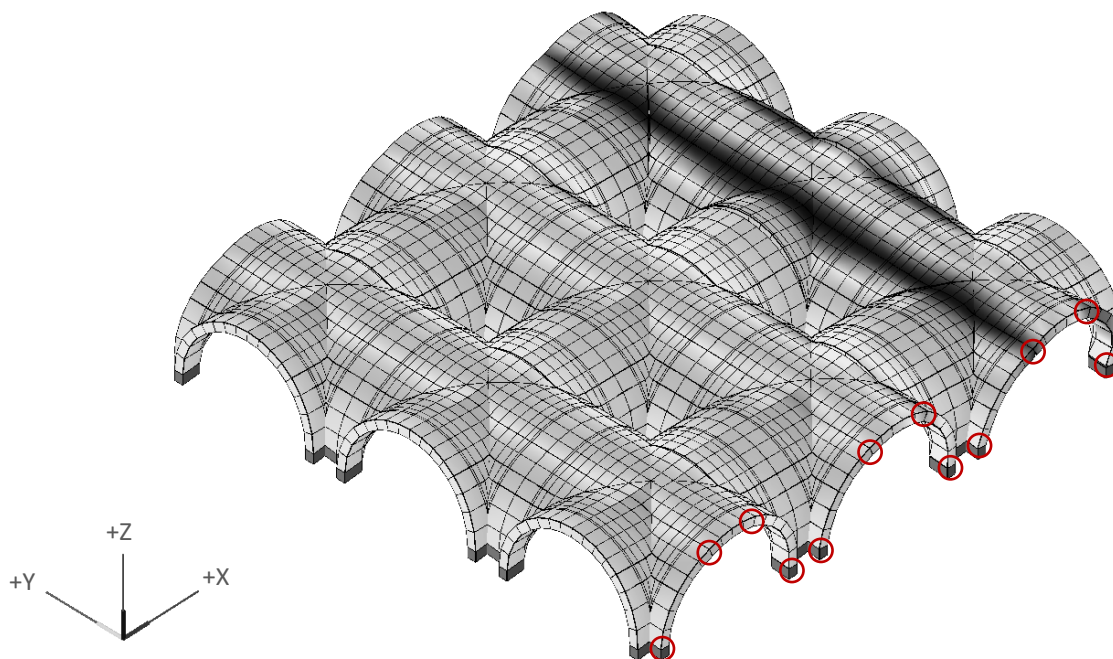
**Figure 5.29** – 3D view of C1 – CV1 with pushover in the +X direction.



**Figure 5.30** – 3D view of C1 – CV1 in the +Y direction.



**Figure 5.31** – Plan view of C1 – CV2-R with pushover in the +X direction.



**Figure 5.32** – 3D view of C2 – CV1 with pushover in the +X direction.

### 5.4.2 Hinge Formation

The development of a four-hinge damage mechanism was singled out from the entire set of results as the most commonly occurring mechanism, and in fact was observed in all 14 tests.

From the set of performed tests, only the geometry of the curvature of the arch was adjusted (by changing the aspect ratio, and hence creating elliptical rather than semi-circular arches), while the thickness of the masonry is uniform. As described by Heyman (1995), stability in masonry arches depends on the position of the thrust line within the confines of the masonry thickness, with the thickness providing the geometrical factor of safety. When subjected to a horizontal load, the ‘invisible’ thrust line distorts and intersects the intrados and extrados of the arch causing the hinge formation. The shape of the thrust line, which depends on the load funicular, characterises the amount of lateral thrust exerted on the supporting abutments.

As recalled in the literature, with the formation of three hinges the masonry arch is statically determinate, and only upon the formation of the fourth hinge would the structure collapse. Through the principle of the line of thrust, the formation of the hinges in the tested models is asymmetrical and alternate in the direction towards which they open to, in the same manner as outlined by Heyman (1995) (refer to Figures 5.23 & 5.24). In cases where the capacity curves reached the target displacement, further hinges formed, signifying a total collapse of the vault. Furthermore, the four-hinge phenomenon is repeatedly mentioned and noted in most of the available and reviewed literature (in Chapter 3) on the performance of cross-vaults subject to seismic load, and hence the observed behaviour in this study can be regarded as valid.

For the purpose of further distinguishing between tests, reference is made to the symmetrical properties of the models:

- A. Models with four lines of symmetry: *CV1, CV1-R, C2 – CV1 & C2 – CV1-R*
- B. Models with two lines of symmetry: *CV2, CV3, CV2-R, CV3-R and all tests of Configuration 1 (C1 – CV1, etc.)*

For type A models, the four-hinge mechanism occurred equally in both X and Y directions as expected, and despite being three-dimensional geometrically complex models, the global behaviour is fundamentally equivalent to that of a two-dimensional masonry arch.

For type B models, the development of this mechanism was not identical when comparing the deformation in the two principal directions. In this regard, the singular vault models have the shorter dimension in the Y while for the models of *Configuration 1*, the X-direction possesses the shorter length. In spite of this, the deformation in the X-direction of all type B models was equivalent to the four-hinge mechanism, with a strong resemblance to the behaviour of the 2D arch. This behaviour is common since the X dimension is 4m for all type B models, regardless of the aspect ratio of the base vault.

In the Y-direction, the behaviour differs slightly despite being representative of the four-hinge mechanism, and such variation is evident in the cracking of the masonry as is outlined in

subsection 5.4.3. In both *Configuration 1* and 2 models, four hinges form in each vaulting bay at the same approximate locations, but as rationalised in section 5.3, the edge vault is more heavily deformed than the rest.

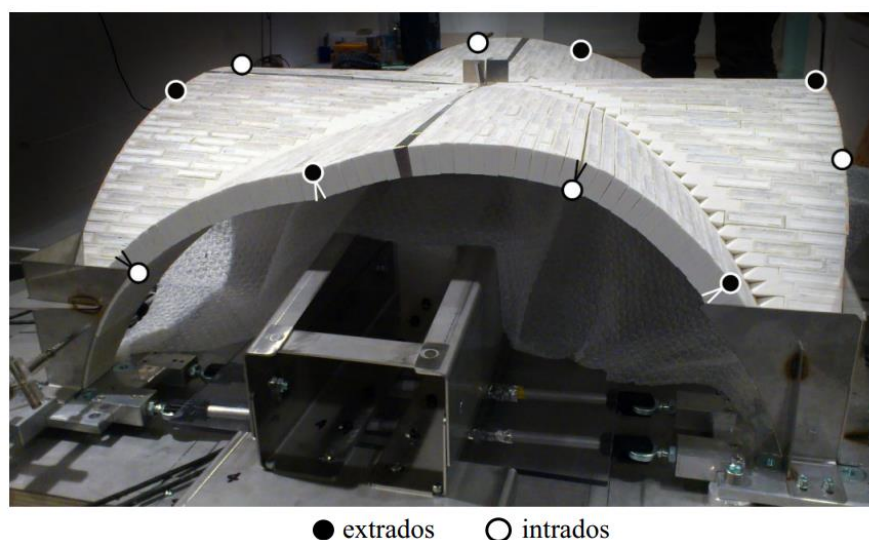
### 5.4.3 Cracking

As a result of the formation of hinges in the cross-vault section, cracks propagate through the vault's webs within the mortar joints between the masonry units. Since the software implements a macro-modelling strategy, the cracking between each element does not represent cracking between individual masonry units, but rather between homogenised blocks, which still provides a tolerable level of accuracy.

Despite the crack formation being relatively similar between all tests, the results provide insight into particular differences between local and global behaviour of the cross-vaults. For all performed tests, the cracking in the negative direction (-) was a mirror image of that in the positive direction (+), and only differed between the X and Y directions. As classified in the previous subsection, the cracking was identical in both directions for the type A models owing to their symmetrical properties.

For the singular vault models, the common X-direction characterised by the four-hinge formation displays straight cracks at the position of the central two hinges. These cracks run directly from one side of the vault to the other, passing over the groins (refer to Figure 5.25). Furthermore, cracking at the crown and at the diagonal groins is visible, as a result of the lateral movement of the vault (refer to Figure 5.22). This is comparable to the behaviour noted in Rossi et al. (2015) and Alforno et al. (2022), further underlining the effect of the limited self-weight of the vault at the crown contributing to the lateral instability (refer to Figure 5.33).

Equally, such behaviour was also observed in the X-direction for the C1 tests with the cracks propagating longitudinally through all three adjacent cross-vaults, but with a more intense



**Figure 5.33** – Observed hinging and damage in shake-table test. Reproduced from Rossi et al. (2015).

concentration of vertical displacement at the central vault (refer to Figure 5.29). In the C2 tests, the cracking is identical as above, in both X and Y (refer to Figure 5.32).

In the Y-direction however, the behaviour is different between the un-ribbed singular vault models and the aggregate *Configuration 1* models. In the former, the cracking is wider and more pronounced at the groins, and cracks formed perpendicularly to the hinging cracks, extending to the lateral arch of the X-direction (refer to Figure 5.26). As addressed in the literature review, the groins are the most stressed areas of the cross-vault since they receive thrusts from two directions, and hence it is sensible that cracking at the groins is of a greater degree. However, this may also be dependent on the stereotomy. In this case, the observed fracturing is more typical of the local cross-vault behaviour rather than the global.

For the ribbed singular vaults, although the behaviour is predominantly the same, a squarish or rectangular crack formed at the crown, possibly due to the geometry and the position of the ribs in the model. The cracks at the groins are less noticeable due to the geometry of the diagonal ribs (refer to Figures 5.27 & 5.28). Similar results were obtained in the study conducted by Lengyel et al. (2019), who concludes that the difference in stiffness of the cross-vault's web and rib materialises in local defects, with cracking at the crown and between the masonry webs and ribs.

For the C1 tests, the crack formation of the pushover in the Y-direction is straight at the position of the hinges and repeated for each bay of vaulting. As remarked in subsection 5.3.1, the cracking at the edge vault (considering the load direction) is appreciably more visible (refer to Figures 5.30 & 5.32).

In certain instances of both singular vault models and of *Configuration 1*, the appearance of *Sabouret* cracks as mentioned by Heyman (1995) was noted, with the lateral arch completely detaching from the webs of the cross-vault (refer to Figure 5.31). This specific type of cracking emerged parallel to the cracks resulting from the hinge formation, and which indicates to the vulnerability of the edges of cross-vaults due to secondary effects.

#### **5.4.4 Post-peak Behaviour**

Upon close examination, a definite amount of pushover curves suggest a degree of post-peak masonry residual strength. In many instances, a plateau can be observed, with sharp drops in the  $\lambda$  value, followed by a subsequent gradual rise and further sinking of the  $\lambda$  value. In some cases, it was also observed that the  $\lambda$  reaches a peak at a plateau, with a drop in value occurring at a further point in the curve. The drops in the plateau could have been caused either by numerical instability during the computation of the solution or by the vault experiencing increasing damage. The latter signifies that the vault stiffens slightly, and hence possibly sustains more load.

In this manner, this behaviour corresponds to quasi-ductile behaviour. Ductility is consistently desired by engineers when designing structures as compelled by the design codes. This

ductility denotes the provision of a warning before collapse, which in beam design is represented by plastic hinge formation. Despite masonry being considered as a brittle material, the warning in this case is in the bi-linear nature of the rotation at cracked mortar joints.

When the  $\lambda$  value drops below 80% of the peak value, the structure is presumed as failed and collapse would occur upon formation of the four hinges. In general, the curves were pushed beyond this point, reaching substantial horizontal displacements. There were multiple cases however, of the capacity curve not displaying a plateau due to convergence failure during the analysis.

## **5.5 Seismic Vulnerability Assessment**

### **5.5.1 Deflection Results**

The defined seismic vulnerability criteria allowed for the capacity and demand displacements to be computed automatically by the software, and whose ratio determine the FoS. A safety factor value of less than 1 ( $\alpha < 1$ ), implies failure of the vault for the pushover analysis in a particular direction for any of the three limit states. Tables 5.7 – 5.11 present the deflection results of all tests for the '*Pushover Modal*' analyses.

| Test | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| CV1  | +X Modal                    | DL          | 0.089                      | 0.093                    | 0.96                       |
|      |                             | SD          | 0.973                      | 0.168                    | 5.80                       |
|      |                             | NC          | 1.300                      | 0.791                    | 1.64                       |
|      | -X Modal                    | DL          | 0.089                      | 0.093                    | 0.95                       |
|      |                             | SD          | 0.919                      | 0.168                    | 5.47                       |
|      |                             | NC          | 1.230                      | 0.790                    | 1.56                       |
|      | +Y Modal                    | DL          | 0.093                      | 0.086                    | 1.09                       |
|      |                             | SD          | 0.743                      | 0.147                    | 5.05                       |
|      |                             | NC          | 0.994                      | 0.744                    | 1.34                       |
|      | -Y Modal                    | DL          | 0.093                      | 0.086                    | 1.09                       |
|      |                             | SD          | 0.807                      | 0.147                    | 5.50                       |
|      |                             | NC          | 1.083                      | 0.744                    | 1.46                       |
| CV2  | +X Modal                    | DL          | 0.083                      | 0.071                    | 1.16                       |
|      |                             | SD          | 2.400                      | 0.122                    | 19.69                      |
|      |                             | NC          | 3.203                      | 0.667                    | 4.80                       |
|      | -X Modal                    | DL          | 0.079                      | 0.071                    | 1.11                       |
|      |                             | SD          | 2.972                      | 0.123                    | 24.07                      |
|      |                             | NC          | 3.964                      | 0.668                    | 5.93                       |
|      | +Y Modal                    | DL          | 0.089                      | 0.070                    | 1.27                       |
|      |                             | SD          | 0.460                      | 0.110                    | 4.19                       |
|      |                             | NC          | 0.614                      | 0.648                    | 0.95                       |
|      | -Y Modal                    | DL          | 0.085                      | 0.070                    | 1.22                       |
|      |                             | SD          | 0.595                      | 0.112                    | 5.29                       |
|      |                             | NC          | 0.798                      | 0.651                    | 1.23                       |
| CV3  | +X Modal                    | DL          | 0.069                      | 0.056                    | 1.24                       |
|      |                             | SD          | 4.279                      | 0.092                    | 46.72                      |
|      |                             | NC          | 5.708                      | 0.573                    | 9.96                       |
|      | -X Modal                    | DL          | 0.071                      | 0.056                    | 1.27                       |
|      |                             | SD          | 5.097                      | 0.092                    | 55.65                      |
|      |                             | NC          | 6.800                      | 0.573                    | 11.87                      |
|      | +Y Modal                    | DL          | 0.082                      | 0.058                    | 1.42                       |
|      |                             | SD          | 1.124                      | 0.083                    | 13.60                      |
|      |                             | NC          | 1.502                      | 0.572                    | 2.62                       |
|      | -Y Modal                    | DL          | 0.081                      | 0.058                    | 1.41                       |
|      |                             | SD          | 1.124                      | 0.083                    | 13.55                      |
|      |                             | NC          | 1.500                      | 0.573                    | 2.62                       |

Table 5.7 – Safety factor ( $\alpha$ ) values for CV1, CV2 & CV3.



| Test  | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|-------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| CV1-R | +X Modal                    | DL          | 0.090                      | 0.058                    | 1.54                       |
|       |                             | SD          | 1.239                      | 0.083                    | 14.85                      |
|       |                             | NC          | 1.655                      | 0.566                    | 2.93                       |
|       | -X Modal                    | DL          | 0.090                      | 0.058                    | 1.54                       |
|       |                             | SD          | 1.190                      | 0.083                    | 14.26                      |
|       |                             | NC          | 1.589                      | 0.566                    | 2.81                       |
|       | +Y Modal                    | DL          | 0.093                      | 0.054                    | 1.72                       |
|       |                             | SD          | 1.043                      | 0.078                    | 13.45                      |
|       |                             | NC          | 1.392                      | 0.528                    | 2.64                       |
|       | -Y Modal                    | DL          | 0.093                      | 0.054                    | 1.72                       |
|       |                             | SD          | 1.064                      | 0.077                    | 13.76                      |
|       |                             | NC          | 1.419                      | 0.528                    | 2.69                       |
| CV2-R | +X Modal                    | DL          | 0.079                      | 0.049                    | 1.62                       |
|       |                             | SD          | 1.790                      | 0.070                    | 25.59                      |
|       |                             | NC          | 2.390                      | 0.503                    | 4.75                       |
|       | -X Modal                    | DL          | 0.081                      | 0.049                    | 1.64                       |
|       |                             | SD          | 1.807                      | 0.070                    | 25.79                      |
|       |                             | NC          | 2.412                      | 0.504                    | 4.79                       |
|       | +Y Modal                    | DL          | 0.082                      | 0.044                    | 1.86                       |
|       |                             | SD          | 1.337                      | 0.063                    | 21.11                      |
|       |                             | NC          | 1.784                      | 0.459                    | 3.88                       |
|       | -Y Modal                    | DL          | 0.082                      | 0.044                    | 1.86                       |
|       |                             | SD          | 1.339                      | 0.063                    | 21.12                      |
|       |                             | NC          | 1.789                      | 0.460                    | 3.89                       |
| CV3-R | +X Modal                    | DL          | 0.065                      | 0.040                    | 1.62                       |
|       |                             | SD          | 1.080                      | 0.057                    | 18.85                      |
|       |                             | NC          | 1.440                      | 0.450                    | 3.20                       |
|       | -X Modal                    | DL          | 0.070                      | 0.041                    | 1.72                       |
|       |                             | SD          | 1.049                      | 0.058                    | 18.05                      |
|       |                             | NC          | 1.401                      | 0.446                    | 3.14                       |
|       | +Y Modal                    | DL          | 0.064                      | 0.034                    | 1.87                       |
|       |                             | SD          | 1.098                      | 0.049                    | 22.40                      |
|       |                             | NC          | 1.466                      | 0.392                    | 3.74                       |
|       | -Y Modal                    | DL          | 0.067                      | 0.034                    | 1.94                       |
|       |                             | SD          | 0.939                      | 0.049                    | 19.18                      |
|       |                             | NC          | 1.256                      | 0.388                    | 3.24                       |

**Table 5.8** – Safety factor ( $\alpha$ ) values for CV1-R, CV2-R & CV3-R.

| Test     | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|----------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| C1 - CV1 | +X Modal                    | DL          | 0.084                      | 0.097                    | 0.86                       |
|          |                             | SD          | 0.686                      | 0.172                    | 3.99                       |
|          |                             | NC          | 0.918                      | 0.793                    | 1.16                       |
|          | -X Modal                    | DL          | 0.089                      | 0.096                    | 0.93                       |
|          |                             | SD          | 0.457                      | 0.170                    | 2.69                       |
|          |                             | NC          | 0.610                      | 0.791                    | 0.77                       |
|          | +Y Modal                    | DL          | 0.067                      | 0.048                    | 1.39                       |
|          |                             | SD          | 0.087                      | 0.068                    | 1.28                       |
|          |                             | NC          | 0.120                      | 0.513                    | 0.23                       |
|          | -Y Modal                    | DL          | 0.074                      | 0.048                    | 1.55                       |
|          |                             | SD          | 0.110                      | 0.068                    | 1.61                       |
|          |                             | NC          | 0.148                      | 0.512                    | 0.29                       |
| C1 - CV2 | +X Modal                    | DL          | 0.075                      | 0.072                    | 1.04                       |
|          |                             | SD          | 0.782                      | 0.132                    | 5.93                       |
|          |                             | NC          | 1.047                      | 0.681                    | 1.54                       |
|          | -X Modal                    | DL          | 0.076                      | 0.072                    | 1.05                       |
|          |                             | SD          | 0.785                      | 0.132                    | 5.96                       |
|          |                             | NC          | 1.048                      | 0.681                    | 1.54                       |
|          | +Y Modal                    | DL          | 0.067                      | 0.039                    | 1.74                       |
|          |                             | SD          | 0.236                      | 0.055                    | 4.26                       |
|          |                             | NC          | 0.317                      | 0.431                    | 0.73                       |
|          | -Y Modal                    | DL          | 0.066                      | 0.038                    | 1.74                       |
|          |                             | SD          | 0.226                      | 0.054                    | 4.19                       |
|          |                             | NC          | 0.303                      | 0.426                    | 0.71                       |
| C1 - CV3 | +X Modal                    | DL          | 0.067                      | 0.057                    | 1.16                       |
|          |                             | SD          | 1.031                      | 0.100                    | 10.31                      |
|          |                             | NC          | 1.374                      | 0.589                    | 2.33                       |
|          | -X Modal                    | DL          | 0.068                      | 0.057                    | 1.19                       |
|          |                             | SD          | 1.014                      | 0.100                    | 10.14                      |
|          |                             | NC          | 1.353                      | 0.589                    | 2.30                       |
|          | +Y Modal                    | DL          | 0.062                      | 0.031                    | 2.02                       |
|          |                             | SD          | 0.103                      | 0.044                    | 2.34                       |
|          |                             | NC          | 0.137                      | 0.360                    | 0.38                       |
|          | -Y Modal                    | DL          | 0.060                      | 0.031                    | 1.97                       |
|          |                             | SD          | 0.120                      | 0.044                    | 2.73                       |
|          |                             | NC          | 0.160                      | 0.361                    | 0.44                       |

**Table 5.9** - Safety factor ( $\alpha$ ) values for Aggregate Configuration 1 of CV1, CV2 & CV3.

| Test       | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|------------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| C1 - CV1-R | +X Modal                    | DL          | 0.083                      | 0.063                    | 1.32                       |
|            |                             | SD          | 0.770                      | 0.098                    | 7.88                       |
|            |                             | NC          | 1.028                      | 0.610                    | 1.69                       |
|            | -X Modal                    | DL          | 0.083                      | 0.064                    | 1.30                       |
|            |                             | SD          | 0.822                      | 0.099                    | 8.34                       |
|            |                             | NC          | 1.099                      | 0.613                    | 1.79                       |
|            | +Y Modal                    | DL          | 0.061                      | 0.034                    | 1.83                       |
|            |                             | SD          | 0.135                      | 0.048                    | 2.82                       |
|            |                             | NC          | 0.181                      | 0.392                    | 0.46                       |
|            | -Y Modal                    | DL          | 0.068                      | 0.037                    | 1.85                       |
|            |                             | SD          | 0.207                      | 0.053                    | 3.94                       |
|            |                             | NC          | 0.281                      | 0.411                    | 0.68                       |
| C1 - CV2-R | +X Modal                    | DL          | 0.075                      | 0.053                    | 1.42                       |
|            |                             | SD          | 1.145                      | 0.076                    | 15.01                      |
|            |                             | NC          | 1.530                      | 0.545                    | 2.81                       |
|            | -X Modal                    | DL          | 0.074                      | 0.053                    | 1.40                       |
|            |                             | SD          | 1.146                      | 0.076                    | 15.03                      |
|            |                             | NC          | 1.531                      | 0.545                    | 2.81                       |
|            | +Y Modal                    | DL          | 0.051                      | 0.025                    | 2.01                       |
|            |                             | SD          | 0.126                      | 0.036                    | 3.51                       |
|            |                             | NC          | 0.169                      | 0.320                    | 0.53                       |
|            | -Y Modal                    | DL          | 0.051                      | 0.025                    | 2.03                       |
|            |                             | SD          | 0.141                      | 0.036                    | 3.92                       |
|            |                             | NC          | 0.189                      | 0.319                    | 0.59                       |
| C1 - CV3-R | +X Modal                    | DL          | 0.068                      | 0.043                    | 1.57                       |
|            |                             | SD          | 1.154                      | 0.061                    | 18.78                      |
|            |                             | NC          | 1.154                      | 0.472                    | 2.44                       |
|            | -X Modal                    | DL          | 0.068                      | 0.043                    | 1.58                       |
|            |                             | SD          | 1.286                      | 0.062                    | 20.74                      |
|            |                             | NC          | 1.286                      | 0.475                    | 2.70                       |
|            | +Y Modal                    | DL          | 0.043                      | 0.019                    | 2.29                       |
|            |                             | SD          | 0.161                      | 0.027                    | 5.96                       |
|            |                             | NC          | 0.159                      | 0.255                    | 0.62                       |
|            | -Y Modal                    | DL          | 0.043                      | 0.019                    | 2.25                       |
|            |                             | SD          | 0.169                      | 0.027                    | 6.22                       |
|            |                             | NC          | 0.169                      | 0.255                    | 0.66                       |

**Table 5.10** – Safety factor ( $\alpha$ ) values for Aggregate Configuration 1 of CV1-R, CV2-R & CV3-R.

| Test       | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|------------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| C2 - CV1   | +X Modal                    | DL          | 0.062                      | 0.050                    | 1.24                       |
|            |                             | SD          | 0.153                      | 0.086                    | 1.78                       |
|            |                             | NC          | 0.205                      | 0.543                    | 0.38                       |
|            | -X Modal                    | DL          | 0.062                      | 0.050                    | 1.23                       |
|            |                             | SD          | 0.114                      | 0.083                    | 1.38                       |
|            |                             | NC          | 0.159                      | 0.541                    | 0.29                       |
|            | +Y Modal                    | DL          | 0.065                      | 0.046                    | 1.42                       |
|            |                             | SD          | 0.215                      | 0.066                    | 3.26                       |
|            |                             | NC          | 0.291                      | 0.503                    | 0.58                       |
|            | -Y Modal                    | DL          | 0.064                      | 0.046                    | 1.39                       |
|            |                             | SD          | 0.217                      | 0.067                    | 3.27                       |
|            |                             | NC          | 0.291                      | 0.505                    | 0.58                       |
| C2 - CV1-R | +X Modal                    | DL          | 0.068                      | 0.021                    | 3.24                       |
|            |                             | SD          | 0.246                      | 0.030                    | 8.17                       |
|            |                             | NC          | 0.334                      | 0.212                    | 1.57                       |
|            | -X Modal                    | DL          | 0.067                      | 0.023                    | 2.85                       |
|            |                             | SD          | 0.222                      | 0.034                    | 6.62                       |
|            |                             | NC          | 0.298                      | 0.253                    | 1.18                       |
|            | +Y Modal                    | DL          | 0.011                      | 0.044                    | 0.26                       |
|            |                             | SD          | 0.018                      | 0.079                    | 0.23                       |
|            |                             | NC          | 0.025                      | 0.369                    | 0.07                       |
|            | -Y Modal                    | DL          | 0.012                      | 0.044                    | 0.27                       |
|            |                             | SD          | 0.018                      | 0.078                    | 0.23                       |
|            |                             | NC          | 0.024                      | 0.368                    | 0.07                       |

**Table 5.11** – Safety factor ( $\alpha$ ) values for Aggregate Configuration 2 of CV1 & CV1-R (refer to note in section 5.1).

### 5.5.2 Limit State Analysis

The deflection results explicitly present the Near Collapse limit state as the most critical from all performed tests. Although much less frequent, only the CV1 and C1 – CV1 models failed in the Damage Limitation limit state. Excluding when  $\alpha < 1$ , the  $\alpha$  values are generally symmetrical between the negative and positive load application for a particular pushover direction.

Failure in the Damage Limitation LS was only observed in the square shaped cross-vaults (CV1 and its variants). This LS implicates that an existing structure is only lightly damaged, and hence, failure in this LS means that a local PGA value of 0.07g would result in damage in the square vaults. Such results demonstrate further the inferiority of the square geometry in comparison to a rectangular vault.

The singular vault models of CV1-R, CV2-R, CV3 and CV3-R all resulted in  $\alpha > 1$  for all three limit states, further manifesting the strengthening effect of the diagonal ribs, and the enhanced stiffness of the rectangular cross-vault over the square one. Despite these outcomes suggesting that local PGA values can be sustained without failure in any of the limit states, these models represent independent cross-vaults, and if forming part of an aggregate, this may not be necessarily true.

It can also be noted that for the rectangular cross-vaults (CV2, CV3, CV2-R and CV3-R), the safety factor of the pushover analysis in the X-direction is generally of a larger magnitude than in the Y-direction, and hence the lateral displacement capacity is greater. However, this does not imply that the stiffness and resistance in the X-direction is greater, since in this direction, lower values of the base shear were sustained as explained in Section 5.2.

For all *Configuration 1* tests apart from C1-CV1, in the  $\pm$  Y-direction,  $\alpha_{NC} < 1$ , signifying failure. The Near Collapse LS denotes heavy damage of the structure, and hence suggests that in aggregate, the global behaviour of cross-vaults is more susceptible to damage in the longitudinal direction. This can be compared to the local behaviour of the vaults, where in the Y-direction, the singular vaults displayed greater stiffness and strength.

## 6 Conclusions and Recommendations for Future Research Work

The primary aim of this dissertation was to assess through numerical modelling, the seismic vulnerability of unreinforced masonry cross-vaults. This research study investigates through parametric variation, the effects of geometrical and constructional parameters on the seismic performance of unreinforced masonry cross-vaults subjected to seismic load based on EC8 guidelines. The studied parameters are the aspect ratio of the cross-vault on plan, the addition of diagonal ribs, and the multiplication of the base cross-vault to simulate an aggregate. Additionally, the study sought to outline masonry shell behaviour, damage and collapse mechanisms and structural stability issues.

The reviewed literature in Chapter 2 provided a good basis for understanding the masonry cross-vault as a three-dimensional shell structure, in terms of its form, geometry and construction techniques, and was essential for the definition of the geometry of the tested numerical models. Furthermore, the static structural behaviour of cross-vaults was also considered before delving into the more complex dynamic behaviour as a result of seismic load. The literature of Chapter 3 additionally highlights certain seismic vulnerability aspects and their importance in comprehending the damage mechanisms that cross-vaults may sustain during earthquakes, allowing for adequate interpretation of results.

The assessment was conducted by adopting a DMEM numerical modelling approach provided by the HiStrA software, within which a total of fourteen models were generated and subsequently tested via pushover analysis. The aforementioned methodology and analysis, substantiated by a thorough literature review in Chapters 2 and 3 ensured that the research goals set out at the inception of this study have been attained.

### 6.1 Concluding Remarks

The results from each test were compiled and analysed, from which behavioural differences and comparisons were established. Based on the findings from the executed pushover analyses, a number of salient conclusions can be made.

Through the comparison of the various models, the findings evidently assert that the aspect ratio on plan inadvertently affects the stiffness and strength of a cross-vault subjected to horizontal seismic load. Contrary to what was assumed, the sway stiffness increases when the cross-vault is rectangular. For the shorter dimension in the direction of the load application (in this case the Y), the recorded base shear was larger than in the longer X coordinate, and the gradient of the pushover curves was steeper. Such behaviour was paralleled to the plate buckling phenomenon as observed in thin steel plates. The results visibly indicate that this behaviour is a repercussion of the change in curvature between the different shaped cross-vaults. By virtue of its geometry and the produced thrust line, a semicircular arch is more easily compressible in comparison to an elliptical arch, and the hypothesis that an elliptical arch is stiffer, can be deemed satisfactory.

It is apparent that the addition of the diagonal ribs to the numerical models resulted in the ribbed vaults sustaining a greater base shear force. Although the derived results highlight the strengthening and reinforcing effect of the ribs (within the limits and capability of the software), whether these are structural or constructional elements remains open for discussion.

In part, the findings underline certain distinctions between local and global cross-vault behaviour. Globally, the aggregate configuration tests show the structural significance of diaphragm action, even in curved geometry. The results also demonstrate that in an aggregate, the edge vault (opposite to the point of load application) is the most deformed, which however, was interpreted in the context of the static incremental nature of the load. Locally, the cracking is more defined, and pronounced fracturing at the groins and the crown was noted.

In both local and global context, the asymmetric four-hinge damage mechanism was perceived as the main cross-vault pathology when subjected to horizontal loads. The paths traced by the cracking between the macro-elements originate from the location of the hinges, and in general the cracking was linear. Despite still developing the four hinges in the Y-direction, for the rectangular vaults (CV2, CV3 and their variants), the cracking varied between the X and Y-directions. In certain instances, secondary fractures were observed, either perpendicular to the straight cracks, square-shaped cracking at the crown of ribbed vaults or *Sabouret* type cracking.

Despite a cross-vault being by nature a complex three-dimensional curved structure, the results show that its behaviour can be assimilated to that of the simple two-dimensional masonry arch. Unmistakably, the findings confirm that masonry tensile strength is limited and that its compressive strength has a negligible effect on the stability of curved masonry shells. Moreover, it can be concluded that the results further confirm the significance of the pressure line principle in understanding seismic performance of curved masonry structures.

From the analysis of the pushover curves, a degree of post-peak masonry behaviour was observed. Although masonry is considered as brittle, this behaviour was reviewed in terms of ductility, with the cross-vaults displaying quasi-ductile behaviour by cracking.

The limit state analysis determined that the *Near Collapse* LS was the least satisfied LS from all the performed tests, characterised by the highest PGA. A more relevant LS in the context of safeguarding the architectural value of heritage and monumental buildings (where cross-vaults are typically found) is the *Damage Limitation* LS, where failure in this LS denotes that a PGA of 0.07g with a return period of 225 years would not be sustained by the tested models.

To summarise, cross-vaults represent a structurally intricate and extraordinary roofing typology, and although not as common as in other countries, local examples exist. This dissertation highlighted various aspects of the seismic vulnerability of unreinforced masonry cross-vaults through parametric variation, in terms of masonry shell behaviour.

Notwithstanding, further research could benefit this field of study, and local studies could propose retrofitting interventions as part of the effort to safeguard Malta's diverse architectural heritage in the event of an earthquake.

## **6.2 Recommendations for Future Research Work**

The available literature in this field of study is at present considerably limited, and this study highlighted various areas in which additional research can be supplemented. Within this context, the following recommendations suggest potential research which may be undertaken and explored:

- As a continuation of this research, a new set of models can be generated by varying the rise of the cross-vaults; a geometrical parameter which was kept constant within this study. Hence, the effect of two (or more) geometrical parameters applied in conjunction, can be investigated, allowing for comparison with the results presented in this study.
- Considering that locally we have a number of cross-vaulted structures, it would be interesting to assess the seismic performance of local case studies, by means of physical (shake table) or numerical models. Comparisons in behaviour can be outlined, and different retrofitting techniques could subsequently be proposed to minimise damage upon the event of an earthquake.
- A more detailed study of the aggregate effect could be conducted by implementing a non-linear dynamic analysis rather than a static pushover analysis, whereby the seismic load constantly changes direction. Insight could be gained on vibrational damage and results could potentially provide a clearer understanding of the global and local behaviour when cross-vaults are in aggregate.
- Cross-vaults require some form of buttressing, and in Gothic cathedrals this is typically provided by flying buttresses, which in their own way are another arched masonry structure. Hence, this study could be extended to investigate the effect of buttressing and its impact on the seismic vulnerability of cross-vaults. Additionally, the behaviour of the buttress could be analysed and strengthening techniques could be proposed.
- For the various models tested in this study, the effect of applied retrofits such as steel tie rods, composite fibre-reinforced polymer (CFRP) strips and textile-reinforced mortar can be examined. HiStrA tolerates the definition of such retrofits and materials, and hence the enhancement effect on strength and stiffness of the cross-vaults can be



analysed and directly compared to the results of the unreinforced masonry versions obtained in this study.

### 6.3 Study Limitations

In addition to the methodology limitations that were identified and rationalised in Chapter 4 of this dissertation, the following limitations are also to be acknowledged.

Prior to the commencement of this study, the researcher had no experience using the HiStrA software and the type of modelling it implements. Considering the limited amount of time available to grasp its functions, it is recognised that the author may have not made efficient use of the software, resulting in loss of time that could have been used to produce further tests and results.

A tangible constraint in this study was the computational power available to the researcher, which was required to produce the output of results. Since the *Configuration 2* tests involved a largely increased number of macro-elements, the analysis on the first run did not execute. Thus, the analysis settings had to be adjusted as remarked in section 4.3.5 and resulted in certain analysis (particularly the  $\pm Y$  pushover) to not be executed properly. The computational power required was underestimated in particular for the *Configuration 2* tests, and in this case the use of a device with stronger computational power would have been preferable.

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# Appendices

## Appendix A – Local Cross-vault Examples

## Appendix B – Numerical Models

- B.1 Singular Cross-Vaults
- B.2 Aggregate Configuration 1
- B.3 Aggregate Configuration 2

## Appendix C – Pushover Analysis Results

- C.1 Pushover '*Massa*' Curves
- C.2 Pushover '*Modal*' Curves

## Appendix D – Deformation Images

- D.1 CV1
- D.2 CV2
- D.3 CV3
- D.4 CV1-R
- D.5 CV2-R
- D.6 CV3-R
- D.7 C1 – CV1
- D.8 C1 – CV2
- D.9 C1 – CV3
- D.10 C1 – CV1-R
- D.11 C1 – CV2-R
- D.12 C1 – CV3-R
- D.13 C2 – CV1
- D.14 C2 – CV1-R

## Appendix E – Seismic Vulnerability Criteria – Pushover '*Massa*'





## Appendix A: Local Cross-Vault Examples

**Note:** All images are in black and white for uniformity since not all of the original images were in colour. Figures A3, A4 & A6 have been enhanced using an AI image enhancing tool due to their original low quality.

*Figure credits (where not stated in caption):*

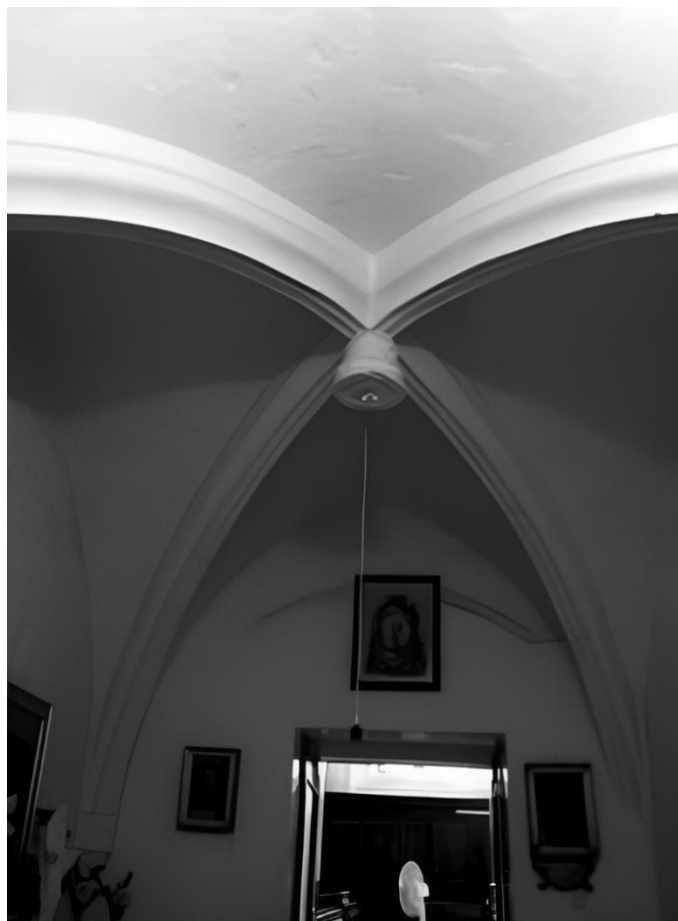
- Figure A1: <https://timesofmalta.com/article/tale-of-a-church-and-a-priory.801735>
- Figure A2: [https://en.wikipedia.org/wiki/Chapel\\_of\\_St\\_Anne,\\_Fort\\_St\\_Angelo](https://en.wikipedia.org/wiki/Chapel_of_St_Anne,_Fort_St_Angelo)
- Figure A7: <https://heritagemalta.mt/explore/the-inquisitors-palace/>
- Figure A11: <https://wirtizzejtun.com/churches/>
- Figure A13: <https://timesofmalta.com/article/A-palace-for-an-office.496783>
- Figure A14: <https://themaltaexperience.com/la-sacra-infermeria/>
- Figure A16: <https://www.carmelitepriory.org/the-priory/spaces-to-visit/>
- Figure A17: <https://ginniehart.blogspot.com/2016/05/malta-addolorata-cemetery.html>
- Figure A18: <https://www.beyondthebay.co/malta/ta-pinu-shrine-guide-gozo-malta>



**Figure A1** – Cloister at Augustinians Friary in Rabat, Malta.



**Figure A2** – Cross-vaults at St Anne's Church in Fort St Angelo, Birgu.



**Figure A3** – Santa Scholastica, Birgu. Reproduced from Galea (2016).



**Figure A4** – Grandmaster's staircase in Fort St Angelo, Birgu. Reproduced from Galea (2016).



**Figure A5** – Ribbed vault at Convent of Franciscan Minor Observants in Rabat, Malta. Reproduced from Galea (2016).



**Figure A6** – Cloister at Convent of Franciscan Minor Observants in Rabat, Malta. Reproduced from Galea (2016).



**Figure A7** – Courtyard of Inquisitor's Palace, Birgu.



**Figure A8** – Ribbed vault of Inquisitor's Palace, Birgu. Reproduced from Brincat (2020).



**Figure A9** – Ribbed vault at the Lodge, Cospicua. Reproduced from Galea (2016).



**Figure A10** – Ribbed vault at Law Courts Annex, Valletta. Reproduced from Galea (2016).





**Figure A11** – Ribbed vault at Old church of St Catherine, Žejtun.



**Figure A12** – Ribbed vault at Spinola Palace, Valletta. Reproduced from Galea (2016).



**Figure A13** – Cloister of courtyard at Spinola Palace, Valletta.



**Figure A14** – Magazine Ward at the Sacra Infermeria, Valletta.



**Figure A15** – Ribbed vault at Carmelite crypts, Mdina. Reproduced from Galea (2016).



**Figure A16** – Groin vaults of the kitchen at Carmelite Priory, Mdina.



**Figure A17** – Neo-Gothic cloister at Addolorata Cemetery, Paola.



**Figure A18** – Nave of Basilica of National Shrine of the Blessed Virgin of Ta' Pinu in Għarb, Gozo.

# Appendix B: Numerical Models

## B.1 Singular Cross-vaults

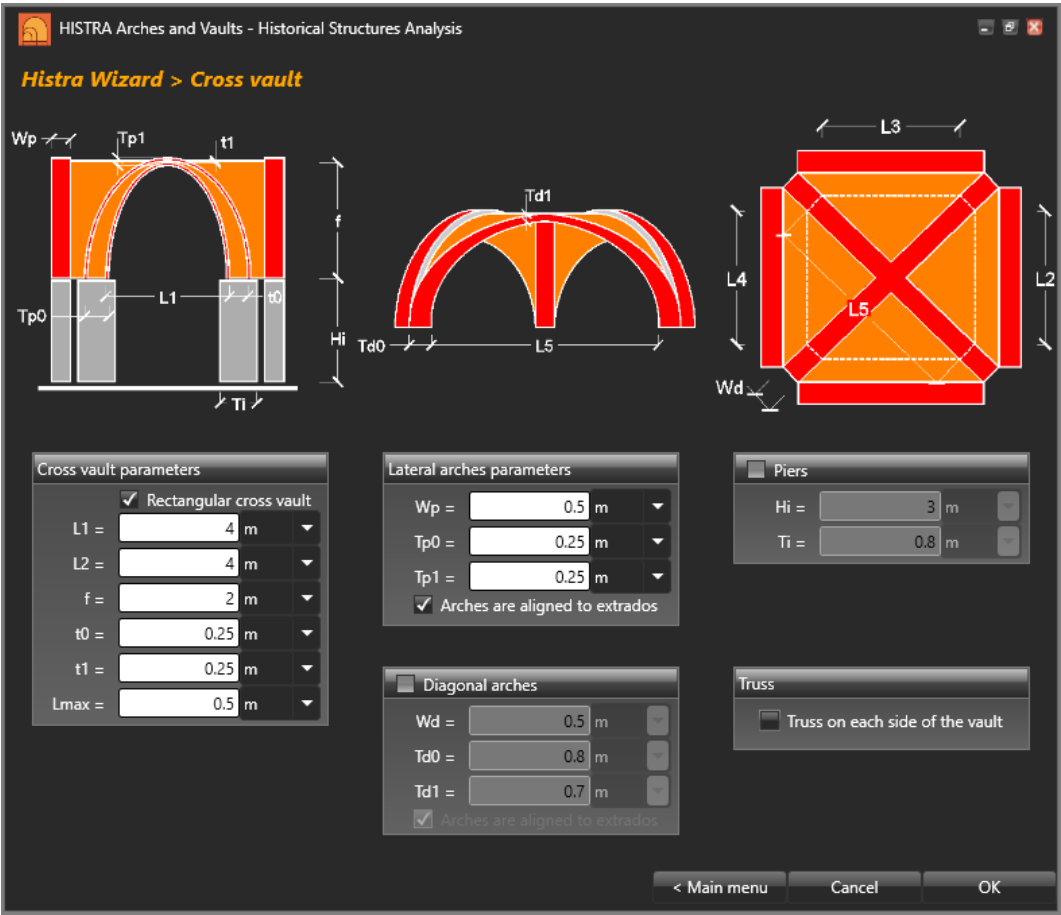


Figure B1 – Geometric input of CV1 in HiStrA Wizard.

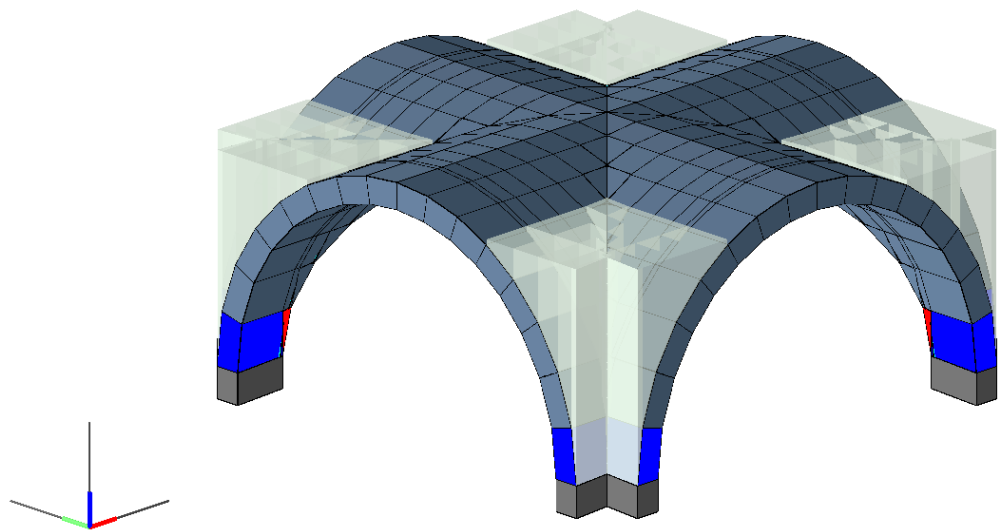


Figure B2 – Computational model of CV1.

HISTRA Arches and Vaults - Historical Structures Analysis

**Histra Wizard > Cross vault**

**Cross vault parameters**

☒ Rectangular cross vault

L1 = 4 m

L2 = 3 m

f = 2 m

t0 = 0.25 m

t1 = 0.25 m

Lmax = 0.5 m

**Lateral arches parameters**

Wp = 0.5 m

Tp0 = 0.25 m

Tp1 = 0.25 m

☒ Arches are aligned to extrados

**Diagonal arches**

☐ Diagonal arches

Wd = 0.5 m

Td0 = 0.8 m

Td1 = 0.7 m

☒ Arches are aligned to extrados

**Piers**

Hi = 3 m

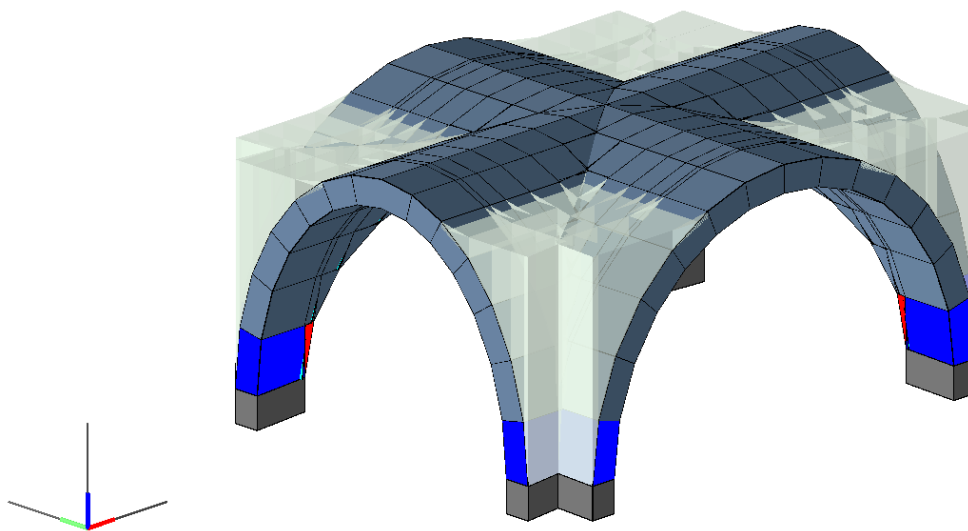
Ti = 0.8 m

**Truss**

☐ Truss on each side of the vault

< Main menu Cancel OK

**Figure B3** – Geometric input of CV2 in HiStrA Wizard.



**Figure B4** – Computational model of CV2.



HISTRA Arches and Vaults - Historical Structures Analysis

**Histra Wizard > Cross vault**

**Cross vault parameters**

- ☒ Rectangular cross vault
- L1 = 4 m
- L2 = 2 m
- f = 2 m
- t0 = 0.25 m
- t1 = 0.25 m
- Lmax = 0.5 m

**Lateral arches parameters**

- Wp = 0.5 m
- Tp0 = 0.25 m
- Tp1 = 0.25 m
- ☒ Arches are aligned to extrados

**Diagonal arches**

- ☐ Diagonal arches
- Wd = 0.5 m
- Td0 = 0.8 m
- Td1 = 0.7 m
- ☒ Arches are aligned to extrados

**Piers**

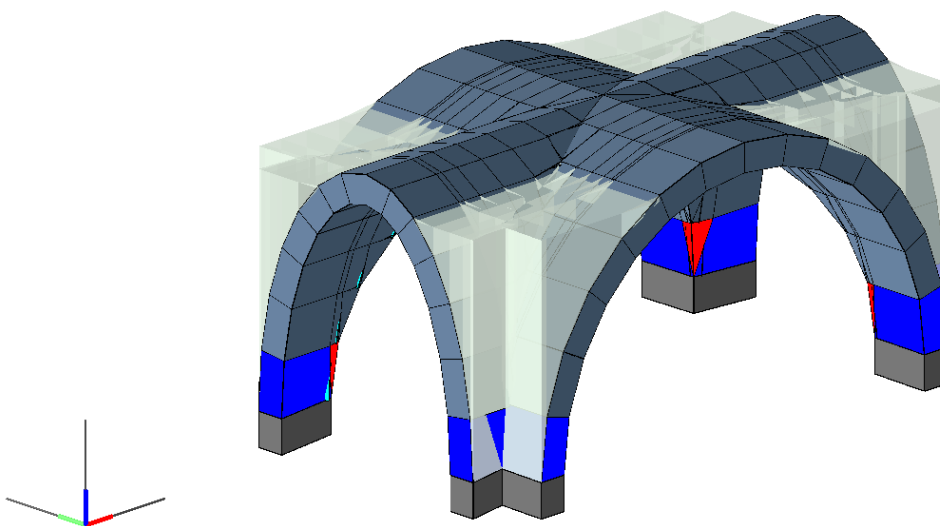
- Hi = 3 m
- Ti = 0.8 m

**Truss**

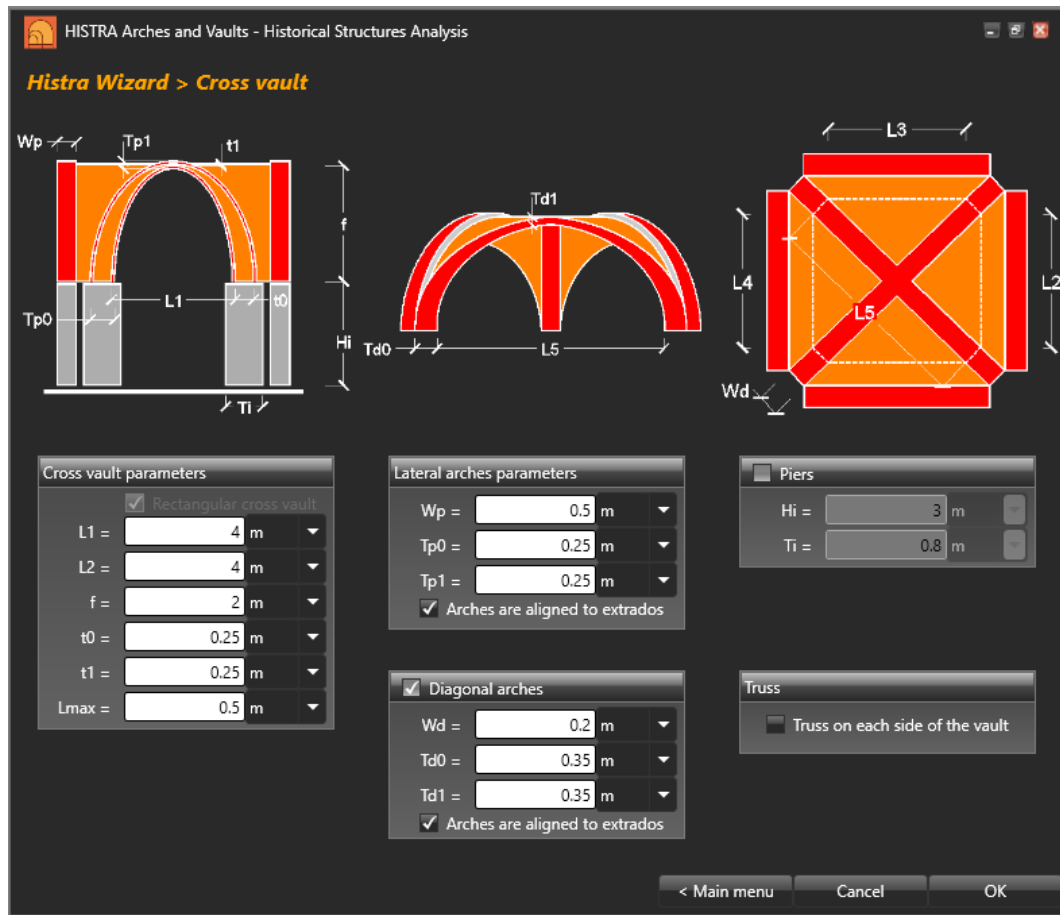
- ☐ Truss on each side of the vault

< Main menu Cancel OK

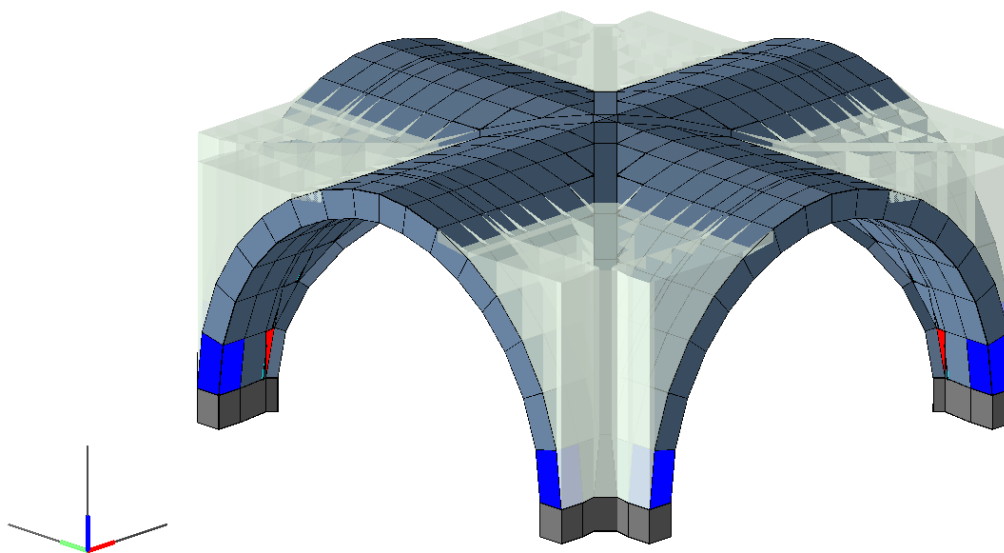
**Figure B5** – Geometric input of CV3 in HiStra Wizard.



**Figure B6** – Computational model of CV3.



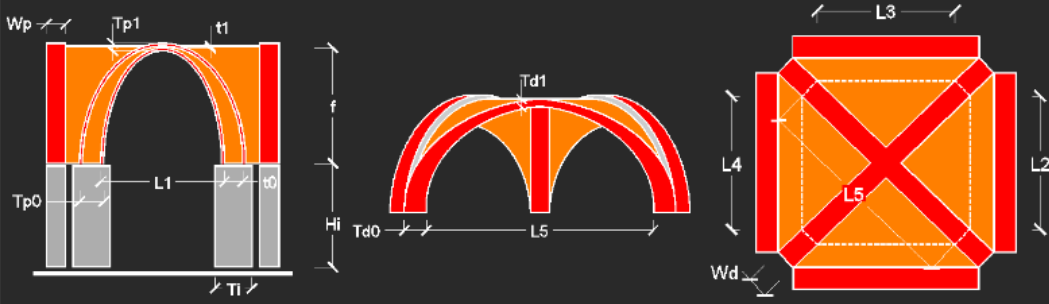
**Figure B7** – Geometric input of CV1-R in HiStrA Wizard.



**Figure B8** – Computational model of CV1-R.

HISTRA Arches and Vaults - Historical Structures Analysis

**Histra Wizard > Cross vault**



**Cross vault parameters**

☒ Rectangular cross vault

L1 = 4 m

L2 = 3 m

f = 2 m

t0 = 0.25 m

t1 = 0.25 m

Lmax = 0.5 m

**Lateral arches parameters**

Wp = 0.5 m

Tp0 = 0.25 m

Tp1 = 0.25 m

☒ Arches are aligned to extrados

**Diagonal arches**

☒ Diagonal arches

Wd = 0.2 m

Td0 = 0.35 m

Td1 = 0.35 m

☒ Arches are aligned to extrados

**Piers**

Hi = 3 m

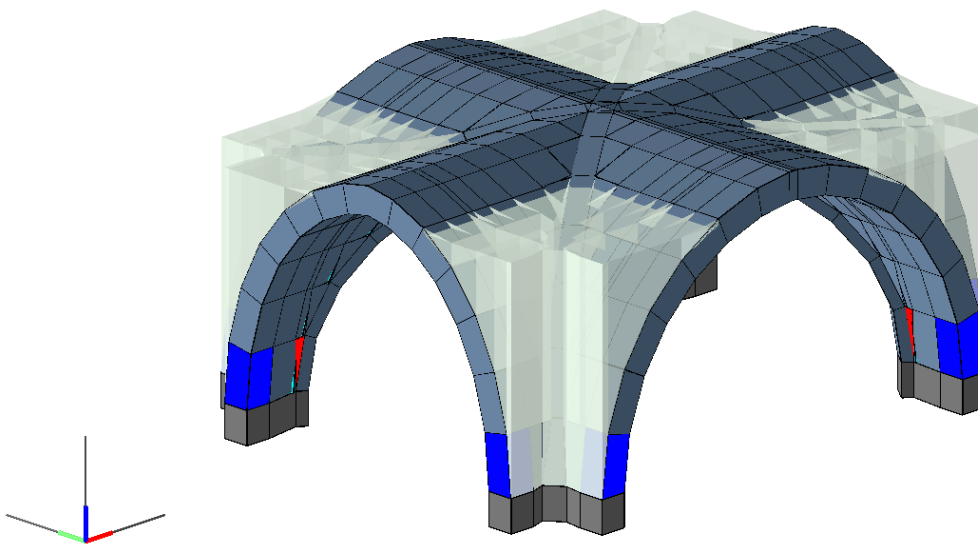
Ti = 0.8 m

**Truss**

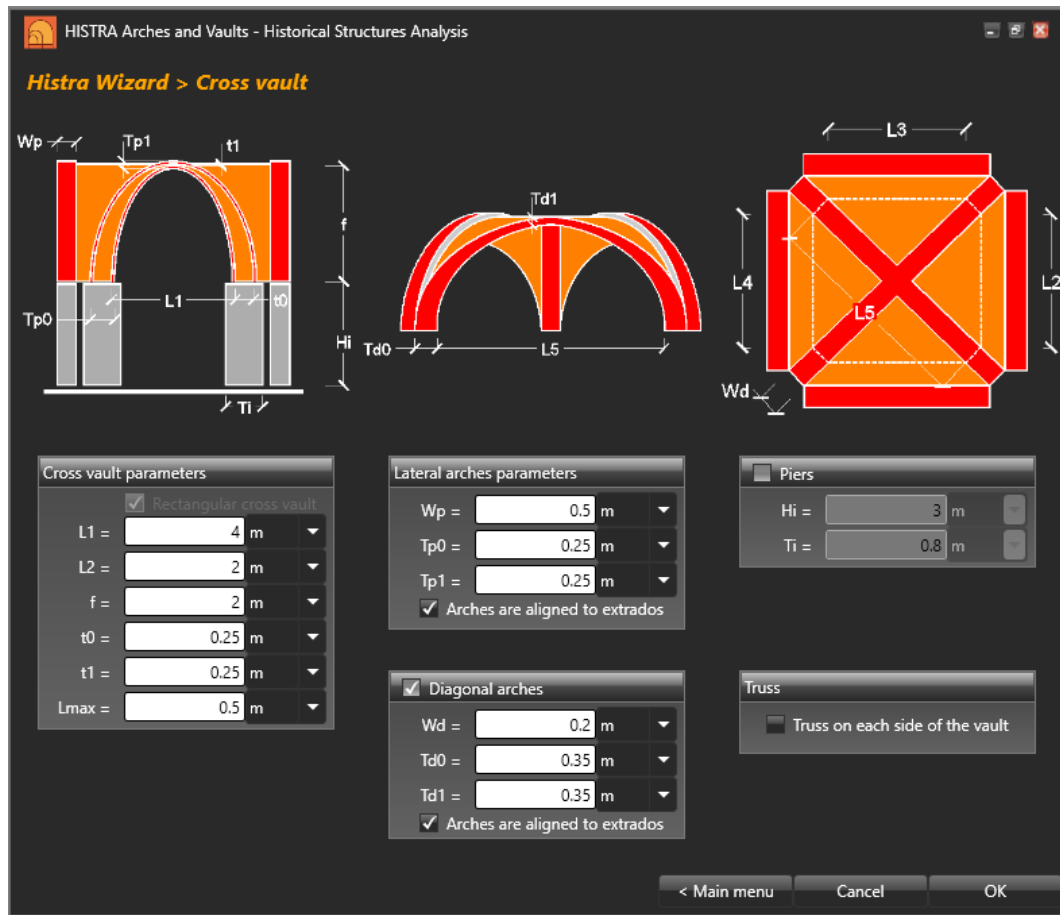
☐ Truss on each side of the vault

< Main menu Cancel OK

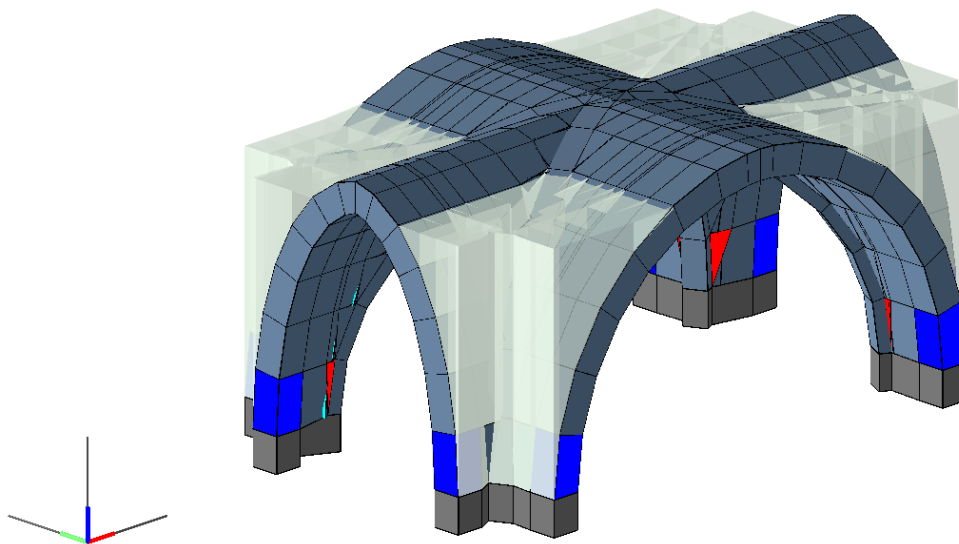
**Figure B9** – Geometric input of CV2-R in HiStrA Wizard.



**Figure B10** – Computational model of CV2-R.

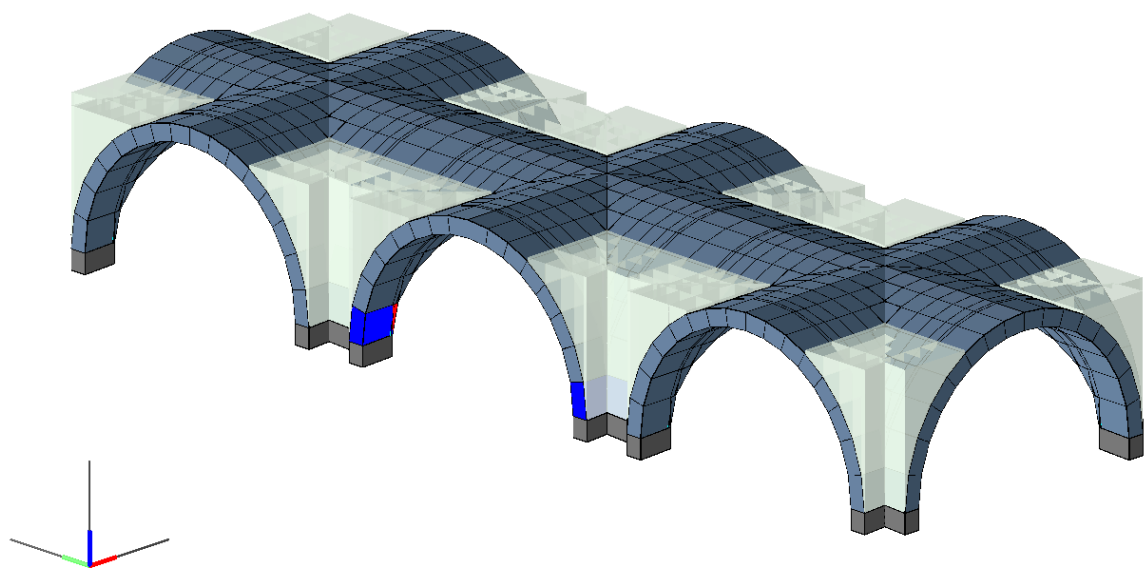


**Figure B11** – Geometric input of CV3-R in HiSTRA Wizard.

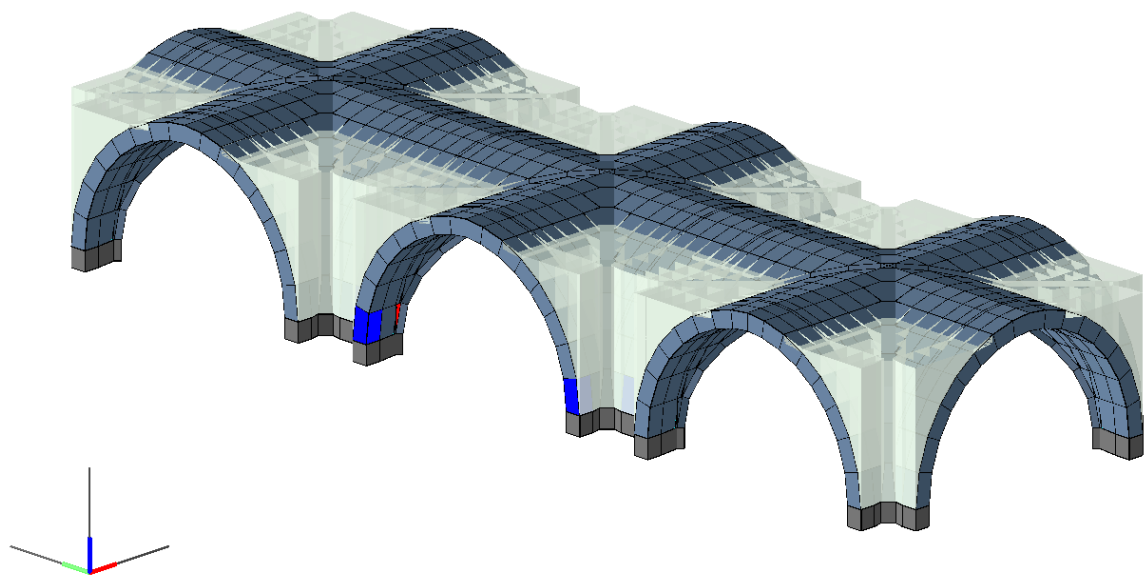


**Figure B12** – Computational model of CV3-R.

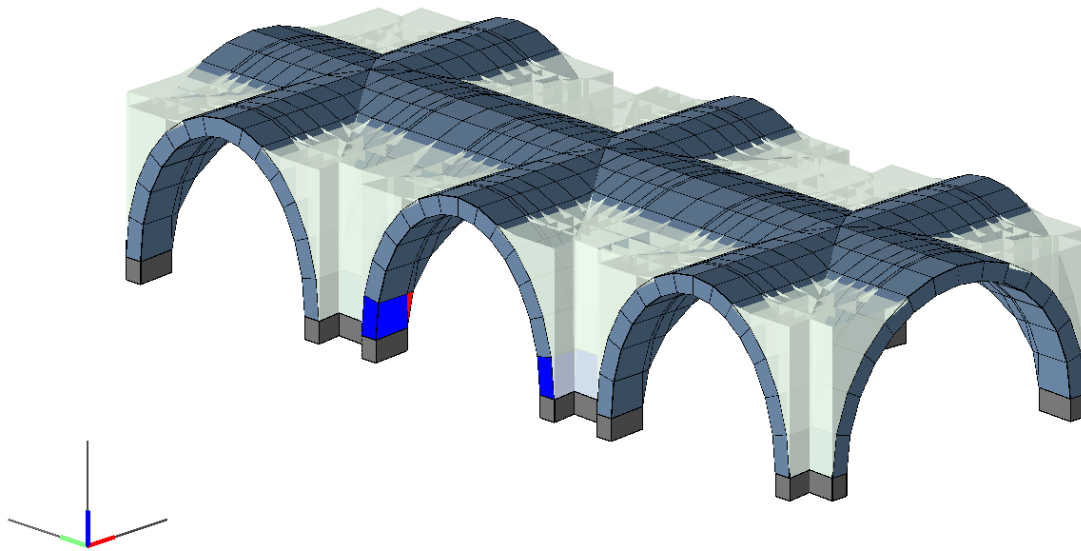
**B.2    Aggregate Configuration 1**



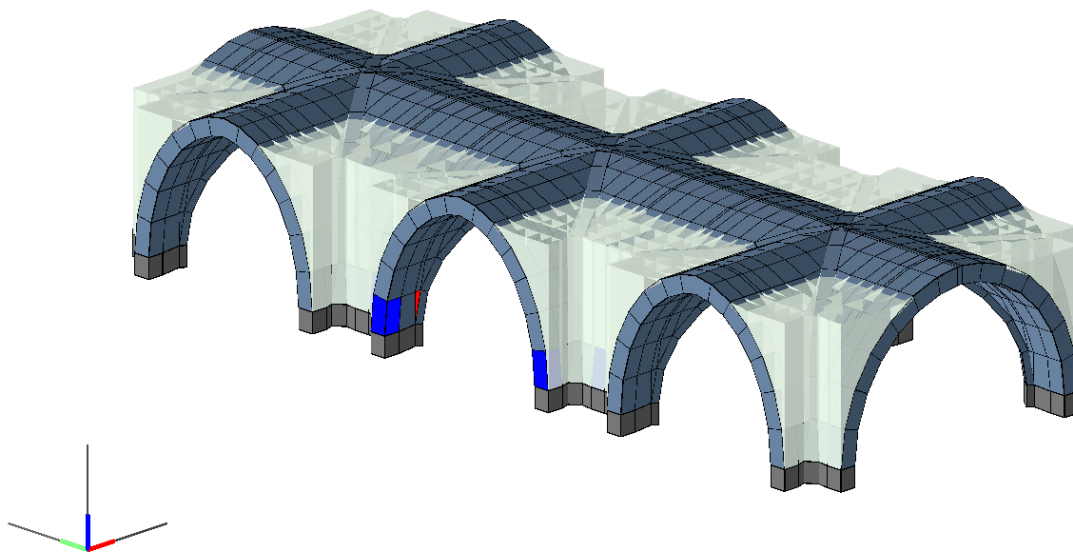
**Figure B13** – Computational model of C1 – CV1.



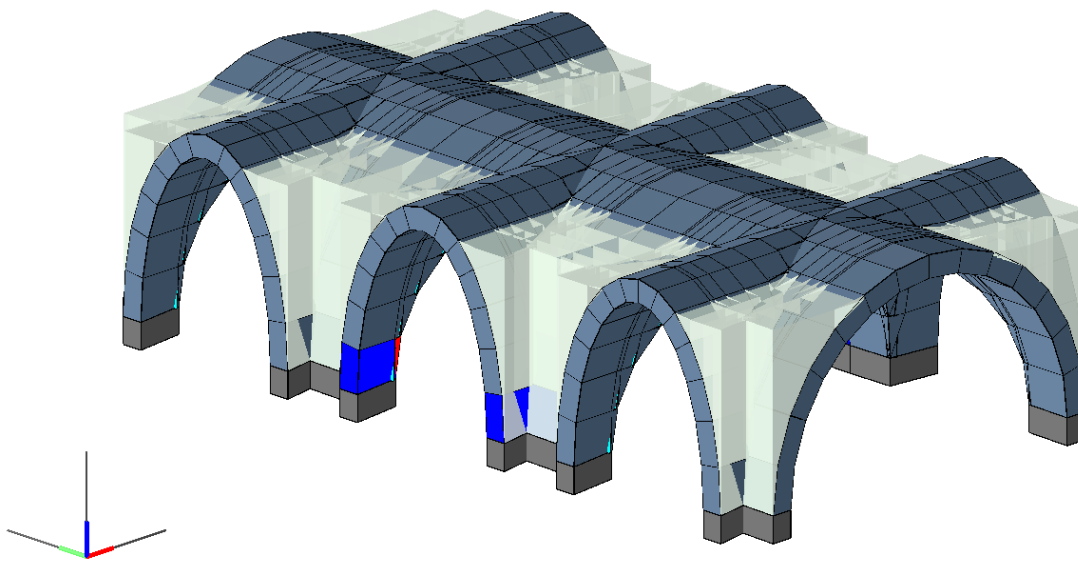
**Figure B14** – Computational model of C1 – CV1-R.



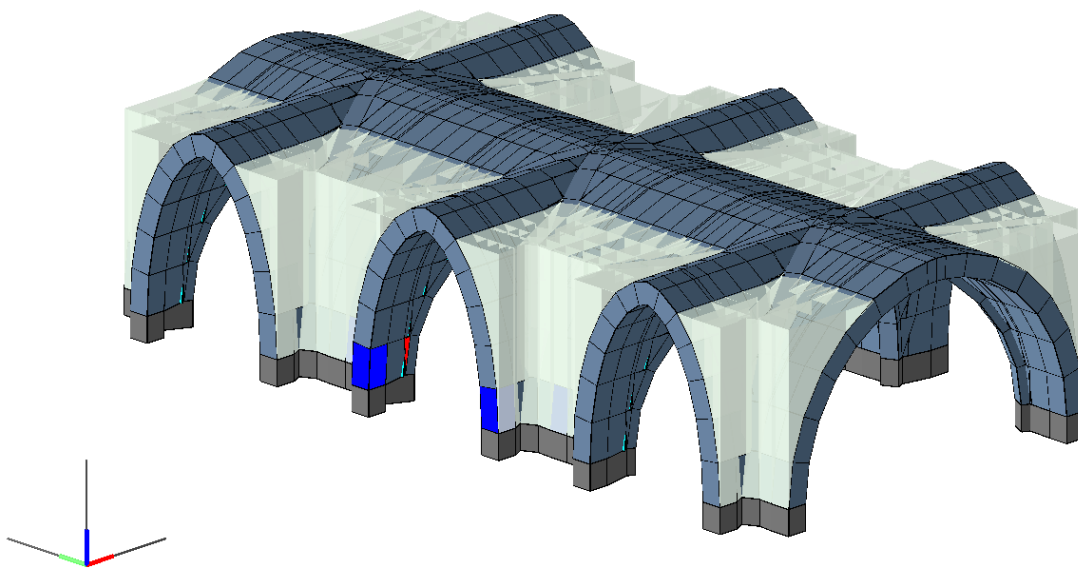
**Figure B15** – Computational model of C1 – CV2.



**Figure B16** – Computational model of C1 – CV2-R.

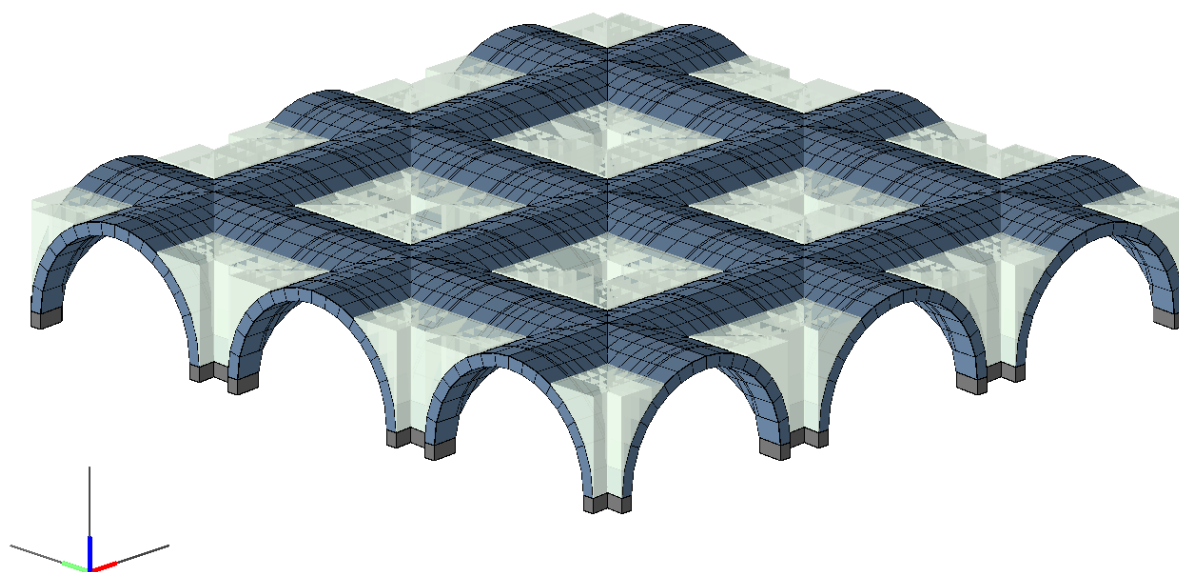


**Figure B17** – Computational model of C1 – CV3.

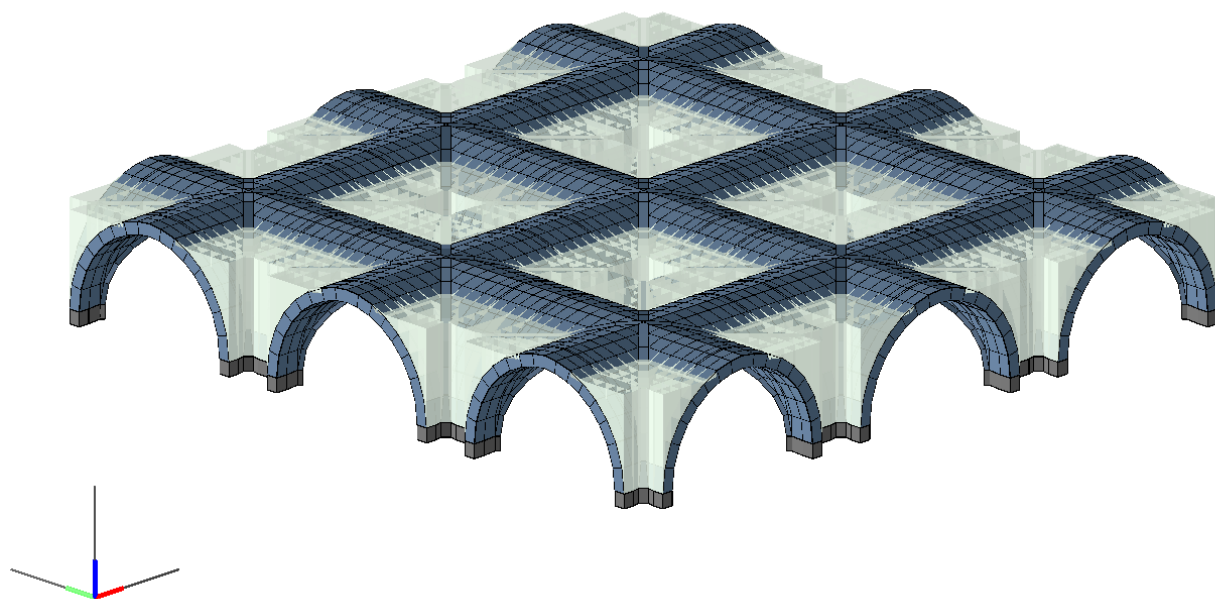


**Figure B18** – Computational model of C1 – CV3-R.

### B.3 Aggregate Configuration 2



**Figure B19** – Computational model of C2 – CV1.

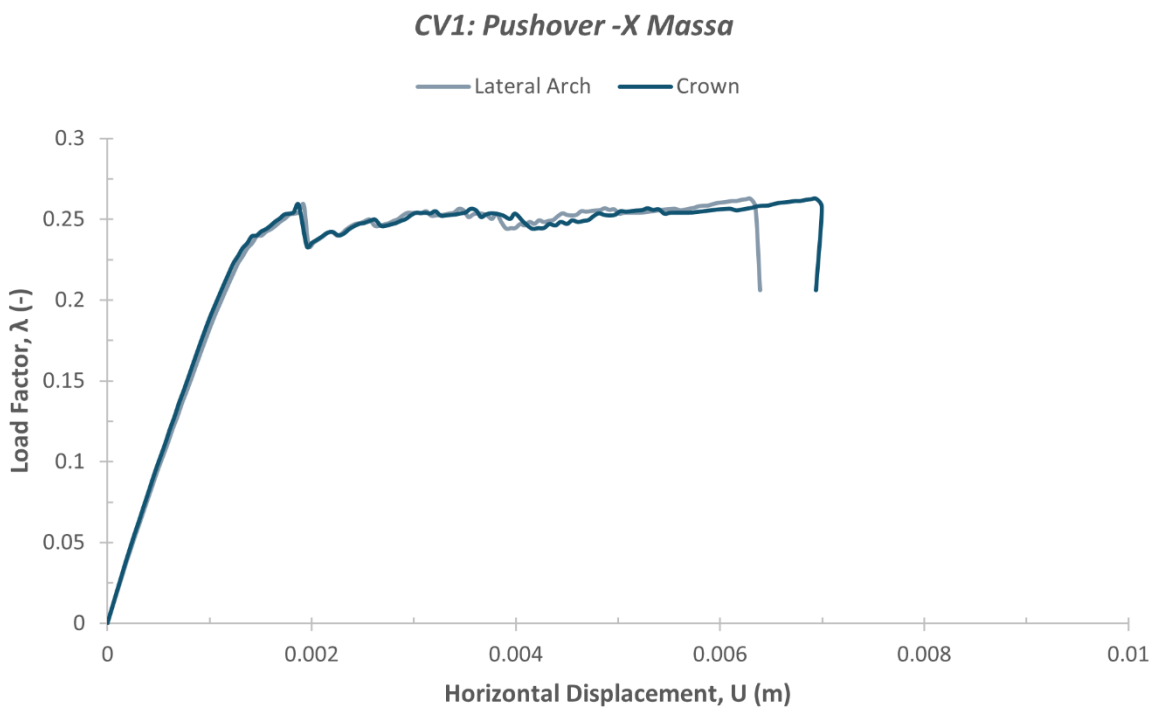
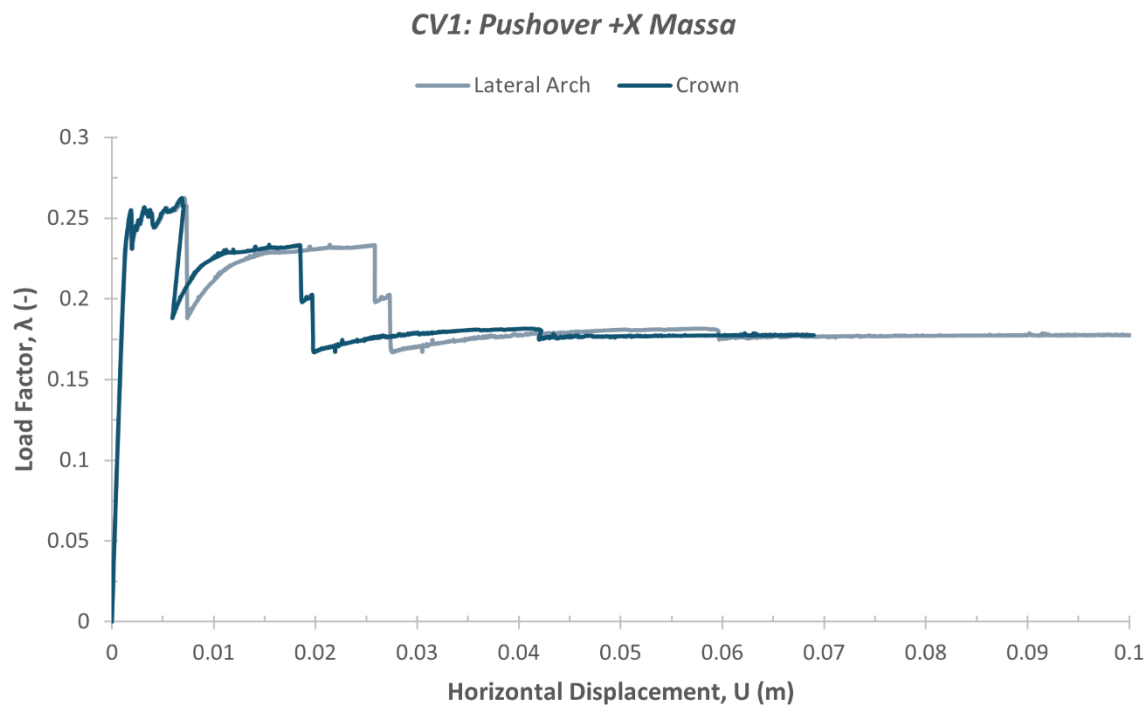


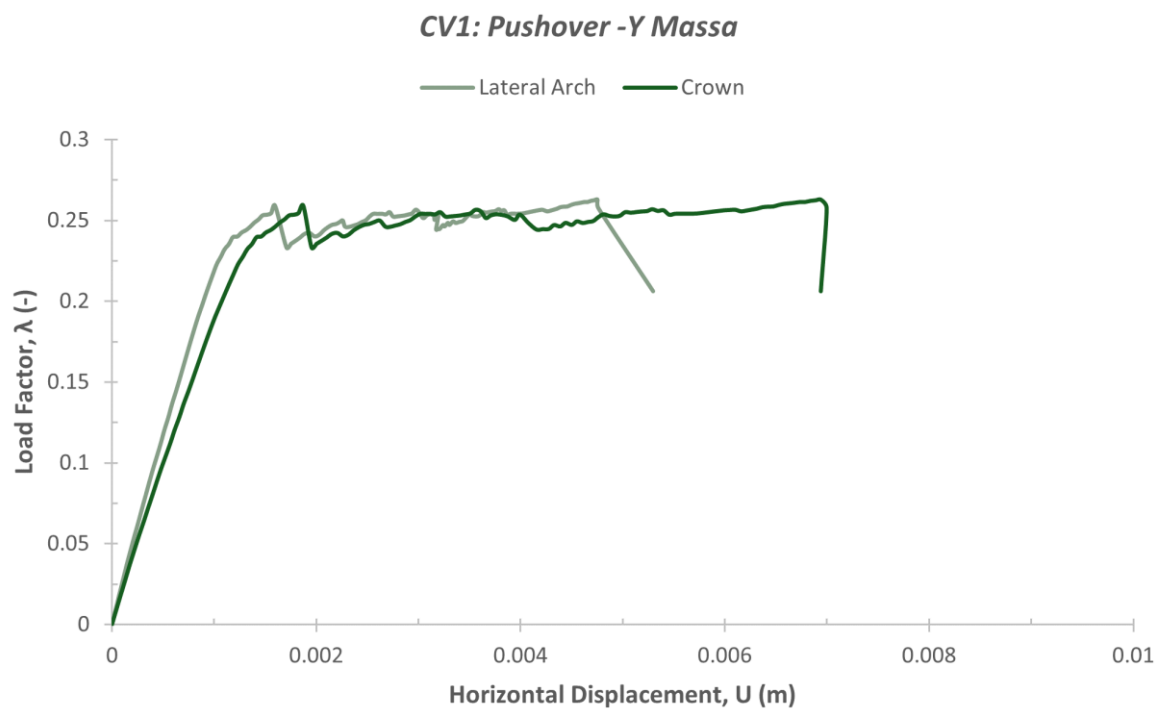
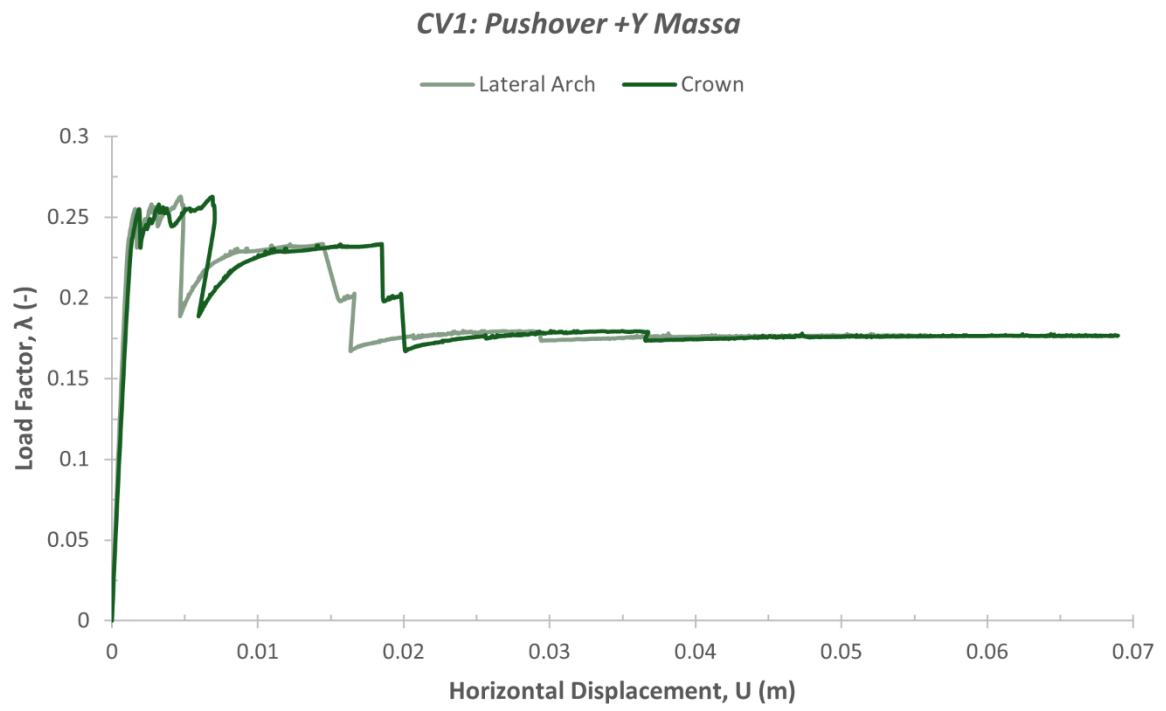
**Figure B20** – Computational model of C2 – CV1-R.



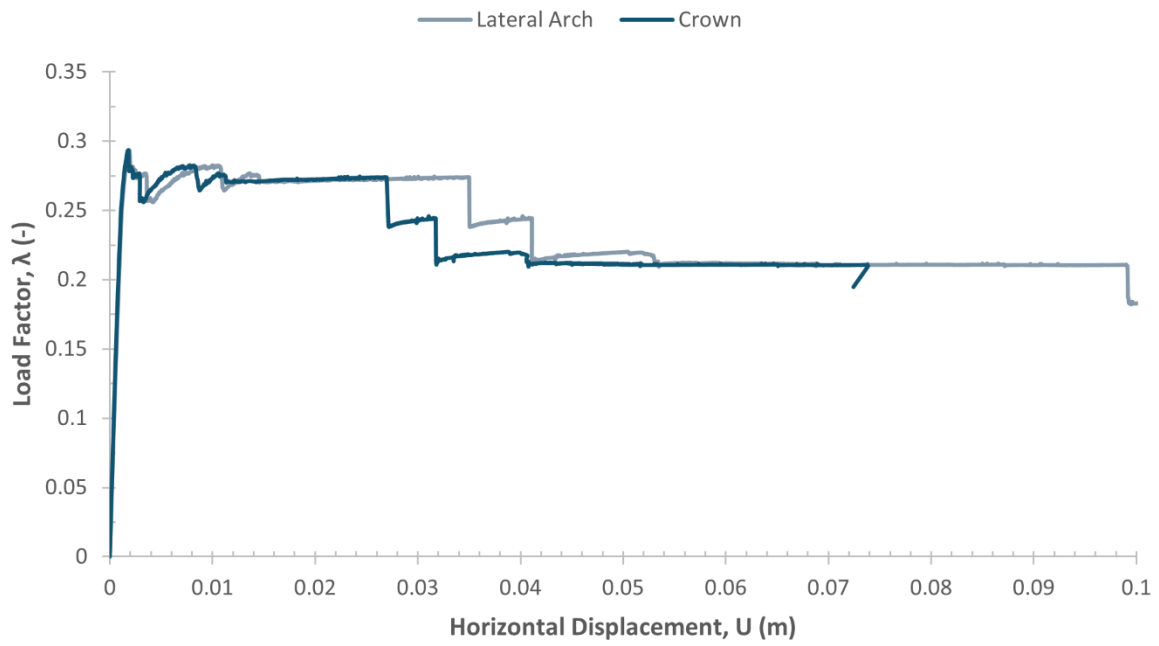
Appendix C: Pushover Analysis Results

C.1 Pushover ‘Massa’ Curves

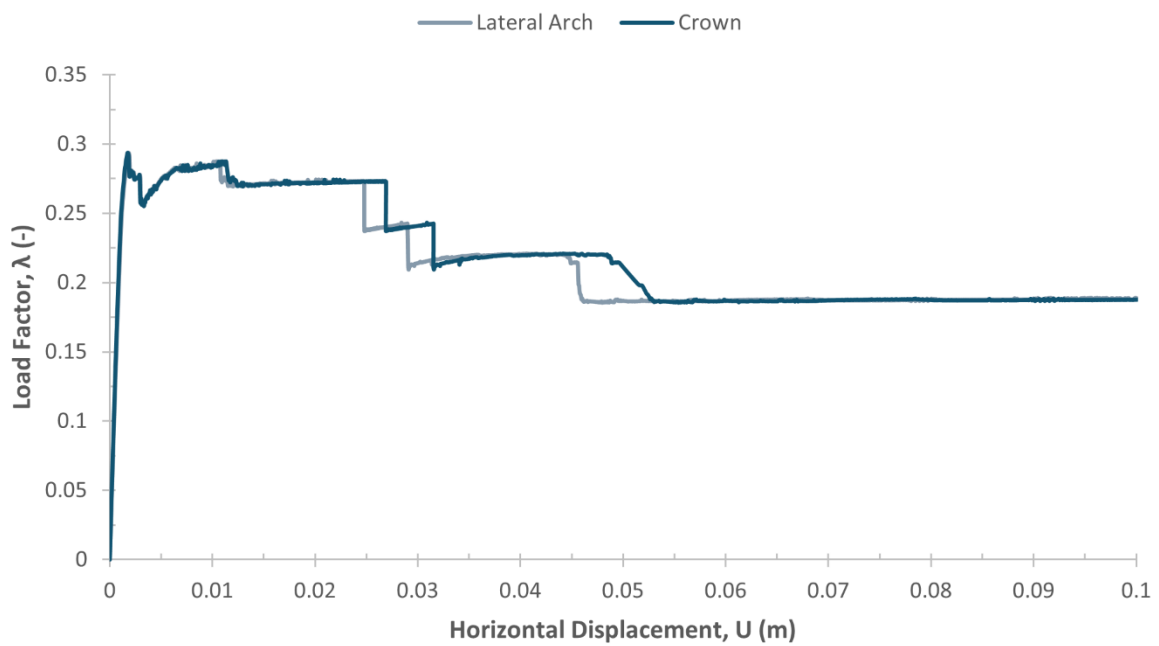




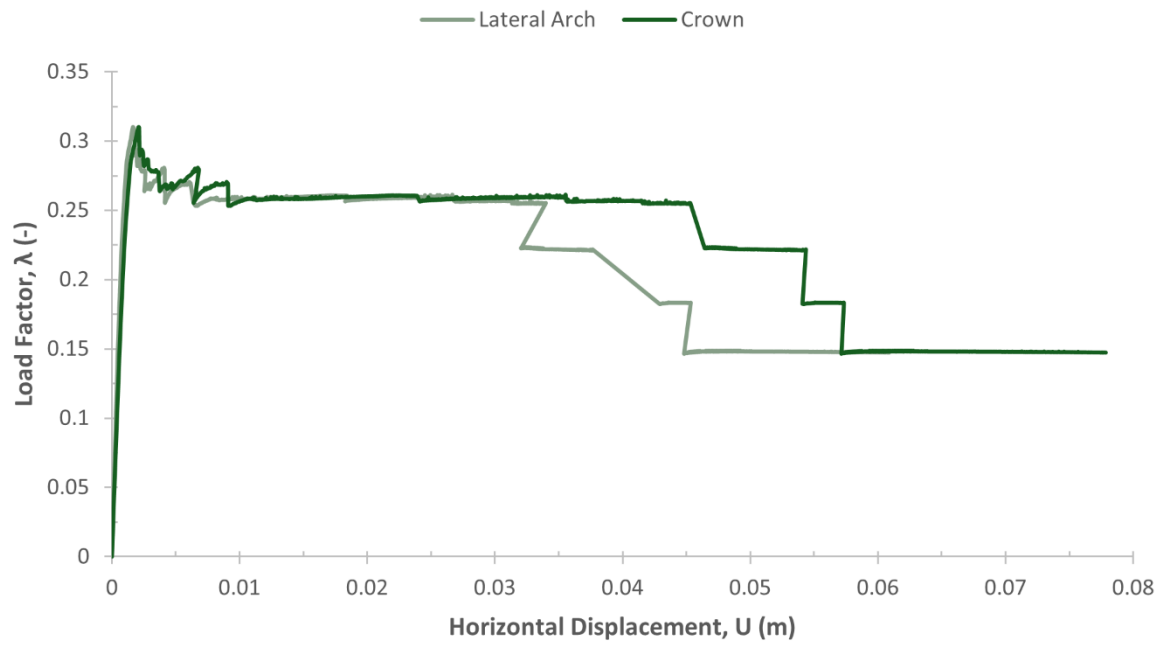
**CV2: Pushover +X Massa**



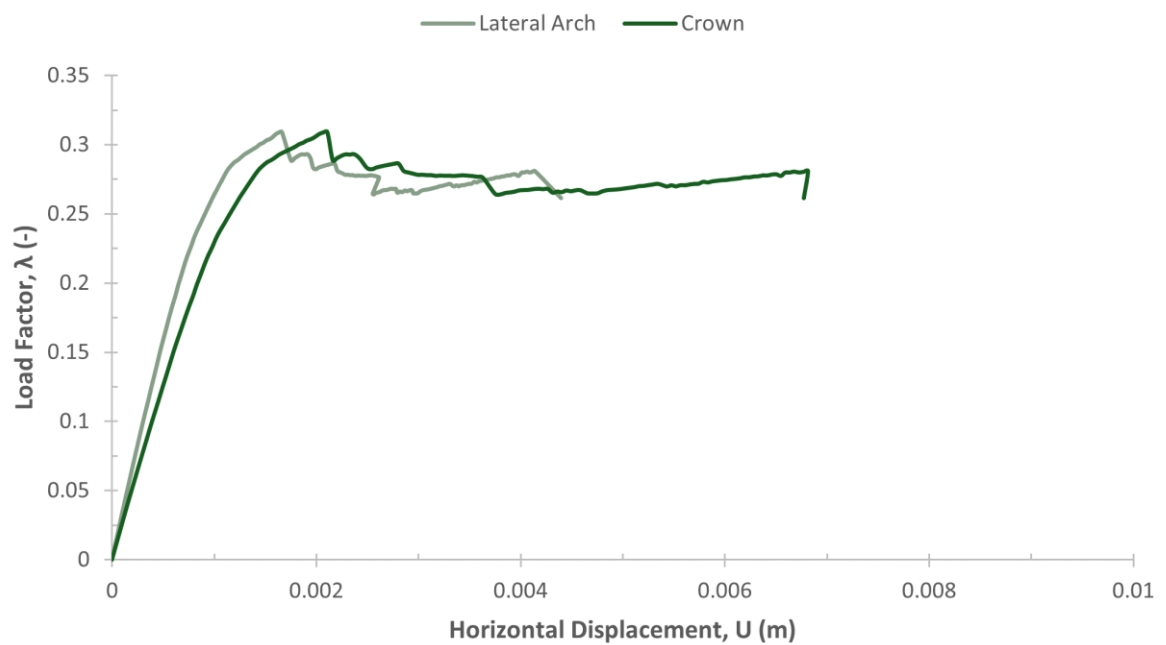
**CV2: Pushover -X Massa**



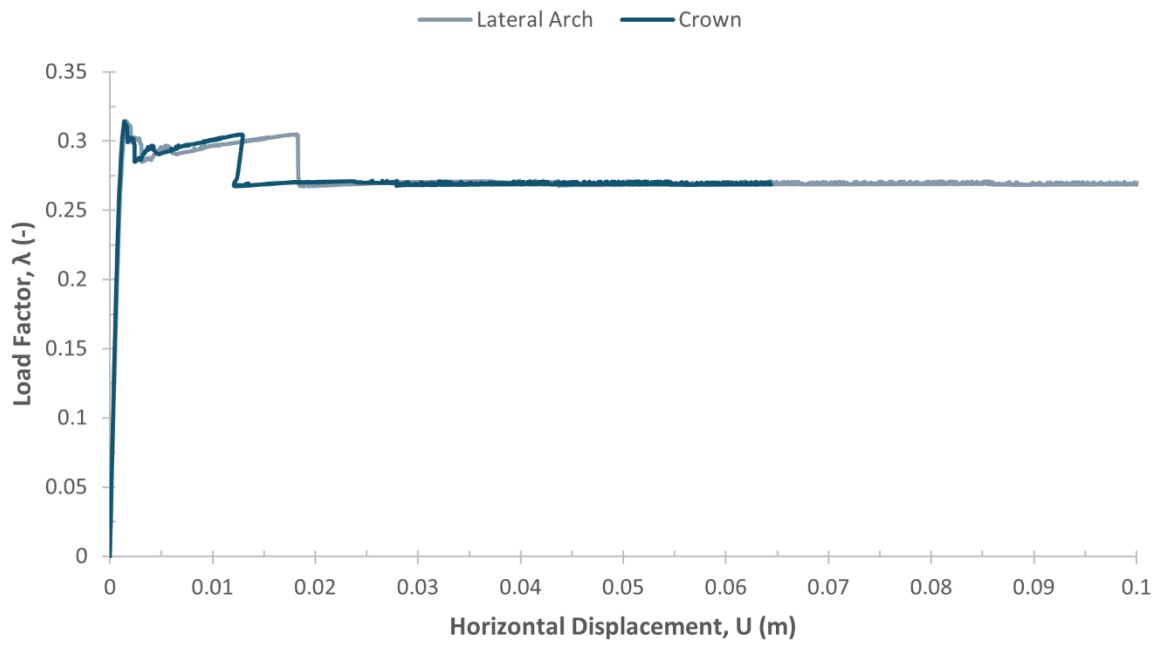
**CV2: Pushover +Y Massa**



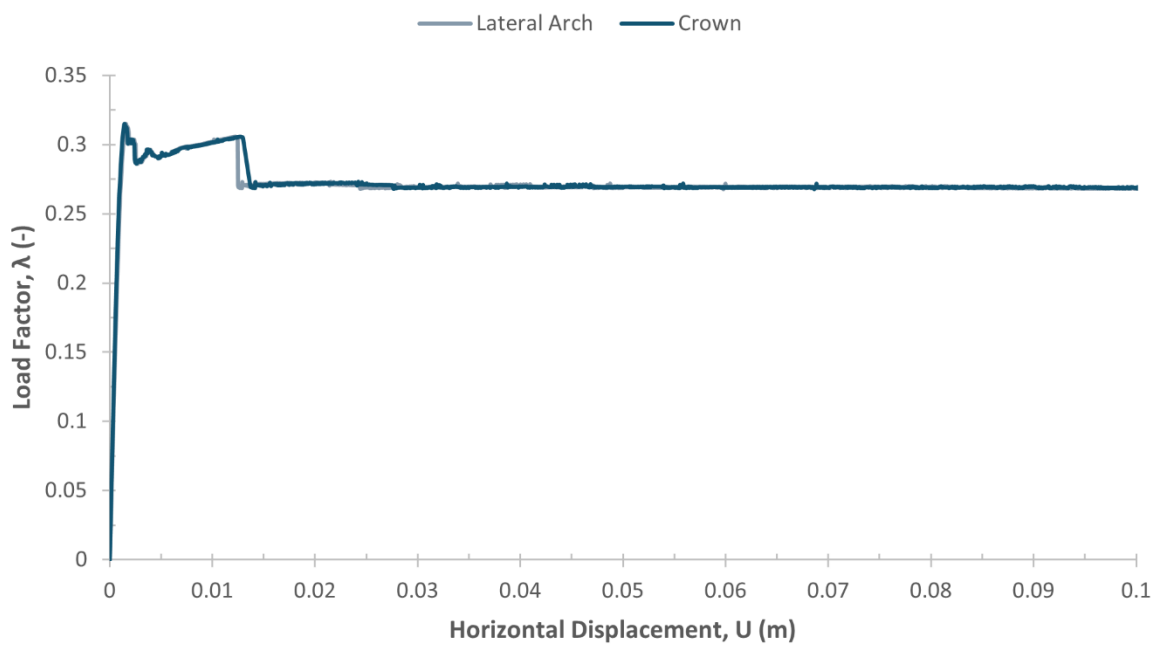
**CV2: Pushover -Y Massa**



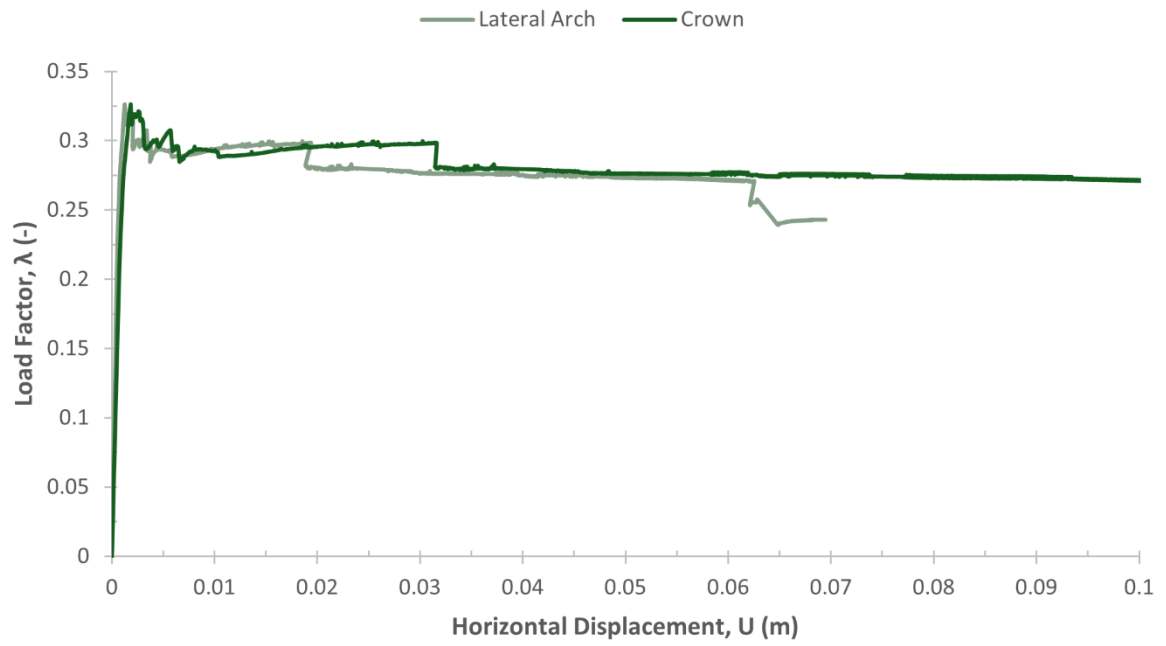
**CV3: Pushover +X Massa**



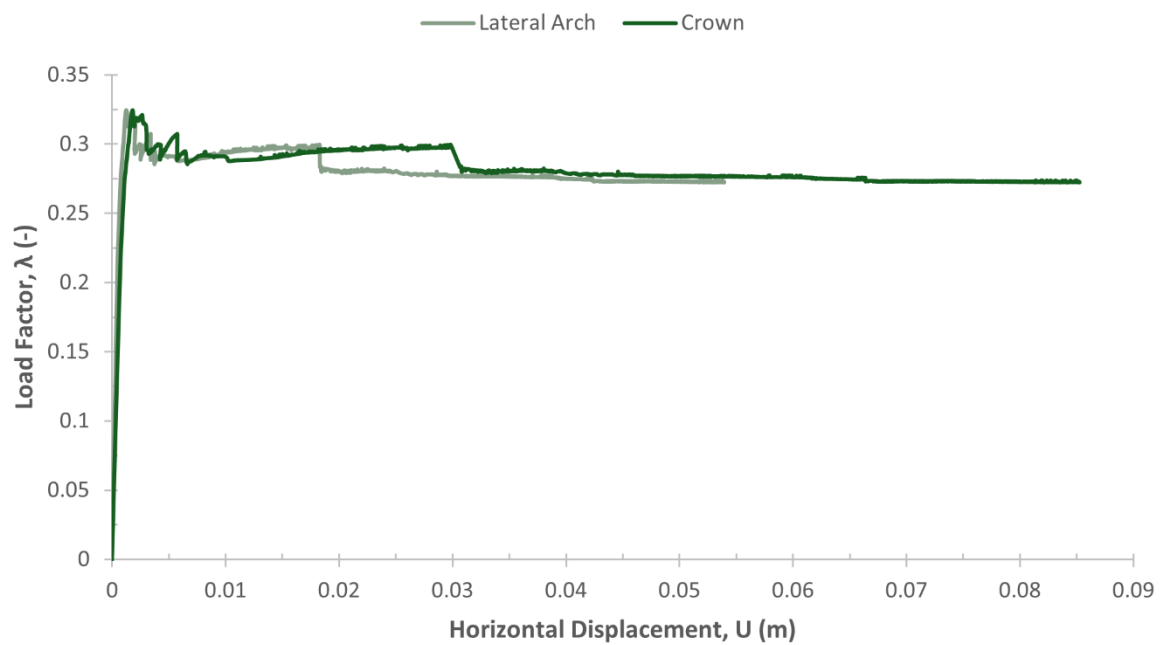
**CV3: Pushover -X Massa**



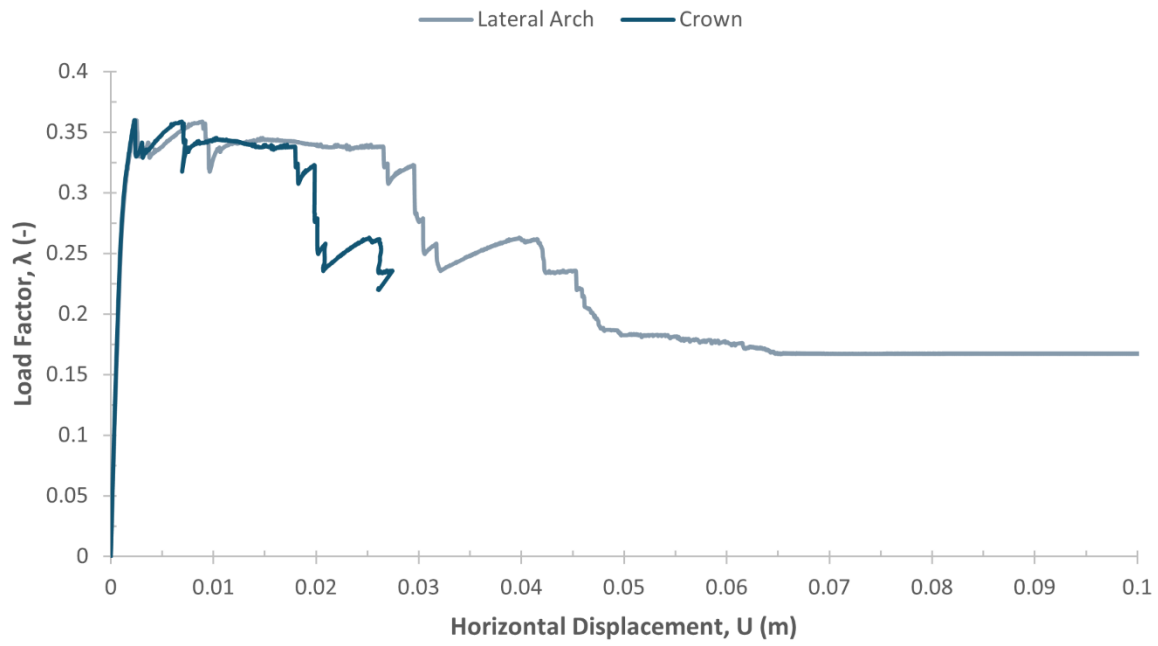
**CV3: Pushover +Y Massa**



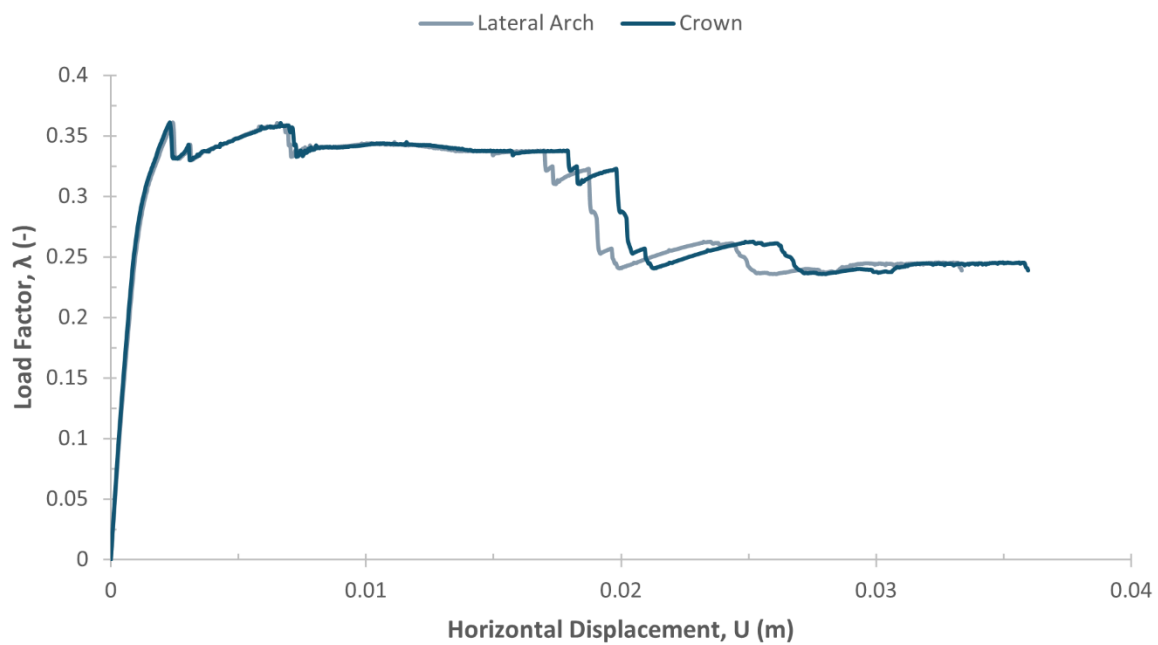
**CV3: Pushover -Y Massa**



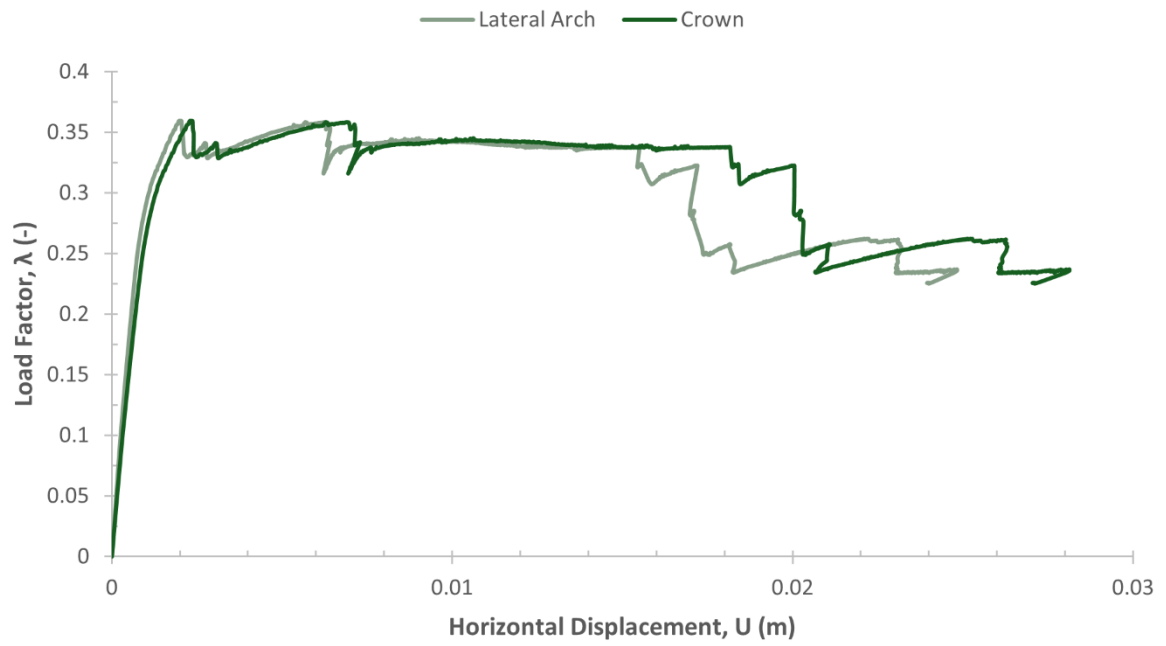
**CV1-R: Pushover +X Massa**



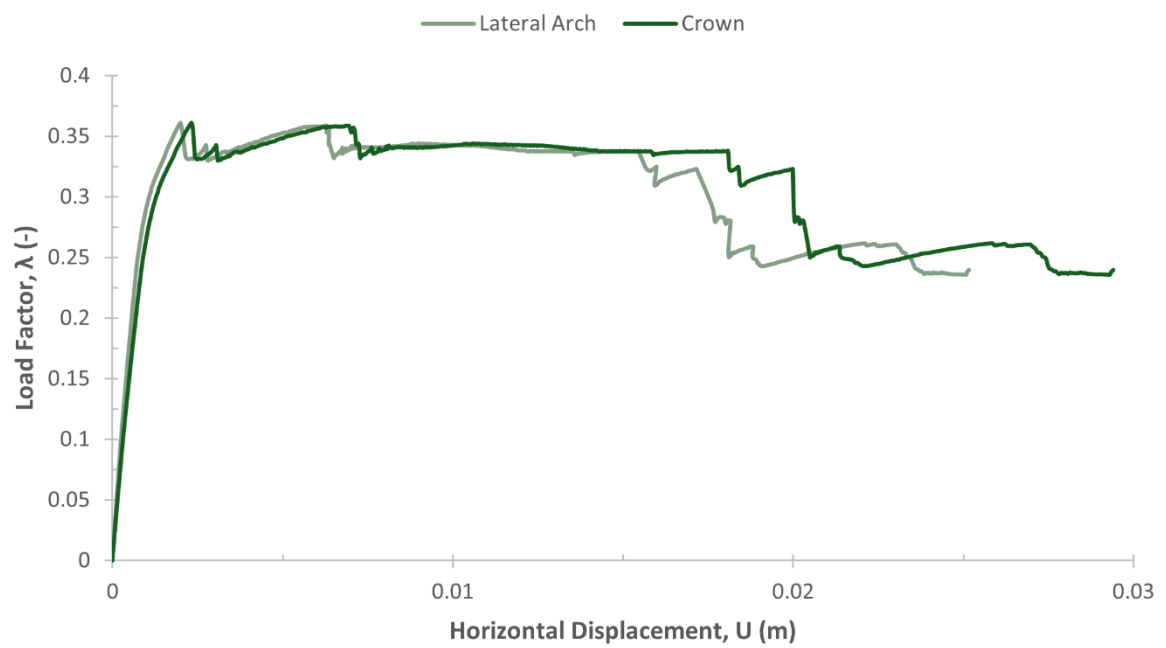
**CV1-R: Pushover -X Massa**



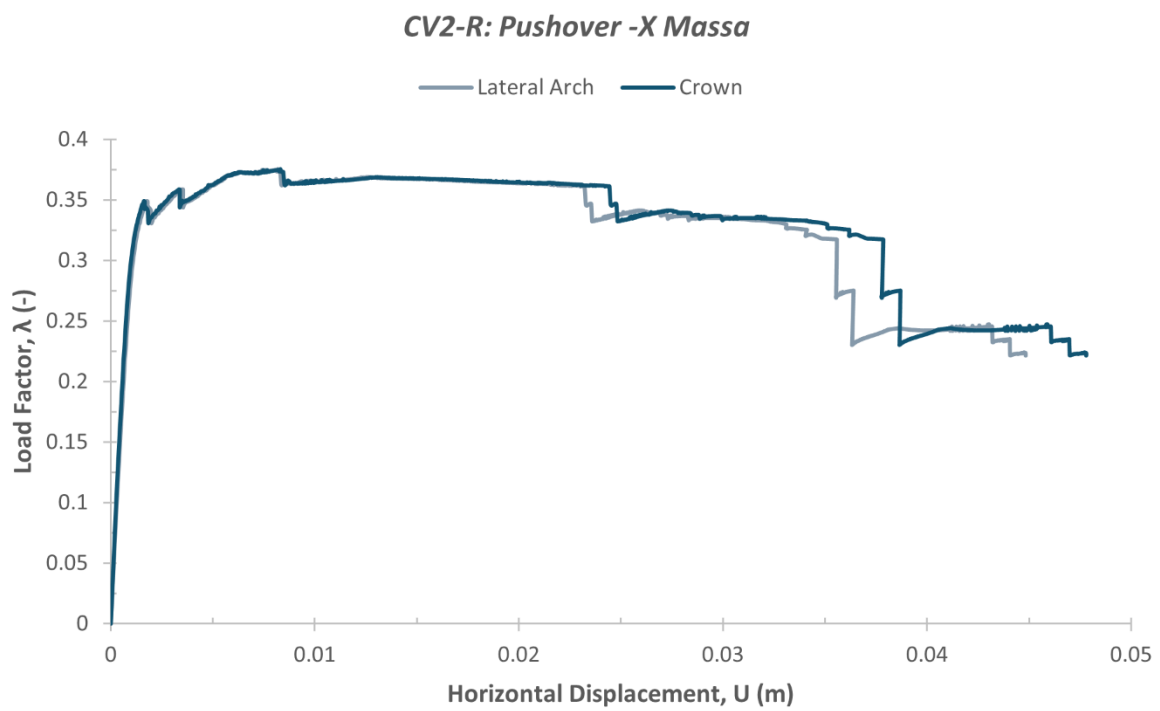
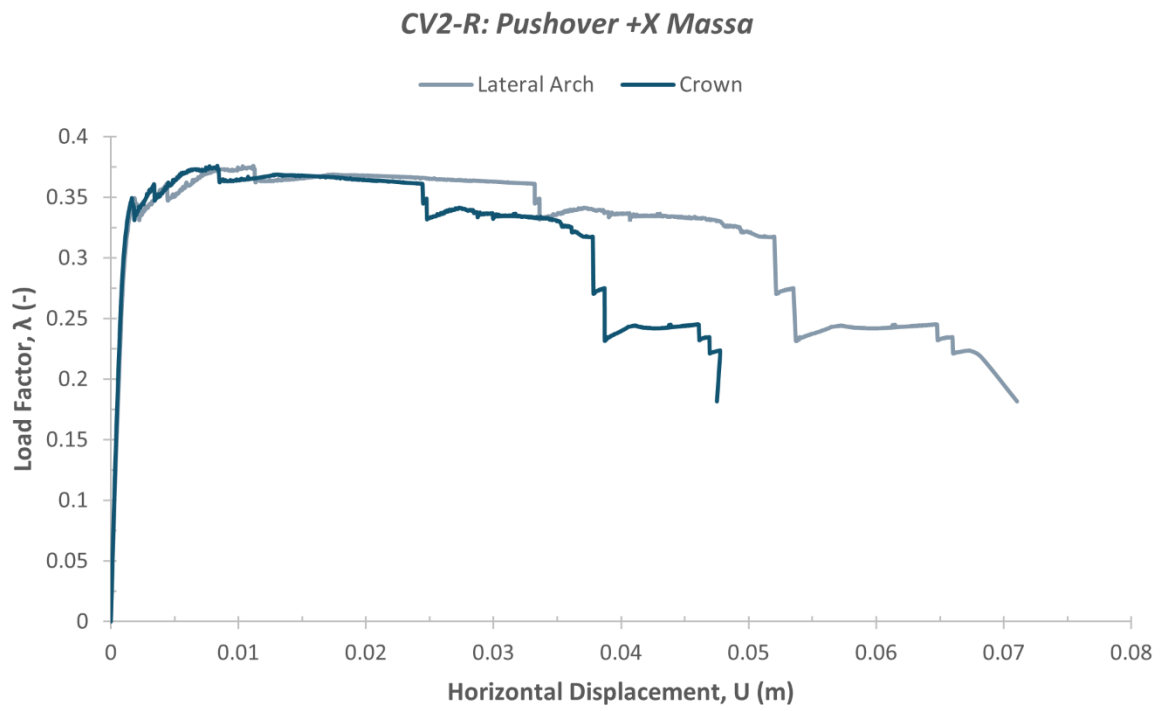
**CV1-R: Pushover +Y Massa**

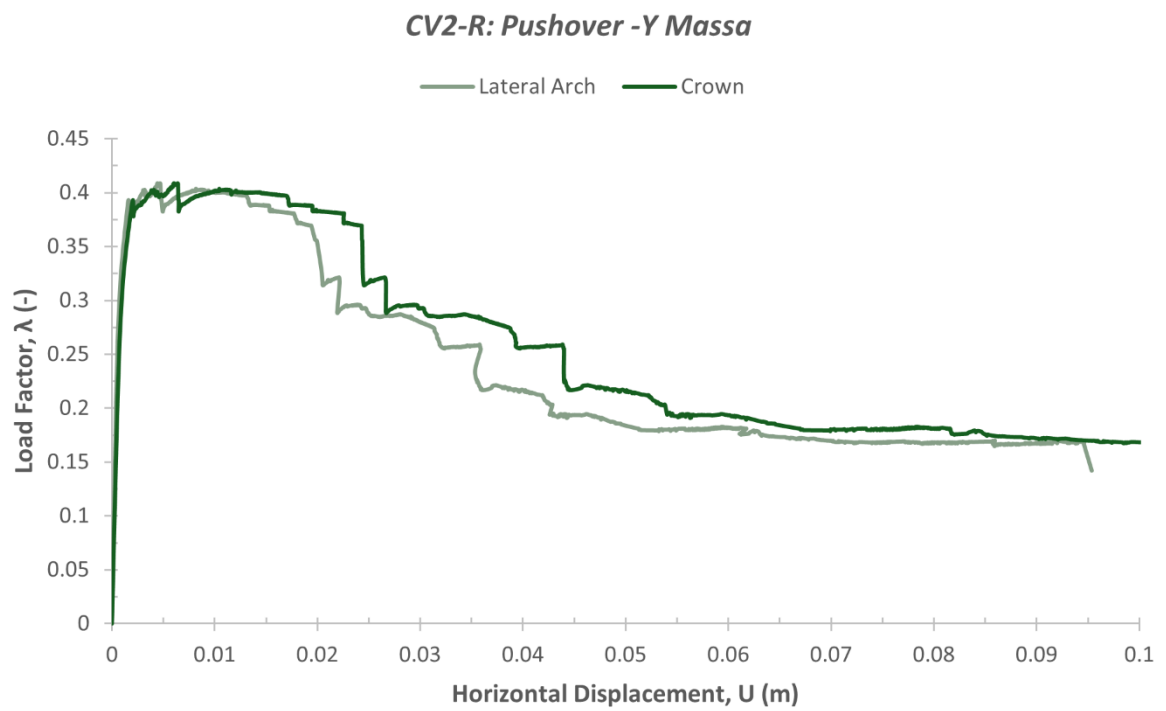
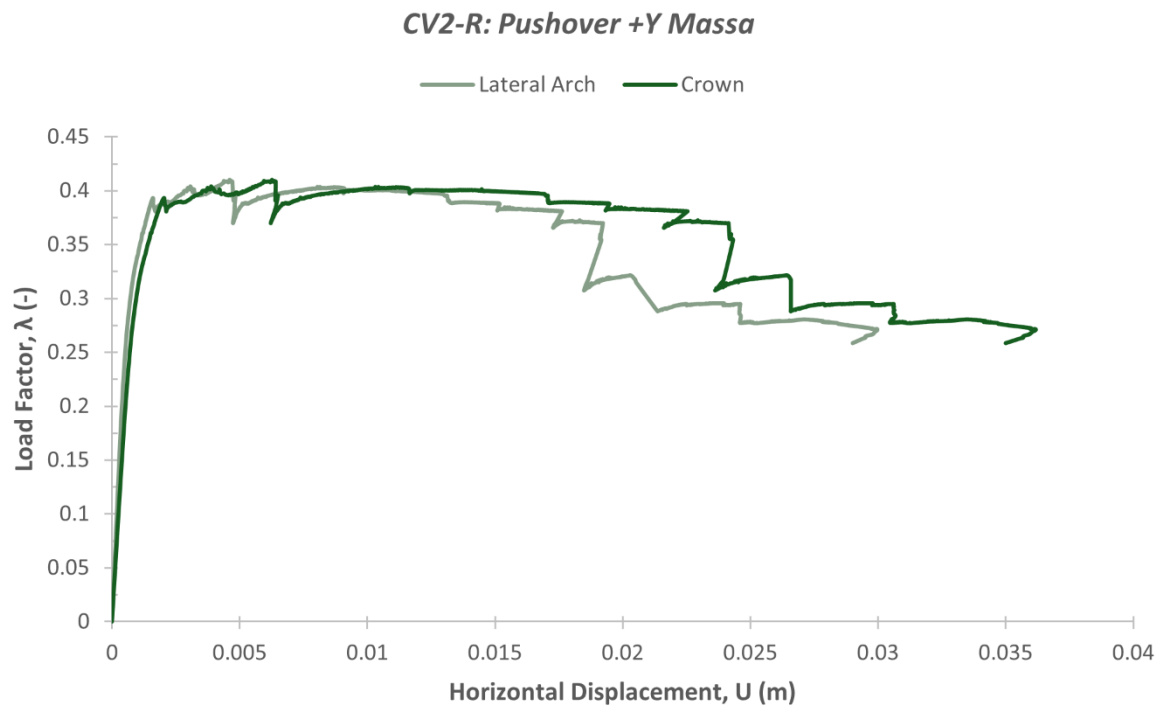


**CV1-R: Pushover -Y Massa**

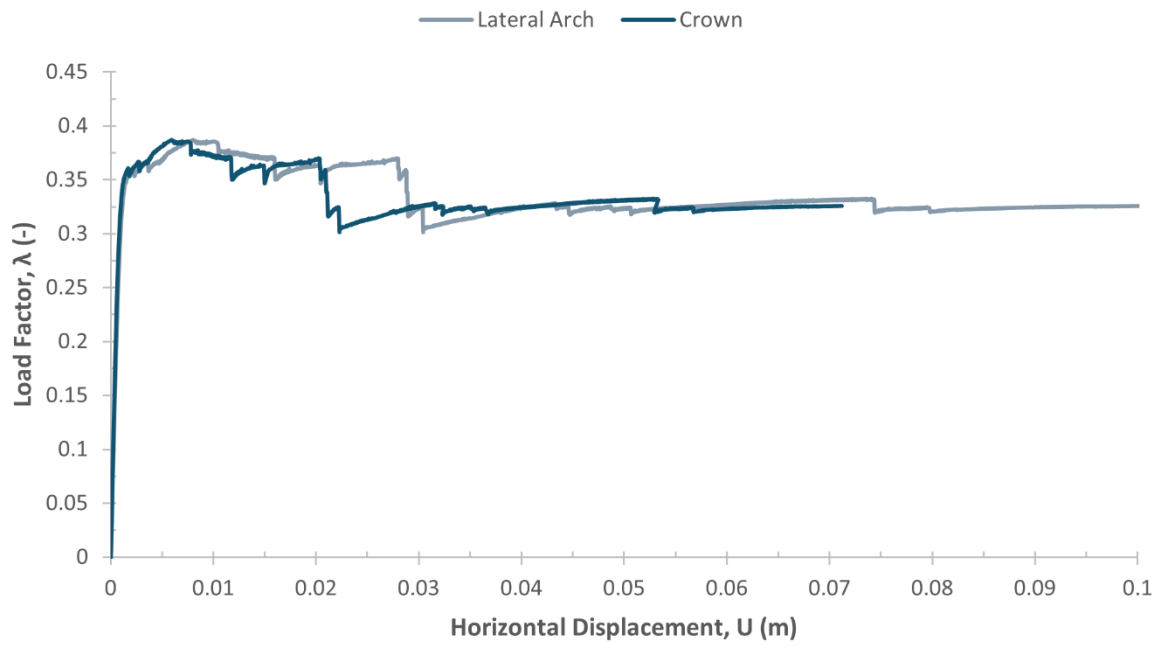




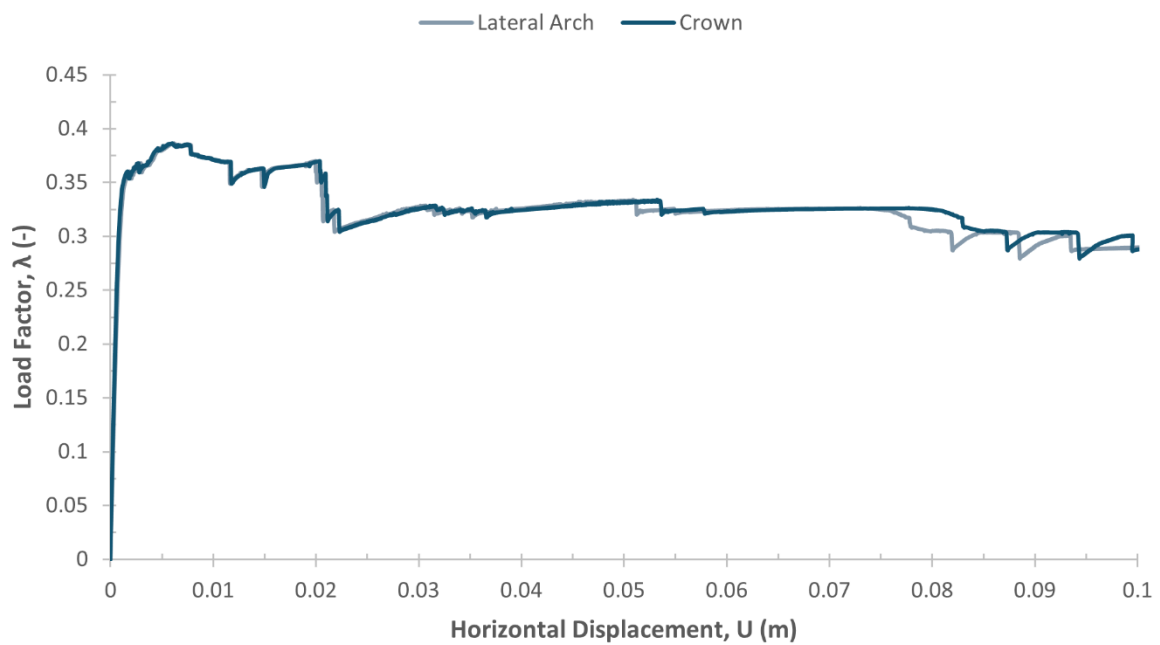


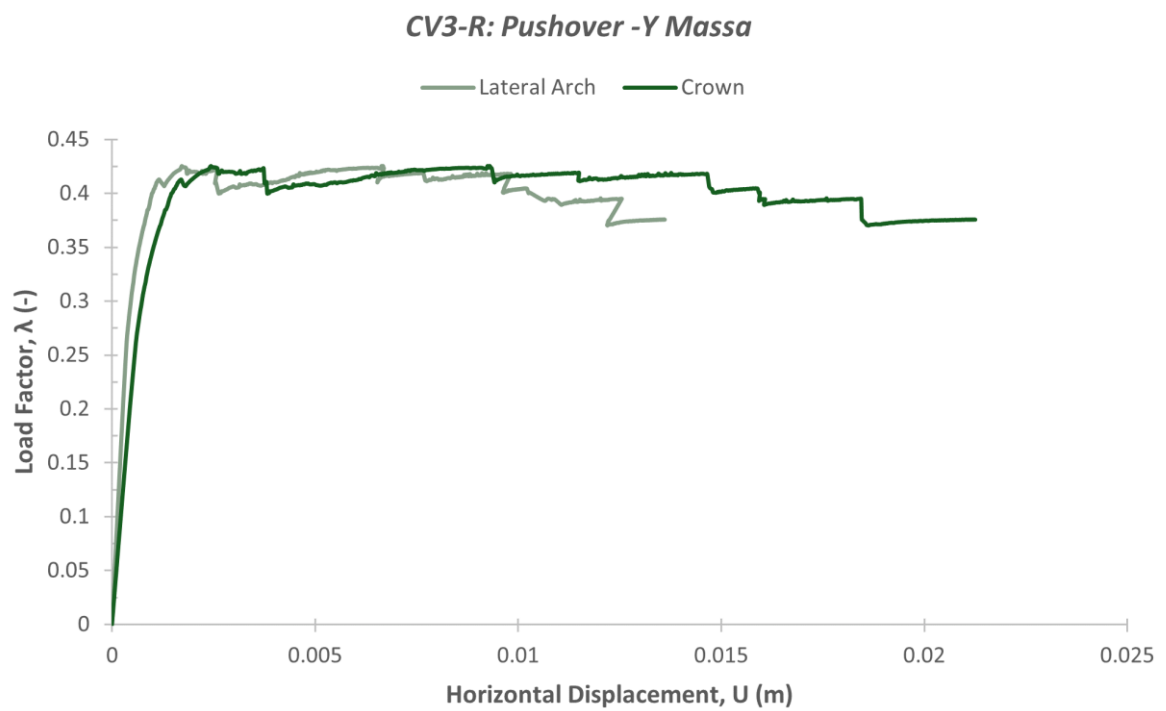
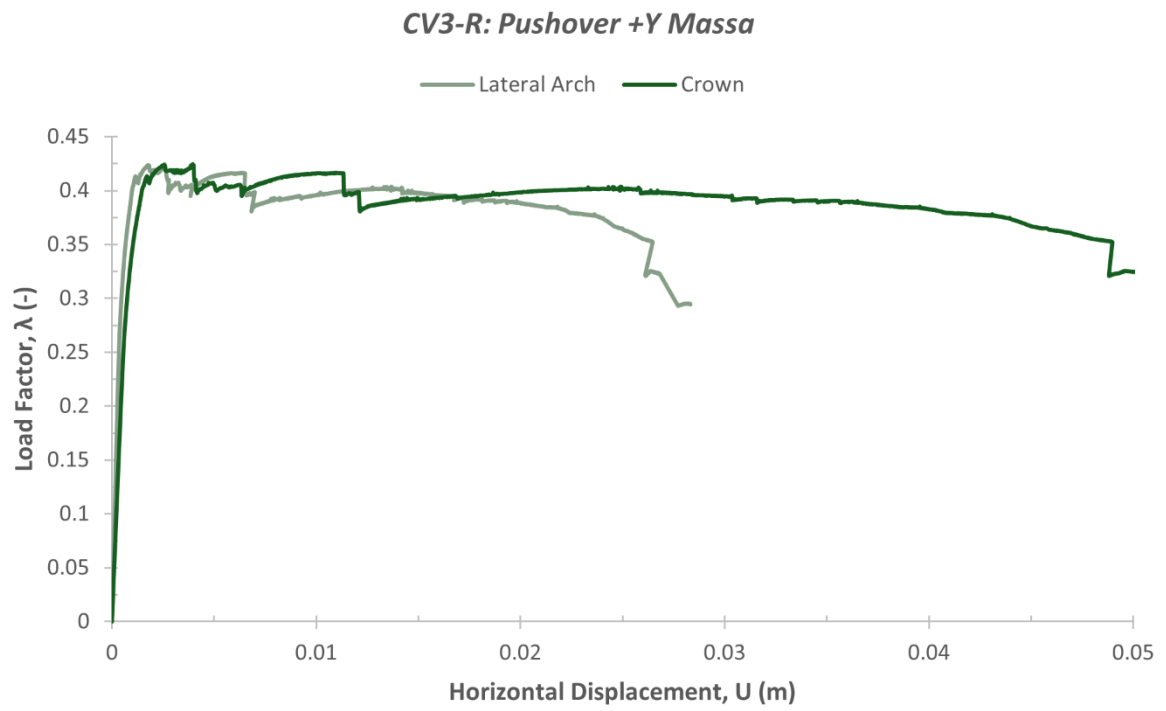


**CV3-R: Pushover +X Massa**

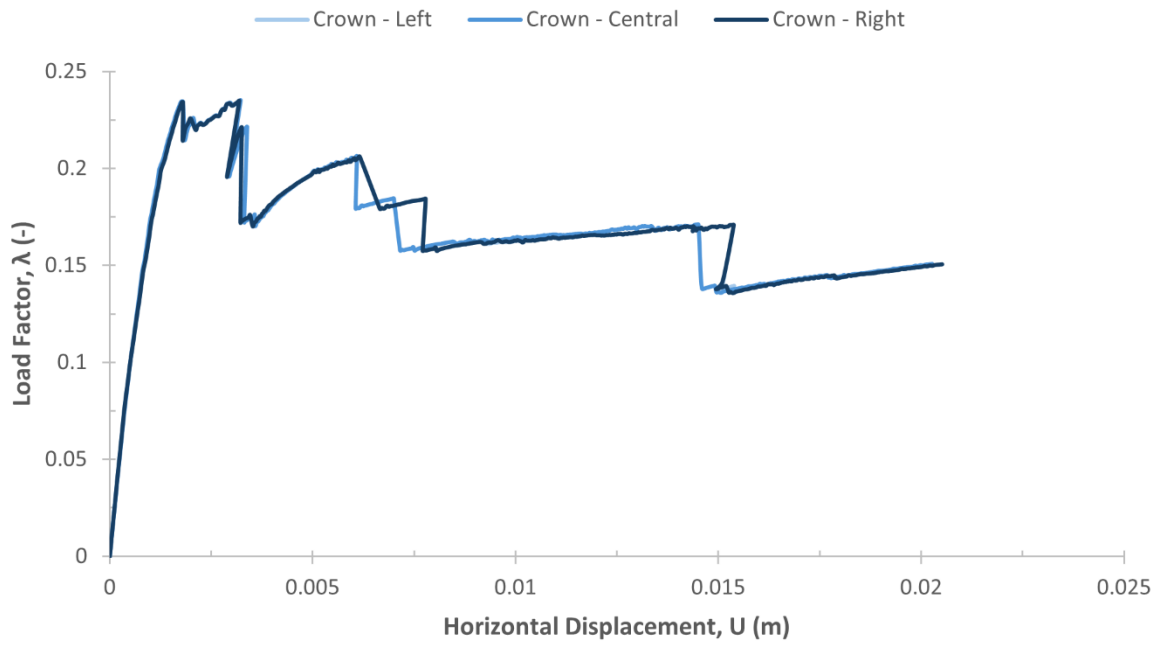


**CV3-R: Pushover -X Massa**

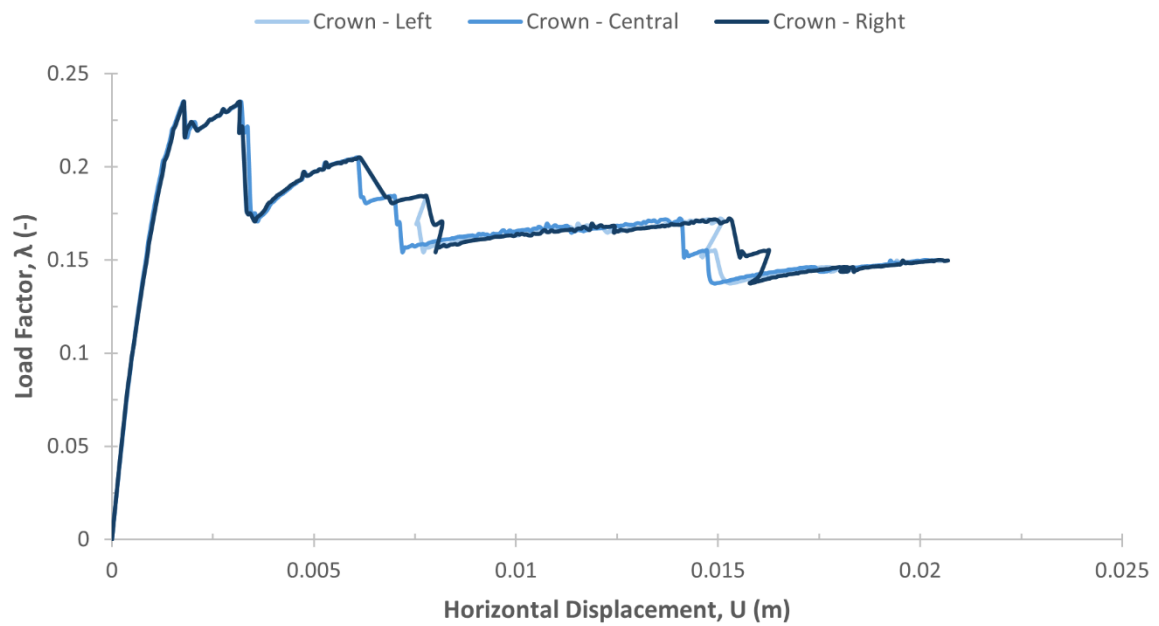


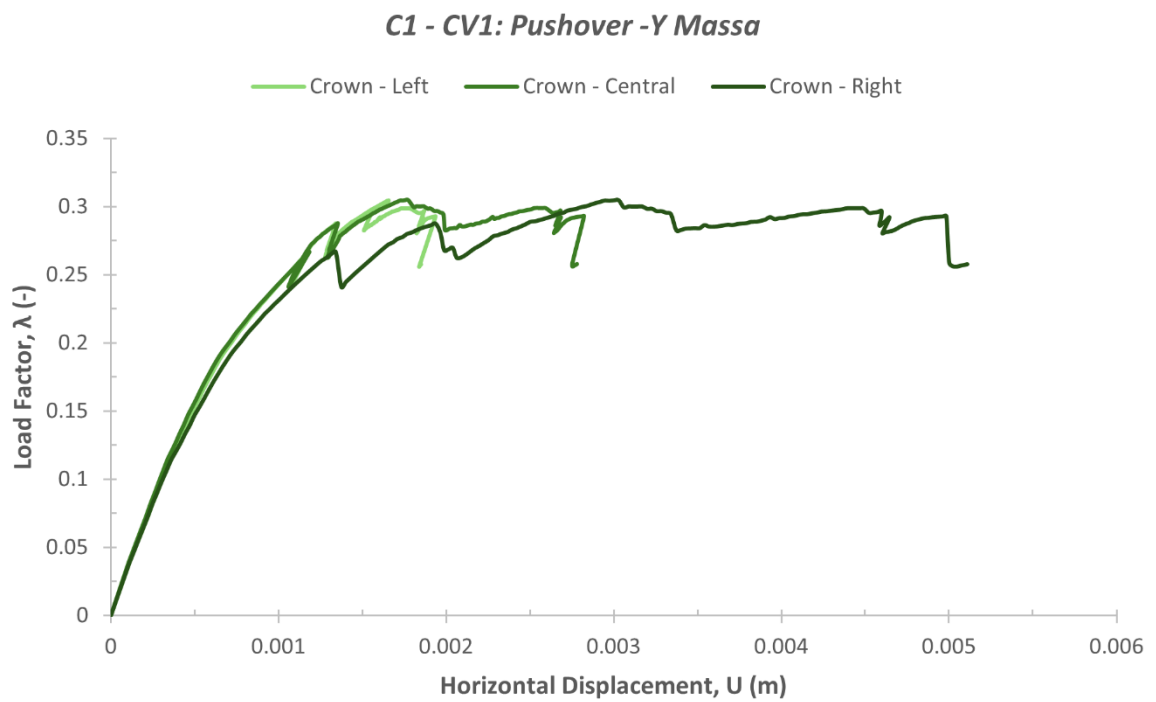
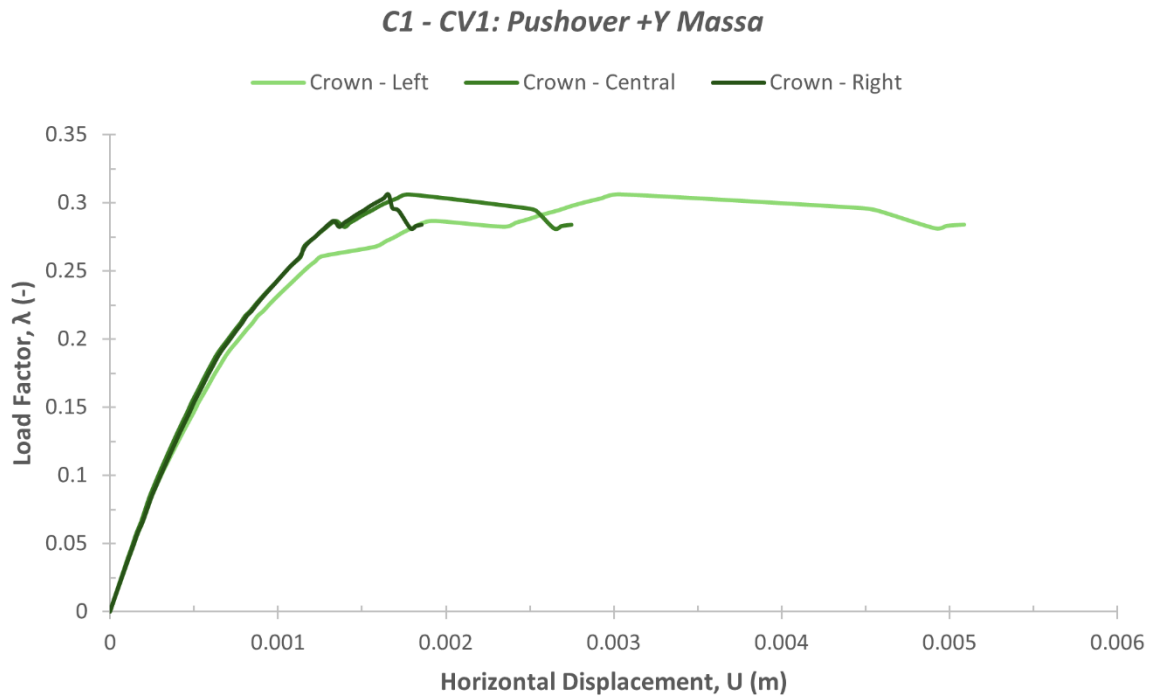


***C1 - CV1: Pushover +X Massa***

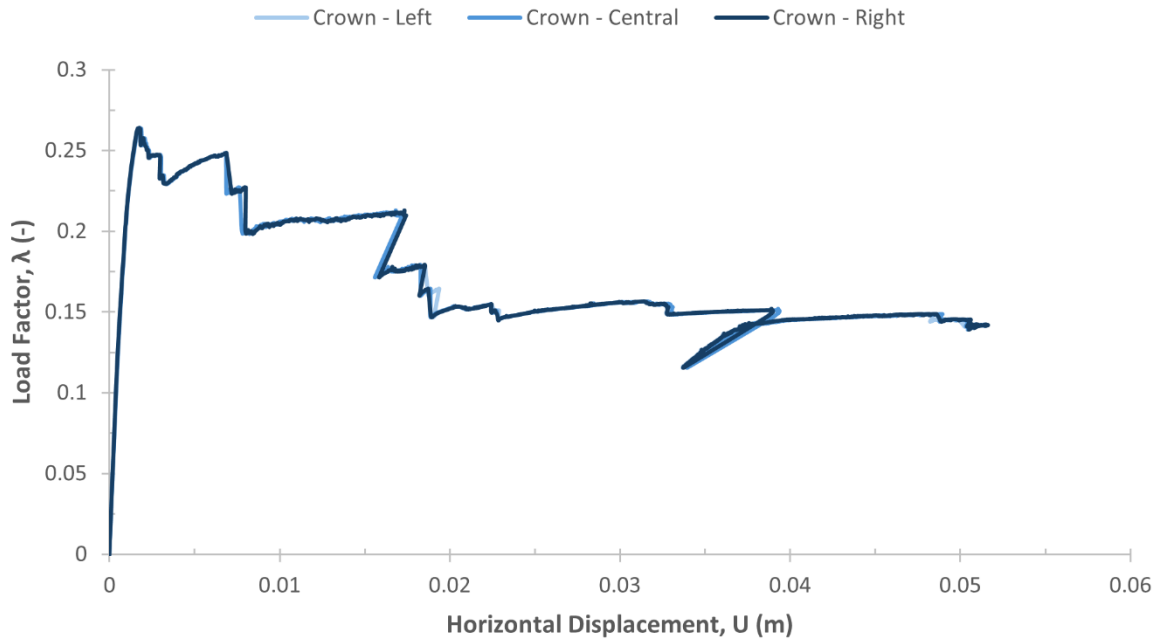


***C1 - CV1: Pushover -X Massa***

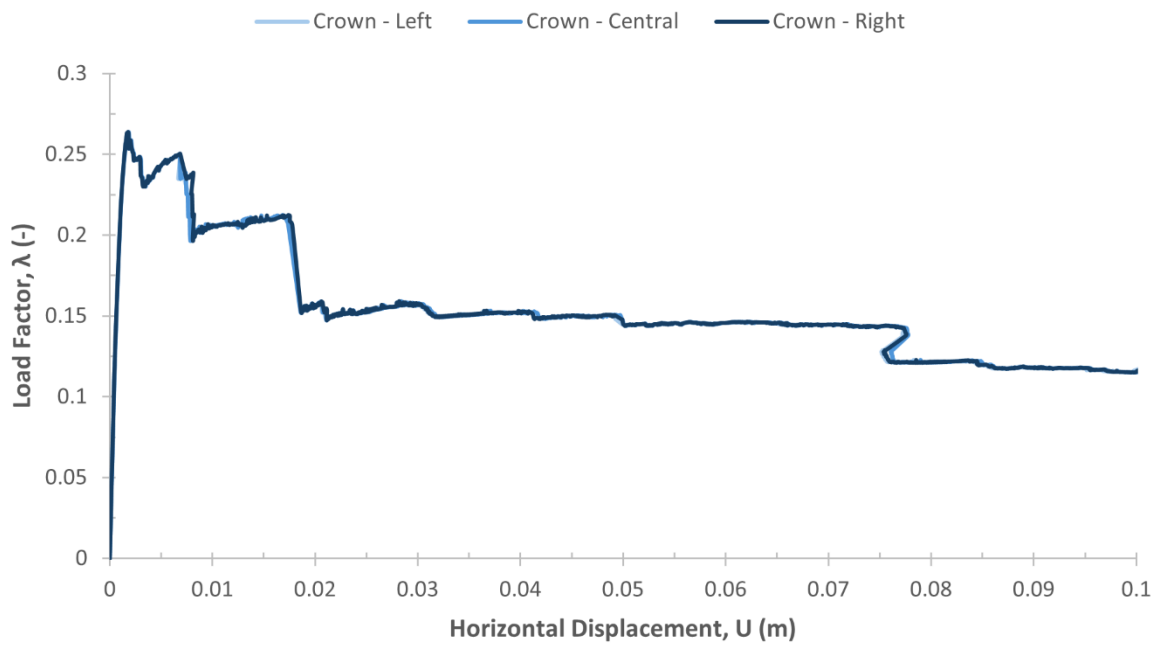


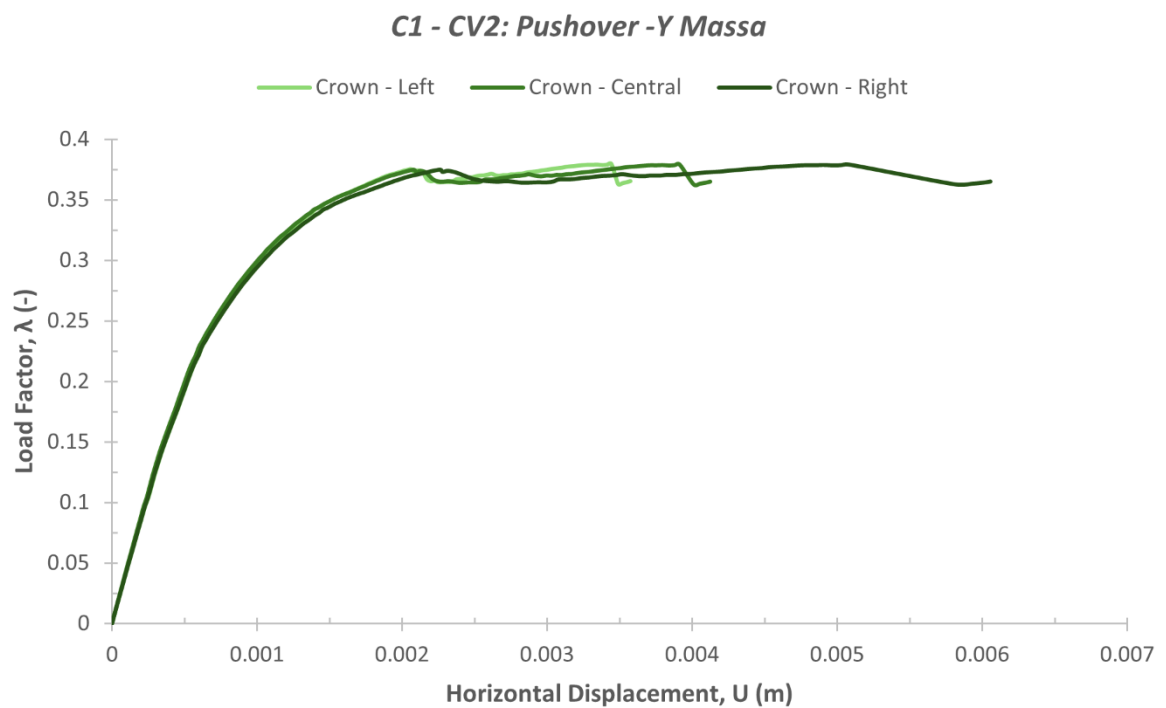
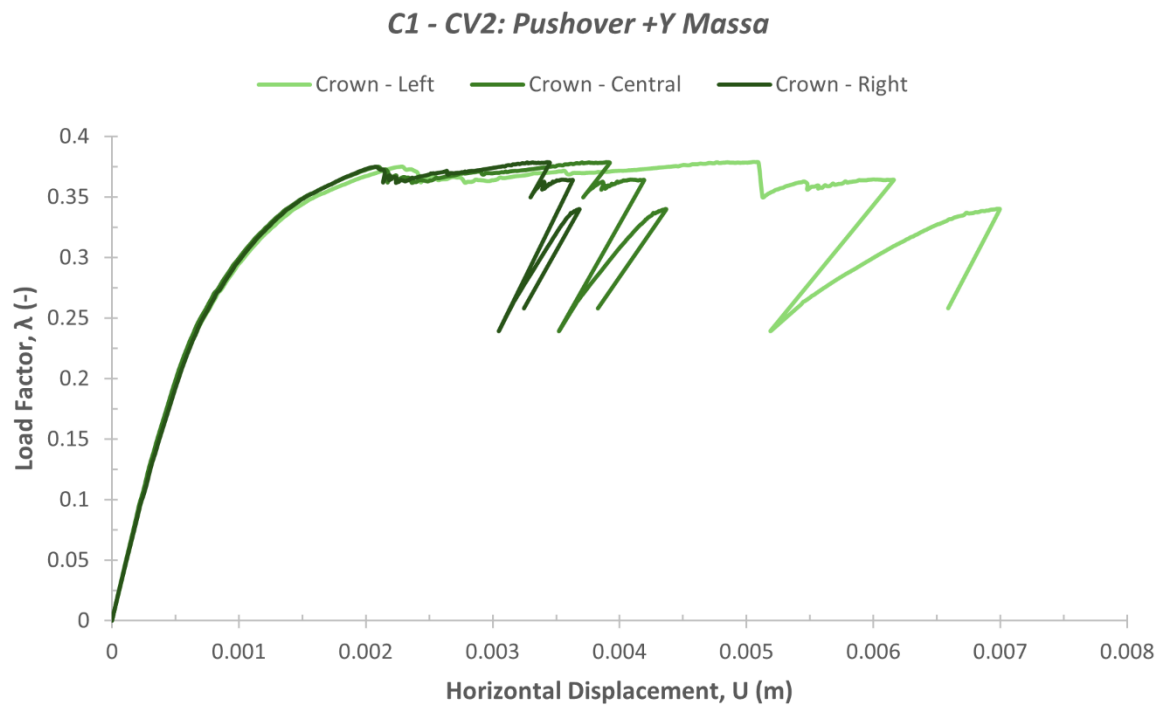


***C1 - CV2: Pushover +X Massa***



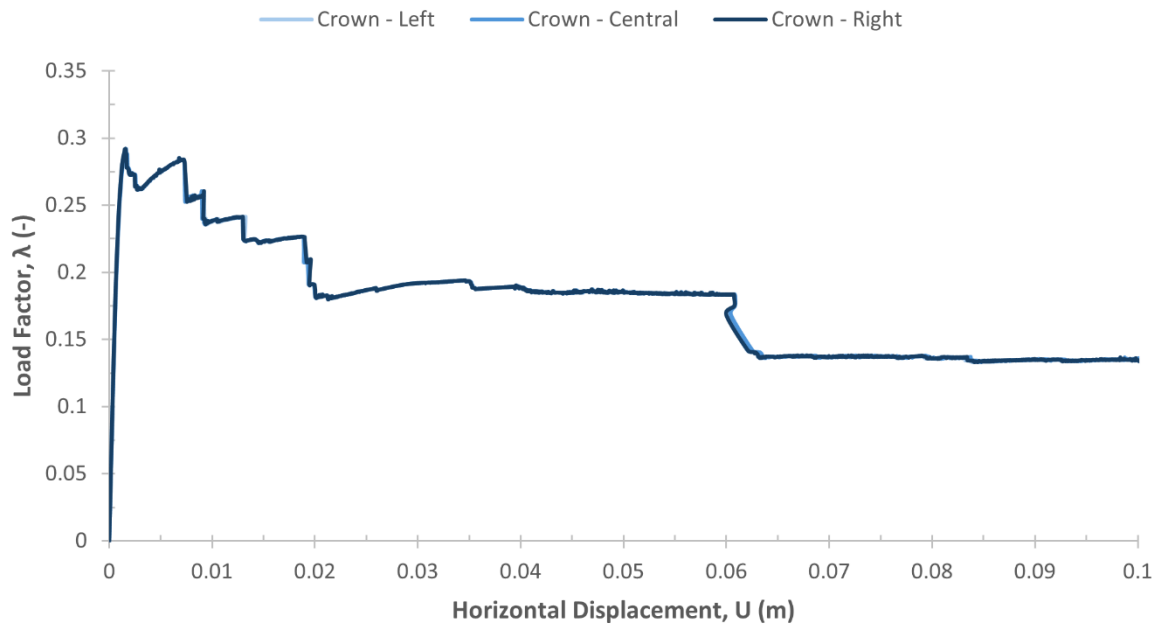
***C1 - CV2: Pushover -X Massa***



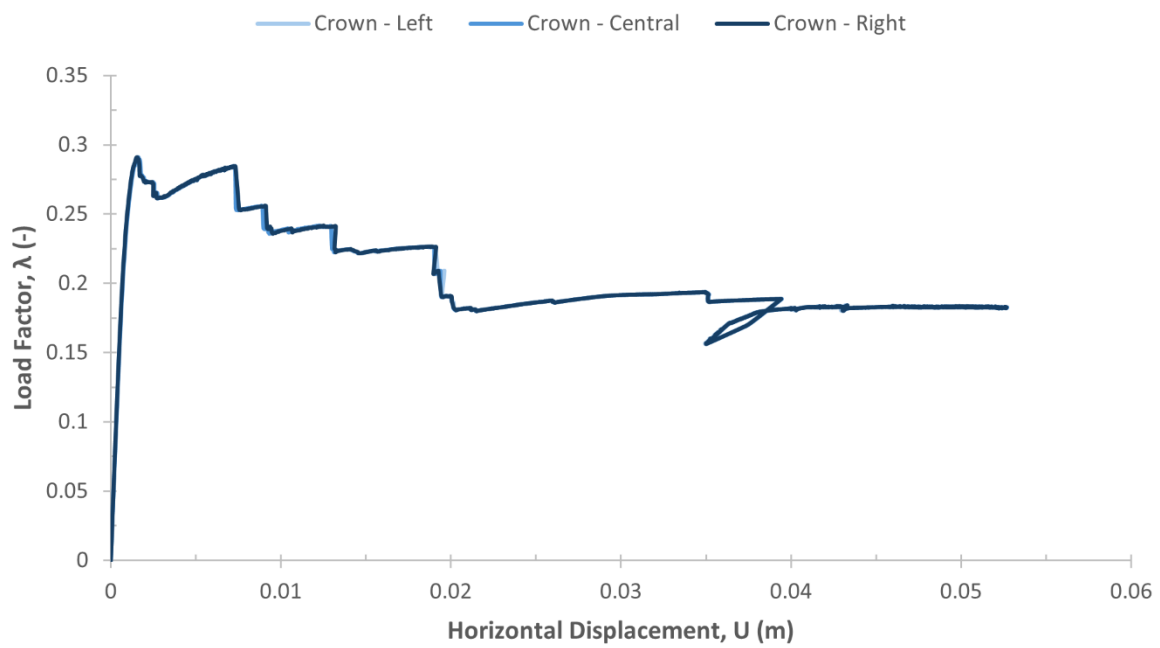


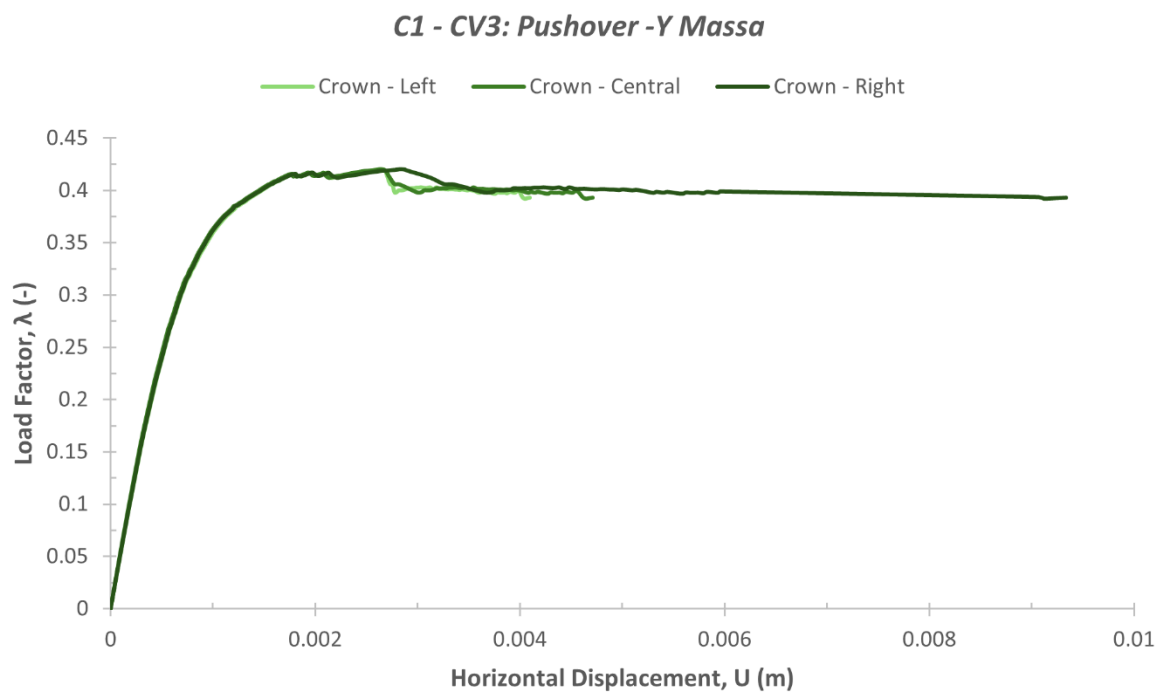
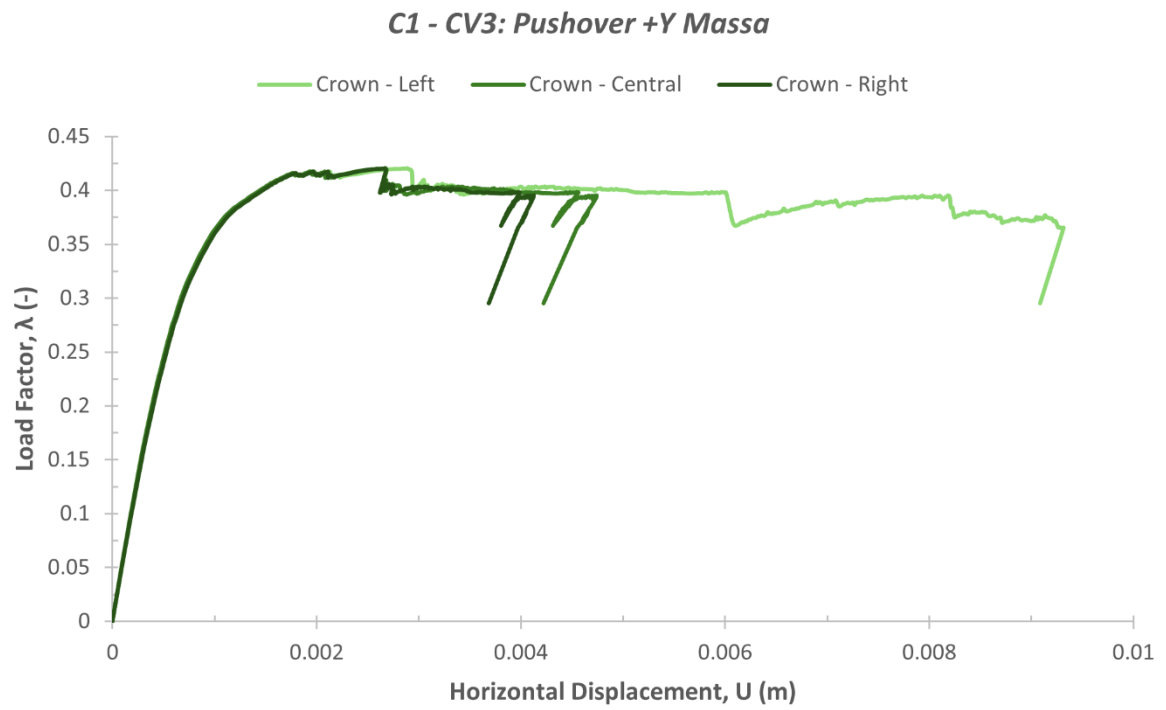


### *C1 - CV3: Pushover +X Massa*

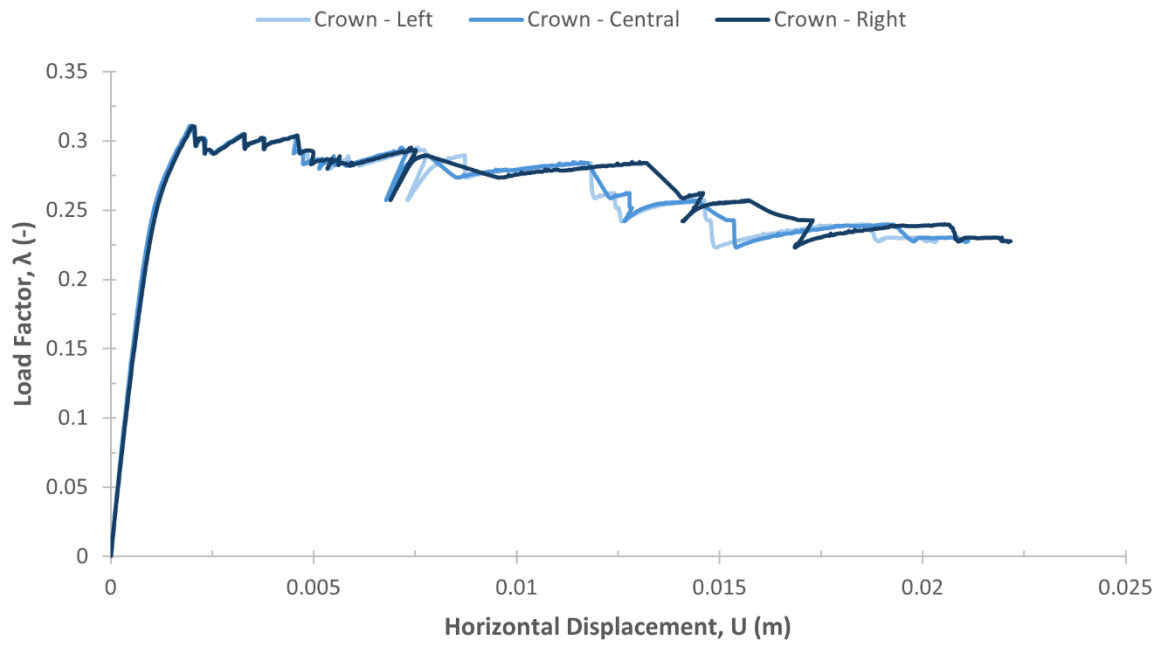


### *C1 - CV3: Pushover -X Massa*

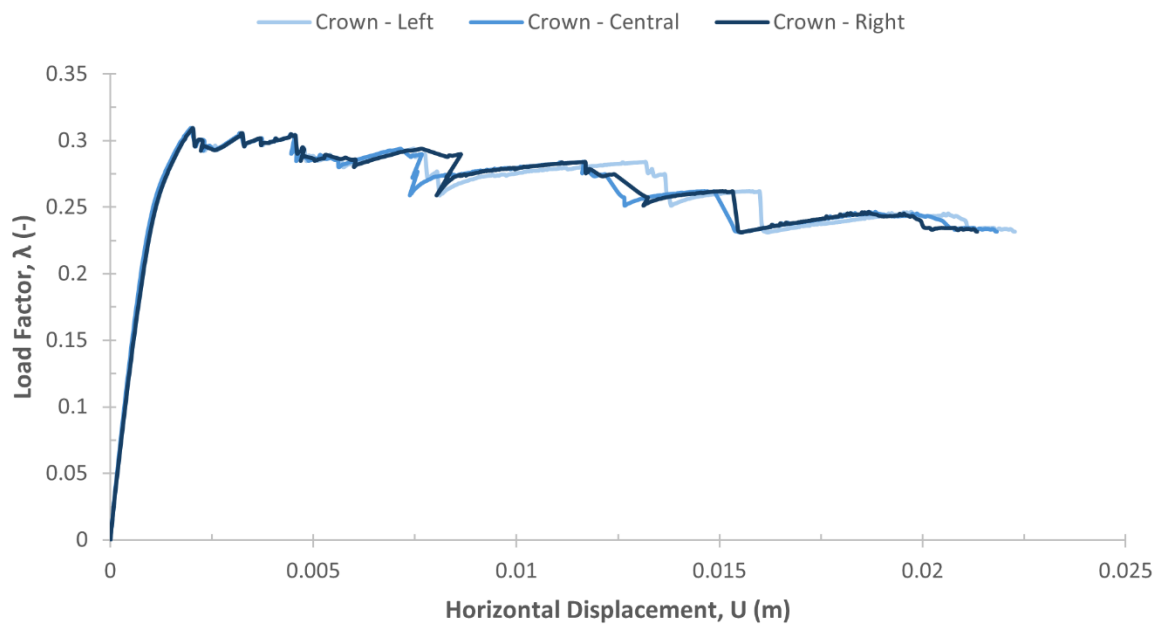


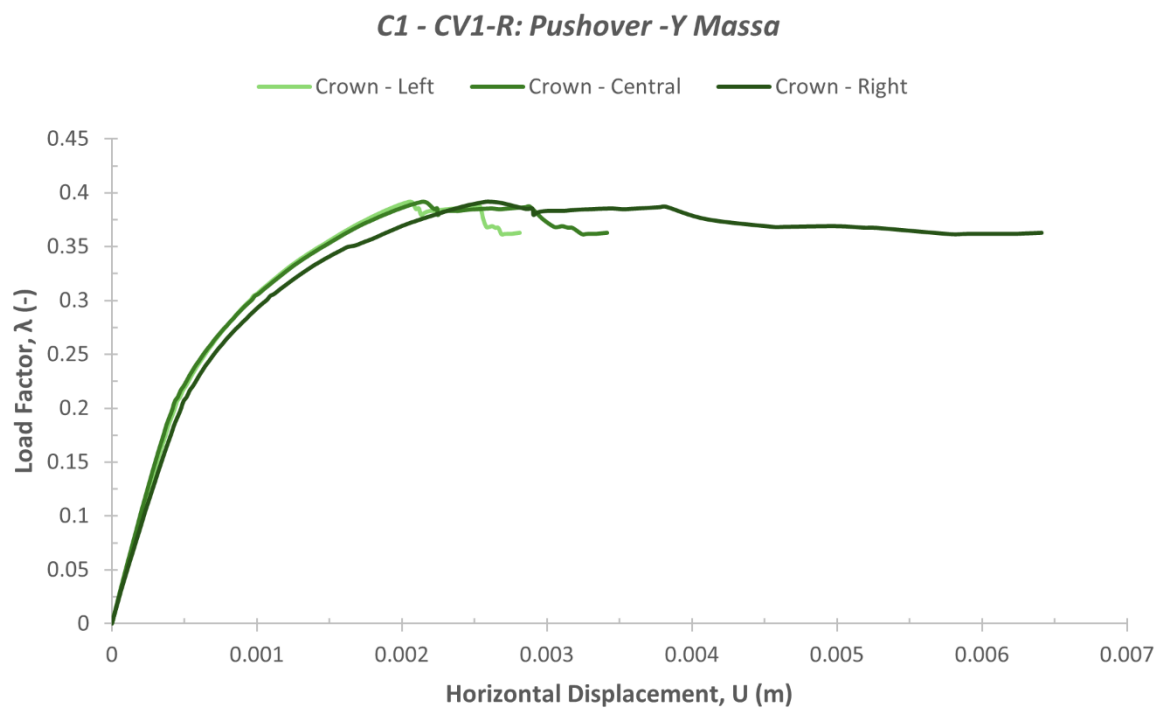
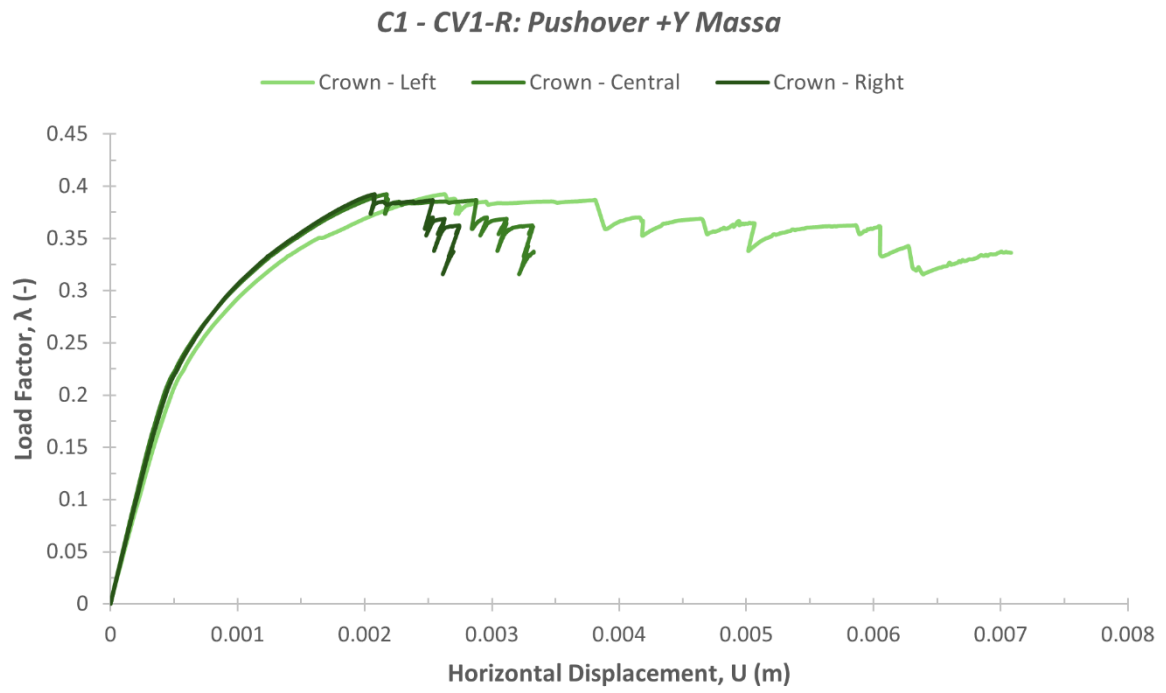


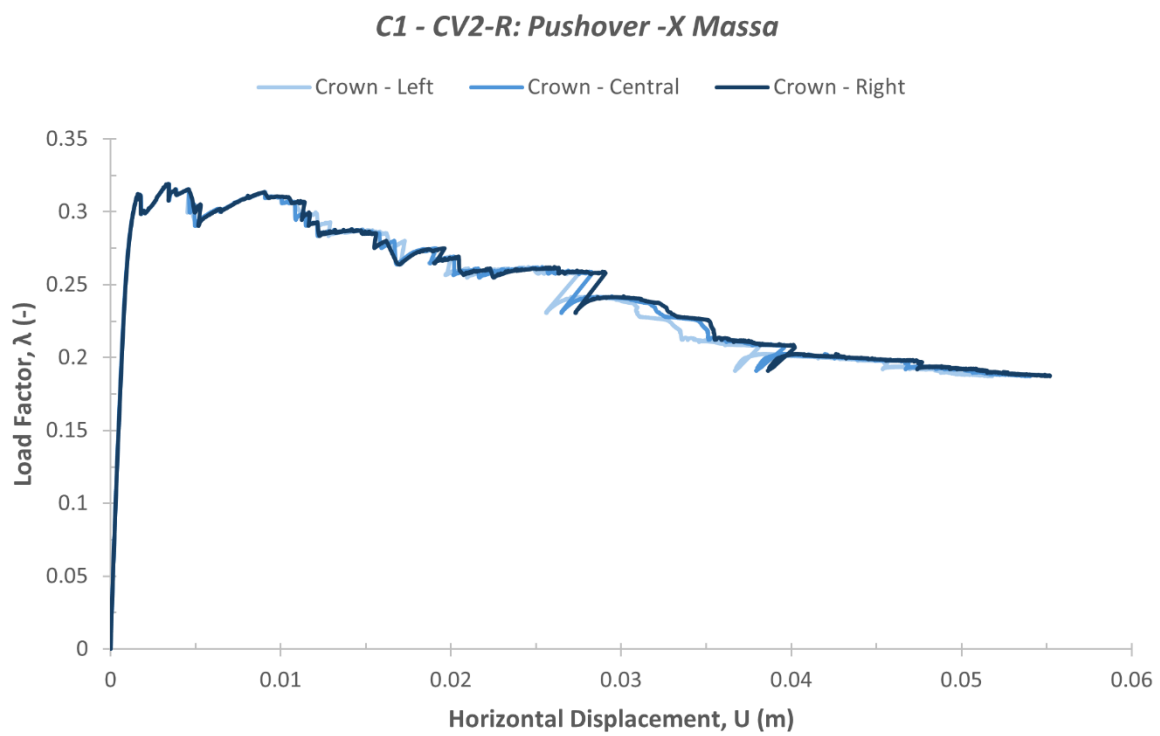
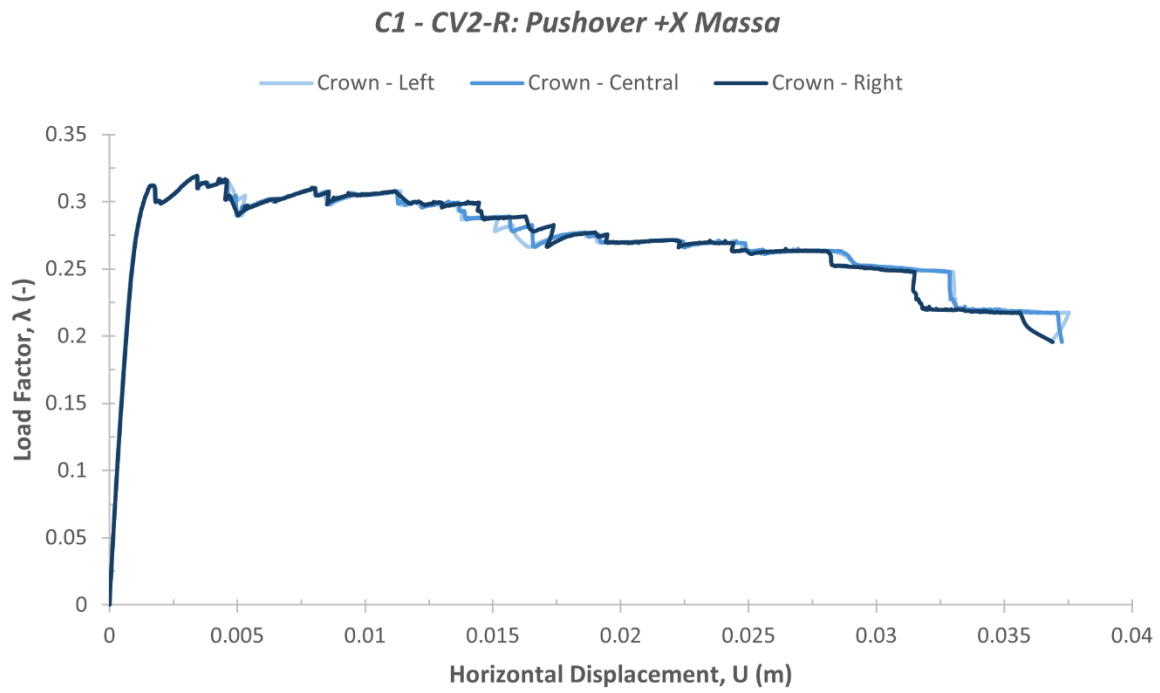
***C1 - CV1-R: Pushover +X Massa***

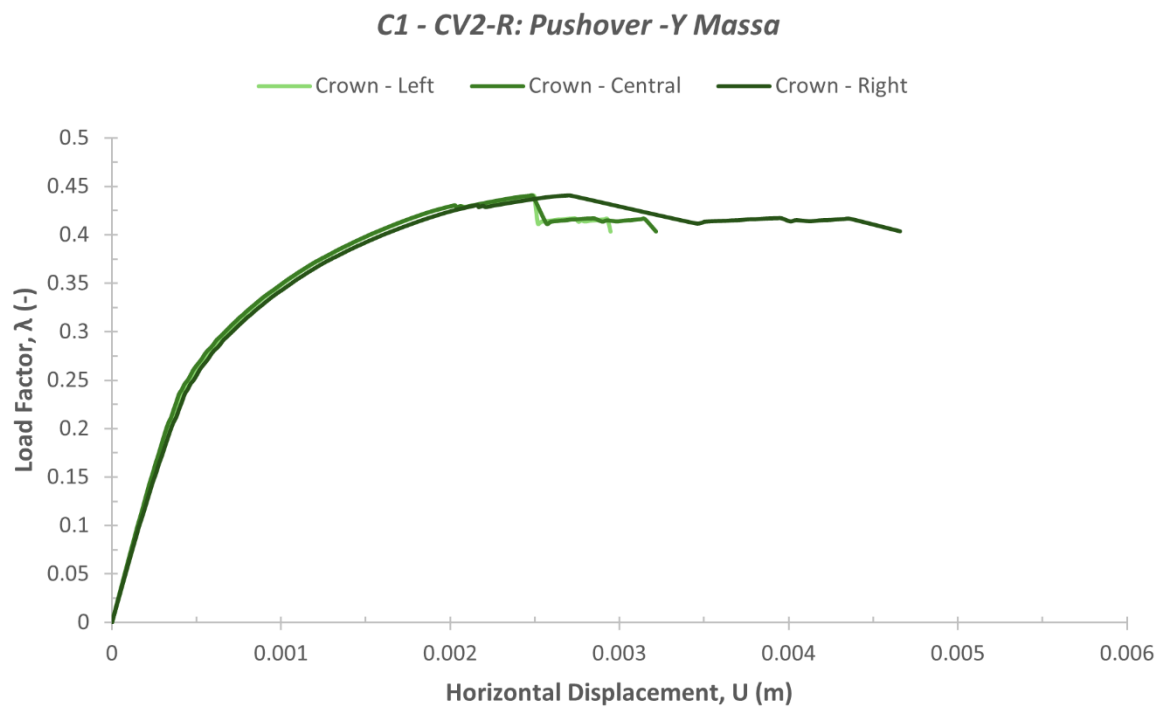
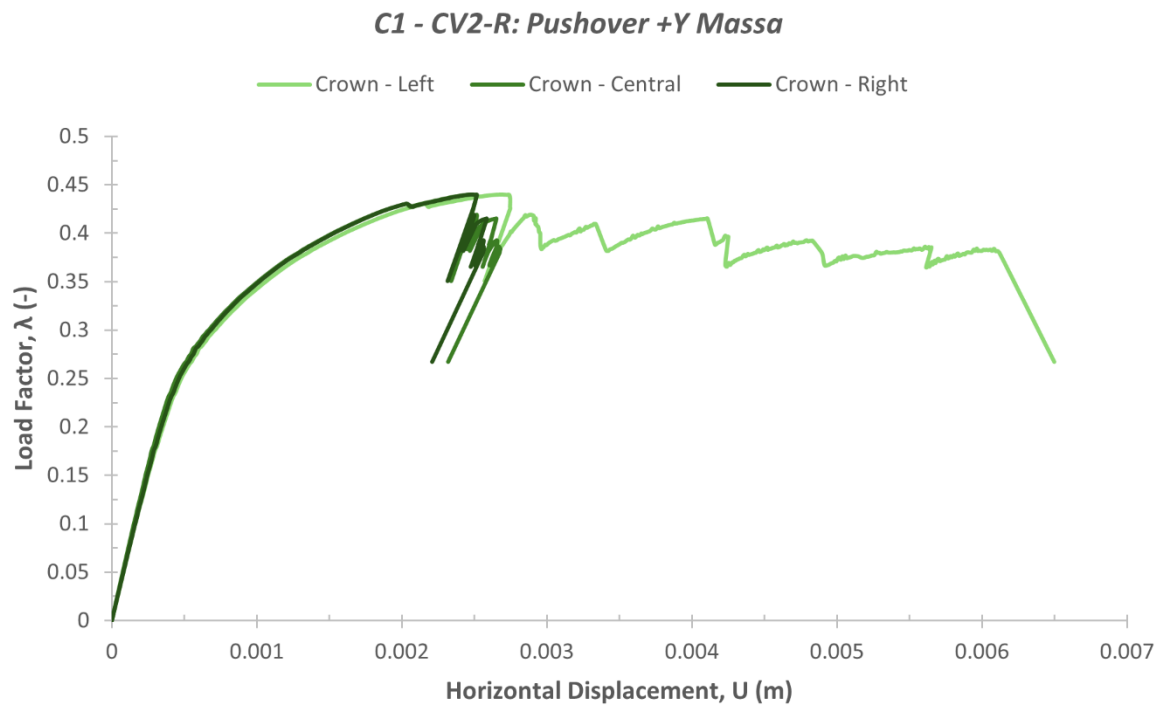


***C1 - CV1-R: Pushover -X Massa***

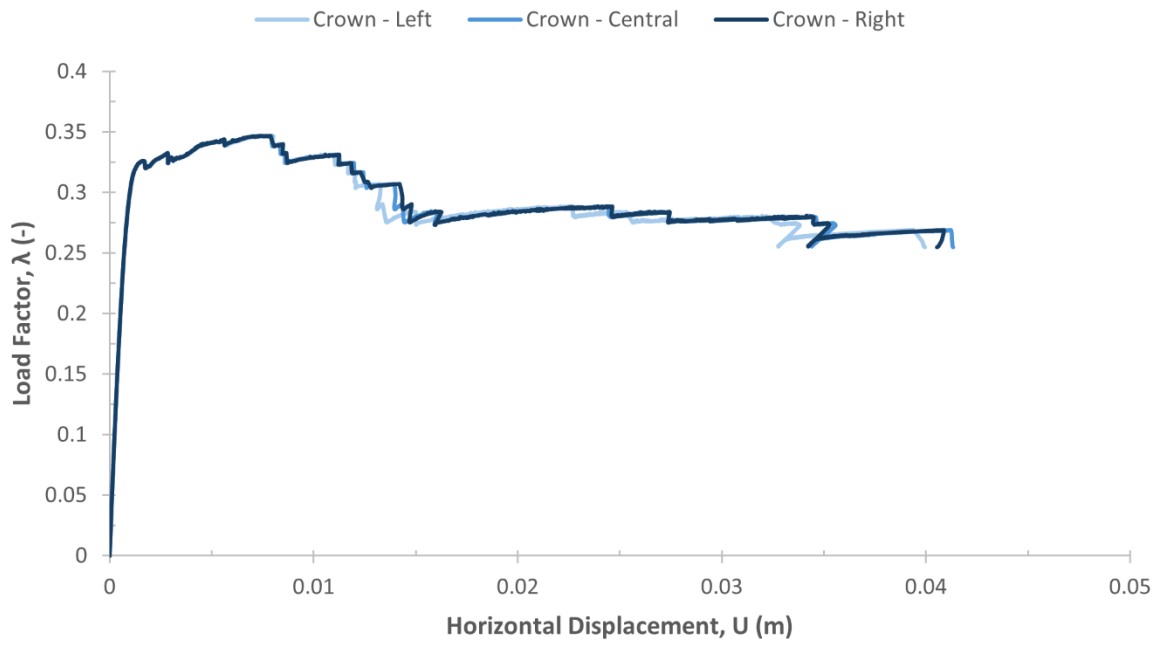




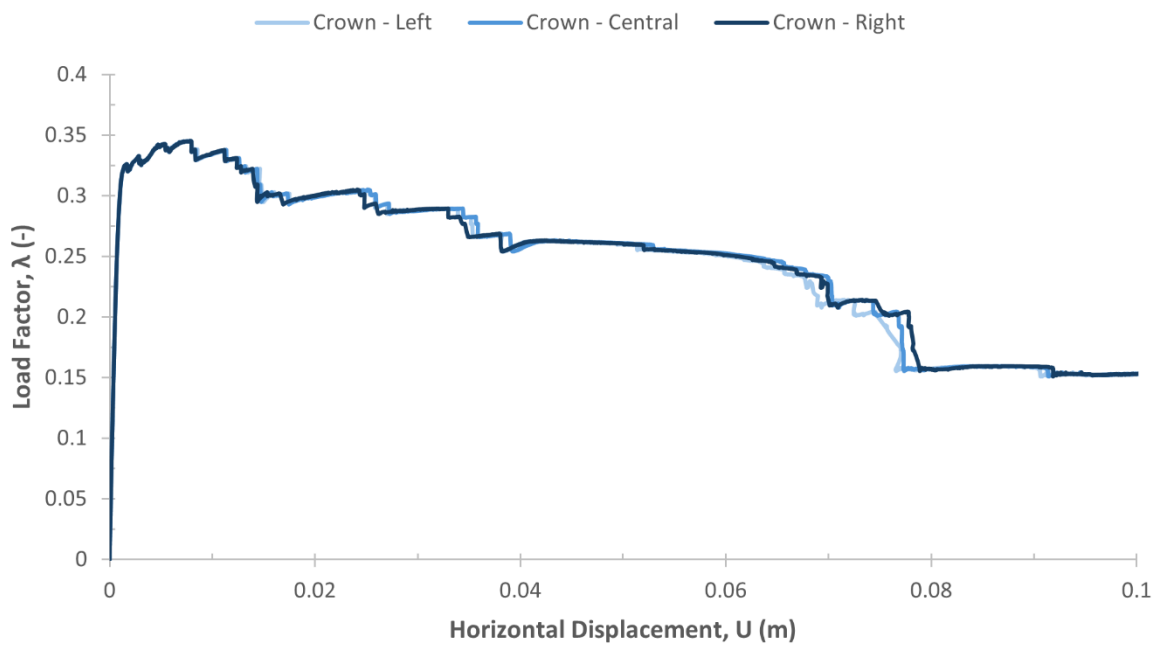




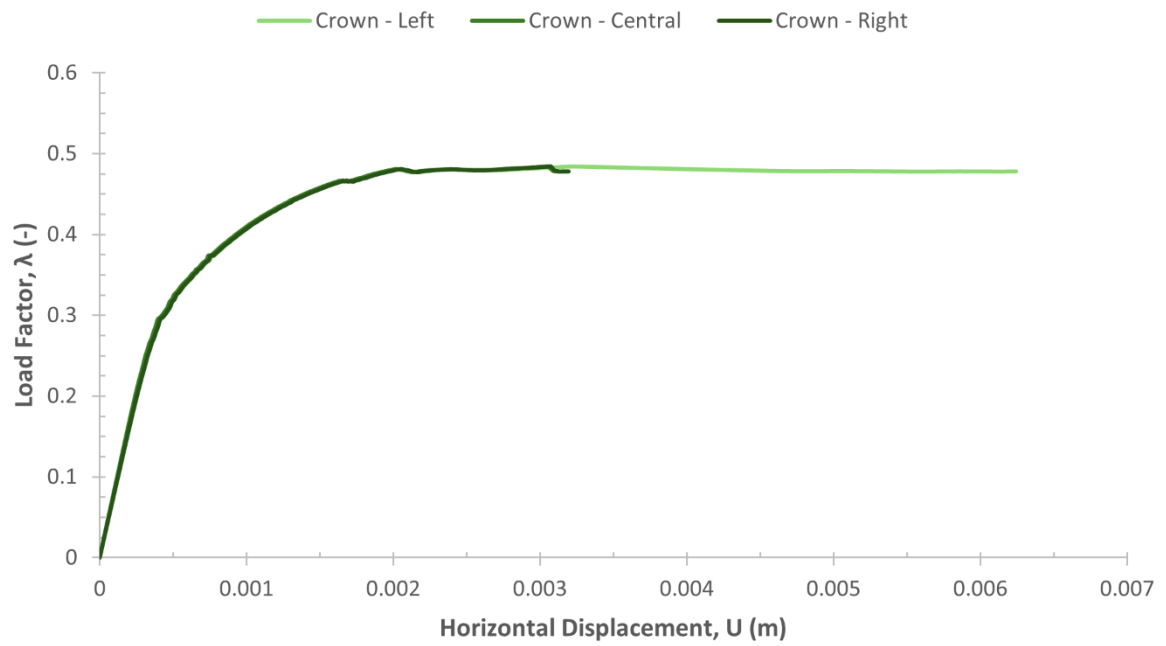
***C1 - CV3-R: Pushover -X Massa***



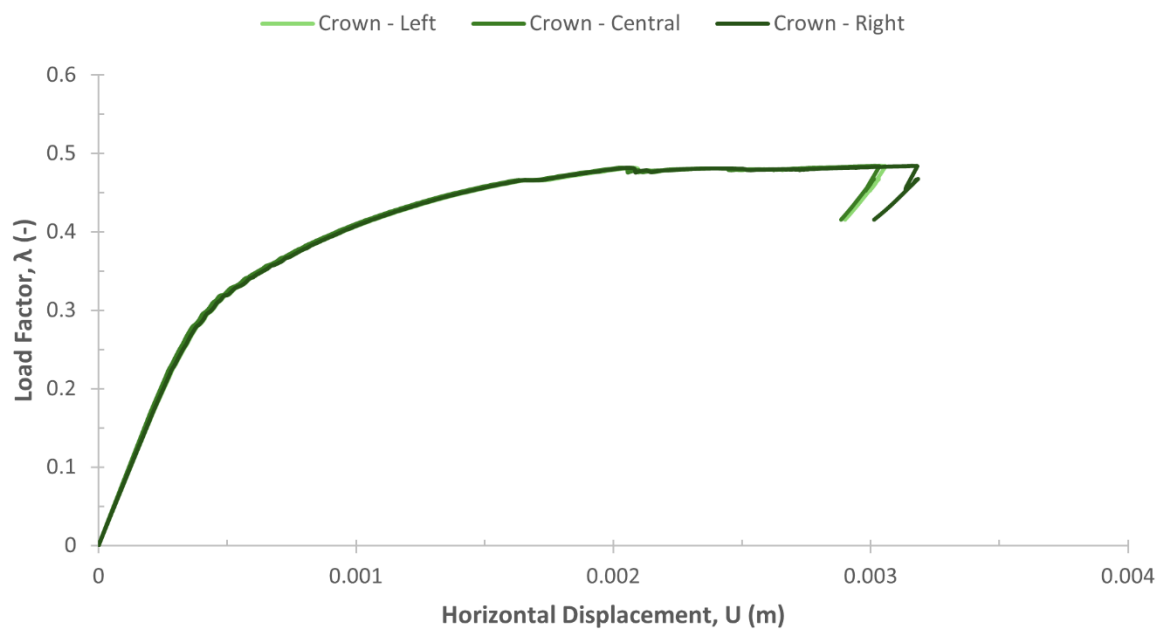
***C1 - CV3-R: Pushover -X Massa***



***C1 - CV3-R: Pushover +Y Massa***

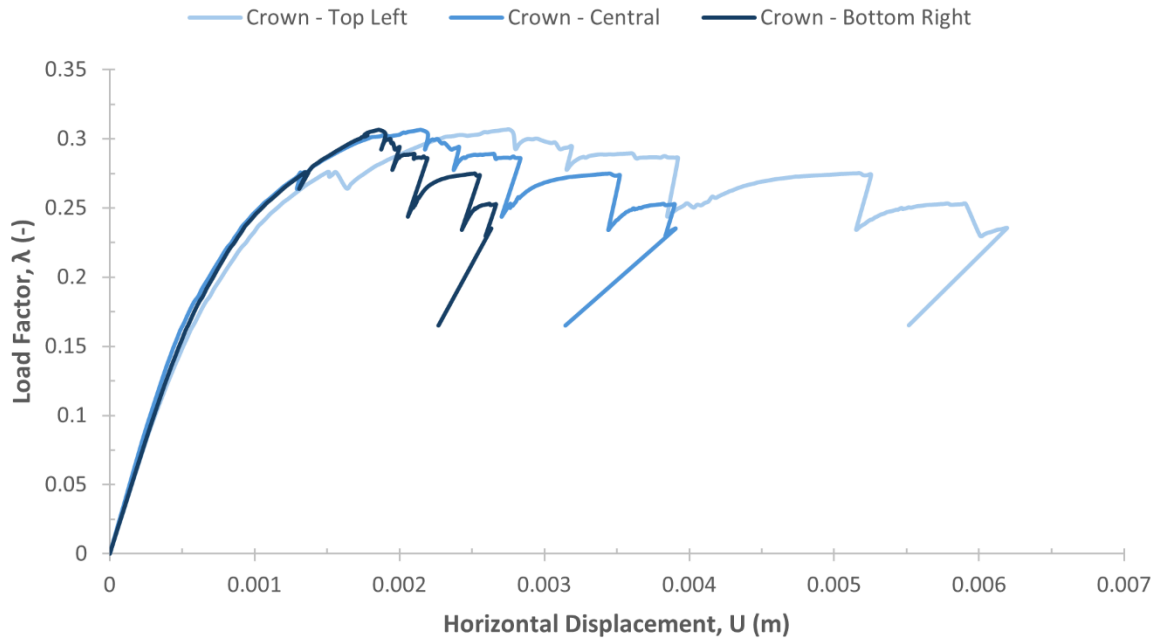


***C1 - CV3-R: Pushover -Y Massa***

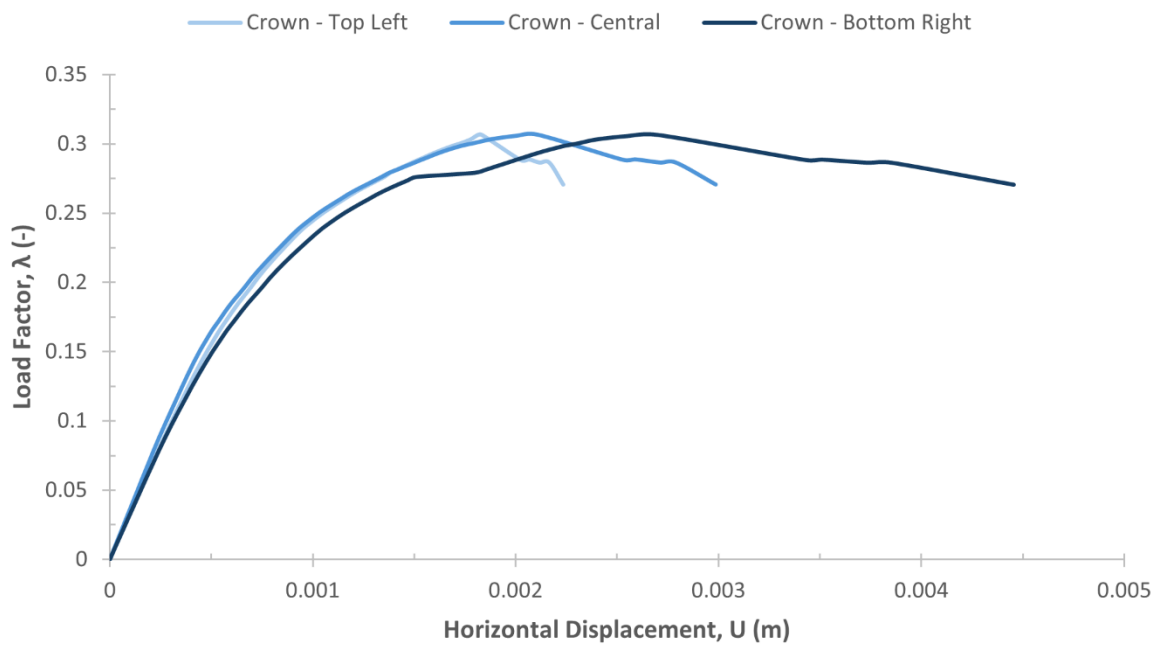


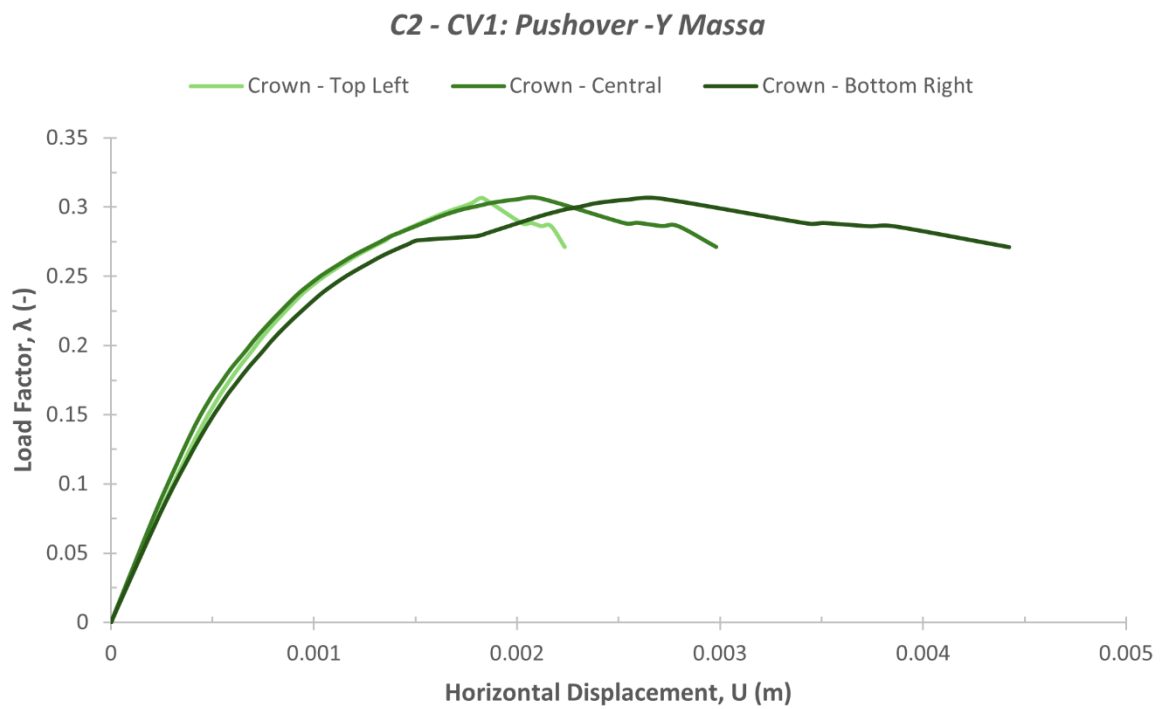
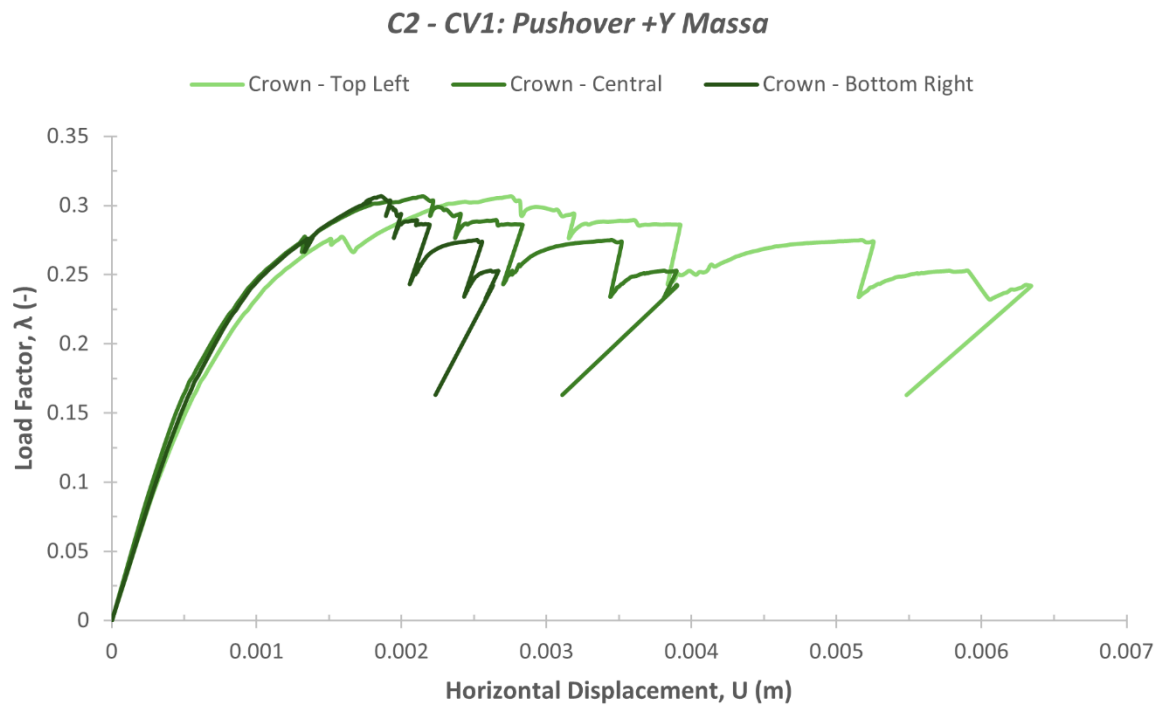


***C2 - CV1: Pushover +X Massa***

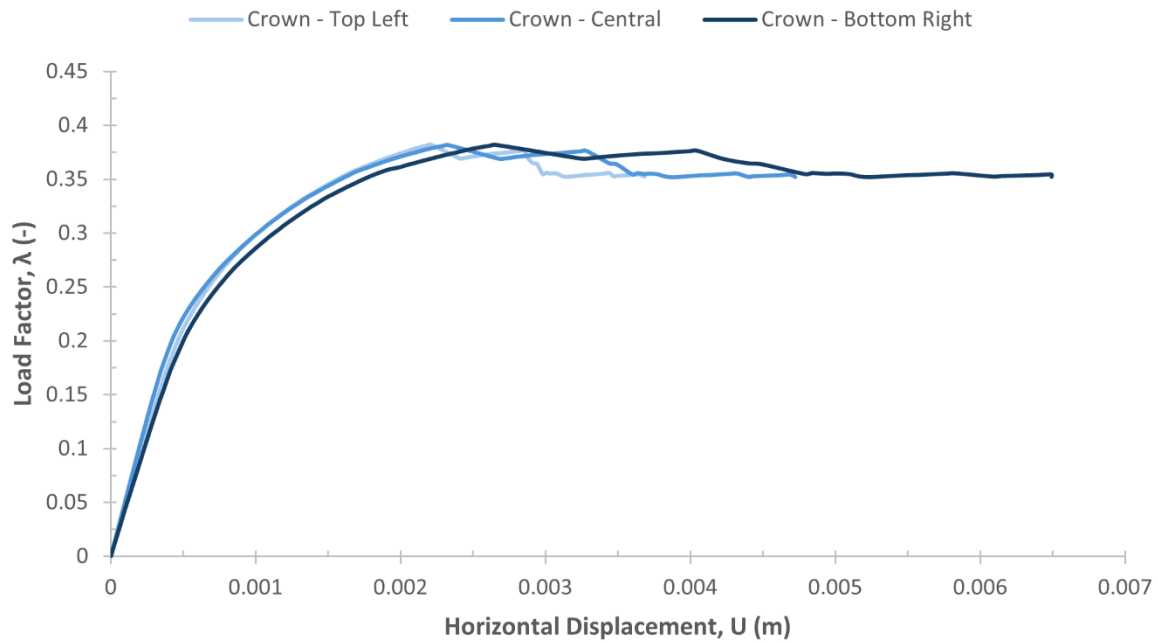


***C2 - CV1: Pushover -X Massa***

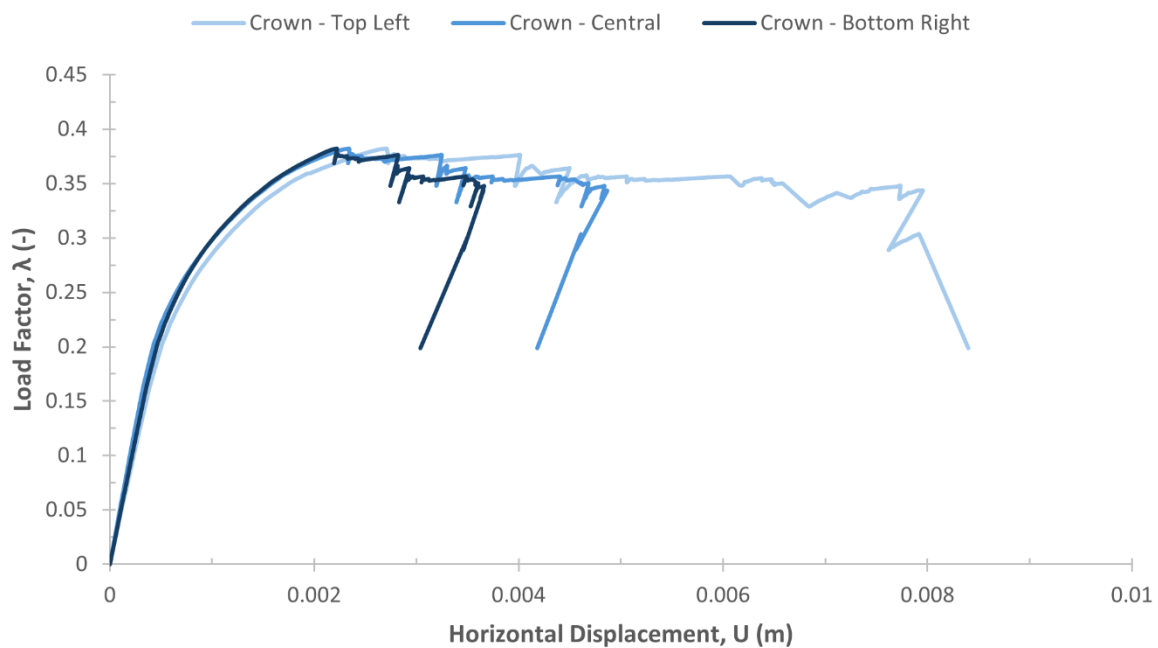


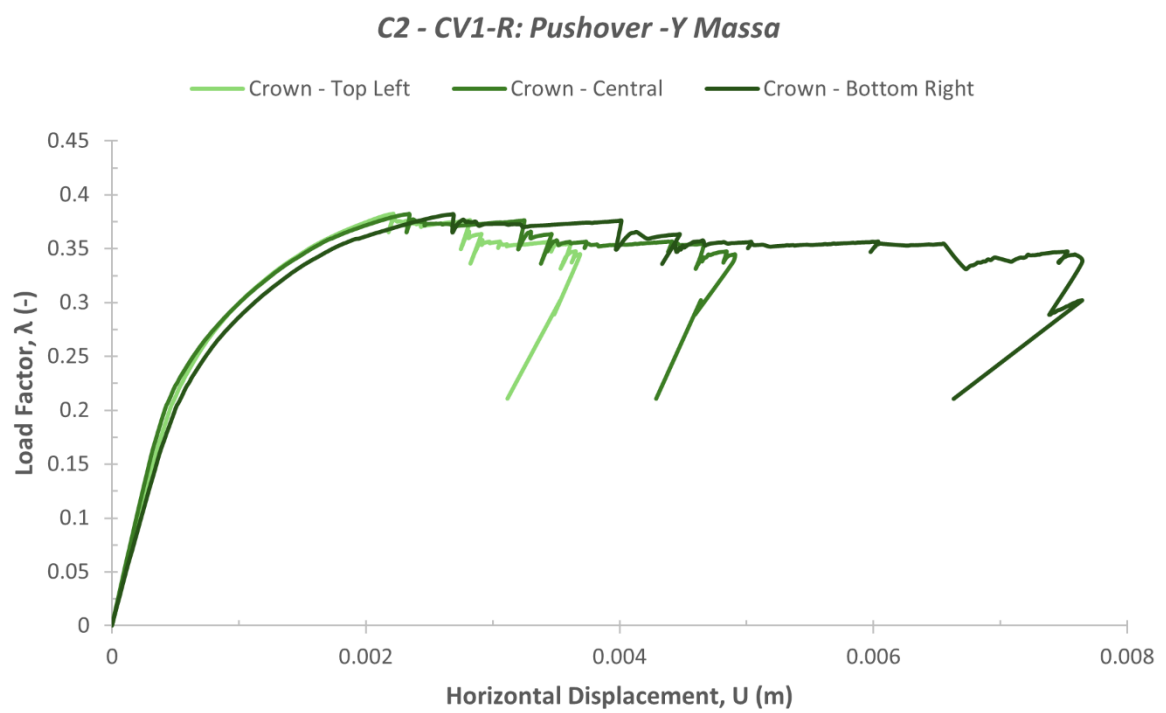
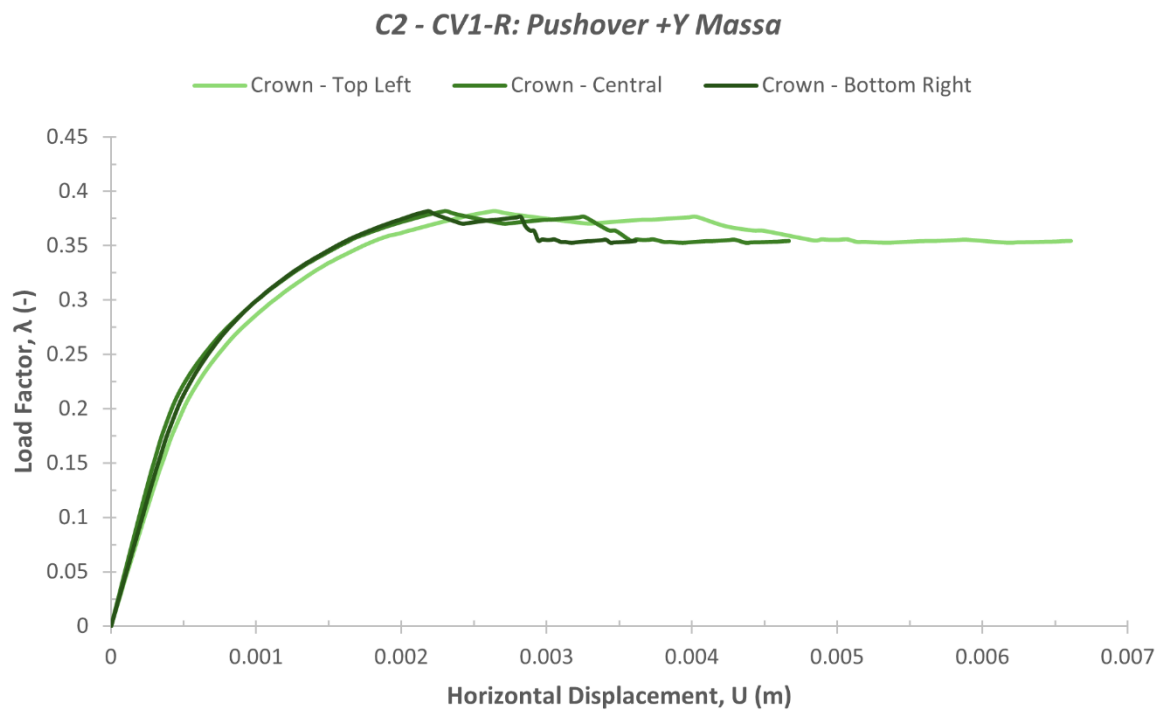


***C2 - CV1-R: Pushover +X Massa***

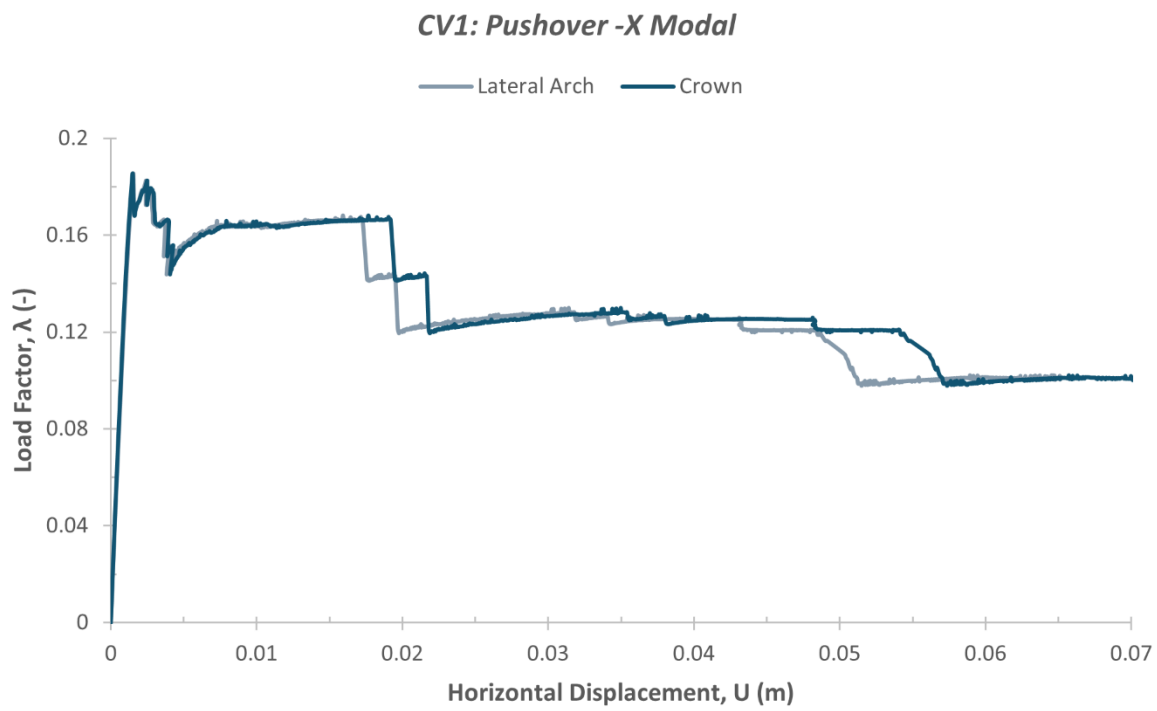
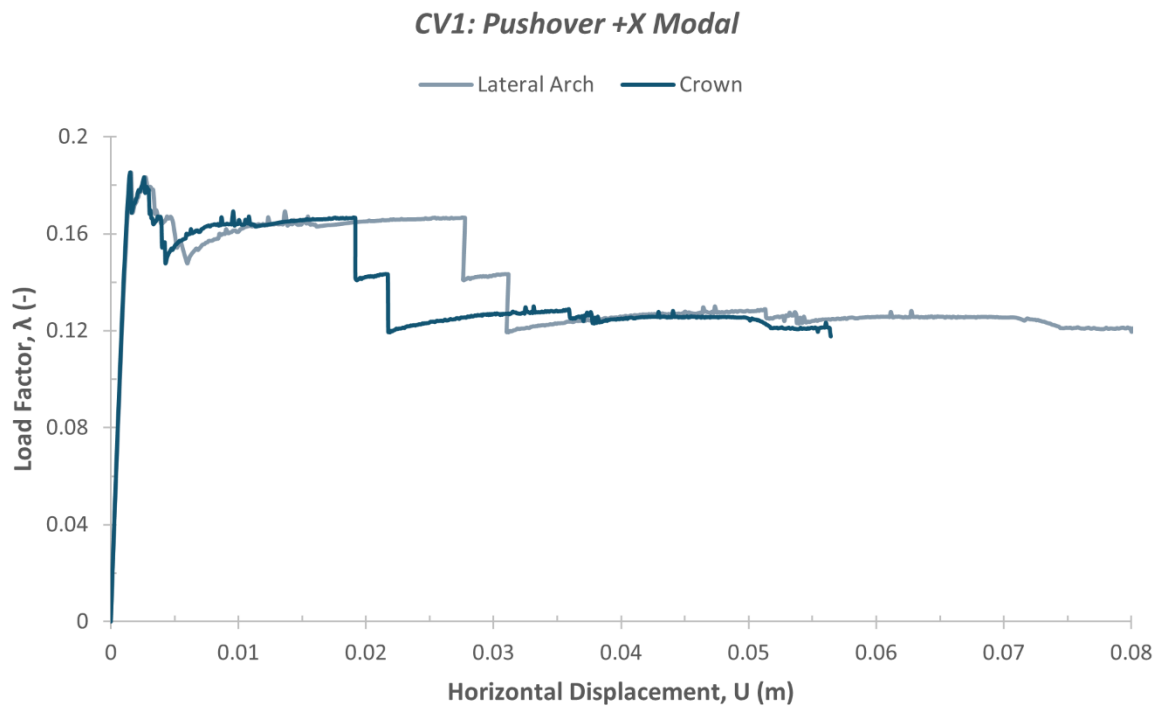


***C2 - CV1-R: Pushover -X Massa***

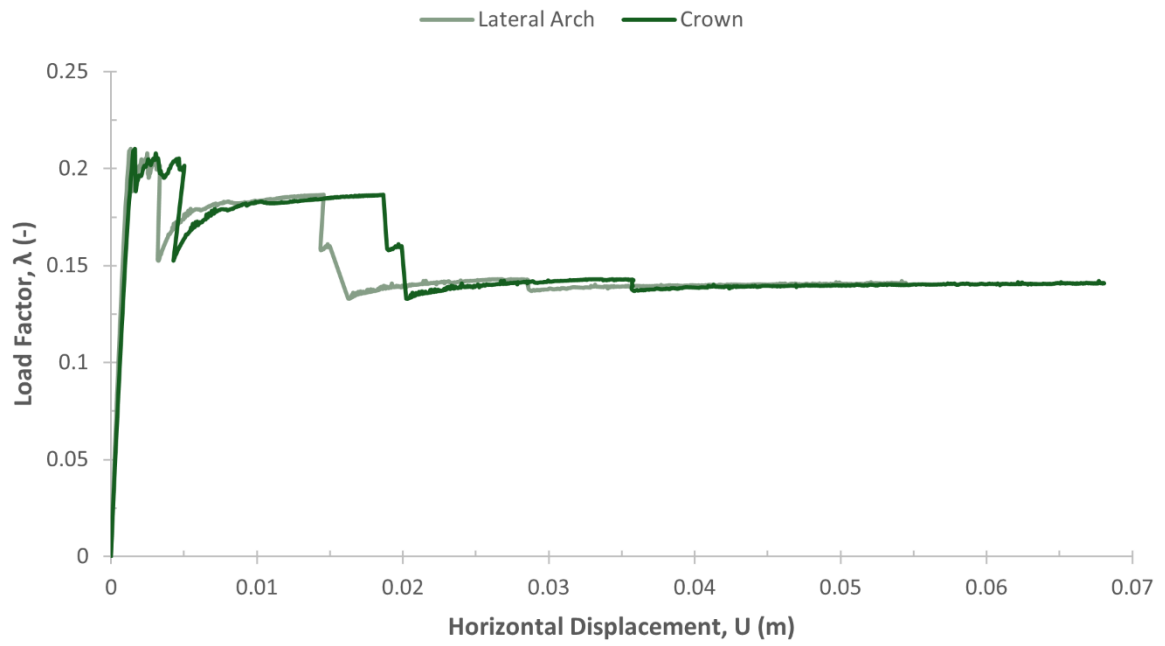




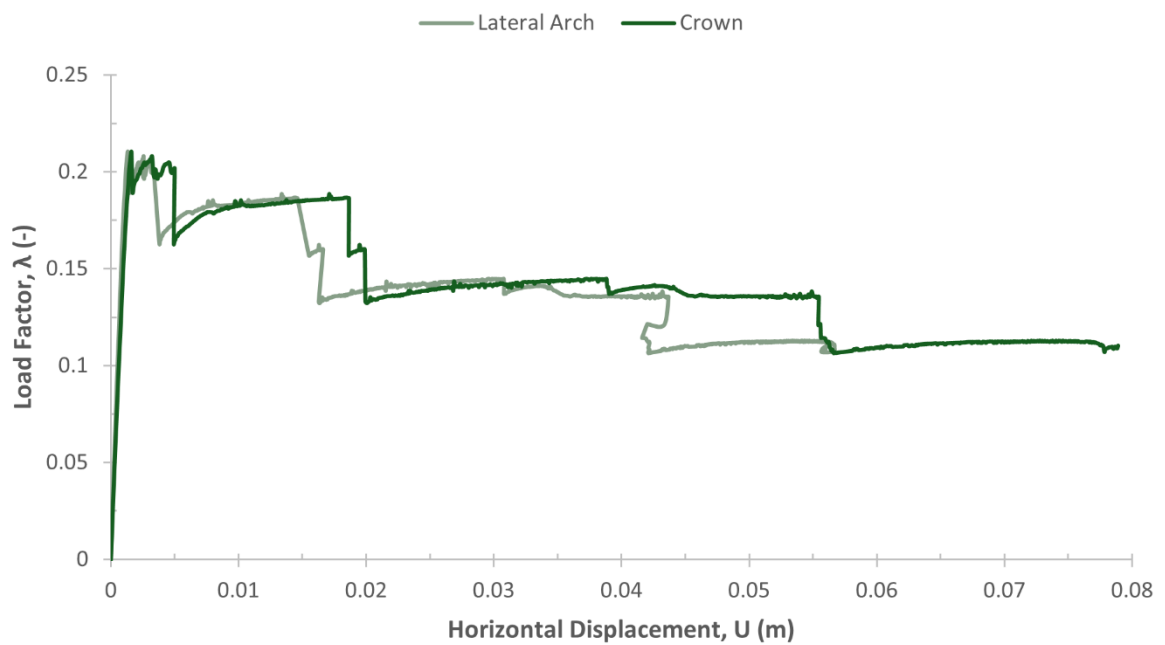
C.2 Pushover 'Modal' Curves



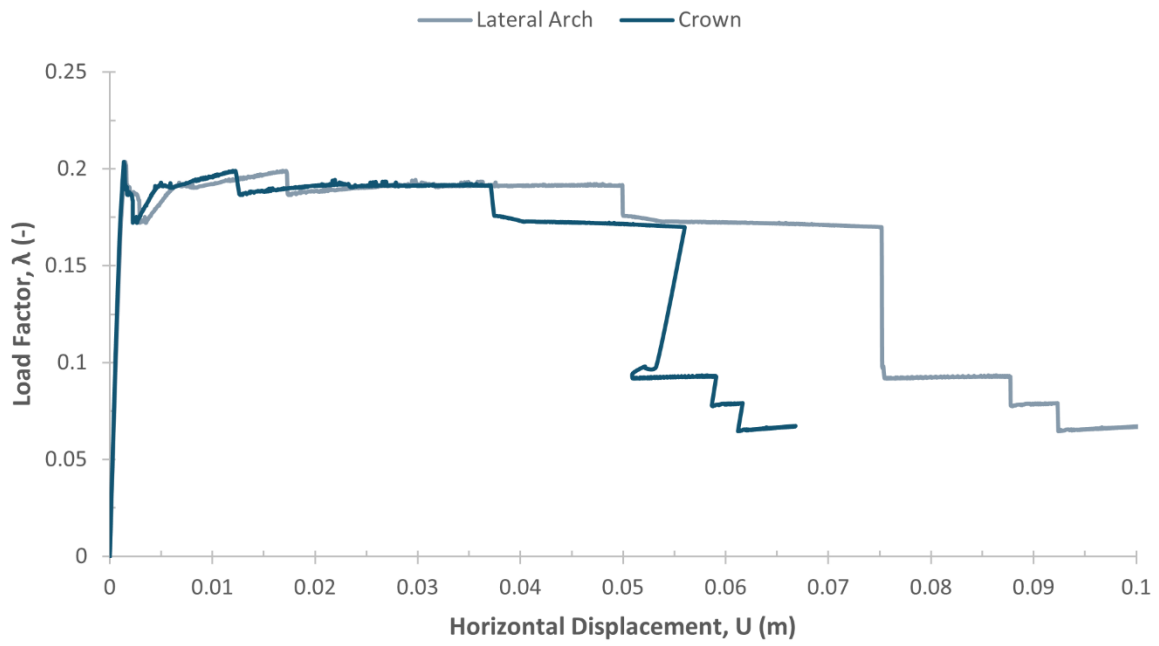
**CV1: Pushover +Y Modal**



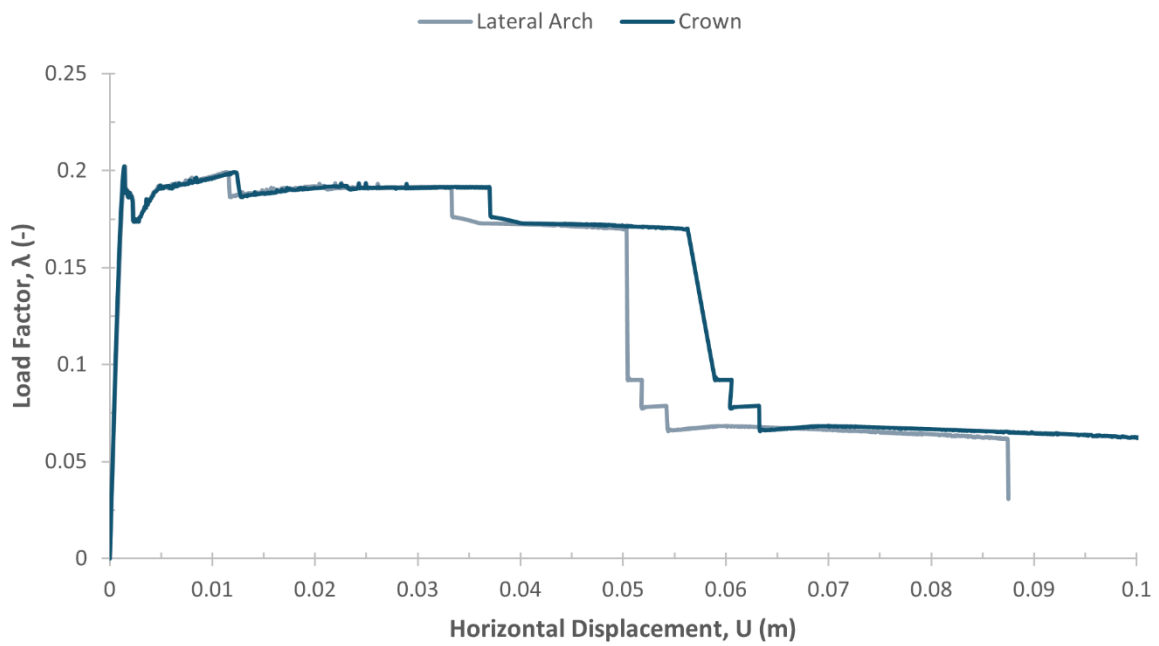
**CV1: Pushover -Y Modal**



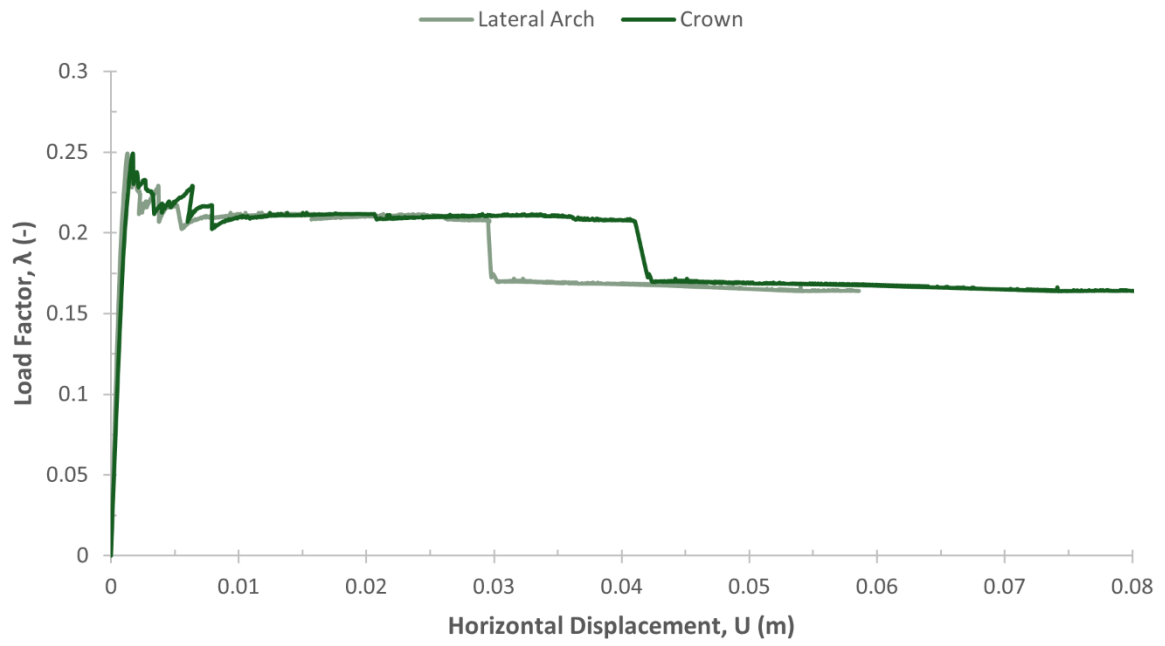
**CV2: Pushover +X Modal**



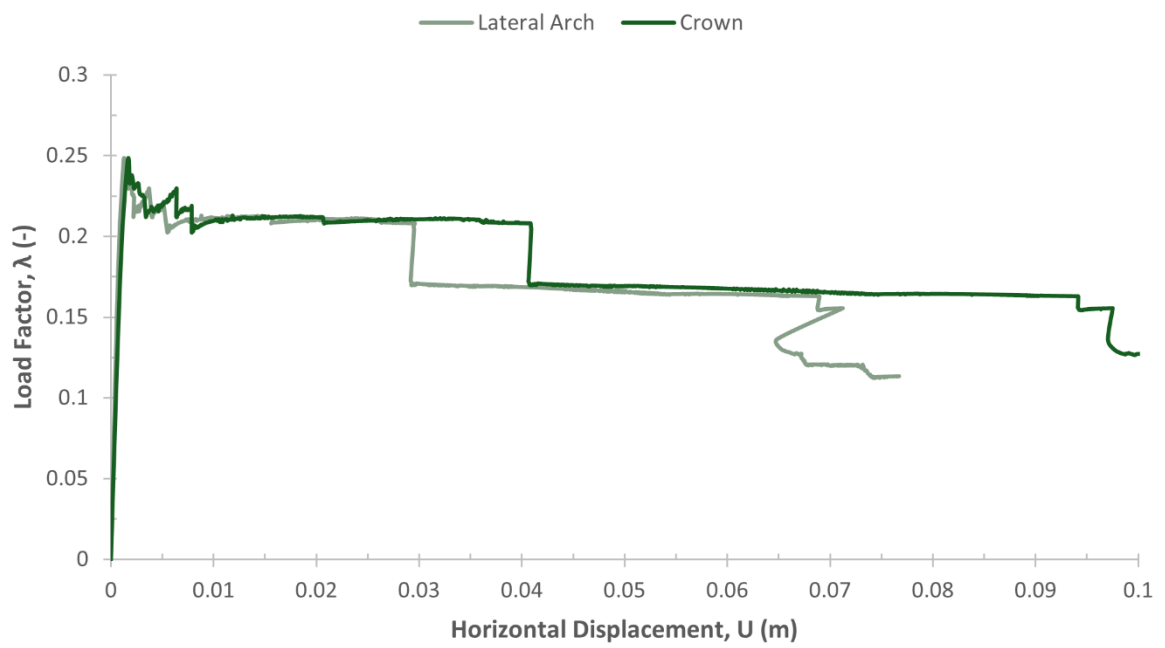
**CV2: Pushover -X Modal**



**CV2: Pushover +Y Modal**

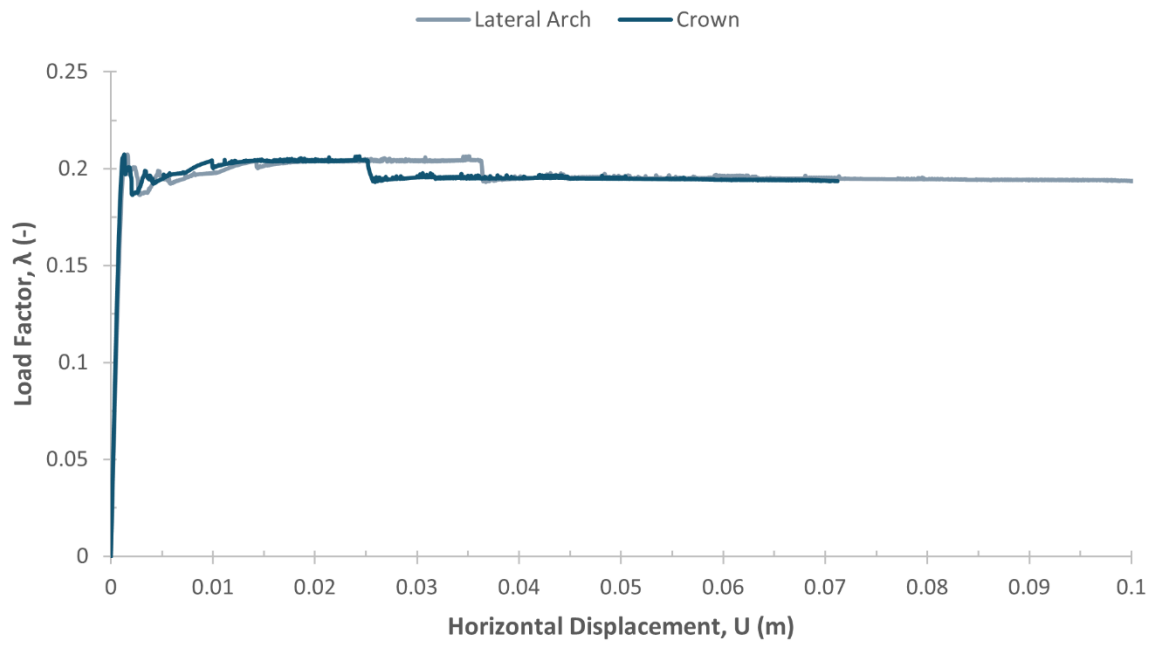


**CV2: Pushover -Y Modal**

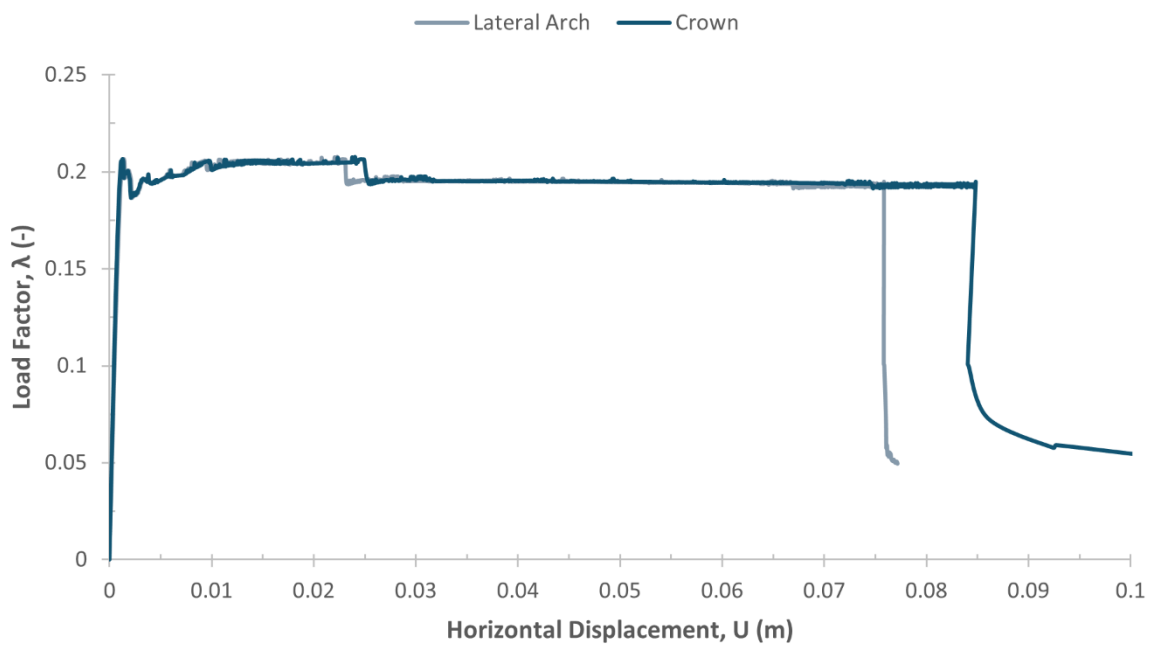




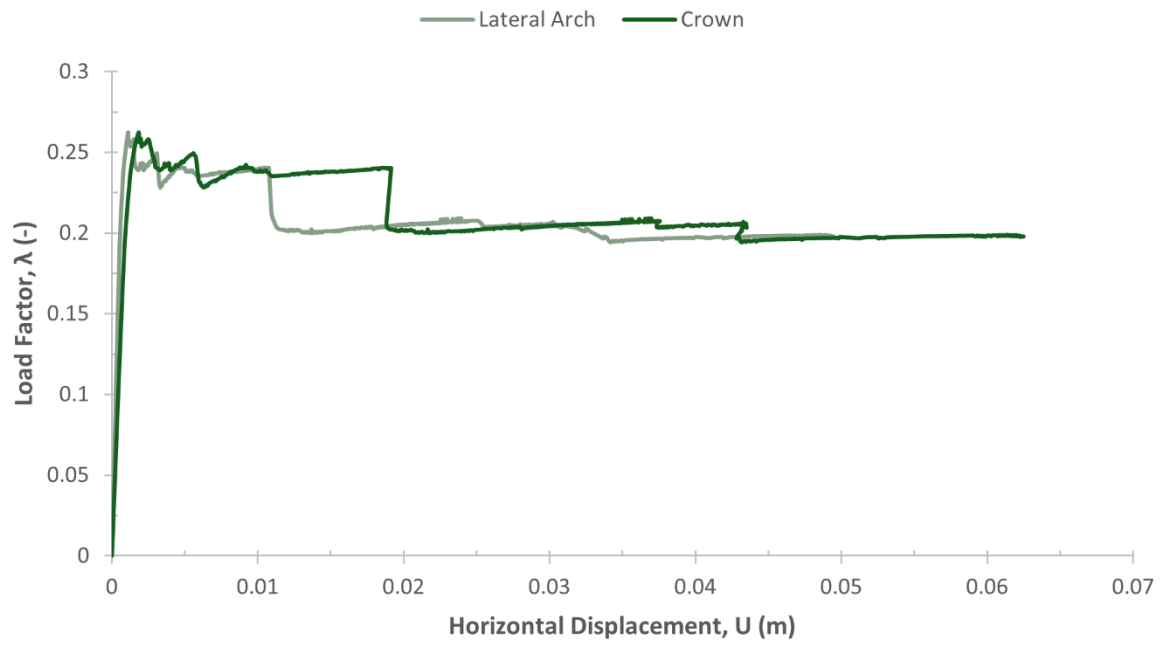
**CV3: Pushover +X Modal**



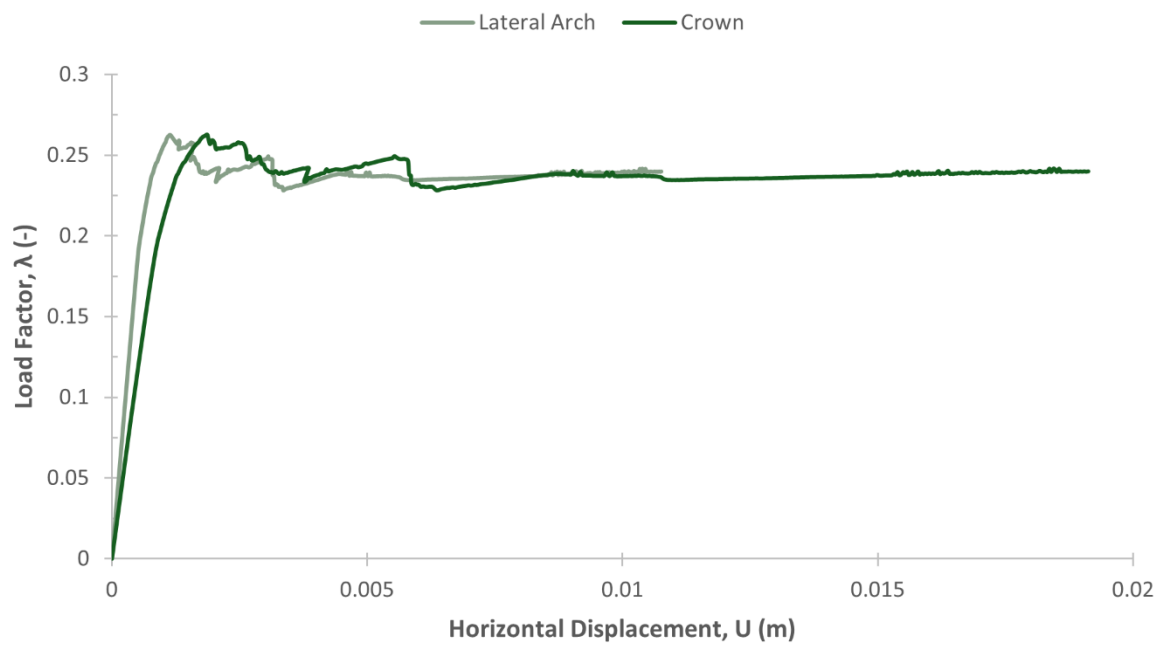
**CV3: Pushover -X Modal**



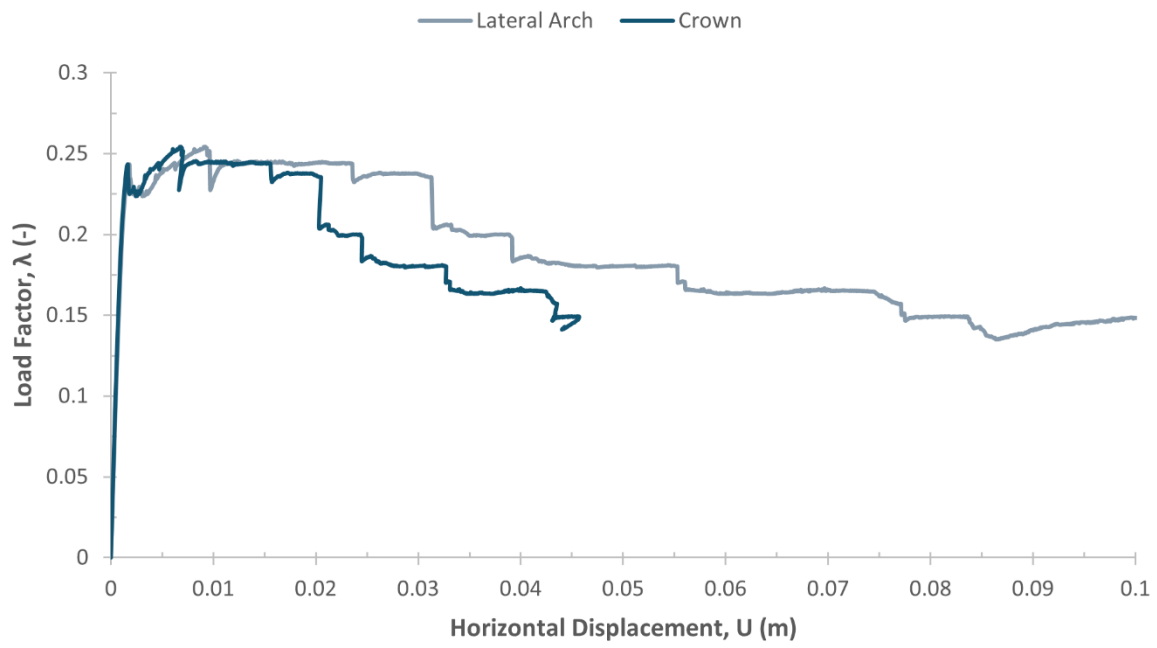
**CV3: Pushover +Y Modal**



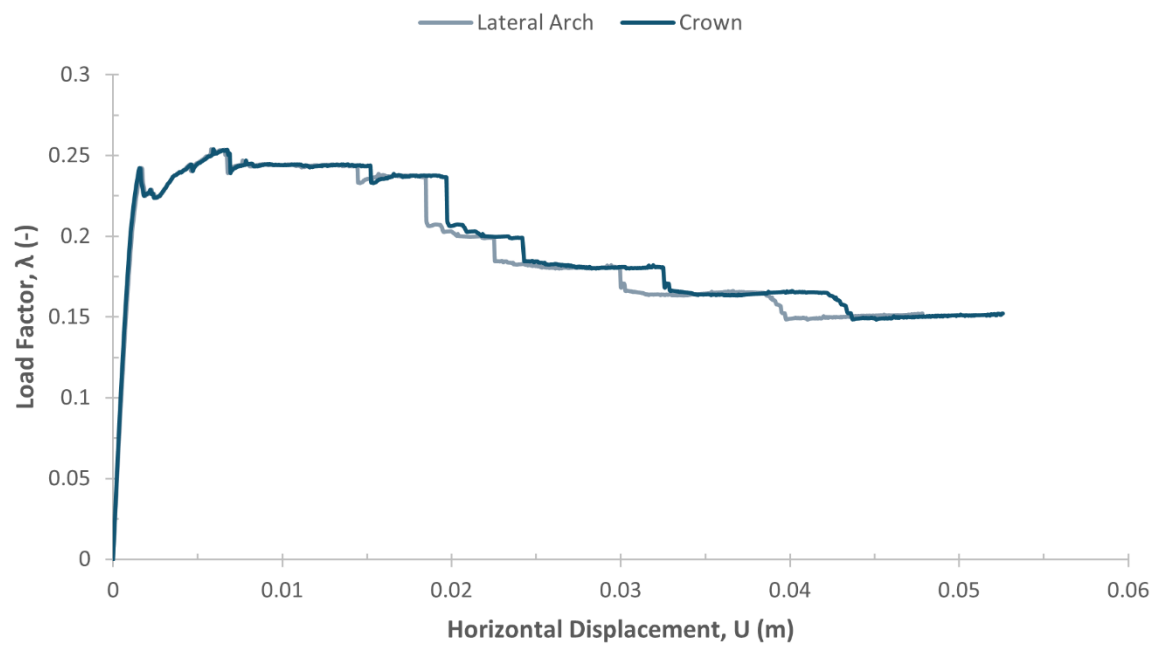
**CV3: Pushover -Y Modal**



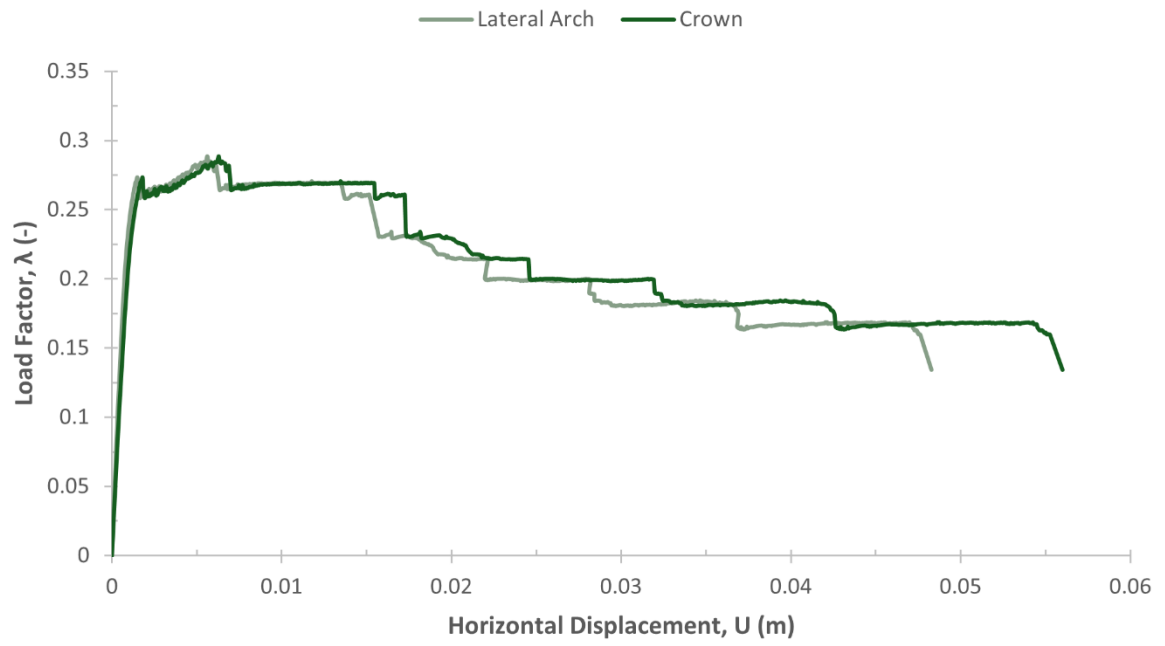
**CV1-R: Pushover +X Modal**



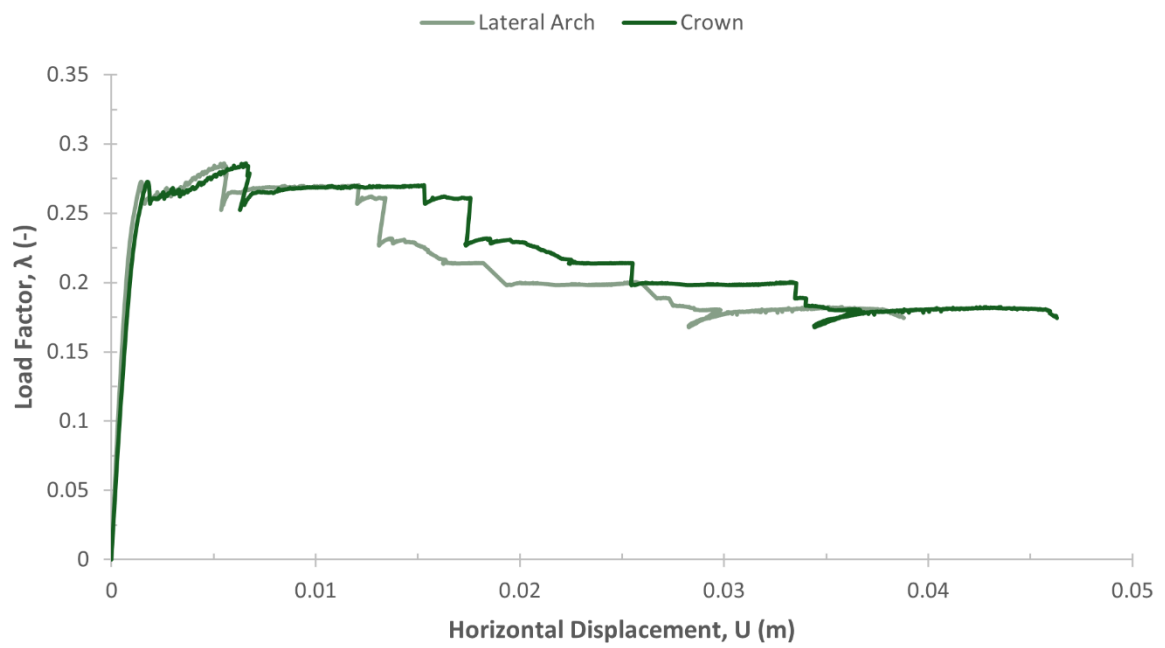
**CV1-R: Pushover -X Modal**



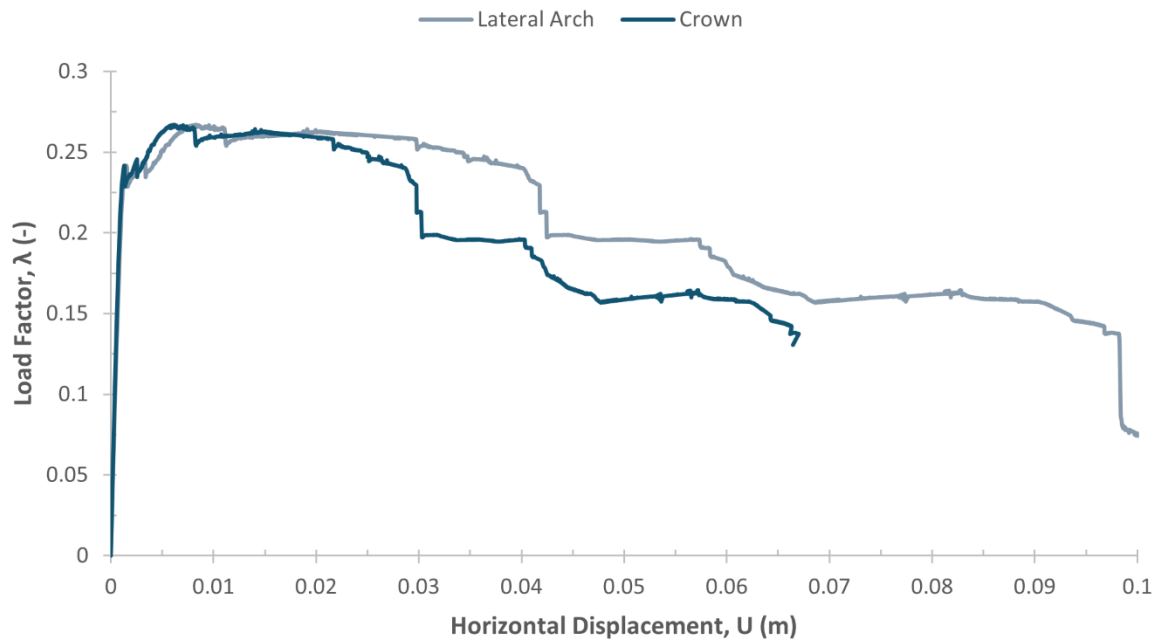
**CV1-R: Pushover +Y Modal**



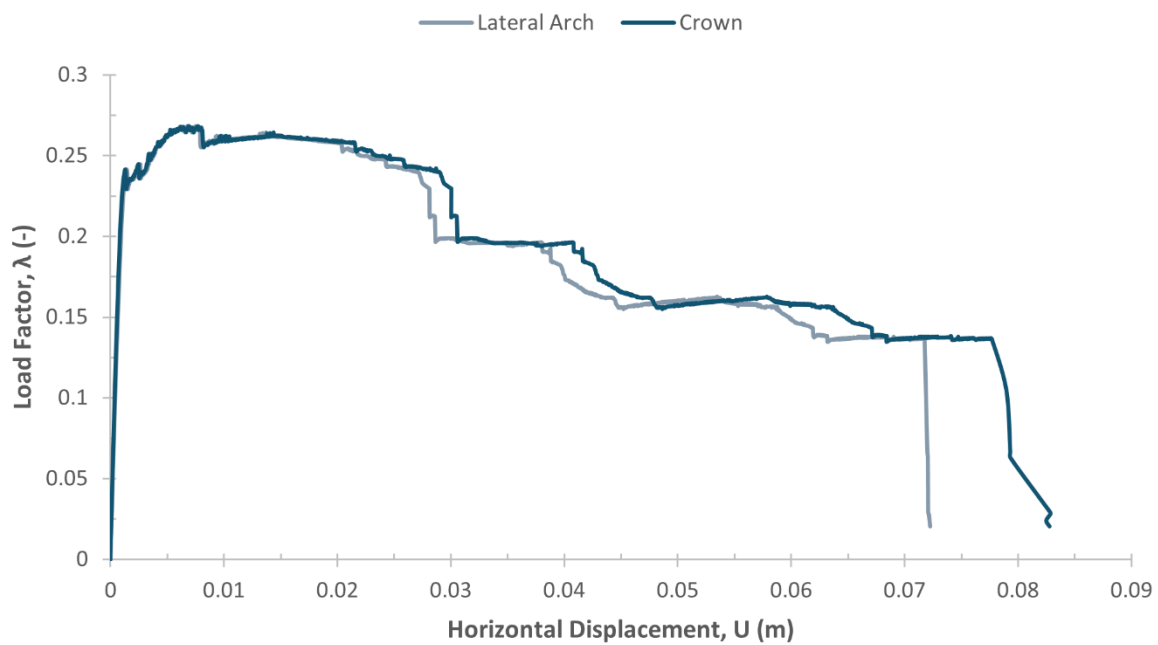
**CV1-R: Pushover -Y Modal**



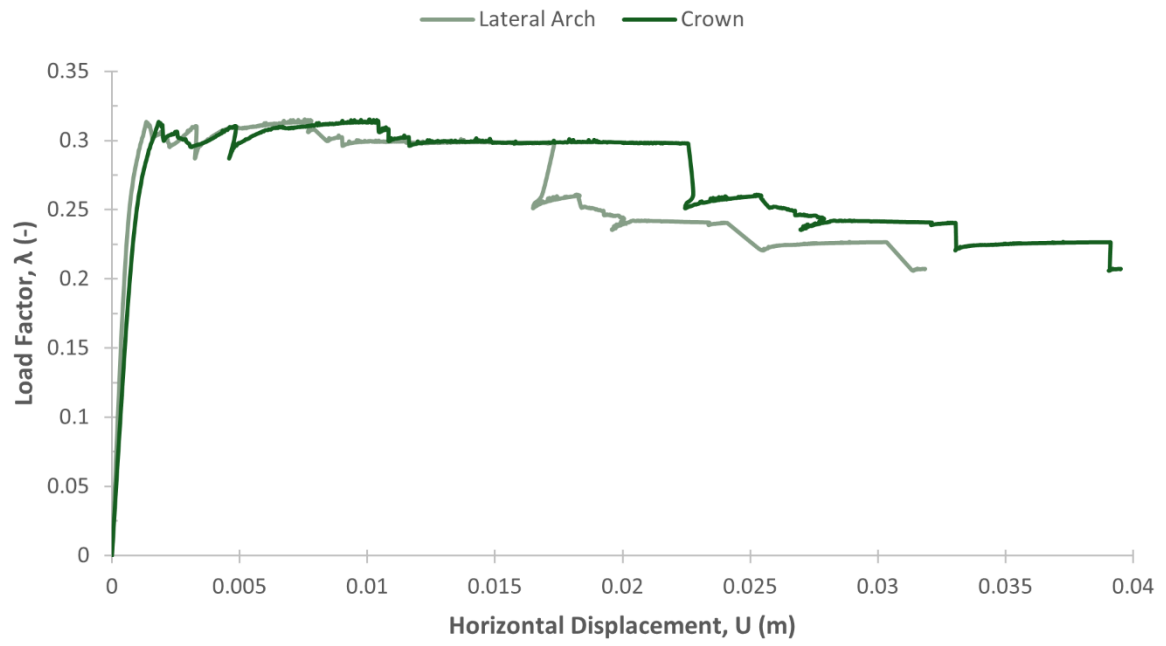
**CV2-R: Pushover +X Modal**



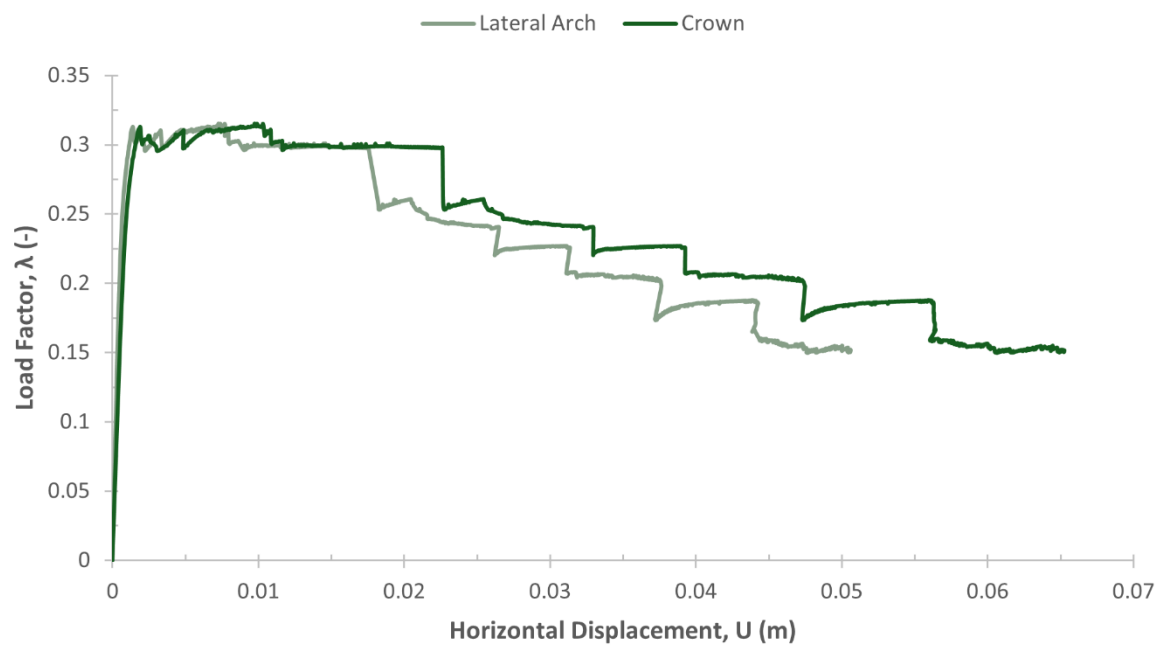
**CV2-R: Pushover -X Modal**



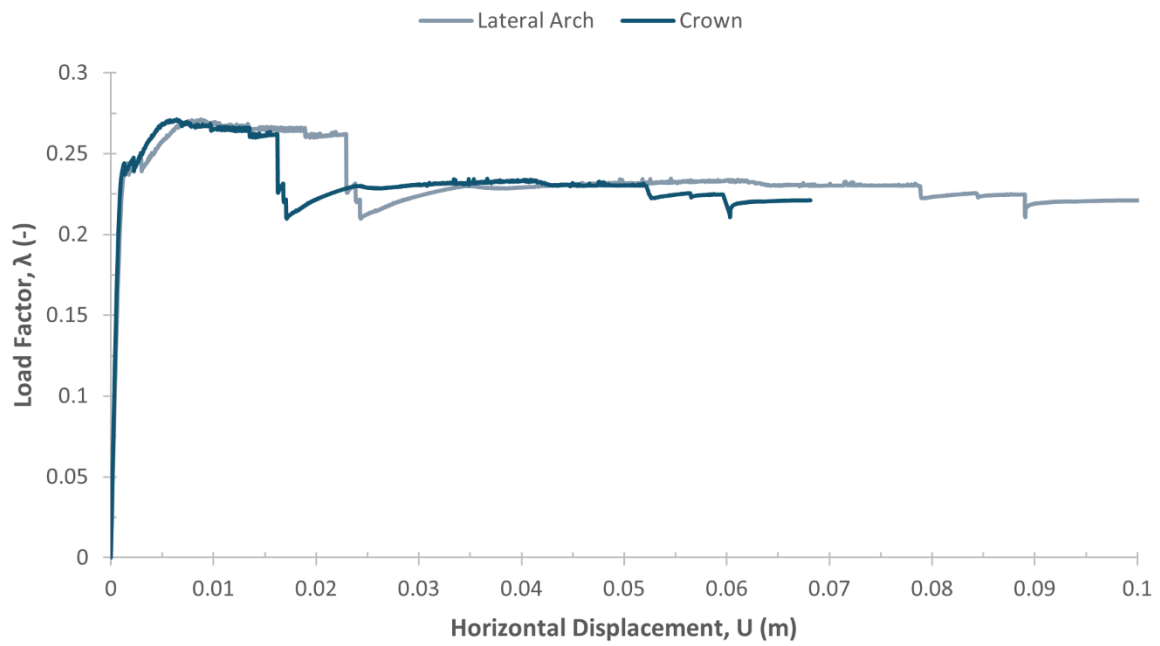
**CV2-R: Pushover +Y Modal**



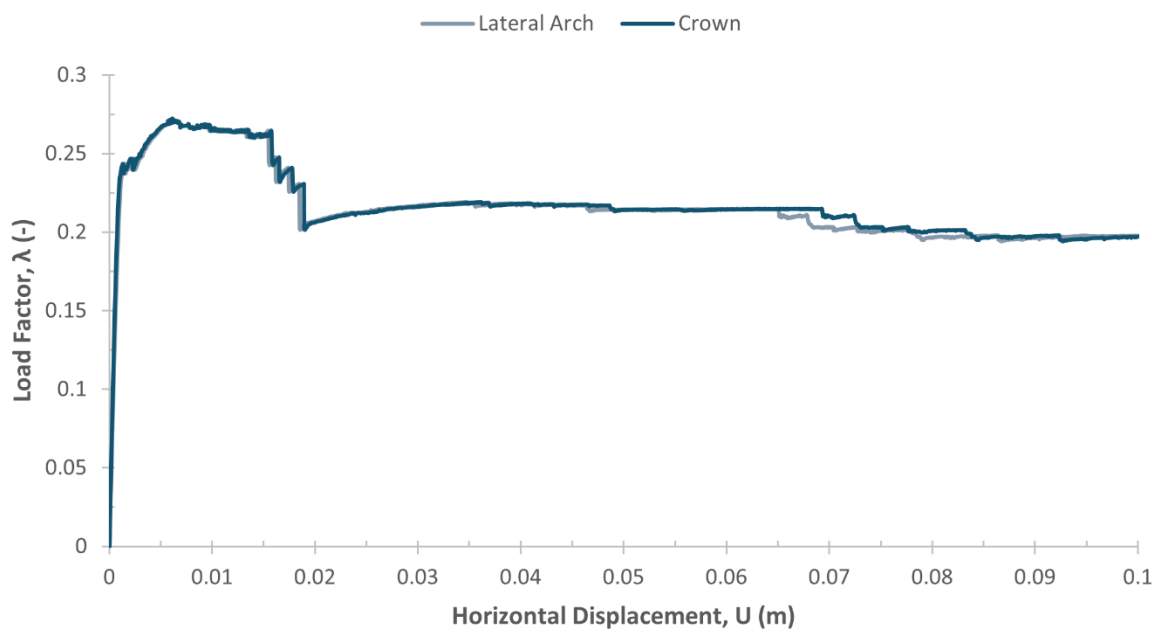
**CV2-R: Pushover -Y Modal**



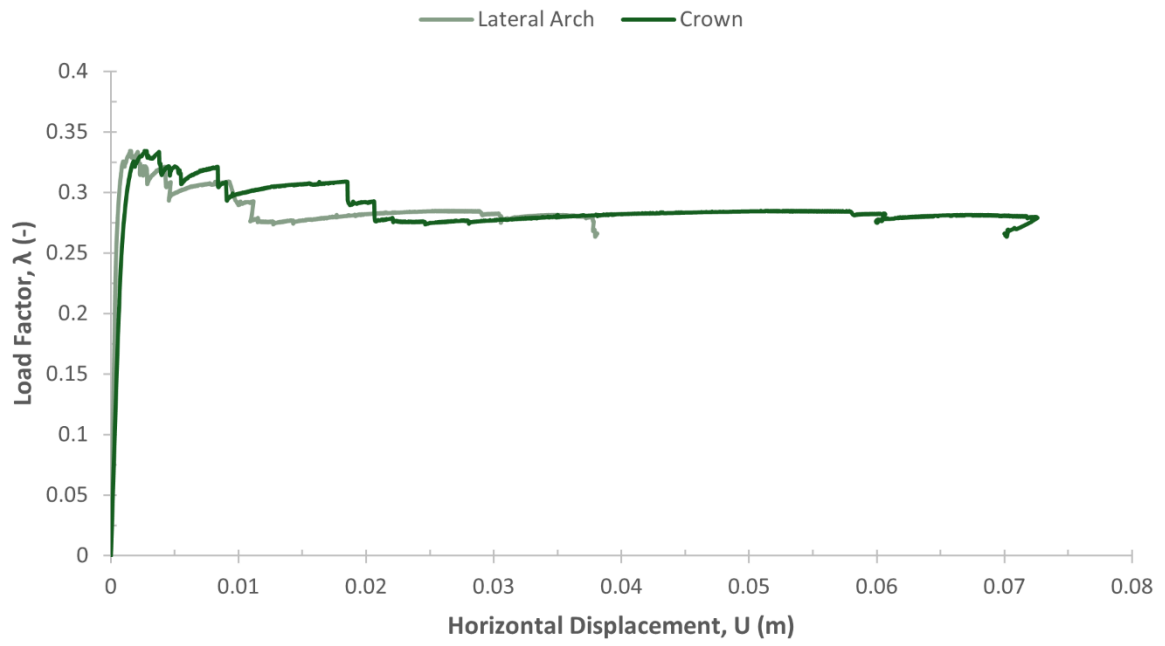
**CV3-R: Pushover +X Modal**



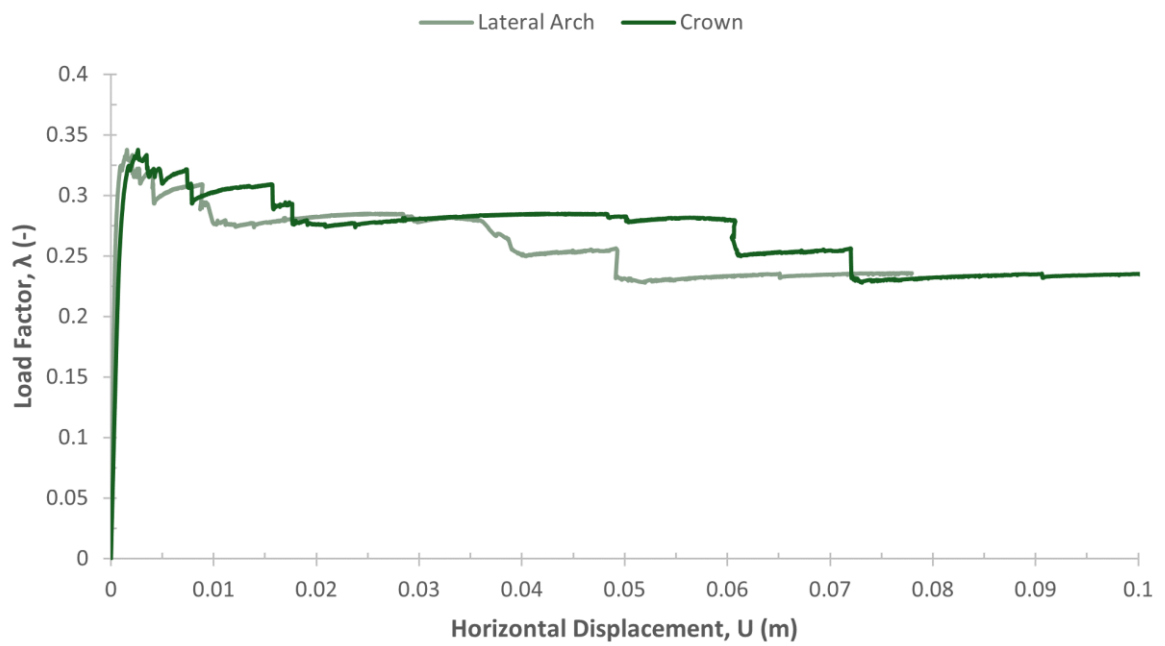
**CV3-R: Pushover -X Modal**



**CV3-R: Pushover +Y Modal**

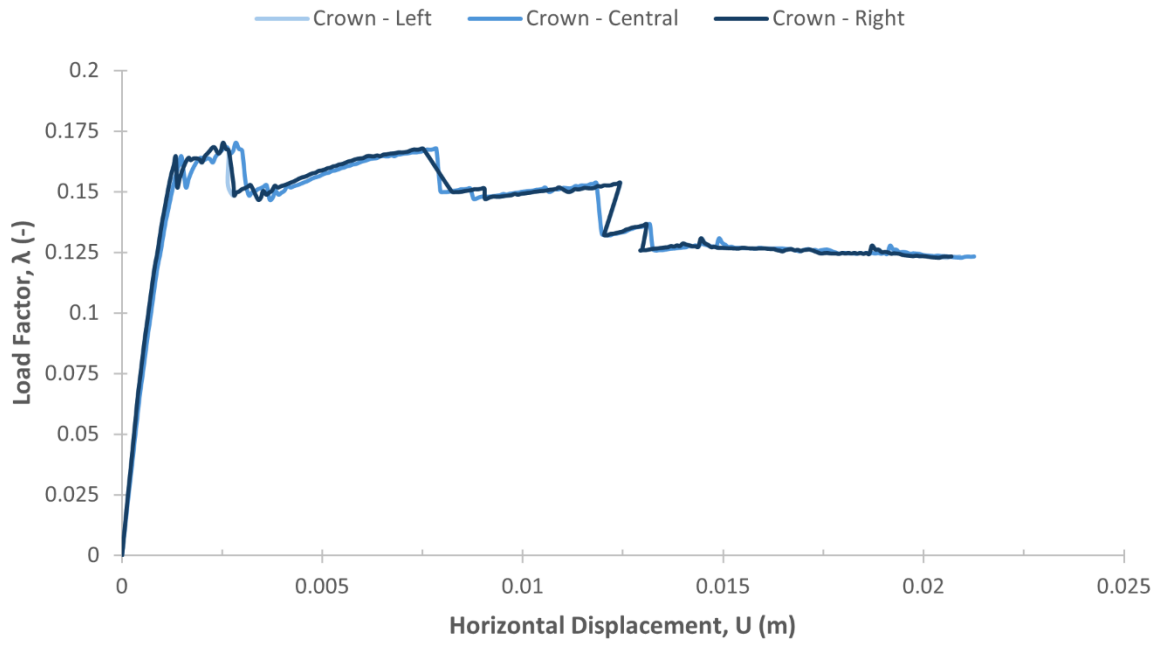


**CV3-R: Pushover -Y Modal**

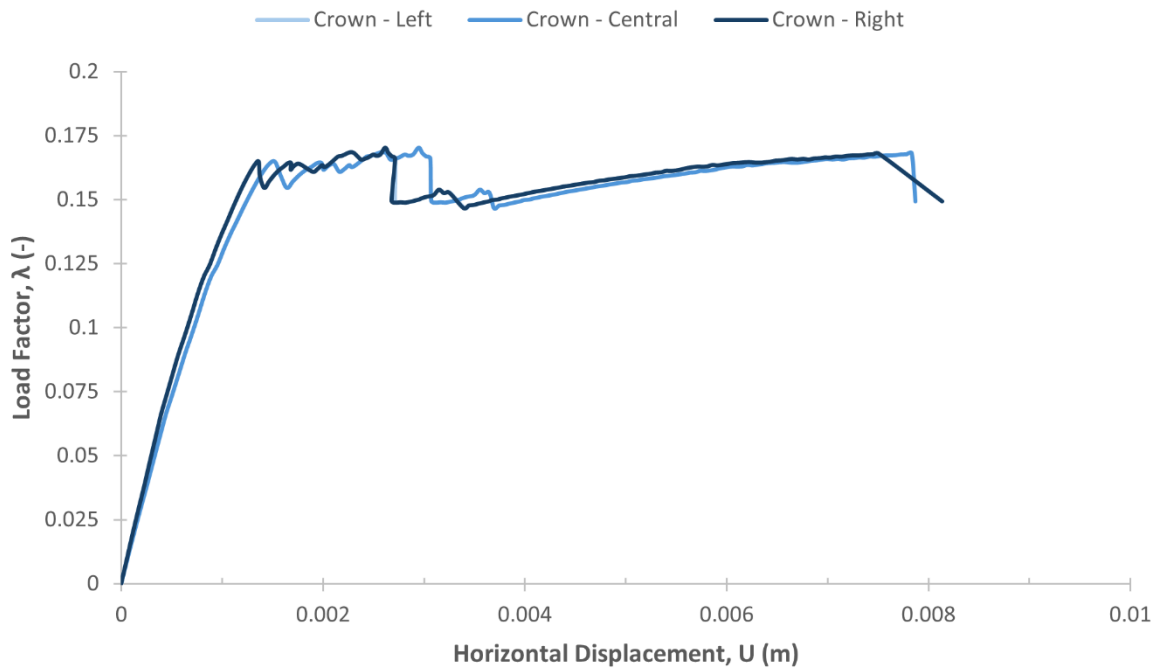


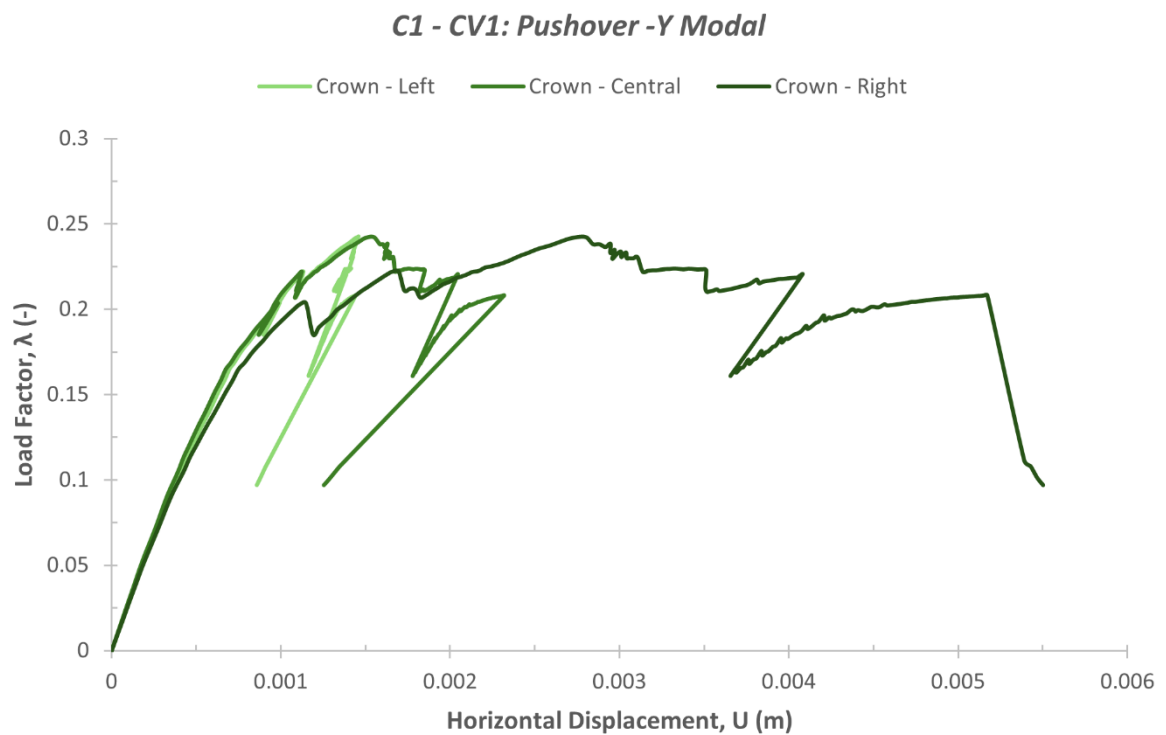
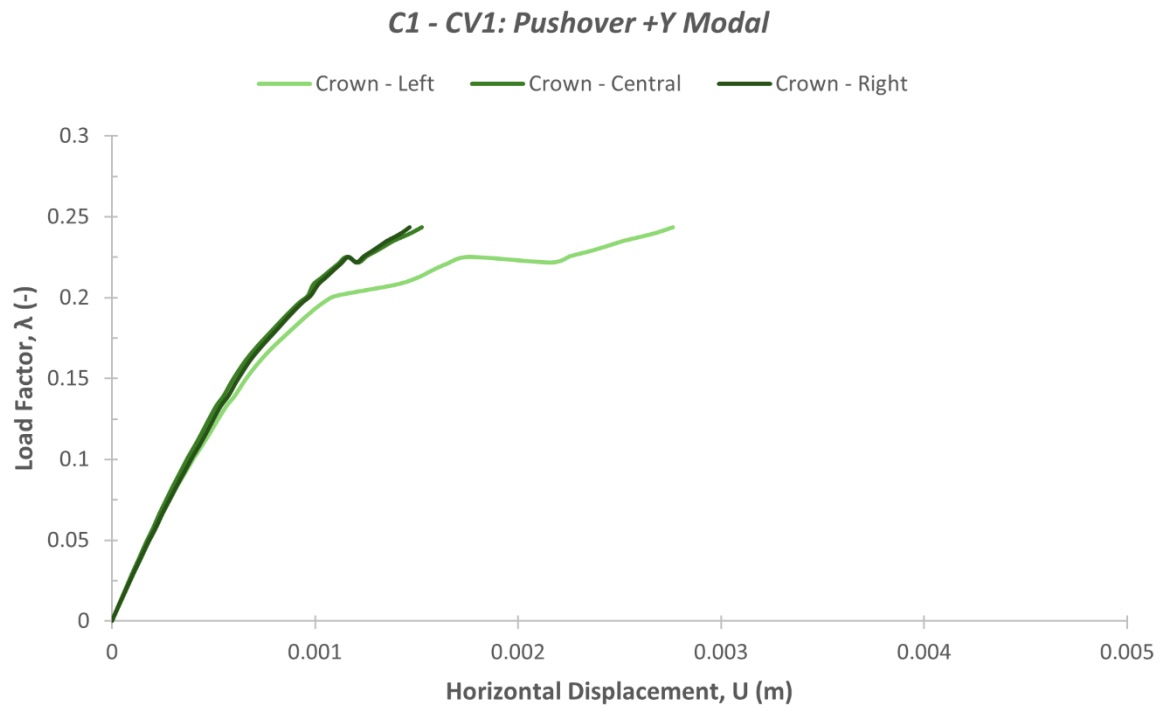


***C1 - CV1: Pushover +X Modal***

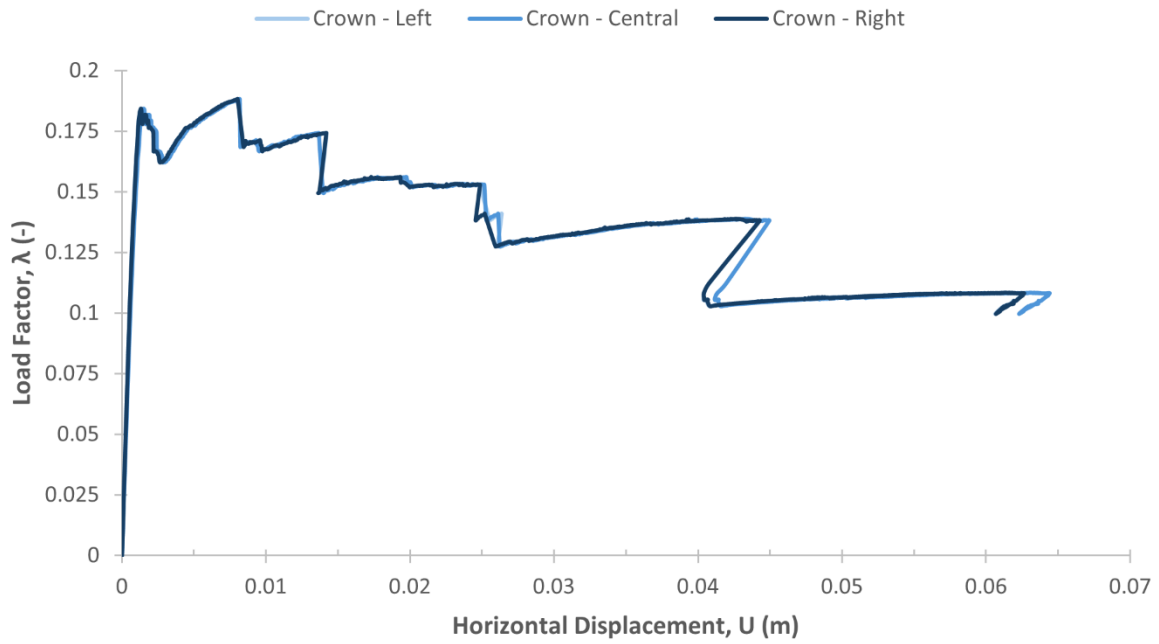


***C1 - CV1: Pushover -X Modal***

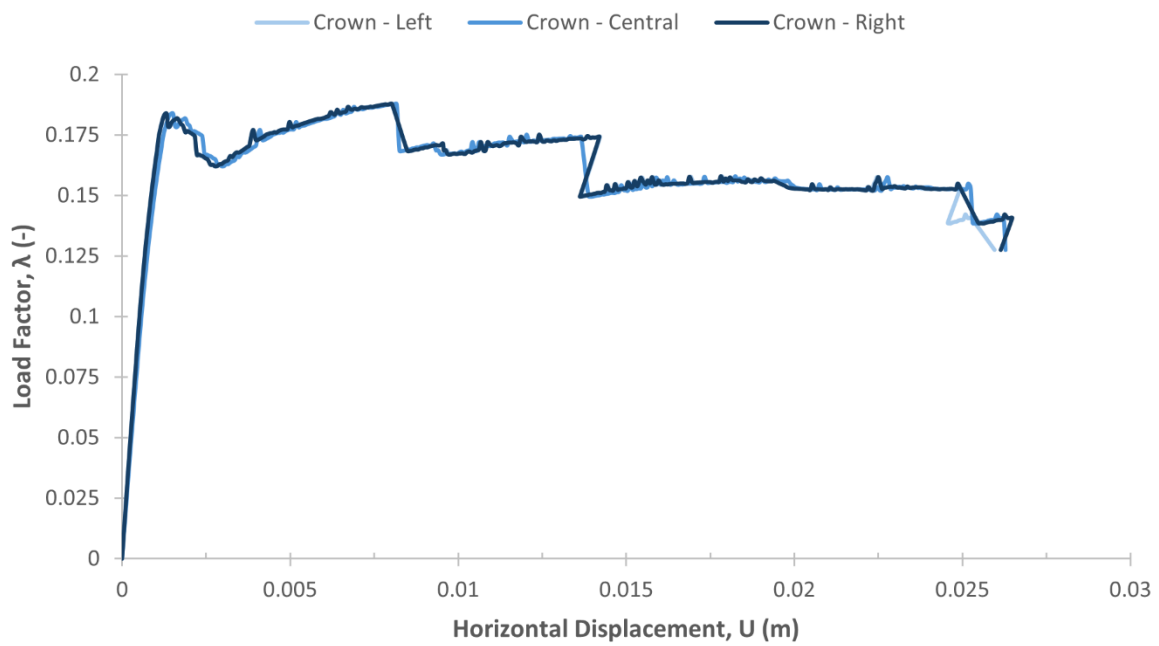


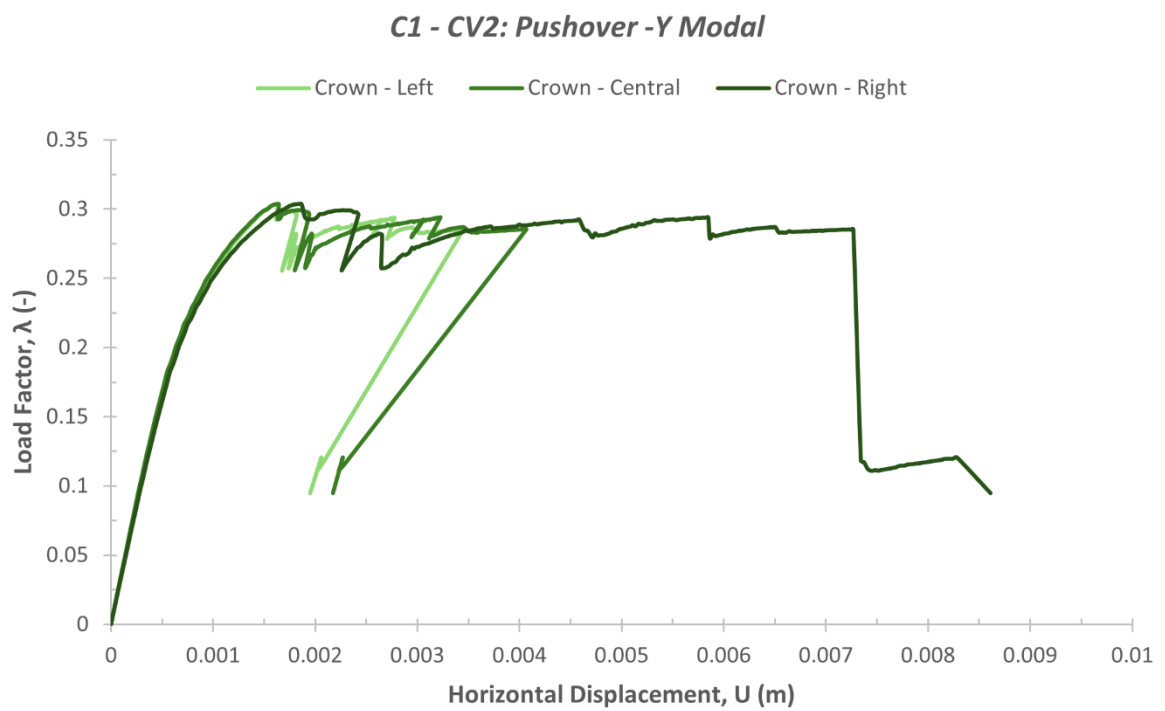
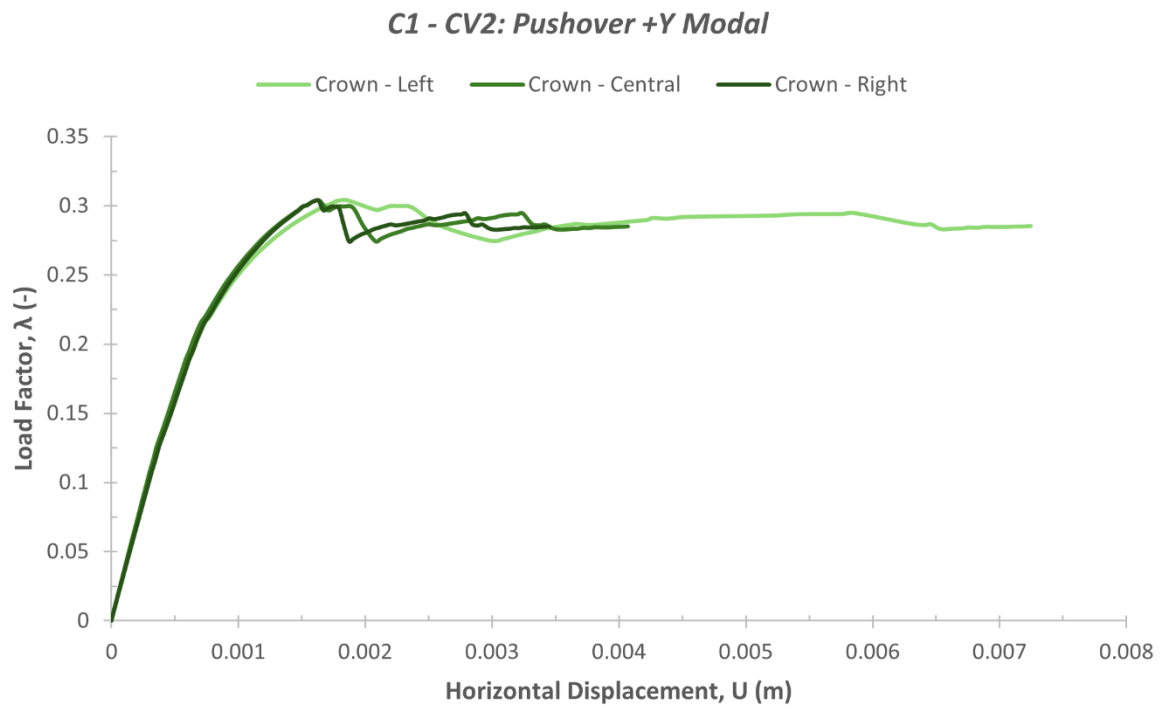


***C1 - CV2: Pushover +X Modal***

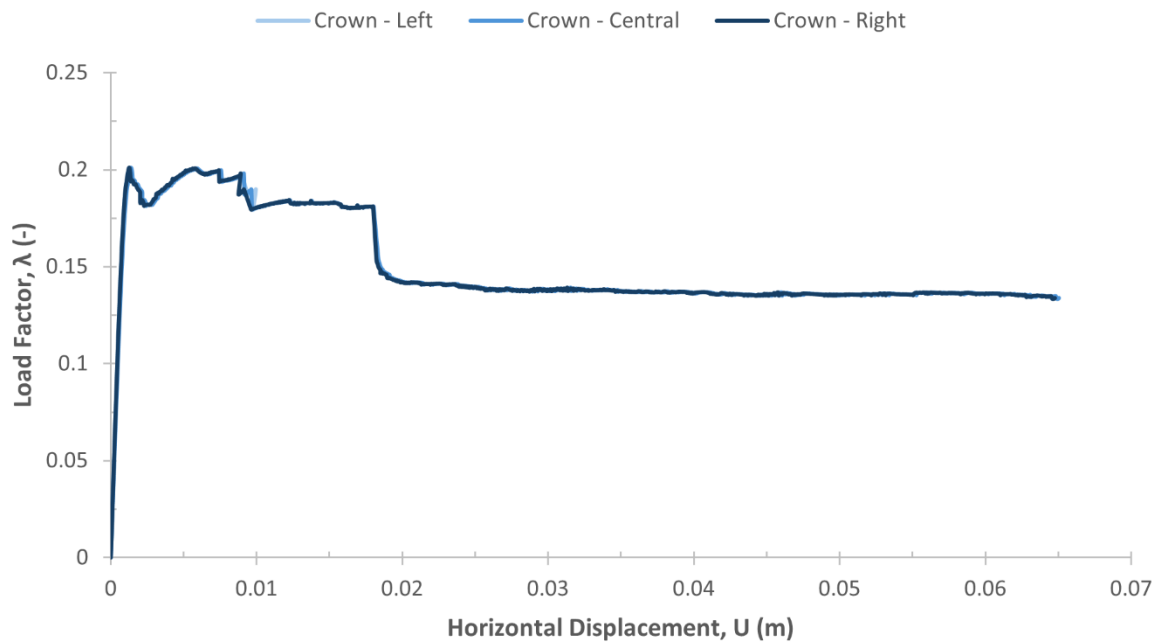


***C1 - CV2: Pushover -X Modal***

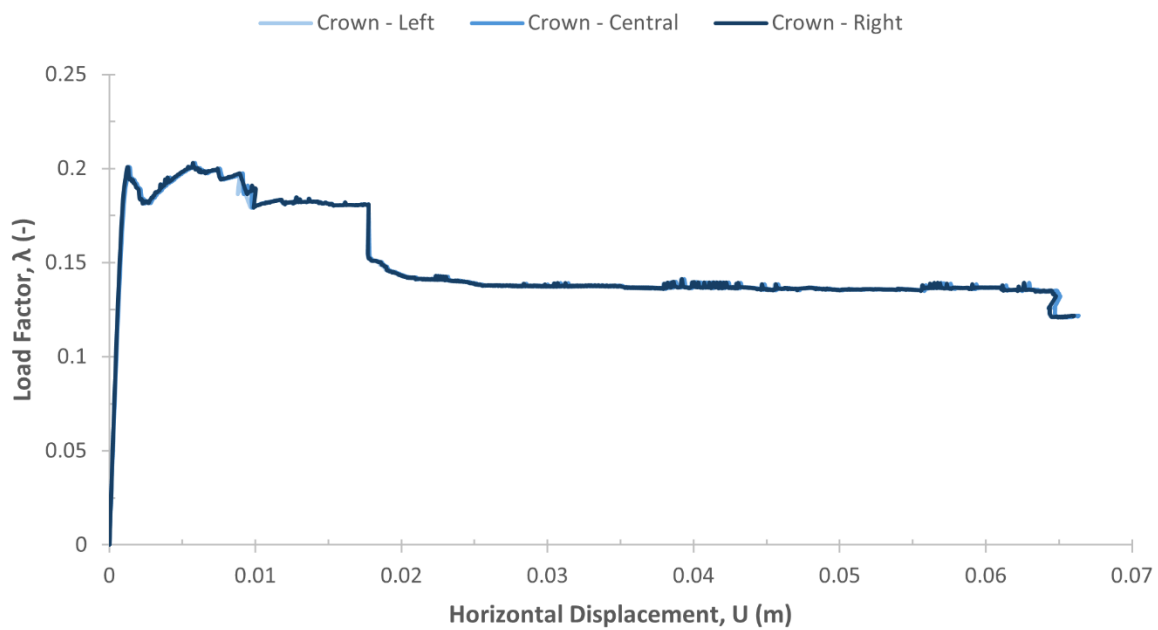


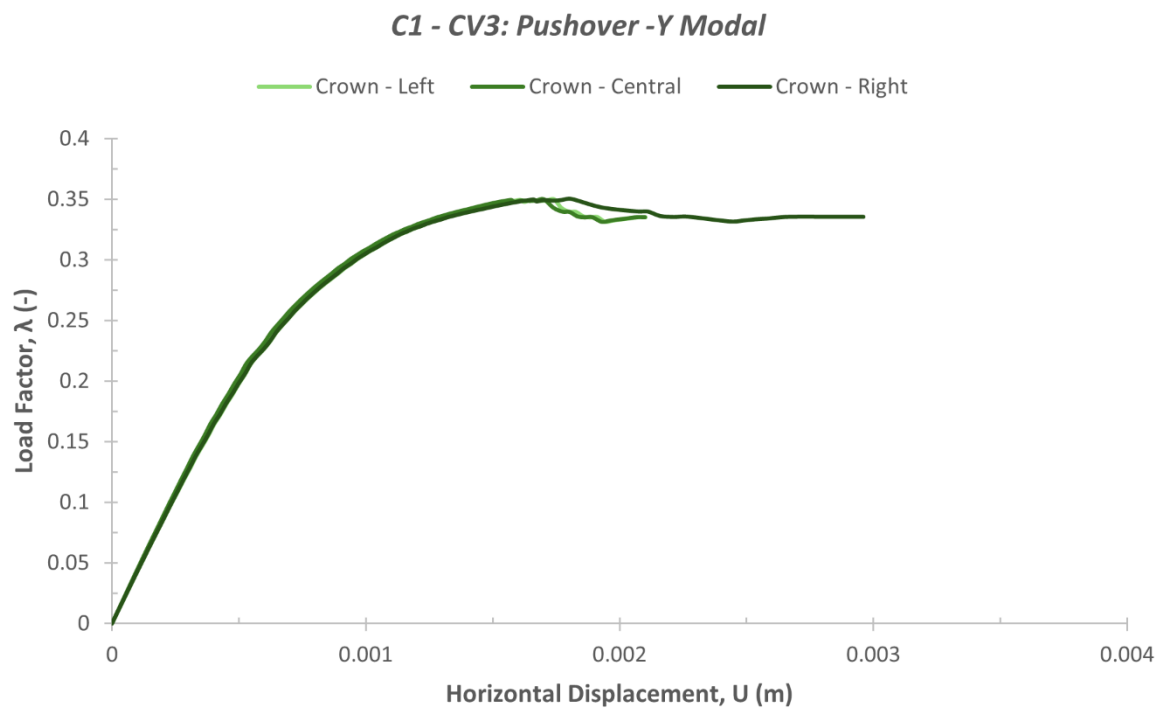
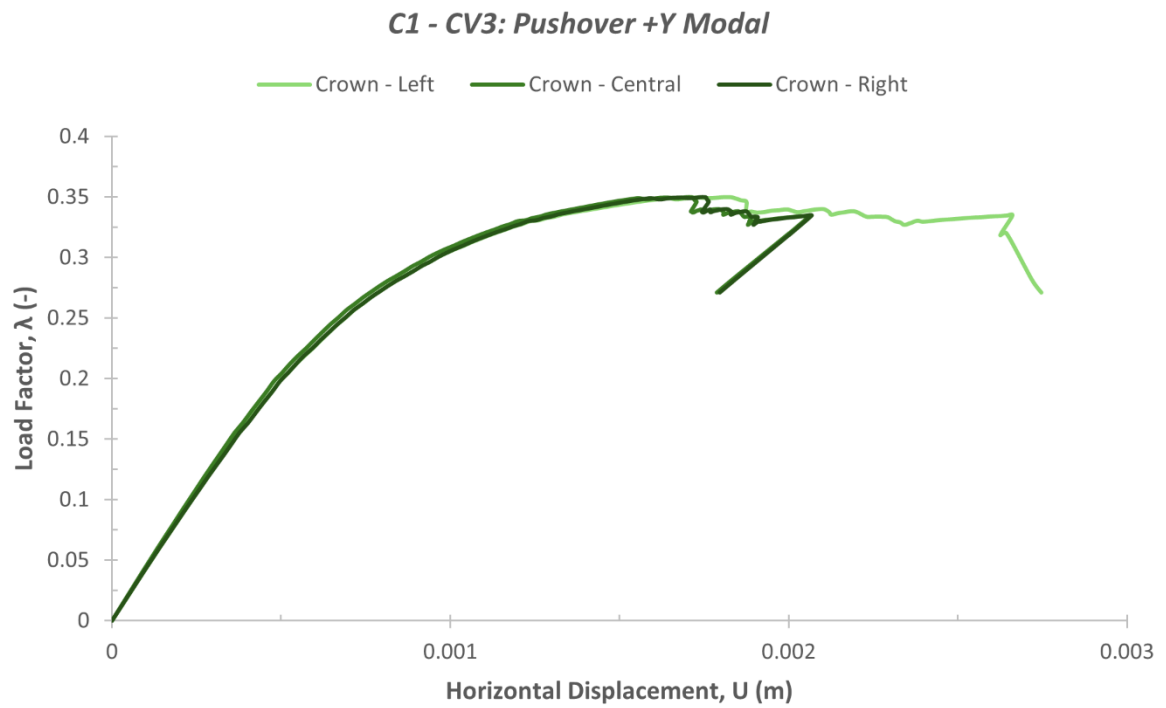


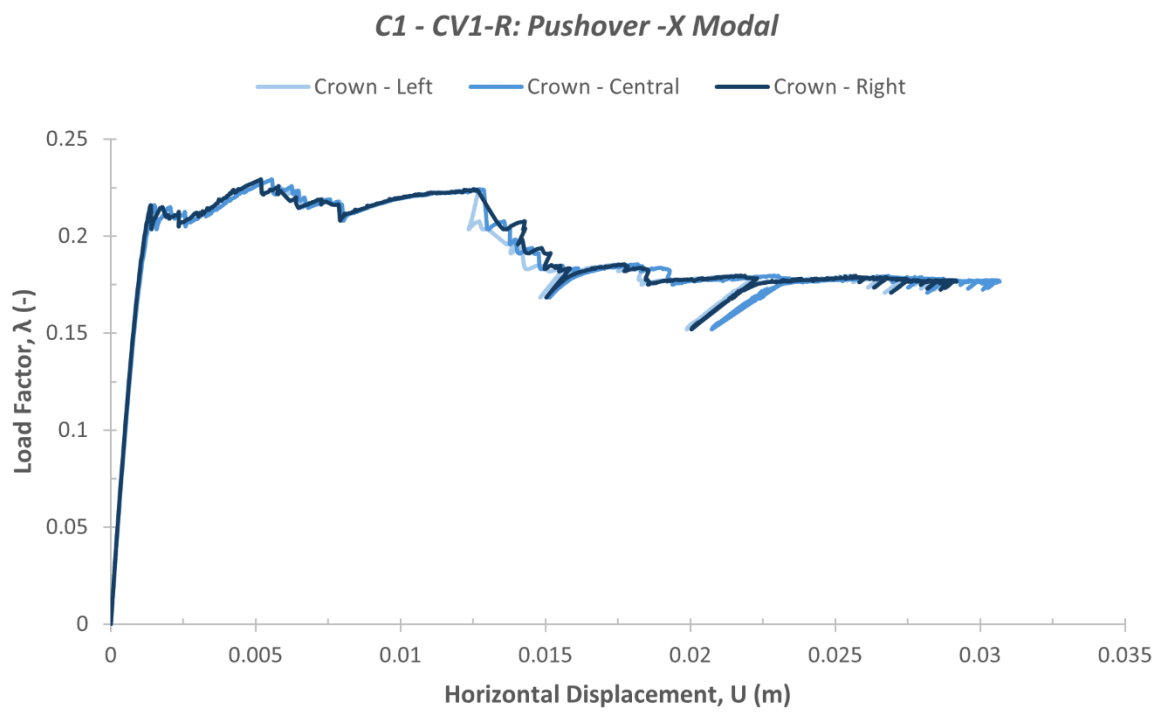
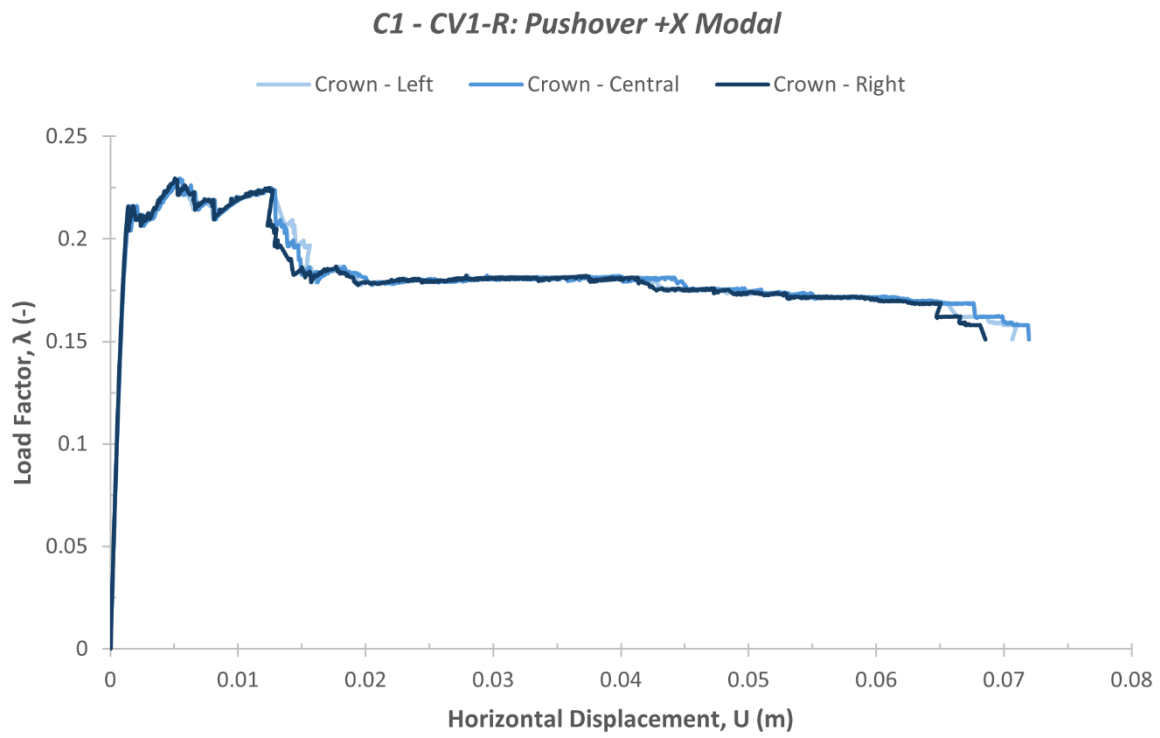
### ***C1 - CV3: Pushover +X Modal***



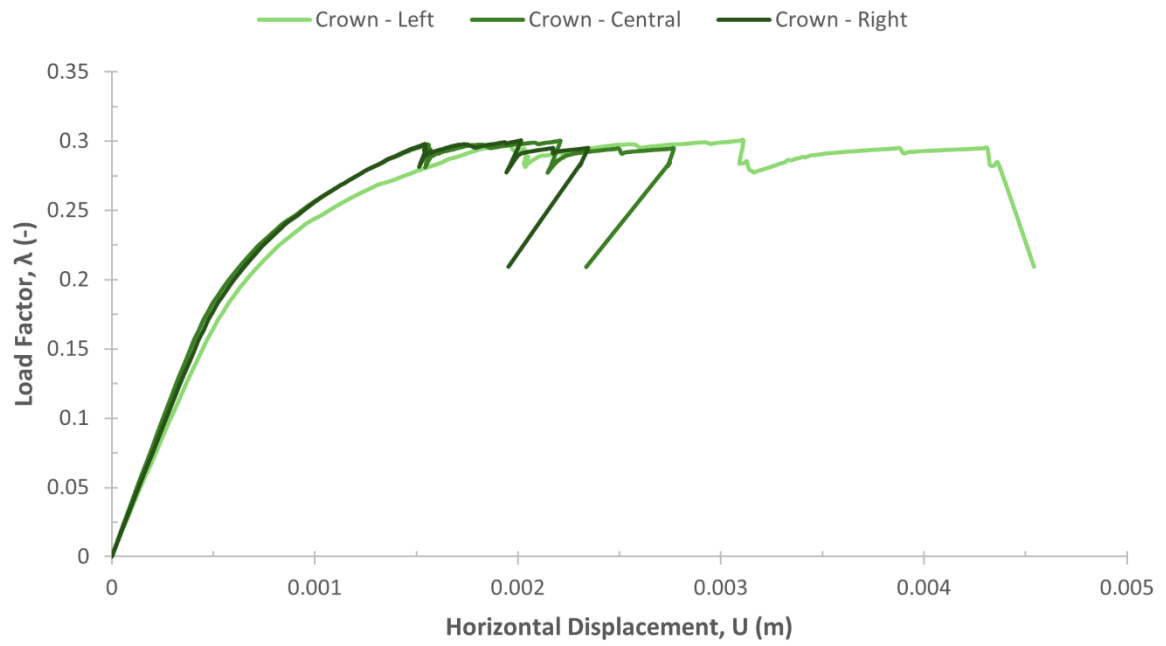
### ***C1 - CV3: Pushover -X Modal***



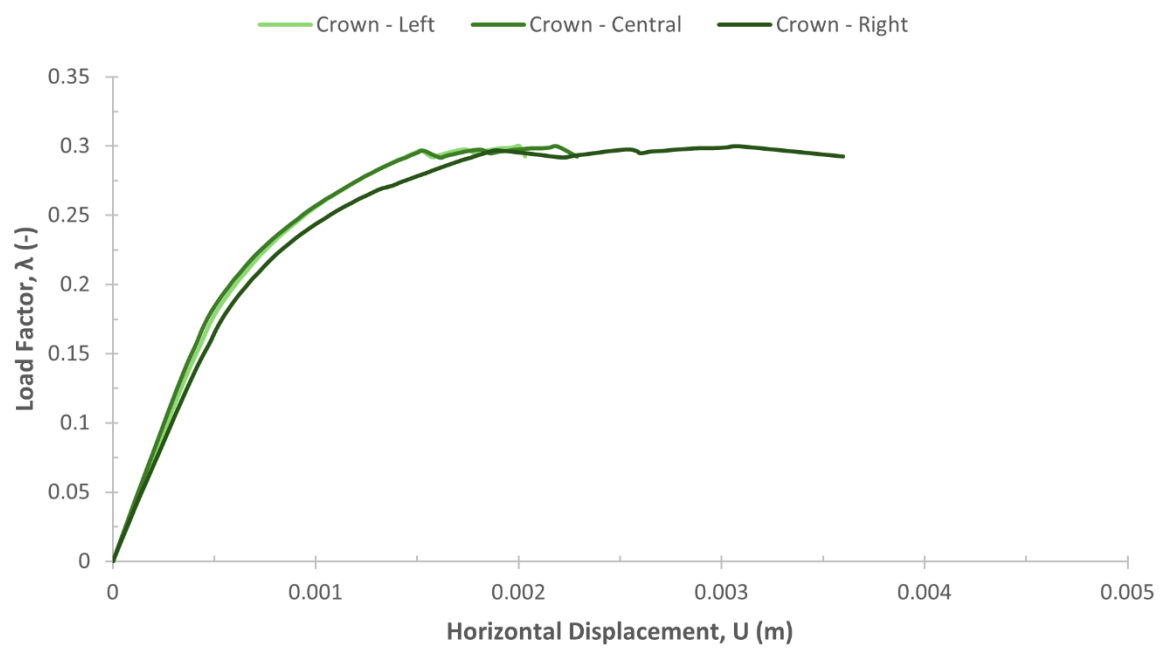




***C1 - CV1-R: Pushover +Y Modal***

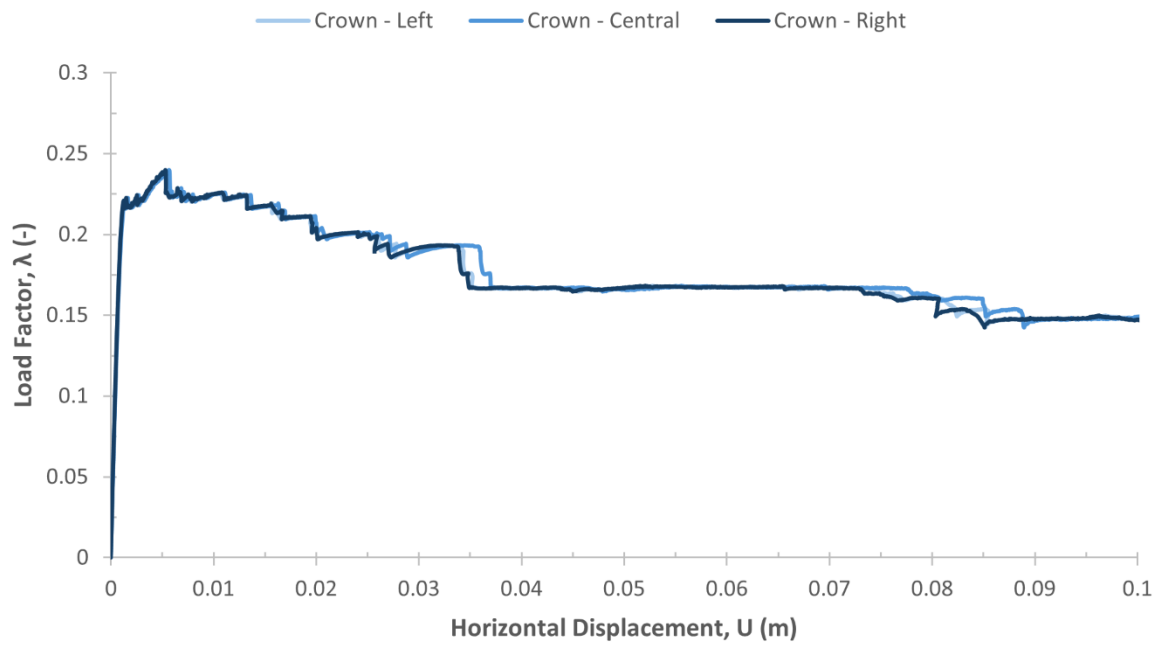


***C1 - CV1-R: Pushover -Y Modal***

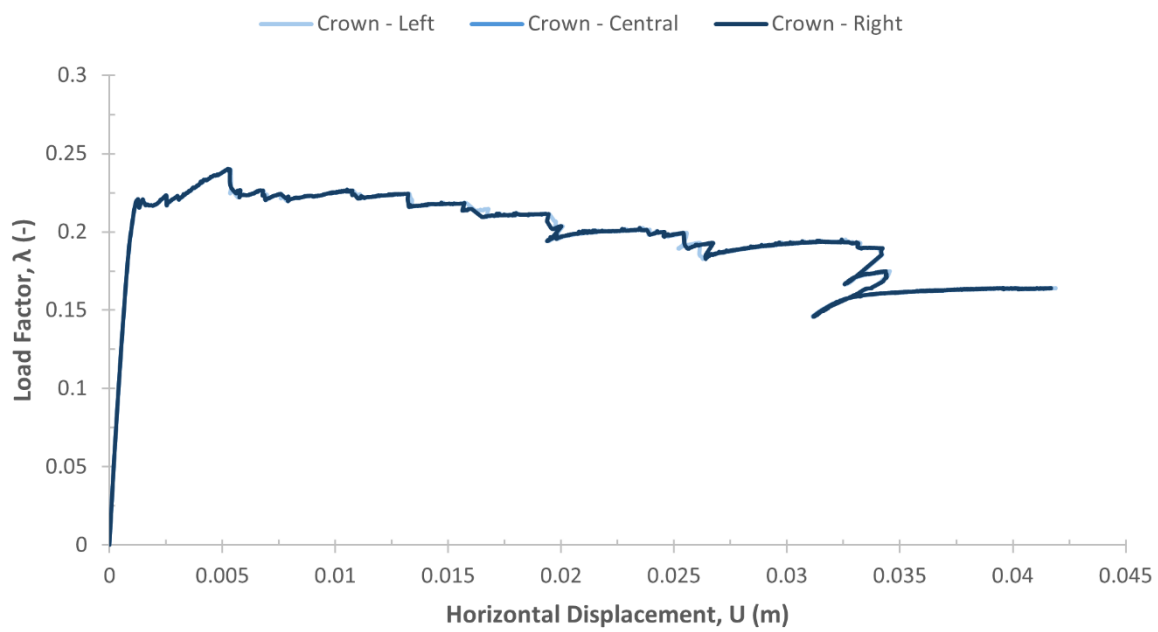




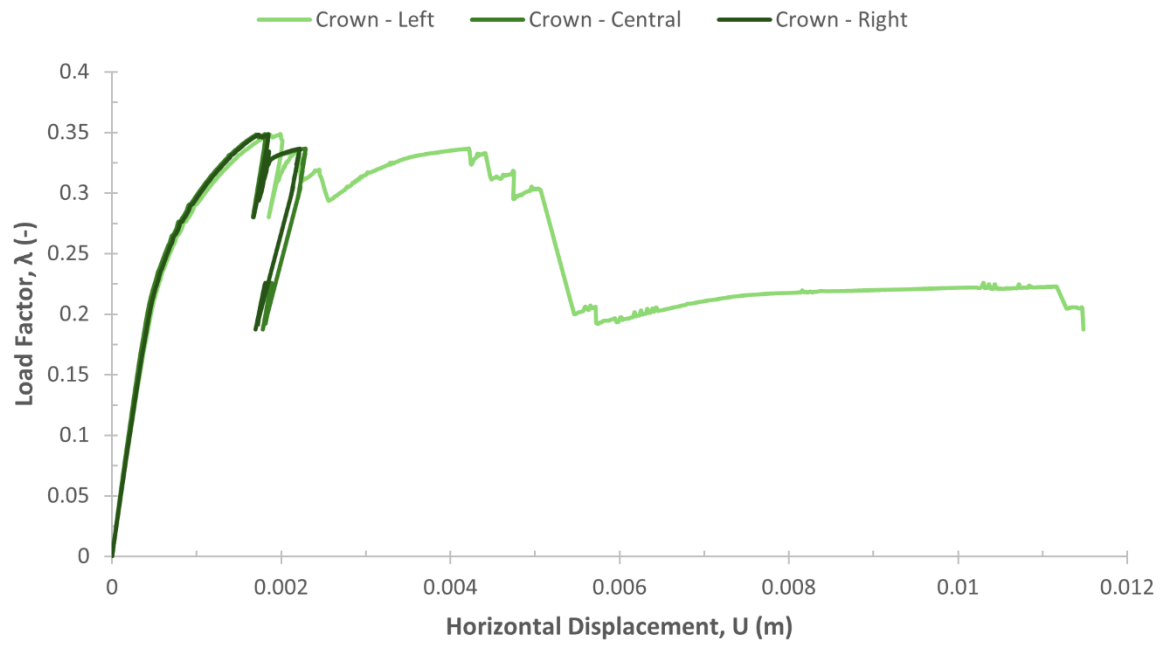
***C1 - CV2-R: Pushover +X Modal***



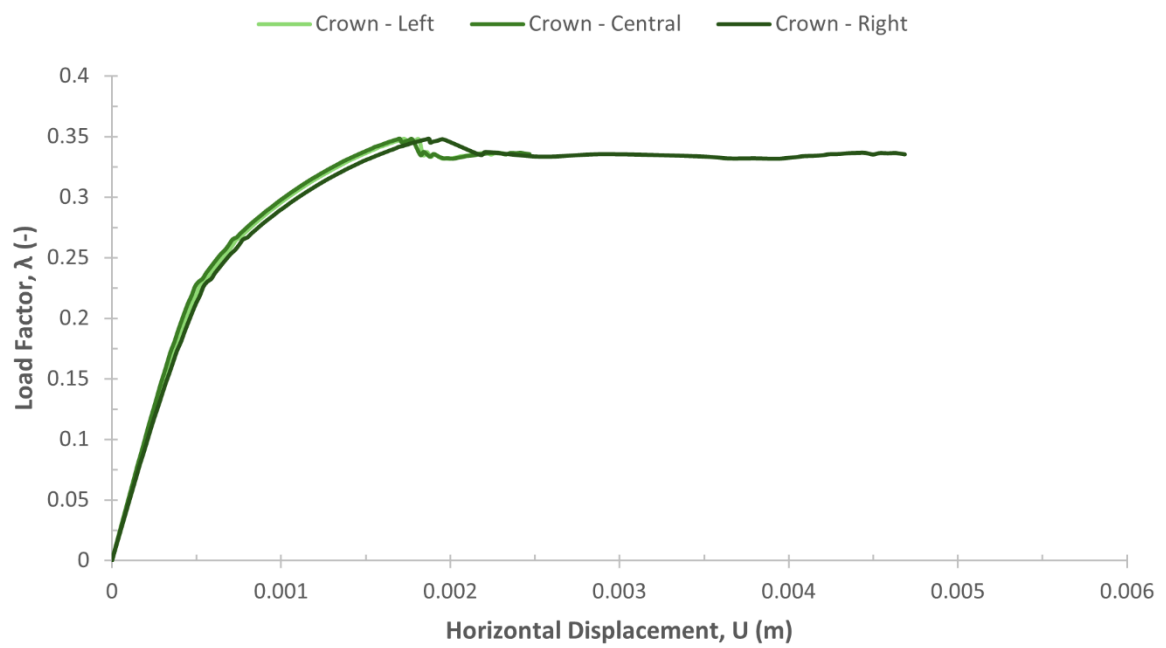
***C1 - CV2-R: Pushover -X Modal***



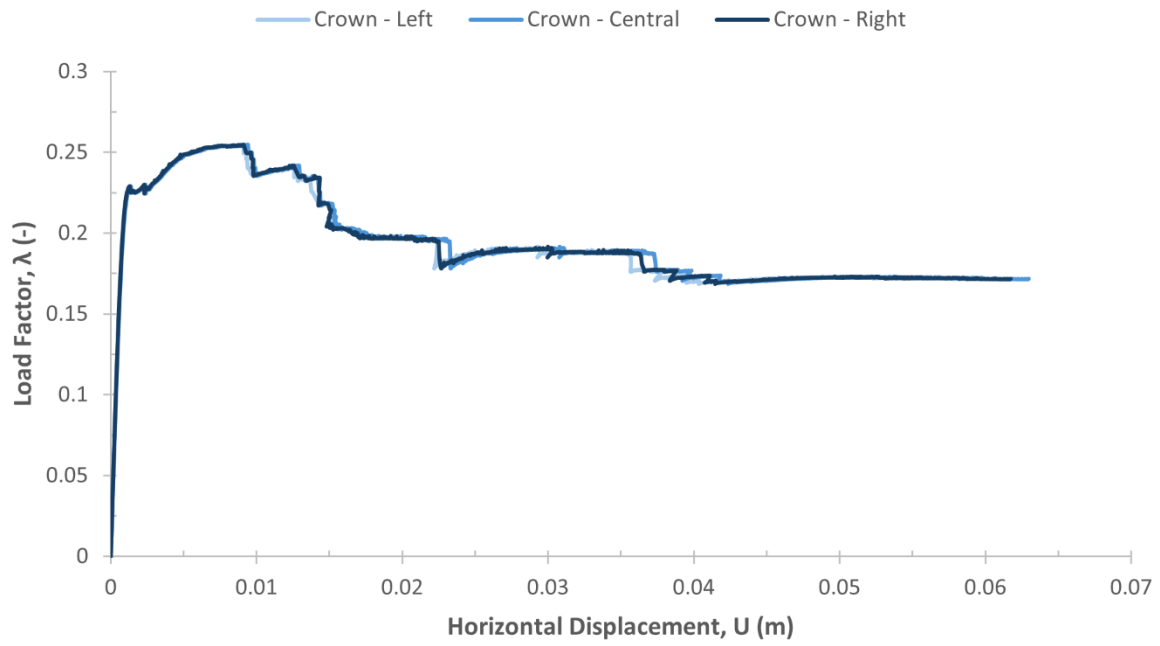
***C1 - CV2-R: Pushover +Y Modal***



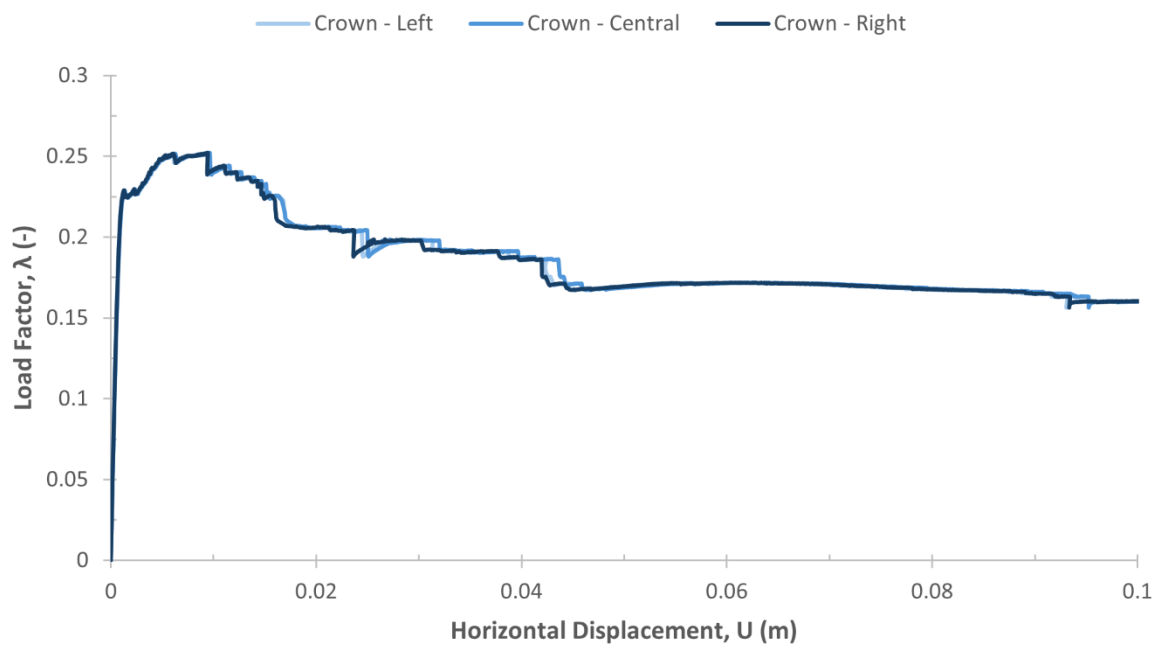
***C1 - CV2-R: Pushover -Y Modal***

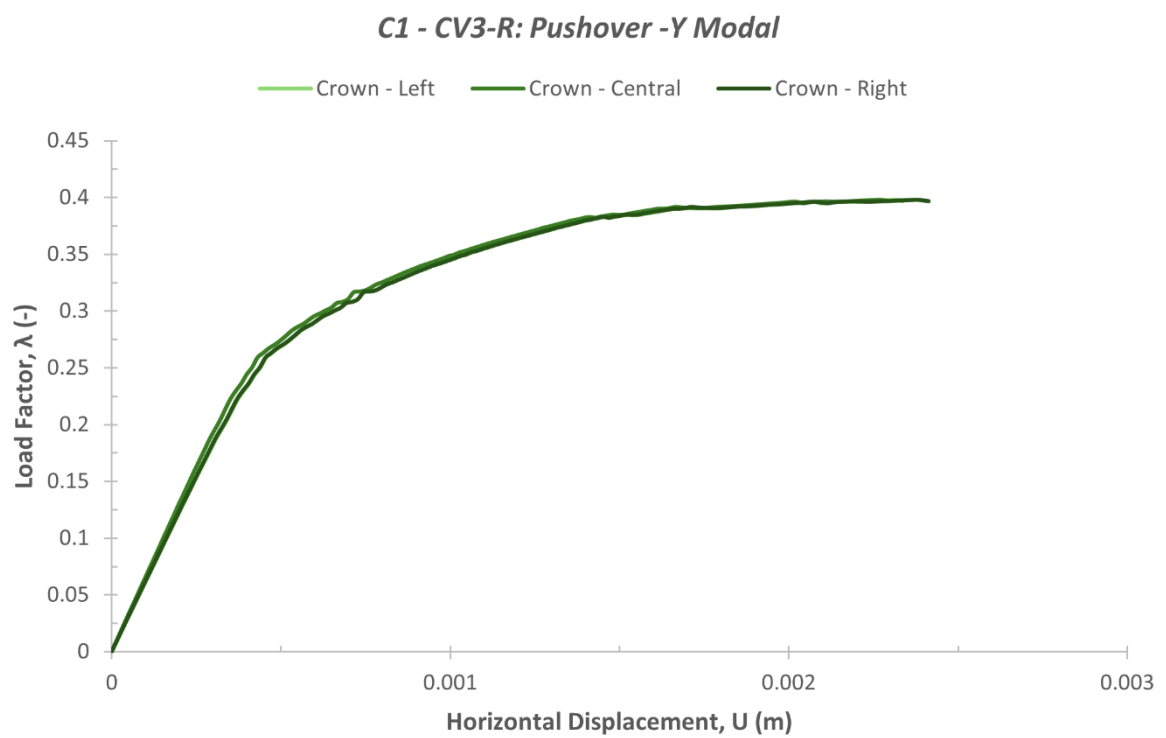
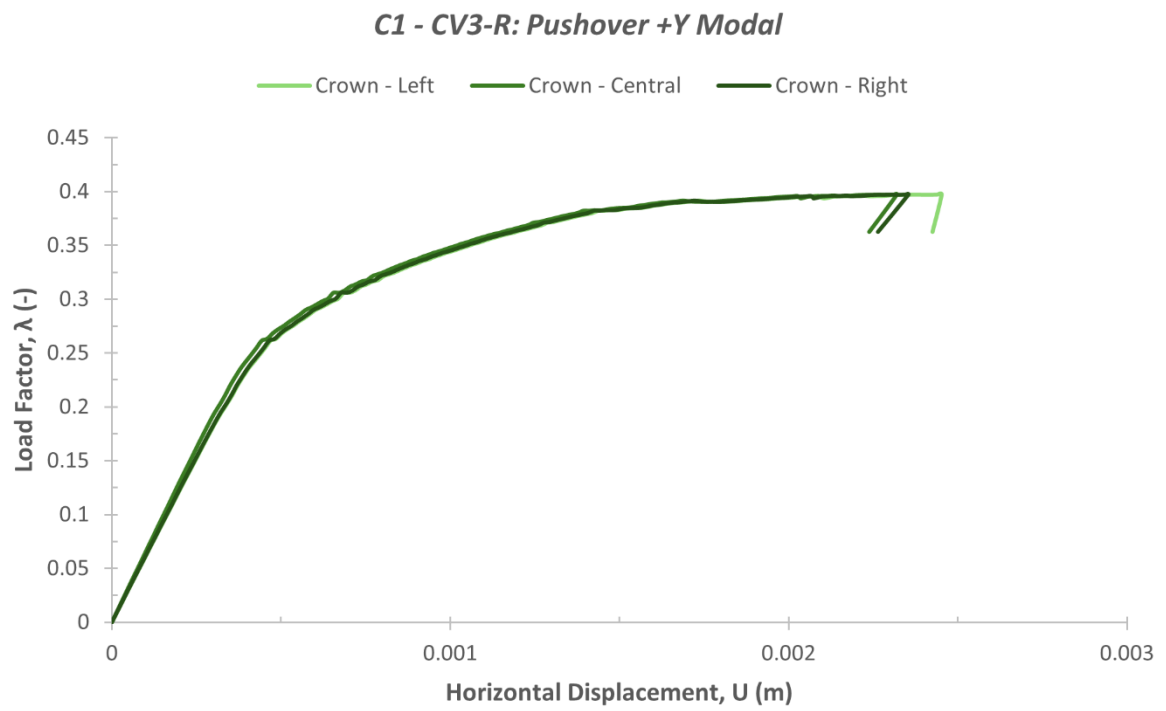


***C1 - CV3-R: Pushover +X Modal***

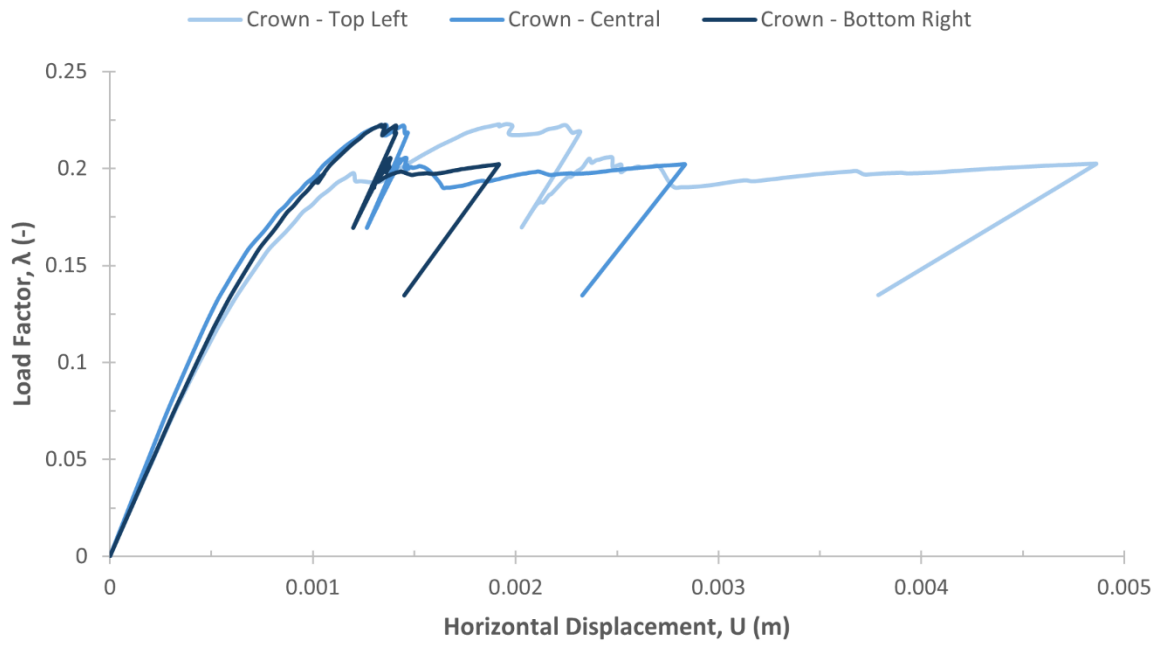


***C1 - CV3-R: Pushover -X Modal***

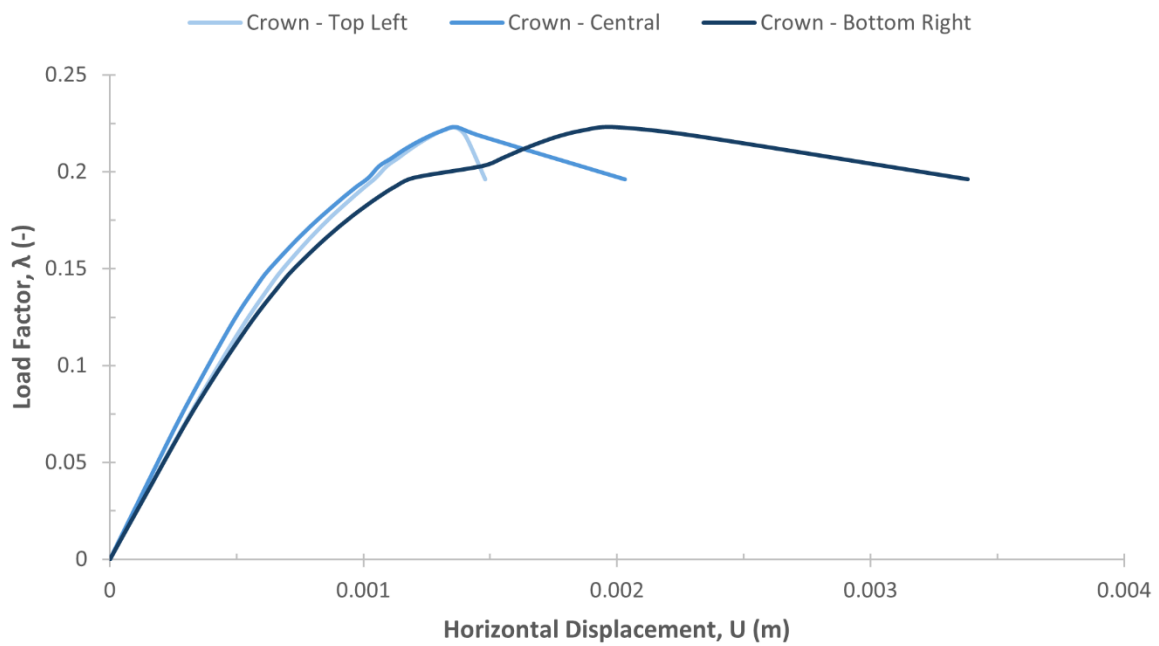


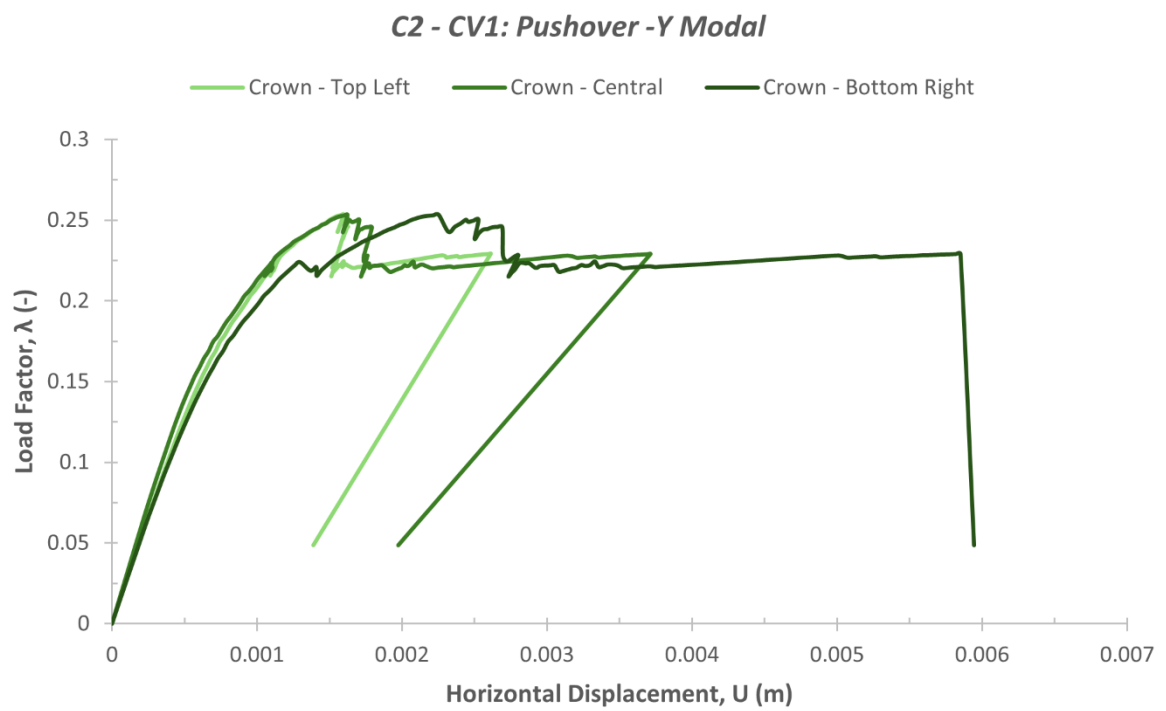
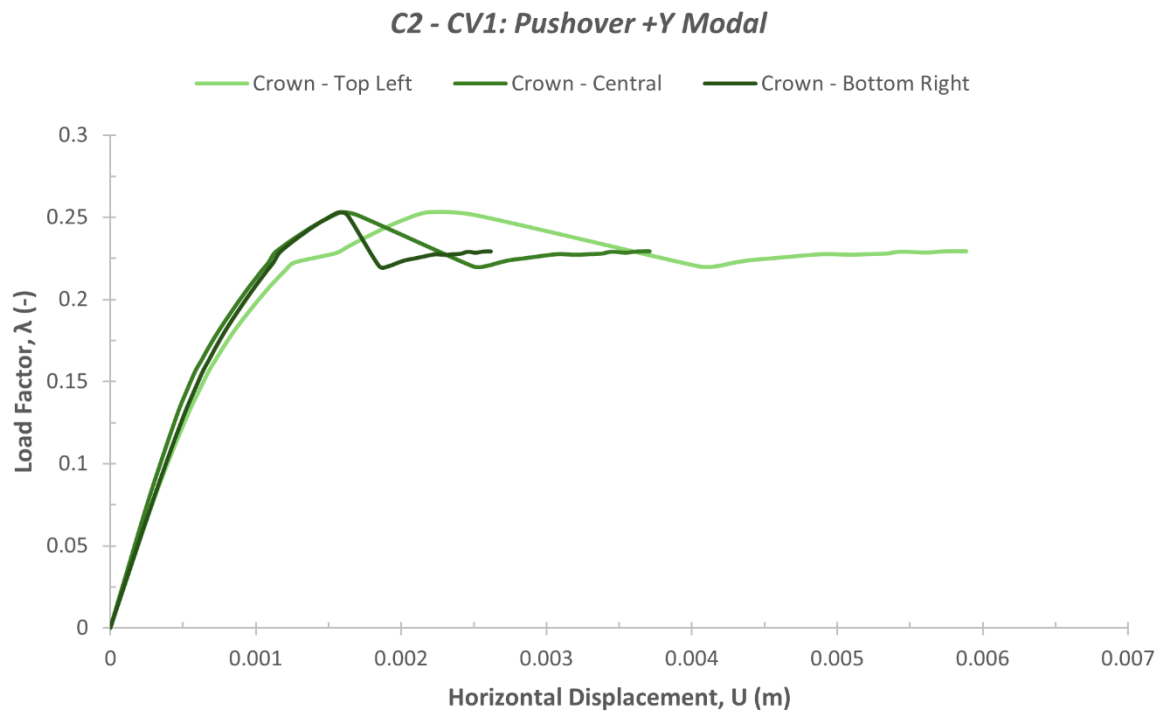


### ***C2 - CV1: Pushover +X Modal***

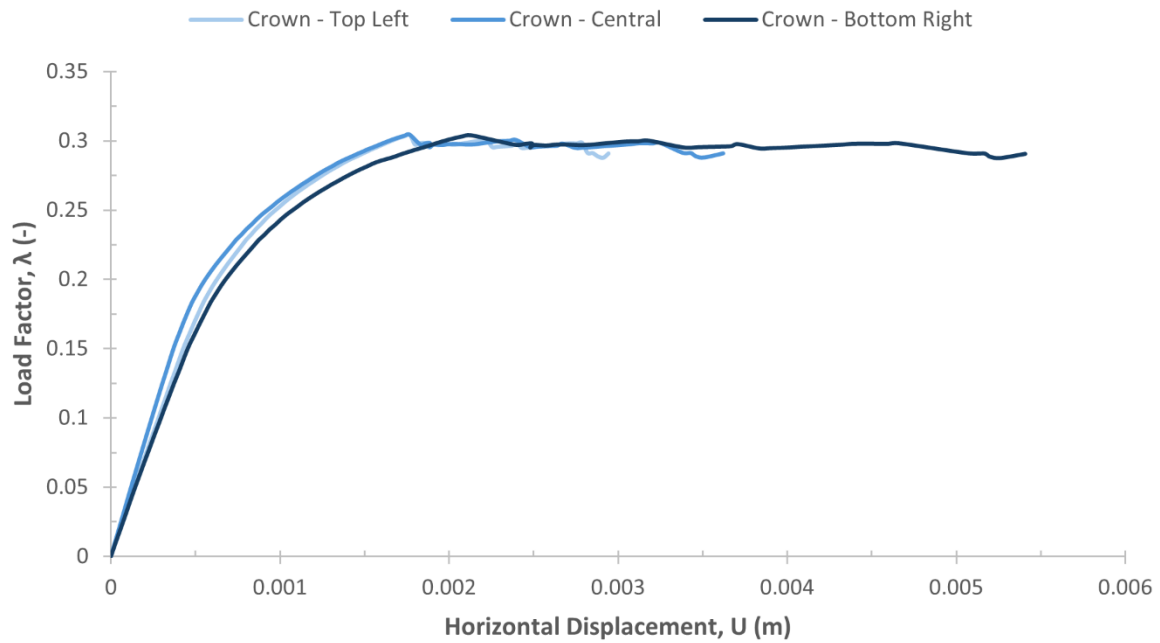


### ***C2 - CV1: Pushover -X Modal***

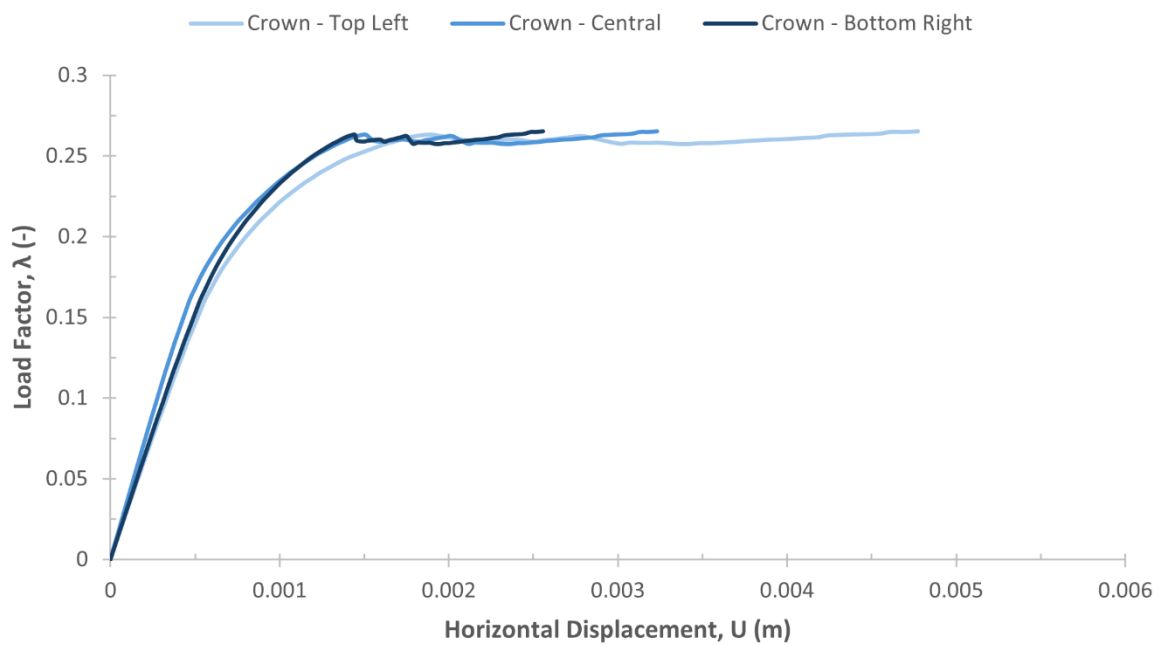




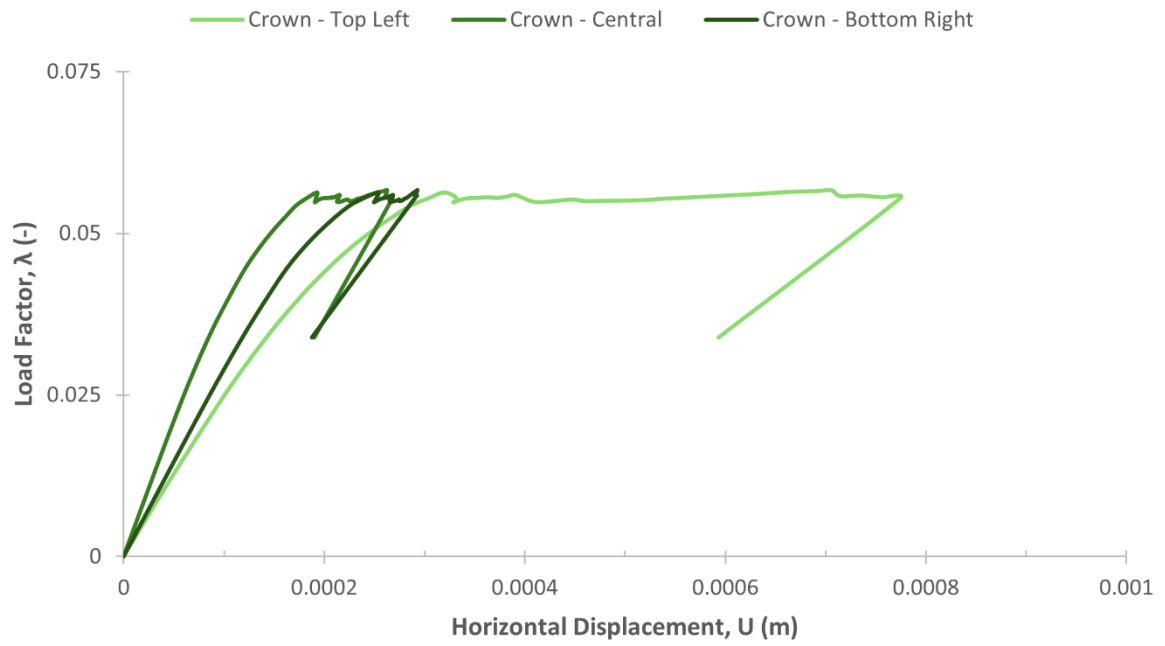
***C2 - CV1-R: Pushover +X Modal***



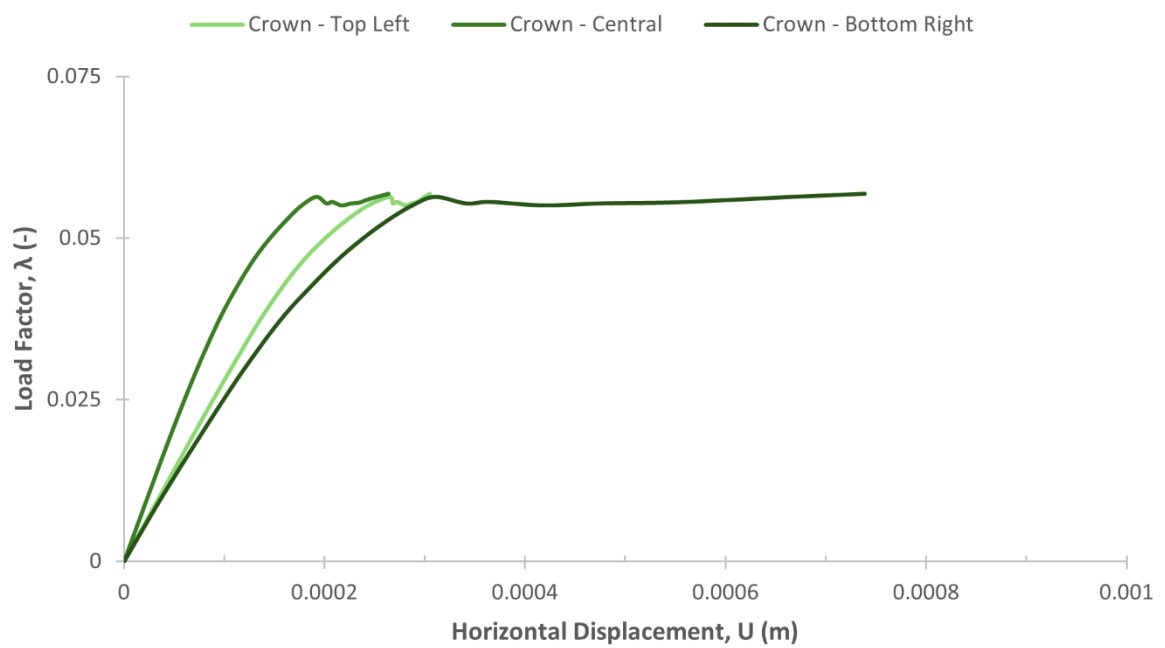
***C2 - CV1-R: Pushover -X Modal***



### ***C2 - CV1-R: Pushover +Y Modal***



### ***C2 - CV1-R: Pushover -Y Modal***





## Appendix D: Deformation Images

### Notes:

- In this appendix, only images extracted from pushover '*Modal*' analyses are presented since the deformation is identical for pushover '*Massa*' analyses (the pushover curves are different).
- The vault load which is shown in the Figures of Appendix B has been removed from the images of this appendix for clarity.
- The colormap of the figures presented in this appendix is in terms of the vertical displacement in the global Z-direction (blue line of the axis at the bottom left of each image).
- Each image was taken at different horizontal displacement (U) values as indicated. In this regard, the scale factor of the deformation varies to ensure clear and readable images.
- Images are separated by test (numerical model) and follow the order of  $\pm X$  and  $\pm Y$ .
- For C2 – CV1-R only, the deformation images of  $\pm Y$  were taken from the pushover '*Massa*' analysis due to the error that occurred in the pushover '*Modal*' analysis for that direction.

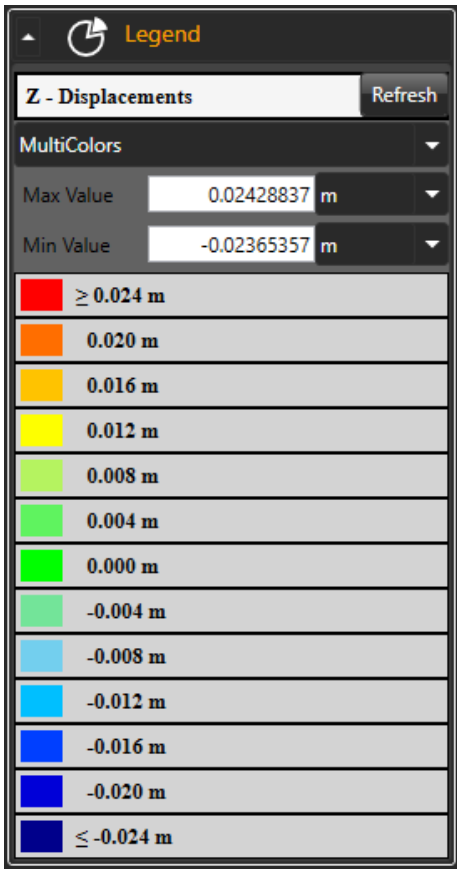


Figure D1 – Legend for +X direction. Taken at  $U = 0.03m$ .

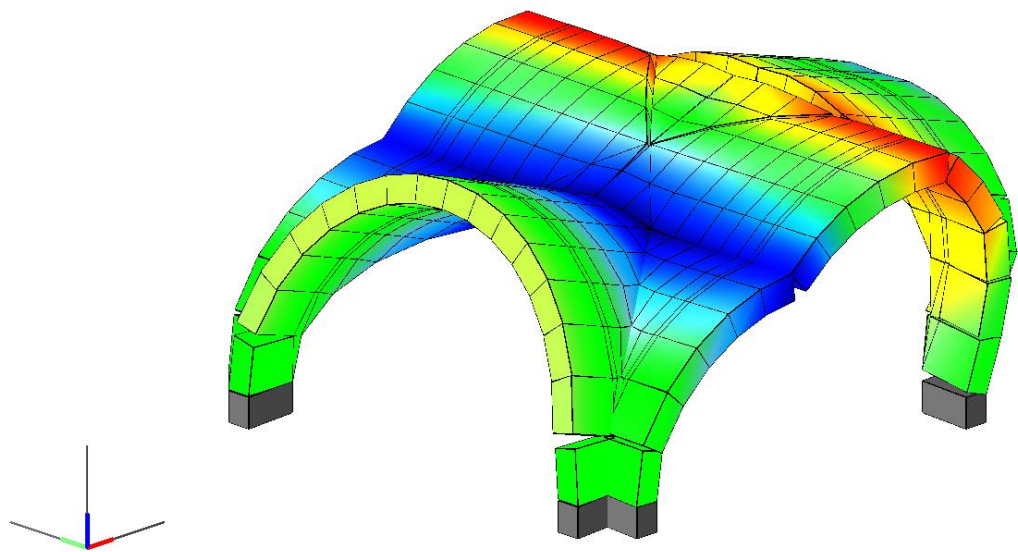
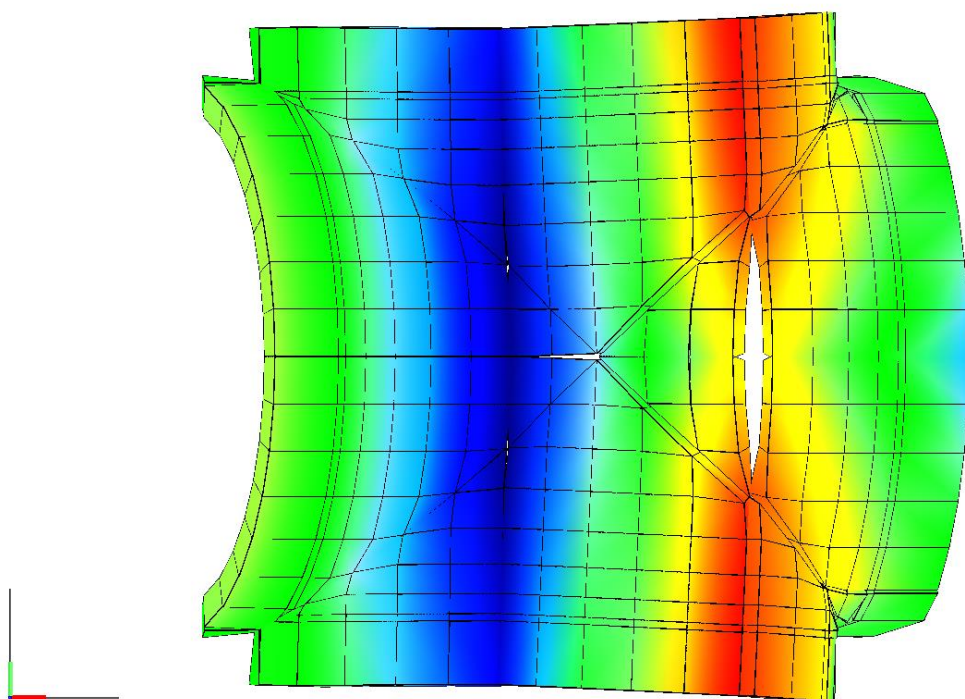
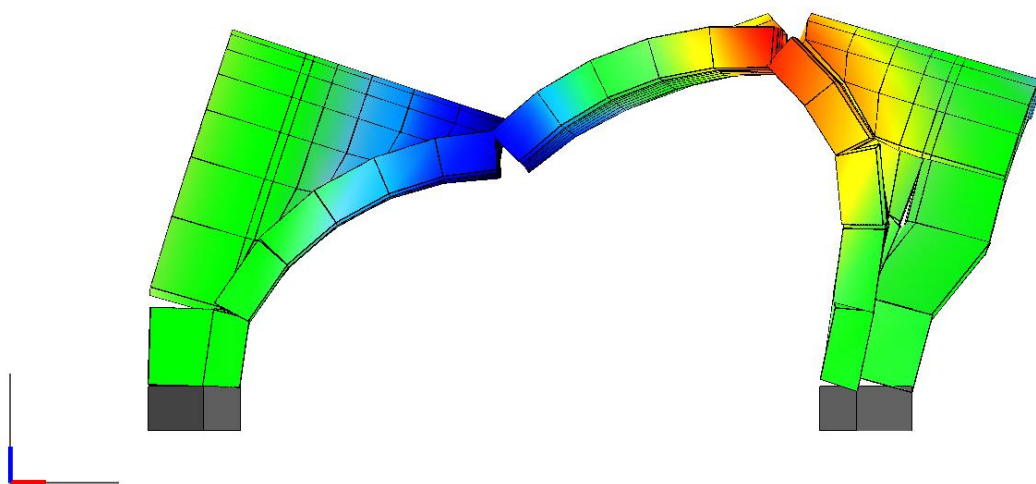


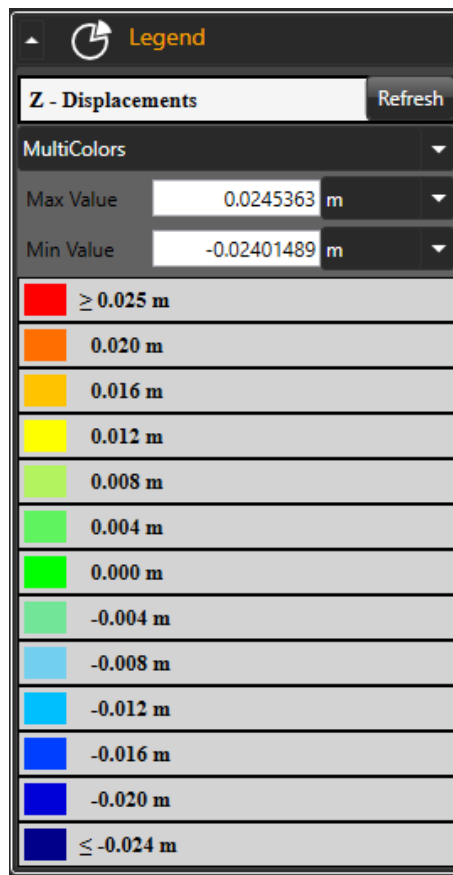
Figure D2 – 3D view of deformation in +X direction.



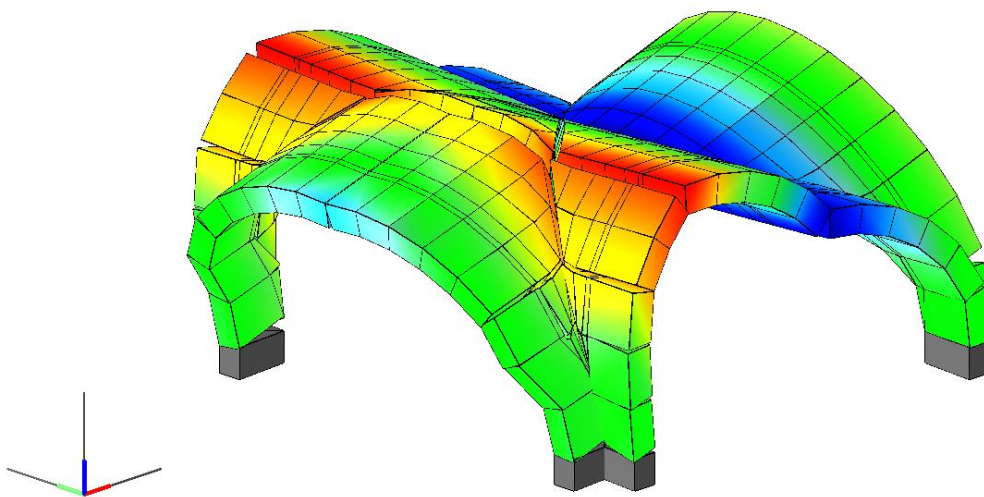
**Figure D3** – Plan view of deformation in +X direction.



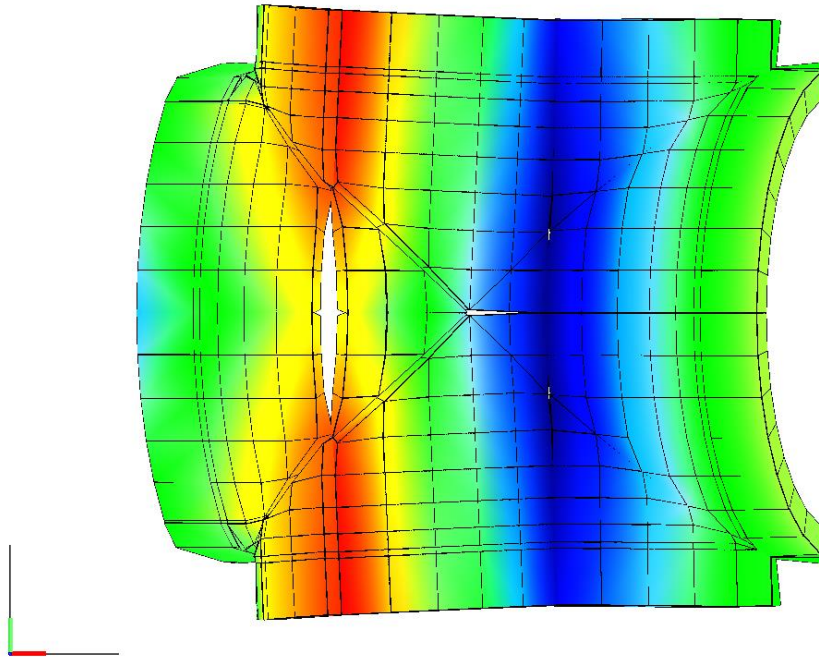
**Figure D4** – Front view of deformation in +X direction.



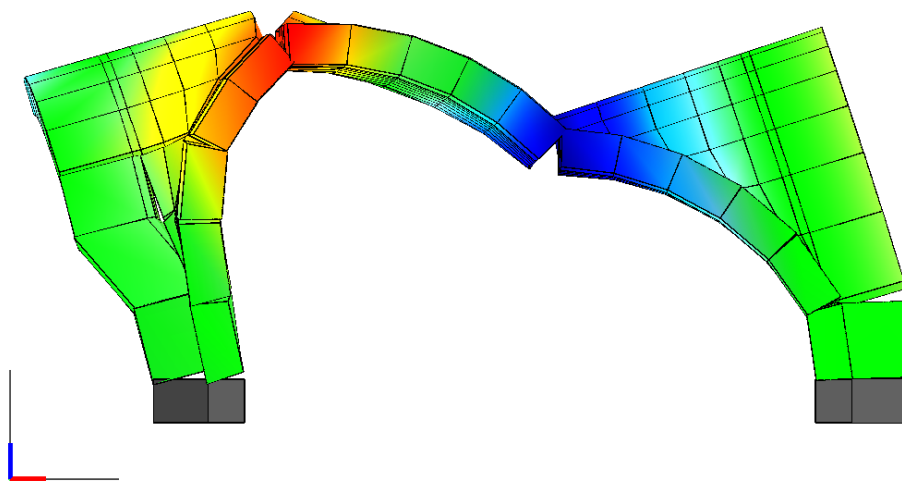
**Figure D5** – Legend for -X direction. Taken at  $U = 0.03m$ .



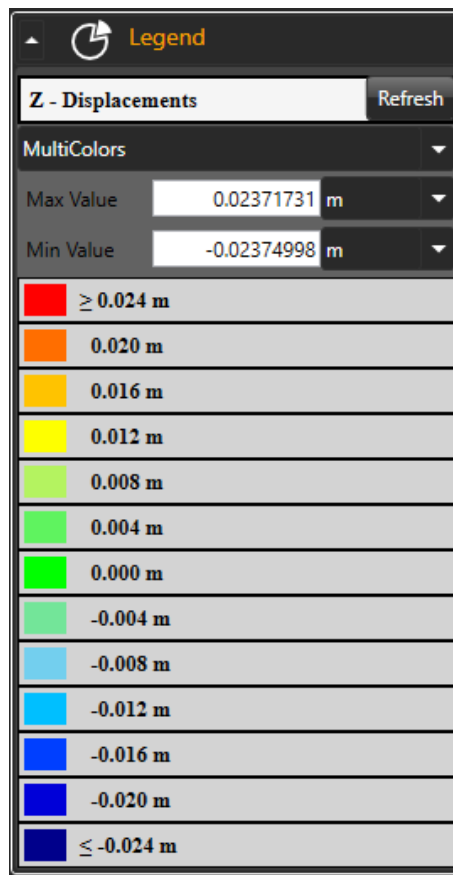
**Figure D6** – 3D view of deformation in -X direction.



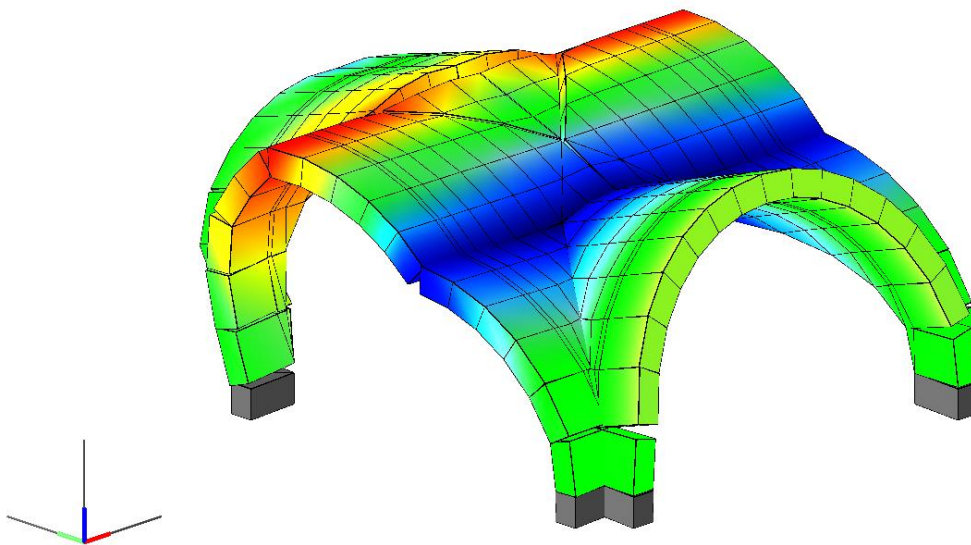
**Figure D7** – Plan view of deformation in -X direction.



**Figure D8** – Front view of deformation in -X direction.

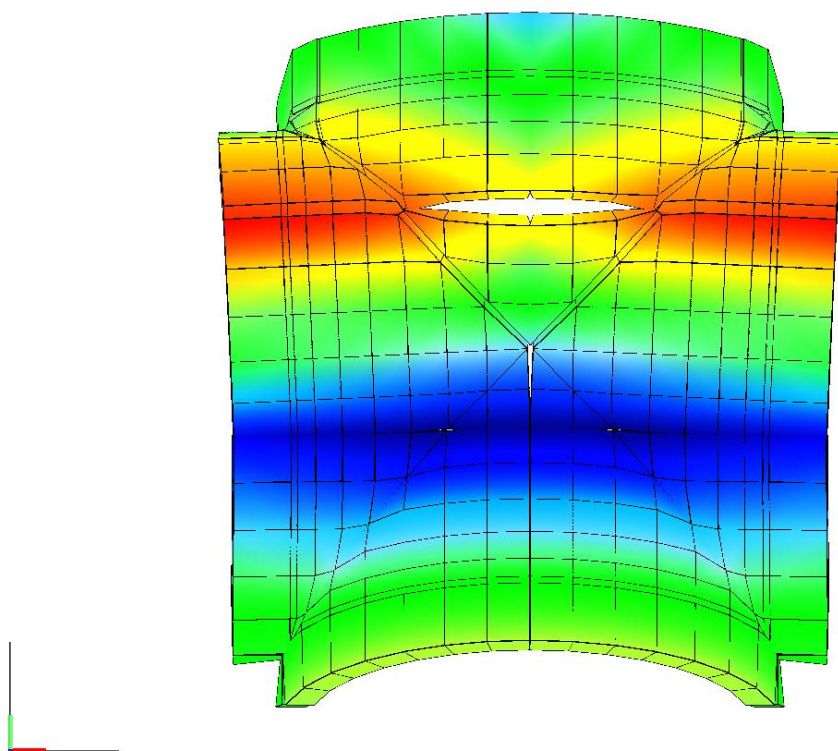


**Figure D9** – Legend for +Y direction. Taken at  $U = 0.03m$ .

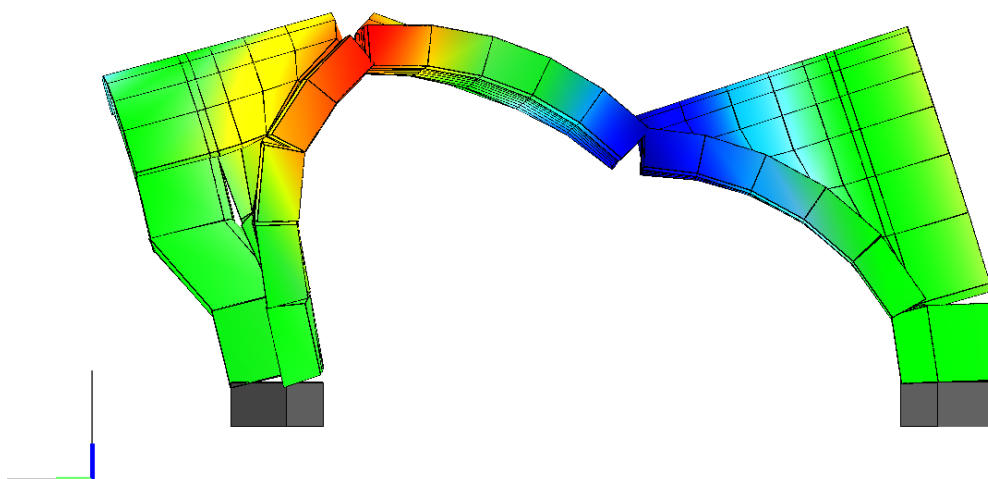


**Figure D10** – 3D view of deformation in +Y direction.

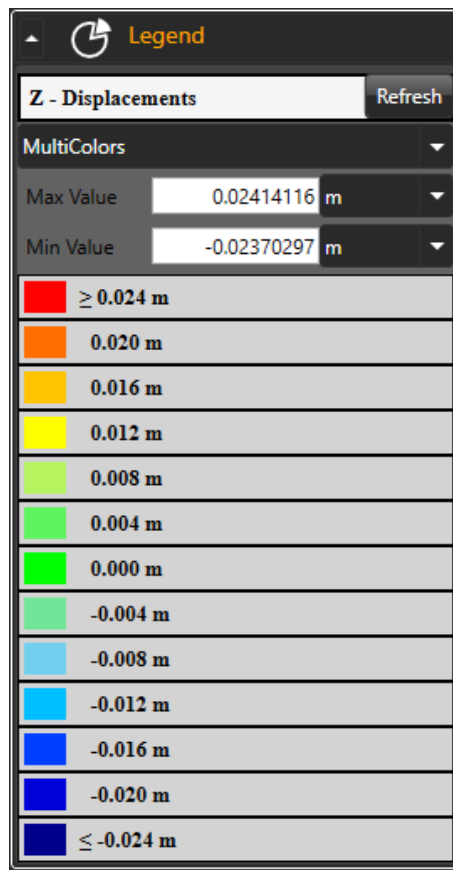




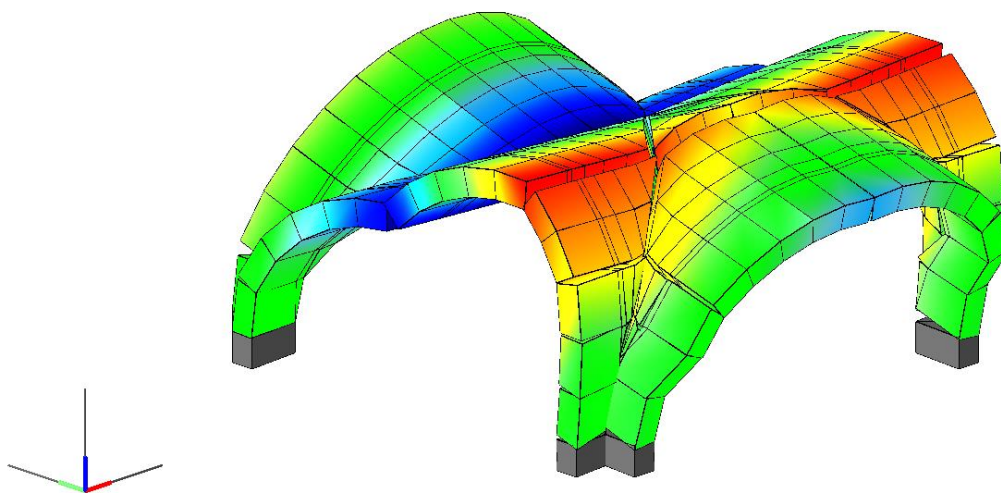
**Figure D11** – Plan view of deformation in +Y direction.



**Figure D12** – Side view of deformation in +Y direction.

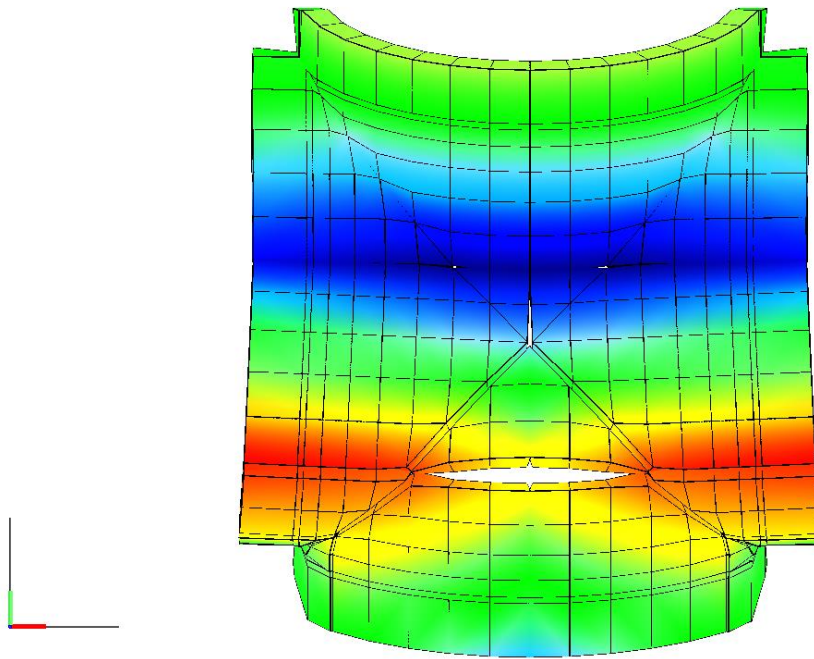


**Figure D13** – Legend for -Y direction. Taken at  $U = 0.03m$ .

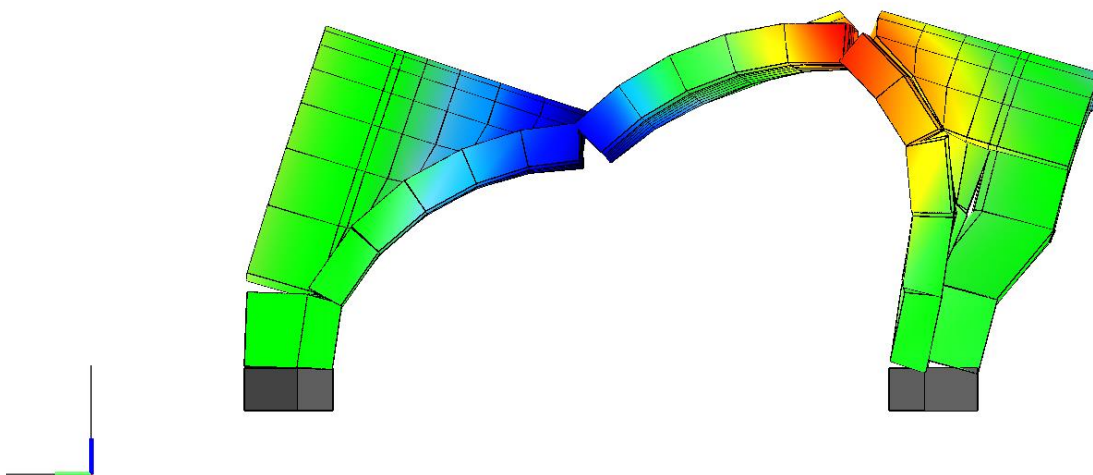


**Figure D14** – 3D view of deformation in -Y direction.





**Figure D15** – Plan view of deformation in -Y direction.



**Figure D16** – Side view of deformation in +Y direction.

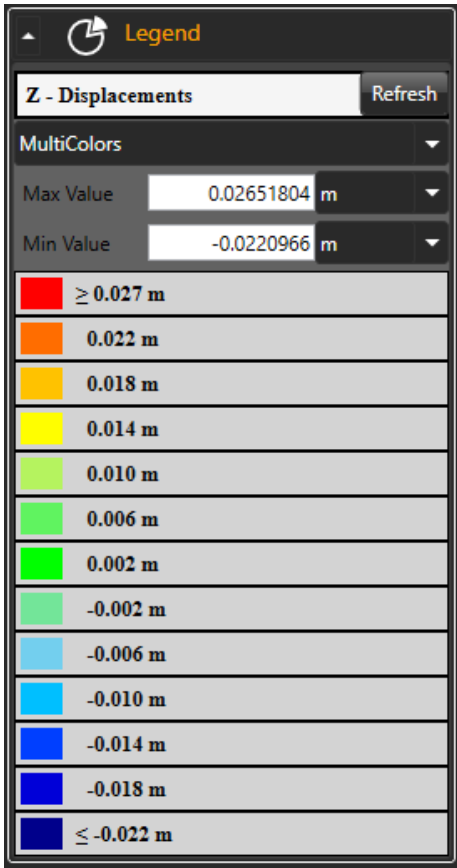


Figure D17 – Legend for +X direction. Taken at U = 0.03m.

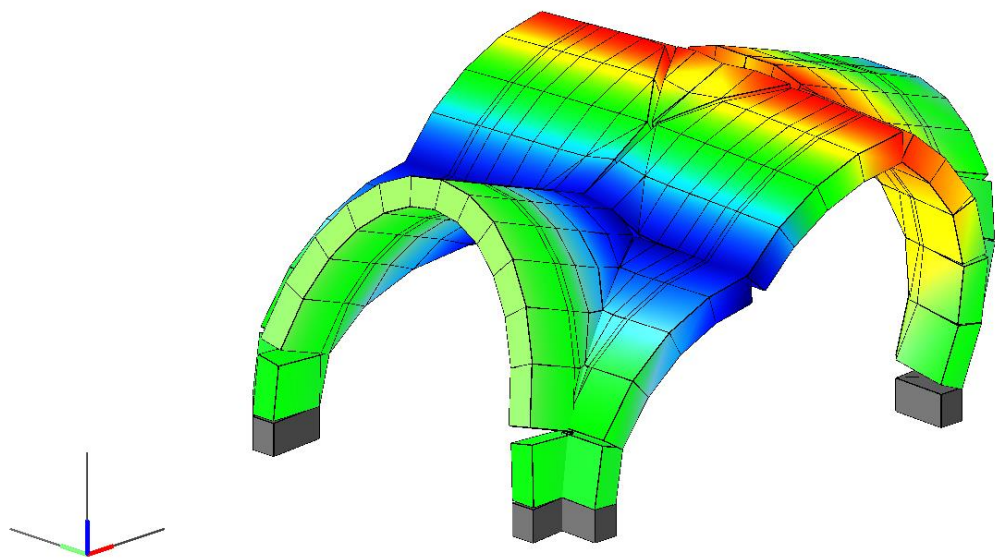
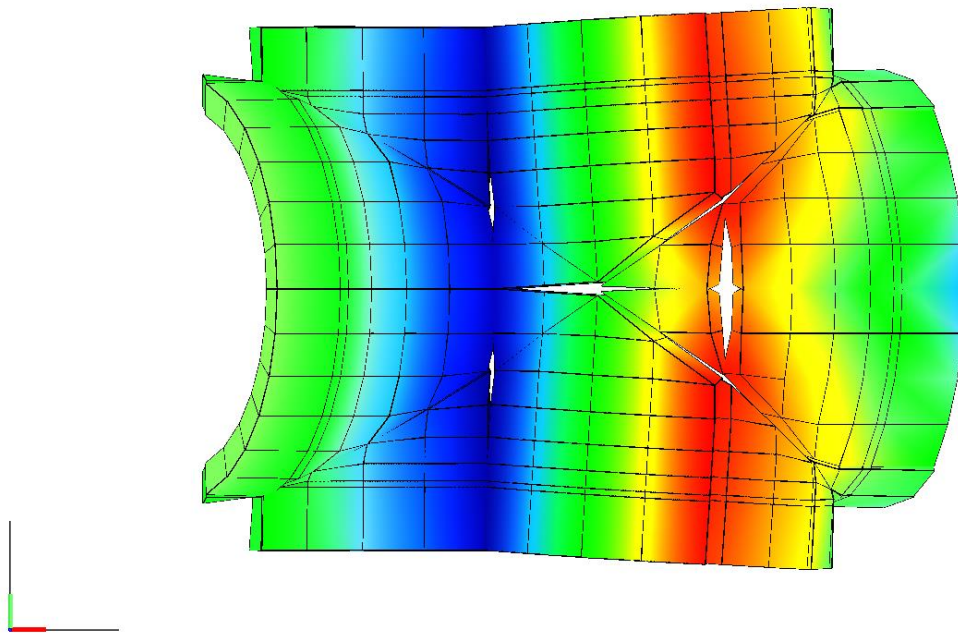
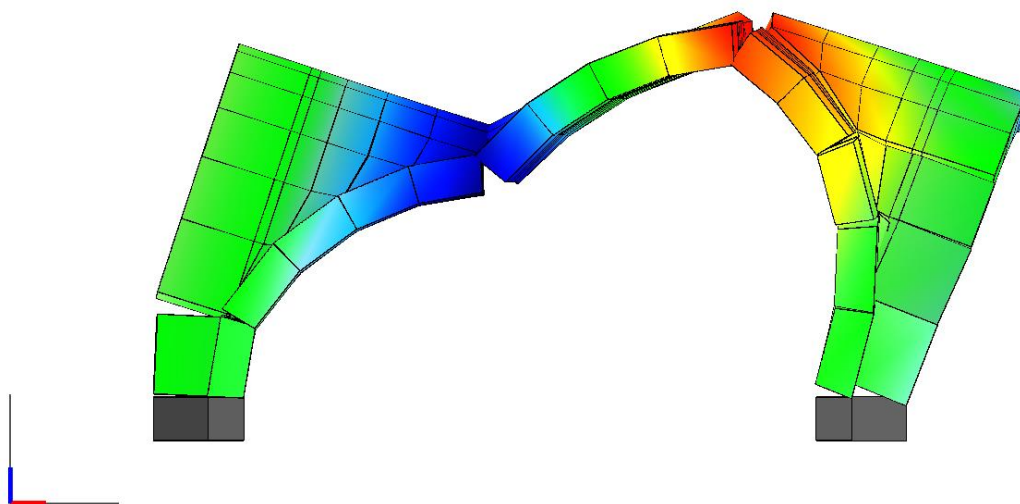


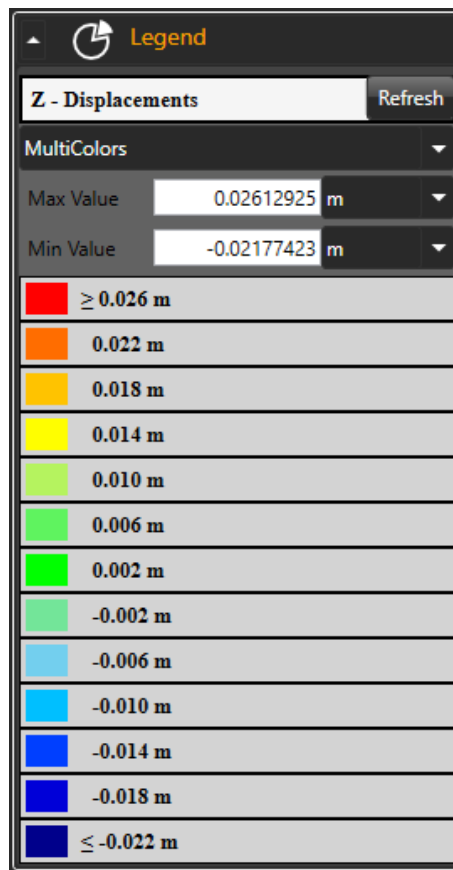
Figure D18 – 3D view of deformation in +X direction.



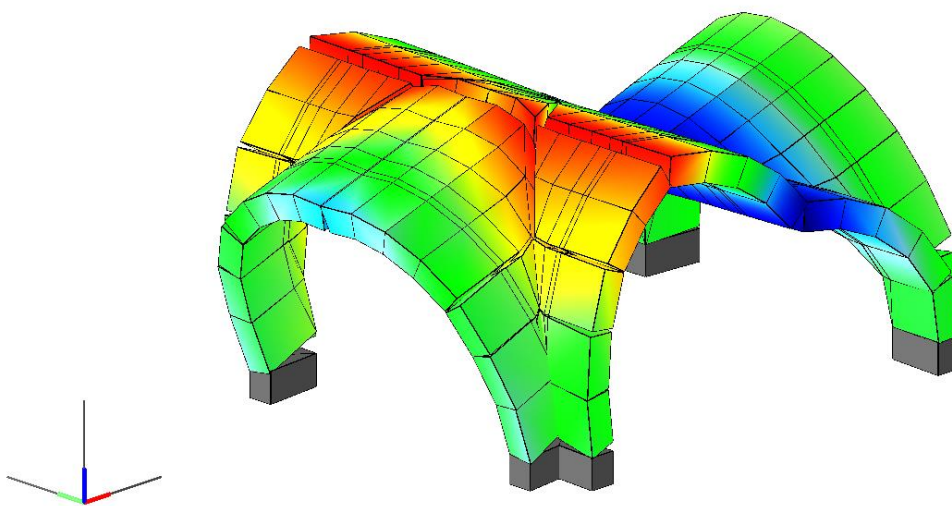
**Figure D19** – Plan view of deformation in +X direction.



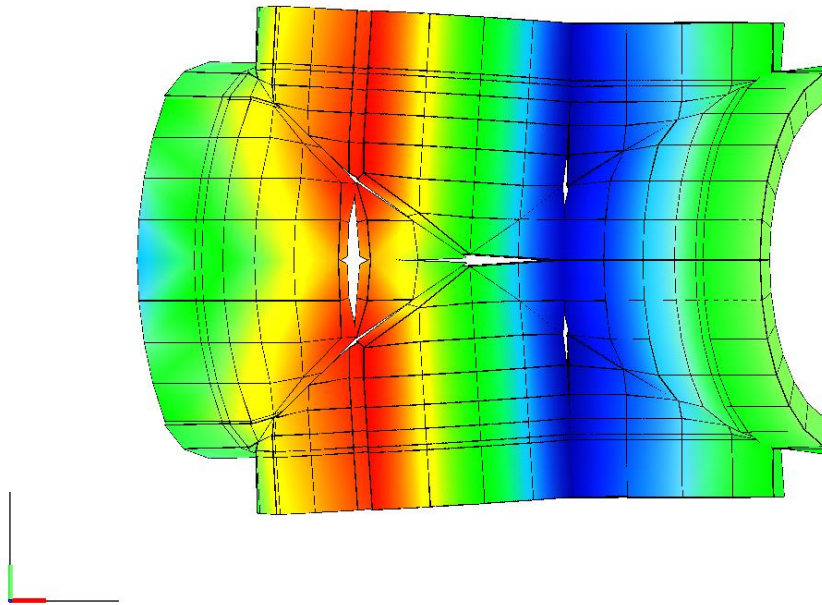
**Figure D20** – Front view of deformation in +X direction.



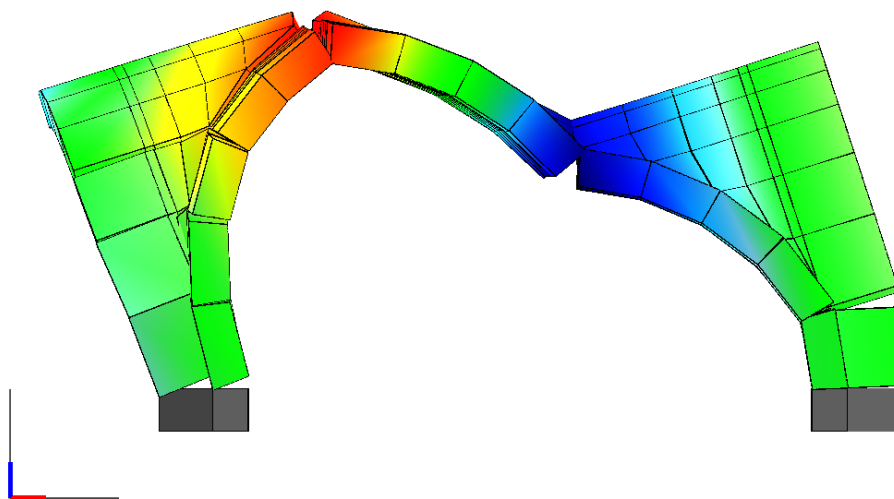
**Figure D21** – Legend for -X direction. Taken at  $U = 0.03\text{m}$ .



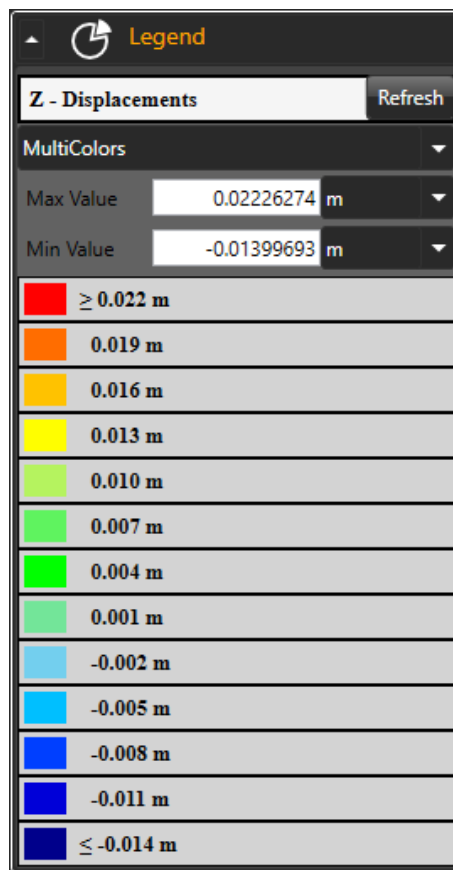
**Figure D22** – 3D view of deformation in -X direction.



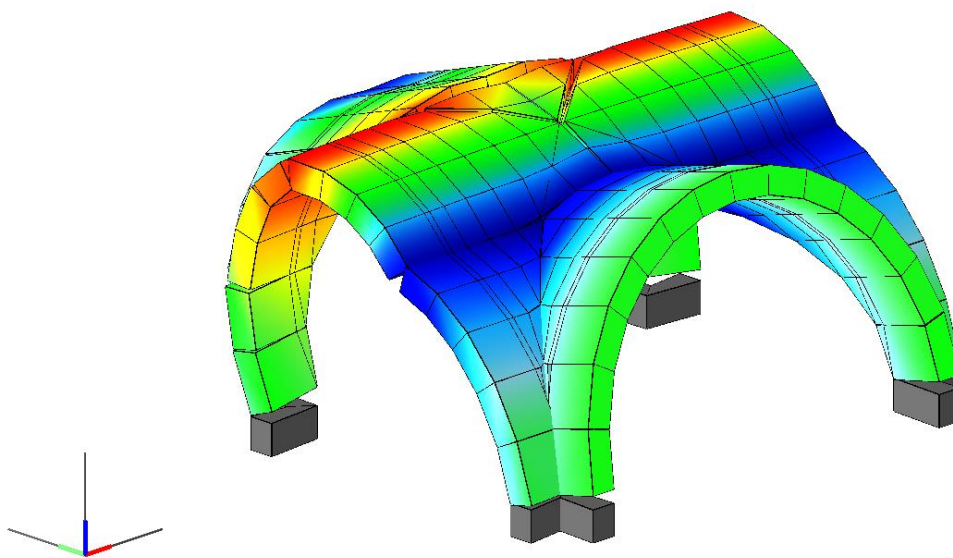
**Figure D23** – Plan view of deformation in -X direction.



**Figure D24** – Front view of deformation in -X direction.

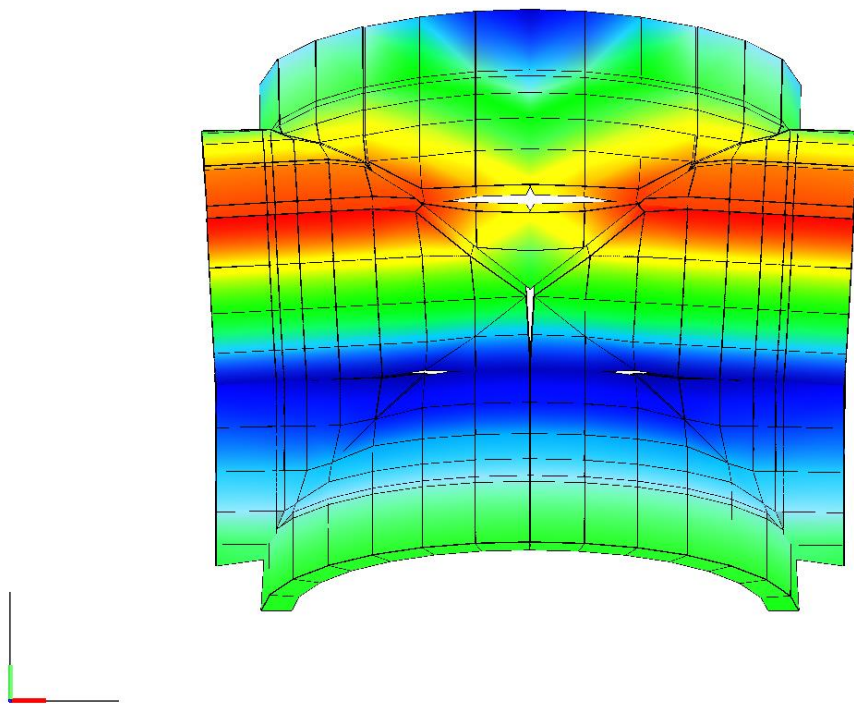


**Figure D25** – Legend for +Y direction. Taken at  $U = 0.03m$ .

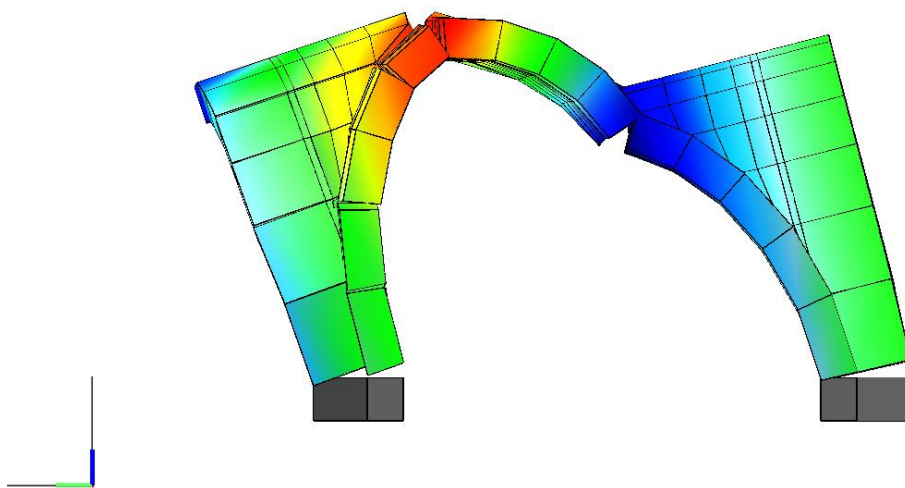


**Figure D26** – 3D view of deformation in +Y direction.

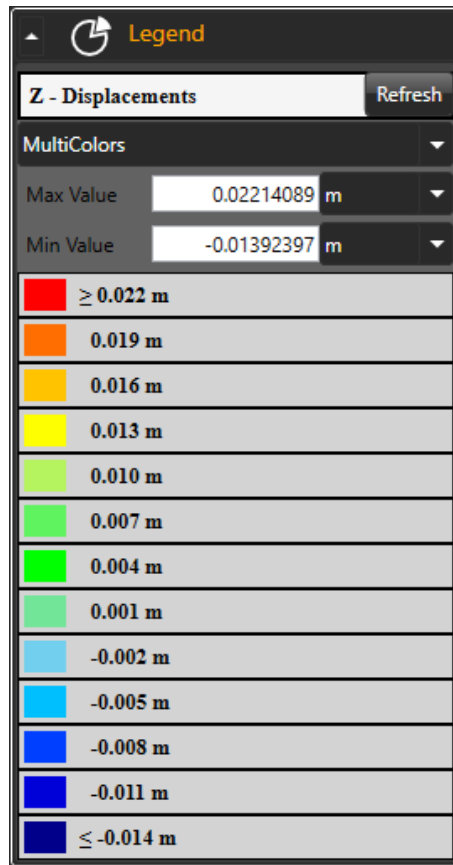




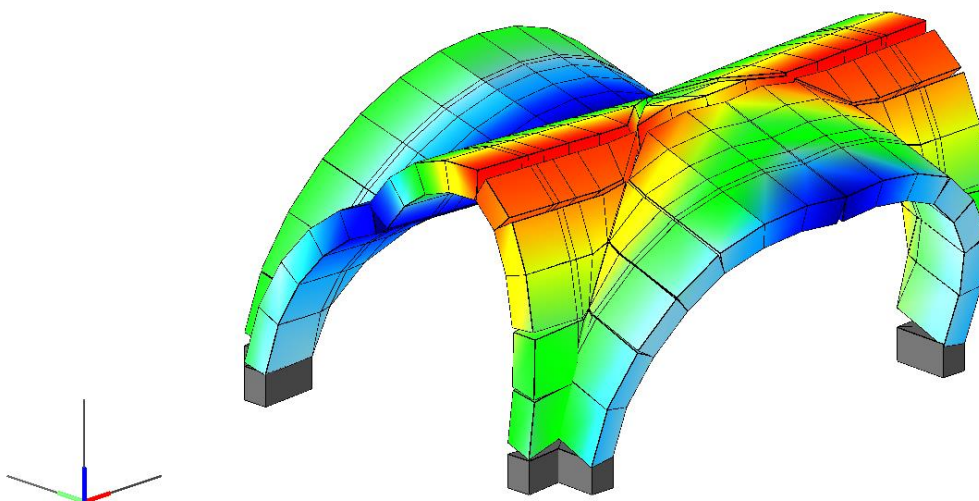
**Figure D27** – Plan view of deformation in +Y direction.



**Figure D28** – Side view of deformation in +Y direction.

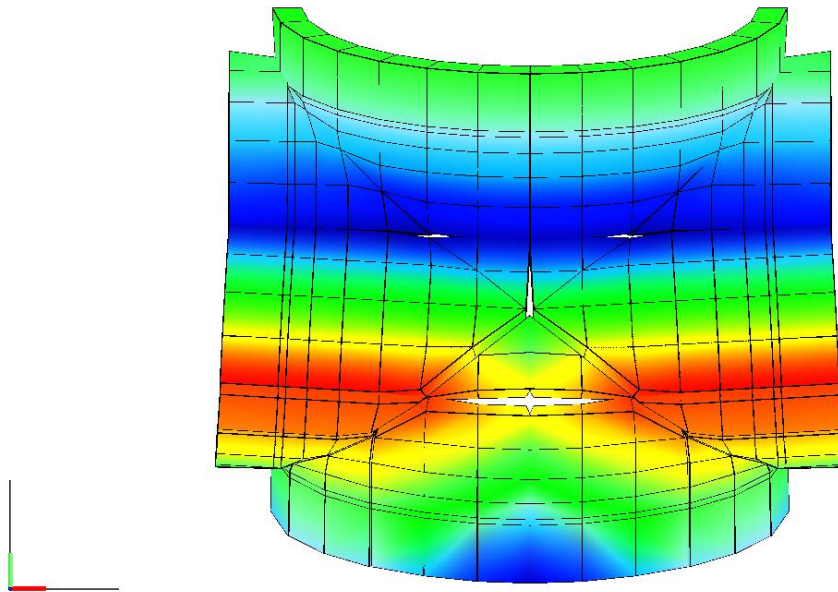


**Figure D29** – Legend for -Y direction. Taken at  $U = 0.03\text{m}$ .

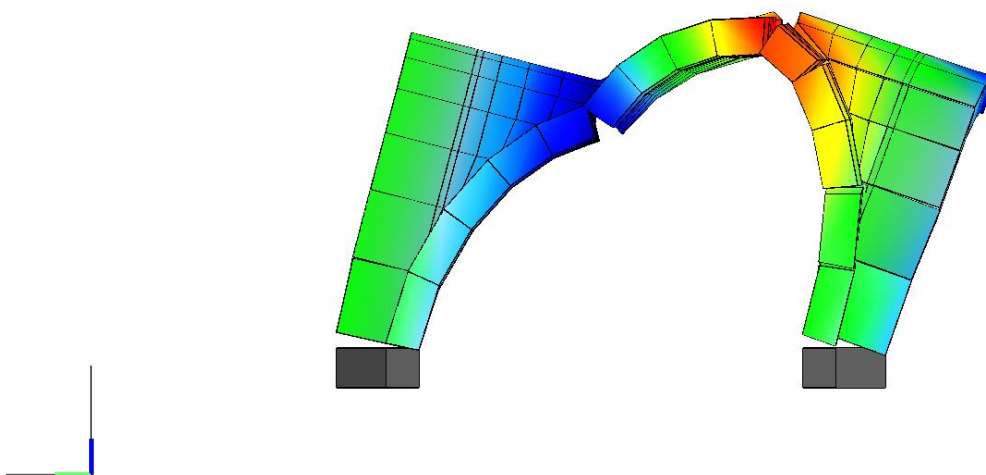


**Figure D30** – 3D view of deformation in -Y direction.





**Figure D31** – Plan view of deformation in -Y direction.



**Figure D32** – Side view of deformation in -Y direction.

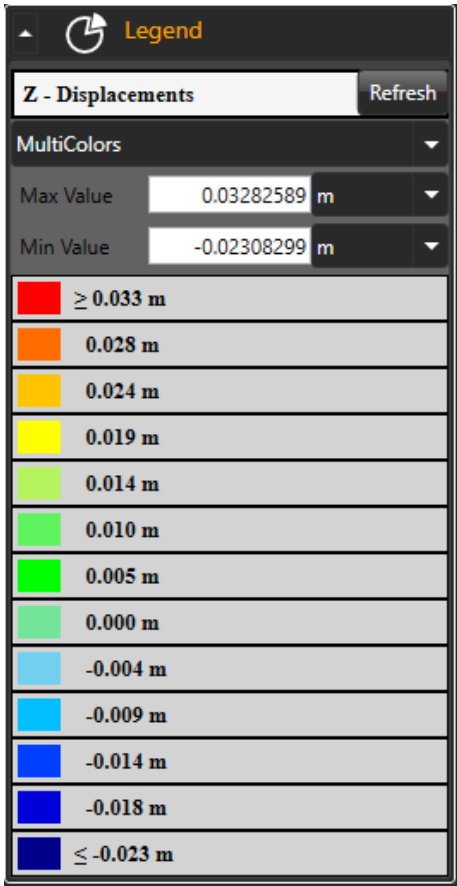


Figure D33 – Legend for +X direction. Taken at U = 0.03m.

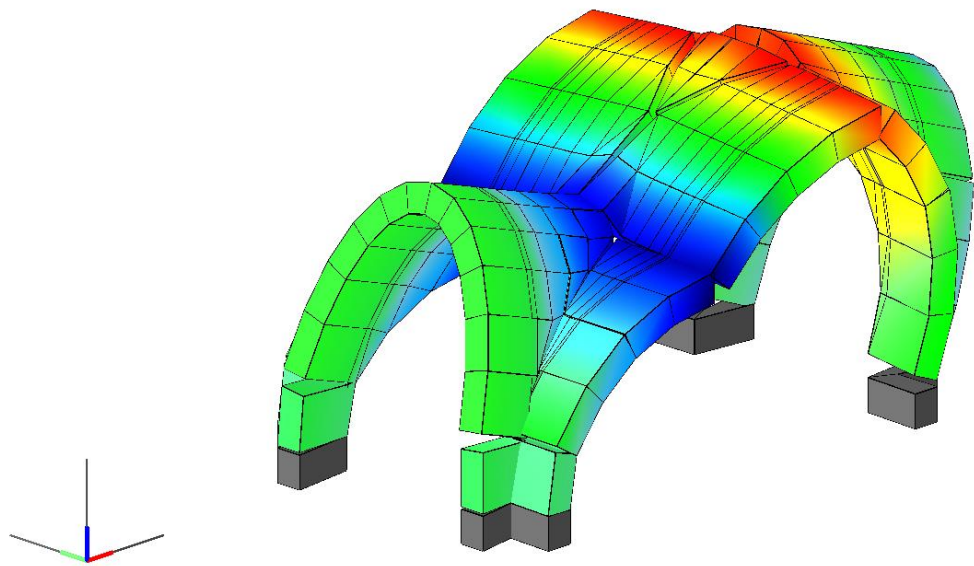
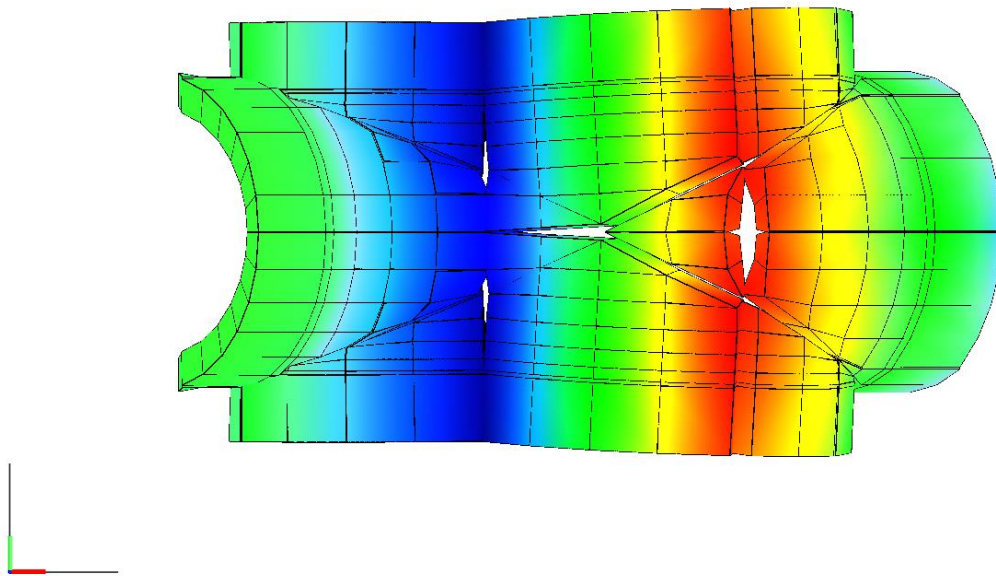
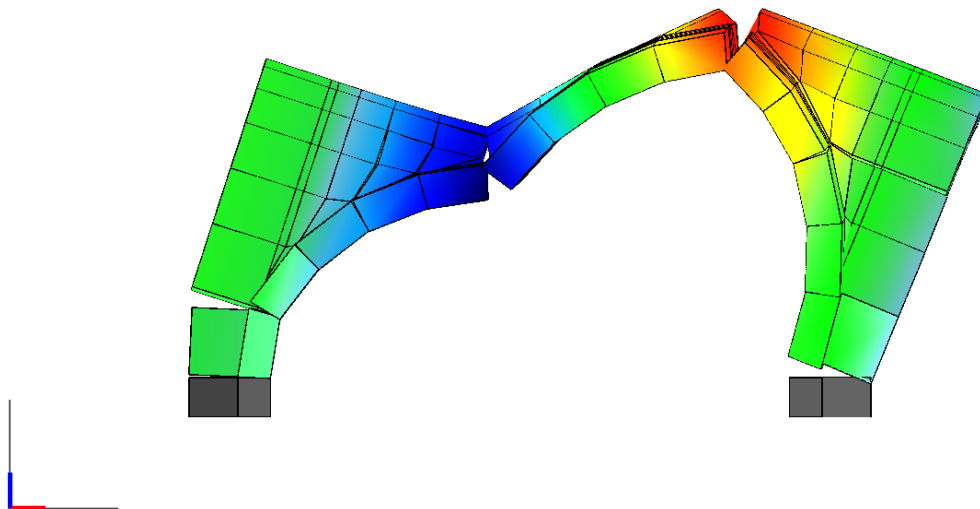


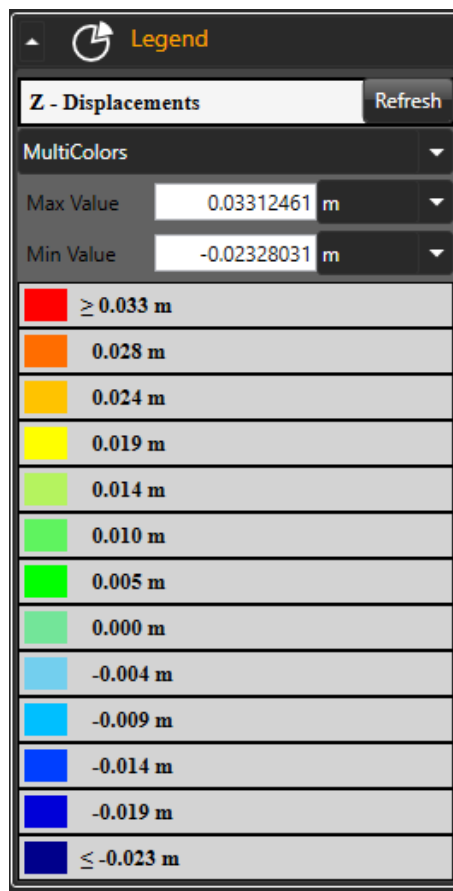
Figure D34 – 3D view of deformation in +X direction.



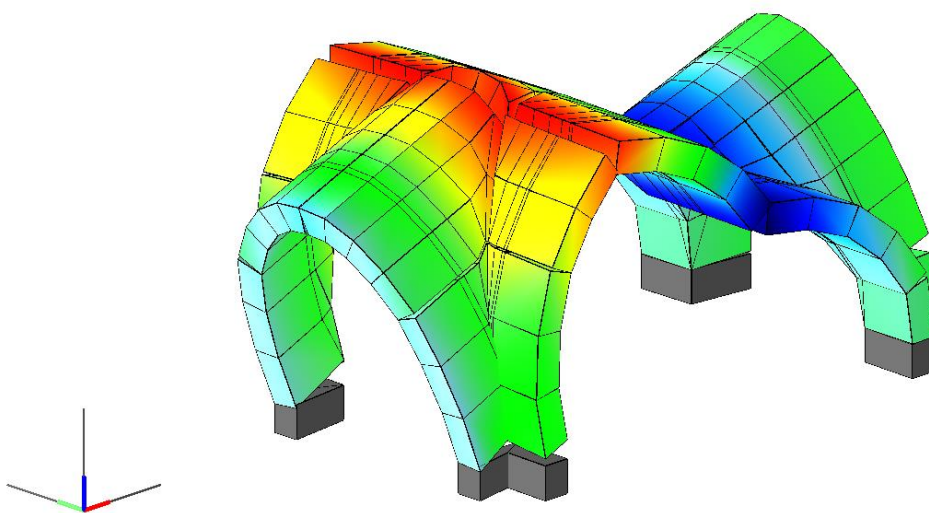
**Figure D35** – Plan view of deformation in +X direction.



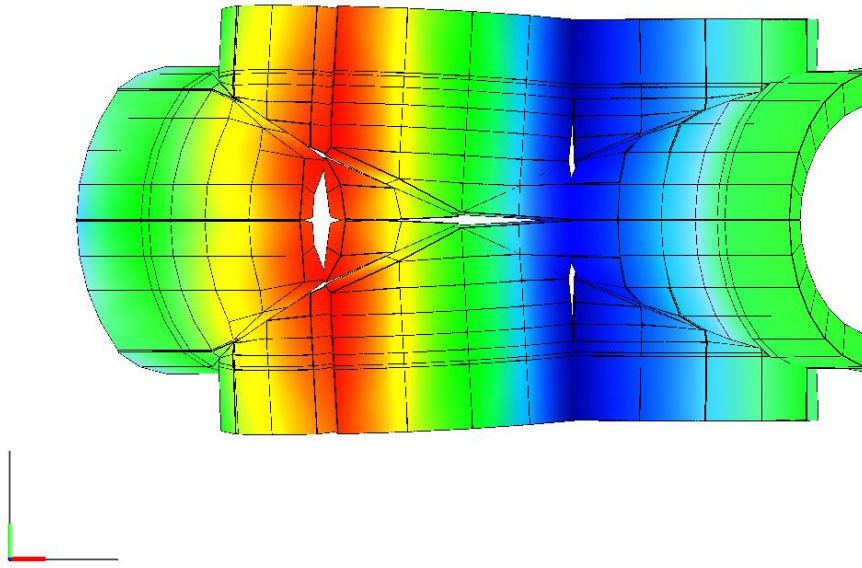
**Figure D36** – Front view of deformation in +X direction.



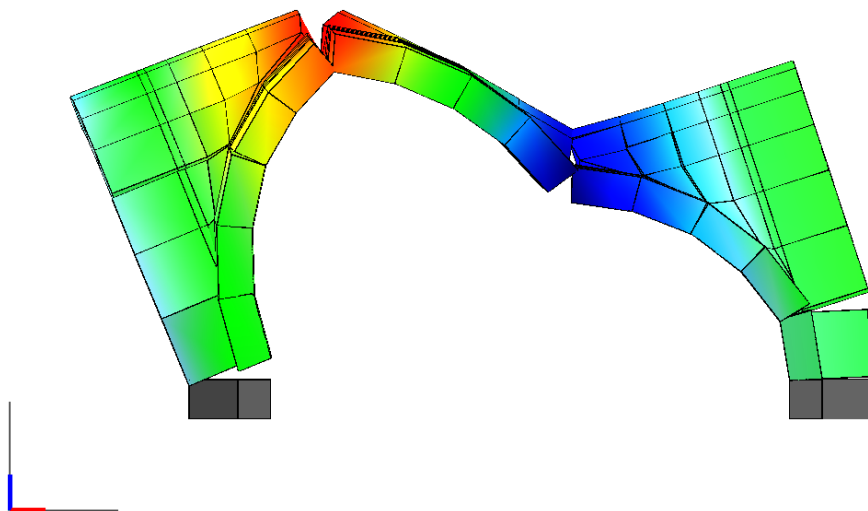
**Figure D37** – Legend for -X direction. Taken at  $U = 0.03$  m.



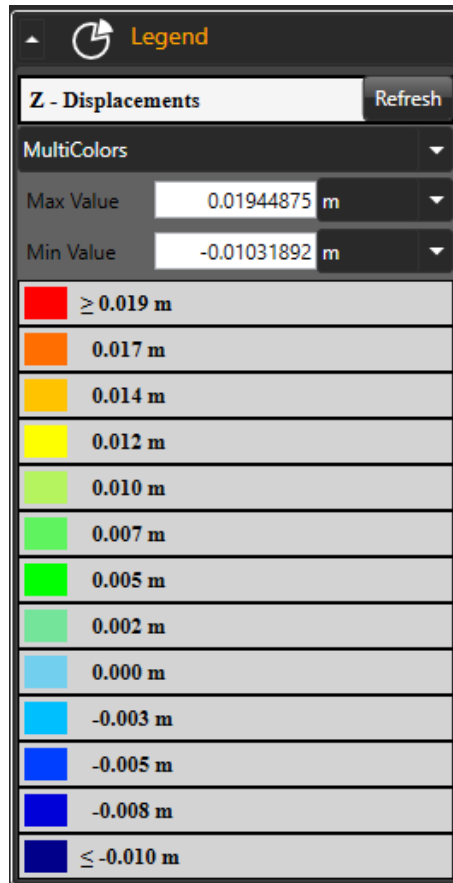
**Figure D38** – 3D view of deformation in -X direction.



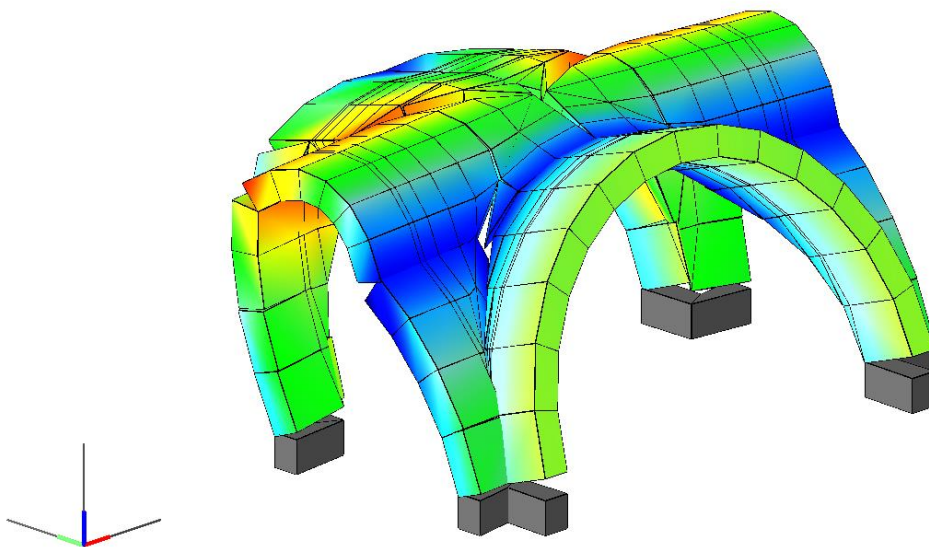
**Figure D39** – Plan view of deformation in -X direction.



**Figure D40** – Front view of deformation in -X direction.

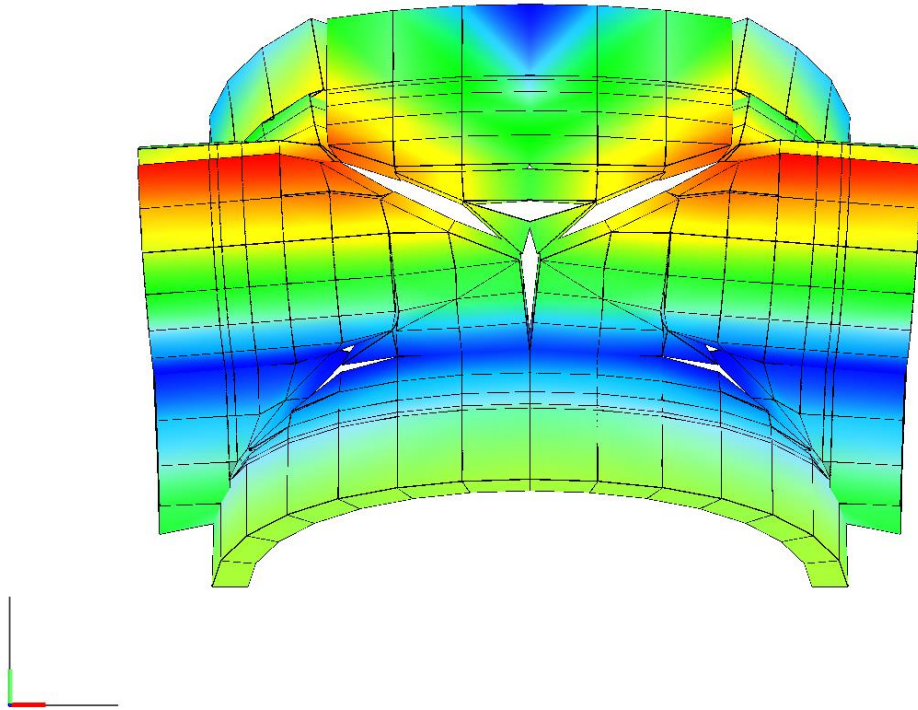


**Figure D41** – Legend for +Y direction. Taken at  $U = 0.04$  m.

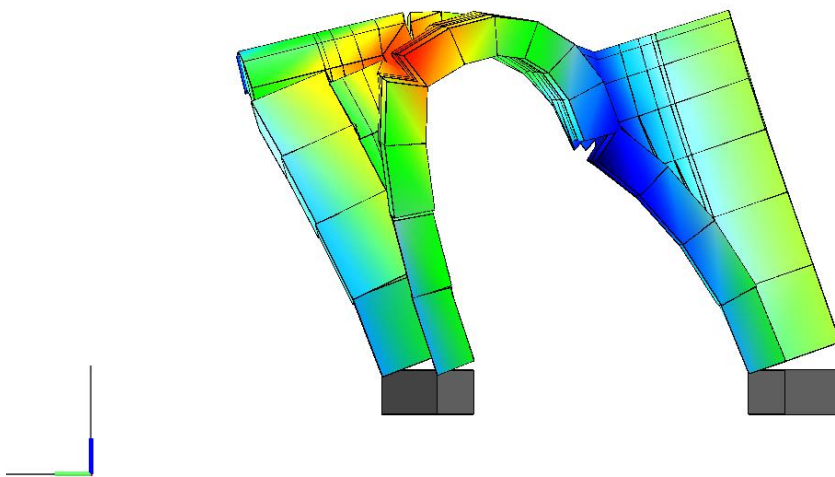


**Figure D42** – 3D view of deformation in +Y direction.

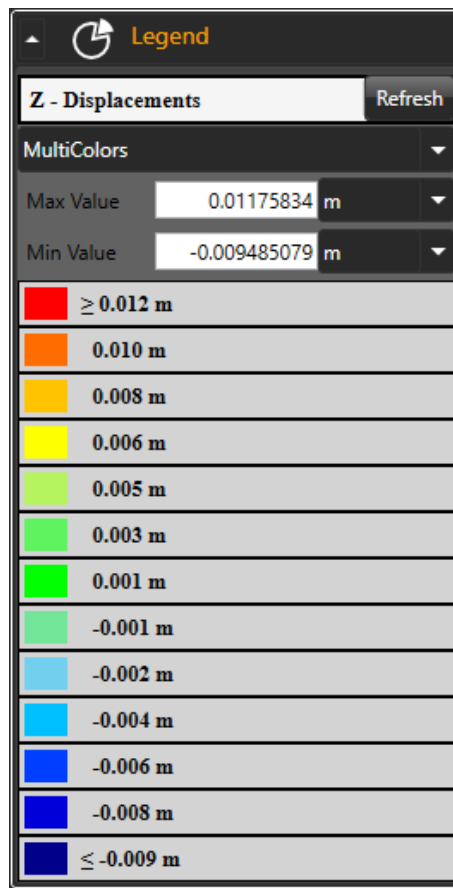




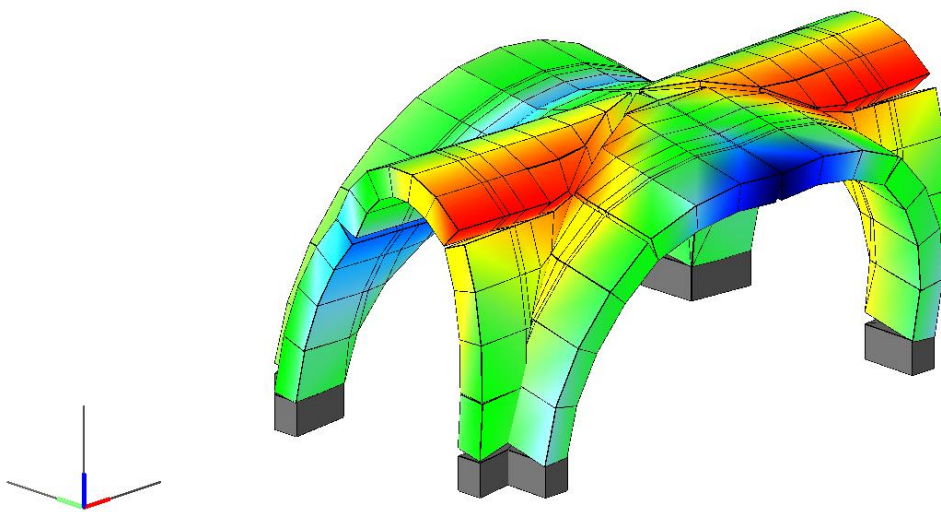
**Figure D43** – Plan view of deformation in +Y direction.



**Figure D44** – Side view of deformation in +Y direction.

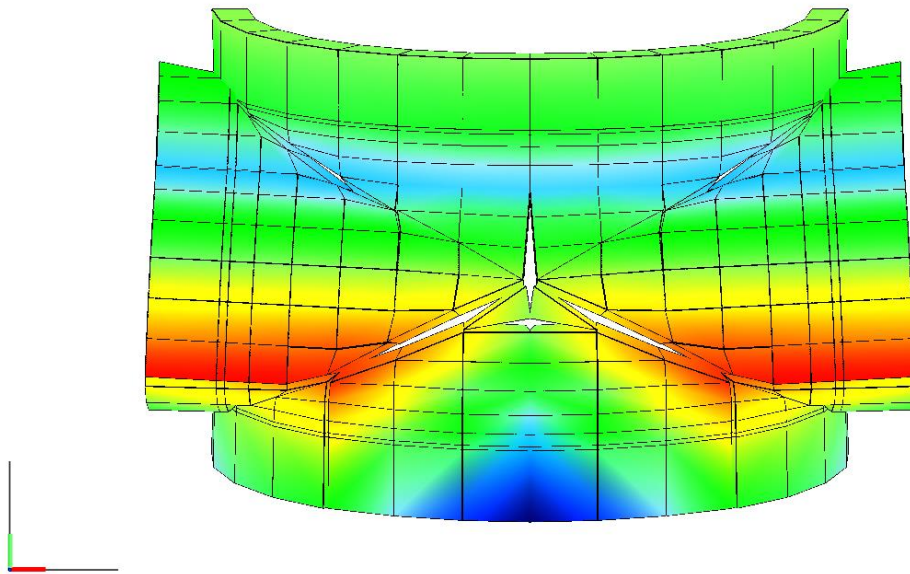


**Figure D45** – Legend for -Y direction. Taken at  $U = 0.02m$ .

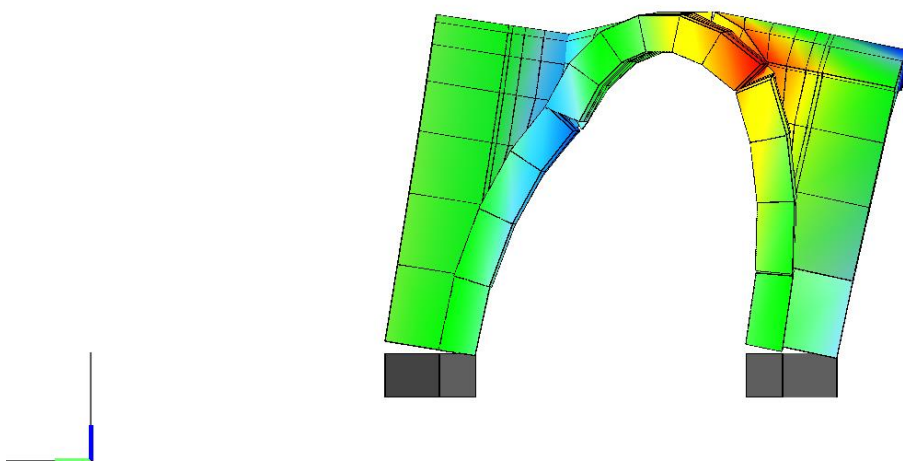


**Figure D46** – 3D view of deformation in -Y direction.





**Figure D47** – Plan view of deformation in -Y direction.



**Figure D48** – Side view of deformation in -Y direction.

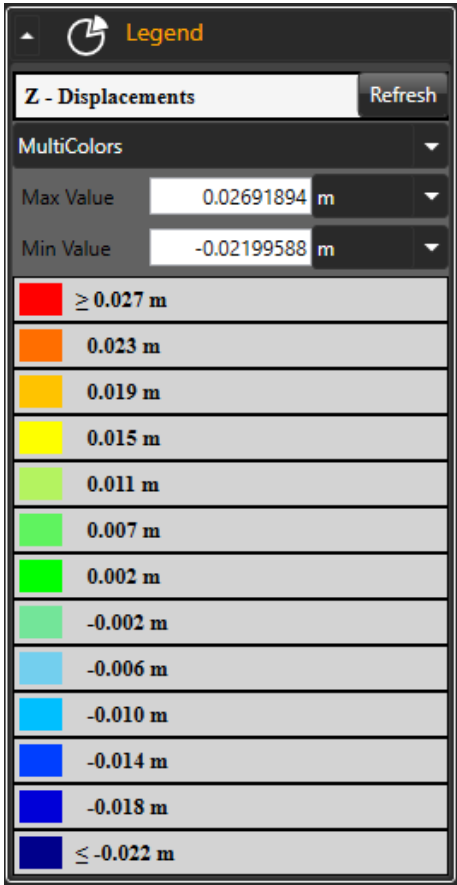


Figure D49 – Legend for +X direction. Taken at  $U = 0.03$  m.

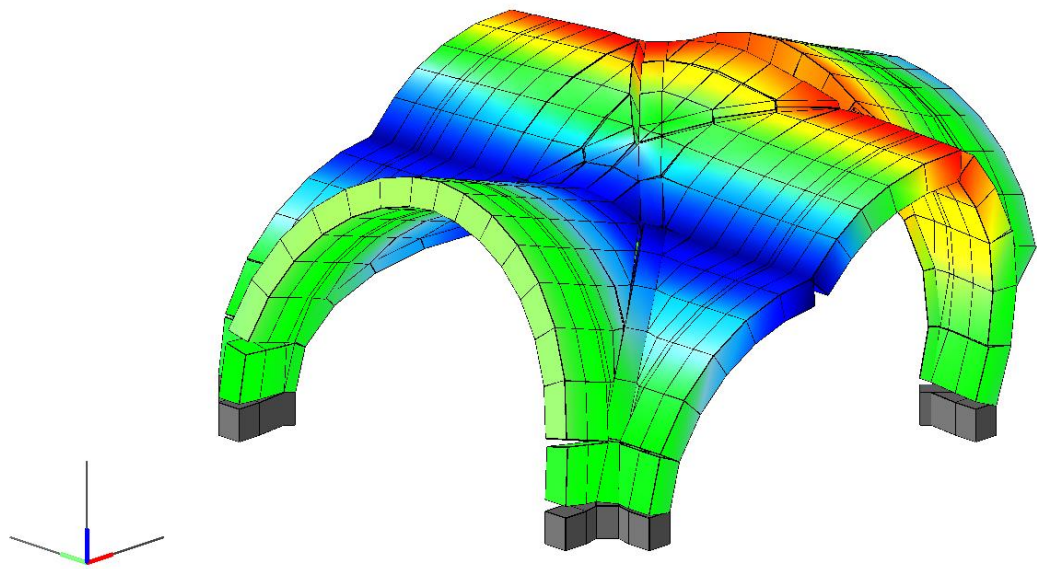
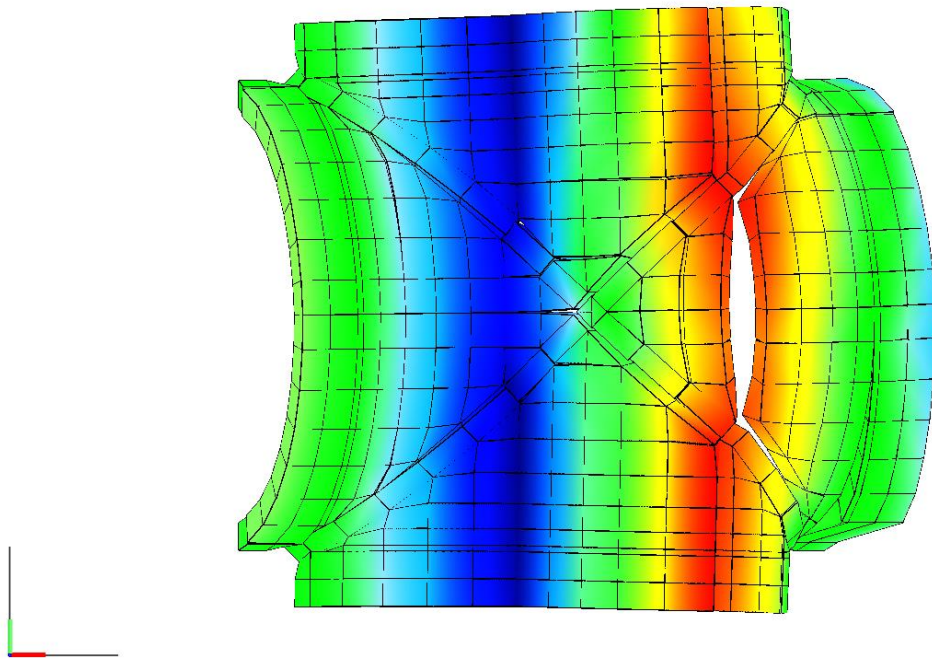
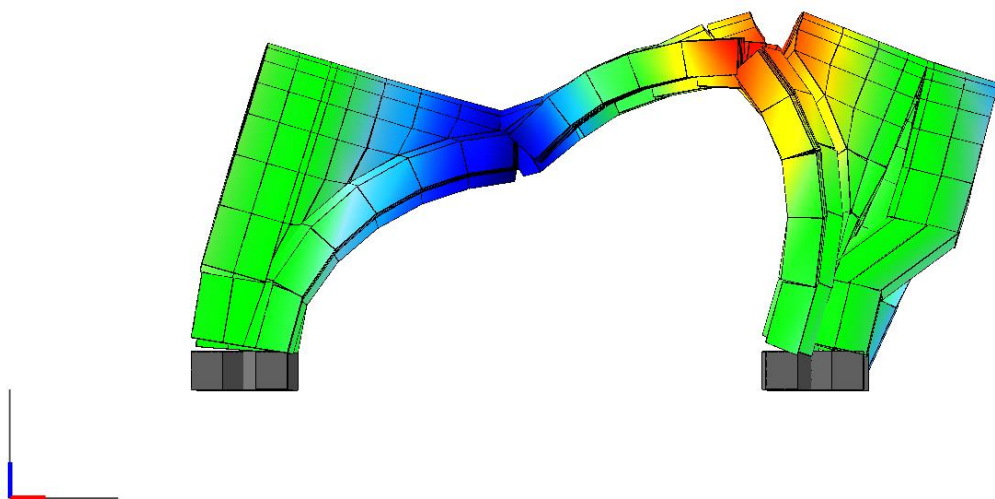


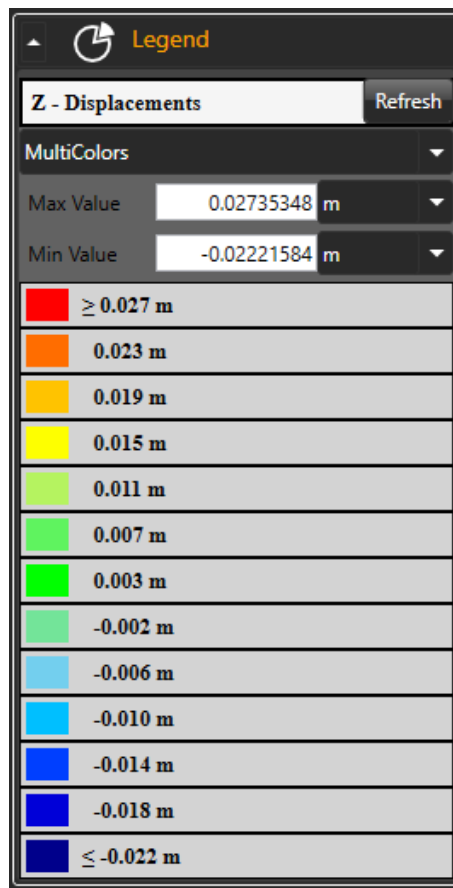
Figure D50 – 3D view of deformation in +X direction.



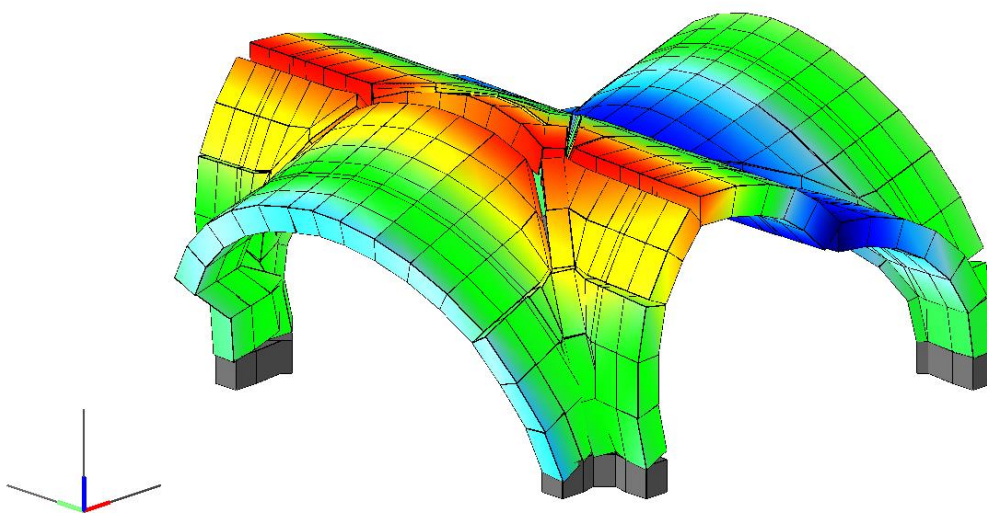
**Figure D51** – Plan view of deformation in +X direction.



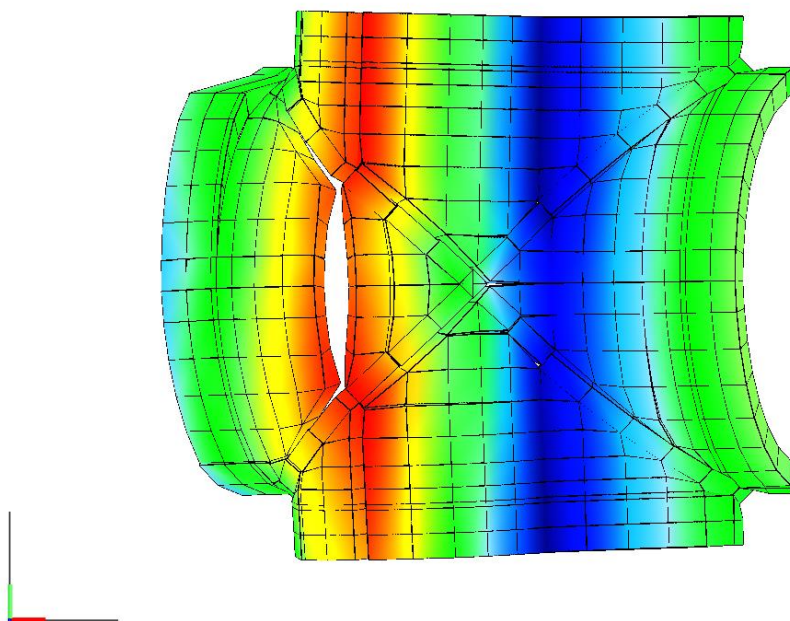
**Figure D52** – Front view of deformation in +X direction.



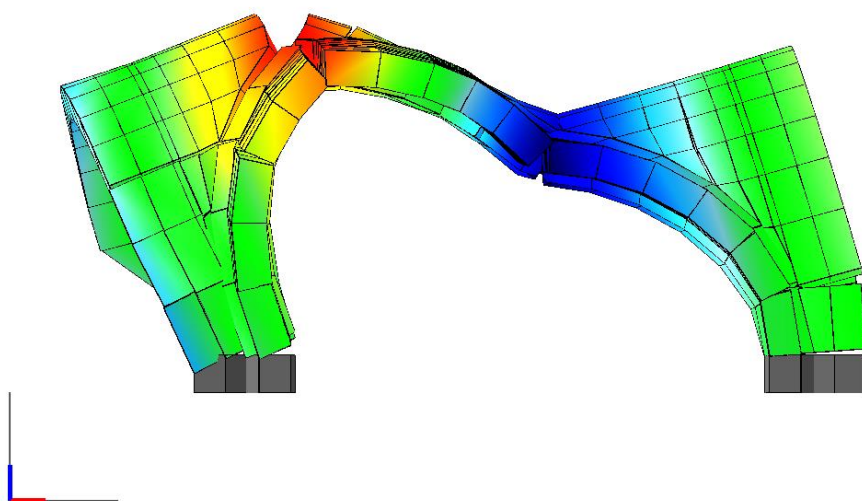
**Figure D53** – Legend for -X direction. Taken at  $U = 0.03\text{m}$ .



**Figure D54** – 3D view of deformation in -X direction.

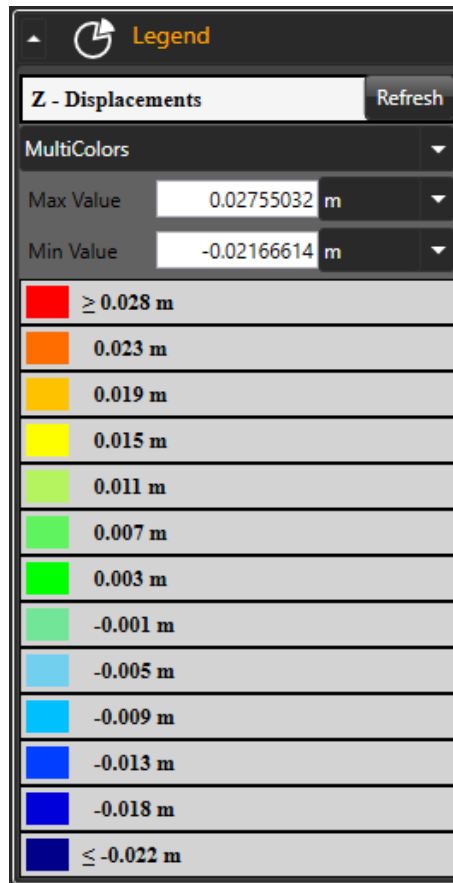


**Figure D55** – Plan view of deformation in -X direction.

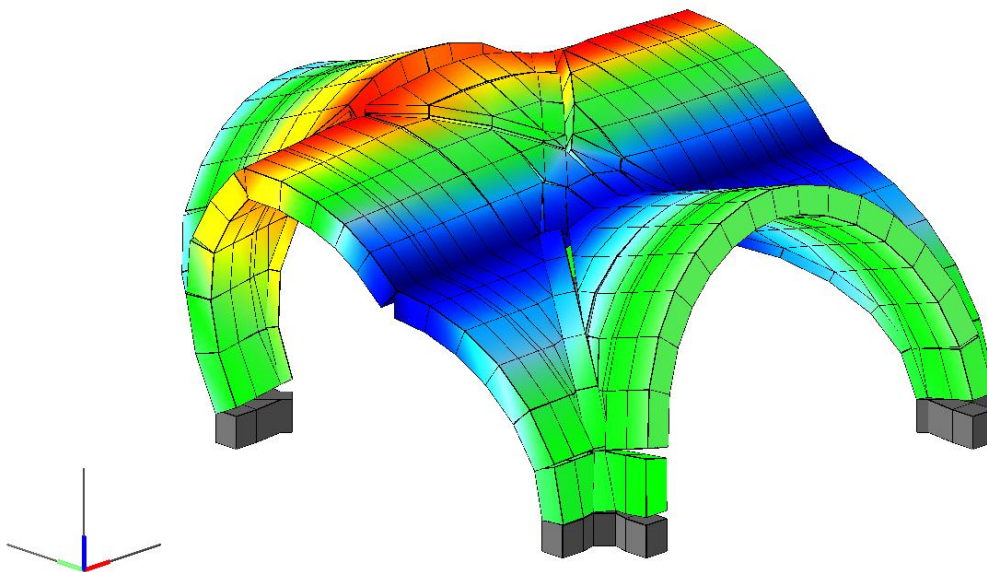


**Figure D56** – Front view of deformation in -X direction.

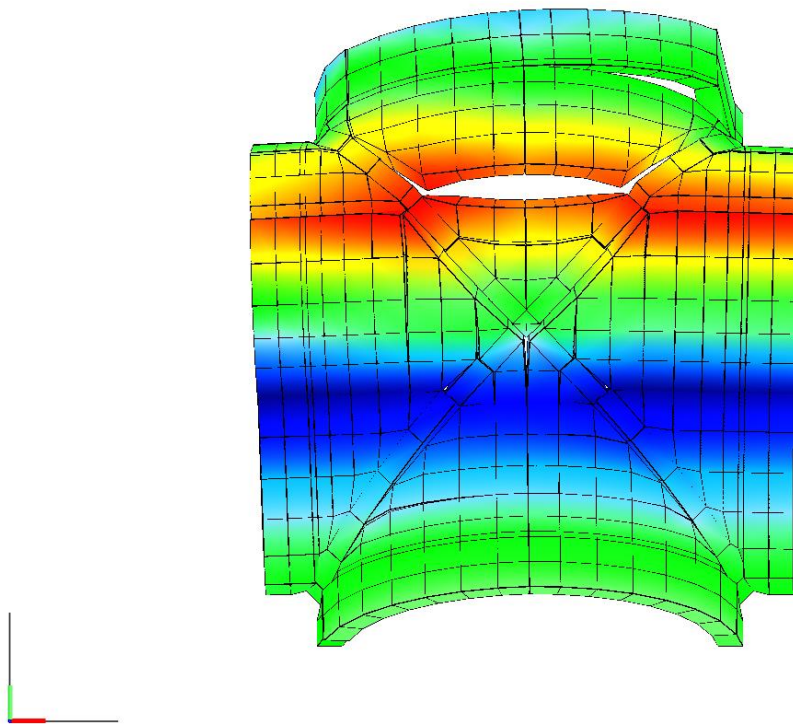




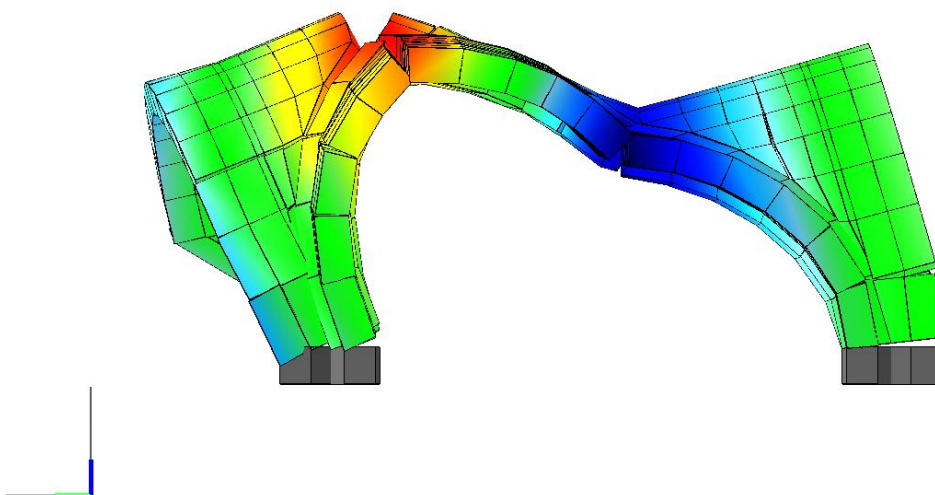
**Figure D57** – Legend for +Y direction. Taken at  $U = 0.03$  m.



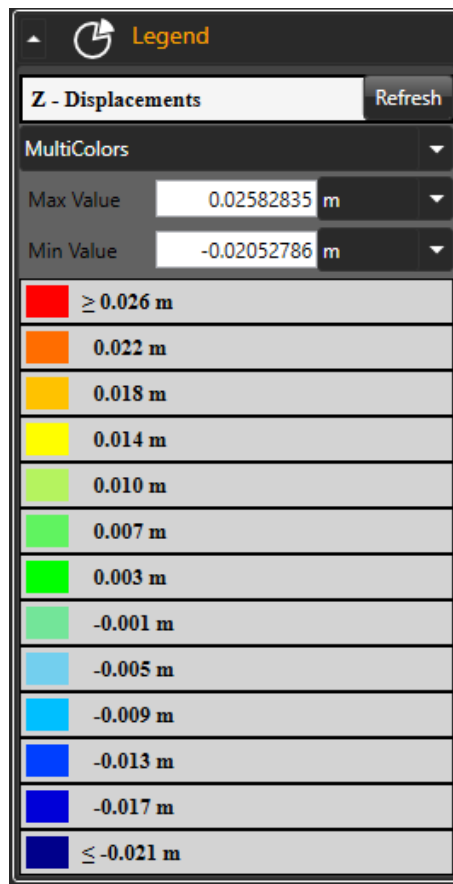
**Figure D58** – 3D view of deformation in +Y direction.



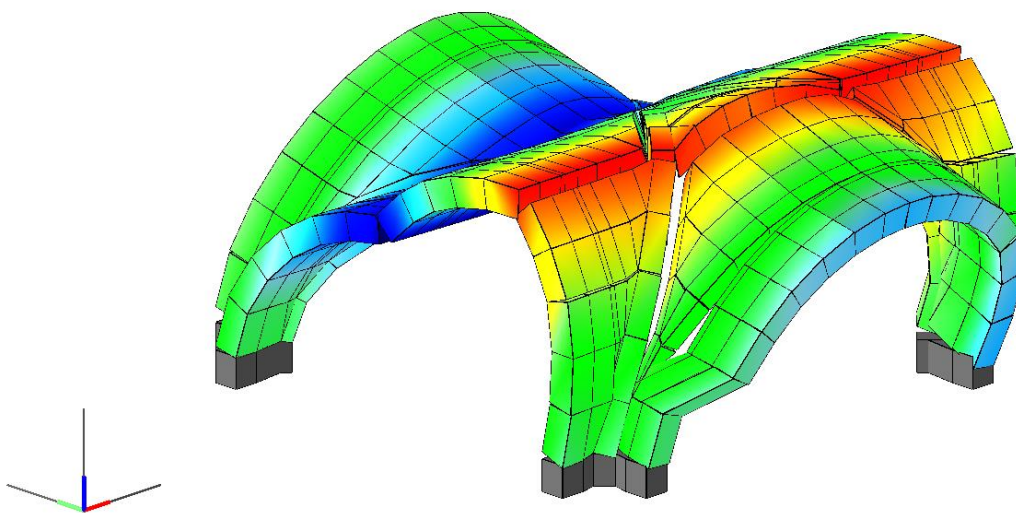
**Figure D59** – Plan view of deformation in +Y direction.



**Figure D60** – Side view of deformation in +Y direction.

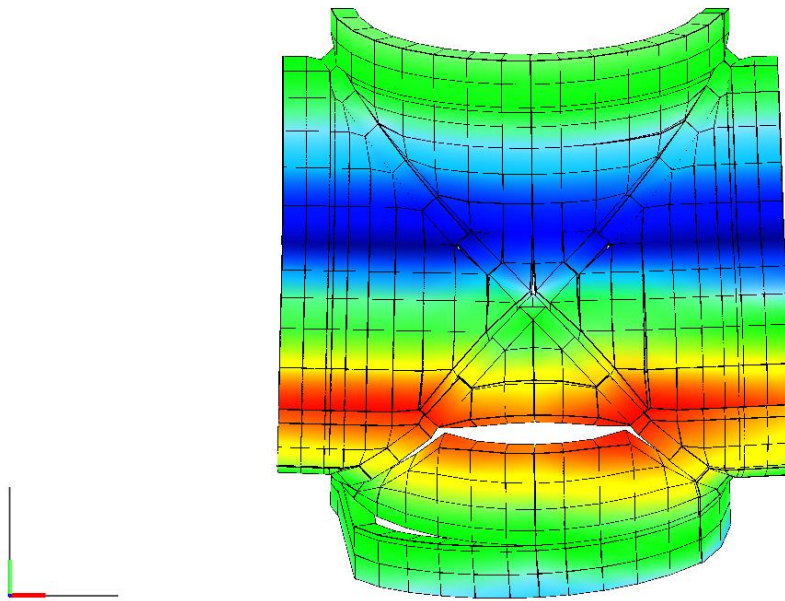


**Figure D61** – Legend for -Y direction. Taken at  $U = 0.03$ m.

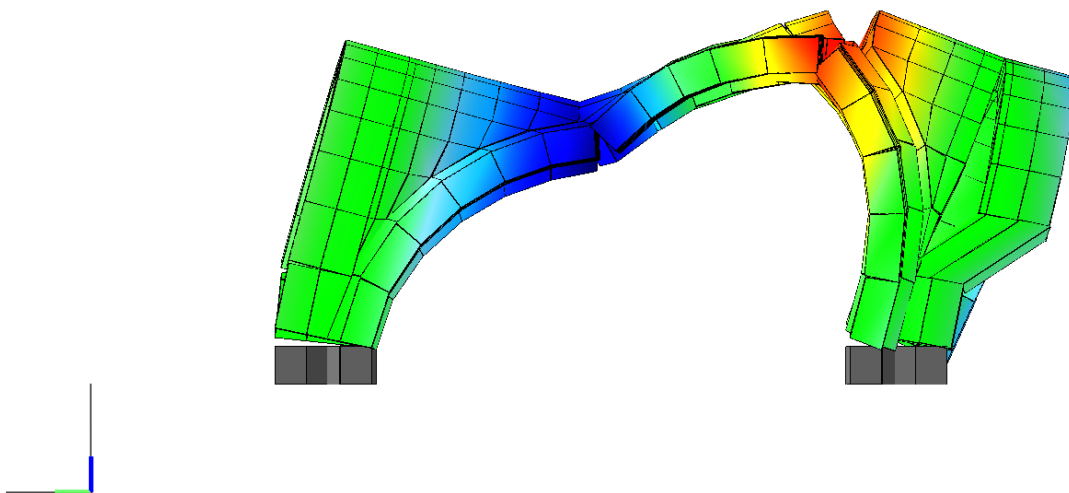


**Figure D62** – 3D view of deformation in -Y direction.





**Figure D63** – Plan view of deformation in -Y direction.



**Figure D64** – Side view of deformation in -Y direction.

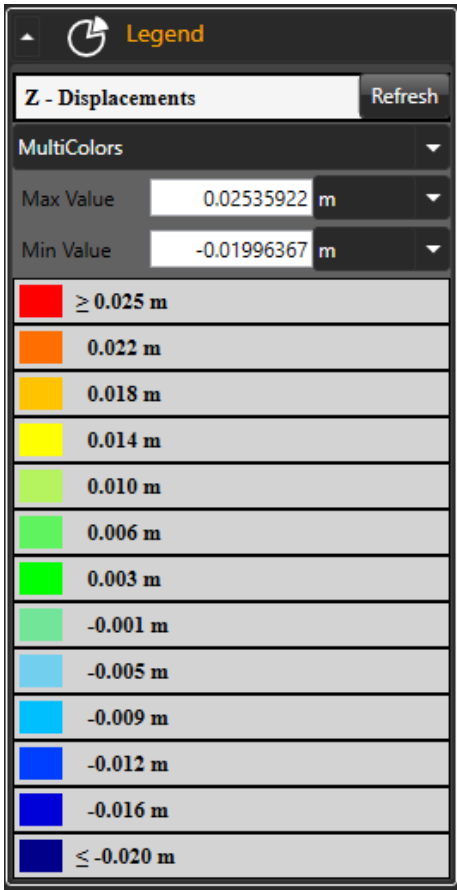


Figure D65 – Legend for +X direction. Taken at U = 0.03m.

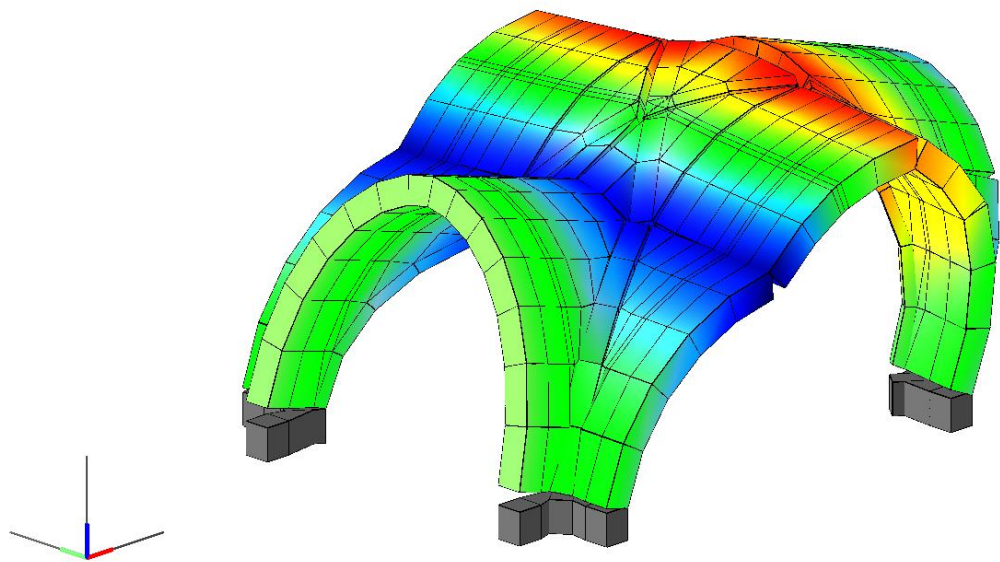
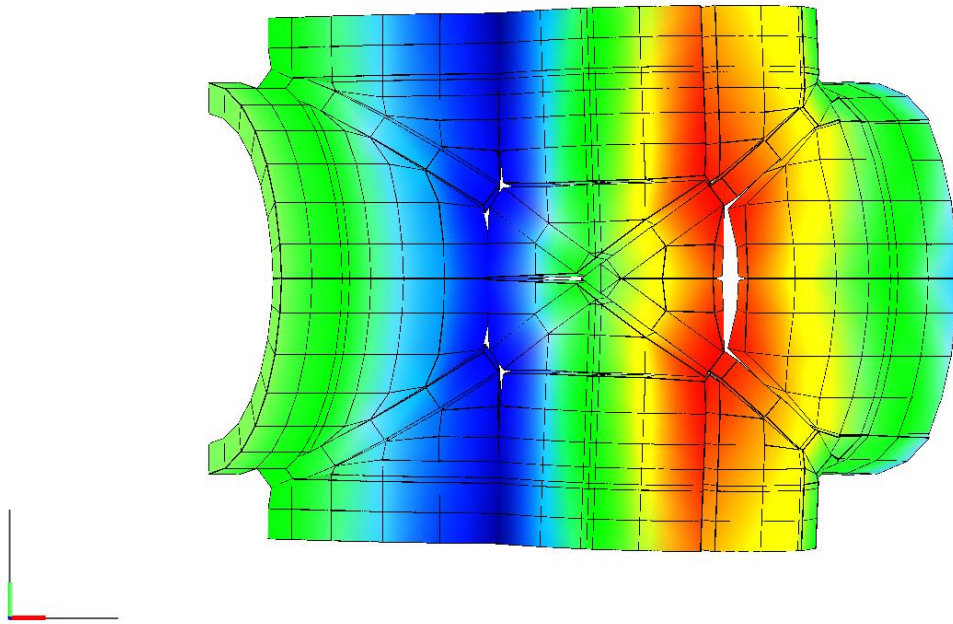
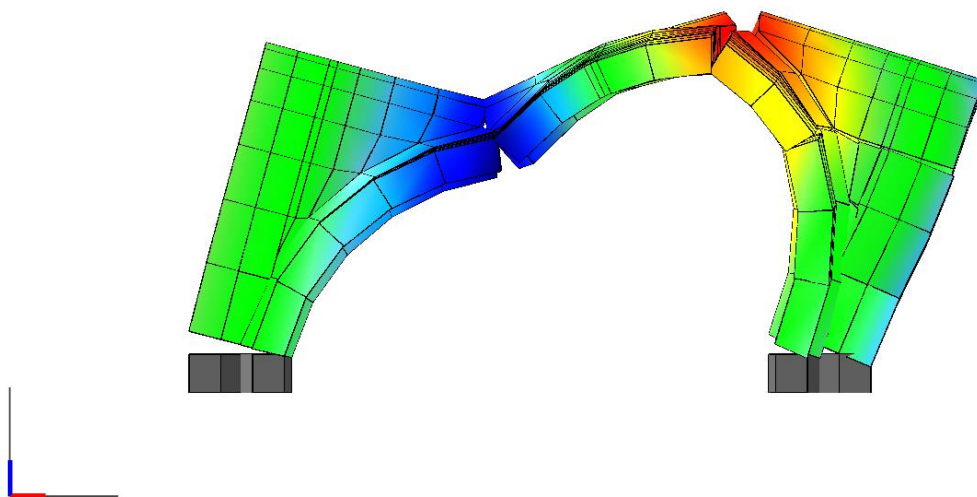


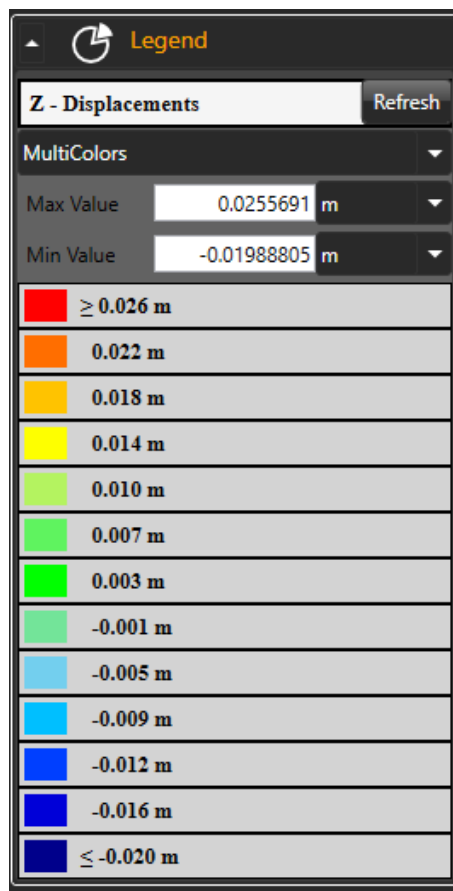
Figure D66 – 3D view of deformation in +X direction.



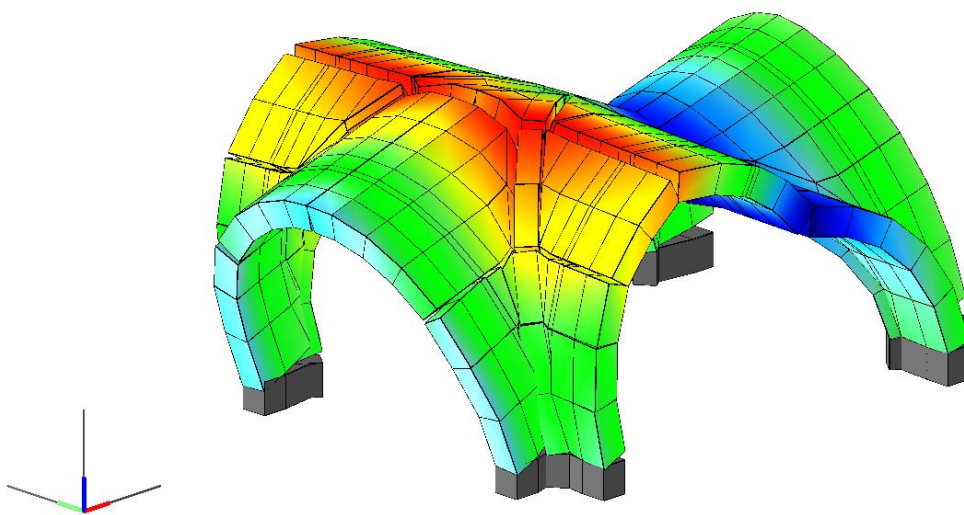
**Figure D67** – Plan view of deformation in +X direction.



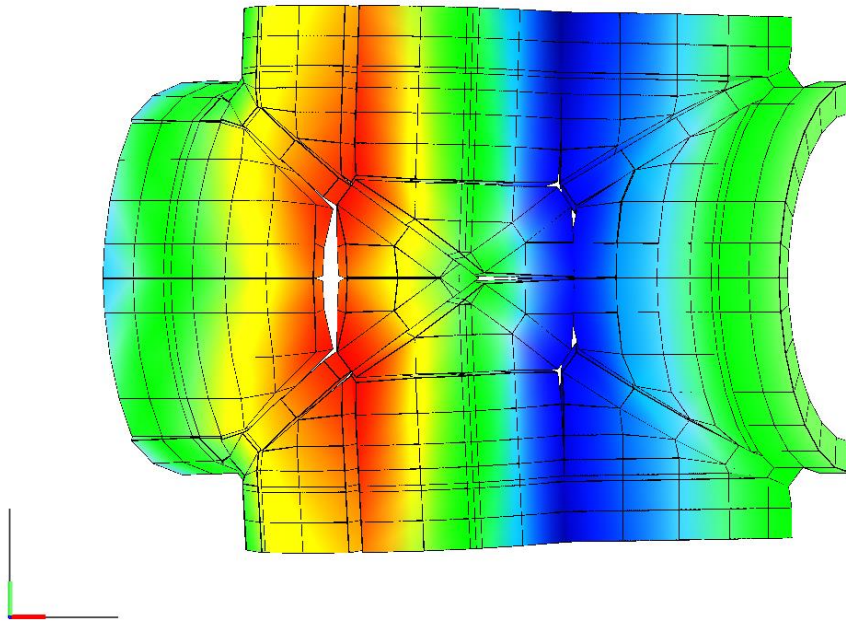
**Figure D68** – Front view of deformation in +X direction.



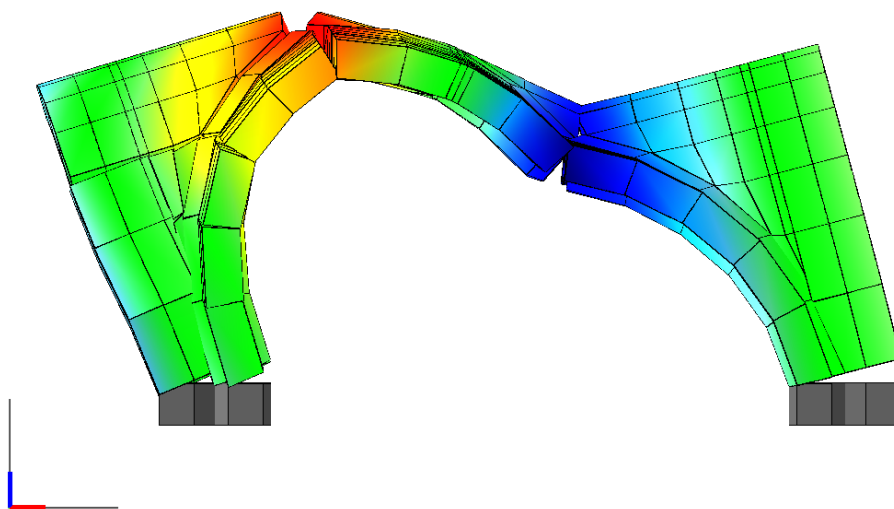
**Figure D69** – Legend for -X direction. Taken at  $U = 0.03$  m.



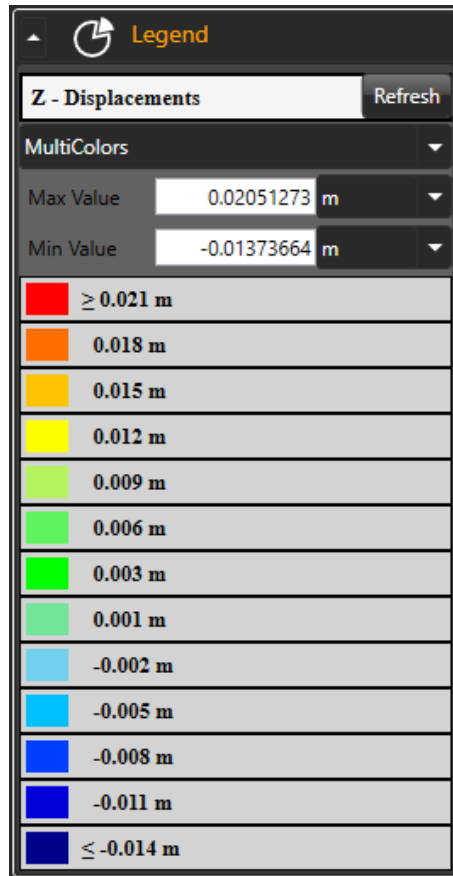
**Figure D70** – 3D view of deformation in -X direction.



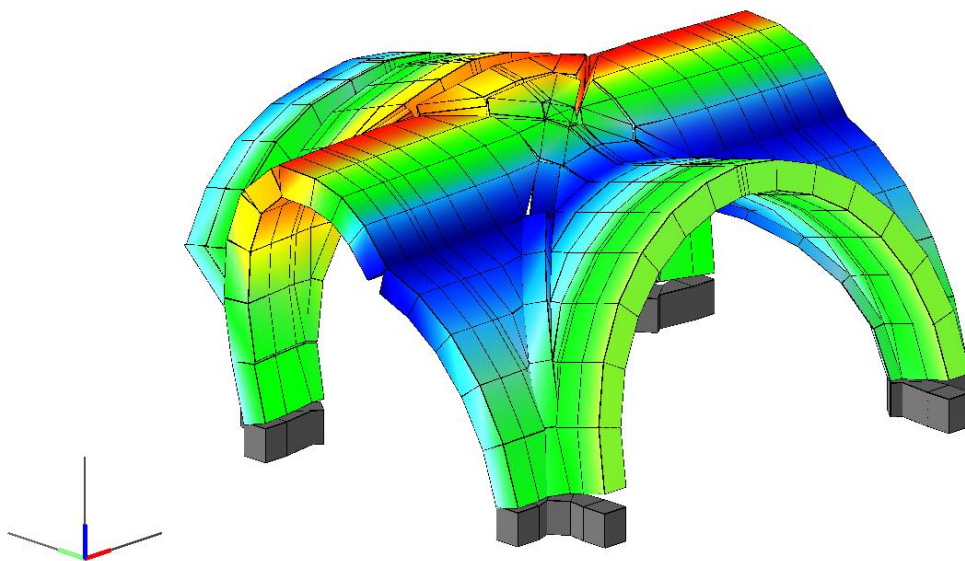
**Figure D71** – Plan view of deformation in -X direction.



**Figure D72** – Front view of deformation in -X direction.

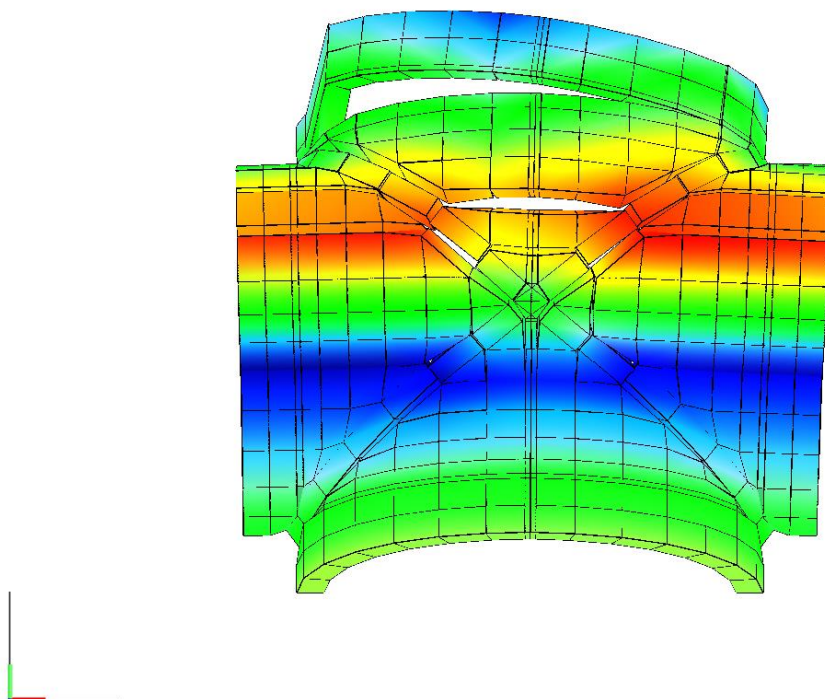


**Figure D73** – Legend for +Y direction. Taken at  $U = 0.03m$ .

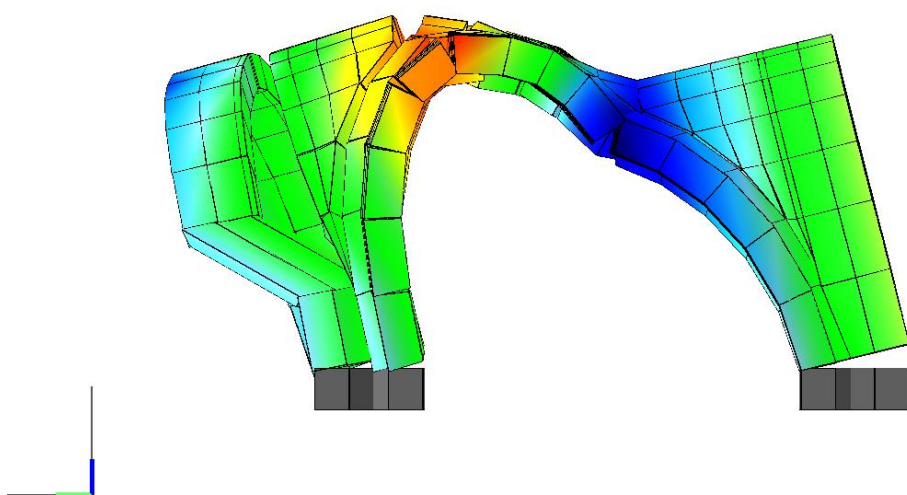


**Figure D74** – 3D view of deformation in +Y direction.

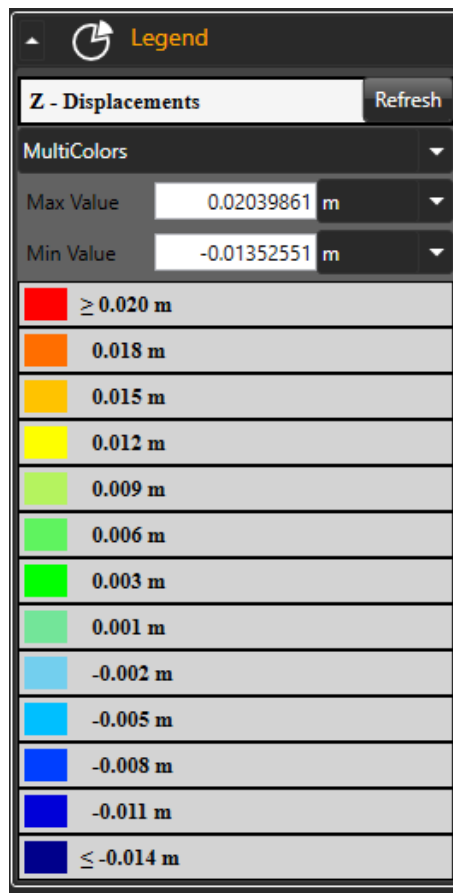




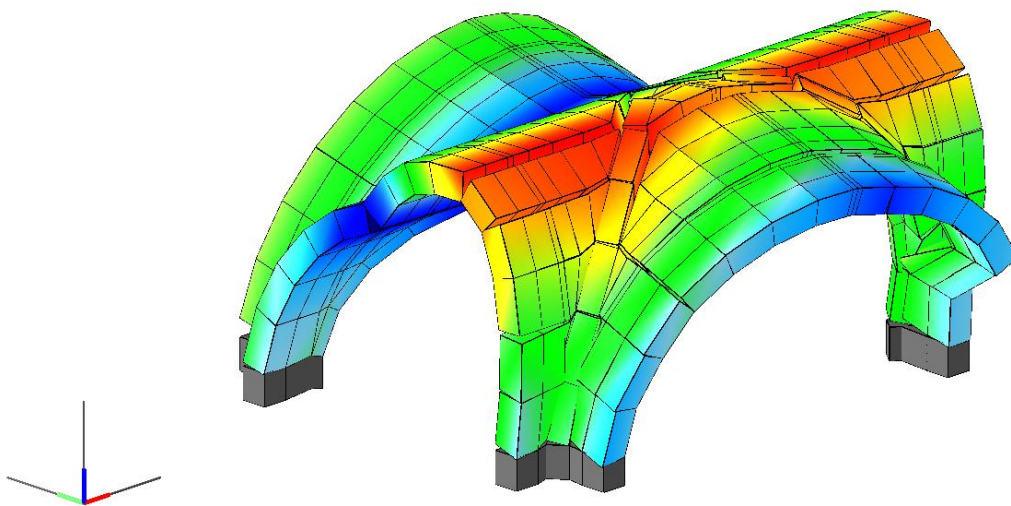
**Figure D75** – Plan view of deformation in +Y direction.



**Figure D76** – Side view of deformation in +Y direction.

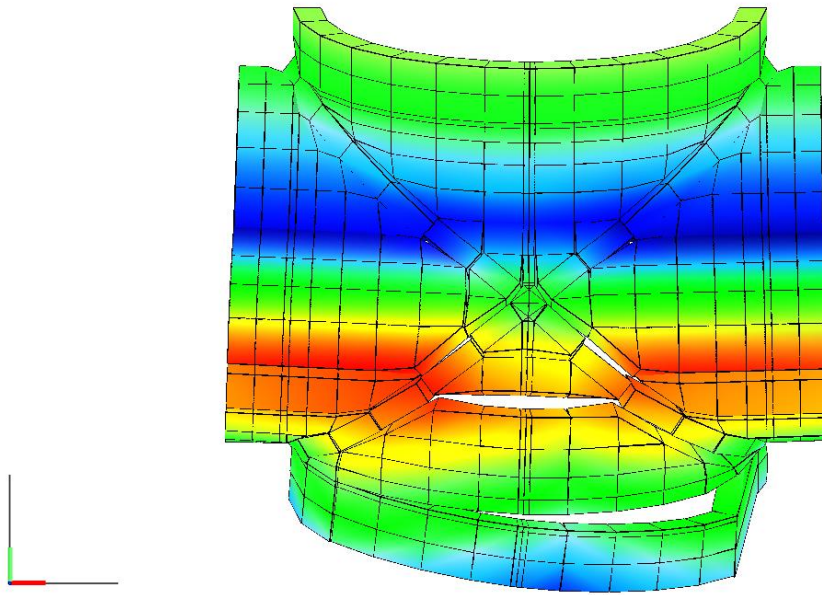


**Figure D77** – Legend for -Y direction. Taken at  $U = 0.03$  m.

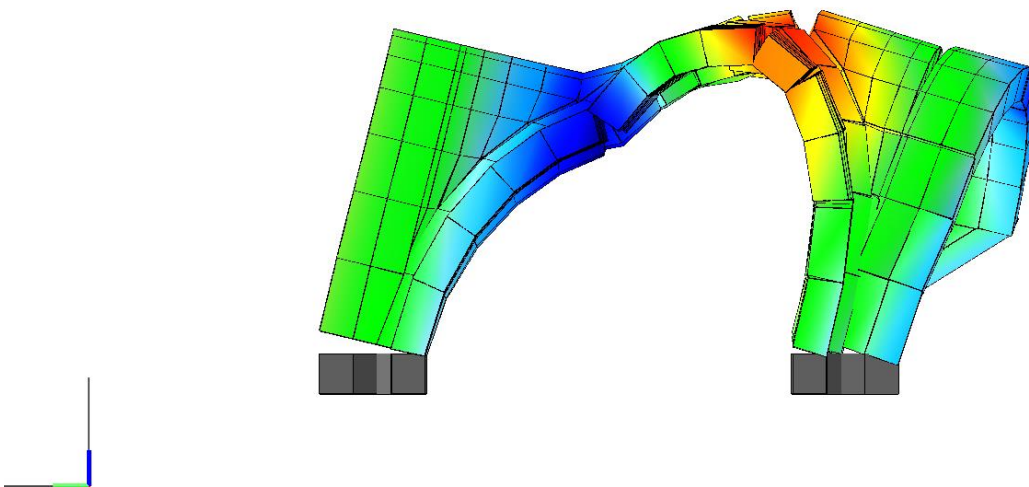


**Figure D78** – 3D view of deformation in -Y direction.





**Figure D79** – Plan view of deformation in -Y direction.



**Figure D80** – Side view of deformation in -Y direction.

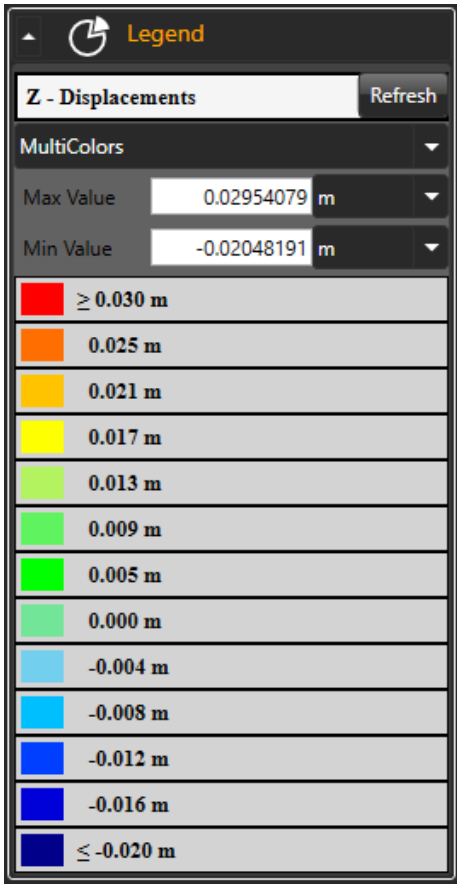


Figure D81 – Legend for +X direction. Taken at  $U = 0.03m$ .

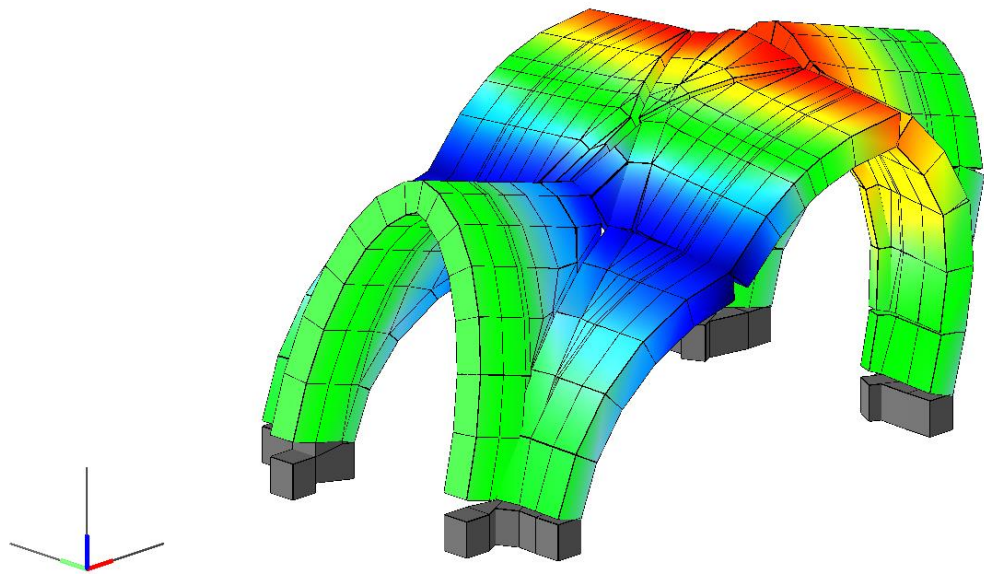
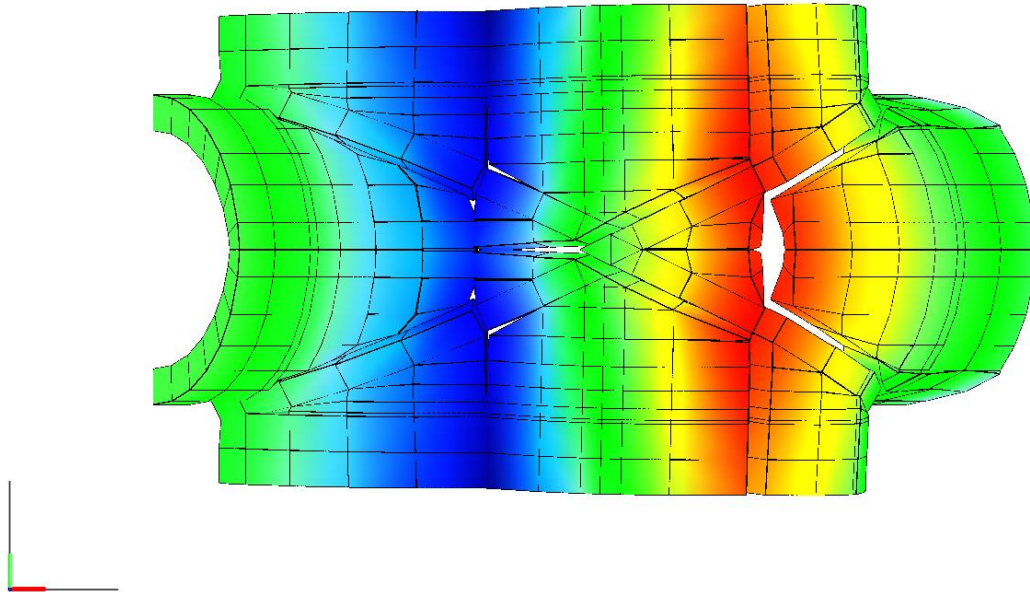
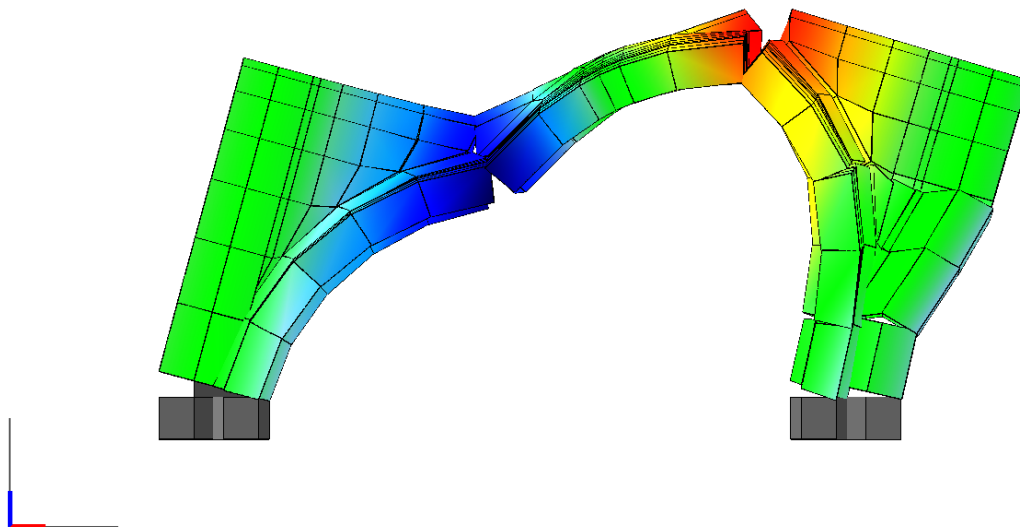


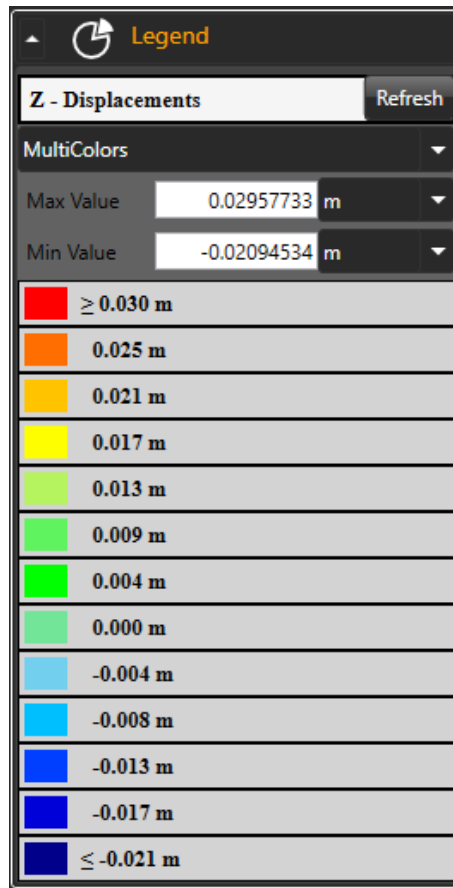
Figure D82 – 3D view of deformation in +X direction.



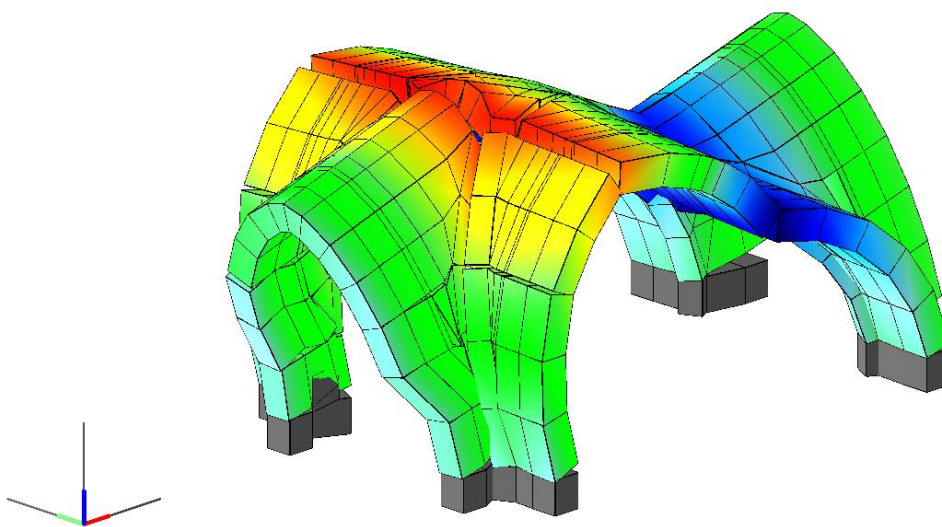
**Figure D83** – Plan view of deformation in +X direction.



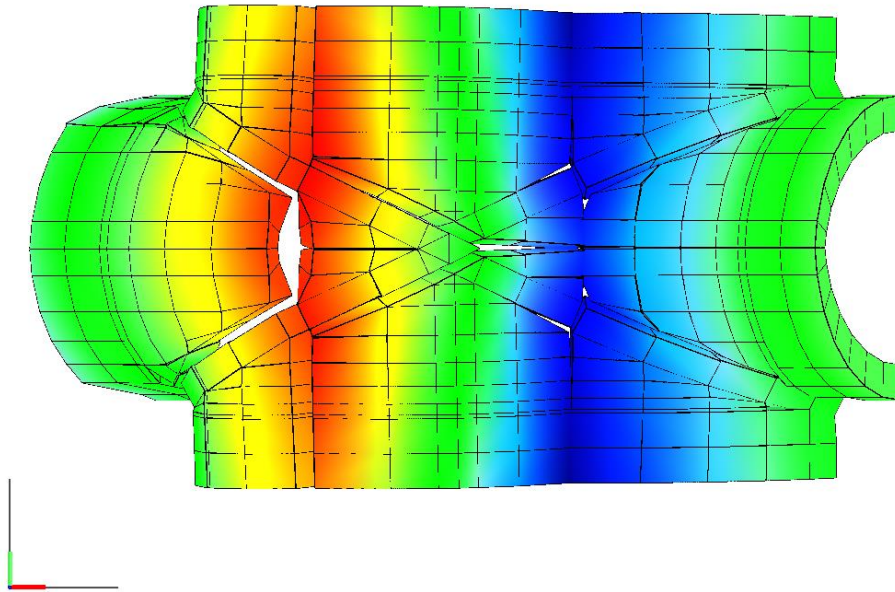
**Figure D84** – Front view of deformation in +X direction.



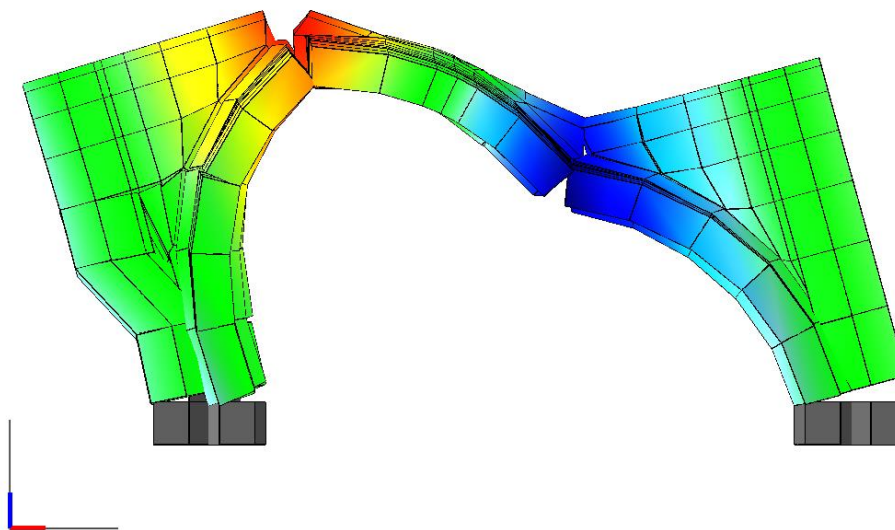
**Figure D85** – Legend for -X direction. Taken at  $U = 0.03$  m.



**Figure D86** – 3D view of deformation in -X direction.

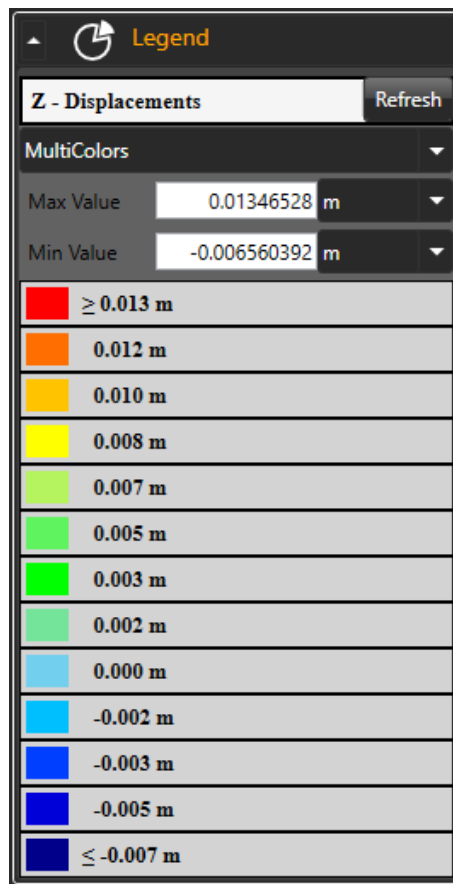


**Figure D87** – Plan view of deformation in -X direction.

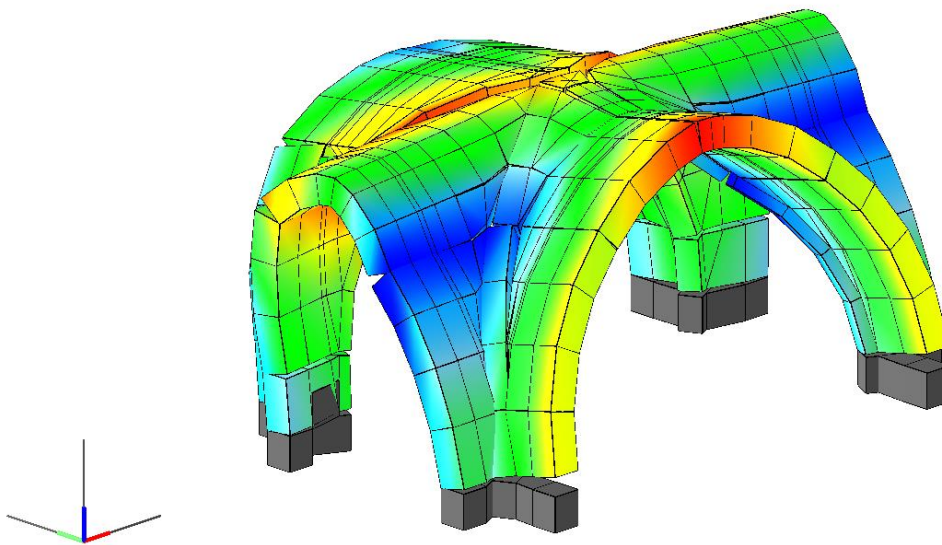


**Figure D88** – Front view of deformation in -X direction.

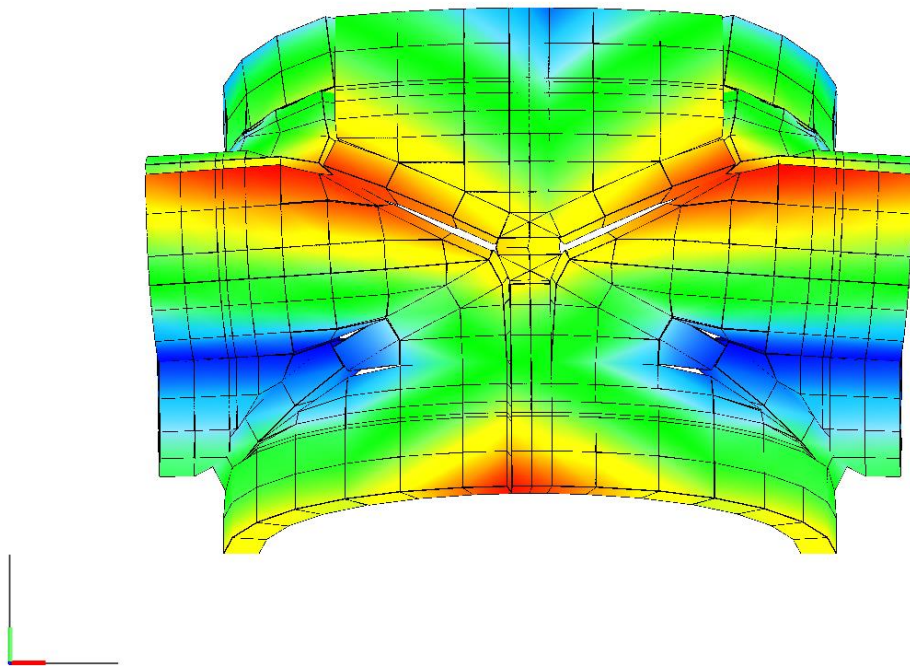




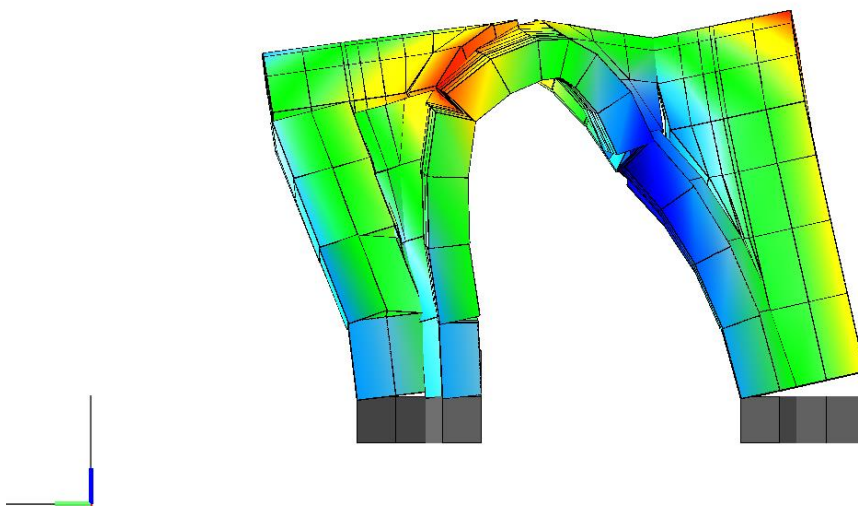
**Figure D89** – Legend for +Y direction. Taken at  $U = 0.03\text{m}$ .



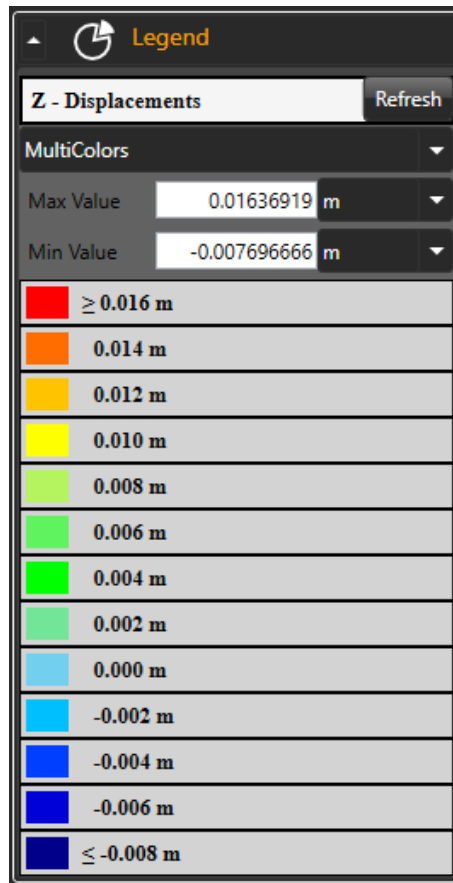
**Figure D90** – 3D view of deformation in +Y direction.



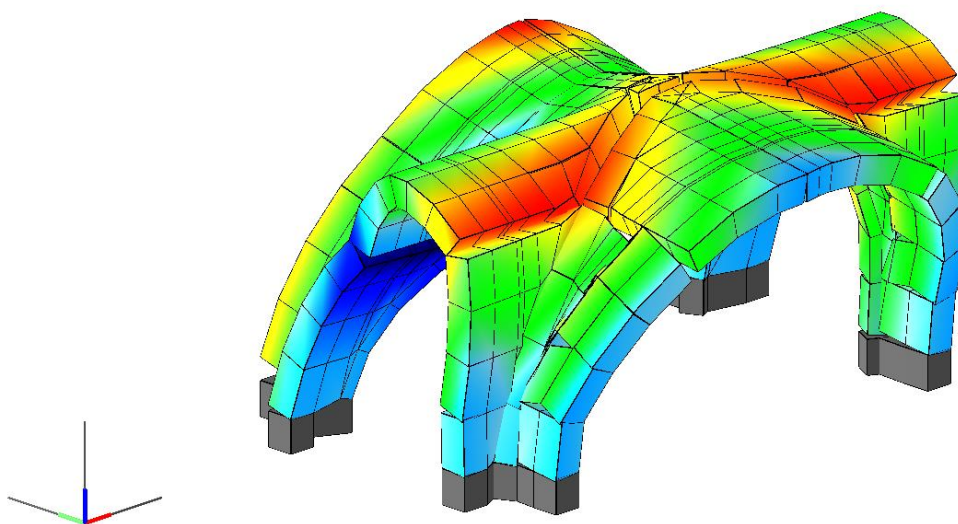
**Figure D91** – Plan view of deformation in +Y direction.



**Figure D92** – Side view of deformation in +Y direction.

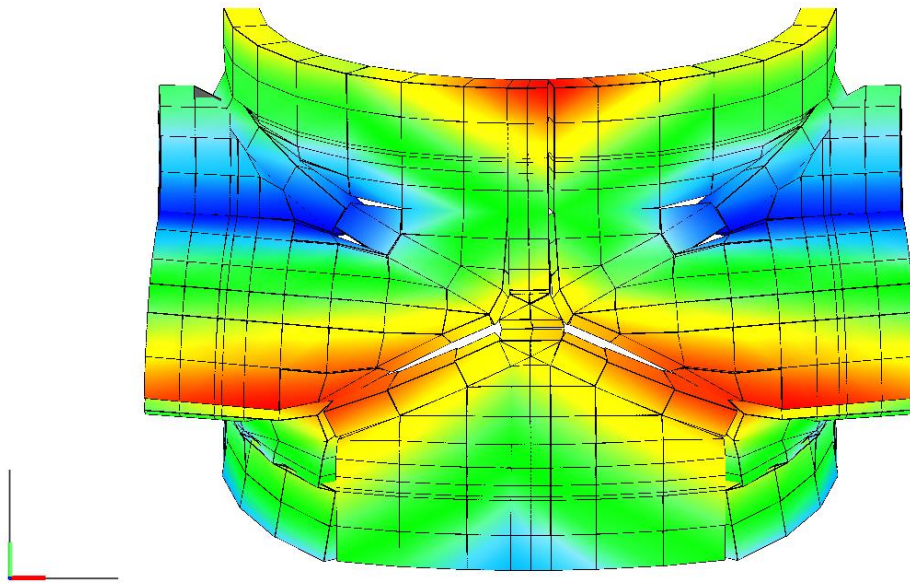


**Figure D93** – Legend for -Y direction. Taken at  $U = 0.03$  m.

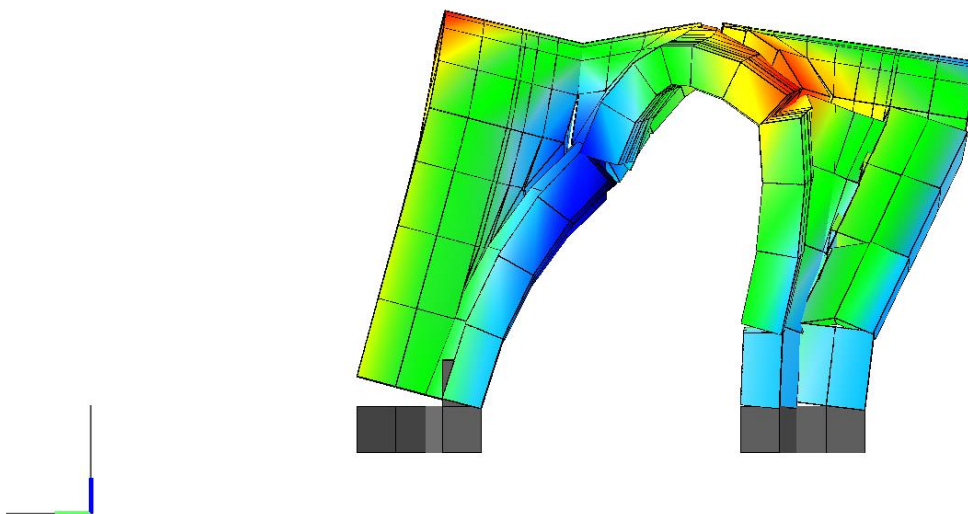


**Figure D94** – 3D view of deformation in -Y direction.





**Figure D95** – Plan view of deformation in -Y direction.



**Figure D96** – Side view of deformation in -Y direction.

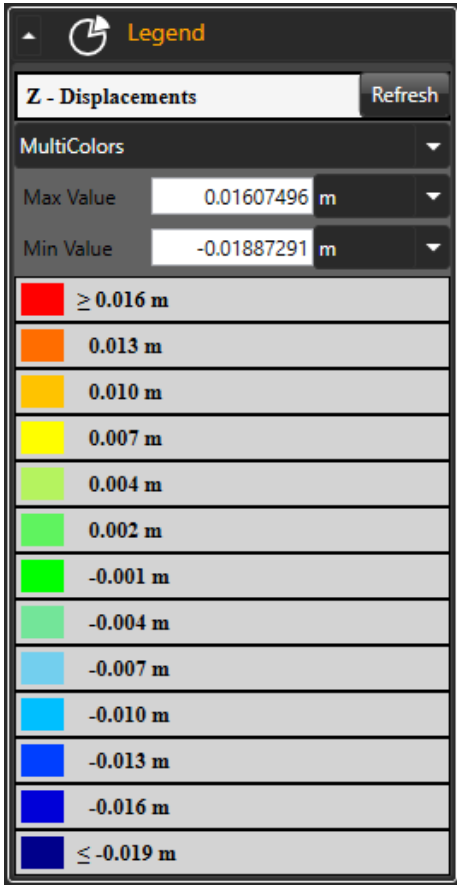


Figure D97 – Legend for +X direction. Taken at U = 0.021m.

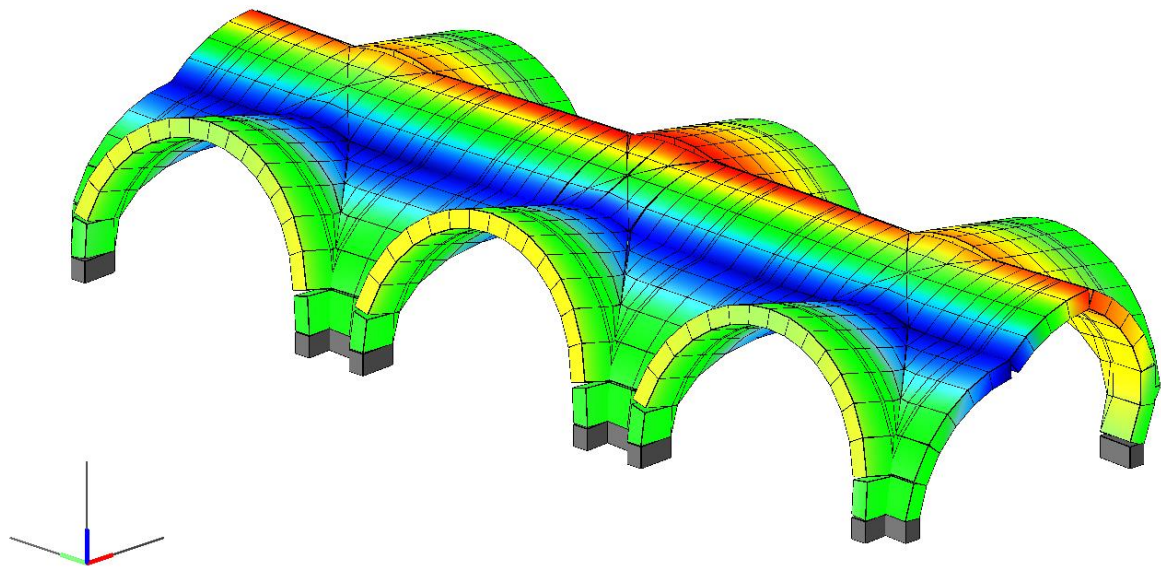
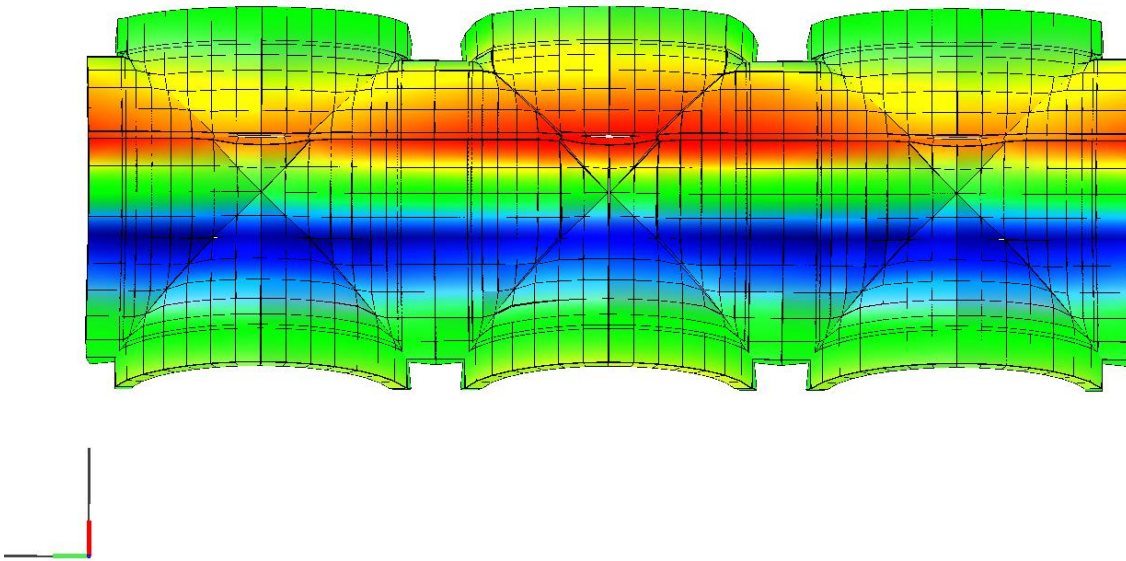
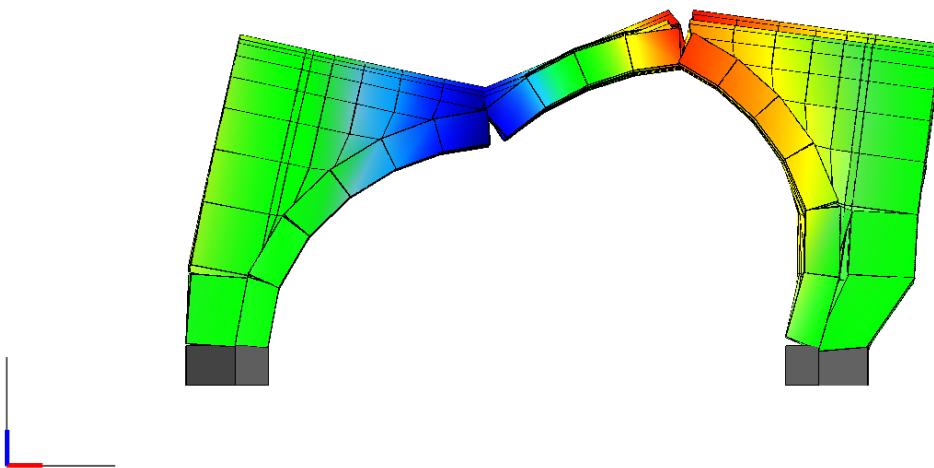


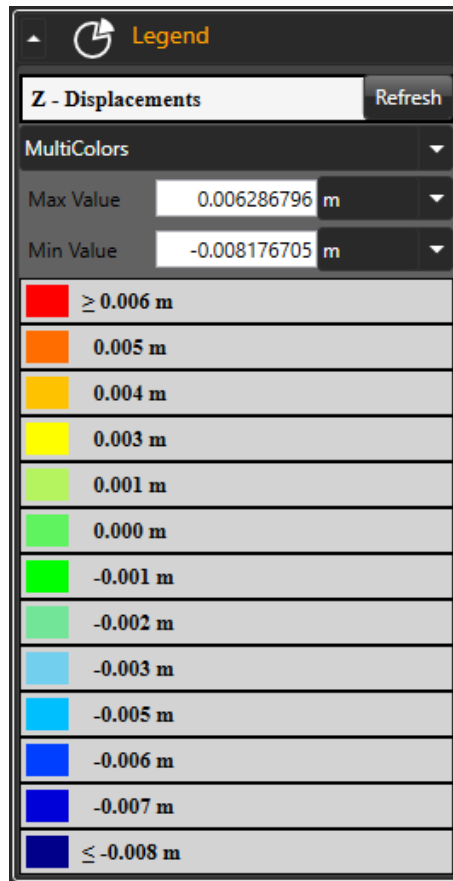
Figure D98 – 3D view of deformation in +X direction.



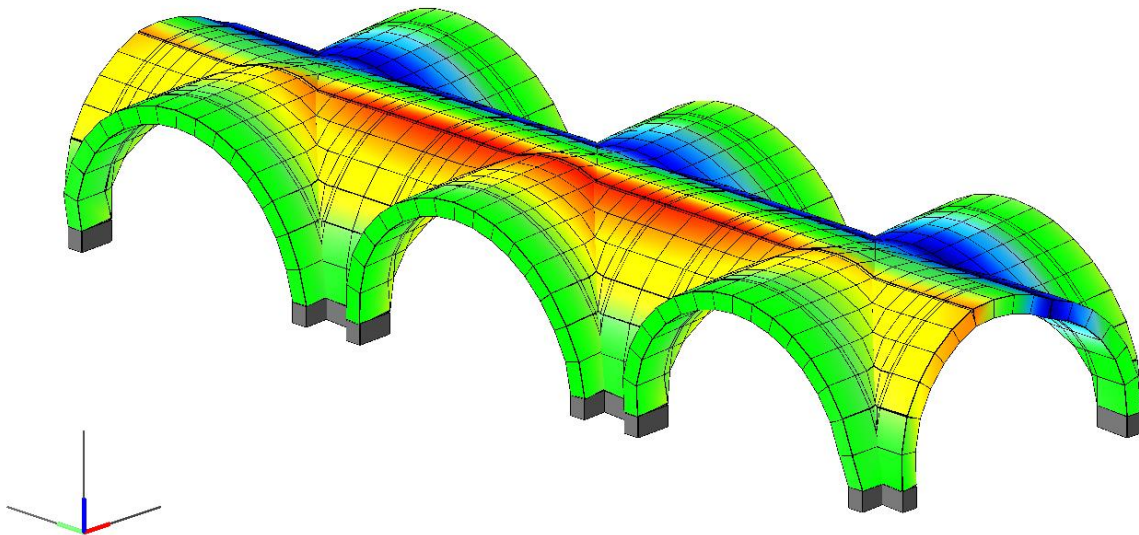
**Figure D99** – Plan view of deformation in +X direction.



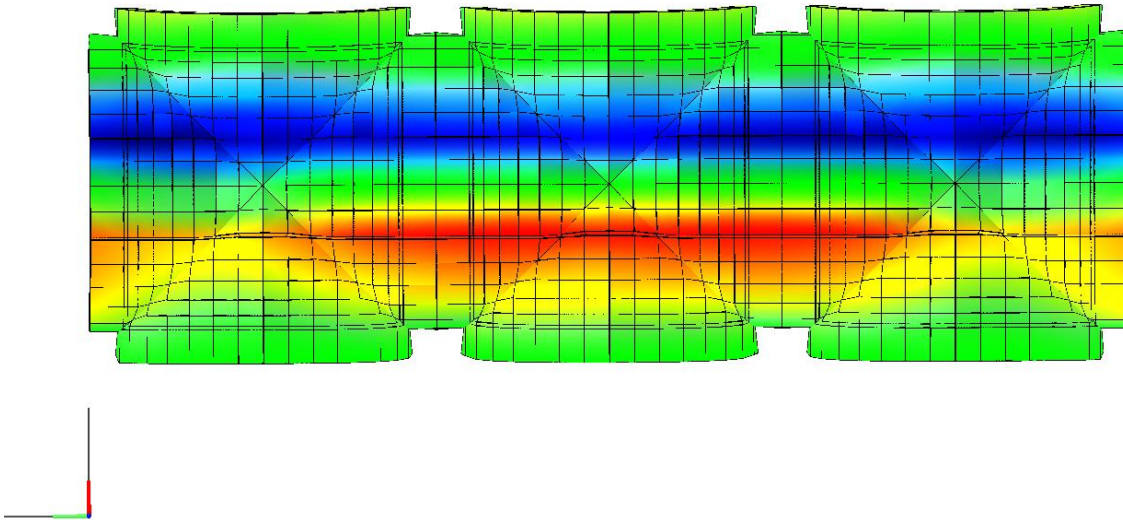
**Figure D100** – Front view of deformation in +X direction.



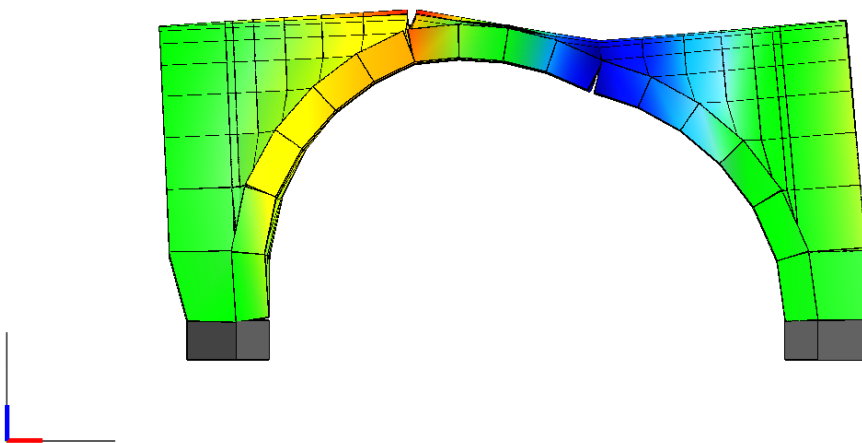
**Figure D101** – Legend for -X direction. Taken at  $U = 0.008$  m.



**Figure D102** – 3D view of deformation in -X direction.

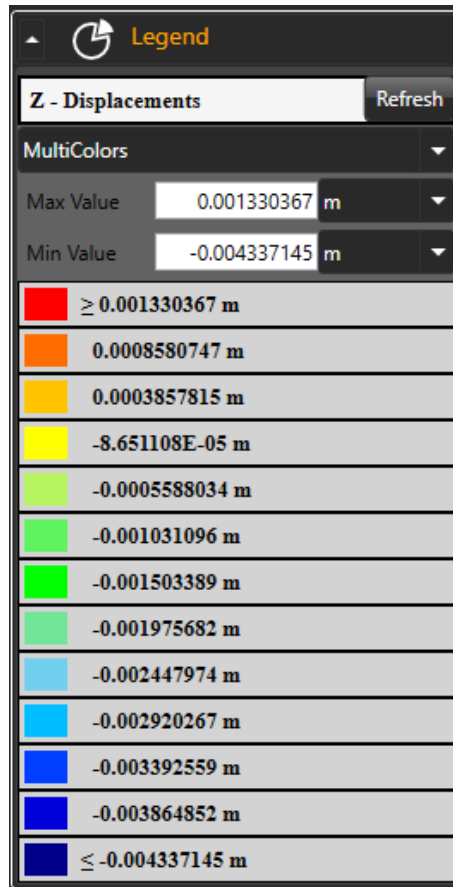


**Figure D103** – Plan view of deformation in -X direction.

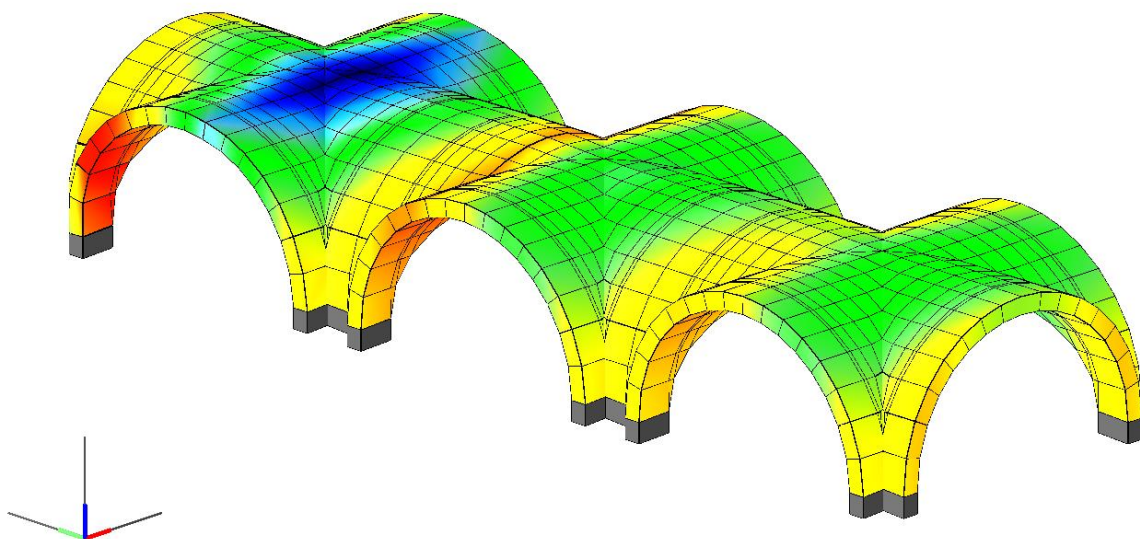


**Figure D104** – Front view of deformation in -X direction.

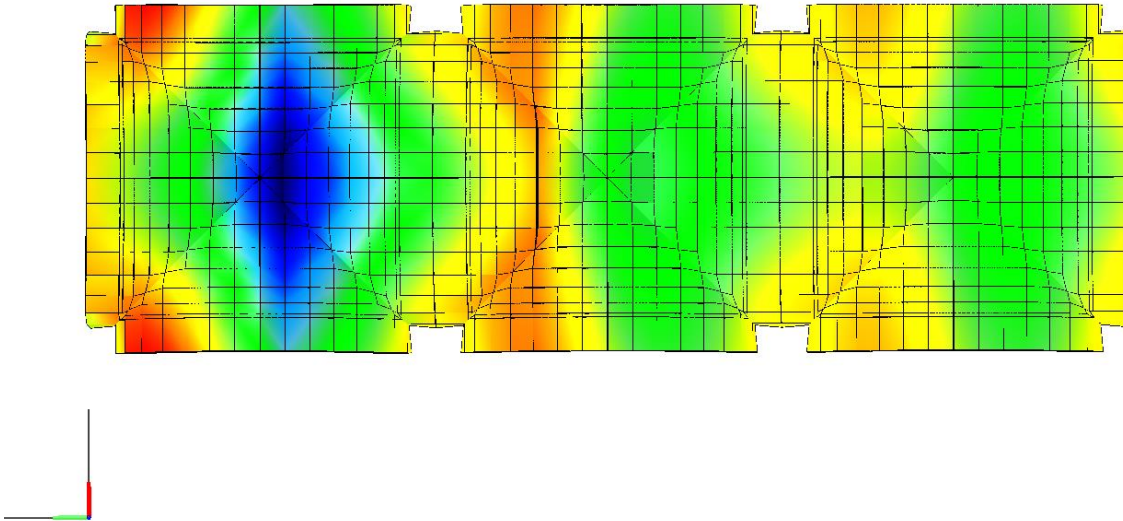




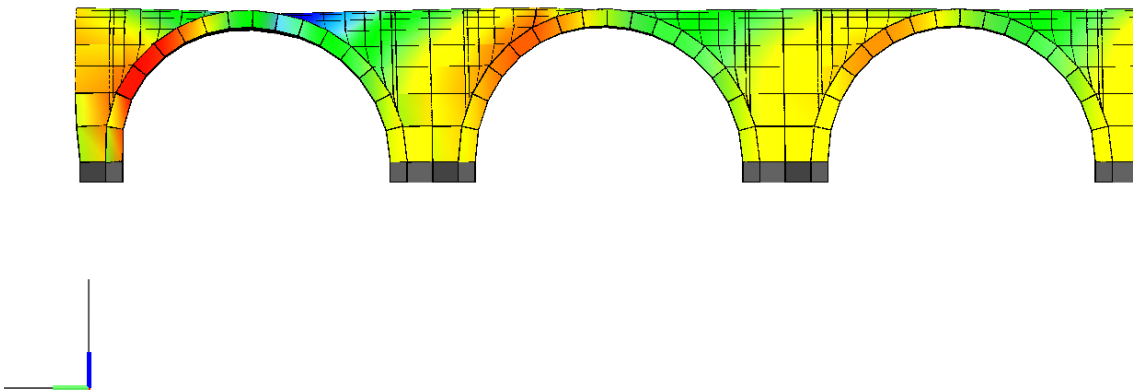
**Figure D105** – Legend for +Y direction. Taken at  $U = 0.002$  m.



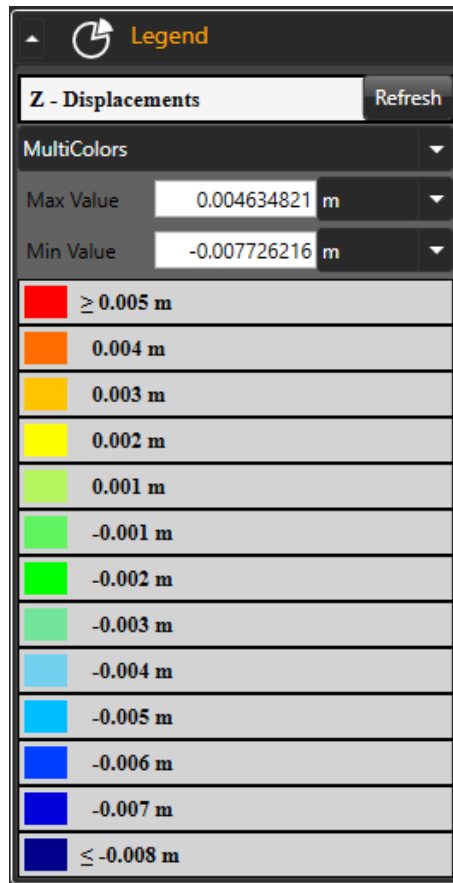
**Figure D106** – 3D view of deformation in +Y direction.



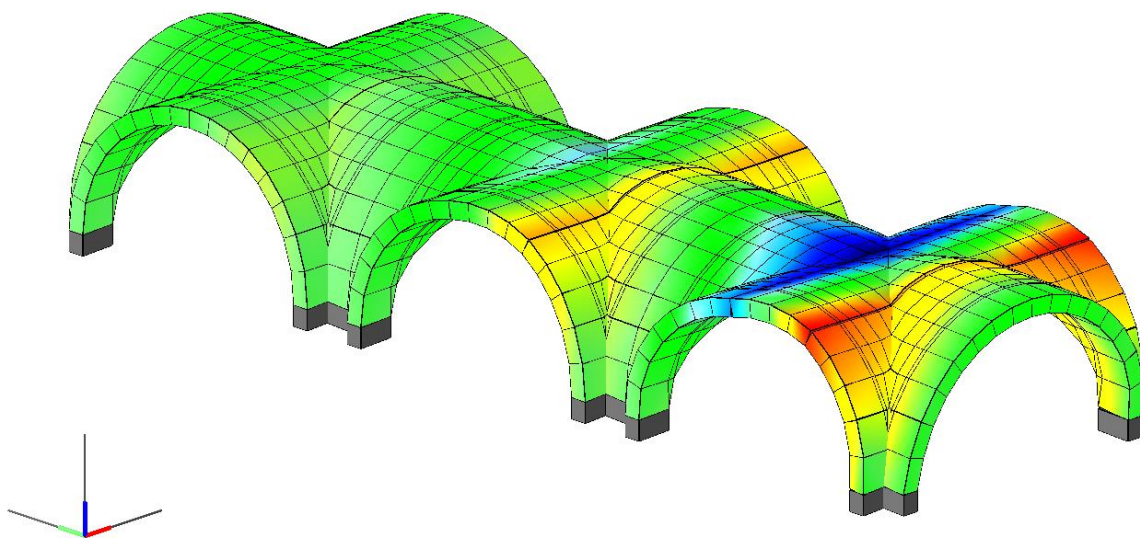
**Figure D107** – Plan view of deformation in +Y direction.



**Figure D108** – Side view of deformation in +Y direction.

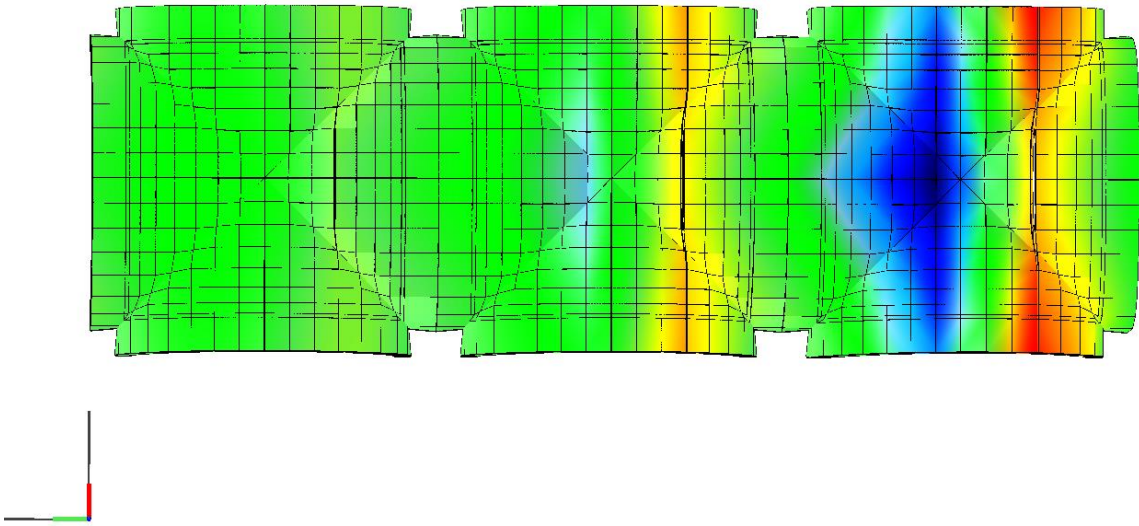


**Figure D109** – Legend for -Y direction. Taken at  $U = 0.002$  m.

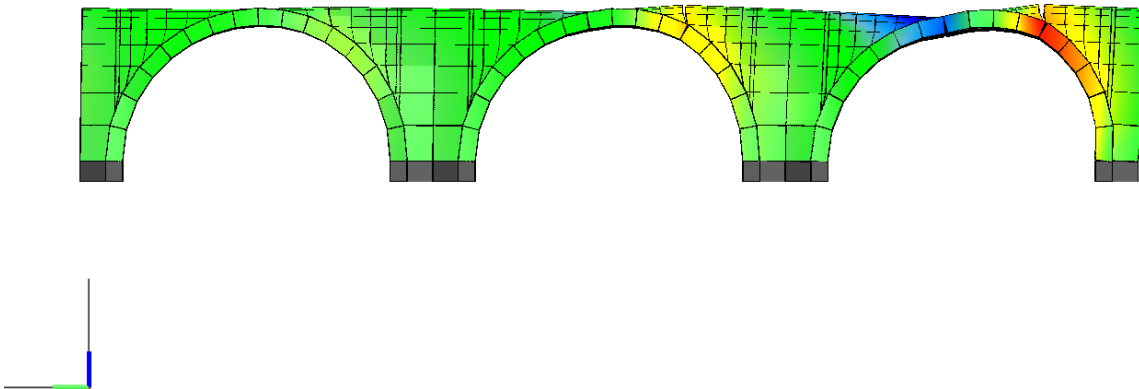


**Figure D110** – 3D view of deformation in -Y direction.





**Figure D111** – Plan view of deformation in -Y direction.



**Figure D112** – Side view of deformation in -Y direction.

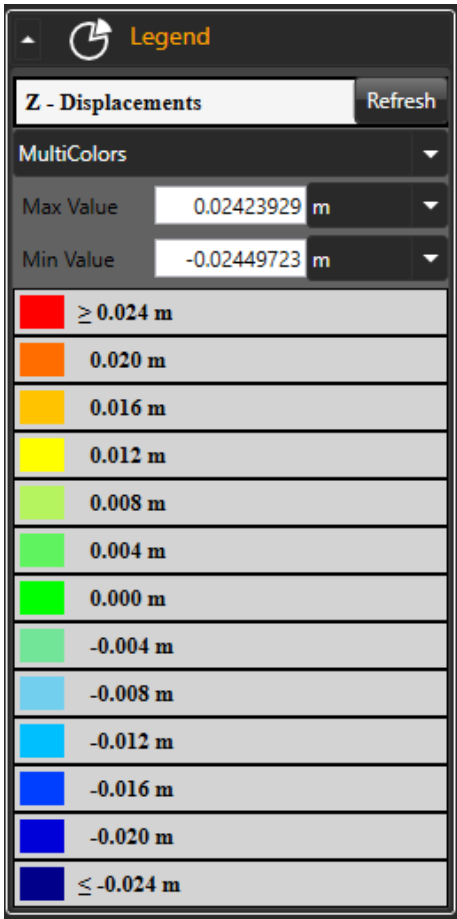


Figure D113 – Legend for +X direction. Taken at  $U = 0.03m$ .

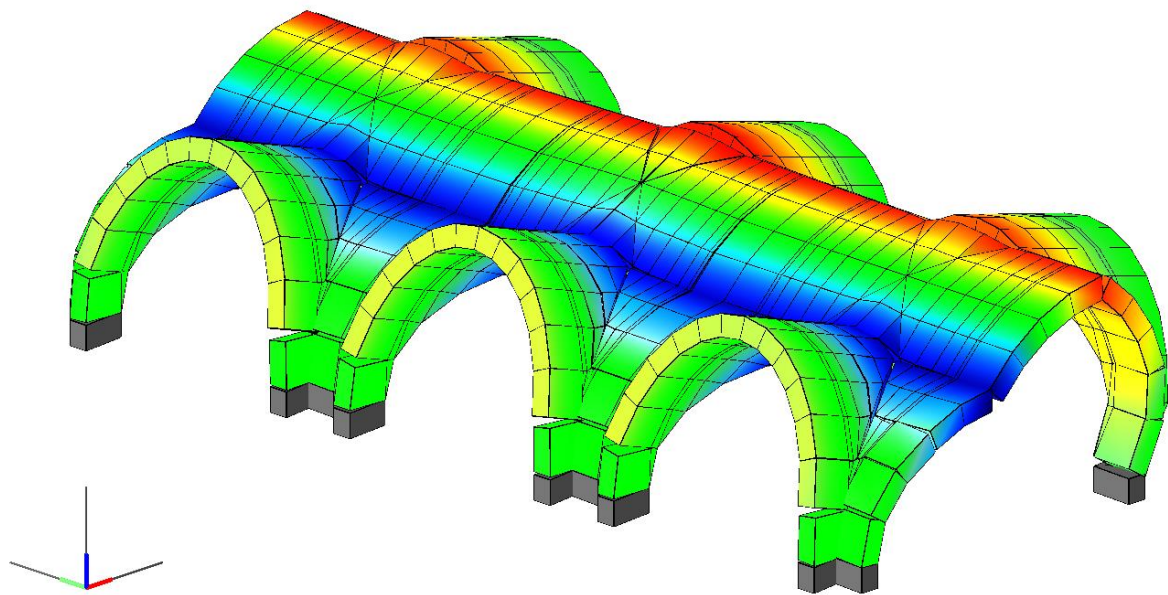
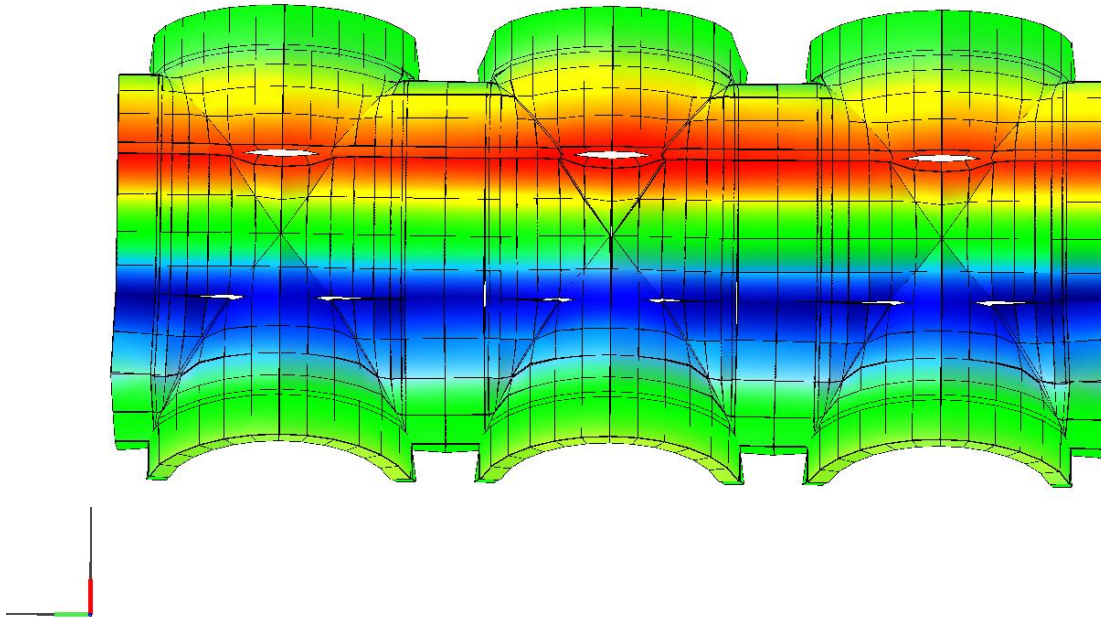
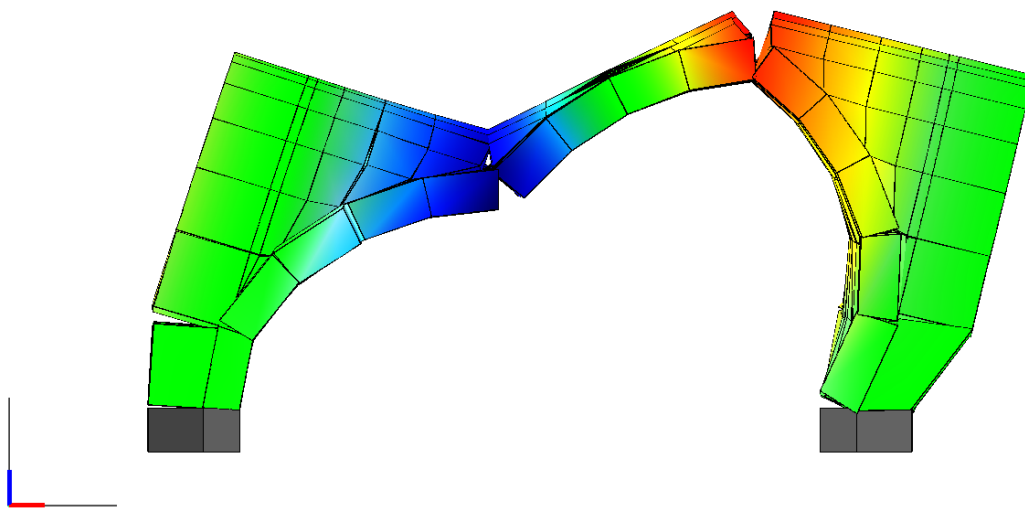


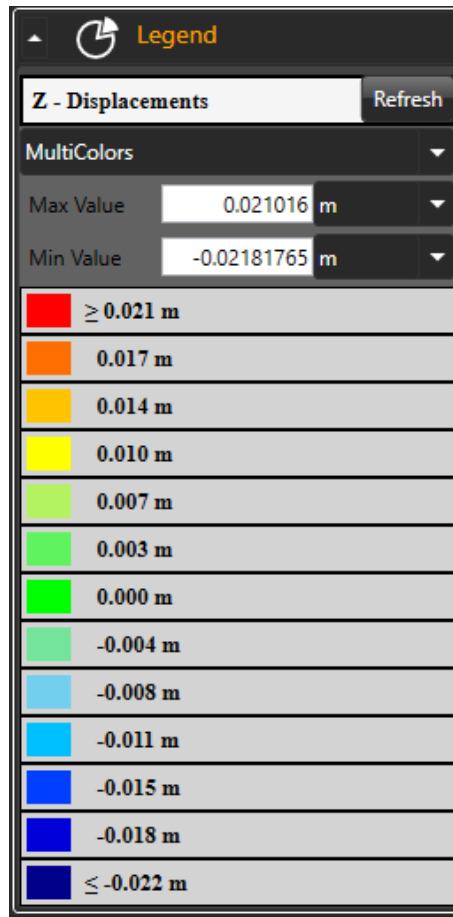
Figure D114 – 3D view of deformation in +X direction.



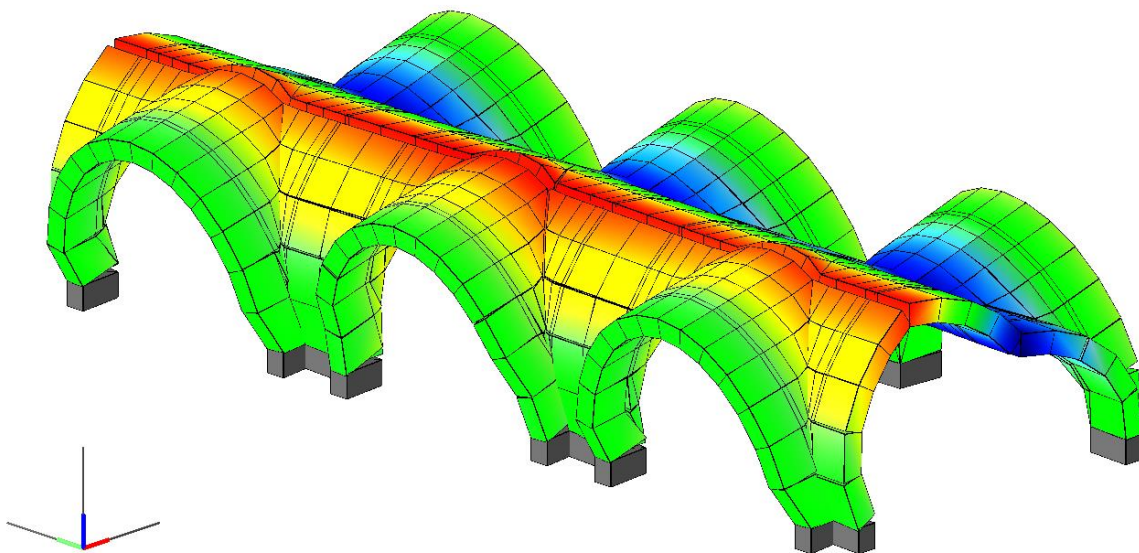
**Figure D115** – Plan view of deformation in +X direction.



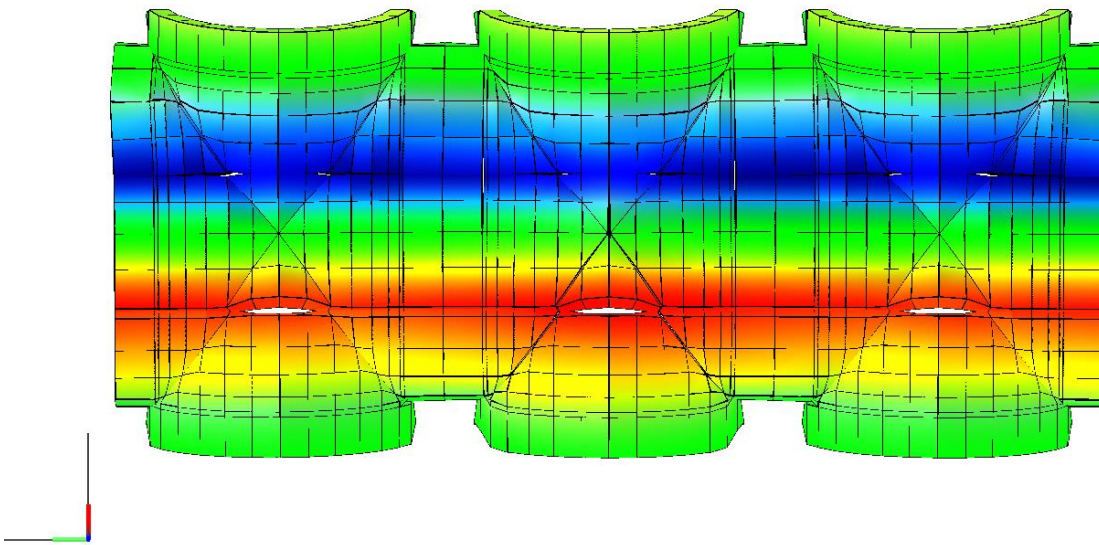
**Figure D116** – Front view of deformation in +X direction.



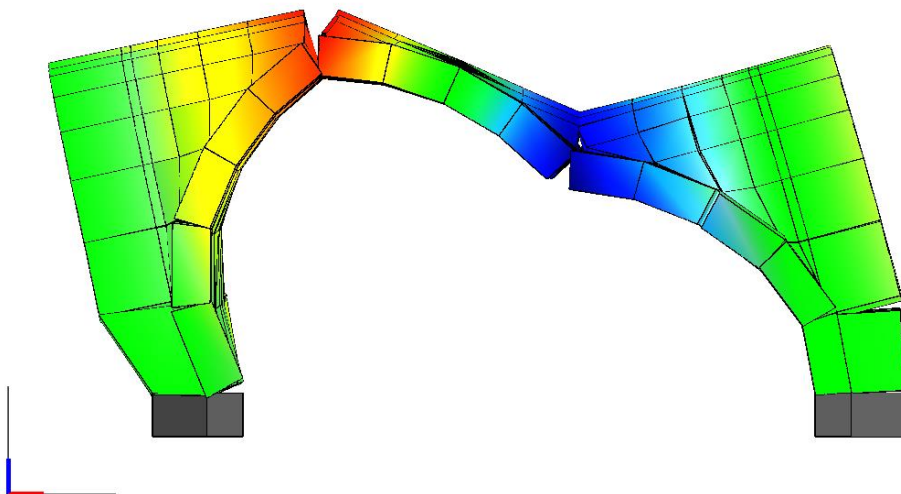
**Figure D117** – Legend for -X direction. Taken at  $U = 0.026$  m.



**Figure D118** – 3D view of deformation in -X direction.

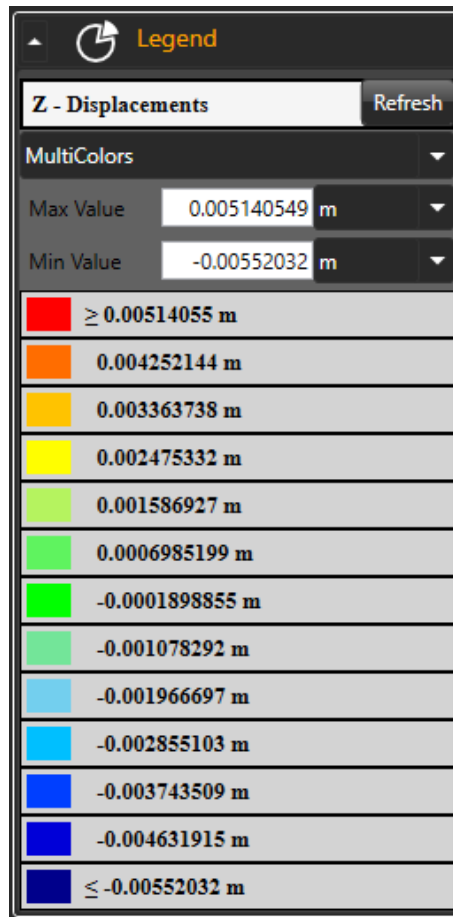


**Figure D119** – Plan view of deformation in -X direction.

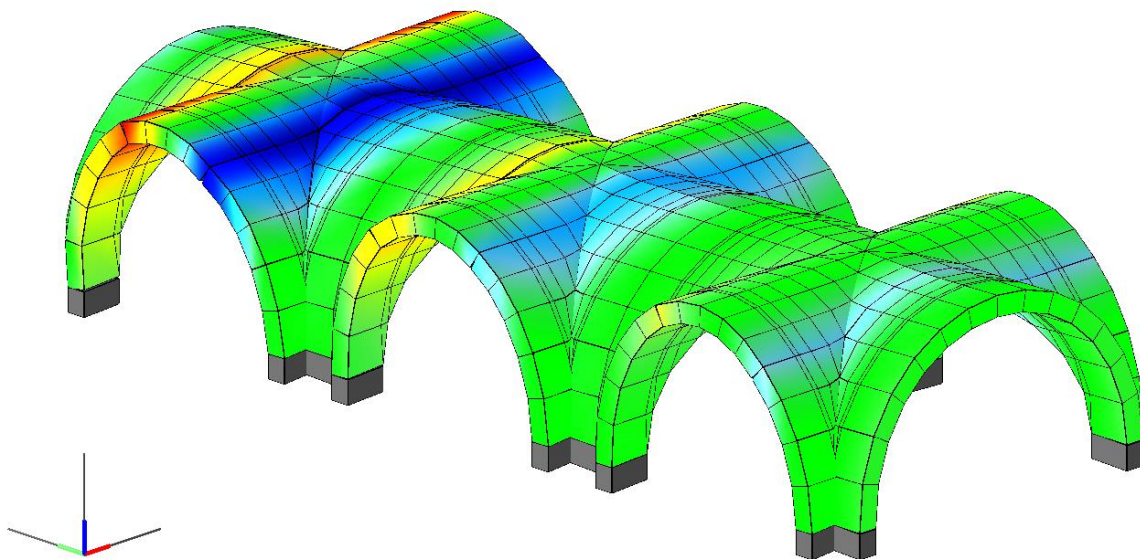


**Figure D120** – Front view of deformation in -X direction.

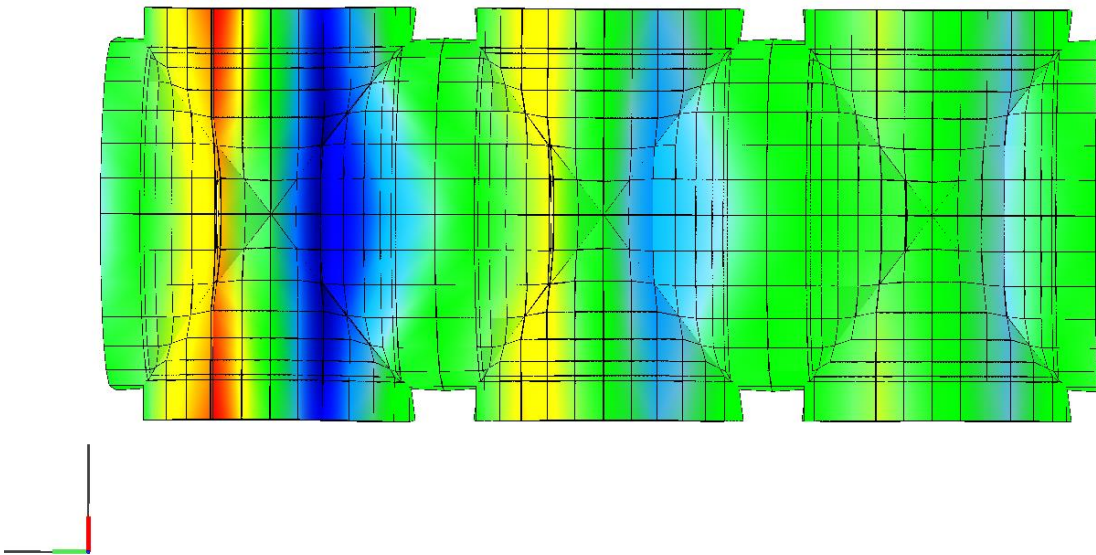




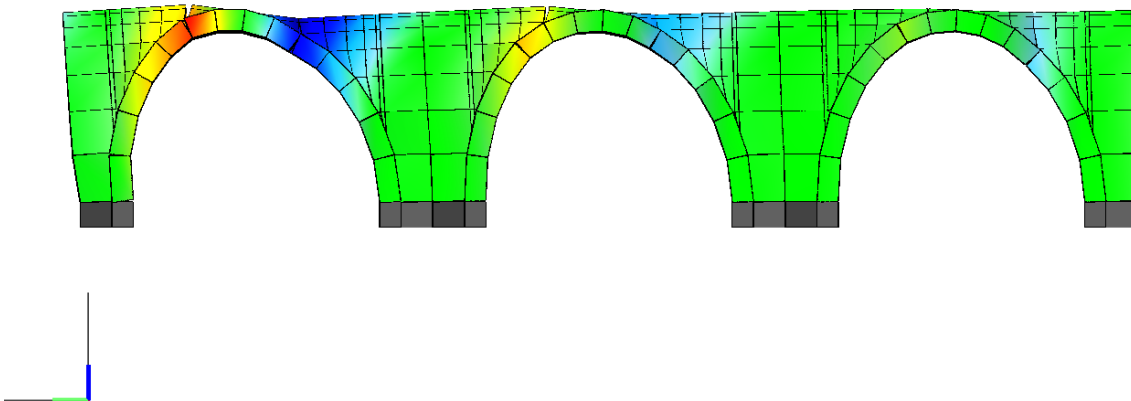
**Figure D121** – Legend for +Y direction. Taken at  $U = 0.004$ m.



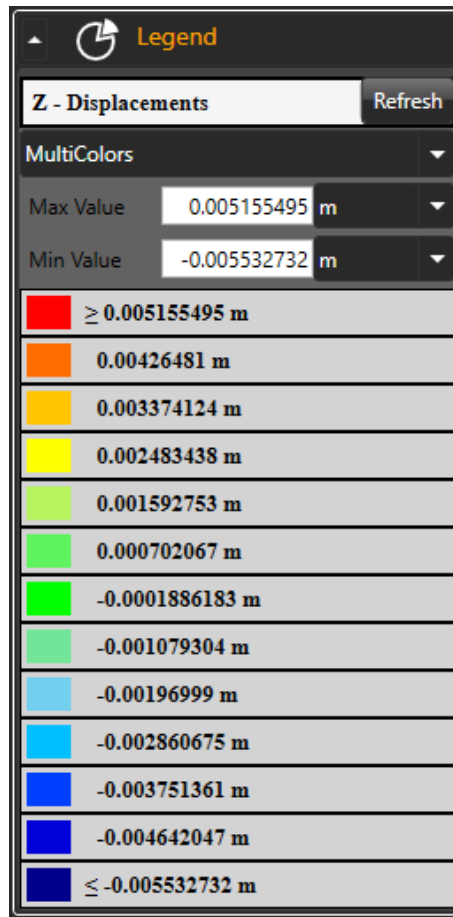
**Figure D122** – 3D view of deformation in +Y direction.



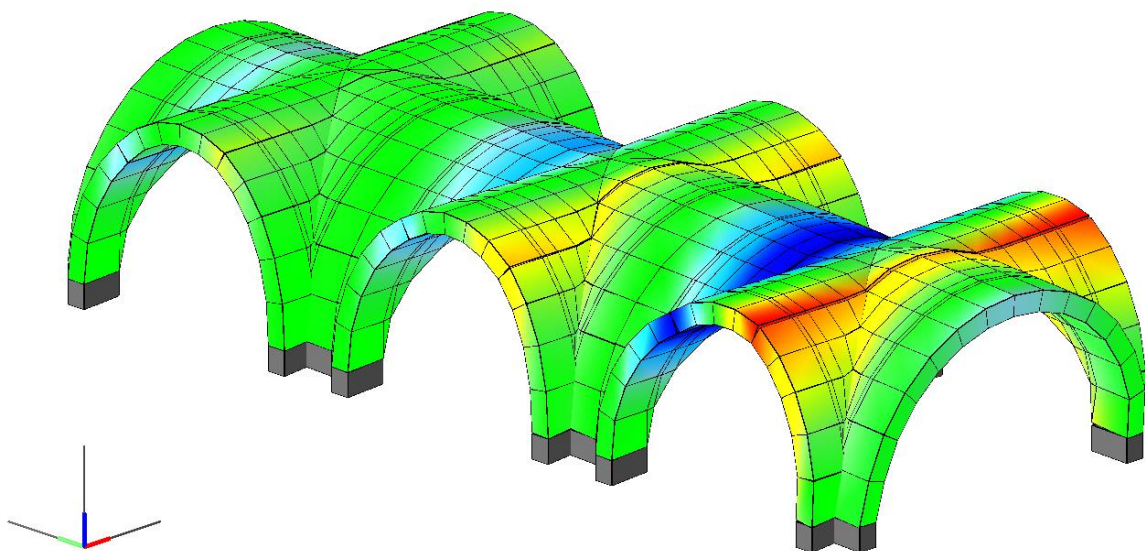
**Figure D123** – Plan view of deformation in +Y direction.



**Figure D124** – Side view of deformation in +Y direction.

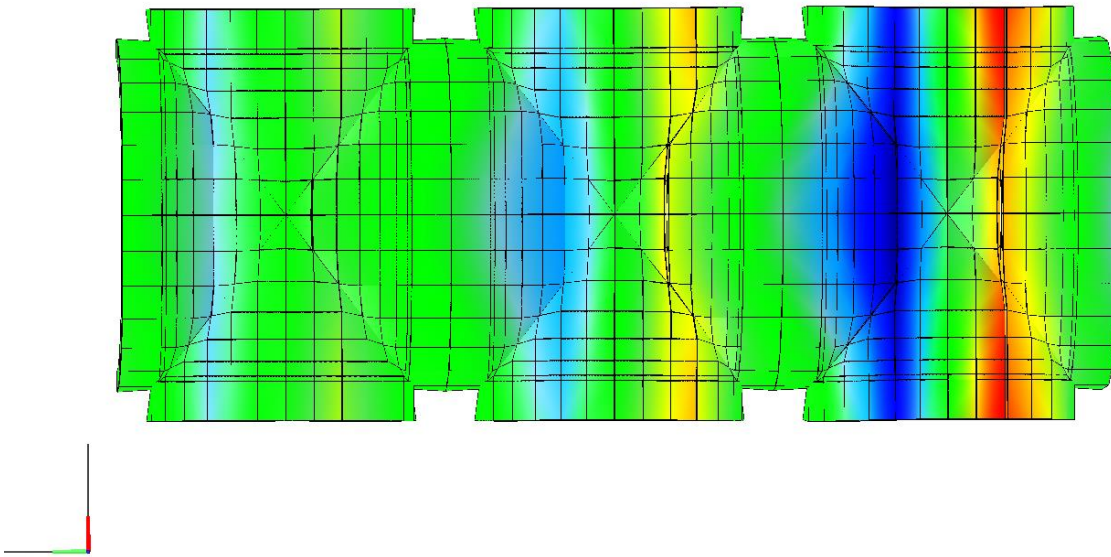


**Figure D125** – Legend for -Y direction. Taken at  $U = 0.004\text{m}$ .

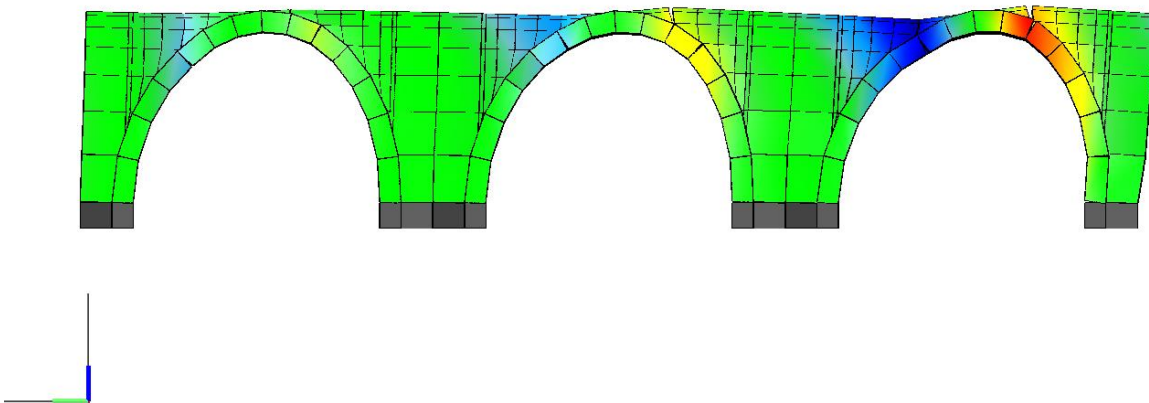


**Figure D126** – 3D view of deformation in -Y direction.





**Figure D127** – Plan view of deformation in -Y direction.



**Figure D128** – Side view of deformation in -Y direction.

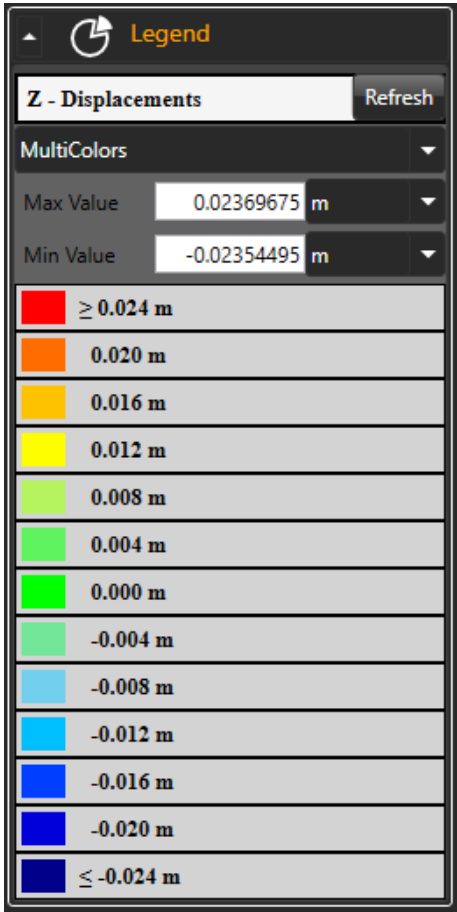


Figure D129 – Legend for +X direction. Taken at  $U = 0.03m$ .

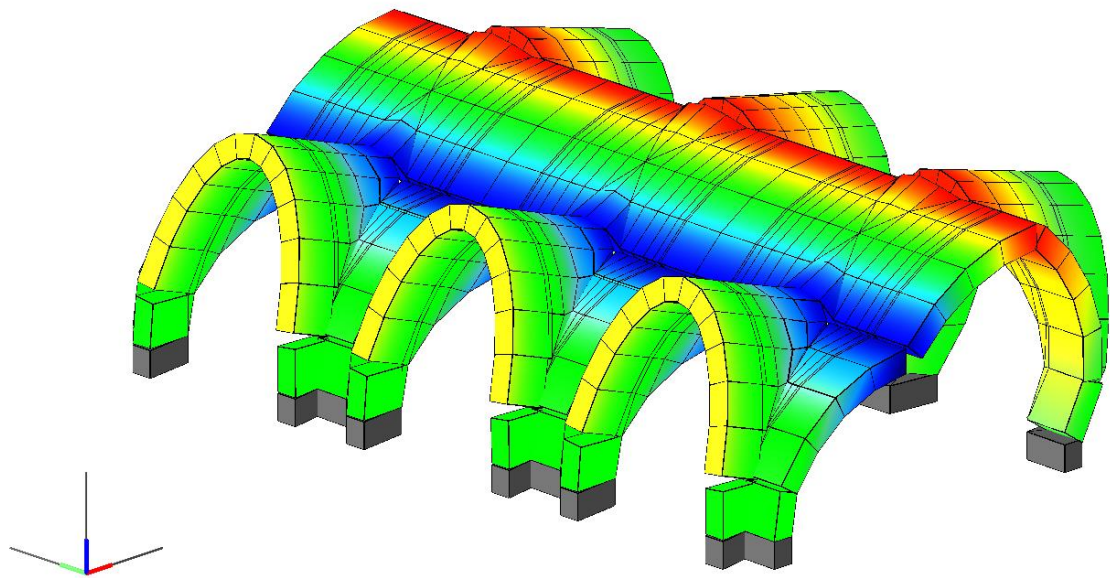
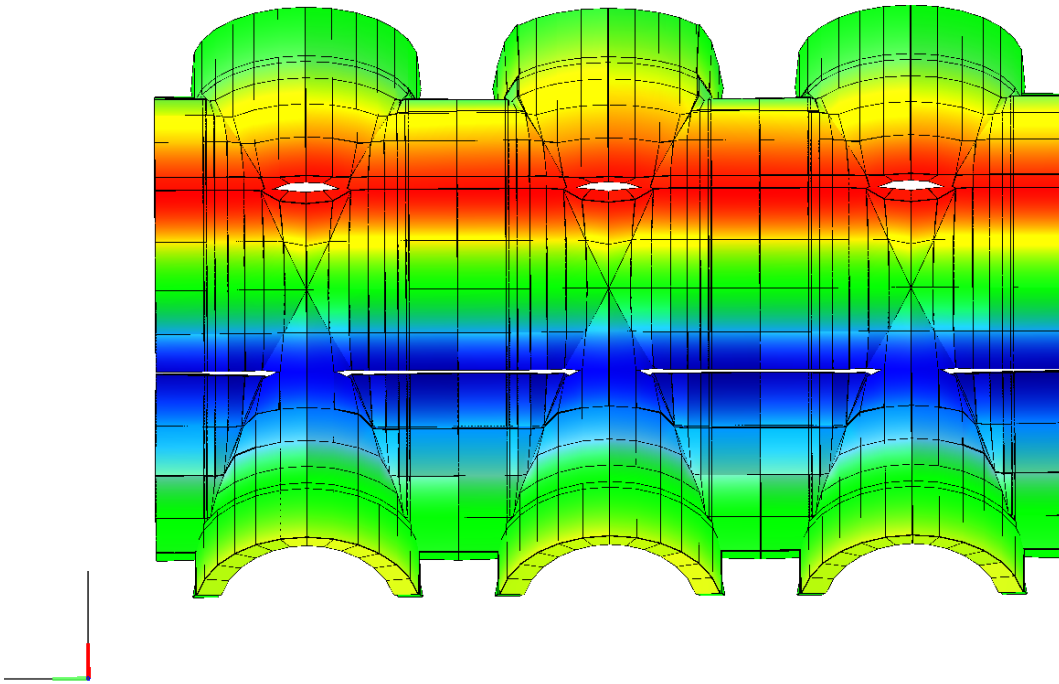
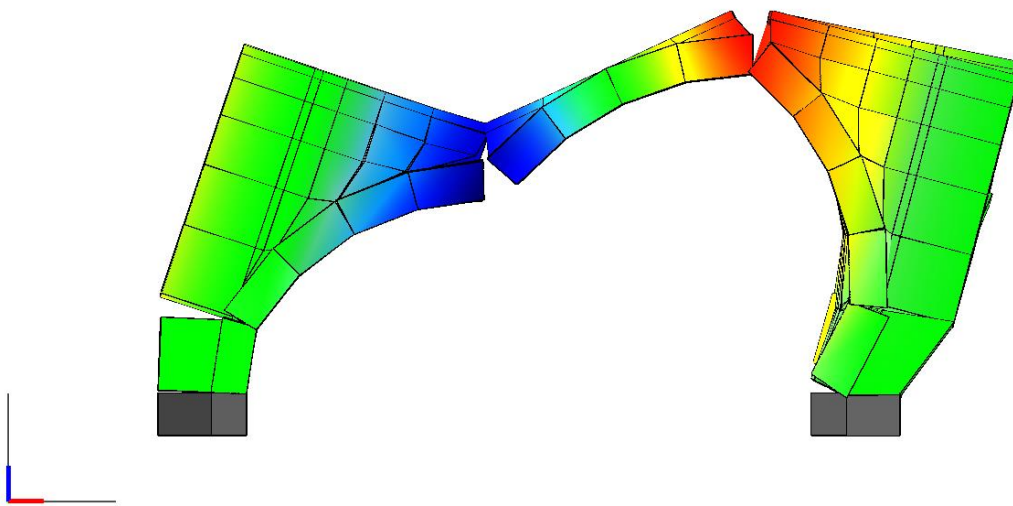


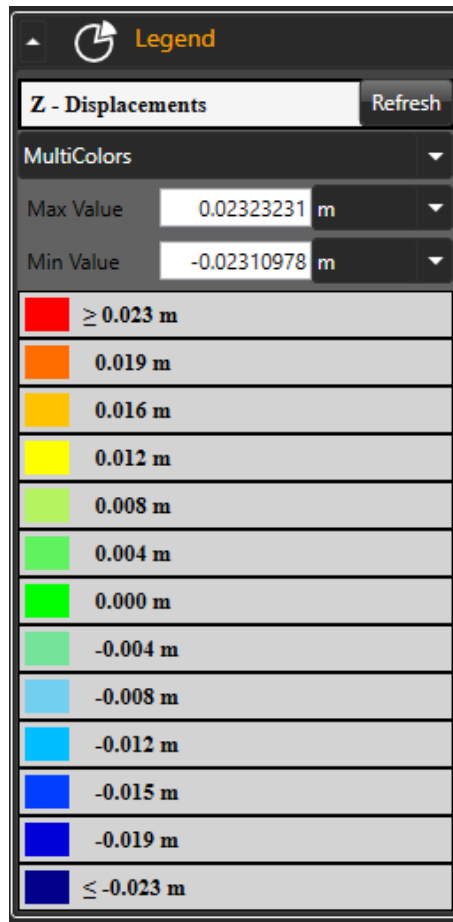
Figure D130 – 3D view of deformation in +X direction.



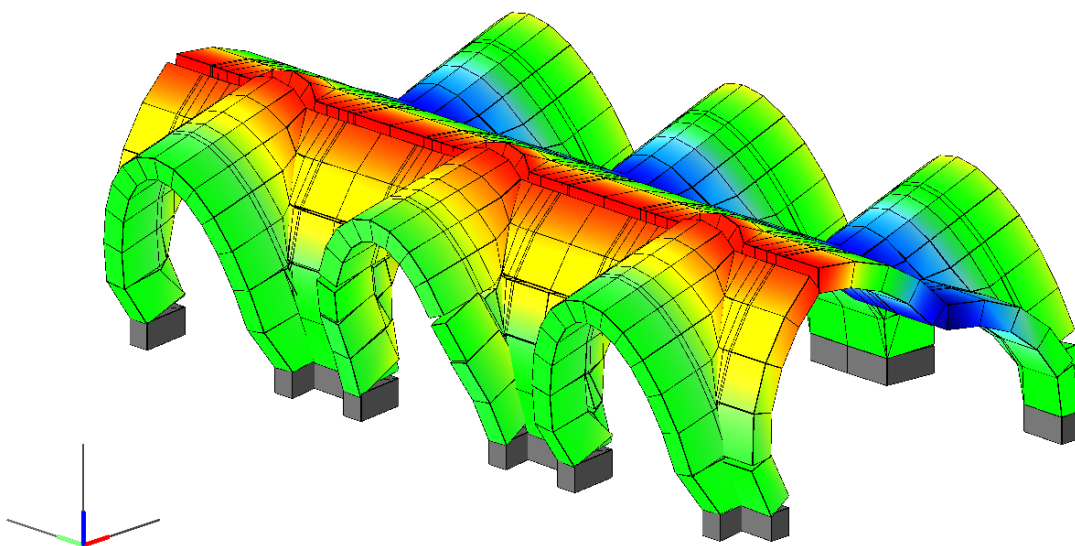
**Figure D131** – Plan view of deformation in +X direction.



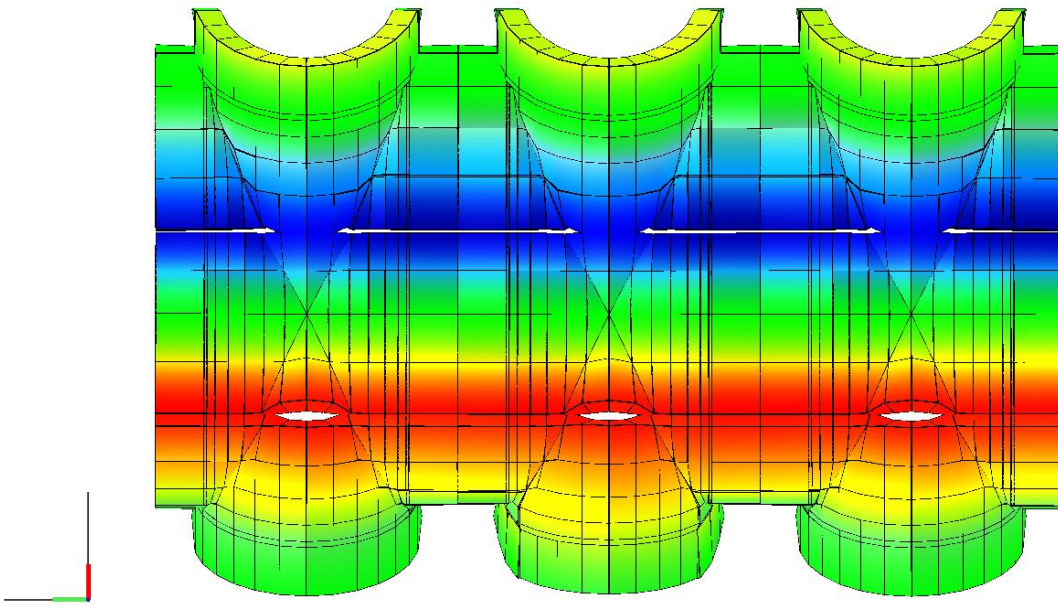
**Figure D132** – Front view of deformation in +X direction.



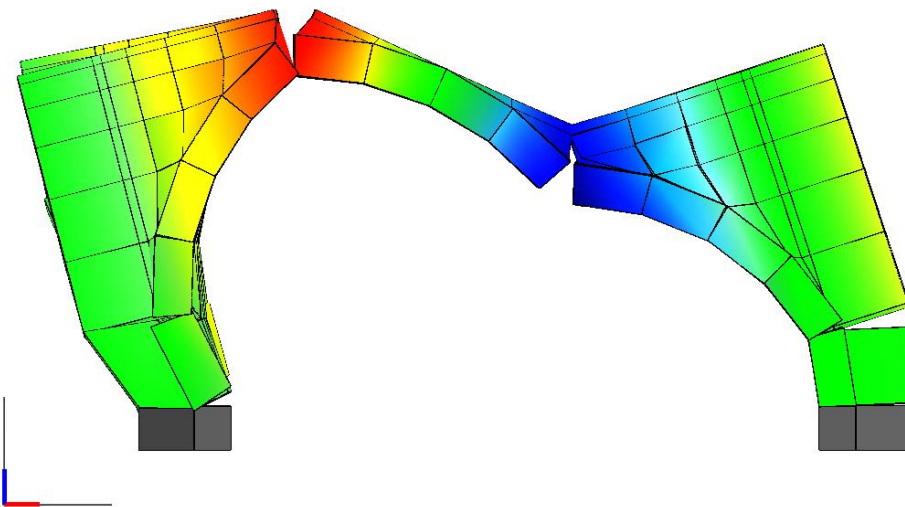
**Figure D133** – Legend for -X direction. Taken at  $U = 0.03m$ .



**Figure D134** – 3D view of deformation in -X direction.

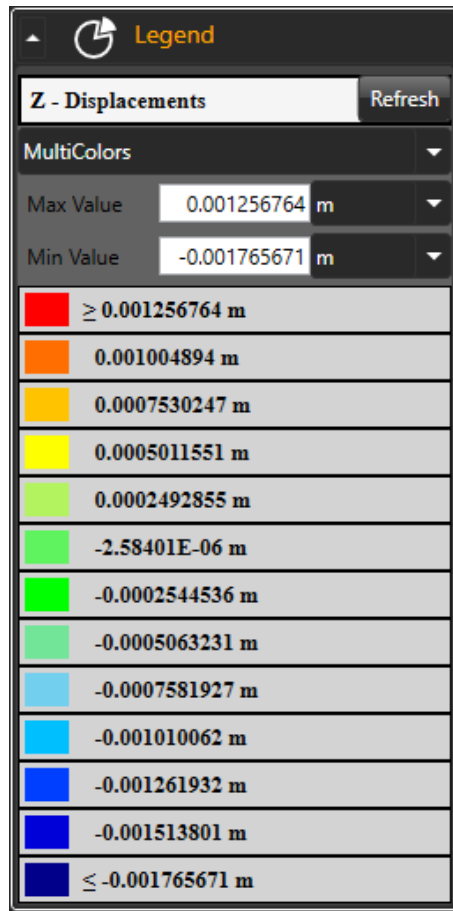


**Figure D135** – Plan view of deformation in -X direction.

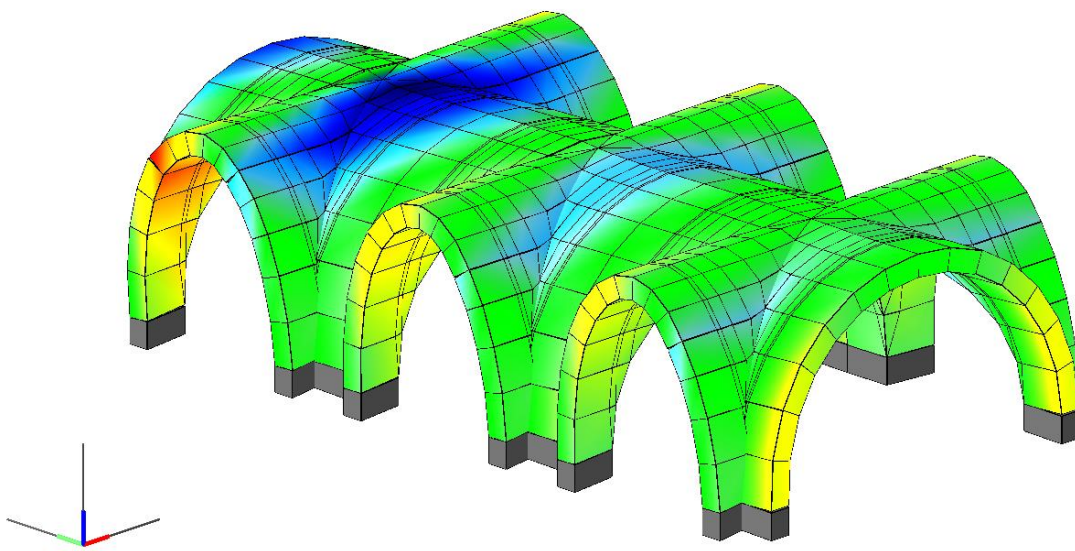


**Figure D136** – Front view of deformation in -X direction.

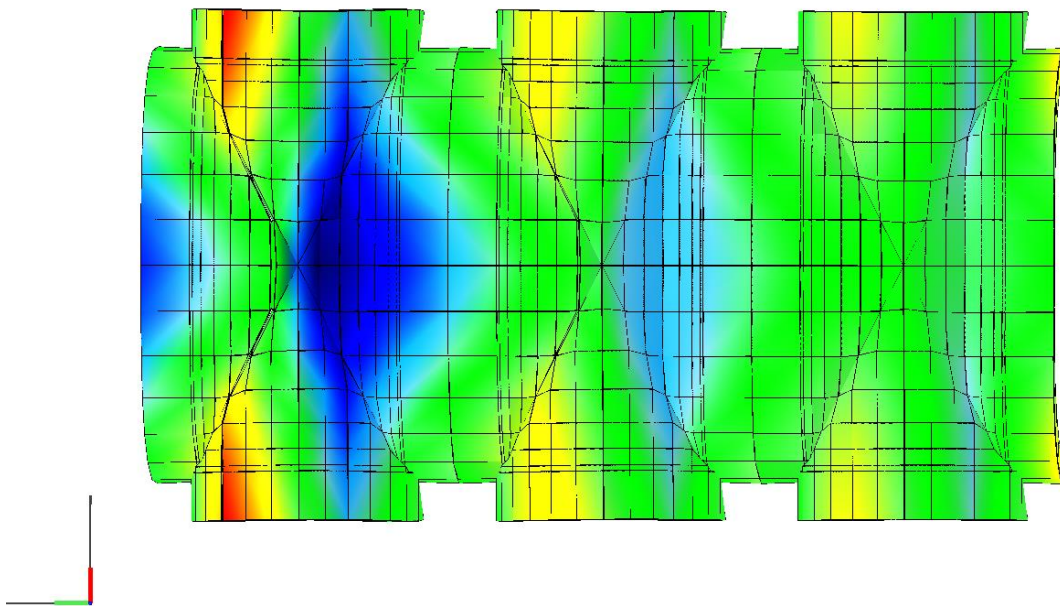




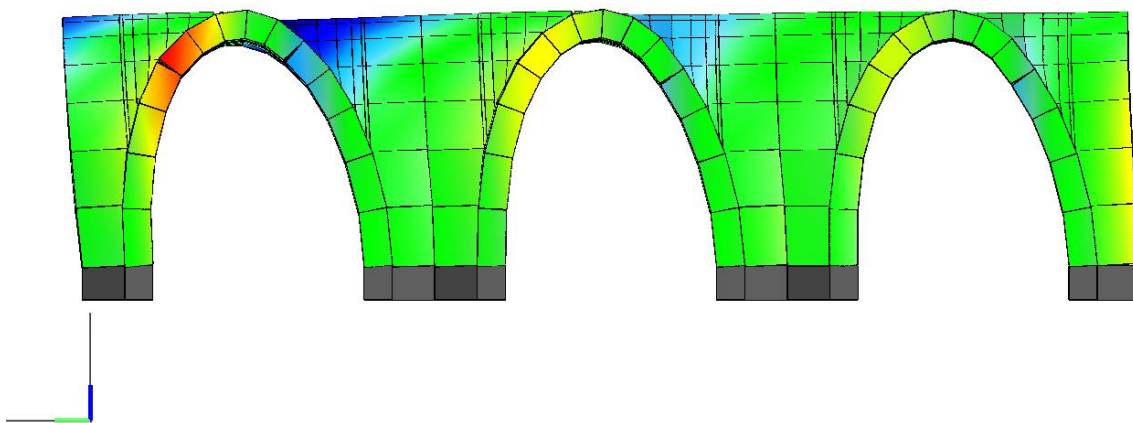
**Figure D137** – Legend for +Y direction. Taken at  $U = 0.002\text{m}$ .



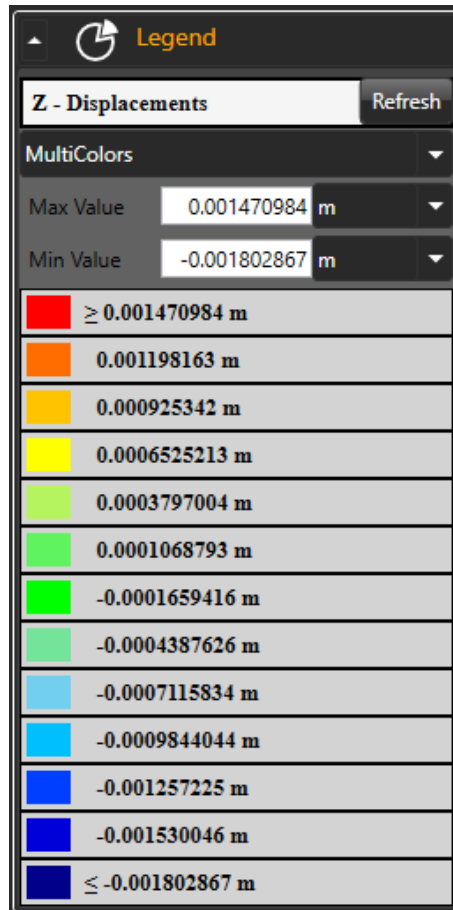
**Figure D138** – 3D view of deformation in +Y direction.



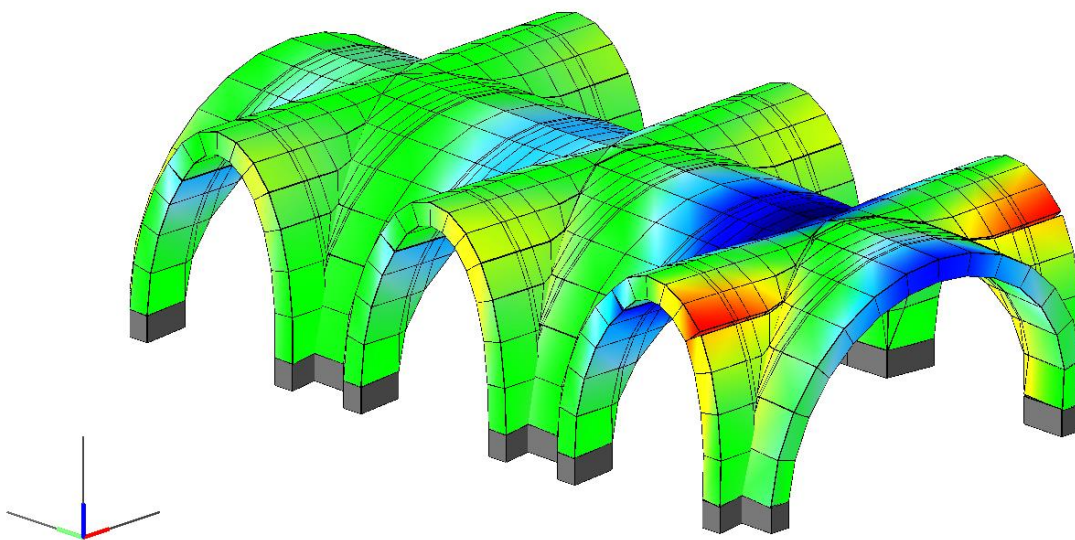
**Figure D139** – Plan view of deformation in +Y direction.



**Figure D140** – Side view of deformation in +Y direction.

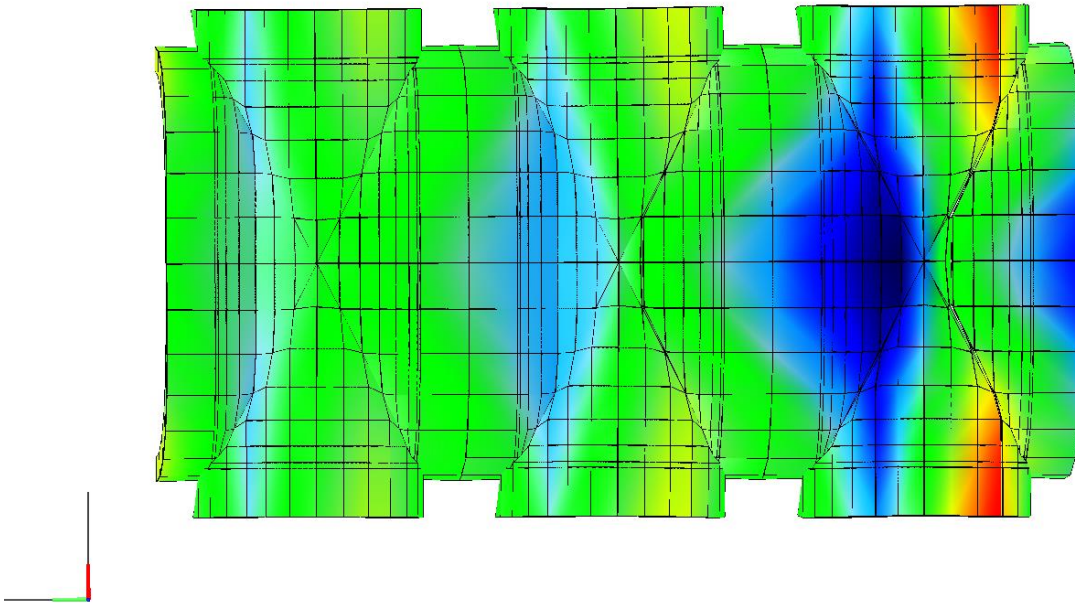


**Figure D141** – Legend for -Y direction. Taken at  $U = 0.002$ m.

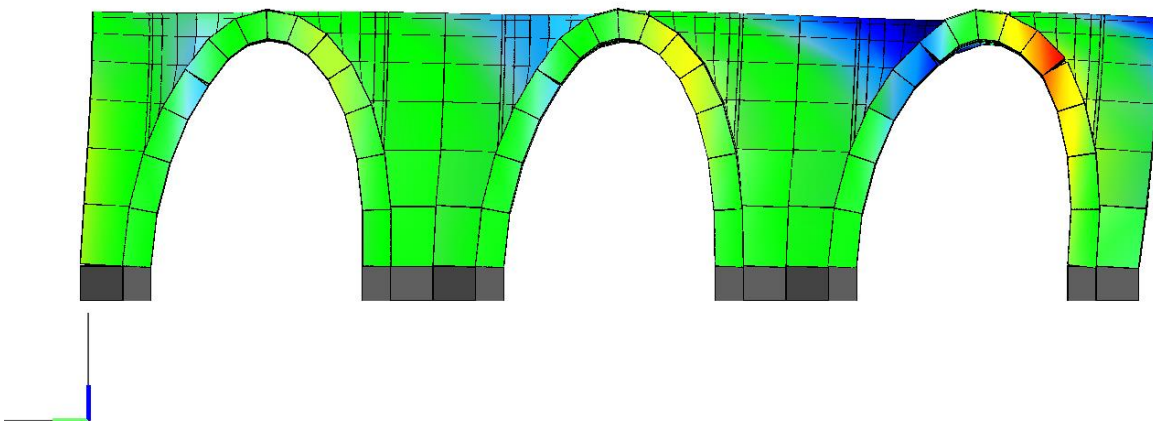


**Figure D142** – 3D view of deformation in -Y direction.





**Figure D143** – Plan view of deformation in -Y direction.



**Figure D144** – Side view of deformation in -Y direction.

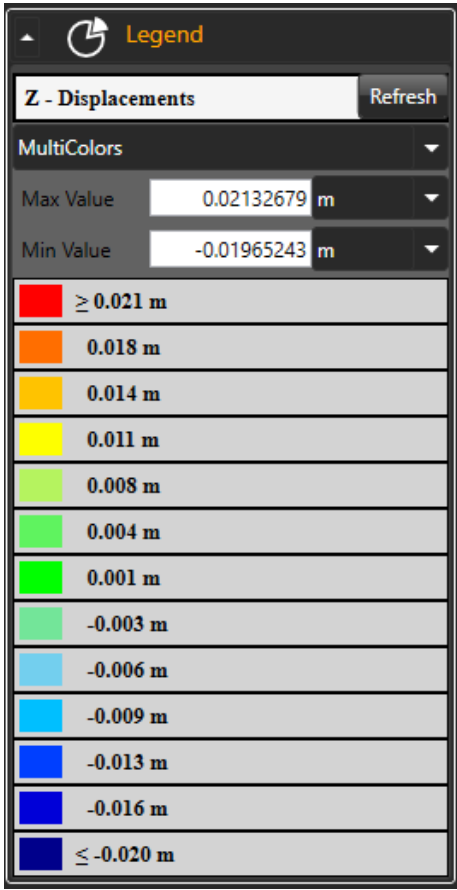


Figure D145 – Legend for +X direction. Taken at  $U = 0.025m$ .

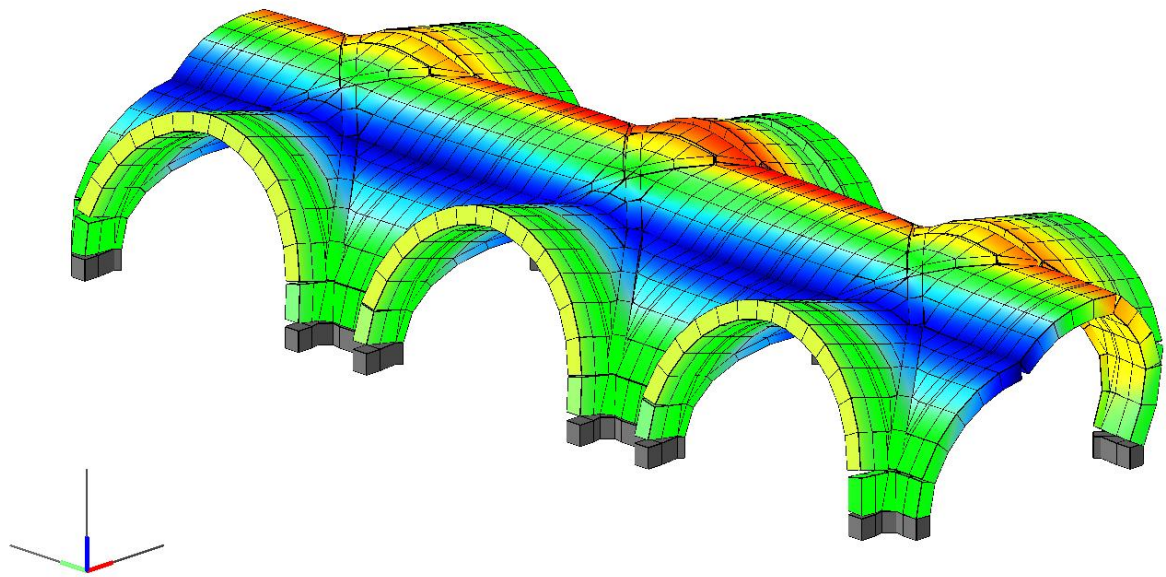
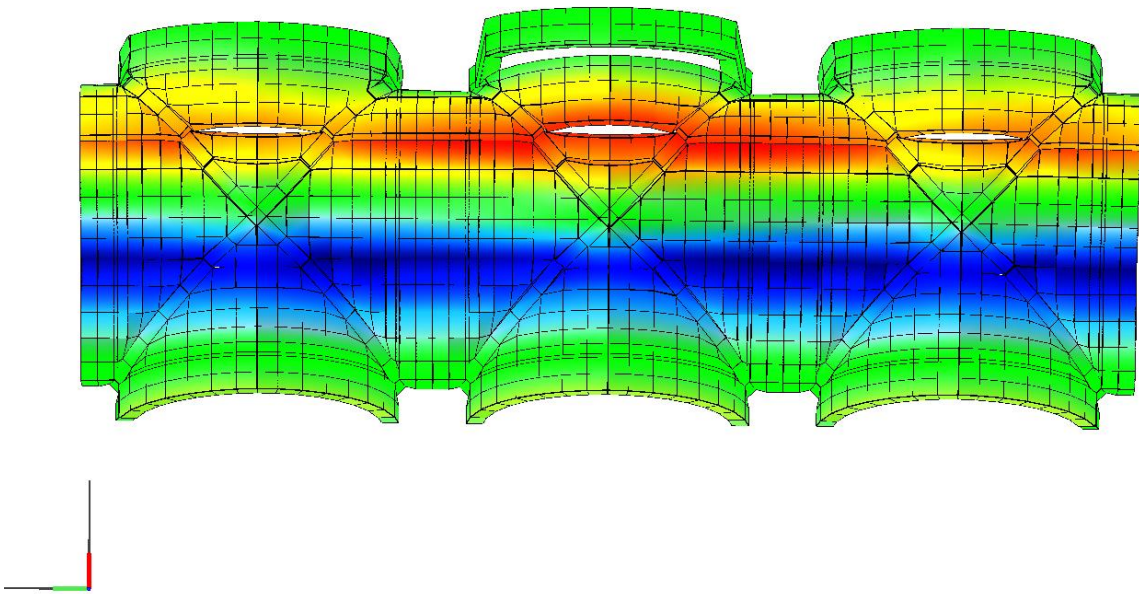
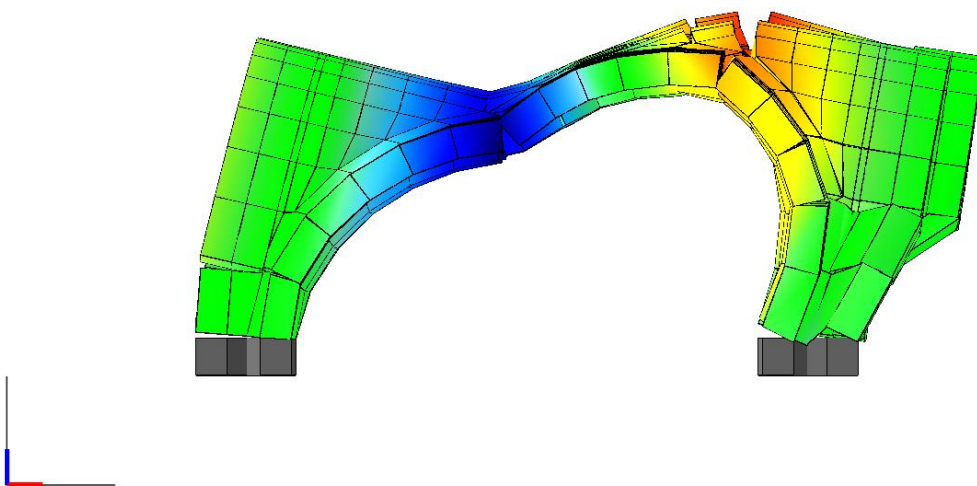


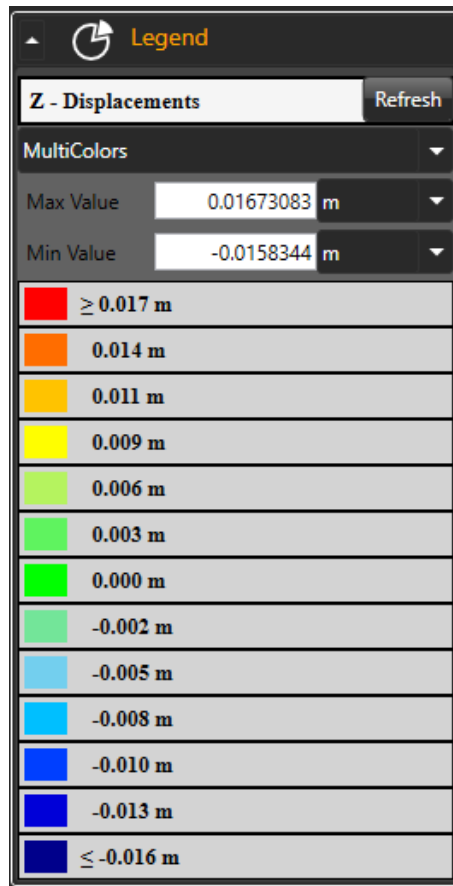
Figure D146 – 3D view of deformation in +X direction.



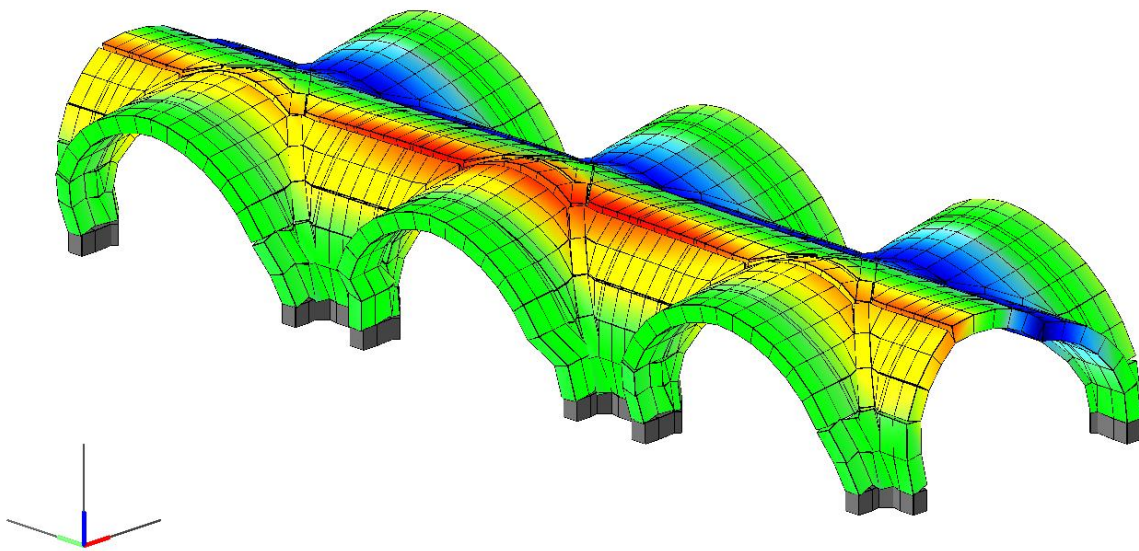
**Figure D147** – Plan view of deformation in +X direction.



**Figure D148** – Front view of deformation in +X direction.

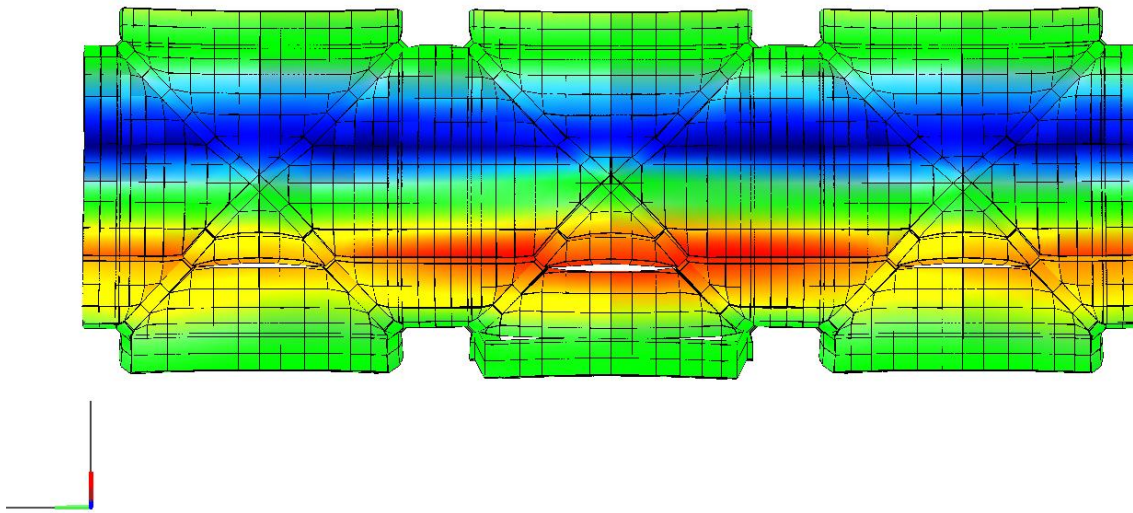


**Figure D149** – Legend for -X direction. Taken at  $U = 0.02$  m.

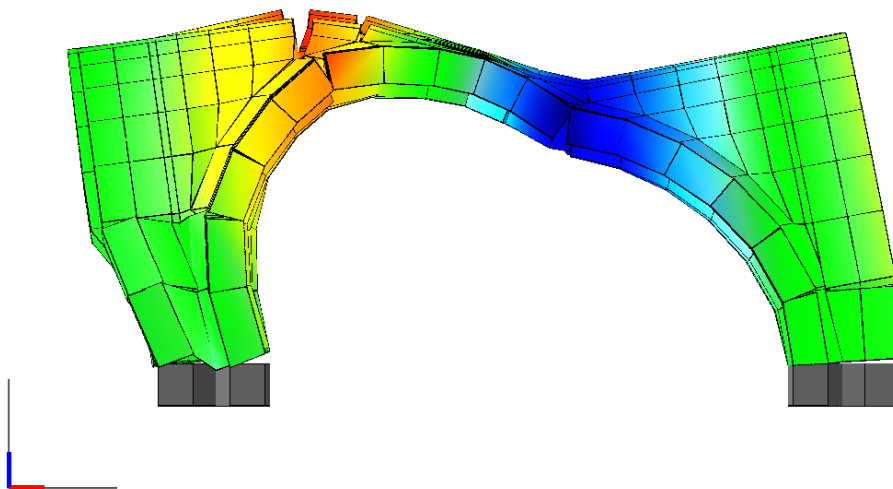


**Figure D150** – 3D view of deformation in -X direction.

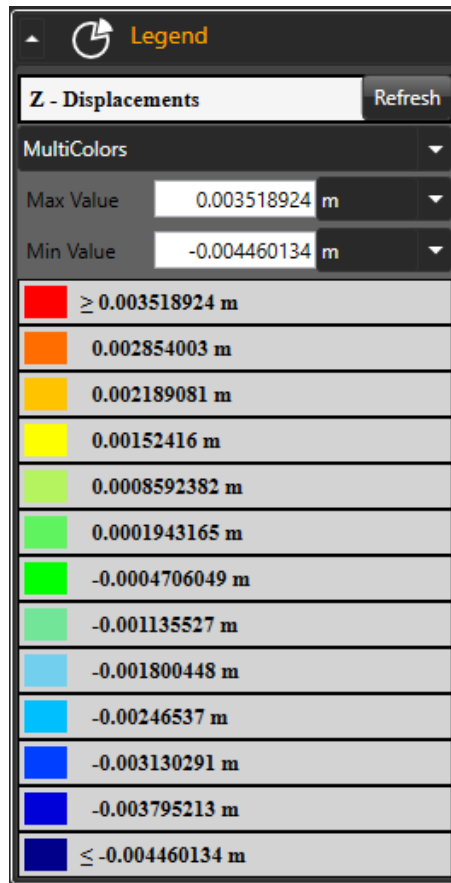




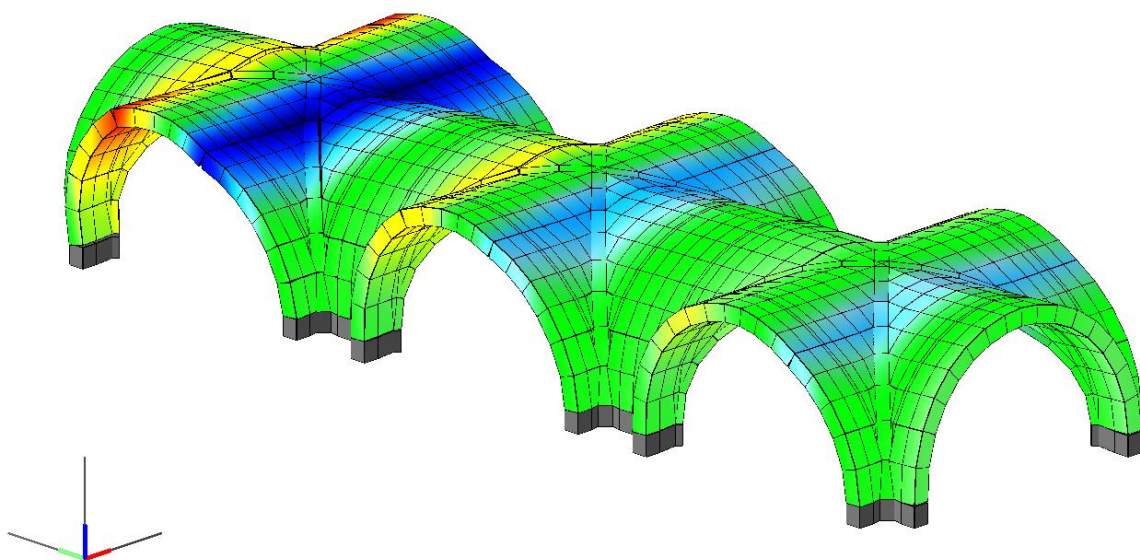
**Figure D151** – Plan view of deformation in -X direction.



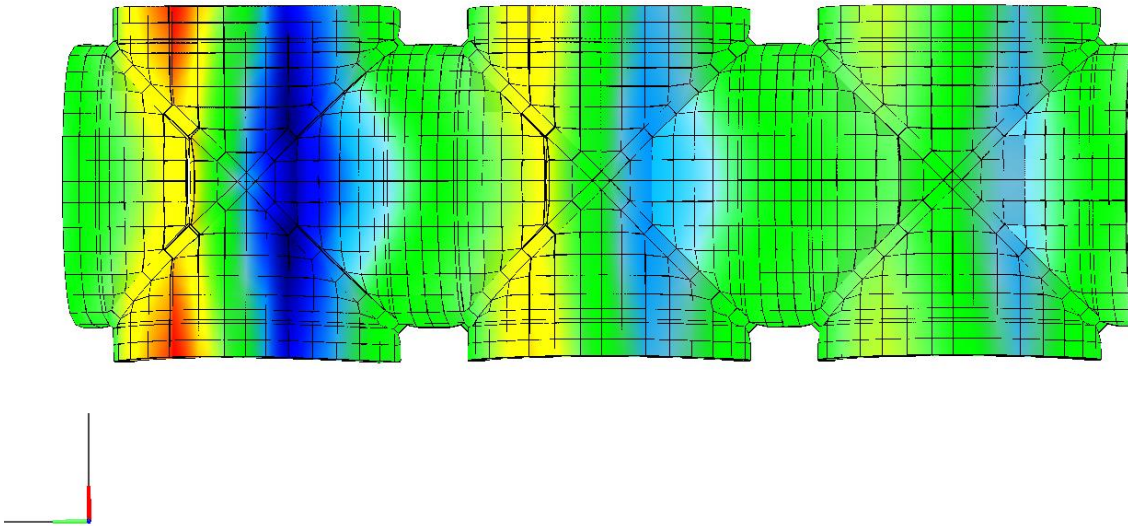
**Figure D152** – Front view of deformation in -X direction.



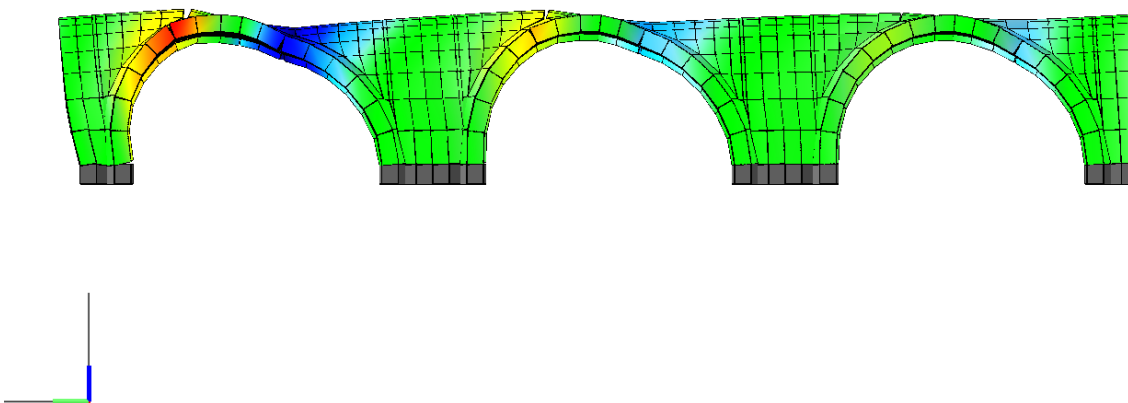
**Figure D153** – Legend for +Y direction. Taken at  $U = 0.002\text{m}$ .



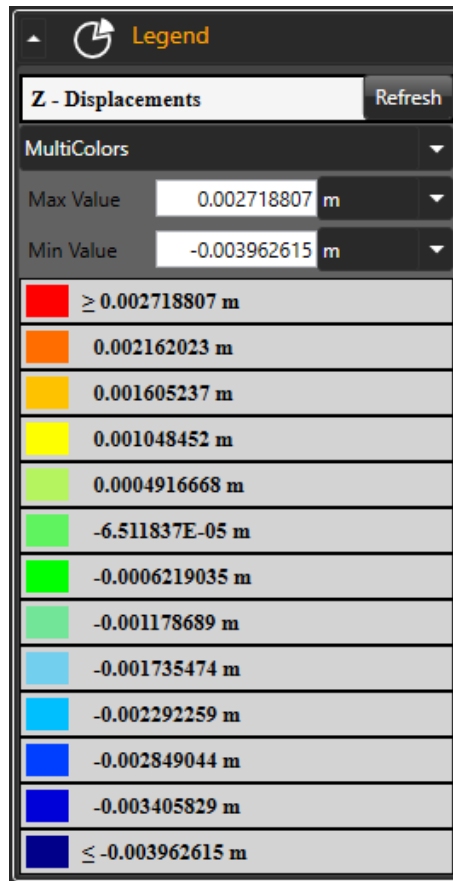
**Figure D154** – 3D view of deformation in +Y direction.



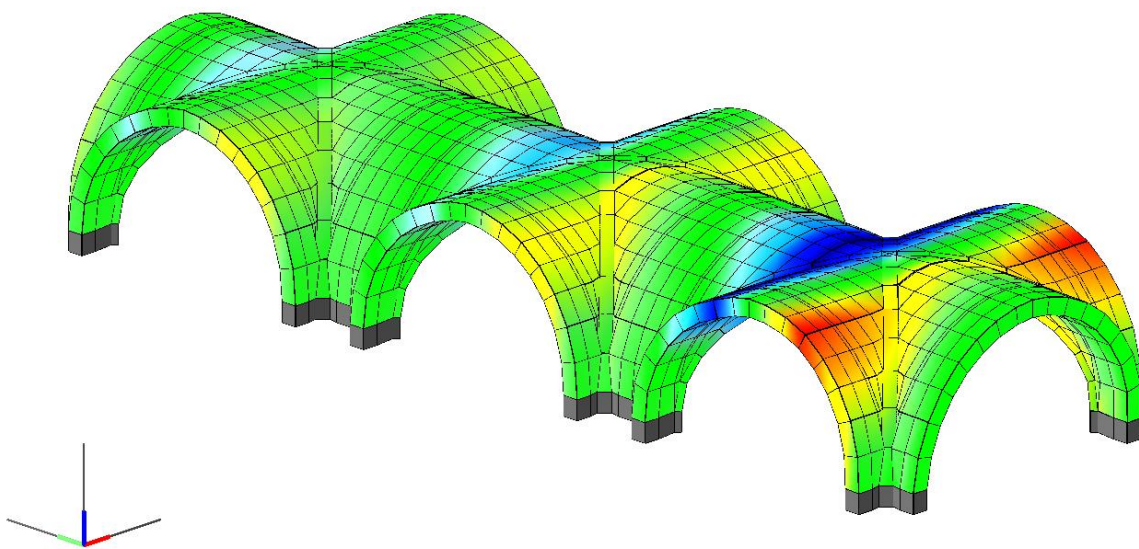
**Figure D155** – Plan view of deformation in +Y direction.



**Figure D156** – Side view of deformation in +Y direction.

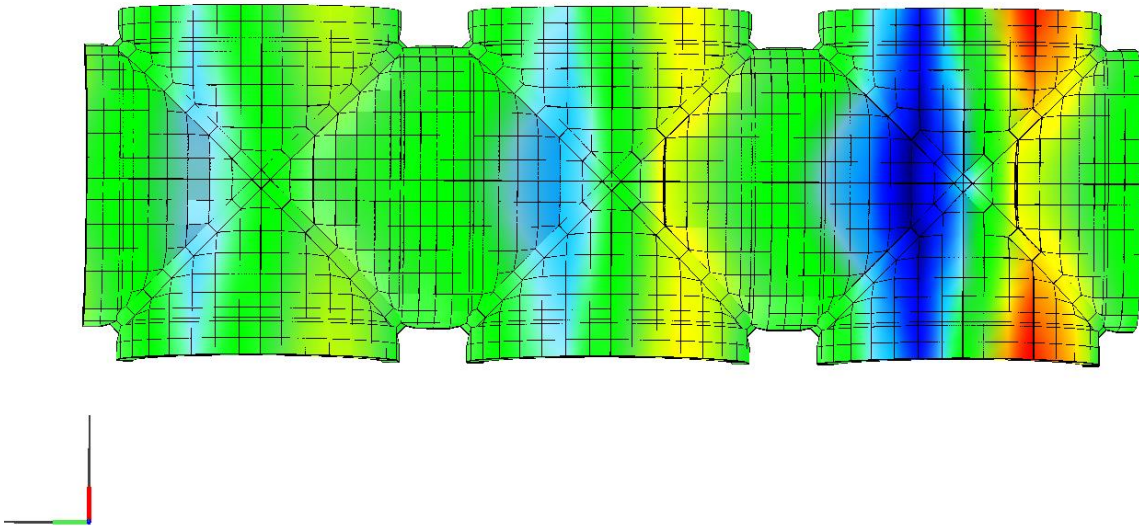


**Figure D157** – Legend for -Y direction. Taken at  $U = 0.004$ m.

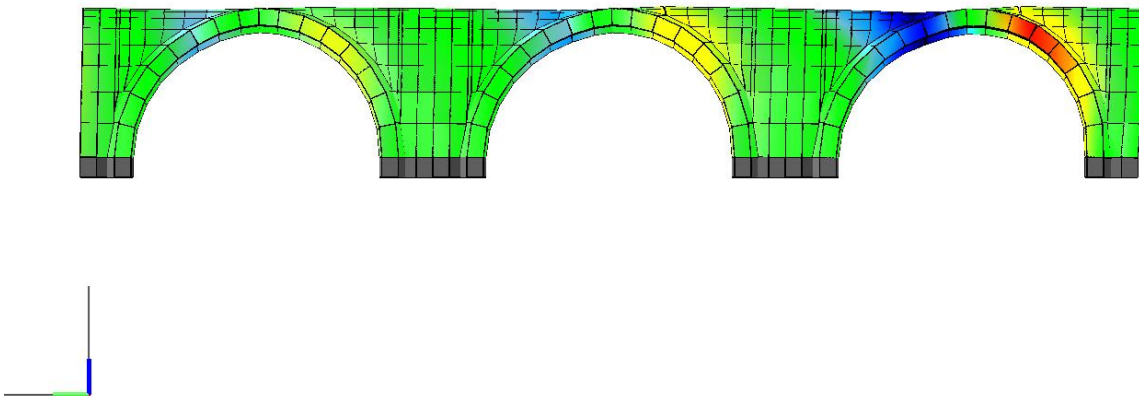


**Figure D158** – 3D view of deformation in -Y direction.





**Figure D159** – Plan view of deformation in -Y direction.



**Figure D160** – Side view of deformation in -Y direction.

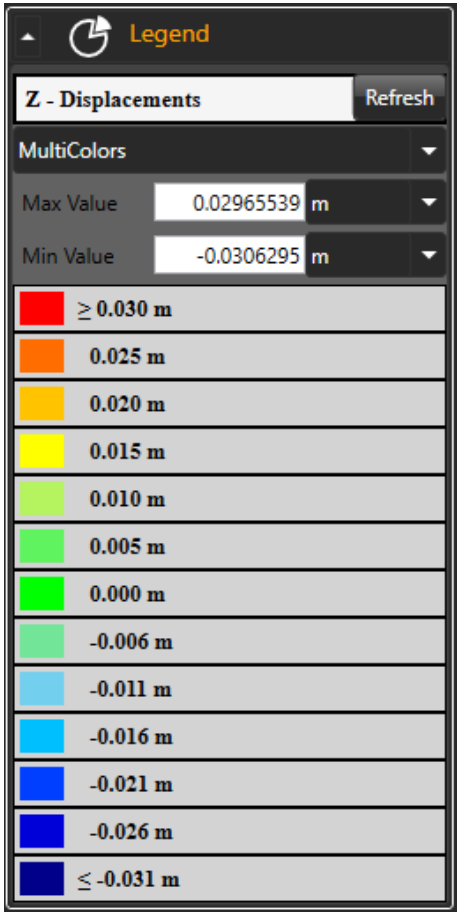


Figure D161 – Legend for +X direction. Taken at  $U = 0.04m$ .

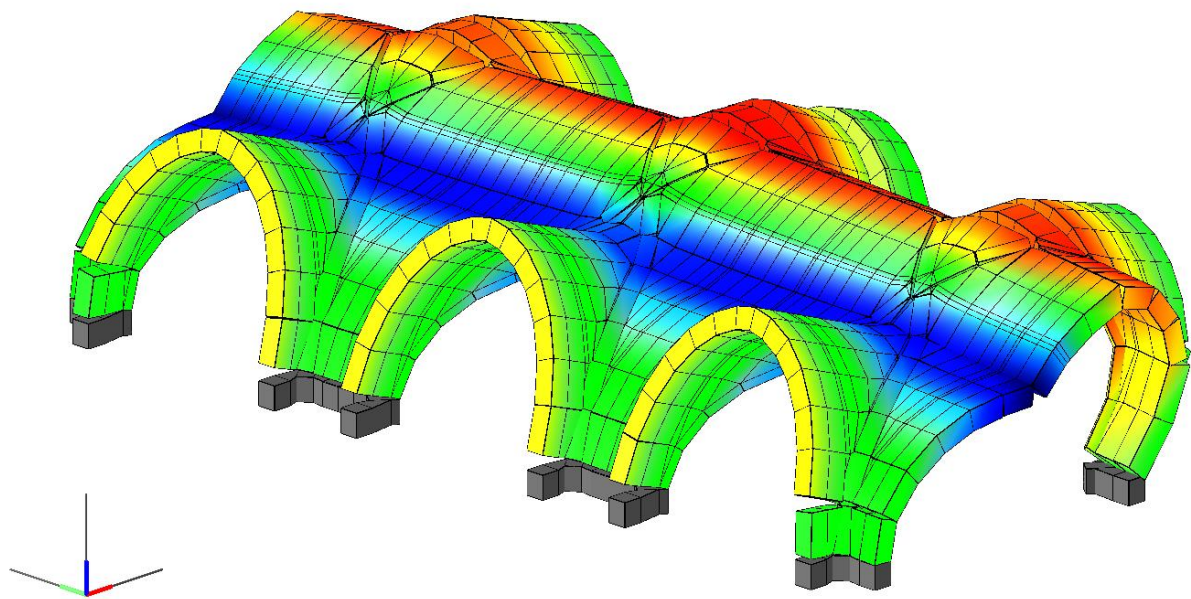
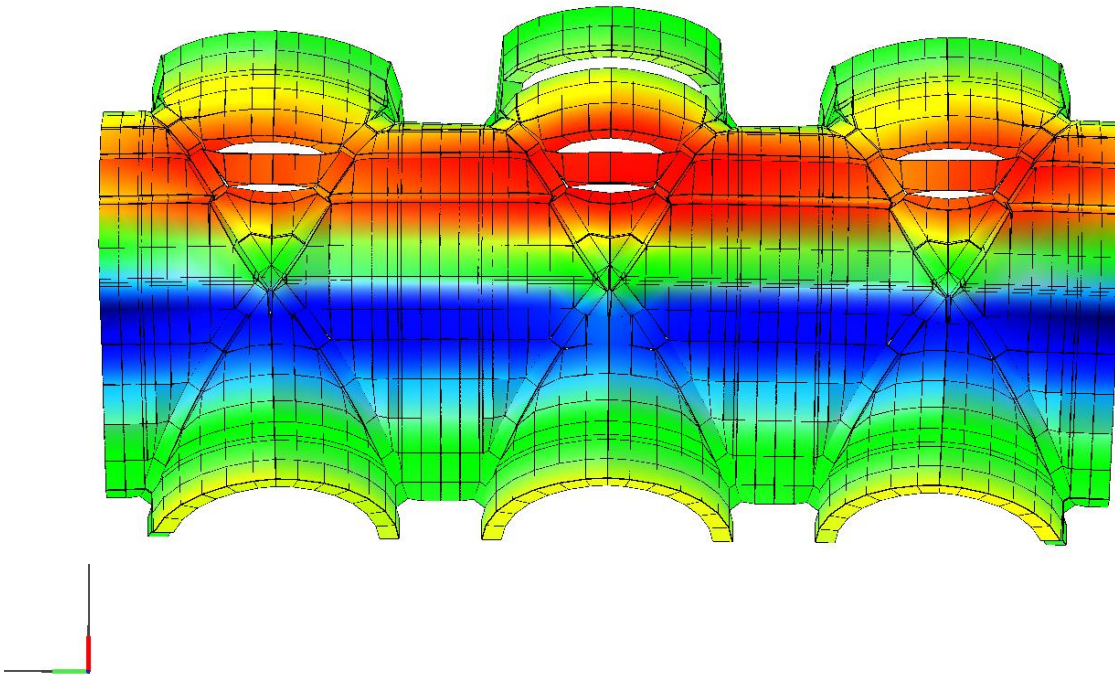
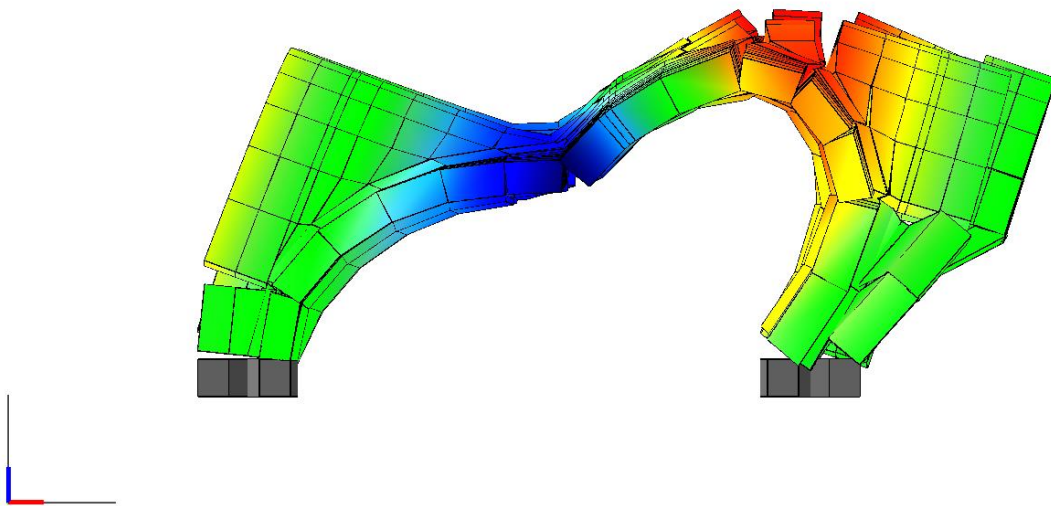


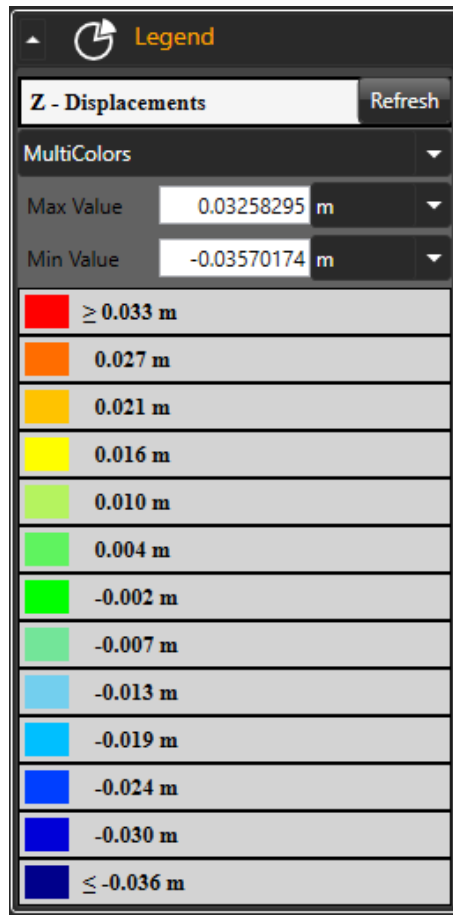
Figure D162 – 3D view of deformation in +X direction.



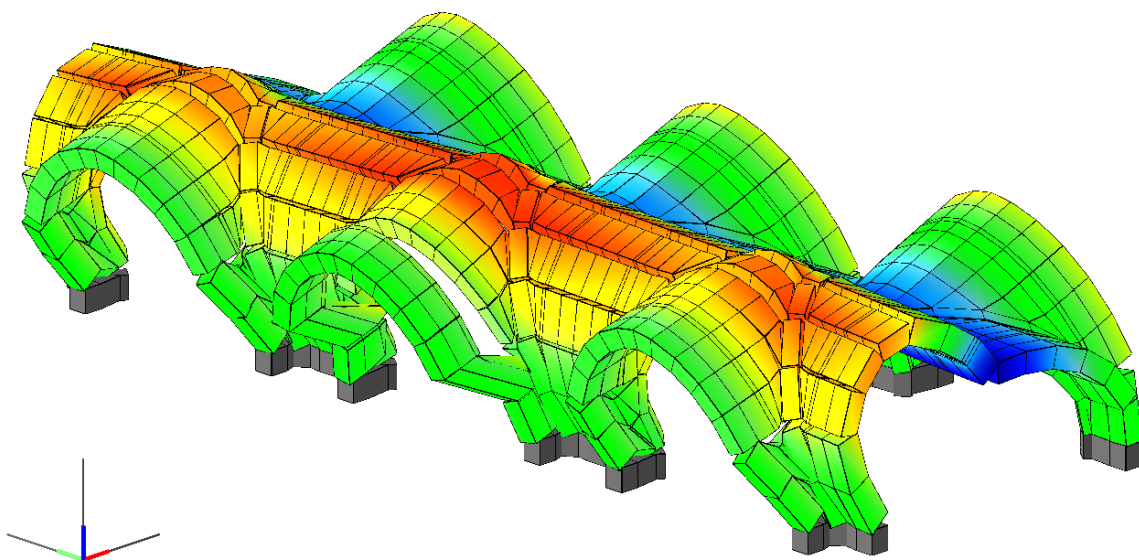
**Figure D163** – Plan view of deformation in +X direction.



**Figure D164** – Front view of deformation in +X direction.

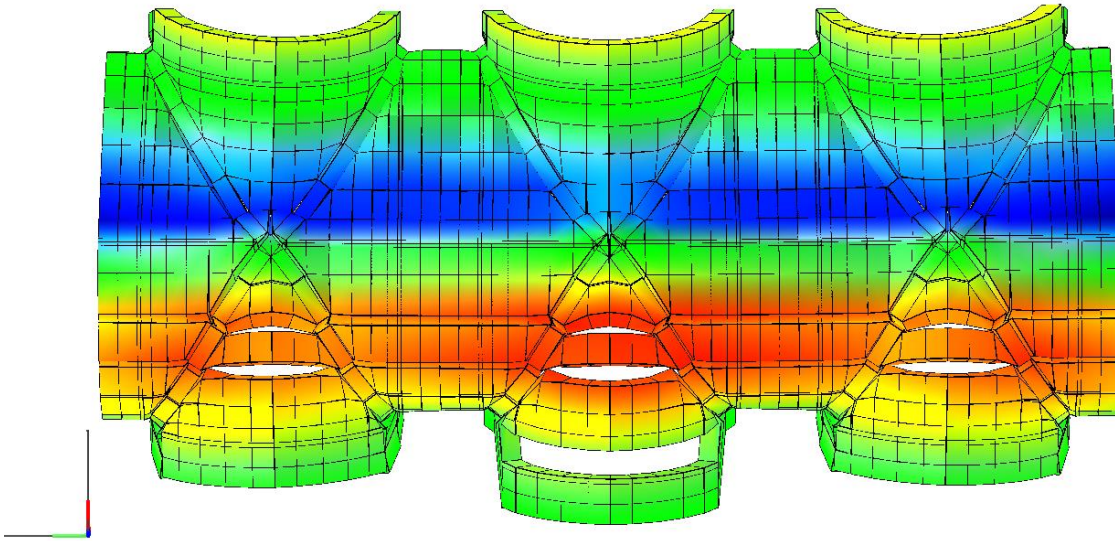


**Figure D165** – Legend for -X direction. Taken at  $U = 0.04m$ .

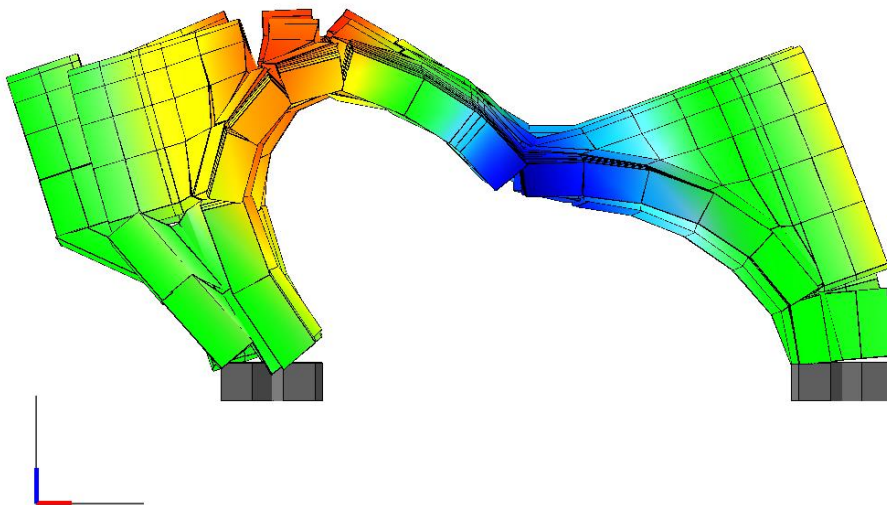


**Figure D166** – 3D view of deformation in -X direction.

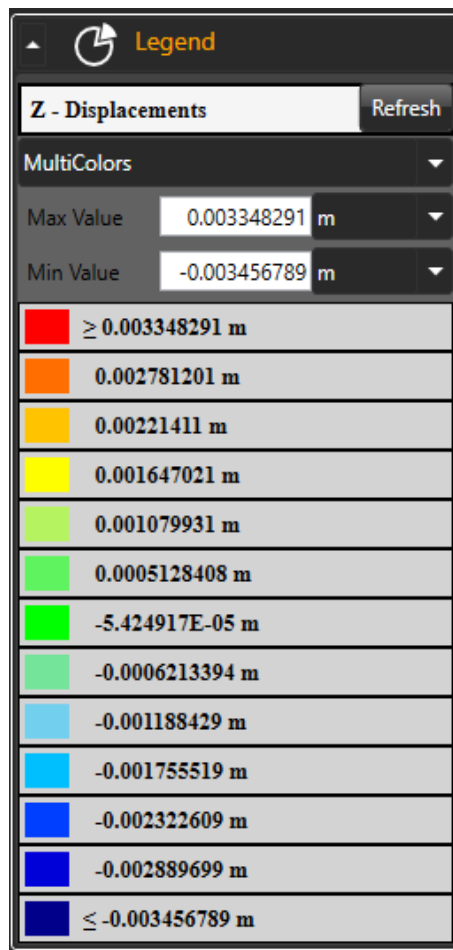




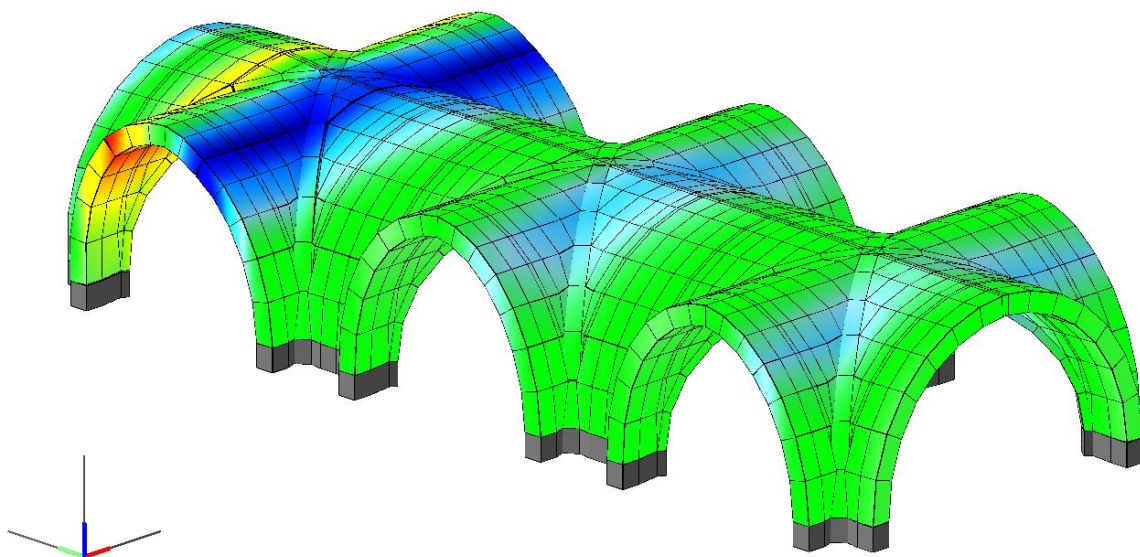
**Figure D167** – Plan view of deformation in -X direction.



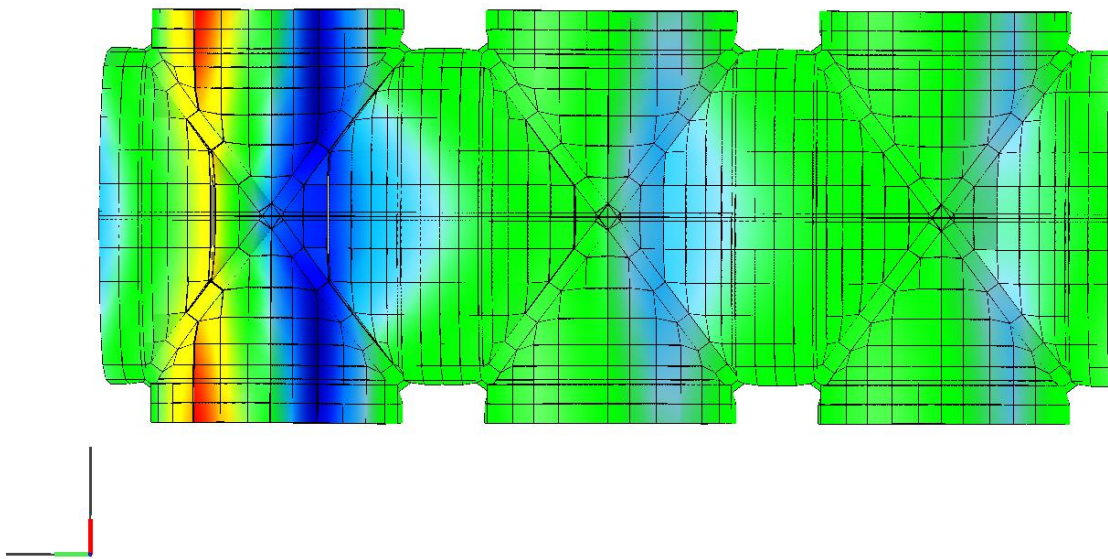
**Figure D168** – Front view of deformation in -X direction.



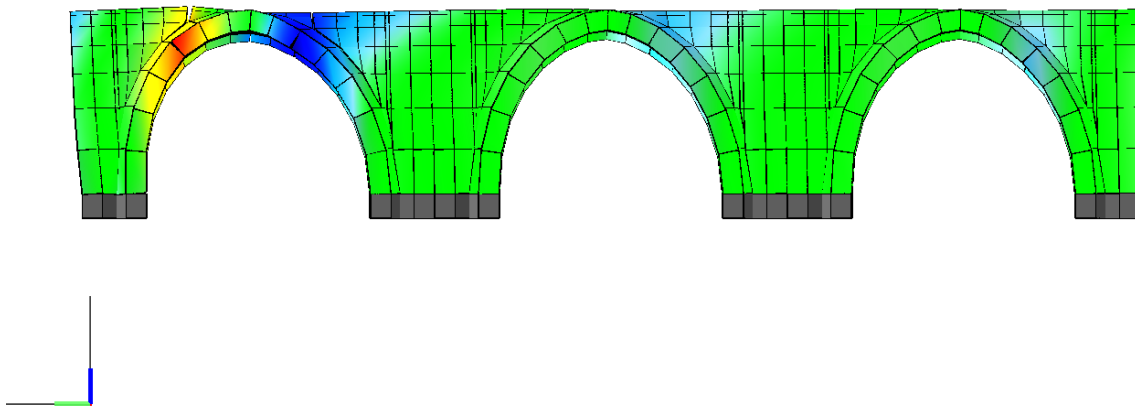
**Figure D169** – Legend for +Y direction. Taken at  $U = 0.002$  m.



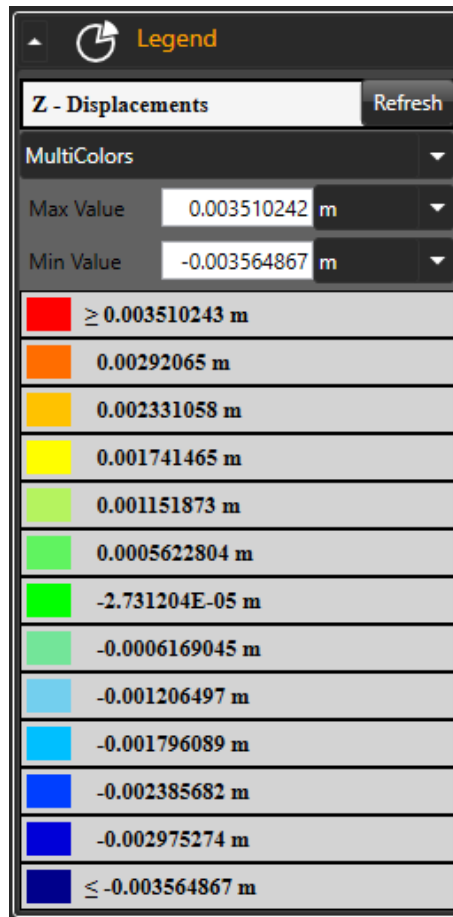
**Figure D170** – 3D view of deformation in +Y direction.



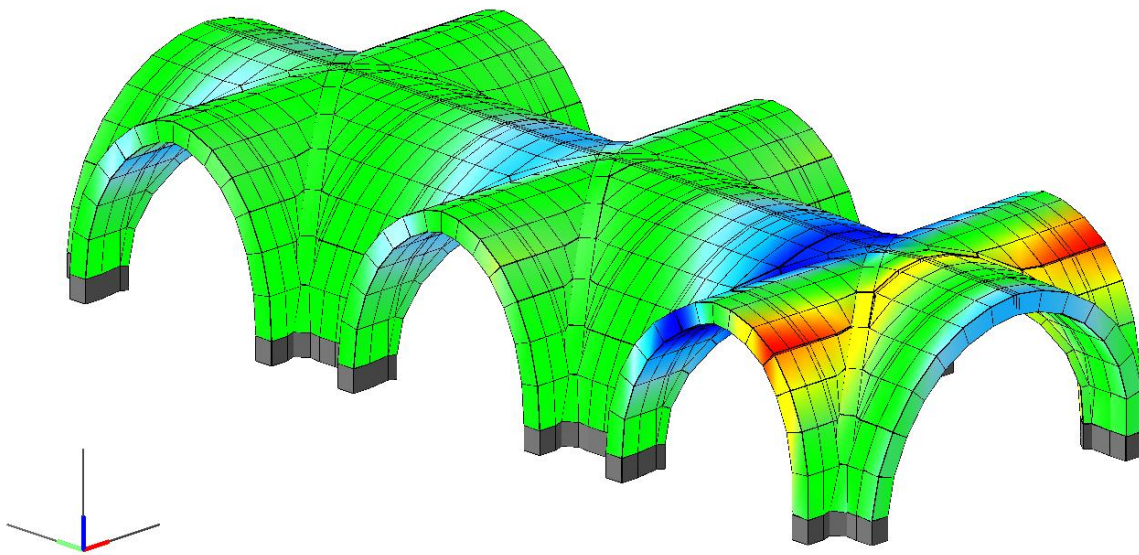
**Figure D171** – Plan view of deformation in +Y direction.



**Figure D172** – Side view of deformation in +Y direction.

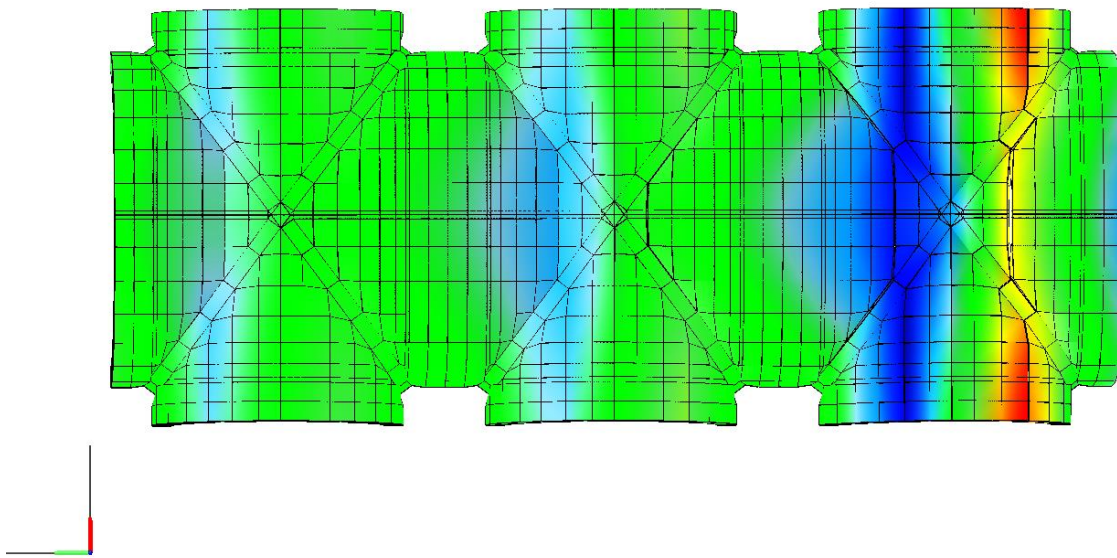


**Figure D173** – Legend for -Y direction. Taken at  $U = 0.002$  m.

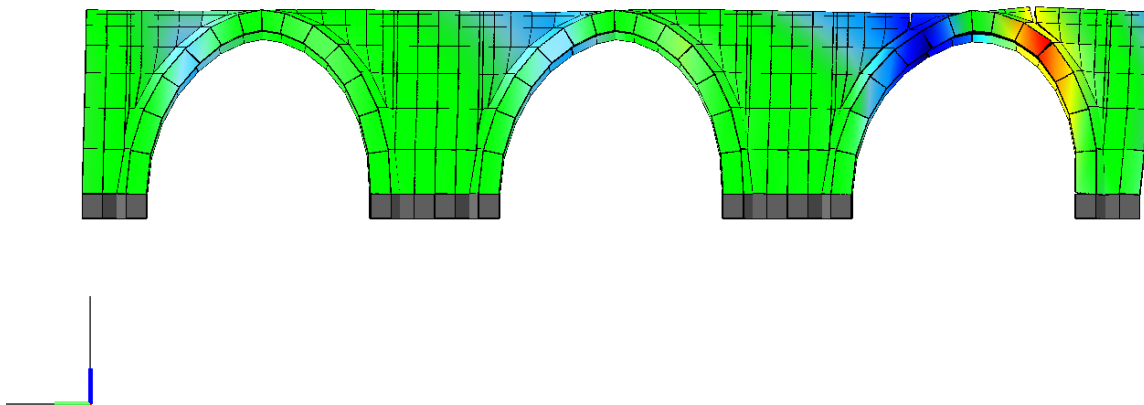


**Figure D174** – 3D view of deformation in -Y direction.





**Figure D175** – Plan view of deformation in -Y direction.



**Figure D176** – Side view of deformation in -Y direction.

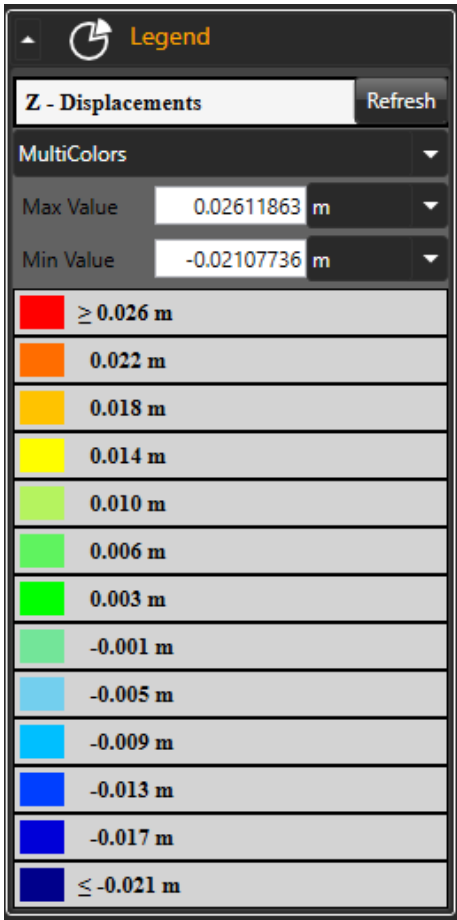


Figure D177 – Legend for +X direction. Taken at U = 0.03m.

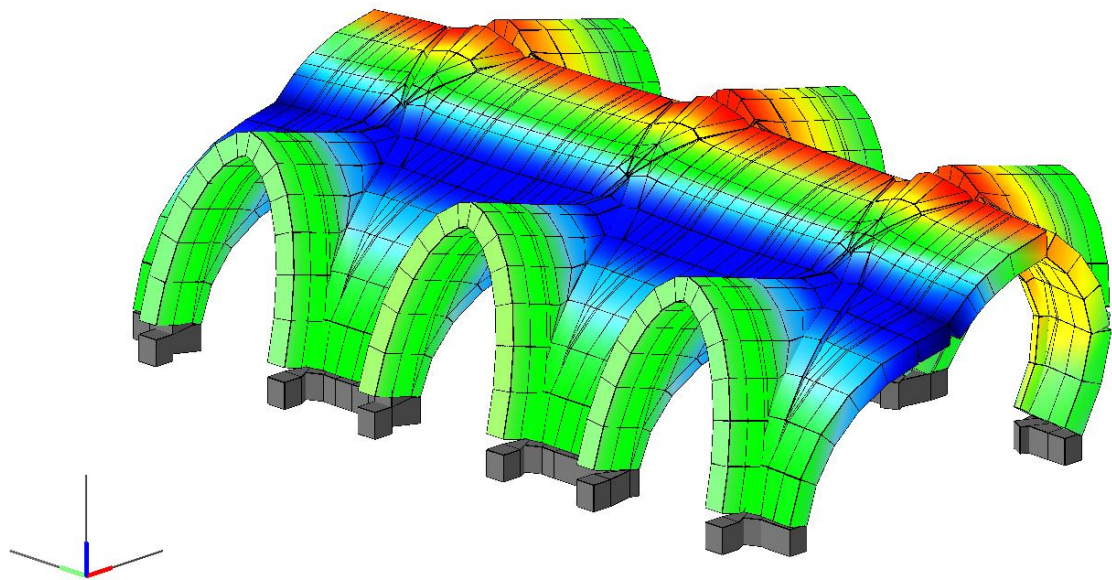
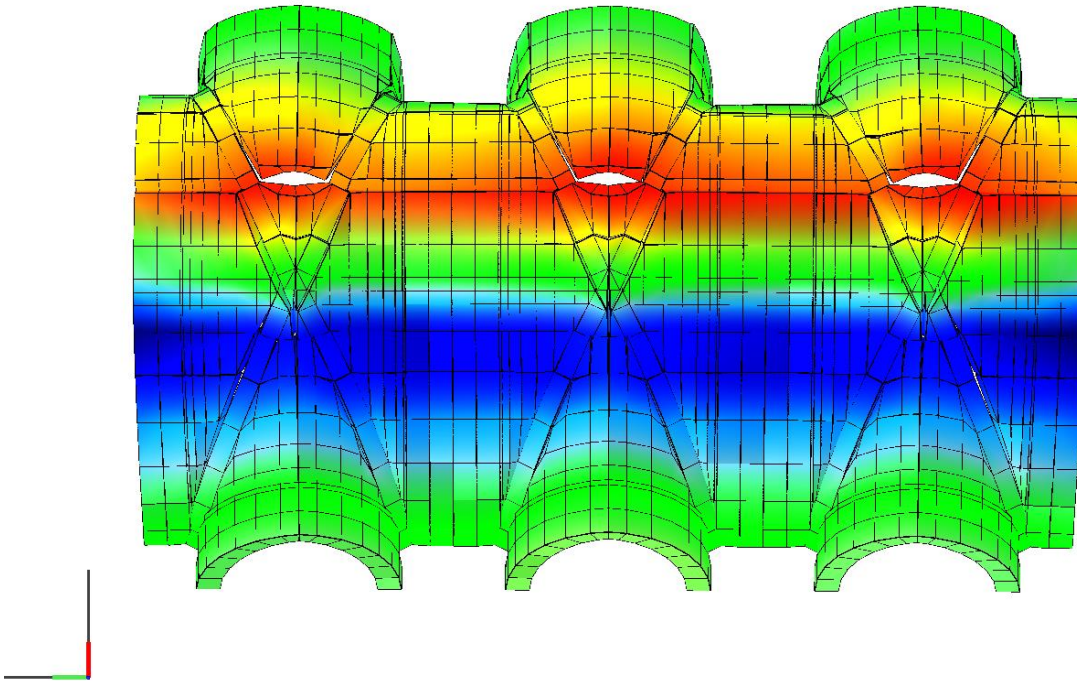
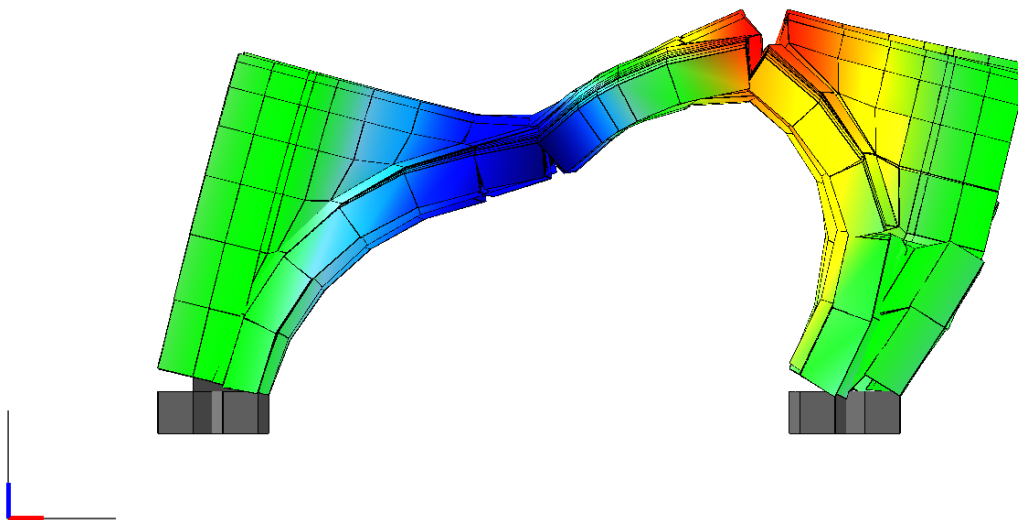


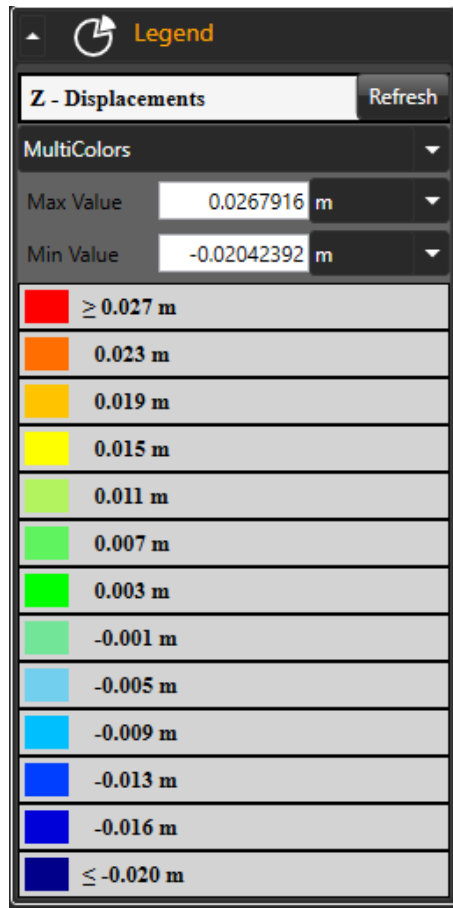
Figure D178 – 3D view of deformation in +X direction.



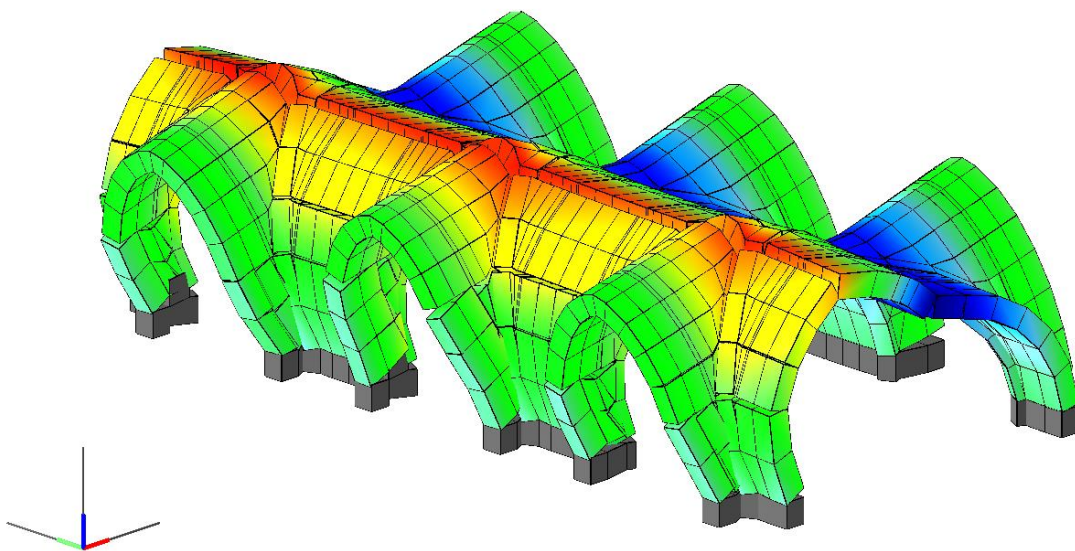
**Figure D179** – Plan view of deformation in +X direction.



**Figure D180** – Front view of deformation in +X direction.

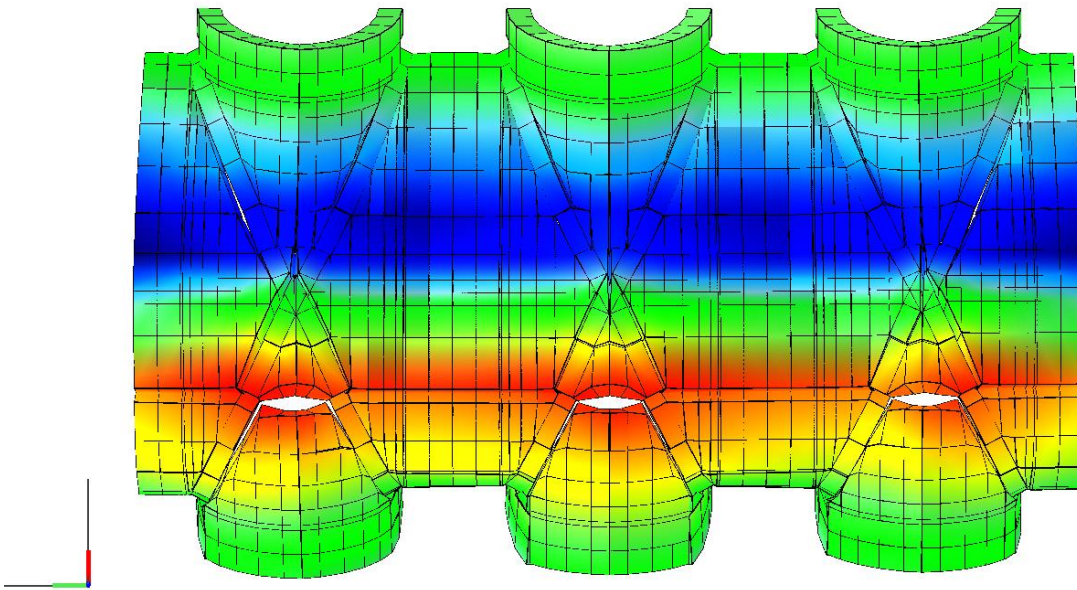


**Figure D181** – Legend for -X direction. Taken at  $U = 0.03$ m.

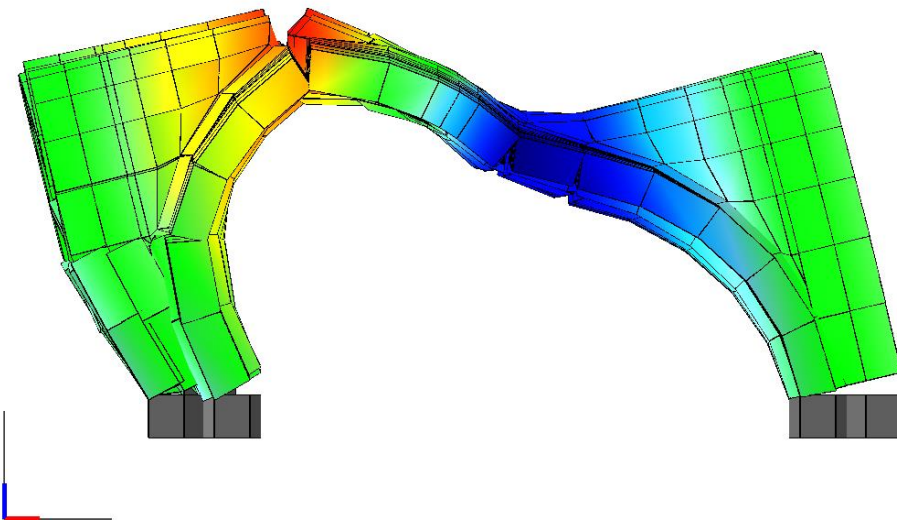


**Figure D182** – 3D view of deformation in -X direction.

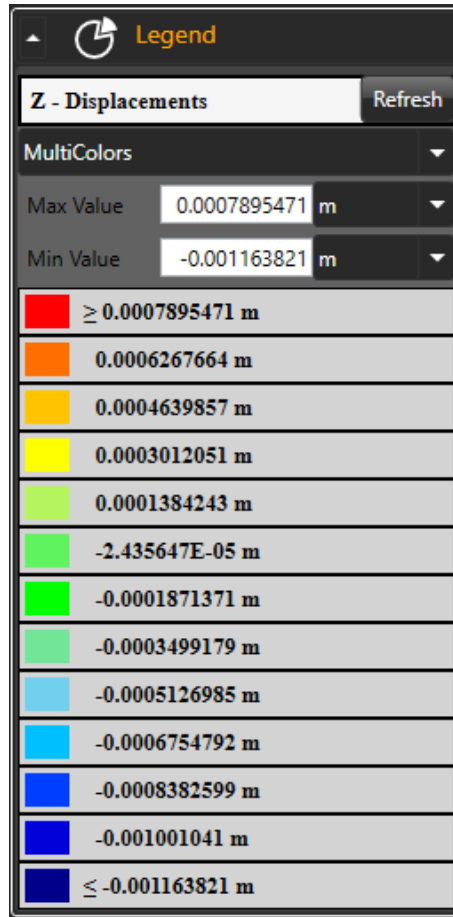




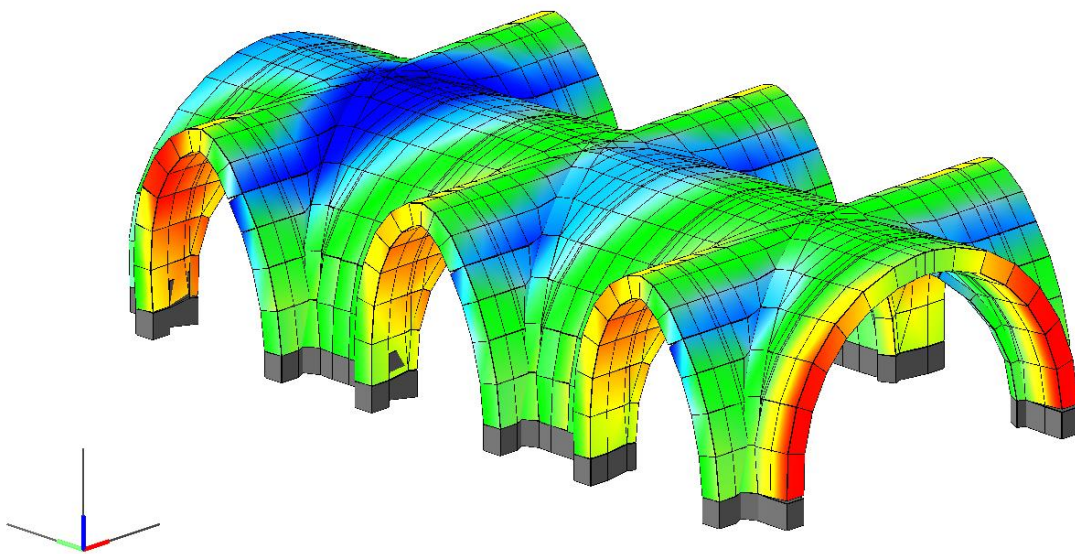
**Figure D183** – Plan view of deformation in -X direction.



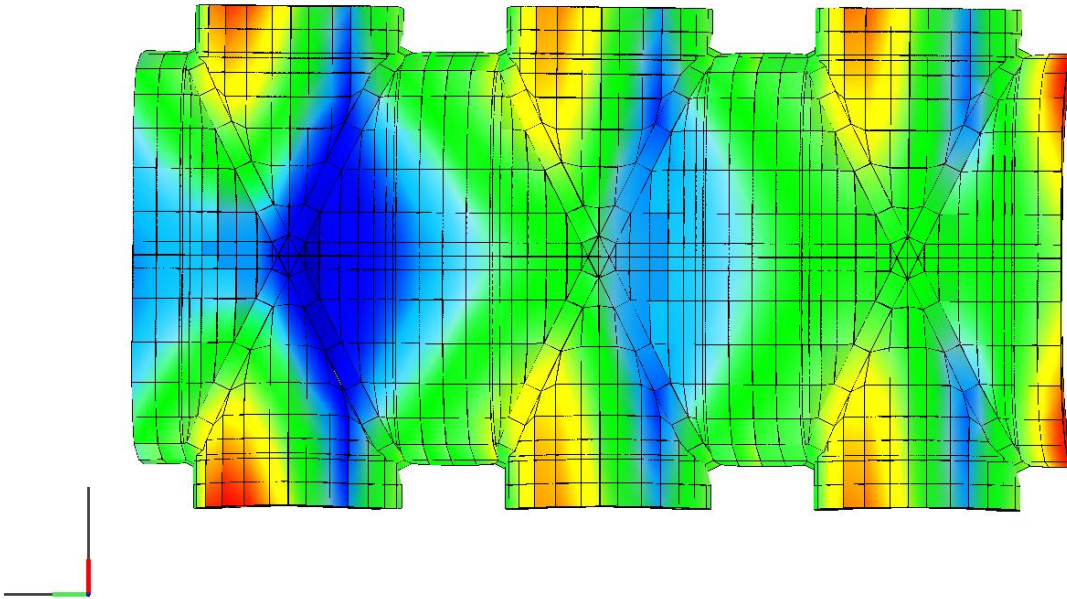
**Figure D184** – Front view of deformation in -X direction.



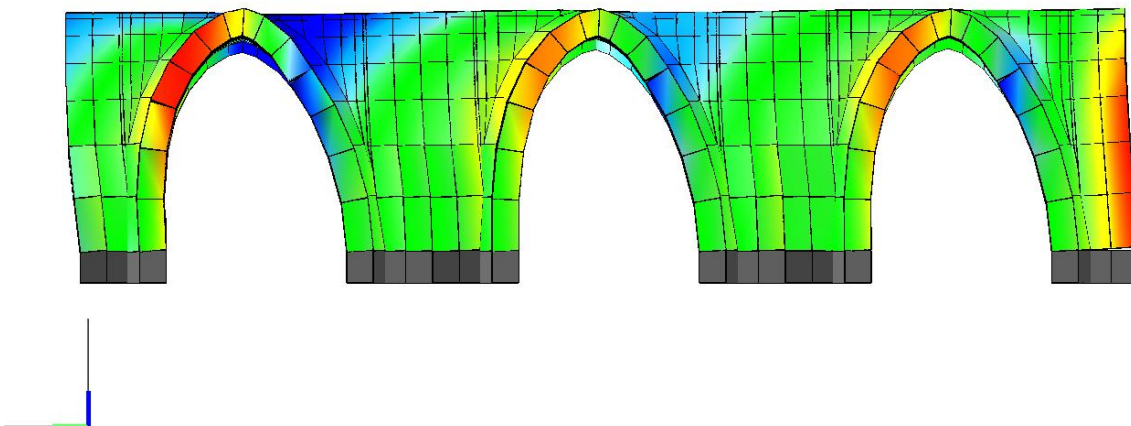
**Figure D185** – Legend for +Y direction. Taken at  $U = 0.002$ m.



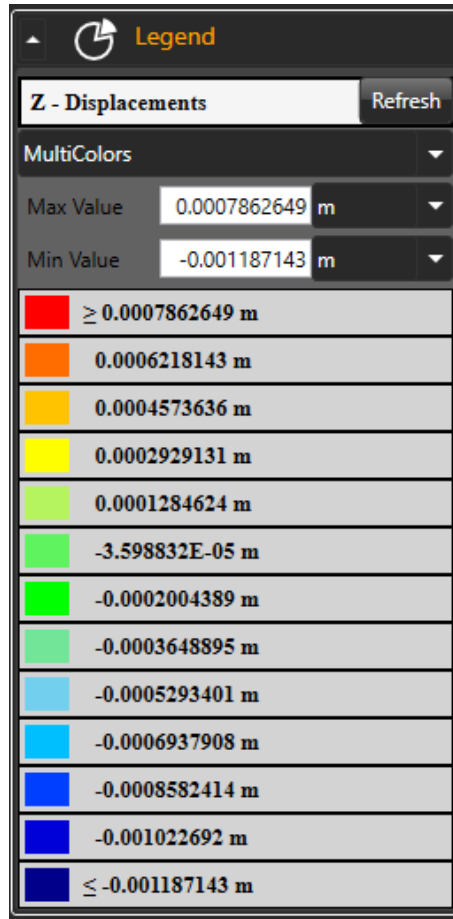
**Figure D186** – 3D view of deformation in +Y direction.



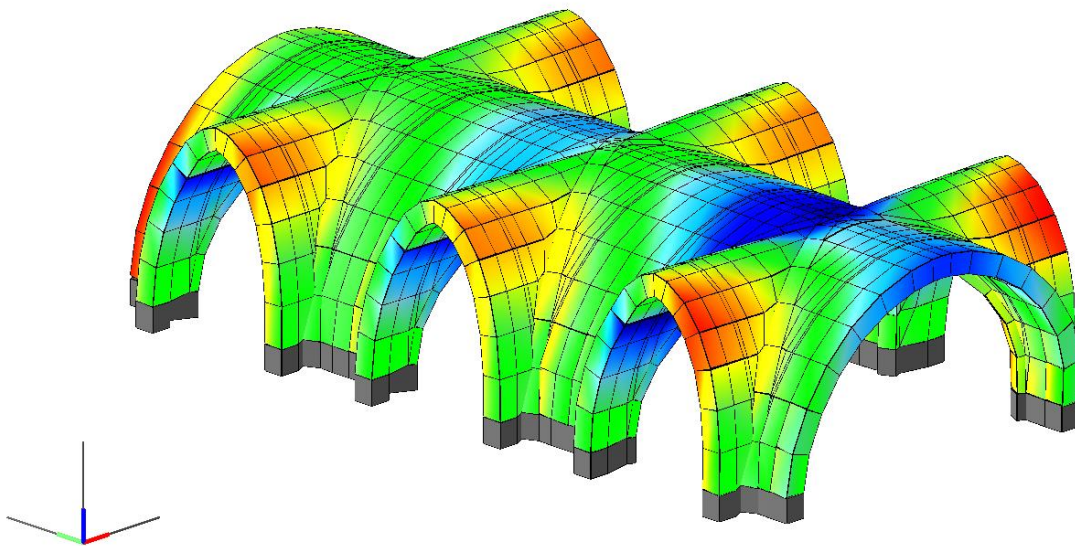
**Figure D187** – Plan view of deformation in +Y direction.



**Figure D188** – Side view of deformation in +Y direction.

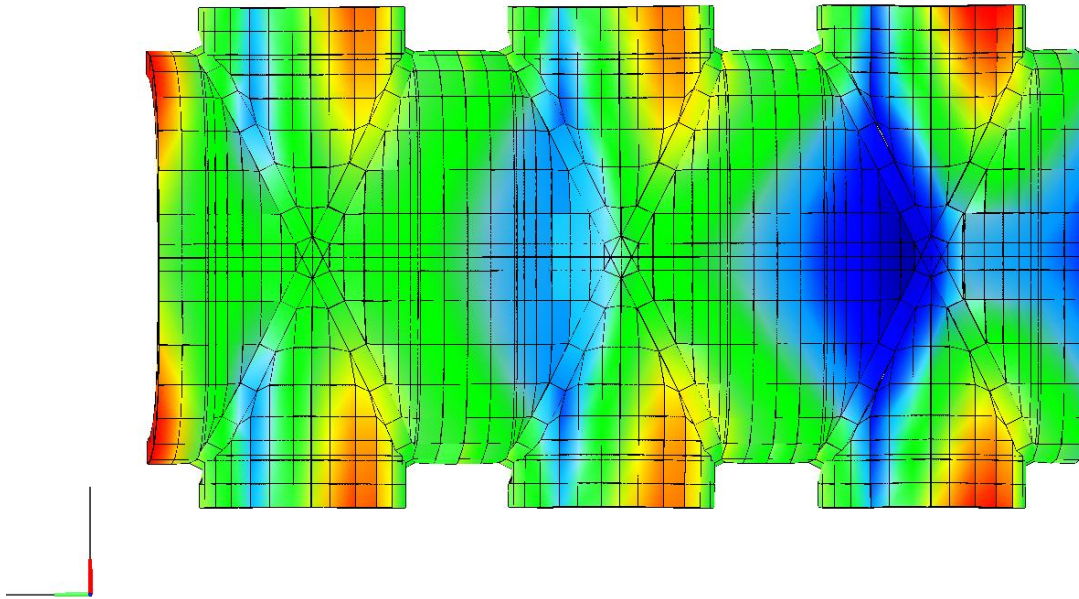


**Figure D189** – Legend for -Y direction. Taken at  $U = 0.002$  m.

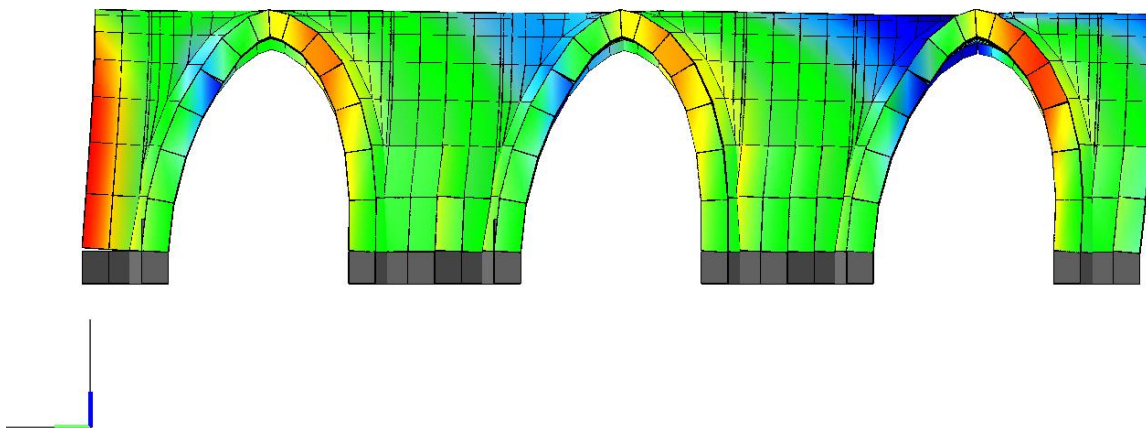


**Figure D190** – 3D view of deformation in -Y direction.





**Figure D191** – Plan view of deformation in -Y direction.



**Figure D192** – Side view of deformation in -Y direction.

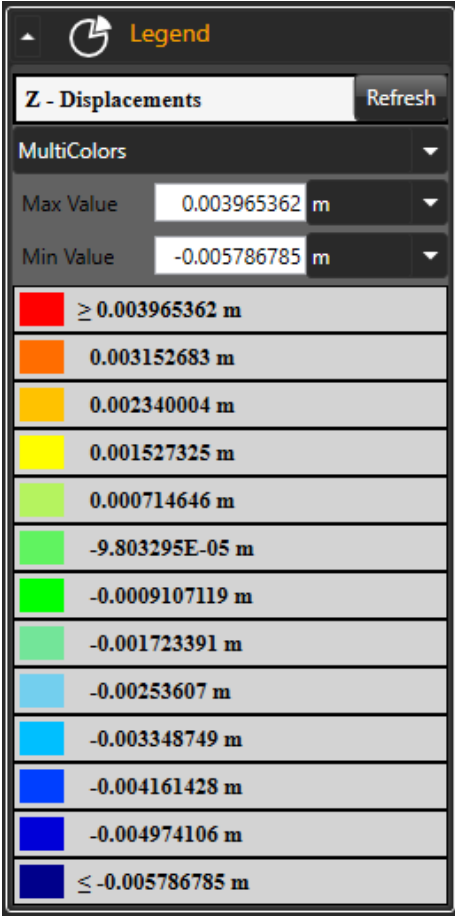


Figure D193 – Legend for +X direction. Taken at  $U = 0.002\text{m}$ .

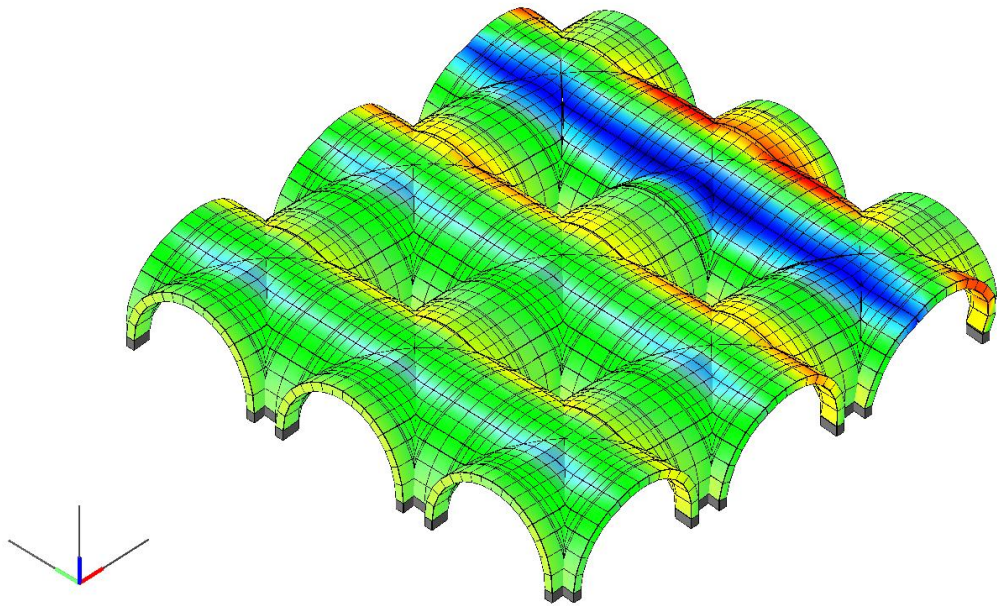
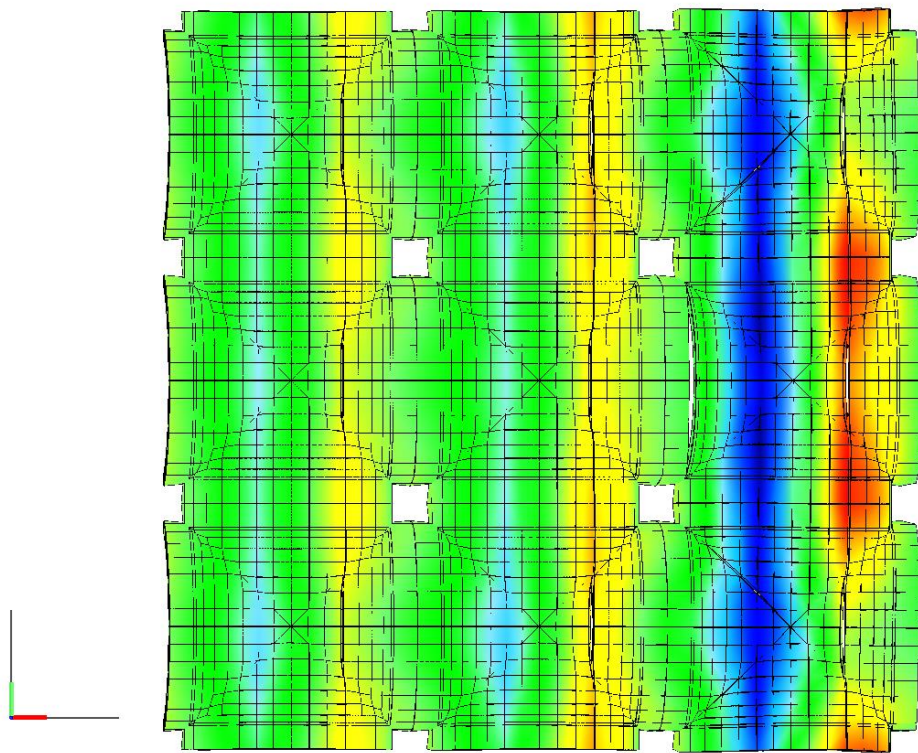
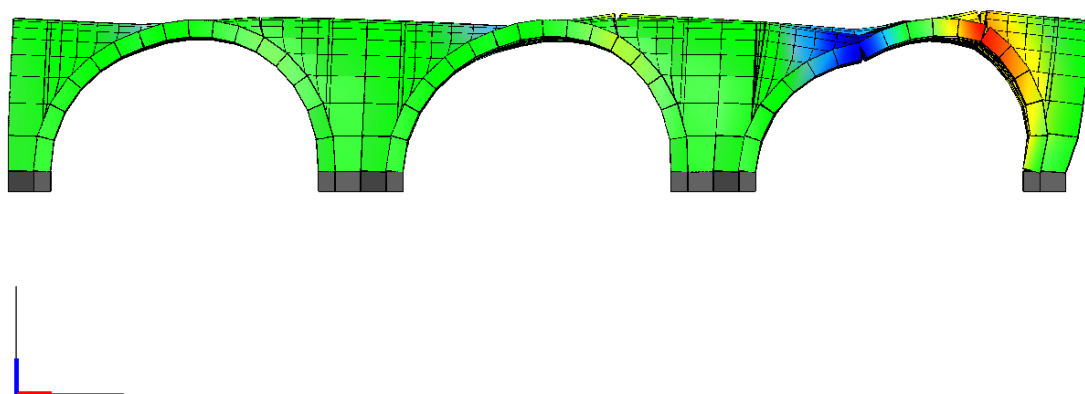


Figure D194 – 3D view of deformation in +X direction.

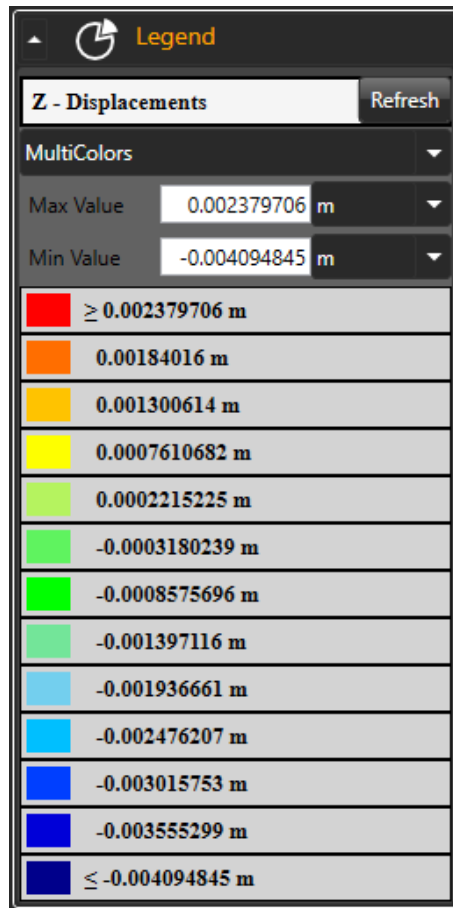


**Figure D195** – Plan view of deformation in +X direction.

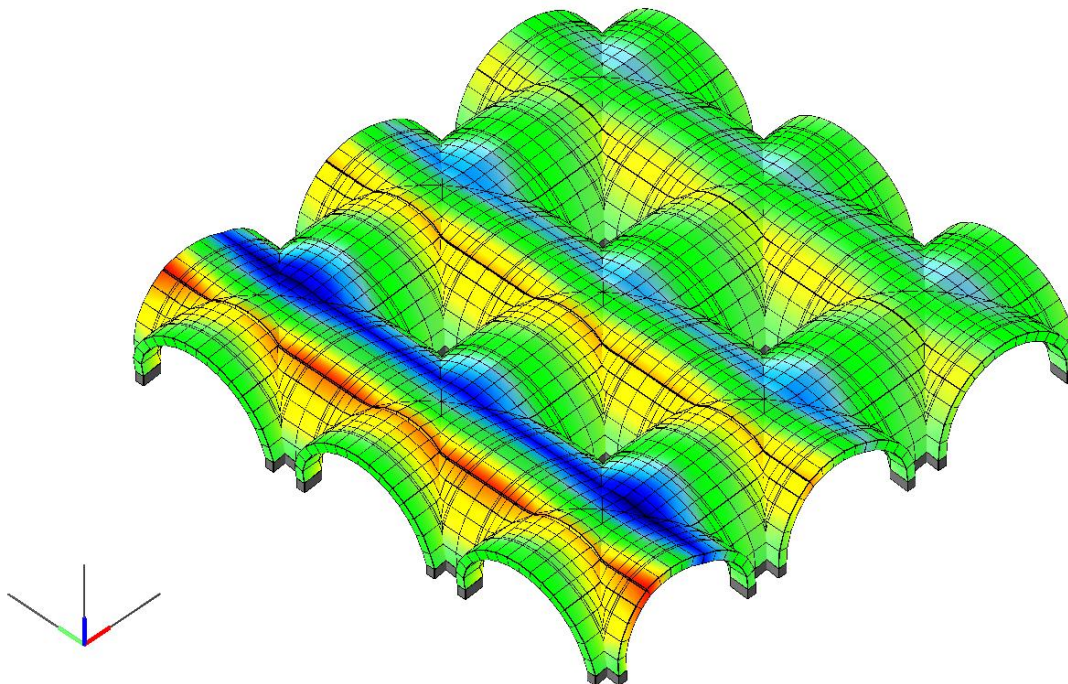


**Figure D196** – Front view of deformation in +X direction.





**Figure D197** – Legend for -X direction. Taken at  $U = 0.002$  m.



**Figure D198** – 3D view of deformation in -X direction.

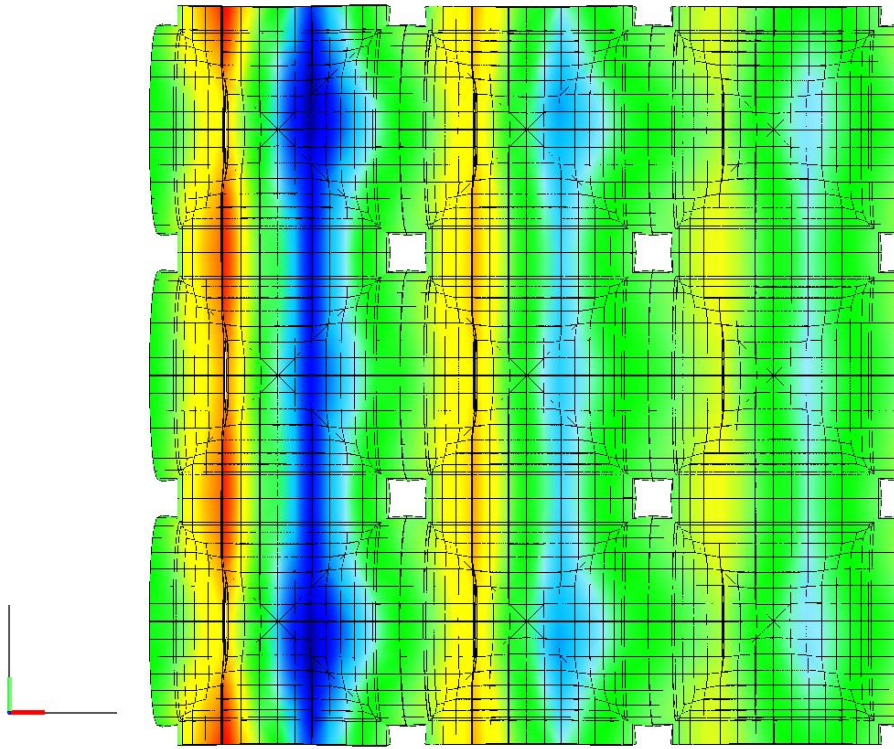


Figure D199 – Plan view of deformation in -X direction.

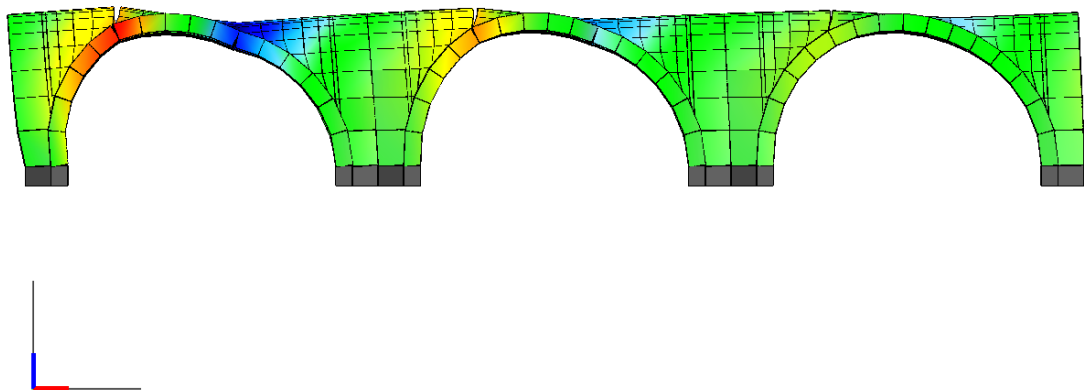
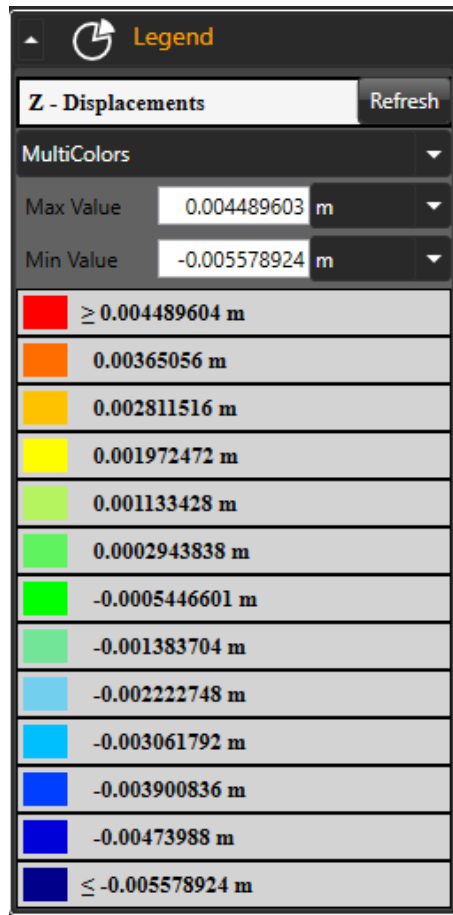
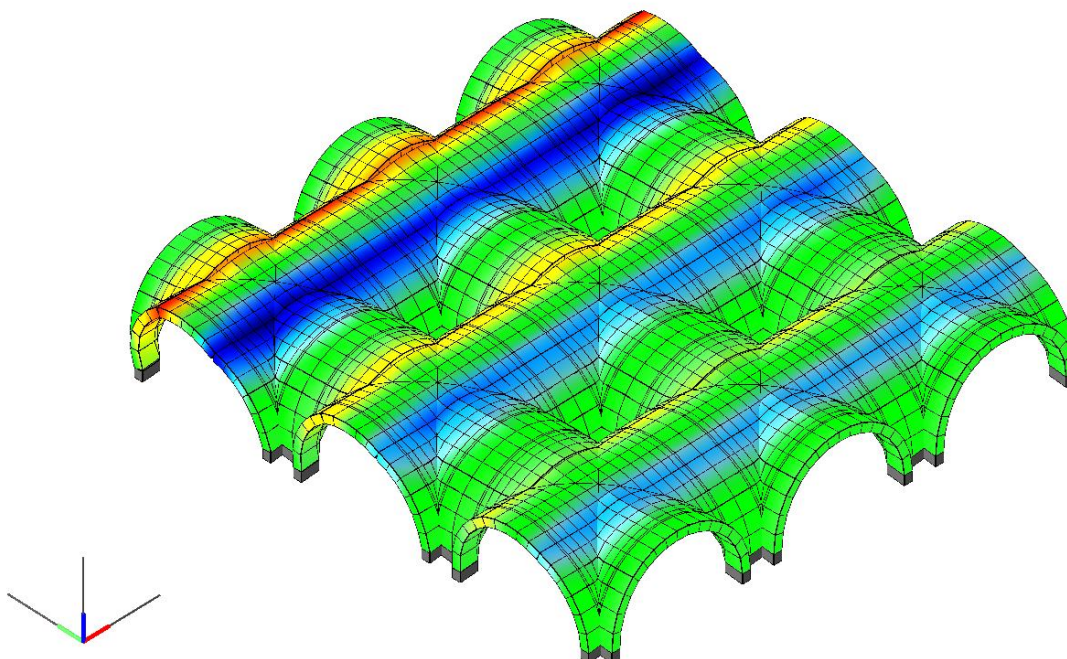


Figure D200 – Front view of deformation in -X direction.

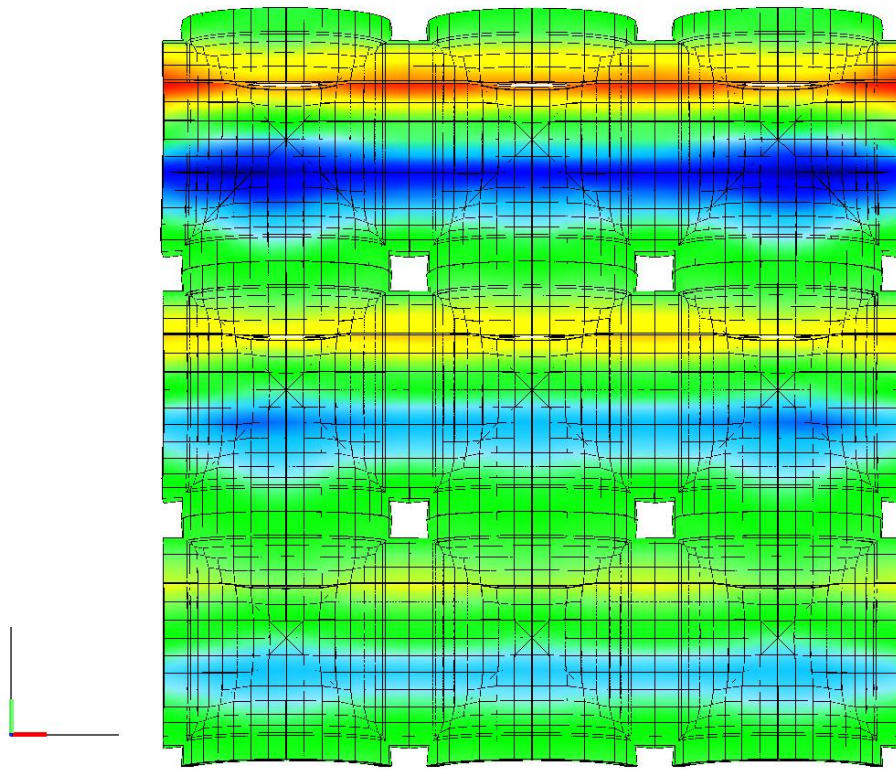


**Figure D201** – Legend for +Y direction. Taken at  $U = 0.004m$ .

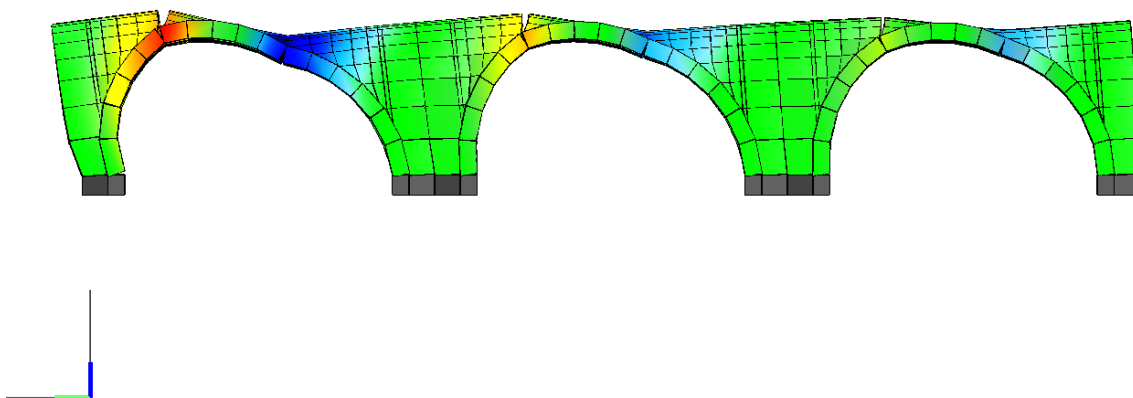


**Figure D202** – 3D view of deformation in +Y direction.

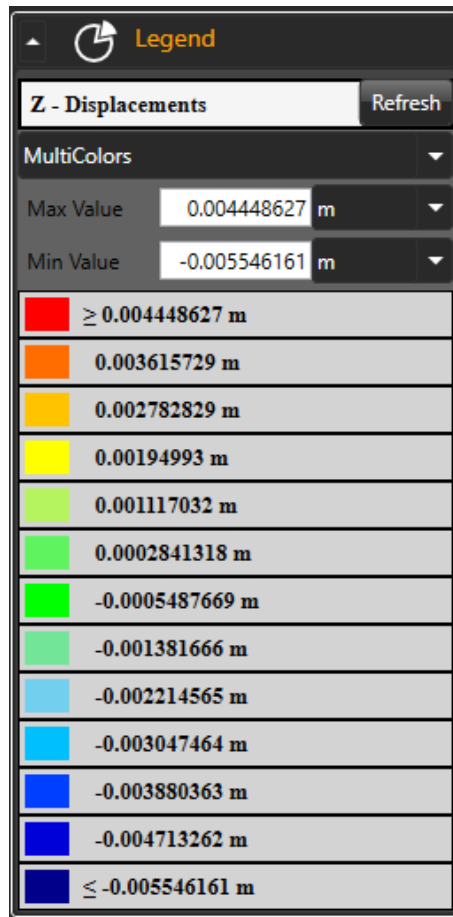




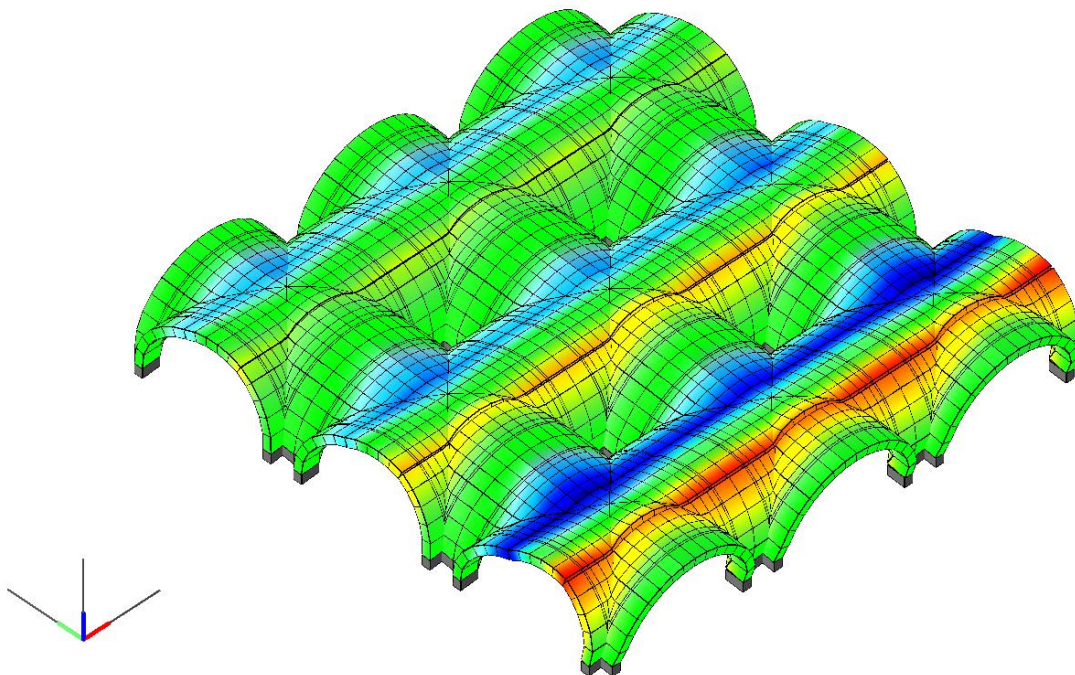
**Figure D203** – Plan view of deformation in +Y direction.



**Figure D204** – Side view of deformation in +Y direction.

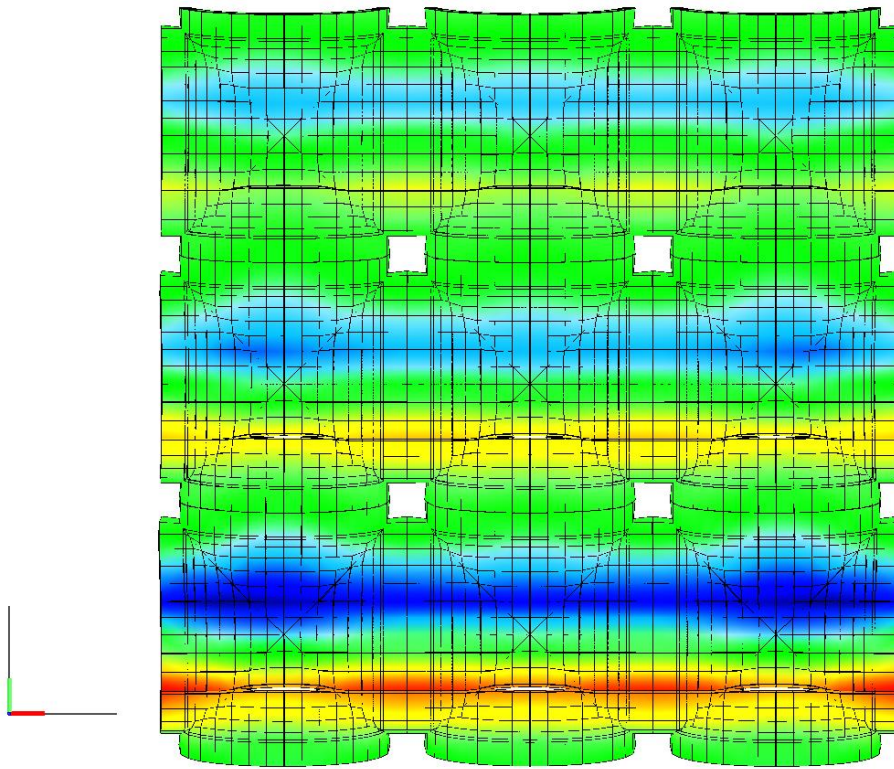


**Figure D205** – Legend for -Y direction. Taken at  $U = 0.004\text{m}$ .

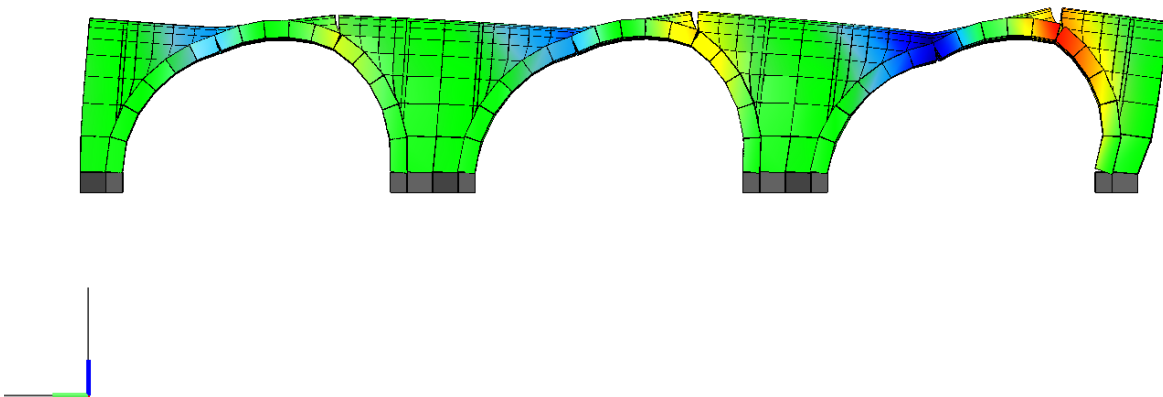


**Figure D206** – 3D view of deformation in -Y direction.





**Figure D207** – Plan view of deformation in -Y direction.



**Figure D208** – Side view of deformation in -Y direction.

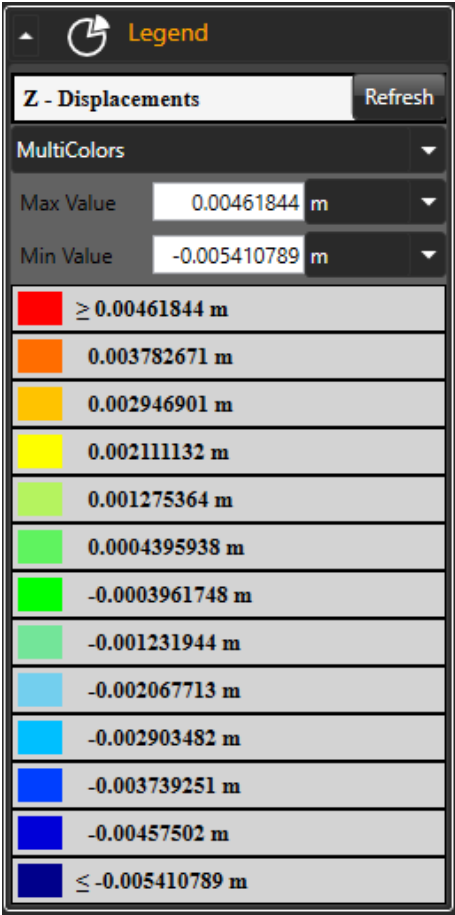


Figure D209 – Legend for +X direction. Taken at  $U = 0.004\text{m}$ .

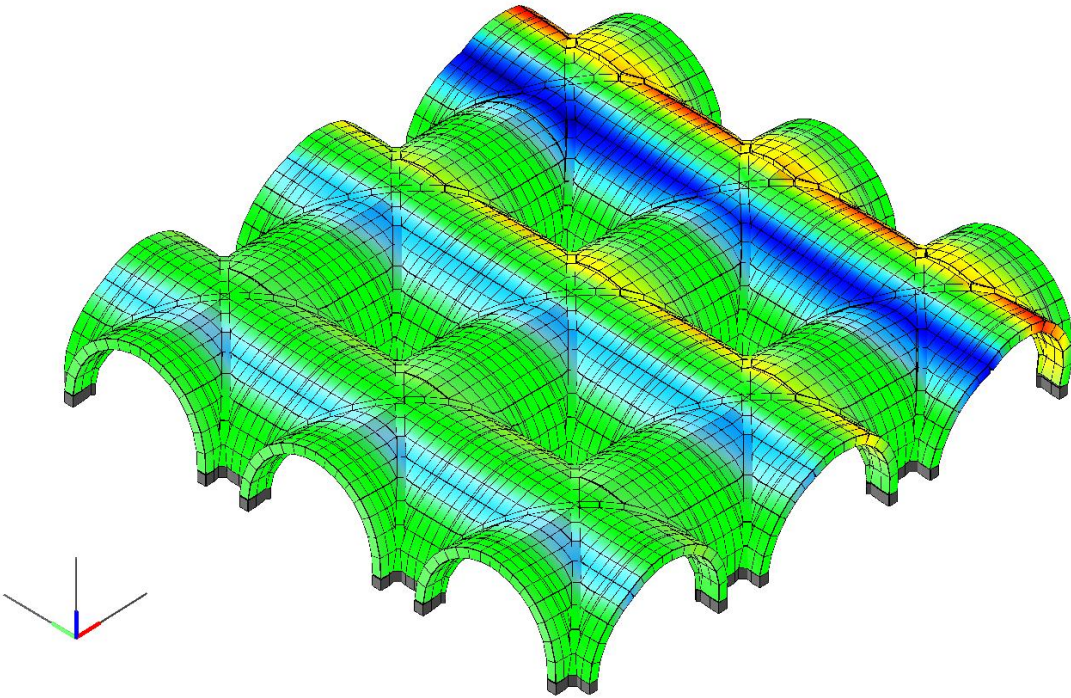
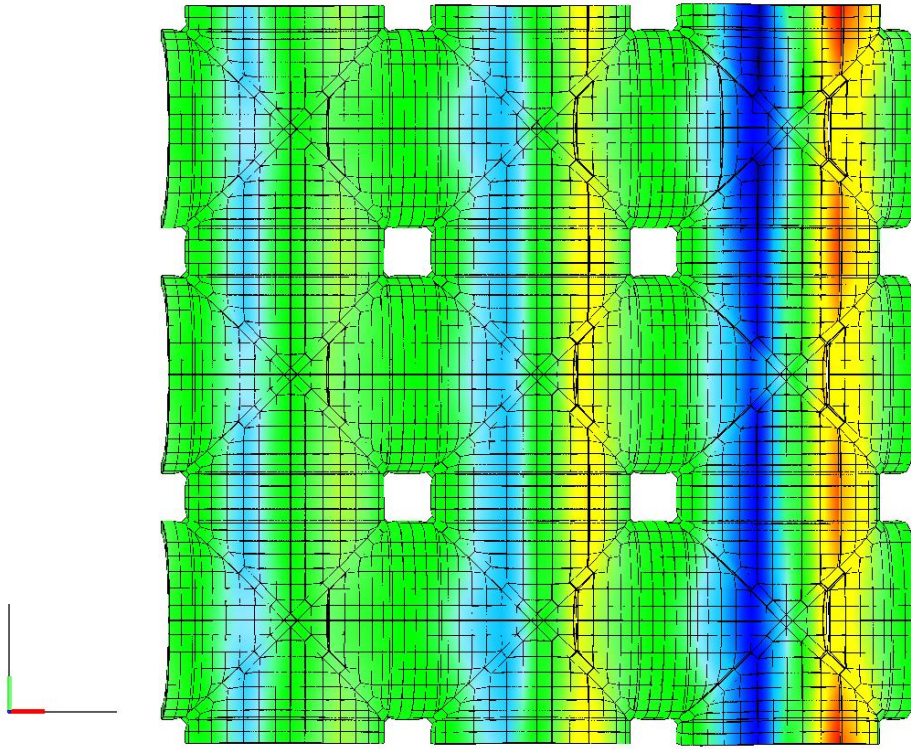
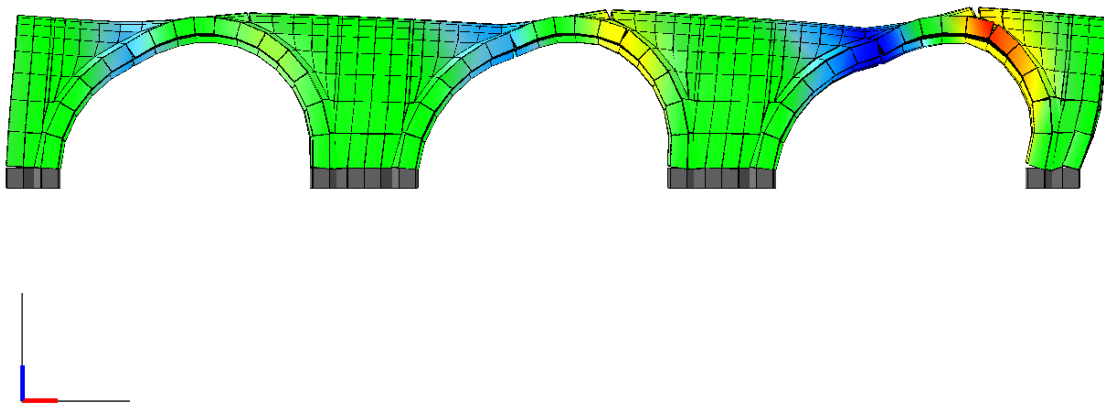


Figure D210 – 3D view of deformation in +X direction.

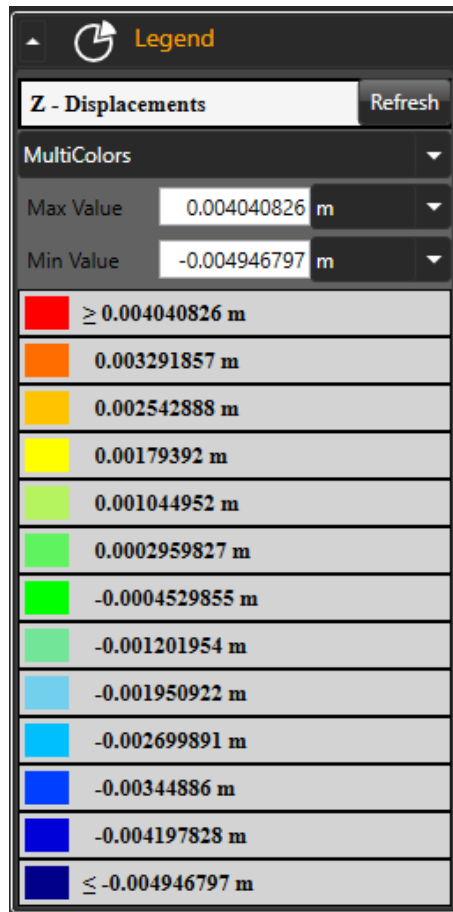


**Figure D211** – Plan view of deformation in +X direction.

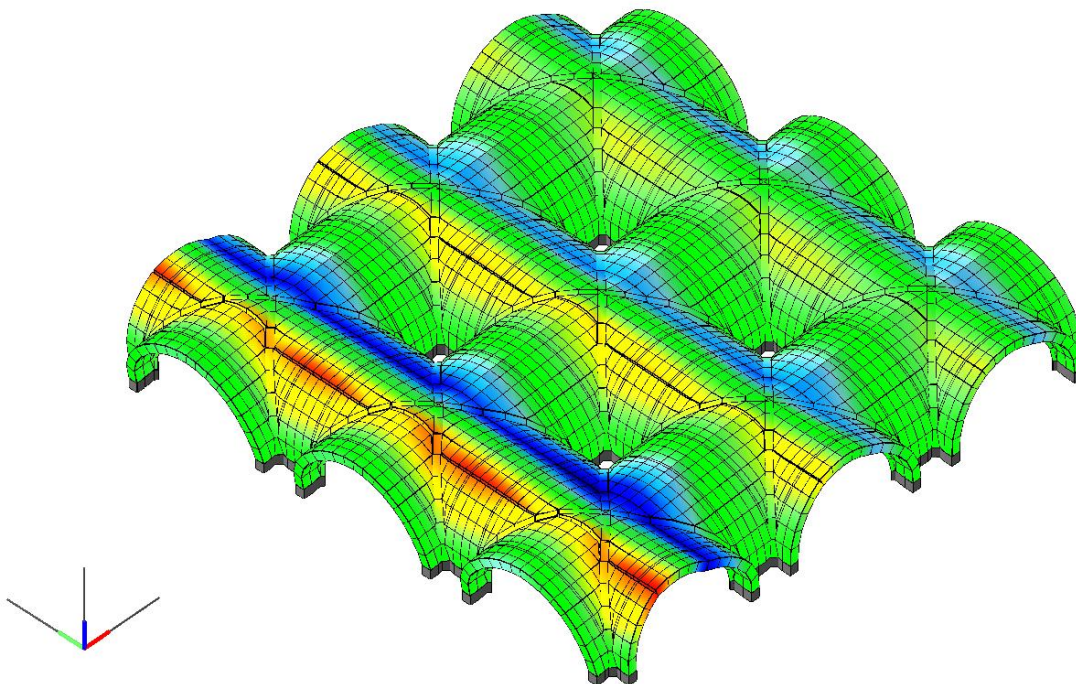


**Figure D212** – Front view of deformation in +X direction.

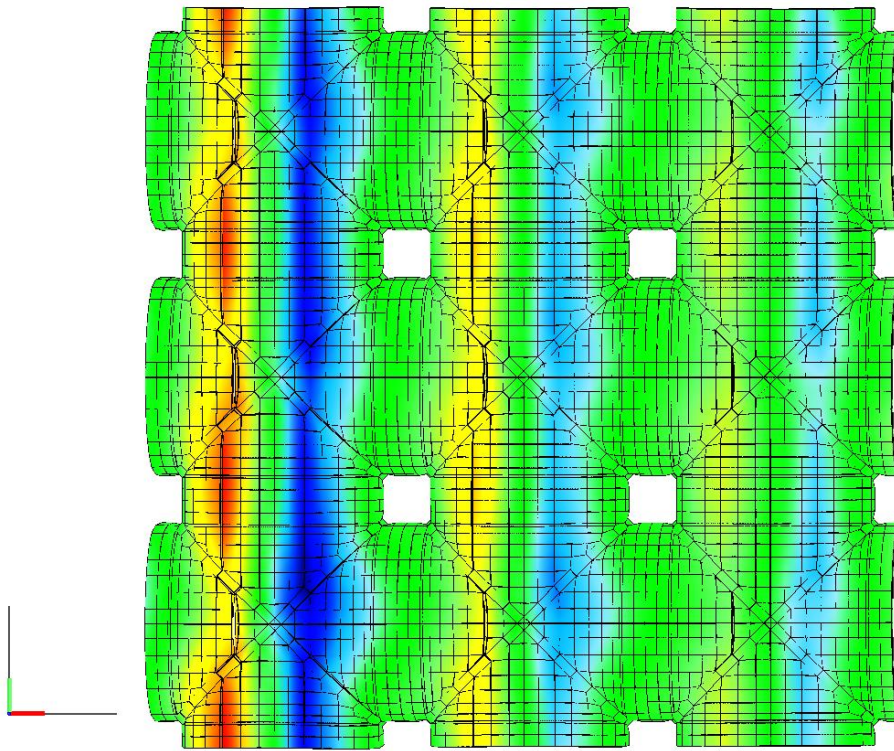




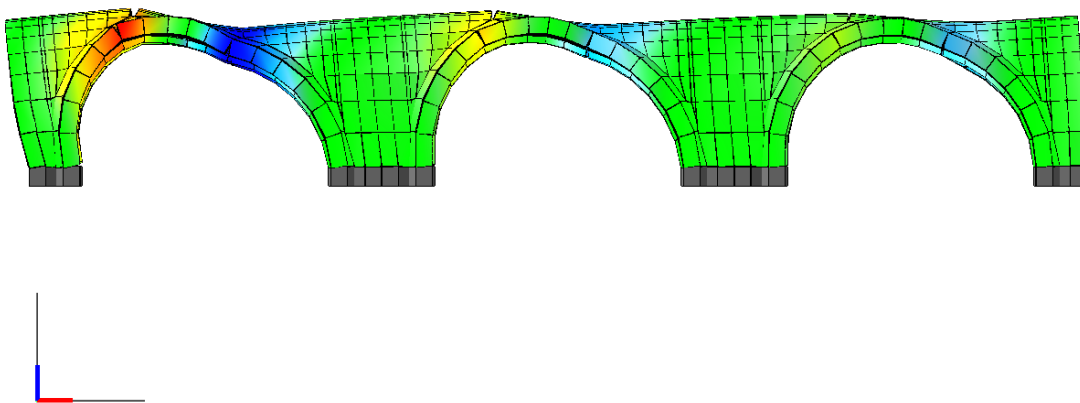
**Figure D213** – Legend for -X direction. Taken at  $U = 0.003$  m.



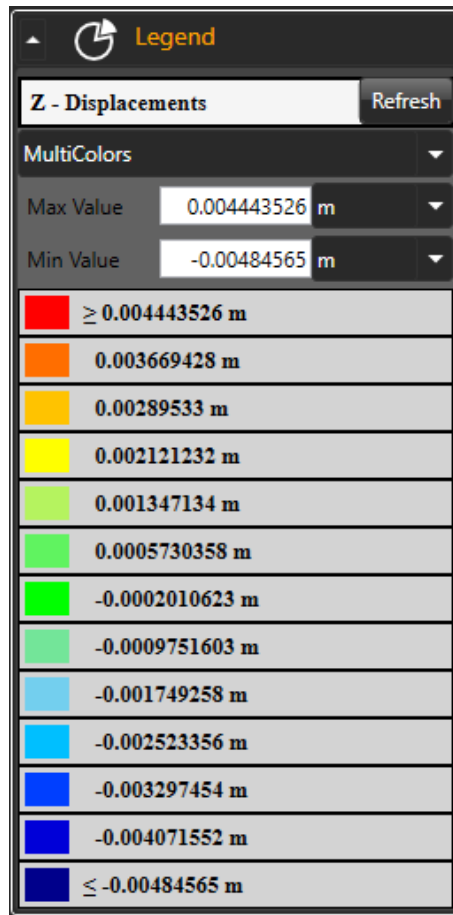
**Figure D214** – 3D view of deformation in -X direction.



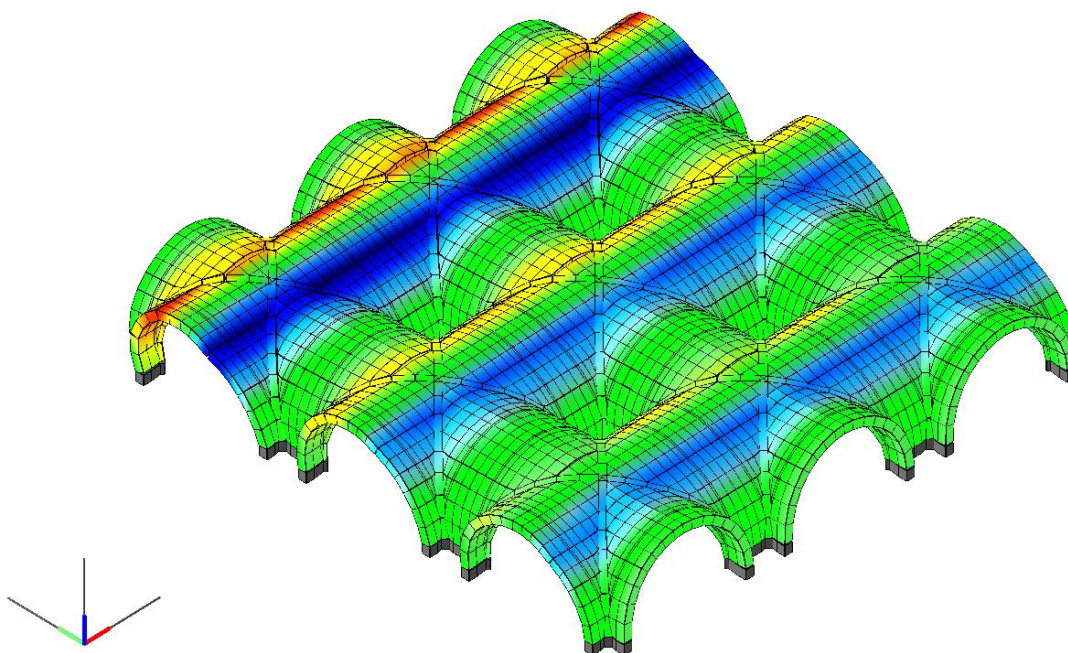
**Figure D215** – Plan view of deformation in -X direction.



**Figure D216** – Front view of deformation in -X direction.

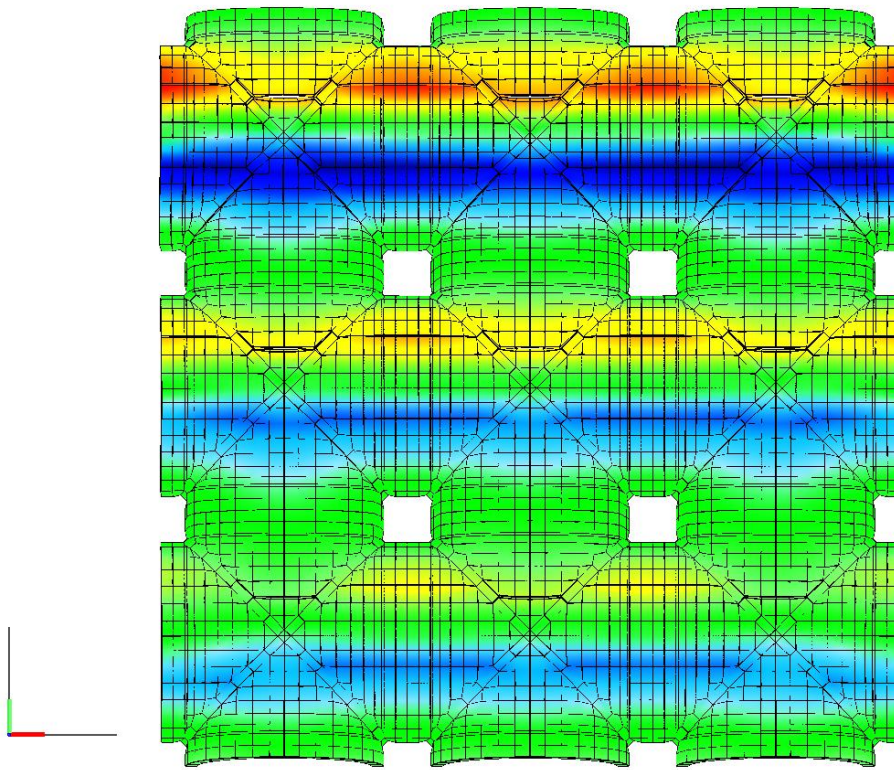


**Figure D217** – Legend for +Y direction. Taken at  $U = 0.004$ m.

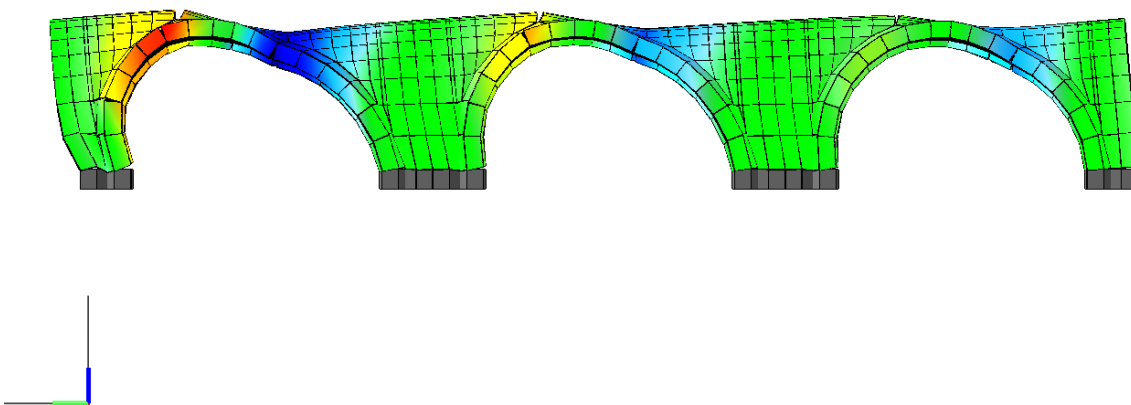


**Figure D218** – 3D view of deformation in +Y direction.

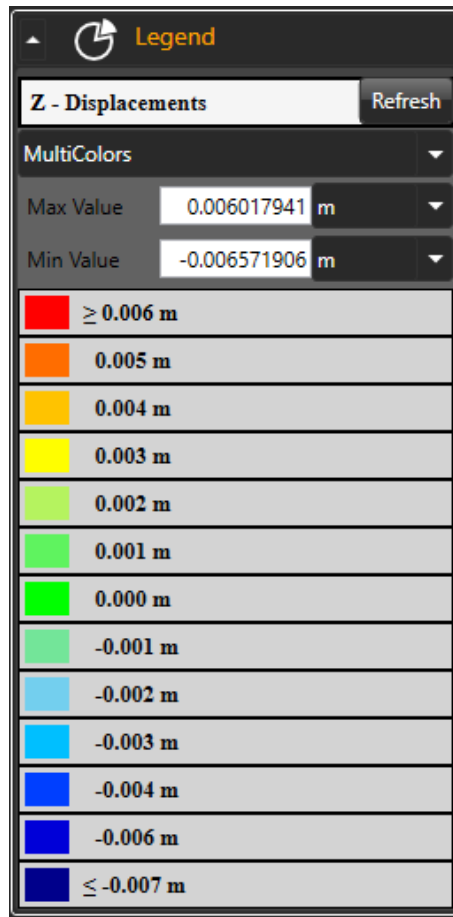




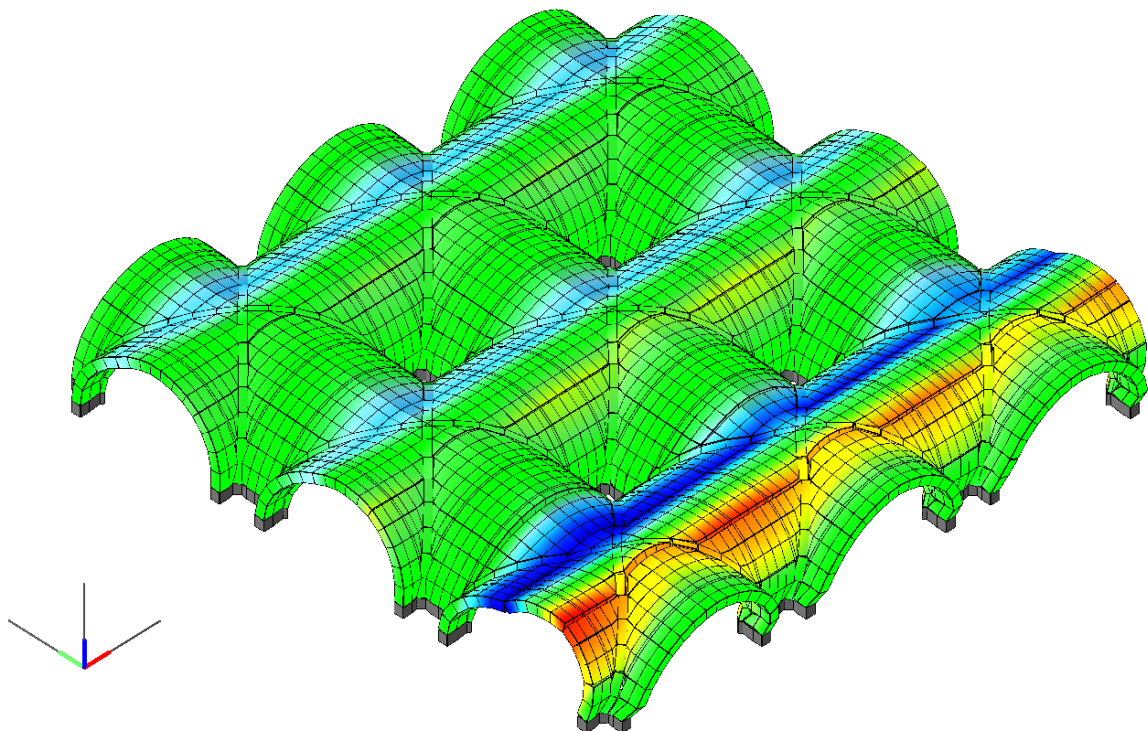
**Figure D219** – Plan view of deformation in +Y direction.



**Figure D220** – Side view of deformation in +Y direction.

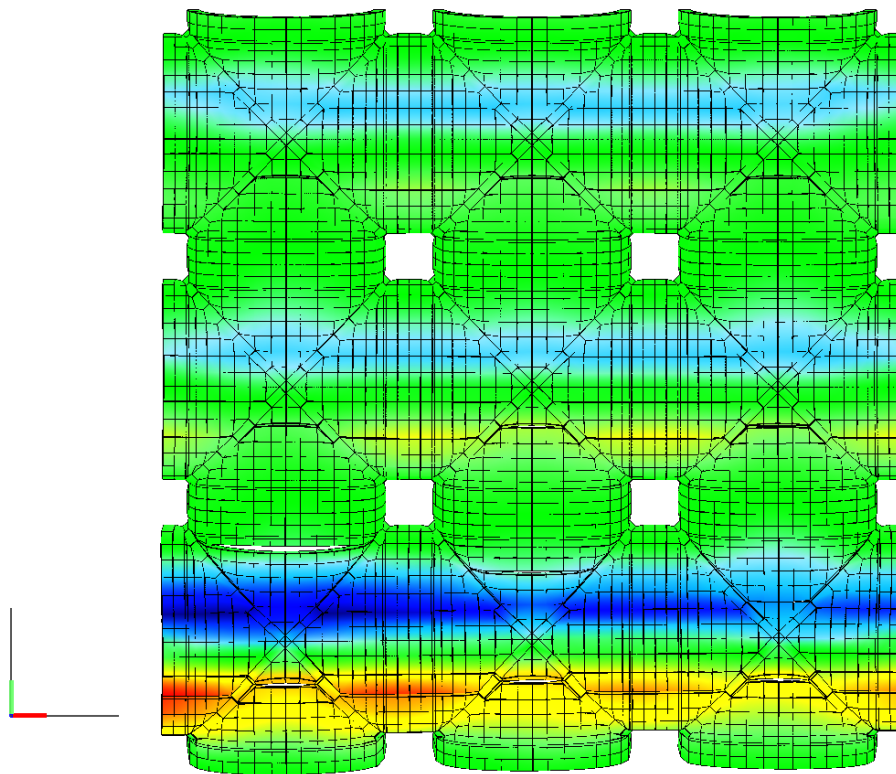


**Figure D221** – Legend for -Y direction. Taken at  $U = 0.004\text{m}$ .

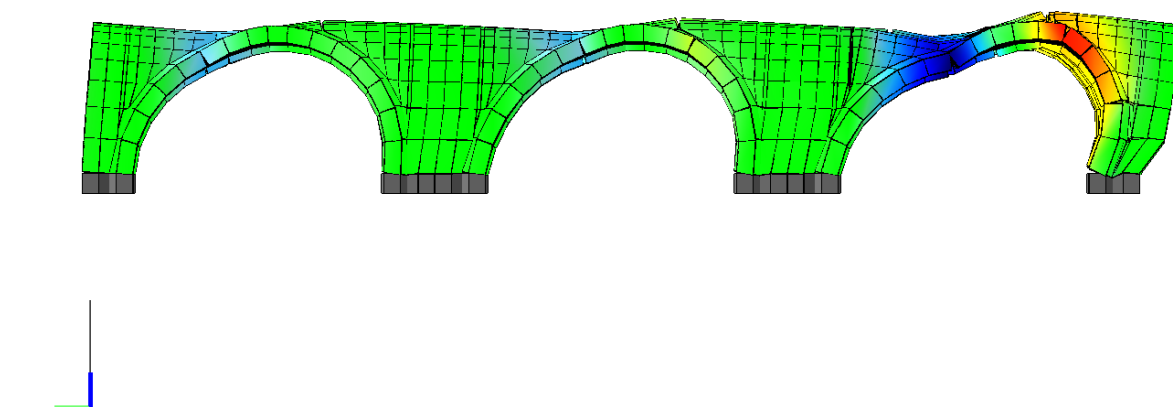


**Figure D222** – 3D view of deformation in -Y direction.





**Figure D223** – Plan view of deformation in -Y direction.



**Figure D224** – Side view of deformation in -Y direction.

## Appendix E: Seismic Vulnerability Criteria – Pushover ‘Massa’

**Note:** Only results of pushover ‘Massa’ are presented since pushover ‘Modal’ are presented in subsection 5.5.1.

| Test | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| CV1  | +X Massa                    | DL          | 0.098                      | 0.071                    | 1.39                       |
|      |                             | SD          | 0.702                      | 0.101                    | 6.94                       |
|      |                             | NC          | 0.938                      | 0.643                    | 1.46                       |
|      | -X Massa                    | DL          | 0.106                      | 0.071                    | 1.49                       |
|      |                             | SD          | 0.432                      | 0.101                    | 4.28                       |
|      |                             | NC          | 0.006                      | 0.635                    | 0.91                       |
|      | +Y Massa                    | DL          | 0.098                      | 0.071                    | 1.39                       |
|      |                             | SD          | 0.703                      | 0.101                    | 6.96                       |
|      |                             | NC          | 0.938                      | 0.642                    | 1.46                       |
|      | -Y Massa                    | DL          | 0.106                      | 0.071                    | 1.49                       |
|      |                             | SD          | 0.432                      | 0.101                    | 4.28                       |
|      |                             | NC          | 0.577                      | 0.635                    | 0.91                       |
| CV2  | +X Massa                    | DL          | 0.089                      | 0.053                    | 1.69                       |
|      |                             | SD          | 1.647                      | 0.075                    | 21.93                      |
|      |                             | NC          | 2.197                      | 0.521                    | 4.21                       |
|      | -X Massa                    | DL          | 0.089                      | 0.053                    | 1.70                       |
|      |                             | SD          | 1.642                      | 0.075                    | 21.87                      |
|      |                             | NC          | 2.193                      | 0.520                    | 4.22                       |
|      | +Y Massa                    | DL          | 0.090                      | 0.058                    | 1.56                       |
|      |                             | SD          | 0.464                      | 0.082                    | 5.64                       |
|      |                             | NC          | 0.621                      | 0.557                    | 1.11                       |
|      | -Y Massa                    | DL          | 0.093                      | 0.058                    | 1.62                       |
|      |                             | SD          | 0.416                      | 0.082                    | 5.06                       |
|      |                             | NC          | 0.559                      | 0.555                    | 1.01                       |
| CV3  | +X Massa                    | DL          | 0.070                      | 0.039                    | 1.77                       |
|      |                             | SD          | 3.869                      | 0.056                    | 68.78                      |
|      |                             | NC          | 5.160                      | 0.435                    | 11.87                      |
|      | -X Massa                    | DL          | 0.068                      | 0.039                    | 1.73                       |
|      |                             | SD          | 7.010                      | 0.056                    | 124.91                     |
|      |                             | NC          | 9.347                      | 0.435                    | 21.48                      |
|      | +Y Massa                    | DL          | 0.076                      | 0.045                    | 1.68                       |
|      |                             | SD          | 2.682                      | 0.065                    | 41.33                      |
|      |                             | NC          | 3.577                      | 0.477                    | 7.50                       |
|      | -Y Massa                    | DL          | 0.076                      | 0.045                    | 1.68                       |
|      |                             | SD          | 3.593                      | 0.065                    | 55.40                      |
|      |                             | NC          | 4.791                      | 0.478                    | 10.01                      |

**Table E1** - Safety factor ( $\alpha$ ) values for CV1, CV2 & CV3.

| Test  | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|-------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| CV1-R | +X Massa                    | DL          | 0.092                      | 0.043                    | 2.15                       |
|       |                             | SD          | 1.200                      | 0.061                    | 19.56                      |
|       |                             | NC          | 1.601                      | 0.426                    | 3.76                       |
|       | -X Massa                    | DL          | 0.092                      | 0.043                    | 2.15                       |
|       |                             | SD          | 1.200                      | 0.061                    | 19.58                      |
|       |                             | NC          | 1.602                      | 0.426                    | 3.76                       |
|       | +Y Massa                    | DL          | 0.093                      | 0.043                    | 2.15                       |
|       |                             | SD          | 1.213                      | 0.062                    | 19.59                      |
|       |                             | NC          | 1.618                      | 0.429                    | 3.78                       |
|       | -Y Massa                    | DL          | 0.093                      | 0.043                    | 2.15                       |
|       |                             | SD          | 1.207                      | 0.062                    | 19.54                      |
|       |                             | NC          | 1.613                      | 0.428                    | 3.77                       |
| CV2-R | +X Massa                    | DL          | 0.081                      | 0.036                    | 2.28                       |
|       |                             | SD          | 2.207                      | 0.051                    | 43.42                      |
|       |                             | NC          | 2.944                      | 0.372                    | 7.92                       |
|       | -X Massa                    | DL          | 0.081                      | 0.036                    | 2.28                       |
|       |                             | SD          | 2.220                      | 0.051                    | 43.75                      |
|       |                             | NC          | 2.960                      | 0.372                    | 7.96                       |
|       | +Y Massa                    | DL          | 0.085                      | 0.036                    | 2.40                       |
|       |                             | SD          | 1.441                      | 0.051                    | 28.41                      |
|       |                             | NC          | 1.923                      | 0.364                    | 5.29                       |
|       | -Y Massa                    | DL          | 0.085                      | 0.035                    | 2.40                       |
|       |                             | SD          | 1.443                      | 0.051                    | 28.49                      |
|       |                             | NC          | 1.927                      | 0.363                    | 5.30                       |
| CV3-R | +X Massa                    | DL          | 0.065                      | 0.028                    | 2.33                       |
|       |                             | SD          | 1.245                      | 0.040                    | 31.09                      |
|       |                             | NC          | 1.661                      | 0.317                    | 5.23                       |
|       | -X Massa                    | DL          | 0.066                      | 0.028                    | 2.33                       |
|       |                             | SD          | 1.246                      | 0.041                    | 30.76                      |
|       |                             | NC          | 1.662                      | 0.320                    | 5.20                       |
|       | +Y Massa                    | DL          | 0.062                      | 0.025                    | 2.48                       |
|       |                             | SD          | 2.496                      | 0.036                    | 70.02                      |
|       |                             | NC          | 3.329                      | 0.287                    | 11.61                      |
|       | -Y Massa                    | DL          | 0.064                      | 0.025                    | 2.54                       |
|       |                             | SD          | 1.135                      | 0.036                    | 31.76                      |
|       |                             | NC          | 1.512                      | 0.283                    | 5.35                       |

Table E2 - Safety factor ( $\alpha$ ) values for CV1-R, CV2-R & CV3-R.

| Test     | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|----------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| C1 - CV1 | +X Massa                    | DL          | 0.087                      | 0.069                    | 1.25                       |
|          |                             | SD          | 0.192                      | 0.111                    | 1.73                       |
|          |                             | NC          | 0.259                      | 0.648                    | 0.40                       |
|          | -X Massa                    | DL          | 0.087                      | 0.070                    | 1.24                       |
|          |                             | SD          | 0.195                      | 0.113                    | 1.73                       |
|          |                             | NC          | 0.260                      | 0.653                    | 0.40                       |
|          | +Y Massa                    | DL          | 0.075                      | 0.040                    | 1.86                       |
|          |                             | SD          | 0.139                      | 0.057                    | 2.43                       |
|          |                             | NC          | 0.215                      | 0.432                    | 0.50                       |
|          | -Y Massa                    | DL          | 0.074                      | 0.040                    | 1.84                       |
|          |                             | SD          | 0.165                      | 0.057                    | 2.88                       |
|          |                             | NC          | 0.221                      | 0.433                    | 0.51                       |
| C1 - CV2 | +X Massa                    | DL          | 0.079                      | 0.054                    | 1.46                       |
|          |                             | SD          | 0.413                      | 0.077                    | 5.37                       |
|          |                             | NC          | 0.553                      | 0.545                    | 1.01                       |
|          | -X Massa                    | DL          | 0.079                      | 0.054                    | 1.47                       |
|          |                             | SD          | 0.434                      | 0.077                    | 5.63                       |
|          |                             | NC          | 0.582                      | 0.544                    | 1.07                       |
|          | +Y Massa                    | DL          | 0.074                      | 0.033                    | 2.25                       |
|          |                             | SD          | 0.223                      | 0.047                    | 4.72                       |
|          |                             | NC          | 0.297                      | 0.359                    | 0.83                       |
|          | -Y Massa                    | DL          | 0.071                      | 0.033                    | 2.18                       |
|          |                             | SD          | 0.239                      | 0.047                    | 5.09                       |
|          |                             | NC          | 0.321                      | 0.359                    | 0.89                       |
| C1 - CV3 | +X Massa                    | DL          | 0.068                      | 0.040                    | 1.70                       |
|          |                             | SD          | 0.513                      | 0.057                    | 8.93                       |
|          |                             | NC          | 0.686                      | 0.444                    | 1.54                       |
|          | -X Massa                    | DL          | 0.067                      | 0.040                    | 1.68                       |
|          |                             | SD          | 0.507                      | 0.057                    | 8.84                       |
|          |                             | NC          | 0.679                      | 0.444                    | 1.53                       |
|          | +Y Massa                    | DL          | 0.062                      | 0.026                    | 2.42                       |
|          |                             | SD          | 0.260                      | 0.037                    | 7.05                       |
|          |                             | NC          | 0.347                      | 0.297                    | 1.17                       |
|          | -Y Massa                    | DL          | 0.063                      | 0.026                    | 2.44                       |
|          |                             | SD          | 0.269                      | 0.037                    | 7.30                       |
|          |                             | NC          | 0.359                      | 0.297                    | 1.21                       |

**Table E3** - Safety factor ( $\alpha$ ) values for Aggregate Configuration 1 of CV1, CV2 & CV3.

| Test       | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|------------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| C1 - CV1-R | +X Massa                    | DL          | 0.087                      | 0.050                    | 1.74                       |
|            |                             | SD          | 0.744                      | 0.072                    | 10.37                      |
|            |                             | NC          | 0.994                      | 0.503                    | 1.98                       |
|            | -X Massa                    | DL          | 0.086                      | 0.050                    | 1.73                       |
|            |                             | SD          | 0.679                      | 0.071                    | 9.51                       |
|            |                             | NC          | 0.906                      | 0.501                    | 1.81                       |
|            | +Y Massa                    | DL          | 0.067                      | 0.029                    | 2.35                       |
|            |                             | SD          | 0.154                      | 0.041                    | 3.75                       |
|            |                             | NC          | 0.205                      | 0.322                    | 0.64                       |
|            | -Y Massa                    | DL          | 0.076                      | 0.031                    | 2.42                       |
|            |                             | SD          | 0.361                      | 0.045                    | 8.05                       |
|            |                             | NC          | 0.501                      | 0.337                    | 1.48                       |
| C1 - CV2-R | +X Massa                    | DL          | 0.072                      | 0.038                    | 1.89                       |
|            |                             | SD          | 1.044                      | 0.054                    | 19.26                      |
|            |                             | NC          | 1.394                      | 0.416                    | 3.35                       |
|            | -X Massa                    | DL          | 0.071                      | 0.038                    | 1.88                       |
|            |                             | SD          | 0.996                      | 0.054                    | 18.39                      |
|            |                             | NC          | 1.329                      | 0.416                    | 3.19                       |
|            | +Y Massa                    | DL          | 0.056                      | 0.022                    | 2.52                       |
|            |                             | SD          | 0.150                      | 0.032                    | 4.69                       |
|            |                             | NC          | 0.201                      | 0.266                    | 0.75                       |
|            | -Y Massa                    | DL          | 0.056                      | 0.022                    | 2.52                       |
|            |                             | SD          | 0.183                      | 0.032                    | 5.73                       |
|            |                             | NC          | 0.246                      | 0.265                    | 0.93                       |
| C1 - CV3-R | +X Massa                    | DL          | 0.065                      | 0.030                    | 2.14                       |
|            |                             | SD          | 1.063                      | 0.044                    | 24.43                      |
|            |                             | NC          | 1.063                      | 0.348                    | 3.06                       |
|            | -X Massa                    | DL          | 0.063                      | 0.031                    | 2.07                       |
|            |                             | SD          | 1.965                      | 0.044                    | 44.93                      |
|            |                             | NC          | 1.959                      | 0.354                    | 5.54                       |
|            | +Y Massa                    | DL          | 0.044                      | 0.016                    | 2.82                       |
|            |                             | SD          | 0.234                      | 0.022                    | 10.52                      |
|            |                             | NC          | 0.234                      | 0.198                    | 1.18                       |
|            | -Y Massa                    | DL          | 0.045                      | 0.016                    | 2.82                       |
|            |                             | SD          | 0.206                      | 0.023                    | 9.08                       |
|            |                             | NC          | 0.206                      | 0.199                    | 1.04                       |

**Table E4** - Safety factor ( $\alpha$ ) values for Aggregate Configuration 1 of CV1-R, CV2-R & CV3-R.

| Test       | Pushover Analysis Direction | Limit State | Displacement Capacity (cm) | Displacement Demand (cm) | Safety Factor ( $\alpha$ ) |
|------------|-----------------------------|-------------|----------------------------|--------------------------|----------------------------|
| C2 - CV1   | +X Massa                    | DL          | 0.070                      | 0.040                    | 1.72                       |
|            |                             | SD          | 0.172                      | 0.058                    | 2.98                       |
|            |                             | NC          | 0.230                      | 0.442                    | 0.52                       |
|            | -X Massa                    | DL          | 0.071                      | 0.040                    | 1.75                       |
|            |                             | SD          | 0.166                      | 0.058                    | 2.87                       |
|            |                             | NC          | 0.234                      | 0.440                    | 0.53                       |
|            | +Y Massa                    | DL          | 0.071                      | 0.041                    | 1.77                       |
|            |                             | SD          | 0.169                      | 0.058                    | 2.92                       |
|            |                             | NC          | 0.228                      | 0.443                    | 0.51                       |
|            | -Y Massa                    | DL          | 0.071                      | 0.040                    | 1.75                       |
|            |                             | SD          | 0.166                      | 0.058                    | 2.87                       |
|            |                             | NC          | 0.234                      | 0.440                    | 0.53                       |
| C2 - CV1-R | +X Massa                    | DL          | 0.074                      | 0.019                    | 3.94                       |
|            |                             | SD          | 0.322                      | 0.027                    | 12.01                      |
|            |                             | NC          | 0.435                      | 0.150                    | 2.90                       |
|            | -X Massa                    | DL          | 0.075                      | 0.019                    | 3.98                       |
|            |                             | SD          | 0.336                      | 0.027                    | 12.44                      |
|            |                             | NC          | 0.449                      | 0.152                    | 2.96                       |
|            | +Y Massa                    | DL          | 0.074                      | 0.019                    | 3.95                       |
|            |                             | SD          | 0.321                      | 0.027                    | 12.05                      |
|            |                             | NC          | 0.430                      | 0.149                    | 2.88                       |
|            | -Y Massa                    | DL          | 0.074                      | 0.019                    | 3.98                       |
|            |                             | SD          | 0.338                      | 0.027                    | 12.68                      |
|            |                             | NC          | 0.451                      | 0.150                    | 3.00                       |

**Table E5** - Safety factor ( $\alpha$ ) values for Aggregate Configuration 2 of CV1 & CV1-R.