TIMES of MALTA 90 YEARS

The maths of herd immunity

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Science Community

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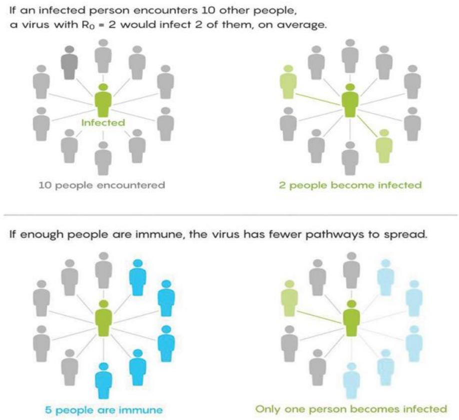

We all hate gossip, but let us say you hear a rumour that you simply cannot keep to yourself. So, you compromise by telling it exactly to one person. No big deal, right? After all, if the person you tell adopts the same policy, the rumour will not spread extremely far. If one new person hears the rumour each day, after one month it will not even have spread to three dozen people.

How bad could it be if, on average, from every two people who hear the rumour there is a naughty one who tells it to two people? Shockingly bad! Roughly, after 32 days the gossip will have reached the whole Maltese population!

How can such a seemingly small change make such a big difference? It all has to do with linear versus exponential functions. Linear functions are characterised by a constant rate of change. The rate of change of exponential functions varies over time.

Infections are spread in much the same way as rumour: Someone picks it up and passes it on to someone else.

Even this most elementary model seems to show that the difference between passing the infection on to one person and passing it on to (on average) one-and-a-half persons can be the difference between a few isolated cases and a pandemic.

Every infectious disease spreads at a rate dependent on a set of complicated factors. Epidemiologists try to summarise the impact of all these factors into one number, the basic reproduction number. This is the average number of new infections each infected person is expected to produce and is denoted by R_0. In the context of our ‘gossiper example’, the basic reproduction numbers were $R_0 = 1$ (in the linear case) and $R_0=1.5$ (in the exponential case).

Is it possible to turn an exponential growth into a linear one?

Here is where vaccination comes in. How many individuals need to be vaccinated to bring a disease’s effective reproduction number down to one?

Let us assume that the vaccination provides complete immunity to the disease. If each infected person contacts N new people per infectious period, we can expect that every person has an R_0/N chance of contracting the disease. But if V of these N people are vaccinated then $(R_0/N)(N-V)$ people will be infected on average.

For the growth to be linear and not exponential, we need this number to be 1. A little algebraic manipulation shows that the total percentage of vaccinated individuals needed to achieve a linear growth is $1 - (1/R_0)$. So, if for example, we have $R_0 = 1.5$, then the total percentage of the population that need to be vaccinated will be 33 per cent since $1 - (1/1.5) = 0.33$.

At this level of vaccination, the population achieves a collective immunity to the disease. This does not mean that no one will get infected, but the outbreak will die. This property is known as herd immunity and the percentage of vaccination required in a population to achieve herd immunity is referred to as the herd immunity threshold.

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Did you know?

• Many mathematicians agree that their passion for mathematics evolved from their irresistibility to puzzle solving. In countries with a strong mathematical tradition, the art of puzzle solving is well cultivated even at young ages. The following is a fairly tricky one, which you might want to try: During World War II, French cities near the front were blacked out. One day, when it was time to darken the window, the parents of the schoolboy Hugues could not find a shade for a window measuring 120cm by 120cm. All that was available was a rectangular sheet of plywood. Its area was correct, but it was 90cm by 160cm. Hugues picked up a ruler and drew quick lines on the plywood. He cut it into two parts along the lines he had drawn. With these parts he made a square covering for the window. How?

For more trivia, see: www.um.edu.mt/think

Sound bites

• Determining how to safely reopen schools and public spaces under social distancing is in part an exercise in geometry: If each person must keep two metres away from everyone else, then figuring out how many people can sit in a classroom or in a church is a question about packing non-overlapping circles into floor plans. Sphere-packing problems have occupied some of the greatest mathematicians, and exciting research is still happening today, particularly in higher dimensions. For example, the Ukrainian mathematician Maryna Viazovska has recently determined the best way to pack spheres in eight-dimensional spaces and (in a joint work with other mathematicians) in 24-dimensional spaces. This is a technique essential for optimising the error-correcting codes used in cell phones.

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