# JOINT LIMITS FOR SYSTOLIC AND DIASTOLIC BLOOD PRESSURE READINGS

F. DEMANUELE
M.D. (MALTA), M.SC.

D. W. B. REID
M.A., M.SC.

*Depart. of Epidemiology and Biometrics, University of Toronto.*

**Summary**

**In the construction of normal limits for a set of variables allowance must be made for the intercorrelation among them. A method of doing this is presented based upon the multivariate normal distribution. It is illustrated for the systolic and diastolic blood pressure readings on a selected group of outpatients. The way in which this approach can be utilized in the classification of patients into different disease states is also discussed.**

Biochemical and physical measurements are generally considered individually in clinical medicine. Often they are compared with similar measurements taken on a group of supposedly healthy subjects, for which 95% normal limits have been set up by adding to and subtracting from the mean twice the standard deviation. If a measured reading on a patient lies within these limits it is felt that as far as that variable is concerned the patient under study is not different from the normal group. Such limits suppose, of course, statistically "normal" distribution for the variables and adequate samples from which to calculate the means and the standard deviations. To overcome these limitations the percentile technique is sometimes recommended since this method does not make any assumption about the form of the distribution (Herrera 1958).

In some cases it may be worthwhile to look at measurements of different characteristics on the same patient simultaneously, taking into account their interrelationship. In the following note a method of doing this will be illustrated for blood pressure readings. It was originally investigated for the analysis of biochemical measurements, though such measurements are on the whole so poorly correlated that probably little advantage is to be gained by considering them in this way. However the intercorrelations among selected sets of biochemical determinations may be large enough to make the following treatment useful.

**Presentation of Blood Pressure Readings**

Systolic and diastolic blood pressure readings are usually presented together. Their normal limits are customarily stated as 120 ± 30 mm. of Hg. for the systolic, and 80 ± 20 mm. of Hg. for the diastolic, and can be represented on a linear scale as two separate intervals. As estimates of closely associated functions, these measurements are definitely correlated. Attempts are sometimes made to take this association into consideration by combining them into an index, as for example, in the pulse pressure or the mean arterial pressure.

The two blood pressure readings can be examined jointly, by plotting them as points in a plane defined by two rectangular axes, one axis standing for the systolic scale and the other axis for the diastolic scale, as illustrated in Figure 1. The resulting swarm of points forms an ellipse with tilted axis; the higher the correlation between such variables the greater the departure of the ellipse from the circular form. A curve can then be drawn superimposed on the scatter diagram to include, say 95% of the points within its confines, and in this way define a region of points which may be considered typical of the group. The points outside this region can be interpreted as extreme or atypical values. This will result in misclassifying 5% of normal individuals as abnormal.

These ellipses can be readily drawn if it is possible to assume a normal bivariate distribution of the blood pressure readings. A computer program has been developed for this purpose. It will calculate the means and standard deviations of the two variables, together with the correlation coefficient, and using these will then draw curves to include any given percentage. The curves are called centour ellipses of equal frequency (Rulon *et al.* 1965, Cooley *et al.* 1962), the term centour being derived from the words percentile and contour. It indicates the proportion of individuals with blood pressure readings which occur less frequently than the one considered, and in this sense more atypical. The point specified by the two means, i.e. the centroid, is then the 100 centour. Only 5% of cases will fall outside a cenour of 5 and these can then be considered as not belonging to the normal group.

A table of centour equivalents can also be constructed. The table is entered with readings for the two variables and the centour scores read.

**Out-Patient Study**

To illustrate the above ideas a group of 314 patients were sifted from a study done on about 1000 out-patients at the Toronto General Hospital (Young *et al.*, 1965). Only those patients were chosen who were found after extensive physical examination and laboratory tests, to be suffering from no major systemic illnesses. From this group 196 patients between the ages of 15 and 44 years were selected since there was no substantial increase of blood pressure with age over this interval. The resulting patients were thought to represent "healthy" out-patients.

From the blood pressure values of this group, the basic statistical quantities were calculated (Table 1). The data were

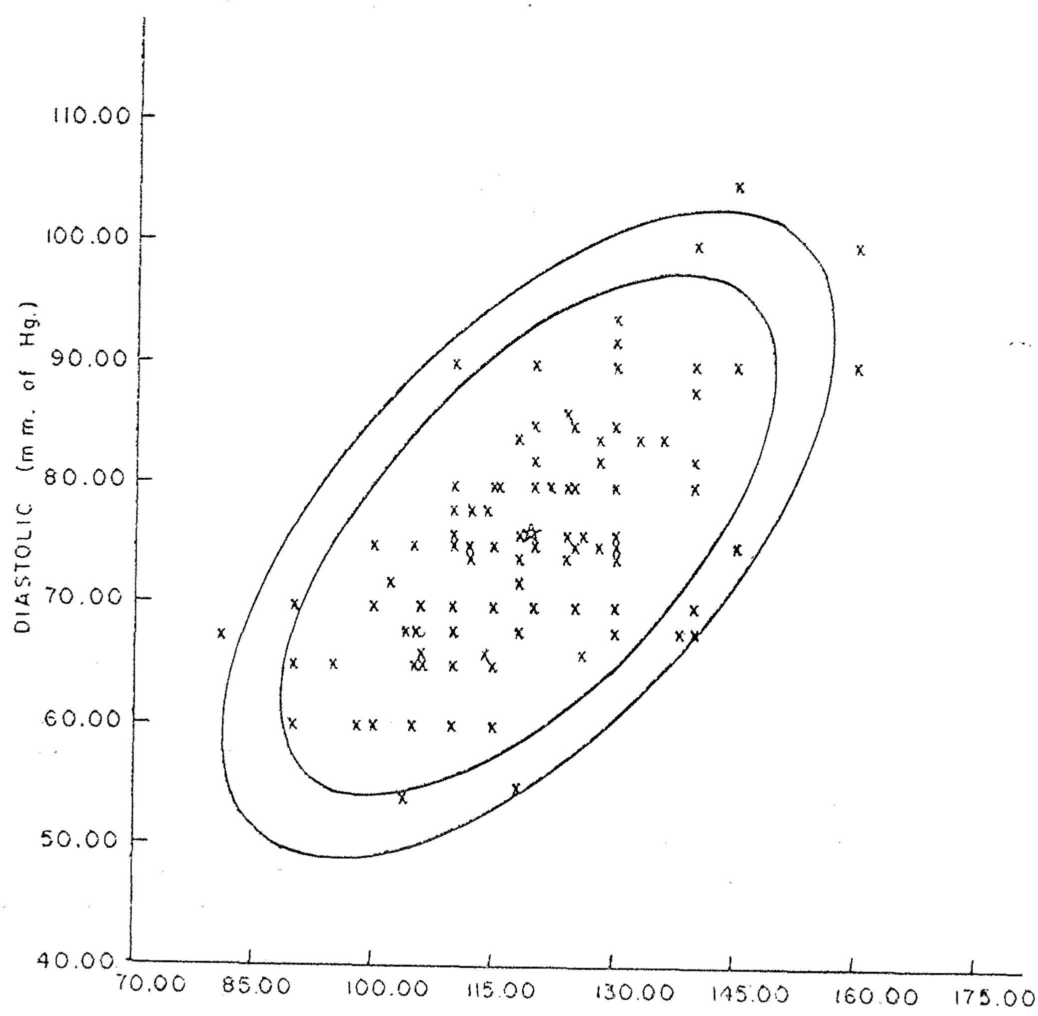
Table 1
**Summary of Blood Pressure Readings on 196 "Healthy" Out-Patients**

(Age, 15 to 44 years)

| | Blood Pressure (mm. of Hg.) | |
|---|---|---|
| | Systolic | Diastolic |
| Mean | 119.8 | 76.0 |
| Standard deviation | 13.9 | 9.2 |
| Correlation coefficient | + 0.59 | |

Scatter Diagram

1st. AND 5th. CENTOUR ELLIPSES ON "HEALTHY" OUTPATIENTS




then plotted, together with the 1st and 5th centours, as shown in Figure 1. An estimated 1% of cases fell outside the 1st centour and 5% outside the second. Three aberrant values were omitted from the figure, as the main purpose of this note is to illustrate this method and not to supply standards of reference.

A table of centour equivalents was also constructed from the above data, for more convenient use (Table 2). From it, for example, an individual with a blood pressure of 135/85 can be seen to lie on the 42nd centour. Similarly, an individual with a blood pressure of 135/65 can be seen to lie on the 1st centour ellipse, outside of which only 1% of blood pressure values occur. In this case the individual is unusual, though both blood pressure readings lie within normal limits. If based

**TABLE 2**

**Centour Equivalents of Blood Pressure Reading on "Healthy" Outpatients (Age 15 to 44 years)**

| SYSTOLIC (mm. of Hg.) | DIASTOLIC (mm. of Hg) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 | 100 | 105 |
| 80 | | | | | | | | | | | | | |
| 85 | | | 1 | 2 | 1 | | | | | | | | |
| 90 | | 1 | 3 | 6 | 6 | 3 | 1 | | | | | | |
| 95 | | 1 | 5 | 11 | 15 | 12 | 5 | 1 | | | | | |
| 100 | | 1 | 6 | 17 | 29 | 28 | 16 | 5 | 1 | | | | |
| 105 | | | 5 | 20 | 42 | 53 | 38 | 17 | 4 | | | | |
| 110 | | | 4 | 18 | 47 | 74 | 69 | 38 | 12 | 2 | | | |
| 115 | | | 2 | 12 | 40 | 80 | 93 | 65 | 27 | 6 | | | |
| 120 | | | | 6 | 26 | 66 | 98 | 86 | 44 | 13 | 2 | | |
| 125 | | | | 2 | 13 | 42 | 78 | 87 | 57 | 22 | 5 | | |
| 130 | | | | | 5 | 20 | 48 | 67 | 56 | 27 | 8 | 1 | |
| 135 | | | | | 1 | 7 | 22 | 40 | 42 | 26 | 9 | 2 | |
| 140 | | | | | | 2 | 8 | 18 | 24 | 19 | 9 | 2 | |
| 145 | | | | | | | 2 | 6 | 10 | 10 | 6 | 2 | |
| 150 | | | | | | | | 1 | 3 | 4 | 3 | 1 | |
| 155 | | | | | | | | | | 1 | 1 | | |
| 160 | | | | | | | | | | | | | |

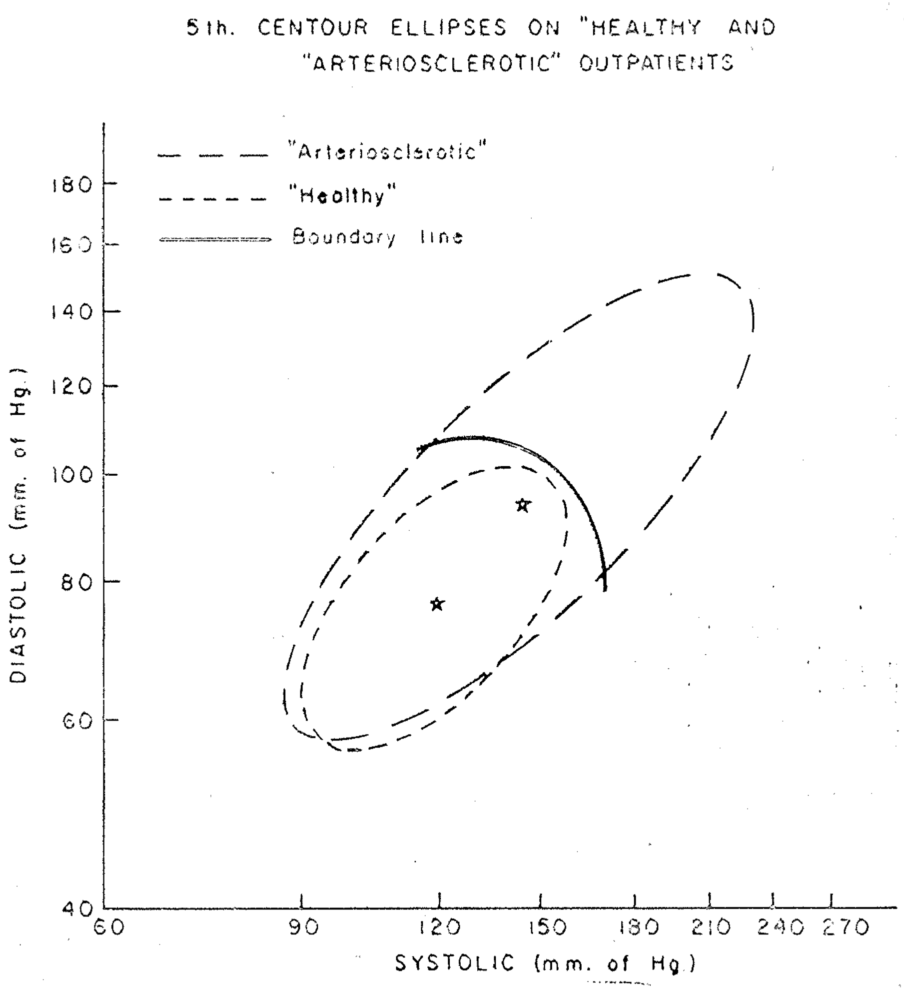
on more substantial data such diagrams or tables could be used in an out-patient setting as an aid in evaluating blood pressure readings on patients.

### Several Groups — Problem of Classification

Centour ellipses can also be constructed for data collected from any group of patients suffering from a particular disease, for comparison with the standard healthy group. If a patient is suspected of belonging to this group, comparison of the centours — normal and sick — will suggest in which group the individual belongs, provided that the two groups are sufficiently far apart that the overlap between the ellipses is not too great. Such comparison should allow, if possible, for the relative frequency with which the two types generally occur. If this can be meaningfully specified, then a curve can be drawn which separates the plane into a region where values would be taken as belonging to the other. The best boundary line is the one which gives the least number of misclassifications. It will pass through the points of intersection of equivalent centour ellipses selected so that the frequency of occurrence of individuals on the adjusted contour lines is the same in the two groups.

To illustrate these points, a further group of patients who were suffering from a number of cardiovascular conditions was isolated from the out-patient data. An effort was made to keep the group as uniform as possible by excluding cases with other non-vascular disease. For comparisons with the previous group of "healthy" out-patients, those patients in the same age bracket, i.e. 15 to 44 years, were selected. There were only 17 patients in this group, obviously too small a size for making worthwhile inferences. However it can be used to illustrate this extension of the general idea.

The resulting 5% centour ellipse for the "arteriosclerotic" group is shown in Figure 2 along with the equivalent ellipse for the "healthy" group. A logarithmic




scale has been used since the distribution of the readings for the "arteriosclerotic" group tends to be asymmetrical. Even though the centour for the "arteriosclerotic" group is poorly determined, it is clear that there is so much overlap between the ellipses that classification obviously cannot be made with confidence using only this information. The relative frequency of the two types, 196:17, can however be added to establish a boundary line between the "healthy" and the "arteriosclerotic" groups (Figure 2). It passes through the points of intersection of those centours for which individuals in the two groups are estimated to occur with equal frequency in the out-patient population. Any values lying on one side of the boundary line can now be classified as normals and those on the other side as abnormals. This would result in minimal misclassification. If based on more substantial body of data this again might be a useful method in the classification of patients into various disease states.

A copy of the program for the above type of analysis can be obtained from the Department of Epidemiology and Biometrics, University of Toronto. When the number of variables is greater than two, the program can be used to determine the centour for each patient, together with tables for the same purpose.

## References

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