USING MULTILEVEL RANDOM COEFFICIENT MODELS TO ASSESS STUDENTS' SPELLING ABILITIES

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Hierarchical nested data, random coefficient model, intra class correlation, multilevel model.

ABSTRACT

This paper presents statistical models that analyze cross-sectional data related to student attainment in English and Maltese spelling. For each spelling test a random sample of 2040 students, whose age ranged from 6.5 to 16 years, was selected to examine the progression of spelling skills over time. The sample comprised equal numbers of male and female students attending state, church and private schools to investigate gender and school bias in students' spelling abilities. This hierarchical nested data can be deemed as a type of two-level data, in which the students spelling scores are level-1 units and schools are the level-2 units. This multilevel approach provides an adequate framework for modelling hierarchical data at several levels of nesting. To inspect the effect of age on student performance in English and Maltese spelling in different schools, a random coefficient model is fitted. This allows the school-specific coefficients describing individual trajectories to vary randomly when the spelling scores are regressed against the student age.

1. DATA COLLECTION

To examine the progression of spelling skills of Maltese students in primary and secondary schools, twenty age cohorts (6½, 7, 7½ … 15, 15½, 16) were identified. All students, who at the time of the test administration were two weeks younger or two weeks older than any of the specified ages, were included in the study. Moreover the selected students were also stratified by gender and school type to guarantee a representative sample. To ensure sufficiently large numbers of participants and proportionate allocations of students in each age group, it was decided that each cohort should include 60 pupils from state schools, 24 students from church schools and 18 pupils from private schools with an equal balance of boys and girls within each group.

Maltese secondary state schools are classified as Junior Lyceums, which take in students who pass the 11+ exams and Area secondary schools, which take in students who fail these exams. To ensure a proportionate allocation of participants, 30 students from Junior Lyceums, 30 pupils from Area Secondary, 24 students from church schools and 18 pupils from private schools were selected with equal numbers of boys and girls within each cohort. Each of the twenty age groups comprised 102 students making an overall total of 2040 participants for each test. The ages of all participants fell within a four week time window centred on the chronological age assessed.

The Maltese and English spelling manual compiled by (Camilleri, Firman, Martinelli 2010) was used to measure spelling skills of school children. To reduce disruptions each test, which consists of a list of 87 words ranked by order of difficulty, was administered to whole classes rather than selecting the few students from each class who fitted the 4-week age window. A sample of approximately 12,000 participants was collected for each test to get the required number of the 2040 students that fall within the required age cohorts. This sample amounted to about 20% of the whole school population in 2009 and guaranteed a maximum margin of error of around 2%. Schools were selected from all the six districts of the Maltese islands to ensure a good geographical representation. For both spelling tests the students were asked to provide the name of school, gender, age and date of birth. This information was essential to identify the students who would fall within the four week time window of each specified age and the type of school s/he attended. After finalizing data collection the scripts were sorted by date of birth. All students who satisfied the stipulated 4-week window criterion were included in the survey sample. The selected scripts were then corrected and marked, where spelling scores ranged from 0 to 87. Scripts that did not provide the required information were excluded.

2. PRELIMINARY ANALYSIS

One of the aims of the study is to inspect and evaluate the progression of spelling attainment with age and compare the spelling scores between male and female students attending different school types.

Figure 1 displays that females tend to attain higher scores in English spelling compared to males. This difference becomes more conspicuous with age. Another interesting fact is that English spelling scores increase more rapidly during primary than secondary school years.

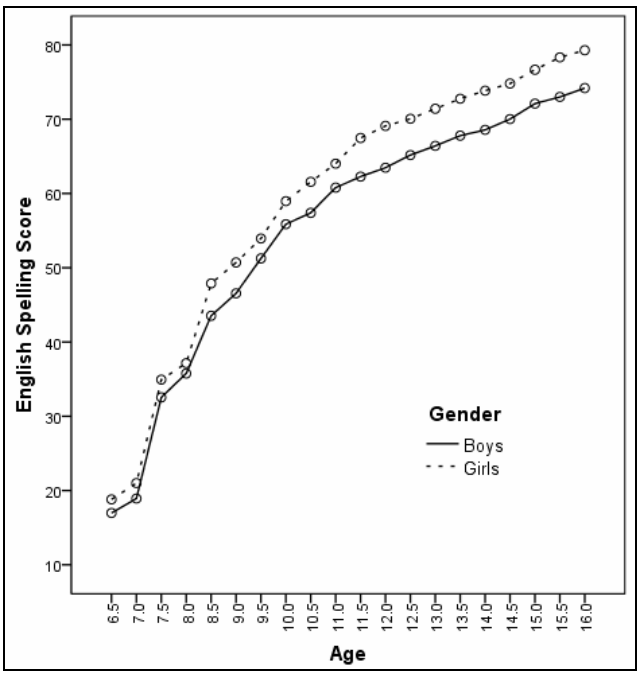

Figure 1: Mean English spelling scores categorized by age and gender

Figure 2 displays a similar attainment in English spelling of primary school children attending private and church schools. However, achievement in English spelling of primary students attending state schools is poorer. At the age of 6 the disparity in the mean spelling scores is small; however, it becomes more evident with age.

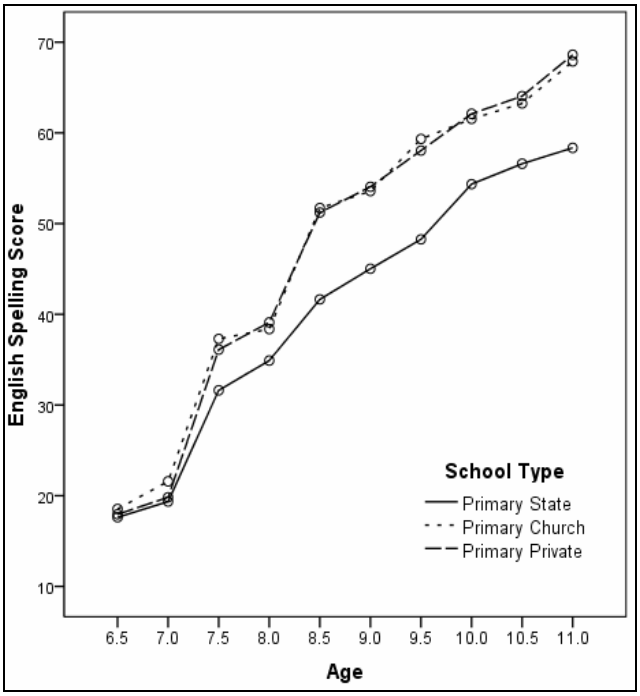

Figure 2: Mean English spelling scores of primary pupils categorized by age and school type

Figure 3 shows that increments in English spelling scores of secondary pupils are linear and less steep than those of primary students. An obvious fact is that attainment of Area secondary students in English spelling is inferior to their counterparts attending other schools. These students failed the 11+ examinations.

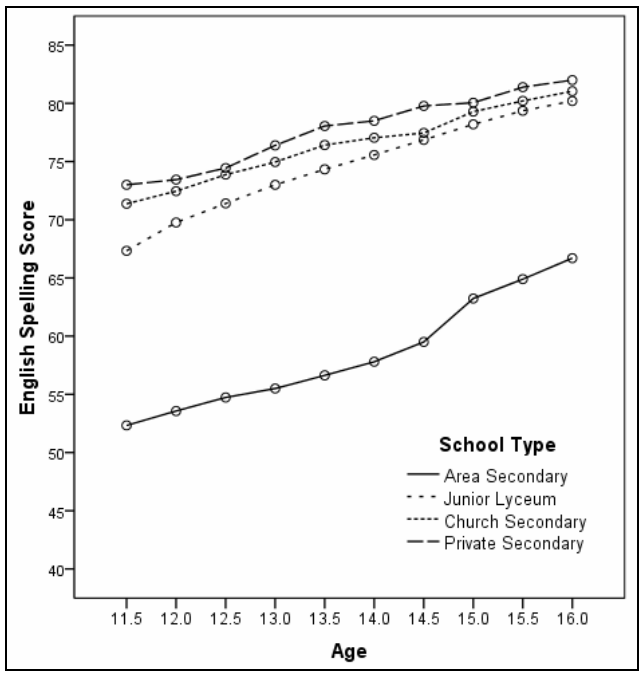

Figure 3: Mean English spelling scores of secondary students categorized by age and school type

Figure 4 displays similar patterns as Figure 1. Girls tend to outperform boys in Maltese spelling at almost all ages and discrepancies in spelling abilities increase with age. The stepwise, rather than linear, increase in the spelling scores is explained by the fact that 7½ year old students are one year ahead academically than 7-year old pupils.

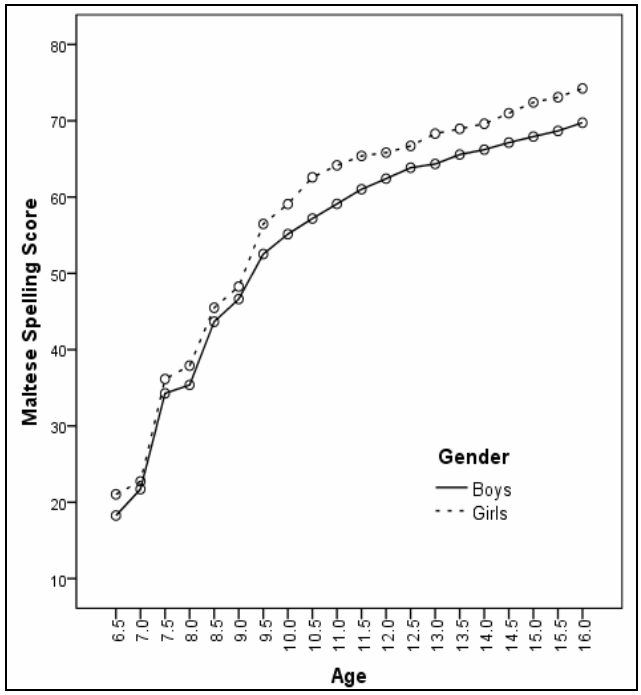

Figure 4: Mean Maltese spelling scores categorized by age and gender

Figure 5 exhibits a contrasting pattern to Figure 2. Pupils attending primary state schools perform better in Maltese spelling but worse in English. This is partly attributed to the fact that church/private schools assign more time for English than Maltese; moreover, pupils are encouraged to speak in English during school hours.

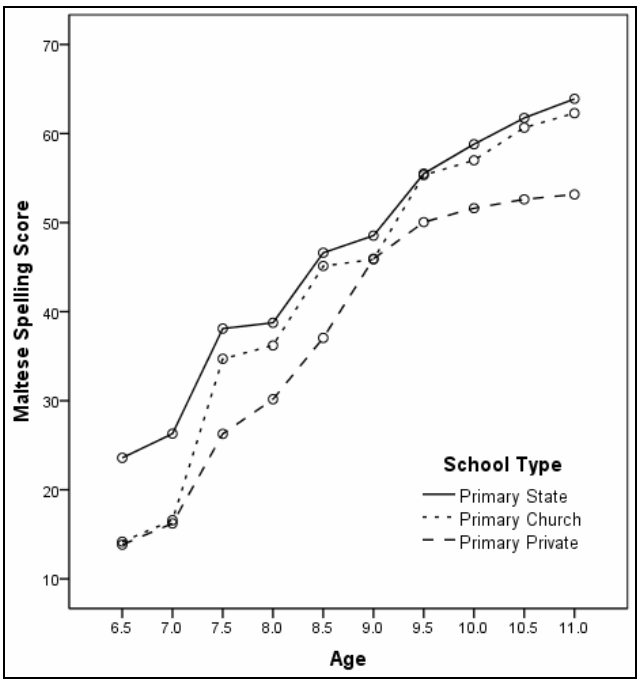

Figure 5: Mean Maltese spelling scores of primary pupils categorized by school type and gender

Figure 6 shows similar trends as Figure 3. Increments in Maltese spelling scores of secondary students are linear and less sharp than those of primary pupils. Differences in Maltese spelling attainment are conspicuous between schools. Pupils attending Junior Lyceums tend to do best, followed by church, private and area secondary schools.

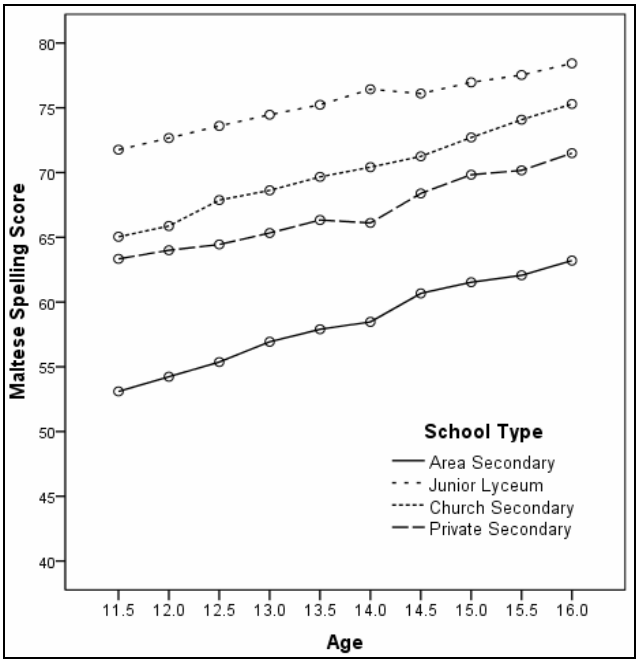

Figure 6: Mean Maltese spelling scores of secondary students categorized by school type and gender

3. A MULTILEVEL MODEL

Generalized linear mixed models are linear in the parameters and the predictors involve a mix of fixed and random effects. Linear mixed models for Normal responses can be written in the form:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\eta} + \boldsymbol{\varepsilon} \tag{1}$$

$\mathbf{y}$ is a vector of responses; whereas, $\mathbf{X}$, $\mathbf{Z}$ are design matrices. $\boldsymbol{\beta}$ are fixed effects, and $\boldsymbol{\eta}$ and $\boldsymbol{\varepsilon}$ are random effects both assumed to be independent and Normally distributed.

$$\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{\Sigma}) \text{ and } \boldsymbol{\eta} \sim N(\mathbf{0}, \boldsymbol{\Psi}) \tag{2}$$

$\mathbf{X}\boldsymbol{\beta}$ is the fixed component and $\mathbf{Z}\boldsymbol{\eta}$ is the random part of the model. Traditional linear regression models are special cases of linear mixed models with $\mathbf{Z} = \mathbf{0}$.

One of the assumptions of linear regression models is that the responses y_i are independent. This assumption is not realistic, particularly when observations are nested within hierarchical structures or are repeated measurements in a longitudinal study. Multilevel modeling is an alternative approach that facilitates the analysis of hierarchical data particularly when observations are nested within higher levels of classification. A two-level linear mixed model can be written in the form:

$$y_{ij} = \underbrace{\mathbf{X}_{ij}^{'}\boldsymbol{\beta}}_{\text{Fixed part}} + \underbrace{\sum_{m=0}^{M} \eta_{mj}^{(2)} z_{mij}^{(2)} + \varepsilon_{ij}}_{\text{Random part}} \tag{3}$$

Unobserved level-2 heterogeneity is accounted for by the inclusion of the random effect $\eta_{mj}^{(2)}$ in the linear predictor. This multilevel model accommodates well the levels of our clustered data set in which students are nested within schools.

In this application, y_{ij} is the Maltese/English spelling score attained by student *i* attending school *j*, X_{1ij} and X_{2ij} respectively stipulate the age and gender of this student and X_{3ij} specify the type of school attended by this pupil. X_{1ij} and X_{2ij} are student related predictors; whereas, X_{3ij} is a school-related explanatory variable.

The student level-1 model is:

$$y_{ij} = \delta_{0j} + \delta_{1j} X_{1ij} + \delta_{2j} X_{2ij} + \varepsilon_{ij} \tag{4}$$

The school level-2 model is:

$$\begin{aligned} \delta_{0j} &= \beta_0 + \beta_3 X_{3ij} + \eta_{0j} \\ \delta_{1j} &= \beta_1 + \eta_{1j} \\ \delta_{2j} &= \beta_2 \end{aligned} \tag{5}$$

The combined model is of the form:

$$y_{ij} = \beta_0 + \beta_1 X_{1ij} + \beta_2 X_{2ij} + \beta_3 X_{3ij} + \eta_{0j} + \eta_{1j} X_{1ij} + \varepsilon_{ij} \tag{6}$$

$\beta_0 + \beta_1 X_{1ij} + \beta_2 X_{2ij} + \beta_3 X_{3ij}$ is the fixed component, while $\eta_{0j} + \eta_{1j} X_{1ij} + \varepsilon_{ij}$ is the random component. This model implies that level-2 units are characterized by two random

effects – intercept and slope, which means that regression lines relating spelling scores to age differ between schools. It is assumed that the random effects ε_{ij}, η_{0j} and η_{1j} are independent and Normally distributed with mean 0 and variances σ_ε^2, σ_0^2 and σ_1^2 respectively.

$$\varepsilon_{ij} \sim N(0,\sigma_\varepsilon^2) \quad \begin{pmatrix}\eta_{0j}\\ \eta_{1j}\end{pmatrix} \sim N_2\left[\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\sigma_0^2 & \sigma_{10}\\ \sigma_{10} & \sigma_1^2\end{pmatrix}\right] \qquad (7)$$

The GLLAMM program is a subroutine of STATA that can estimate generalized linear latent and mixed models. It can fit multilevel latent variable models for (multivariate) responses of mixed type including continuous responses, counts, survival data, dichotomous, ordered and unordered categorical responses. It maximizes the marginal log-likelihood using Newton Raphson algorithm. GLLAMM software uses numerical first and second derivatives of the log-likelihood and produces standard errors as a by-product. The intraclass correlations can be computed using the variances of the random effects.

The intraclass correlation for level-1 units is:

$$\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_0^2+\sigma_1^2} \qquad (8)$$

The intraclass correlations for level-2 units are:

$$\frac{\sigma_0^2}{\sigma_\varepsilon^2+\sigma_0^2+\sigma_1^2} \qquad \frac{\sigma_1^2}{\sigma_\varepsilon^2+\sigma_0^2+\sigma_1^2} \qquad (9)$$

4. RESULTS FOR PRIMARY SCHOOLS

For reliable estimation of the parameters, adaptive quadrature was used instead of ordinary quadrature. Performance of adaptive quadrature is much better, particularly for large cluster sizes and large intraclass correlations. Moreover, adaptive quadrature is likely to give good estimates for normally distributed responses given that a sufficient number of quadrature points are used (Rabe-Hesketh, Skrondal and Pickles 2005). Adaptive quadrature required six iterations to converge. The iterative procedure required a further eight iterations running Newton Raphson to update the parameters while retaining quadrature locations and weights fixed until convergence criteria were met.

Table 1 shows the parameter estimates and standard errors of the multilevel random coefficient models for English and Maltese spelling scores of primary school children. The β_1 estimates indicate that for every 1-year increase in the age of primary school children, the spelling score is expected to increase by 10.36 for English and 9.62 for Maltese. The β_2 estimates indicate that, on average, male primary students score 3 points less than female pupils in both Maltese and English spelling. The β_3 estimates show that the mean English spelling score of students attending primary state schools is respectively 5.78 and 5.37 lower than the mean English spelling score of students attending church and private schools. Conversely, the mean Maltese spelling score of students attending primary state schools is respectively 1.63 and 7.94 higher than the mean Maltese spelling score of students attending church and private schools. Almost all parameters are significant at the 0.05 level of significance.

Primary Schools		English Spelling		Maltese Spelling	
		Est.	S.E.	Est.	S.E.
β_0	Constant	-48.2	5.842	-44.97	3.991
β_1	Age	10.36	0.595	9.62	0.381
β_2	Males	-2.97	0.886	-2.90	0.921
	Females	aliased		aliased	
β_3	State	-5.37	1.677	7.94	1.774
	Church	0.41	1.486	6.31	2.049
	Private	aliased		aliased	
σ_ε^2	$\text{var}(\varepsilon_{ij})$	177.2	8.041	188.7	8.459
σ_0^2	$\text{var}(\eta_{0j})$	171.3	116.2	69.46	79.38
σ_1^2	$\text{var}(\eta_{1j})$	2.37	1.497	0.548	0.738
σ_{10}	$\text{cov}(\eta_{0j},\eta_{1j})$	-20.14	13.04	-6.165	7.653

Table1: Parameter estimates and standard errors (English and Maltese spelling tests for primary school children)

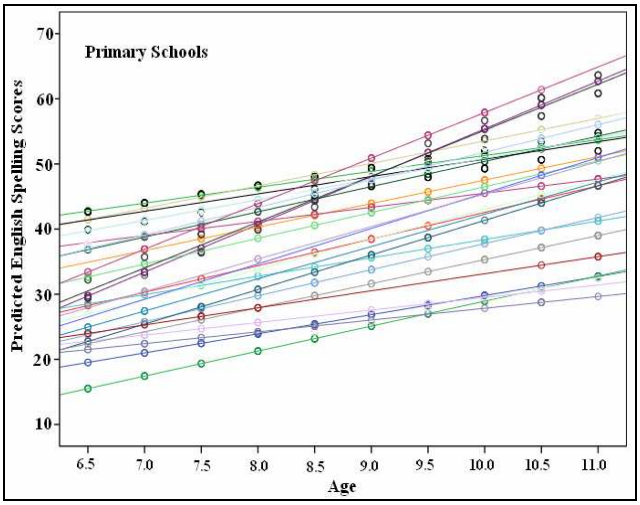

Figure7: Individual primary school trajectories displaying predicted English spelling scores against age.

The student level-1 variances σ_ε^2 for English (177.2) and Maltese (188.7) are respectively larger than the school level-2 variances $\sigma_0^2+\sigma_1^2$ for English (173.7) and Maltese (70.0). This implies that variations in spelling scores, particularly Maltese, are more attributed to differences between students than differences between schools.

For both models, the negative covariances (-20.14, -6.165) suggest that schools with lower intercepts tend to have steeper slopes. The correlations between intercepts and slopes are both close to -1. This is clearly displayed in Figure 7.

Primary Schools	Intraclass correlation	
	English	Maltese
Student level -1	0.505	0.729
School level-2 (intercept)	0.488	0.268
School level-2 (slope)	0.007	0.003

Table2: Intraclass correlations at student and school levels

Table 2 displays the intraclass correlations at student and school levels. At primary level the student level-1 variance explains about 51% and 73% of the total variability in the English and Maltese spelling scores. For both data sets the variability in the intercepts is significantly larger than the variability in the slopes.

5. RESULTS FOR SECONDARY SCHOOLS

Table 3 shows the parameter estimates and standard errors of the multilevel random coefficient models for English and Maltese spelling scores of secondary school children. The β_1 estimates indicate that for every 1-year increase in the age of secondary school children, the spelling score is expected to increase by 2.64 for English and 2.01 for Maltese. The β_2 estimates indicate that, on average, male secondary students score 5 and 4 points less than female pupils in English and Maltese spelling respectively.

Secondary Schools		English Spelling		Maltese Spelling	
		Est.	S.E.	Est.	S.E.
β_0	Constant	43.66	3.428	41.23	4.029
β_1	Age	2.64	0.227	2.01	0.261
β_2	Males	-5.01	0.675	-4.06	0.808
	Females	aliased		aliased	
β_3	Area Sec.	-18.80	1.195	-8.45	1.418
	Junior Ly.	-2.92	1.137	8.28	1.421
	Church	-1.31	1.189	3.54	1.531
	Private	aliased		aliased	
σ_ε^2	$\text{var}(\varepsilon_{ij})$	90.39	4.087	108.9	4.901
σ_0^2	$\text{var}(\eta_{0j})$	36.57	59.99	79.79	83.24
σ_1^2	$\text{var}(\eta_{1j})$	0.139	0.259	0.288	0.348
σ_{10}	$\text{cov}(\eta_{0j},\eta_{1j})$	-2.25	3.944	-4.79	5.388

Table3: Parameter estimates and standard errors (English and Maltese spelling tests for secondary school children)

The β_3 estimates illustrate that the mean English spelling score of students attending area secondary schools is respectively 15.88, 17.49 and 18.8 lower than the mean English scores of students attending Junior Lyceums, church and private schools. Conversely, the mean Maltese spelling score of students attending area secondary schools is respectively 16.73, 11.99 and 8.45 lower than the mean Maltese scores of students attending Junior Lyceums, church and private schools.

Secondary Schools	Intraclass correlation	
	English	Maltese
Student level -1	0.711	0.576
School level-2 (intercept)	0.288	0.422
School level-2 (slope)	0.001	0.002

Table4: Intraclass correlations at student and school levels

For both models, the negative covariances (-2.25, -4.79) indicate that schools with higher intercepts tend to have gentler slopes. The intraclass correlations, shown in table 4, indicate that at secondary level the student level-1 variance explains 71.1% and 57.6% of the total variability in the English and Maltese spelling scores. This implies that variations in spelling scores, particularly English, are more due to differences between students than differences between schools. The fact that the intraclass correlations for school slopes are very small indicate that multilevel random intercept models would have provided comparably good results as well. This implies that the random effects associated with the levels of the random factors enter the model as random intercept rather than random coefficient.

A recommendation for future work is to incorporate latent variables in the model fit such as student engagement. A multilevel structural equation model would then be used to accommodate a hierarchy of nested clusters when some of the variables of interest are latent.

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