

# A New Form Of Primes

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**Abstract:** Let  $p_1, p_2, \dots, p_k, \dots$  denotes the ordered sequence of primes. We will consider primes of the form  $p = 1 \pmod{p_i}$ . Moreover we give a new proof to the converse of W. Isean's Theorem, first proved by Lagrange.

**Definition 1**  $p$  is said to be prime in  $\mathbb{Z}$  if  $p = ab, a, b \in \mathbb{Z} \Rightarrow a = 1$  or  $b = 1$

**Lemma 1** If  $a = b \pmod{m_1 m_2 \dots m_k}$ , then  $a = b \pmod{m_i} \forall i, 1 \leq i \leq k$ , where  $a, b, m_1, \dots, m_k \in \mathbb{Z}$ .

**Proof:**

$$\begin{aligned} a = b \pmod{m_1 m_2 \dots m_k} &\Rightarrow m_1 m_2 \dots m_k / (b - a) \\ &\Rightarrow m_i / (b - a) \forall i, 1 \leq i \leq k \\ &\Rightarrow a = b \pmod{m_i} \forall i, 1 \leq i \leq k \end{aligned} \tag{1}$$

**Lemma 2** If  $\gcd(a, b) = 1$ , then the arithmetic progression  $\langle a + bn \rangle$  contains infinitely many primes

**N.B.** This theorem was first conjectured by Euler in 1785 (with  $a = 1$ ). In 1808, Legendre claimed that he had a proof for this theorem, but later it was found to be false. Finally, in 1837, Dirichlet proved it and it was practically the birth of *Analytic Number Theory*.

**Proposition:** Let  $p_1, p_2, \dots, p_k, \dots$  be the ordered sequence of primes. Then given any prime  $p_k$ , there exists another prime  $p, p > p_k$ , s.t.  $p = 1 \pmod{p_i} \forall i, 1 \leq i \leq k$ .

**Proof:** Consider the sequence  $\langle (p_1 p_2 \dots p_k)n + 1 \rangle$ . This is an arithmetic progression and  $\gcd(p_1 p_2 \dots p_k, 1) = 1$

Hence by Dirichlet's Theorem  $\langle (p_1 p_2 \dots p_k)n + 1 \rangle$  contains infinitely many primes which are of the form  $p = 1 \pmod{p_1 p_2 \dots p_k} \Rightarrow p = 1 \pmod{p_i} \forall i, 1 \leq i \leq k$ .

**Conclusion: Twin primes** are integral pairs  $(p, p + 2)$  in which both members are prime. Examples are 5, 7, 11, 13, and 107, 109. It is conjectured that there are infinitely many twin primes.

Is there a relation between primes of the form  $1 \pmod{p_i}; 1 \leq i \leq k$  and twin primes?