CROSSING LINES

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Abstract: Two questions are tackled. In the first we investigate the regions into which the plane is divided by crossing lines.

1.CROSSING LINES

Rule: Lines must all cross each other at least once from a point distinct from a previous intersection point.

a.. When two lines intersect, the plane is divided into four open regions

b. When three lines intersect, the plane is divided into 7 regions, one of which is enclosed in a triangular shape. (4+3)

c. When four lines intersect, the plane is divided into 3 enclosed regions (two triangles, one quad.) and 8 open regions i.e. 11 regions in all. (7+4)

d. When 5 lines intersect, 10 open regions and 6 closed regions are formed. (16 in all, 11+5)

(i) 10 open regions and 6 closed regions (3 triangle, 3 quad)

(ii) 10 open regions and 6 closed regions (5 triangle, one 5 sided figure)

(iii) 10 open regions and 6 closed regions (4 triangle, 1 quad, one five sided)

Questions

Are these results exhaustive?

Are there more ways to draw five crossing lines?

What happens when more lines intersect?

We can predict how many regions, closed or open, are created using formulae derived from the inherent patterns pointed out. But what about the shapes of the aforesaid regions – is there any specific way we can work out a prediction? Furthermore, how many possible ways are there to draw five or more crossing lines? Are there really only two ways to cross 5 lines? What about higher numbers of crossing lines?

What is the situation if this problem had to be extended to three dimension, lines or planes?

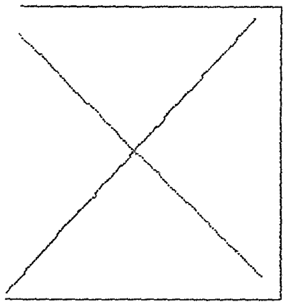
Another interesting area to explore is the number of intersection points – a pattern is immediately discernible.

TABLE OF RESULTS

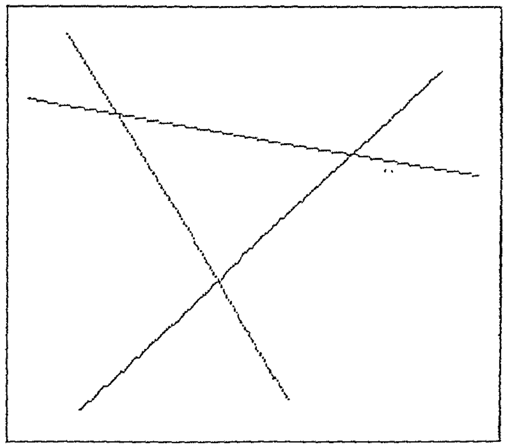
No of lines	Open Regions	Closed Regions	Total No. of Regions
1	2		2
2	4		4
3	6	1	7
4	8	3	11
5	10	6	16

Intersection Points

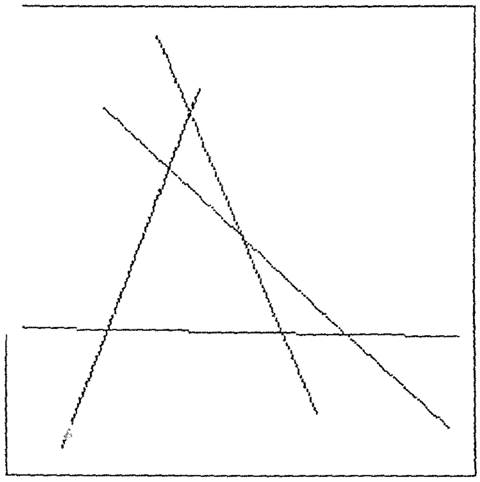
No. of lines	Intersection Points
1	0
2	1
3	3
4	6
5	10



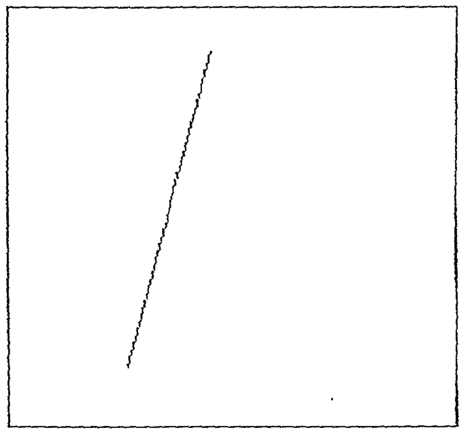
2 INTERSECTING LINES



3 INTERSECTIN G LINES

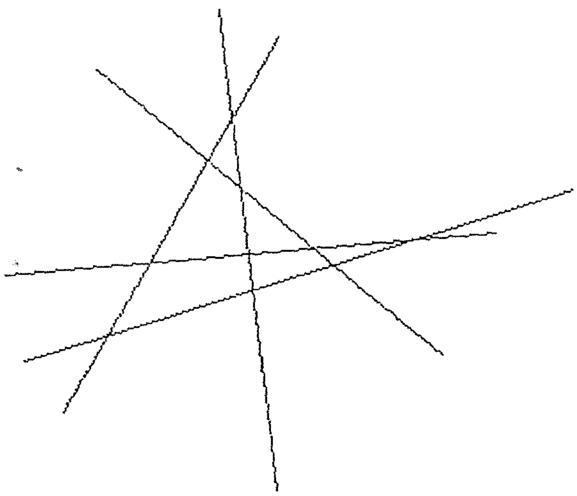


4 INTERSECTING LINES

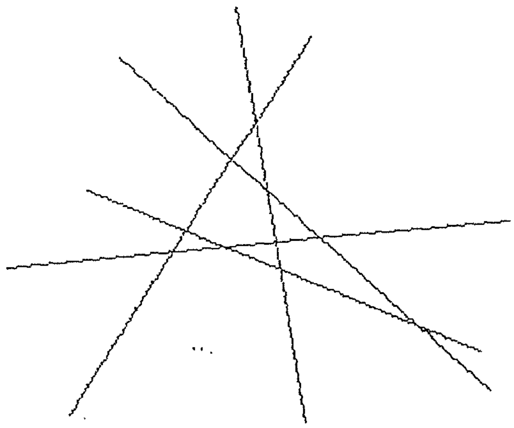


1 LINE

5 INTERSECTING LINES

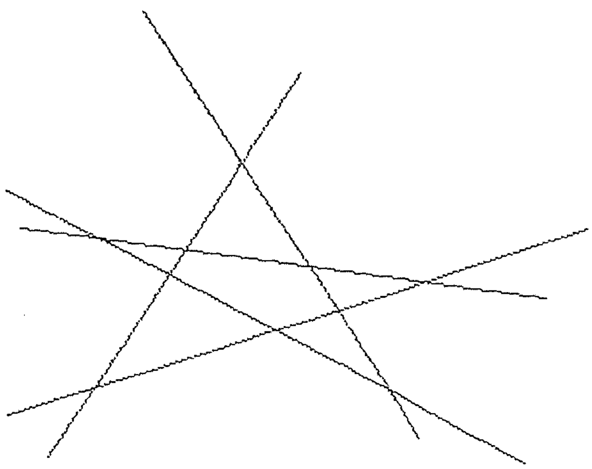


3 TRIANGLES
3 QUADRILATERALS



4 TRIANGLES
1 QUADRILATERAL
ONE 5-SIDED FIGURE

5 TRIANGLES
ONE 5 SIDED FIGURE



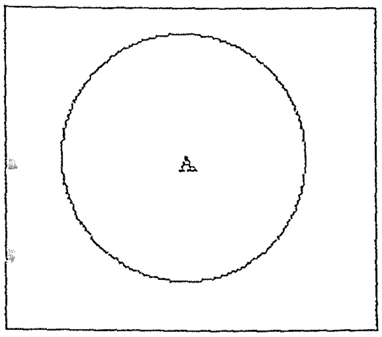
1.INTERSECTING CIRCLES

Rule: The circles must intersect in such a way that the new circle added must take in part of Each and every area of the circles and regions in the former diagram.

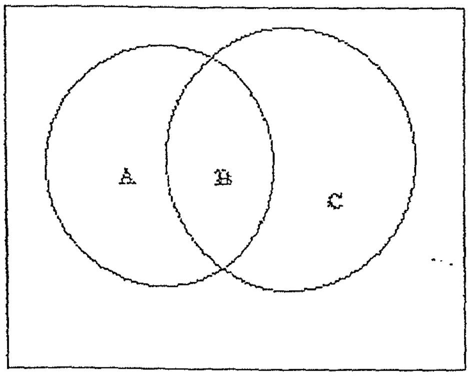
With two circles this is obviously possible, creating three regions.
With three circles, it is also possible, with 7 regions being created.
But with four circles, one region will be enclosed entirely within the circle.

Question: Is it really impossible to draw four circles such that the rule defined above is followed? If not, why is it not possible to draw four circles to fulfill the set condition, when it is possible to do it
with two or three circles? Perhaps this problem is related to the four colour theorem?

Diagrams of Intersecting circles.

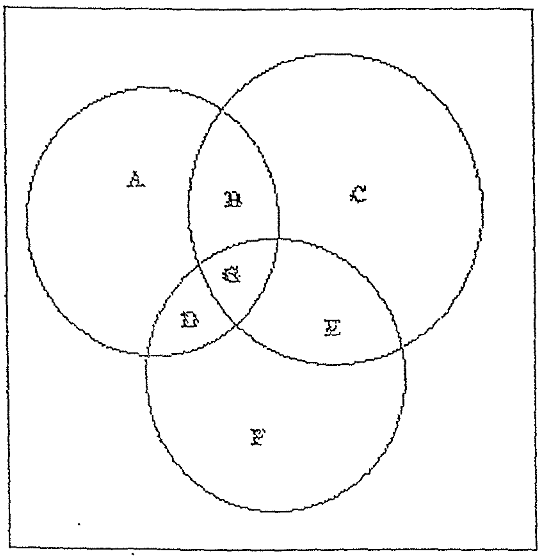


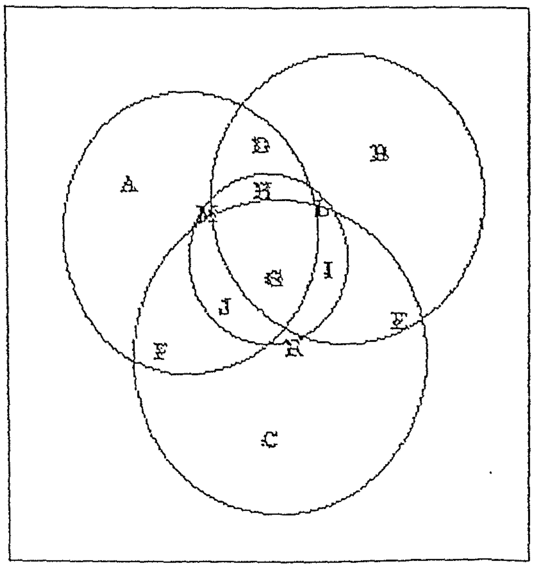
1 CIRCLE (TRIVIAL)



2 INT. CIRCLES

3 ENCLOSED REGIONS





ONE POSSIBILITY FOR FOUR INT. CIRCLES