$\sqrt{2}$ and Eulerian Primes

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1. A simple proof that $\sqrt{2}$ is irrational.

Suppose $\sqrt{2} = \frac{m}{n}$. Then it follows that

$$2n^2 = m^2$$

But this equates an odd product of primes to an even product of primes (because squares of integers have each prime repeated twice). This contradicts the fundamental theorem of arithmetic.

This proof can be generalised to show that $\sqrt{k}$ is irrational for k that has an odd number of primes in its decomposition. For example, $\sqrt{5}, \sqrt{8}$ etc are all irrational.

In fact it can be strengthened even further to show that $\sqrt{k}$ is irrational unless k is already a square. Simply repeat the argument considering any particular prime that divides k.

2. Write down the arithmetic sequence 41, 42, ... in a spiral form; along a diagonal we get the **Eulerian primes.**

57	56	55	54	**53**
58	45	44	**43**	52
59	46	**41**	42	51
60	**47**	48	49	50
61	62	63	...	...

One can find a formula for these numbers: $n^2 - n + 41$. These give a prime for $n = 0 \ldots 40$ (but not for all n).

There do exist polynomials of very high degree and order (approx. 20) whose range is the set of primes.