Infinite Sets

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Abstract

DEFINITIONS

• Two sets M and M' are **equivalent** if there exists a one-to-one correspondence between their elements, i.e. $M \sim M' \Longleftrightarrow \exists f : M \longrightarrow M'$ such that f is bijective.

• A set M is finite if either it is empty or there exists a natural number n such that $M \sim \{1, 2, ..., n\}$; otherwise M is infinite.

• A set M is **infinite** if it is equivalent to a proper subset of itself; otherwise M is finite. M is infinite $\Longleftrightarrow \exists M' \subset M$ s.t. $M \sim M'$.

• An infinite set M is countable if it is equivalent to the set of natural numbers, otherwise it is uncountable. That is, M is countable $\Longleftrightarrow M \sim N$.

THEORETICAL RESULTS

1. (a) A countable union of countable sets is countable.

(b) The set of rational numbers, Q, is countable, i.e. $Q \sim N$.