Education (Malta) Vol. 4 N° 3, pp. 32-37.

Improving Students' Understanding: A Priority In Mathematics Education

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Introduction

The Maltese National Minimum curriculum (NMC) (primary level) speaks of "the satisfaction to be derived from the achievement of success in learning" and lays down that "tests and examinations ... ought to be used by the teachers as tests of understanding and by the children as an opportunity to show their talents" (Education Department, 1990, pp. V-VI), while the secondary level NMC sets as one of the aims of secondary education "the training of the young mind in the pursuit of knowledge and reason" (Department of Information, 1990a, p. 587). Ironically, the NMC's recommendations are expected, rather naively, to function within the existing educational structures characterised as they are by examinations with selectivity in mind[1]. In this article I contend that even though "understanding" may have become today a fashionable educational term, what actually goes on in class is not as rosy as some would like us to believe. Moreover, I suggest that the prevailing present mentality of equating educational success with examinations pass rates should gradually subside in favour of greater emphasis on the processes within.

How real is Children's Mathematical Understanding?

Junior lyceums provide education for some of the more academically inclined students in the Maltese secondary schools.[2] Notwithstanding this, when I recently discussed the concept of area with some form three junior lyceum female classes[3], most students, irrespective of their mathematical achievement as measured by conventional tests, gave unsatisfactory answers when asked to elaborate on the meaning of area. Many opted to recite formulas (eg. length x breath; half base x height; etc.) and only a few were able to associate area to a measure applied to indicate the extent of a surface. None could recall the basic principle involved in area measurement: namely, the selection of a certain area (say, 1 cm^2 or 1 m^2 just to mention some standard measures) called the unit, and the definition of area measurement as the number of such units needed to completely cover the object to be measured.

The situation with male classes should not differ significantly from this[4]. Keeping in mind that area measurement is introduced in the school mathematics curriculum from as early as year IV in the primary school and appears throughout the rest of the primary and secondary mathematics syllabi, the above students' responses give weight to Hart's opinion (1978, p. 38) that "topics which one feels have been covered ad nauseam in school still seem not to be quite understood". Similarly, Buhagiar (1990) concludes that a remarkable number of form one state school students have still not yet come to terms with some basic measurement concepts and skills met at primary level. For instance, he reports that 11% of these students lack the ability to use a ruler: instead of counting the gaps these children count the endpoints, or they start counting at 1 not 0. Such findings should undoubtedly induce the conscientious mathematics teacher to ponder on the real nature of the students' mathematical understanding.

The crux of the matter seems to gravitate on what constitutes understanding. Many mathematical educators have different views about it. For instance, Haylock (1982) maintains that to understand something means to make cognitive connections. The more connections the learner can make between new and past experiences, the greater and more useful the understanding. On his part, Skemp (1976) describes two types of understanding: instrumental and relational. Understanding is instrumental if the learner knows how a skill is performed but not why it works, and relational if both are known. Later, Skemp (1979) adds logical understanding which he distinguishes from relational understanding. Logical understanding is evidenced by the ability to demonstrate that statements follow of logical necessity.

Judging by the previously cited responses, the level of understanding of the concept of area of the vast majority of form three junior lyceum students is apparently in Skemp's (1976) terminology still instrumental. I believe that the situation with area secondary school students would be identical to this, if not worse[5].

Apparently, most students can select the right area formula and work out its algorithm correctly while having only a rather vague idea, if any at all, of what area is. The cognitive connections prescribed in Haylock's (1982) definition of understanding are even less evident. For instance, only a handful of the students with whom I discussed the topic were even remotely aware of the close connection between the area furmula for rectangles (l x b) and one of the area formulas for triangles (0.5 [b x h]).

Understanding as Distinct from Manipulation of Formulas

Area measurement is just a case in point: daily contact with students of different age-groups doing various mathematical topics reveals that they are apparently learning many mathematical skills at the rote manipulation level. Consequently, computational algorithms have frequently little meaning even for those students who can successfully apply them. Mathematical formulas are not seen by students as neat conclusions to mathematical reasoning based on logical steps, but rather as heavenly manna sent to help alleviate their woes. Although it may be that students are often exposed to mathematics in unconnected lumps without any reference to the holistic nature of mathematical knowledge, it appears that formulas are perceived by students in a vacuum irrespective of whether or not these are so presented. The research project "Children's Mathematical Frameworks" (CMF) concludes that even when students aged 10-11 years are guided to arrive at a mathematical formula through concrete experiences and adequate tabulation, in three months time they are unlikely to link the formula to the tabulation (Hart, 1987).

Children's ability to manipulate formulas and work out algorithms correctly without any or little understanding of the underlying concepts is one of the problems which I feel mathematics educators should address. But do mathematics teachers actually look upon this as a problem? I dare say that many teachers consider this more of a strategy than a problem to be solved. Perhaps this is the teachers' way of beating the Maltese educational system which tends to set the same levels for all students irrespective of personal abilities[6]. It may be, as suggested by Brown (1982), that teachers are teaching by rote in an attempt to compensate for children's lack of understanding. While questioning the wisdom of this policy, Brown (1982, p. 460) sustains that the best strategy with low achievers is to "abandon all teaching of routine skills ... (and) ... to concentrate instead on building up a network of mathematical relationships (schemas) ... with the use, where necessary of concrete materials ...". Teachers should keep in mind that although rote learning can lead to instrumental understanding, it neither helps students to make connections nor to develop relational understanding, let alone logical understanding.

I fear that Maltese teachers are unlikely to put into practice Brown's (1982) advice as our highly competitive educational system, geared towards "success" at the 11+ and 16+ examinations, hardly allows time for "... practical ... experience ... the most effective means by which understanding of mathematics can develop ..." (Cockroft Report, 1983, p. 84). The Education Department's "policy of continuous school-based assessment of students, *complemented* (my emphasis) with national end of-year examinations" (Department of Information, 1990b, p. 87), may sound unfamiliar to many a teacher's ear given the present educational set-up which tends to sacrifice understanding for the sake of attainment.

Assessing Understanding

On examinations, the NMC (secondary level) regulates that "every effort should be made to introduce cumulative assessments and to play down the negative aspects of examinations" (Department of Information, 1990a, p. 589). While one can argue that this indicates the education department's dissatisfaction with its own examination system, I believe that our outdated assessment system, which rewards teaching based on drills and rote learning to the detriment of mathematical concept development, is mainly to blame for children's lack of understanding. The continued improvement of teachers' pre-service and in-service education will not have much value unles complemented by a system conducive to concept learning.

As an initial step I suggest that the present conventional assessment methods should be eliminated in favour of the gradual development of diagnostic testing instruments in line with the "Chelsea Diagnostic Mathematics Tests" developed by the "Concepts in Secondary Mathematics and Science" (CSMS) research project (Hart, 1981)[7]. Rather than the computational skills of the students, the CSMS test papers examine the understanding of the processes and underlying ideas. This will offer the mathematics teachers the possibility of concentrating on concept development, relegating computational skills to a definitely secondary role. One may even consider whether oral testing, which the NMC (secondary level) suggests for inclusion in languages and the normative core subjects (ie., religious education; civics and environmental attitudes; and sport) (Department of Information, 1990a), can be adopted in mathematics examinations.

In spite of Skemp's (1982, pp. 25-26) warning that "in general, concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but only by collecting, for him to experience, suitable examples", many mathematics teachers are rushing through the syllabus splashing formulas and definitions all over in a fervent desire to finish it on time for the all important annual examination. All this, when definitions are supposed to add precision to the boundaries of a concept already formed, and to state explicitly its relation to other concepts. Ideally, children should become active learners as they develop their own understanding of those mathematical concepts held by mathematics educators and by the students themselves to be relevant to the present and future needs of the students.

Teachers cannot successfully help students improve their mathematical understanding unless they have insights of students' thinking in relation to the concepts being developed. For this purpose, Bell et al. (1986) recommend the Piagetian method of asking a child probing questions about a carefully chosen situation, which they claim to be very powerful. Woodward (1982) illustrates such a situation: Heidi, described by her teacher as an excellent seventh grade mathematics student, could calculate the area of rectangular shapes by using the appropriate formula and algorithm. The child's ability to manipulate the formula could easily have led her teacher to believe that she had understood the concept of area had Heidi not subsequently uttered a seemingly insignificant statement which suggested otherwise[8]. The teacher wisely acted upon this statement and her follow-up incisive questions revealed that not only did the child not associate the area of a shape to its size, but that she could not even functionally distinguish between area and perimeter.

The "Successful" Mathematics Teacher

In a world where certification does not necessarily imply learning, the "successful" mathematics teachers are regrettably those who manage to develop easily memorizable drill schemes in a bid to coach their students to tackle the usual type problems. A final rush through the past papers, something which students have become accustomed to and are likely to demand, seems to crown their glory. Drills, I would say, do have their place in the mathematics curriculum as "some concepts can be introduced from exercises that ostentatiously have their origins in drill exercises, that initially appear to have no connection to the concept involved" (Olson, 1979, p. 399). It is rather the drill for drill's sake of the traditional curriculum that should be replaced by greater emphasis on understanding.

Teachers who bother to look beyond examinations to delve into the realm of mathematical reasoning hardly ever get the praise they deserve. For one thing, they are unlikely to finish their syllabus by the end of the academic year. Faced with the delicate dilemma of deciding between what they know is right but is unlikely to bear fruit given the system, and what they know to be wrong but is more likely to deliver the goods, an uncomfortable balance between the two extreme positions is usually sought: concepts are presented briefly with formulas and rote computations following almost immediately. What usually ensues is a "superficial coverage of topics ... (which) ... leaves students with little sense of understanding and accomplishment, fewer opportunities for problem solving and less development of skills" (Bybee et al., 1990, p. 93)[9].

The form three students referred to at the beginning of this article, when hard pressed, did vaguely recollect images of themselves finding areas of flat shapes by counting the number of squares. However, they could not relate the counting of the squares to the area formulas. This reflects their uneven transition from the practical and concrete mathematics, the very basis of concept development, to the formal and abstract mathematics, the type of mathematics usually assessed by conventional examinations. The resulting mathematical lacunas, which should have been avoided in the first place, although often identified, are hardly ever remedied later on.

A Mathematics Programme based on Understanding

Richards (1990) cautions that mathematics programmes should be designed so that children work from their own points of understanding. Regrettably, the pedagogical dictum of "moving from the known to the unknown" is often neglected by the many mathematics teachers who consider themselves primarily, if not solely, responsible for the coverage of the present year's syllabus irrespective of students' mathematical background. Are these teachers to blame? Each year's mathematical syllabus is already too vast in itself to allow teachers the necessary time to devote to the much needed remedial work. Teachers' efforts are further hampered by a syllabus which at times exposes children to mathematical topics incompatible with their level of cognitive development.

Actually, this all boils down to the curriculum "depth vs breath" debate. Logic dictates that if children need more time to learn concepts than is presently allotted during the scholastic year, either fewer concepts should be introduced, or else a leveling down of content takes place. While an answer to this delicate dilemma is, I feel, beyond the scope of this article, one augurs that all ensuing debates centre round children's understanding. Educational planners should keep in mind that "less may mean more" if what is meant by "less" is that as fewer topics are introduced the teachers would eventually have more time on their hands as to be able to direct their teaching towards concept development and provide the much needed remedial education. Providing students from the early stages with a good grounding in mathematical knowledge would undoubtedly accelerate their future studies.

The postponement of the first official examinations until the end of the fourth primary year offers teachers in the first three years the necessary time and tranquillity to concentrate on the development of mathematical concepts. It would indeed be unforgivable were these teachers to stick to their traditional methods in presenting mathematics as a collection of often unconnected skills and techniques, as I suspect some still do, even though they are give this opportunity. More than anything else, people's unwillingness to change often emanates from the fear that the unknown holds. The education department can help in this respect by regularly holding in-service courses. Besides presenting the latest methodologies, such courses should function as a medium which encourages debate on the educational, philosophical and sociological aspects of reforms. A consulted and informed teaching force is more likely to accept and work in favour of planned changes.

The primary and secondary mathematics curriculum should seek to address the needs of all students; whilst stretching each individual student's potential to the full, it should guarantee the minimum acceptable level of numeracy for all[10]. A curriculum which is primarily, if not exclusively, geared towards student's preparation for further mathematical activity, may eventually only benefit that minority which actually continues with its mathematical studies. Our educational system, regardless of the well-sounding phrases (eg., the primary and secondary NMC) does not in practice cater for the individual student except for some extreme cases (eg., support teaching in year III). It has for years promoted, maybe unintentionally, the more able students to the detriment of all the rest, even though it may well be that the very gifted succeed in spite of their schooling. For instance, the National Council of Teachers of Mathematics (NCTM) (1980, p. 18) feels that "the student most neglected, in terms of realizing full potential, is the gifted student of mathemtaics", while Sinkinson (1982) argues that the abler students, who usually employ "own methods" when solving mathematics problems instead of the formal "thought" ones, are somewhat penalised by a system which looks unfavourably on "child methods" and is more likely to accept the formal thought methods.

Within state schools, albeit the need for restructuring and reform are so evident, isolated efforts by the individual teacher, however competent and well-meaning, can actually jeopardise students' attainment notwithstanding the considerable gains in mathematical understanding. Of its very nature, Malta's highly centralised educational system forbids individual initiative. In this respect, secondary private schools are at an advantage. They are in a better position to organise their teaching to suit the personal needs of their students, even though they eventually have to face the 16+ examinations, something which I fear tends to dictate their teaching methods in much the same way as it does for the state schools. Primary private schools without access to secondary education within their own system are similarly affected in their educational efforts by the 11+ examinations.

The Reflective Practitioner

The views expressed within this article might sound familiar to many mathematics teachers as they are likely to have been repeatedly debated within the four walls of many a staffroom. The hitch of the situation is that this is where the debate usually ends. The numerous "reforms" thrown down teachers' throats over the years have reduced teachers to a seemingly helpless lot. As a consequence, teachers tend to neglect one of the primary duties towards the community: in conjunction with parents and other interested community members, teachers can form part of a catalyst force urging for reforms. Granted that reforms in Malta only materialise with the Education Department's blessings, history has shown that such reforms stand a better chance of success if they are accepted by and reflect the needs of the parties concerned[11]. Teachers should and ought to be consulted on all matters regarding their immediate and related areas of interest. The "slot-filler" mentality in which teachers are just numbers to be juggled about according to the latest rules of the game ought to stop to be replaced by system based on collective bargaining leading to fruitful agreements.

While eagerly awaiting the implementation of the much publicised 1988 Education Act's calls for school autonomy and decentralisation, a parallel line of action ought to ensue: conscious efforts should be directed at changing teachers' current passive mentality into a more reflective and active one. The reflective practitioner, Van Manen (1991, p. 153) contends, is a "professional who reflects in action through constant decision making ... guided by the theoretical and practical principles of his or her discipline - even though these principles may be operating in a more or less tacit fashion". Reflective participation, even though restricted by the nature of our centralised system, is somehow possible within the four walls of the classroom where the Maltese teacher enjoys considerable autonomy. However, as soon as the teacher steps out of this domain all initiatives of participation are immediately blocked, and any efforts by teachers to break through this barrier are looked upon unfavourably and may be even interpreted as a treat to school authority. Presently, teacher's participation in the running of their schools is limited to yearly elections of their representatives on the school councils. These councils, more often than not, do not, and cannot, function properly handicapped as they are by gross financial limitations and no real decision-making power[12]. This is hardly the ideal setting in which teachers can participate as reflective practitioners.

Planning towards the Future

Although not all Maltese mathematics educators may agree on the direction towards which to steer our energies, I believe that all concur that curricular changes are needed rather urgently. These changes, however, can only be effective if implanted into an educational system conducive to their practical application - something which our system definitely is not. One hopes that frank discussions among all interested parties would lead to national educational policies based on the widest possible consensus geared at improving children's understanding. Richardson (1988) warns that unless the focus is shifted from performance to understanding, teachers will be interfering in, rather than helping with,the development of mathematical concepts in children. It is high time that Maltese curriculum planners and teachers heed this admonition. Basically, it entails a quality leap of looking beyone the question. "What can this child do?" to "What does this child understand?". The forthcoming setting up of the Secondary Education Certificate Examination (SEC) in mathematics by the University of Malta, provided due consideration is given to assessing children's understanding, may eventually prove the right opportunity to start gradually building anew[13].

Notes

1.Bezzina (1991) argues that unless the NMC alters the way schools function, something which he contends it does not, the whole exercies (ie., the implementation of the NMC) would eventually prove futile.

2.Figures published by the Department of Information (1990b, p. 87) show that about 75% of the total year VI population (state & private) applied for the May 1990 junior lyceum entrance examinations, 43.6% of the entrants were successful. This percentage represents approximately 32.7% of total age group.

3.These classes, five in all, are taught mathematics by the author in a female junior lyceum. In this particular school,form three students are grouped into classes according the their option choices. Usually, classes are of mixed ability. The author feels that the classes referred to in this article (5 classes out of a total of 10 form three classes) are representative of the mathematical ability spread of the form three population of this school.

4.Buhagiar (1990) concludes from a study on Maltese form one state school students that the performance of male and female students on area concepts is comparable.

5.Suffice it to mention that the Schools Council Low Attainers in Mathematics Project and HMI school surveys suggest that the practice of routine skills is given the highest priority with low attainers (Brown, 1982).

6.Notwithstanding the rigid streaming meant to permit all students to move at their own pace, by the end of the academic year all students are expected rather illogically to have practically covered the same ground.

7.For detailed information about the purposes and uses of these tests see Hart et al. (1985).

8.Heidi calculated correctly the area of two given gardens and established that the two areas were equal. However, she still thought that one garden was bigger than the other.

9.Although Bybee et al.'s (1990) article actually refers to the science curriculum, the author feels that their conclusions are equally applicable to the mathematics curriculum.

10.The meaning attached to "numeracy" here is as defined by the Cockroft Report (1983, paragraph 39). The report rejects the notion (found in many submissions) that numeracy merely describes the ability to do basic calculations. Instead it should include feeling "at home" with using numbers, and to be able to understand their use by others.

11.Consider the failure of the comprehensive system. Introduced in Malta in 1972 it was definitely dropped in 1981 with the opening of the first junior lyceums heralding the return of selectivity.

12.School councils are regulated by section IV of the 1988 Education act (Department of Information, 1988, pp. 282-283).

13.The first Secondary Education Certificate (SEC) examination in mathematics by the University of Malta is to be held in May/June 1992.

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