On Polynomial Reconstruction of Disconnected Graphs *

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1 Introduction

Abstract

Let H be a disconnected graph with connected components $H_1, H_2, \ldots, H_t$. If the characteristic polynomial of H were **not** reconstructible from the deck of characteristic polynomials of its one-vertex deleted subgraphs, then H would consist of exactly two connected components of the same order. We show that if H has a pendant edge in the component with the larger number of edges or if the smaller component of H is a tree, then H is polynomial reconstructible.

Keywords: *disconnected graphs, Ulam's Reconstruction Conjecture, polynomial reconstruction, p-deck, adjacency matrix, characteristic polynomial, spectrum of eigenvalues, interlacing.*

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2 Introduction

A graph $G(\mathcal{V}, \mathcal{E})$ of order n has a vertex set $\mathcal{V}(G) = \{v_1, v_2, \ldots, v_n\}$ and a set $\mathcal{E}$ of $m(G)$ edges joining distinct pairs of vertices. The **adjacency matrix** $\mathbf{A}(G)$ (or $\mathbf{A}$) of a graph G is an $n \times n$ symmetric matrix $[a_{ij}]$ such that $a_{ij} = 1$ if v_i and v_j are adjacent and 0 otherwise. The adjacency matrix describes G completely (up to isomorphism). The **characteristic polynomial** ϕ, defined as $\phi(G, \lambda) :=\mathrm{Det}(\lambda \mathbf{I_n} - \mathbf{A})$, can be expressed as

$$\phi(G, \lambda) = \prod_{i=1}^{n} (\lambda - \lambda_i). \tag{1}$$

It is a polynomial $\sum_{i=0}^{n} a_i \lambda^i$ with integer coefficients a_i. We refer to the solutions $\lambda_1, \lambda_2, \ldots, \lambda_n$ of the characteristic equation of $\mathbf{A}$ as the **eigenvalues** of G. They are independent of the labelling of G and they are said to form the **spectrum** $Sp(G)$ of G [1, 2, 4].
In 1941, S. M. Ulam and P. J. Kelly formulated what has become known as Ulam's Reconstruction Conjecture (RC) [9, 12]. Alternative statements of the RC were presented independently by Kelly and Harary in 1964. In the latter's form we are presented with a deck $\mathcal{D}$ of n cards, each showing a one-vertex-deleted unlabelled subgraph $G - v$ for each $v \in \mathcal{V}(G)$. The problem is to recover the parent graph from $\mathcal{D}$. Many partial results have been obtained and the RC has been proved for various classes of graphs including regular graphs, trees and disconnected graphs.

A variation of the RC, first posed by D. M. Cvetković in 1973 at the Eighteenth International Scientific Colloquium held in Ilmenau and later considered by I. Gutman and D. M. Cvetković in [7], is the **polynomial reconstruction problem** (PRP) which asks whether we can recover the characteristic polynomial of a graph H of order n from, the **p-deck**, $\mathcal{PD}(H)$, of H, consisting of the n characteristic polynomials of the one-vertex-deleted subgraphs (with multiplicities) [6]. This is referred to as **Problem D** by A. Schwenk in [10]. Since the solutions of $\phi(G - v, \lambda) = 0$ are the eigenvalues of $G - v$, we refer to the information on a card as either a characteristic polynomial or a set of eigenvalues.
S. Simić resolved the problem positively for connected graphs with the smallest eigenvalue of the one-vertex-deleted subgraphs bounded below by -2 [11]. Also D. M. Cvetković and M. Lepović showed that a tree T is polynomial reconstructible in [8] by showing that there is no graph H which is non-isomorphic with T and with the same p-deck as T, such that $(H,\ T)$ is a counter example pair to the PRP.
A major problem that is often the cause why polynomial reconstruction is hard to prove for a particular class of graphs is that disconnected graphs often

present themselves as candidates for a counter example. In specific examples considered so far, non isomorphic graphs of order at least 3, with the same p-deck were found to be cospectral so that they do not provide counter example pairs to the PRP[5]. In this article, we consider the class $\{H\}$ of disconnected graphs. We study the cases where a component belongs to two particular classes in turn and show that certain subclasses of disconnected graphs are polynomial reconstructible.
We shall not give an algorithm to reconstruct $\phi(H)$. Instead, we study the properties that a graph G must have such that for H disconnected, $(H,\ G)$ is a counter example pair to the PRP. This approach establishes certain subclasses whose graphs cannot be counter examples, providing a positive answer to the PRP for all such subclasses.

We list below a set of results which we will be making use of:

Theorem 2.1 (F.H.Clarke)[3, 1]
Let G be a graph and let $\phi(G,\lambda) = \phi(G)$ be its characteristic polynomial. The derivative of the characteristic polynomial is given by:

$$\phi'(G,\lambda) = \sum_{i=1}^{n} \phi(G - v_i, \lambda)$$

Corollary 2.2 *From the polynomial deck of the subspectra, all the coefficients in $\phi(G,\lambda)$ are derived, except for the constant term a_0.* ■

Proof: By integration of $\phi'(G,\lambda)$, the result follows immediately. ■
This much has been obtained independently by A. J. Schwenk [10] and Gutman and Cvetković [7].

Theorem 2.3 The Interlacing Theorem
*Let G be a graph and let $v \in \mathcal{V}(G)$. If $Sp(G)=\{\Lambda_i\}$ where $\Lambda_1 \geq \Lambda_2 \geq \ldots \geq \Lambda_n$ and $Sp(G-v) = \{\mu_i\}$ where $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_{n-1}$,
then the eigenvalues of $(G-v)$ interlace with those of G,* i.e.

$$\Lambda_1 \geq \mu_1 \geq \Lambda_2 \geq \mu_2 \geq \ldots \geq \mu_{n-1} \geq \Lambda_n. \quad \blacksquare$$

In section 3, we discuss the properties of a potential counter example pair $(G,\ H)$ to the PRP when H is disconnected. In section 4, interesting interlacing properties of the eigenvalues of G, H and their subgraphs reveal that certain subclasses of the class of disconnected graphs are polynomial reconstructible. We proceed, in section 5, to compare the number of edges of the components of H, were a counter example to exist. This analysis exhibits various subclasses of the class of disconnected graphs which are polynomial reconstructible. We conclude by reviewing the subclasses of the class of disconnected graphs which we showed to be polynomial reconstructible. We proceed to highlight the remaining subclasses for which the PRP is still open.

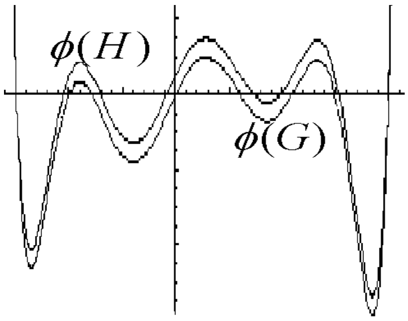

Figure 1: The polynomials $\phi(G)$ and $\phi(H)$.

3 If the PRP had a negative answer for Disconnected Graphs

Remark 3.1 Let H be a disconnected graph with components $H_1 \dot{+} H_2 \dot{+} \ldots \dot{+} H_t$, $2 \leq t \leq n$. Suppose that H is not uniquely reconstructible and that the graph G has the same p-deck as H and the pair (G, H) is a counter example to the PRP. From Corollary 2.2, it follows that $\mathcal{PD}(G) = \mathcal{PD}(H)$ but $a_0(G) \neq a_0(H)$.

Lemma 3.2 *$\phi(H)$ is a polynomial with real roots.*

Proof: The result follows since $\mathbf{A}(H)$ is real and symmetric so that the eigenvalues of H are real. ■

Remark 3.3 If T is a tree and $(H,\ T)$ a counter example pair to the PRP, then H is necessarily disconnected. As a preparation to their proof that T is polynomial reconstructible, D. Cvetković and M. Lepović listed the properties of H as a partner in a counter example pair which we describe in Lemmas 3.4 to 3.7.

Lemma 3.4 *(i)* $n(G) = n(H)$

(ii) $m(G) = m(H)$

(iii) G and H have the same degree sequence

Proof:

(i) G and H agree on the number of verices since the number of polynomials of the one-vertex-deleted subgraphs are the same for both G and H.

(ii) $m(G) = m(H)$ follows since $\phi(G) = \phi(H)$ save for the constant term. Hence, the coefficient of λ^{n-2} is the same for both.

(iii) G and H have the same degree sequence since they have the same p-deck and the degree of the i^{th} vertex is the difference between the coefficients of λ^{n-2} in $\phi(G)$ and of λ^{n-3} in $\phi(G - v_i)$. ■

Lemma 3.5 *G and H have no eigenvalues in common.*

Proof: Suppose that G and H share a common eigenvalue, say λ_i. Then, $\phi(G, \lambda_i) = \phi(H, \lambda_i) = 0$. $\Rightarrow a_0(G) = a_0(H)$ contadicting the result in Remark 3.1. ■

Lemma 3.6 *G is connected.*

Proof: Suppose G is disconnected with $G = G_1 \dot{+} G_2 \dot{+} \ldots \dot{+} G_s,\ 2 \leq s \leq n$. Then, the maximum eigenvalue that appears in $\mathcal{PD}$ is $\lambda_{max}(G)$. Since H is disconnected, the same holds for H. But then H and G share a common eigenvalue λ_{max}. ■

Lemma 3.7 $\lambda_{max}(G) > \lambda_{max}(H)$.

Proof: $\lambda_{max}(H) = \lambda_{max}(H - v_i)$ for some i since H is disconnected. Now, since $\mathcal{PD}(G) = \mathcal{PD}(H)$, $\exists w \in G$ such that $\lambda_{max}(G - w) = \lambda_{max}(H)$. But $\lambda_{max}(G - w) < \lambda_{max}(G)$, since any vertex deletion in a connected graph lowers the maximum eigenvalue. This implies that $\lambda_{max}(H) < \lambda_{max}(G)$ as required. ■

Theorem 3.8 *If the graph H is disconnected, G has the same p-deck as H and $0 < \phi(H, \lambda) - \phi(G, \lambda) = \Delta a_0$, then $a_0(H) = a_0(G) + \Delta a_0$ where $\Delta a_0 > 0$.*

Proof: Since $\phi'(H, \lambda) > 0,\ \forall \lambda > \lambda_{max}(H)$, from Lemma 3.7 and Figure 1, it follows that $\phi(H, \lambda_{max}(G)) = \Delta a_0 > 0$. Thus $\phi(H, \lambda) - \phi(G, \lambda) = \Delta a_0,\ \forall \lambda$. In particular $\phi(H)(0) - \phi(G)(0) = \Delta a_0$. It follows that $a_0(H) - a_0(G) = \Delta a_0 > 0$.■

4 Interlacing Properties of Eigenvalues

Theorem 4.1 *The eigenvalues of H and G interlace in pairs* i.e. *if $\{\Lambda_i\}$ where $\Lambda_1 \geq \Lambda_n \geq \ldots \geq \Lambda_n$ are the eigenvalues of G and if $\{\zeta_i\}$ where $\zeta_1 \geq \zeta_2 \geq \ldots \geq \zeta_n$ are the eigenvalues of H, then:*

$$\Lambda_1 > \zeta_1 \geq \zeta_2 > \Lambda_2 \geq \Lambda_3 > \zeta_3 \geq \zeta_4 > \Lambda_4 \geq \ldots \geq \zeta_n > \Lambda_n.$$

Proof: To understand the strict inequalities in the result, we use Lemmas 3.2, 3.5 and Figure 1. ■

Remark 4.2 In [8], the properties of a disconnected graph H, which is not polynomial reconstructible, are treated to prove that a tree T cannot have a partner which together with T forms a counter example pair to the PRP. Here we reproduce these results since they lead to Theorems 5.5 and 5.6.

Theorem 4.3 *H consists of exactly two connected components H_1 and H_2.*

Proof: Suppose $H = H_1 \dot{+} H_2 \dot{+} \ldots \dot{+} H_t,\ 2 \leq t \leq n$. Let, without any loss of generality, $\lambda_{max}(H)$ be an eigenvalue of H_1.

Case (i) Suppose that the two largest eigenvalues of H_1 are the two largest eigenvalues of H. But then if $v \in H_2$, $H - v$ will still have two eigenvalues of H_1 between the two largest eigenvalues of G. Thus, $\exists w \in \mathcal{V}(G)$ such that $G - w$ also has these two eigenvalues of H_1 between the two largest eigenvalues of G. This contradicts The Interlacing Theorem 2.3. Hence, the two largest eigenvalues of H belong to spectra of different components.

Case (ii) Suppose that H_3 is the third component of H, H_1 and H_2 being the components which have their largest eigenvalues between the two larger eigenvalues of G. Then if $u \in \mathcal{V}(H_3)$, $H - u$ will have two eigenvalues between the two larger eigenvalues of G. The same situation holds for $G - w$ for some $w \in \mathcal{V}(G)$. This is again a contradiction to The Interlacing Theorem 2.3.

Hence, H has two components only. ■

Lemma 4.4 *If $H = H_1 \dot{+} H_2$, then each component has distinct eigenvalues.*

Proof: Suppose there exists some $v \in H_i$ such that $\phi(H - v)$ has a repeated factor $(\lambda - \lambda_0)$. Thus, λ_0 is an eigenvalue of H and is also an eigenvalue of G since G and H have the same $\mathcal{PD}$. This contradicts Lemma 3.5. ■

Corollary 4.5 *If $H = H_1 \dot{+} H_2$ where H_1 is C_k, the circuit on k vertices, then H is polynomial reconstructible.*

Proof: This follows since every circuit has a repeated eigenvalue. ■

Lemma 4.6 *Let $H = H_1 \dot{+} H_2$ be a disconnected graph which is not polynomial reconstructible. Then H_1 and H_2 are of the same order.*

Proof: Suppose $n(H_1) > n(H_2)$. Then, $\forall v \in \mathcal{V}(H_2)$ such that $H - v$ has two eigenvalues of H_1 next to each other on the real line with no eigenvalues of $H - v$ in between. Since G has the same p-deck as H, there exists some $\omega \in G$ such that $G - \omega$ has two eigenvalues between two successive eigenvalues of G and none between two other successive eigenvalues of G, contradicting the Interlacing Theorem 2.3. ■

Theorem 4.7 *Let H be a disconnected graph and $(H,\ G)$ be a counter example pair to the PRP. Then the multiplicity of the eigenvalues of H does not exceed two.*

Proof: Suppose for contradiction that the multiplicity of λ_0 exceeds two. Then $H - v$ has the eigenvalue λ_0 repeated at least twice, a contradicting the existence of G. ■

Remark 4.8 Since $\phi(H) = O(\lambda^n)$, the following result is deduced immediately. It justifies the shape of $\phi(H)$ in Figure 1.

Corollary 4.9 *The characteristic polynomial $\phi(H)$ has $n(= 2k)$ zeros with k minimum values and $k - 1$ maximum values.*

Theorem 4.10 *Let the eigenvalues of G be $\Lambda_1, \Lambda_2, \ldots, \Lambda_{2k}$. If $\lambda_1, \lambda_2, \ldots, \lambda_k$ are the eigenvalues of H_1, then $\Lambda_1 > \lambda_1 > \Lambda_2$; $\Lambda_3 > \lambda_2 > \lambda_4; \ldots \Lambda_{2k-1} > \lambda_k > \Lambda_{2k}$. Similarly, if $\ell_1, \ell_2, \ldots, \ell_k$ are the eigenvalues of H_2, then $\Lambda_1 > \ell_1 > \Lambda_2$; $\Lambda_3 > \ell_2 > \lambda_4; \ldots \Lambda_{2k-1} > \ell_k > \Lambda_{2k}$. Thus if $\phi(G)$ has a minimum value between two successive eigenvalues of G, then H_1 and H_2 both have one eigenvalue in this range.*

Proof: From Lemma 3.2, Theorem 3.8 and Theorem 4.1, we know that there are two successive eigenvalues of H between two eigenvalues of G in which range there is a minimum value of G. Since G and H have the same p-deck, $H_i - v$ must have all its the eigenvalues common with some of those of $G - \omega$ for some $\omega \in G$. These lie in ranges on the real line between successive eigenvalues of G where G has a maximum value. The remaining eigenvalues of G are those belonging to $H_j,\ i \neq j$, and lie in ranges on the real line between successive eigenvalues of G where G has a minimum value. ■

Lemma 4.11 *$H_1 - v$ and H_2 have no eigenvalues in common.*

Proof: Suppose for contradiction, that $H_1 - v$ and H_2 do share a common eigenvalue. Then, $\phi(H - v)$ has a repeated eigenvalue and so does $\phi(G - \omega)$ for some $\omega \in \mathcal{V}(G)$. For a counter example to exist, this is not allowed by Lemma 4.4. ■

Theorem 4.12 *For all vertices $v \in \mathcal{V}(H_1)$, the sets of eigenvalues $\{\mu_i : 1 \leq i \leq k - 1\}$ of $H_1 - v$ and $\{\ell_i : 1 \leq i \leq k\}$ of H_2 strictly interlace each other.*

Proof: There exists $\omega \in G$ such that $\phi(G-\omega) = \phi(H-v)$. Thus, the eigenvalues of $H-v$ interlace with the eigenvalues of G. Also the eigenvalues of H_2 lie in the ranges between consecutive eigenvalues of G where $\phi(G)$ has a minimum value. This implies that the eigenvalues of $H_1 - v$ lie in ranges between consecutive eigenvalues of G where a maximum value of $\phi(G)$ exists. We deduce that
$\ell_1 > \mu_1 > \ell_2 > \ldots > \mu_{k-1} > \ell_k$. ■

Remark 4.13 We refer to the result obtained above as the **Strict Interlacing Theorem.** From several examples we have tried, this strict interlacing among eigenvalues of graphs, of the same order, appears to be very hard to satisfy and could be the key to the successful search for a counter example were one to exist.

Corollary 4.14 $\displaystyle\sum_{i=1}^{k}(l_i)^2 > \sum_{i=1}^{k-1}(\mu_i)^2.$

Proof: For all $v \in \mathcal{V}(H_1)$, there is a matching between the eigenvalues of $H_1 - v$ and $n-1$ of the eigenvalues of H_2, such that for each eigenvalue μ_j of $H-v$, there is an eigenvalue ℓ_t, of H_2, $t = j$ or $j+1$ such that $(\ell_t)^2 \geq (\mu_j)^2$. Since ℓ_{max} lies between the two largest eigenvalues of G and μ_{max} does not, the matching of ℓ_{max} with μ_{max} ensures that the inequality relating the sum of the powers is strict. ■

5 Edges of Components

Remark 5.1 The result of Corollary 4.14 enables us to compare the number of edges of the two components H_1 and H_2 of H. We denote by ρ_{min}, the minimum degree of the component H_1 of H.

Lemma 5.2 *Let $H = H_1 \dot{+} H_2$. If $m(H_1) \geq m(H_2)$ and a counter example pair (H, G) exists, then*

$$m(H_1) - m(H_2) \leq \rho_{min} - 1$$

Proof: By Corollary 4.14

$$\sum_{i=1}^{k}(l_i)^2 > \sum_{i=1}^{k-1}(\mu_i)^2$$

Now no card of the p-deck of H has repeated eigenvalues and interlacing of the $\ell_i s$ and the $\mu_j s$ is strict. Moreover, since $\sum_{i=1}^{k}(l_i)^2 = 2m(H_2)$ and

$\sum_{i=1}^{k-1}(\mu_i)^2 = 2m(H_1 - v)$, then

$$\sum_{i=1}^{k}(l_i)^2 - \sum_{i=1}^{k-1}(\mu_i)^2 \geq 2$$

Thus

$$2[(m(H_2) - m(H_1)) + \rho_{min}] \geq 2$$

Hence

$$0 \leq m(H_1) - m(H_2) \leq \rho_{min} - 1 \blacksquare$$

Remark 5.3 We use Lemma 5.2 repeatedly to determine subclasses of disconnected graphs which are polynomial reconstructible.

Theorem 5.4 *Let $H = H_1 \dot{+} H_2$, and $m(H_1) - m(H_2) \geq \rho_{min}$. Then the PRP has a positive answer for H.*

Proof: This is just the contrapositive of Lemma 5.2. $\blacksquare$

Theorem 5.5 *Let $H = H_1 \dot{+} H_2$ and $m(H_1) > m(H_2)$. If H_1 has a pendant edge, then H is polynomial reconstructible.*

Proof: If counter example (H, G) to the PRP exists, since $\rho_{min} = 1$, then $m(H_1) - m(H_2) = 0$ follows from Lemma 5.2. $\blacksquare$

Theorem 5.6 *Let $H = H_1 \dot{+} H_2$ be a disconnected graph such that the component H_2 with the smaller number of edges, is a tree, then the graph is polynomial reconstructible.*

Proof: By Lemma 4.6, $n(H_1) = n(H_2) = k$ and $m(H_2) = k-1$. Suppose that H were not polynomial reconstructible. Then G exists and $m(H) = m(G) \geq 2k - 1$. Thus $m(H_1) \geq k$. Let H_1 have m edges. By strict interlacing of the eigenvalues of H_2 and $H_1 - v$,

$$\sum_{i=1}^{k-1}(\mu_i)^2 < \sum_{i=1}^{k}(l_i)^2$$

$$2(m - \rho_{min}) < 2(k - 1)$$

$$\Longrightarrow k \leq m < k - 1 + \rho_{min} \qquad (2)$$

But for H_1, $k\rho_{min} \leq 2m$. Thus from inequality (2),

$$k\rho_{min} < 2\rho_{min} + 2k - 2$$

$$\Rightarrow k(\rho_{min} - 2) < 2\rho_{min} - 2$$

Thus for for $\rho_{min} > 2$,

$$k < \frac{2(\rho_{min} - 1)}{\rho_{min} - 2}. \tag{3}$$

Case(i) If $\rho_{min} = 1$, then $m < k$ from inequality (2). Thus $m = k - 1$ and H_1 is a tree. The number of edges in such a graph H precludes the existence of a counter example. Besides, by inequality 2, $\rho_{min} > 1$.

Case(ii) If $\rho_{min} = 2$, then $m < k + 1$ from inequality (2). Then $m = k$ and H_1 is a unicyclic graph, which is a contradiction since a component cannot be a circuit by Corollary 4.5.

Case(iii) If $\rho_{min} = 3$, then $k < 4$ from inequality (3).

Case(iv) If $\rho_{min} > 3$, then $k < 3$ from inequality (3).
But all graphs of order at most 10 have been shown to be polynomial reconstructible [8] and hence a counter example $(G,\ H)$ to the PRP does not exist. ■

6 Conclusion

In the course of the proofs of the results above, it is clear that the following conditions are sufficient for the various subclasses $\{H\}$ of the the class of disconnected graphs to be polynomial reconstructible:

1. The number of components of H is more than two.
2. $H = H_1 \dot{+} H_2$ and $n(H_1) \neq n(H_2)$.
3. $H = H_1 \dot{+} H_2$ where H_i and $H_j - v,\ i \neq j$ have a common eigenvalue.
4. One of the components has a repeated eigenvalue.
5. The second larger eigenvalue of one component H_i is greater than the maximum eigenvalue of $H_j,\ i \neq j$.
6. Each component is a tree.
7. One component is a tree and the other unicyclic [8].
8. $H = H_1 \dot{+} H_2$ and $m(H_1) - m(H_2) \geq \rho_{min}(H_1)$.
9. $H = H_1 \dot{+} H_2$, $m(H_1) > m(H_2)$ and H_1 has degree 1.
10. $H = H_1 \dot{+} H_2$, $m(H_1) > m(H_2)$ and H_2 is a tree.

The PRP is still open for the following subclasses $\{H\}$ of the class of disconnected graphs:

1. $H = H_1 \dot{+} H_2$ and $m(H_1) = m(H_2)$.
2. $H = H_1 \dot{+} H_2$, $0 < m(H_1) - m(H_2) < \rho_{min}$, H_1 has no terminal vertices and H_2 is **not** a tree.

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