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Predictability, Long Memory and Non Linear Dynamics of Stock Returns

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Abstract

The presence of long-range dependence and nonlinear dynamics in stock returns is examined using data from the Athens Stock Exchange. The authors apply (among other well-known time series techniques) the Rescaled Range (R/S) Statistic AND THE modified R/S statistic, as being more appropriate for tracking short and long memory in the stock market and as more robust to other alternative methods. The results support evidence of short memory in all single stock returns in the sample used but there is an agreement between the two types of R/S statistics regarding existence of strong long memory in the squared returns. The findings are in agreement with recent empirical evidence investigating long-range and short-range statistical dependence in other stock exchanges apart from the Greek stock market. Furthermore, nonlinear dynamics are supported in the results from evidence of strong conditional autoregressive evolution in the stock return path. The authors consider these results as indicative of a rapidly developing and strengthening market and not a symptom of inefficiencies in the Greek capital market.

1. Introduction

Extensive empirical work has been undertaken regarding the presence (and predictability) of long memory and nonlinear dynamics in stock return series.

Many papers have as reference the 'random walk hypothesis' (RWH), a benchmark model relevant to purely random and unpredictable behaviour of stock prices in the short- and long-term horizon.

Extant empirical work has documented deviations from independently and identically distributed (iid) conditions in time series and strong indications of long memory although these results do not appear to hold over very long time periods (Fama (1965), Fisher (1966), Lo and Mackinlay (1988), French and Roll (1986), etc).

Likewise, structural changes and financial liberalization in many emerging capital markets as well as the internationalization of the global economic environment, contributed to a fast development in the stock exchanges of those markets and an ever increasing interest on the part of global investors for investment opportunities which may increase performance of their portfolios (Hartmann and Khambata (1993), Hausser et al. (1994), Arjona and Ortiz (1997), etc).

Some authors focused on the presence of nonlinear and non-normal stochastic behaviour of stock market series of emerging markets. In general, such behaviour has resulted in deviations from traditional forms of market efficiency and in efforts for predicting future stock price levels and their volatility (Engle (1982), Engle and Kraft (1983), Bollerslev (1986), Nelson (1990), Arshanapalli and Doukas (1996), Alford and Lussier (1996), etc).

In this paper we will look into the existence of long memory and nonlinear dynamics of a sample of companies listed on the Athens Stock Exchange.

The paper is organized in six sections including the introduction. Section 2 describes the objectives of the paper. Section 3 presents a review of previous empirical work. Section 4 covers methodological issues while Section 5 presents and interprets the results. Finally, section 6 concludes the paper.

2. Objectives

The objective of the paper is to investigate the presence of long memory and non linear dynamics of stock price series of companies quoted on the Athens Stock Exchange. Apart from an extended investigation of stock return processes through widely applied statistical or econometric tools, a relatively new statistic, the Rescaled Range (R/S) Statistic, capable of capturing long-range dependence in time series processes, is applied to our data. The R/S statistic has proved to be superior and more robust compared to other statistics and is specifically designed for tracing time series returns. The aforementioned statistic may be thought of as an important complement to other statistics commonly used when investigating similar financial issues.

3. Review of Extant Empirical Work

Empirically, many tests have been applied in order to assertain if stock market return series are iid or there appear memory patterns (Mandelbrot and Taylor (1967), Mandelbrot and J. Van Ness (1968), Mandelbrot (1971)). The issue of the existence of long memory in stock prices was raised and investigated many years ago by Hurst (1951), Mandelbrot (1972), Mandelbrot and Taqqu (1979), Gewke, and Porter-Hudac (1983) (GPH Test), Brock, Dechert and Scheinkman (1987) (BDS Statistic), etc. The results showed that the RWH should be rejected.

Several authors have investigated dynamic forces in stock returns in the Athens Stock Exchange and found nonlinear deterministic relations in the returns series. Panas (1990), and Koutmos et al. (1993) have studied volatility in the Athens Stock Exchange and found as appropriate models those of an exponential generalized ARCH (E-GARCH) form of volatility. However, ARCH models seem to be more local and not long memory processes. Barcoulas and Travlos (1998) found that the stock return behaviour is better described by nonlinear deterministic models (chaotic miodels). Panas (1999) tested the long memory hypothesis by estimating ARFIMA models (AutoRegressive Fractionally Integrated Moving Average Models) in the Athens Stock Exchange. His results support the hypothesis of long memory and nonlinear characteristics in stock returns.

Shiller (1984) and Summers (1986) advanced the fads approach according to which optimistic / pessimistic trends or fashions appear in the market and affect investors expectations causing persistent deviations, lasting for some time, in the fundamental value of stock prices. The results of most of the papers cited above converge to the evidence of long memory in the daily series of squared returns implying persistence of volatility in the stock returns and possibility for long-term forecasts.

Long-range dependence has been investigated by Granger and Joyeux (1980) and Hosking (1981) who used fractionally differenced time series models (widely known as ARFIMA) models which constitute the long-memory generalisation of ARIMA models) where the stock price enters the following difference equation:

$$(1 - L)^d \,.\, P_t = \varepsilon_t \quad , \quad \varepsilon_t \sim \text{iid}\,(0,\sigma_\varepsilon^2) \tag{1}$$

and L is the lag operator, that is $L \,.\, P_t = P_{t-1}$.

The authors show that when the quantity $(1 - L)^d$ is extended to non-integer powers of d, the result is a well–defined time series being known as 'fractionally differenced of order d' or 'fractionally integrated of order -d'. The characteristics of a fractionally differenced time series are of special interest. Thus, the authors show that P_t is stationary and invertible for $d \in (-1/2, ½)$ and there appears a unique kind of dependence which is positive or negative depending on d being positive or negative, that is, autocorrelation coefficients of P_t have the same sign with d. Autocorrelations decay slowly so that when d is positive their sum tends to infinity and reverts to zero when d is negative. Mandelbrott and other researchers have called the case when $d < 0$ 'antipersistence' and reserve the term 'long-range dependence' for the $d < 0$ case .

Mandelbrott (1971) investigating long-range dependence of asset markets and of economic time series, proposed the use of 'range over standard deviation' or R/S statistic or 'rescaled range' (first developed by Hurst, 1951) which is calculated as the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. In many investigations the R/S statistic has proved superior to more conventional methods relevant to long-range dependence as, to name but a fiew, analysis of autocorrelations, variance ratios and spectral decompositions. It is also possible through the R/S statistic to detect long-range dependence in highly non-Gaussian time series with large skewness and/or kurtosis (Mandelbrott and Wallis, 1969a,b).

The robustness of the R/S statistic is also extended to cases of stochastic processes with infinite variance (Mandelbrott, 1972, 1975) as well as in cases for detecting nonperiodic cycles with periods equal to or greater than the sample period (Mandelbrott, 1972). Many researchers have applied Mandelbrot's R/S analysis (Greene and Fielitz (1977) to common stock returns, Booth, Kaen, and Koveos (1982) in foreign exchange rates, Helms, Kaen, and Rosenman (1984) in futures contracts, etc).

However, traditional tests of the Hurst and Mandelbrot statistics were not designed to distinguish between the short-range and long-range dependence. Lo (1991) describes a modification of the R/S statistic to capture the effects of short-range dependence suggesting that evidence by the earlier literature of long-range dependence in U.S. stock returns may well be the result of quickly decaying short-range dependence (see also Crato and de Lima (1994), Campbell, Lo, and MacKinlay (1997), Lo and MacKinlay (1999). If , as Lo and MacKinley (1999) support, the source of serial correlation is lagged adjustment to new information, the absence of strong dependence in stock returns should not be surprising from an economic standpoint, given the frequency with which financial asset markets clear. However, the authors suggest, fluctuations of aggregate economic output are more likely to display long-run tendencies (long-memory in output) but this form of long-range dependence will not be easily detected by any of the commonly used statistical tools.

Another aspect regards an almost general ascertainment in many empirical works about long-term properties of stock returns (Lo, 1991, Ding et al., 1993) that there is considerable evidence of long memory when some transformations of absolute values of returns are used in the analysis including squared returns. If there is actually long memory in the squares of stock returns then standard statistical tools for inference are not valid (Beran, 1994). Thus, standard errors of estimates of coefficients of conventional ARCH or stochastic volatility models will be misleading and incorrect as will also be the confidence intervals for predictions and dependence for stock returns not measured correctly with autocorrelations.

4. Methodological Issues

To investigate whether share prices follow a RWH we use a number of tests. For serial correlation we use the Ljung-Box Q statistic distributed as X_m^2 (m is the autocorrelation order or the maximum lag considered which practically is estimated up to lag n/4, n = number of sample observations) and used the sum of squared autocorrelations for single and squared returns (the latter implying ARCH process in variances).

We used daily closing prices for 12 large stocks of companies quoted on the Athens Stock Exchange and values of the general stock index (GINDEX) for a time period from 2 January 1989 to 11 May, 1999, i.e. 2679 trading days. The data were selected from the existing files and other computerized material on the Athens Stock Exchange (Yearbooks and Monthly Bulletins).

Stock Returns were calculated using continuously compounded return series and the relation:

$$r_{it} = \ln(P_{it} / P_{i,t-1}) \qquad i = 1,2,...,13 \tag{2}$$

To test for stationarity in the series returns and presence of unit roots we used the Augmented Dickey - Fuller (ADF) (1979) and Phillips - Perron (PP) (1988) tests (the latter allows for serial correlation and heteroscedasticity) (with k = n/4 as the lag order).

We tested for the presence of nonlinear relations and non-normal stochastic behaviour by applying Engle's (1982) Lagrange Multiplier (LM) test and run the following regression equation:

$$e_t^2 = a_0 + a_1 e_{t-1}^2 + u_t \text{ with } e_t = r_t - \hat{r}_t \text{ and } \hat{r}_t = \hat{b}_0 + \hat{b}_1 r_{t-1} \tag{3}$$

and e_t^2 = the residual squared and r_t = stock return in period t.

To check the null H_0: no ARCH(1) in r_t we applied the test statistic nR^2 (where R^2 is the coefficient of determination from the above regression) which is asymptotically distributed as X_1^2.

To test for long-term dependence and predictability of returns we initially used the 'Rescaled Range' or 'Range over Standard Deviation' (R/S statistic) first applied by Hurst (1951) and Mandelbrot (1971) and then used the modified 'R/S' statistic applied by Lo (1991).

The classical R/S statistic is denoted by $\tilde{Q}_n$ and is defined as follows:

$$\tilde{Q}_n \equiv \frac{1}{s_n}[\underset{1\le k\le n}{Max}\sum_{j=1}^{k}(X_j - \bar{X}_n) - \underset{1\le k\le n}{Min}\sum_{j=1}^{k}(X_j - \bar{X}_n)] \tag{4}$$

where s_n is the usual standard deviation estimator estimated as

$$s_n \equiv [\frac{1}{n}\sum(X_j - \bar{X}_n)^2]^{1/2} \tag{5}$$

The term Max(.) in (5) is the maximum (over k) of the partial sums of the first k deviations of X_j from the sample arithmetic mean and is always nonnegative. The term Min(.) in (5) is the minimum (over k) of the same sequence of partial sums and is always nonnegative. The difference inside the brackets is called the 'range' and it

is always nonnegative (that means $\tilde{Q}_n$ is always nonnegative).

Under a simple iid null hypothesis it can be shown that as n (the sample size) increases without bound, the R/S statistic converges in distribution to a well-defined random variable V when properly normalized, i.e.

$$(1/\sqrt{n}).\tilde{Q}_n \Rightarrow V\ , \tag{6}$$

where $\Rightarrow$ denotes weak convergence and V is the range of a Brownian bridge on the unit interval. The mean of V is $\sqrt{\pi}/2 \approx 1.25$ and the standard deviation of V is $\pi^2/6$, whereas the distribution of V is positively skewed and most of its mass falls between 3/4 and 2. We have used critical values for V and conventional significance levels (referred as fractiles of the distribution of $F_V(\upsilon)$) presented in Table 6.2, p. 157 of Lo and MacKinley (1999).

For autoregressive and moving average models (ARMA) and for a number of other stationary stochastic models, $\tilde{Q}_n$ converges to n^H where H = ½. In the case where H = ½ we have a 'random walk' behaviour in the time series. In the case where ½ ≤ H ≤1 , we have indications for persistent long memory (Mandelbrot (1972) and in the case where 0 ≤ H ≤ ½ , we have indications for antipersistent time series. In general, when 1 ≠ H ≠ ½ , we call these cases long-range dependent, since autocorrelations are involved that decay much more slowly than those of more conventional time series (Lo and MacKinley, p. 153, (1999)).

The letter H is called the Hurst coefficient or exponent. In order to estimate the exponent H in a sample of size n, we use the formula

$$\widehat{H} = log\,\tilde{Q}_n \,/\, log\, n \tag{7}$$

We use the Hurst exponent H and Mandelbrot's R/S approach to test for the existence of long memory (H > ½) (positive increments followed by positive increments) in the stock returns and in the squares of the stock returns.

To distinguish between long-range and short-range dependence, we applied the modified R/S statistic (Lo, 1991) having an invariant statistical behaviour over a general class of short memory processes, but deviates for long memory processes. The formula for this statistic is the following:

$$Q_n \equiv \frac{1}{\hat{\sigma}_n(q)}\left[\underset{1\le k\le n}{Max} \sum_{j=1}^{k}(X_j - \bar{X}_n) - \underset{1\le k\le n}{Min} \sum_{j=1}^{k}(X_j - \bar{X}_n)\right] \tag{8}$$

where

$$\hat{\sigma}_n^{\,2}(q) \equiv \dot{\sigma}_x^{\,2} + 2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j\,, \quad \omega_j(q) \equiv 1 - \frac{j}{q+1}, \quad q < n \tag{9}$$

and $\hat{\sigma}_x^{\,2}$ and $\hat{\gamma}_j$ are the usual sample variance and autocovariance estimators of X.

Qn differs from $\tilde{Q}_n$ only in its denominator due to the fact that if {Xt} is subject to short-range dependence, the variances of the partial sum is not simply the sum of the variances of the individual terms, but also includes the autocovariances up to a lag q. The lag of q is not clearly defined and in general little is known about how best to pick q in finite samples. Since, on the presence of short-range dependence, the values of the Qn, described by the Lo, are not considerably diverged among each other and the results are robust to the sample period (see, Lo and MacKinlay, p. 167, 1999), we used in the place of q the complete sample series. We put:

$$\hat{\xi} = \hat{\sigma}_n(q)/\, s_n = \tilde{V}_n / V_n \tag{10}$$

5. Presentation and Interpretation of the Results

The results of the various tests applied in this paper are presented in the Appendix.

Table 1 contains the results of ADF and PP tests for unit roots (non-stationarity) in the stock return processes. The values calculated reject strongly the null hypothesis of unit root existence since in all cases these values are significantly greater than critical values of those tests. Thus, we can conclude that the stock returns follow a stationary process.

Tables 2 and 3 present the results from autocorrelation tests in order to check for linear dependency among stock returns at various times starting from t = 1 and ending with t = 200. The number of autocorrelation and partial autocorrelation coefficients statistically different than zero at the 5% significance level and their highest absolute values are shown for each stock in the sample. In general, we can conclude that regarding single returns, autocorrelations and partial autocorrelations might be viewed as being small enough (highest values 0.174) to say that there is noticeable short-range dependence (short memory) in stock returns. Conversely, regarding the squares of the returns, we could infer that highest values as 0.333 or 0.319 are indicative to some extent of a short-memory effect (but as well as of a possible long-memory effect) in the stock return processes.

Furthermore, in several large stocks in the sample, we observe that the magnitude of autocorrelation and partial autocorrelation coefficients in the single as well as in the squared returns is of considerable magnitude in remote time lags and these coefficients decay rather slowly, an indication of a possible effect from the presence of long memory.

In addition, a careful consideration of autocorrelations in the range of their values leads to the conclusion that there is no seasonality or periodicity in stock returns since values of autocorrelations follow rather random or one-way trendy paths.

Summing up we would argue that there are no strong indications for short-range dependence (short memory) in the stock return series but there are strong indications for some dependencies in the squares of returns (findings interpreted by other researchers as long-memory indications, as for example, in Crato and de Lima, 1994) or in remote time lags of the single stock returns.

In Table 4 we present the values of the Ljung-Box Statistic for overall correlation in the stock return processes as well as in the squared returns. As can be seen from the results of the table, all values of the Q statistic are statistically significant in both cases (single and squared returns with emphasis on the latter), a fact implying evidence of overall correlation in all stocks in the sample and leading to the direction of searching for an ARCH effect (short-term modelled volatility) since there are indications for a change in the conditional distributions of stock returns.

As shown in Table 5, which presents ARCH(1) effects, the LM test values are statistically significant in all cases at 5% level. These results imply heteroscedastic effects and in addition they demonstrate that ARCH(1) forms are suitable to approximate evolution of conditional variance of stock returns and justify results of the Ljung-Box statistic mainly for squared returns.

In Table 6 there are descriptive statistics of the stock returns in the sample in order to study characteristics of stock return distributions and compliance with application of other statistics used in this study.

Regarding skewness, a measure of symmetry in the stock return distributions, it is obvious from the table that in most of the cases (10 out of 13 cases), values are statistically significant at 5% level (greater than zero) denoting that there is positive skewness (non-normal distributions skewed-to-the-right) in stock returns which has been generally observed in empirical evidence.

Also, values for kurtosis for all stocks, a measure of leptocurtic distributions (and of evidence for fat or semi-fat tails in those distributions in case where values are greater than 3), suggest that kurtosis is higher than the zero kurtosis which characterises the normal distribution.

Finally, the Bera-Jarque (BJ) statistic rejects in all cases the assumption of normal distribution in stock returns.

From the results of Tables 1 through 6 we conclude that the stock return series show a non linear behaviour. We will now move to investigate stock return processes through R/S statistic.

Application of the R/S statistic

As stated earlier, application of the R/S statistic is used to detect long-range dependence in highly non-Gaussian time series with large skewness and kurtosis and in stochastic processes with infinite variance. An extreme value for the classical R/S statistic indicates the likelihood of long-range dependence, but a rejection based on this statistic may be also consistent with short-range dependence in the sample data. Lo and MacKinley (1999) urge that because of its sensitivity to short-range dependence, the classical R/S statistic may be used to test for independently and identically distributed variables and it has considerable power against non-iid alternatives. They state that it is well-known that stock market returns are not independently and identically distributed and that aggregate stock market returns exhibit significant serial correlation for short-horizon holding periods and are therefore not independently distributed. If there is considerable short-dependence in stock processes (say autocorrelation over 30%), then the limited distribution of the classical R/S statistic $\tilde{Q}_n / \sqrt{n}$ is not V but (ξ V), where $\xi = \sqrt{(1+\varrho)/(1-\varrho)}$, ϱ is the coefficient of a stationary AR(1) process followed by stock returns with $-1<\varrho<+1$. This should be more evident in cases of some portfolios of common stocks with ϱ as large as 30% or 50%, a fact not observed in our analysis for the single return series.

In our study only the lowest-order autocorrelation coefficients are statistically significant and not succeeding $\varrho = 0.20$ for single return series. Apart from Lo and MacKinley, a further statement of some importance is that their inferences rely only on asymptotic distribution theory and thus, dismissing the possibility of long-range dependence through the R/S statistic may need further consideration based on everyday experience.

To apply the classical R/S statistic we computed the limited distribution of it for the stock returns of our sample using the formula

$$\tilde{V} = \tilde{Q}_n / \sqrt{n} \tag{11}$$

The distribution Fv of V is given in the Table 6.2 of Lo and MacKinley (1999) and a test of the null hypothesis (no long memory) may be performed at the 95% level of confidence by accepting or rejecting Ho according to whether V (with a sample size n) is or is not contained in the intervals of this table which assigns equal probability to each tail of the relevant distribution (two-tailed tests).

The values of V for our sample are listed below in the Table 7.

Based on the results of Table 7, Table 8 presents values for the Hurst (H) exponent and its relevant significance p-values based on the values of V in Table 7. As can be seen at first look, in all cases H > ½. However, working at a 95% confidence interval (5% significance level), we can observe a relatively extended presence of long-memory according to the Mandelbrot's approach (1972) in 7 cases (8 cases if the marginal one of 'MAKED-THRACE' is included), thus, the R/S analysis supports the existence of persistent long memory in a major part of the cases. Furthermore, the test was also performed on the series of squared returns. In all cases of the squared returns there is compelling evidence of long memory and persistent long-run dependence and this finding is in agreement with the works of other researchers (eg, Crato and de Lima (1994), Panas (1999)). Thus, we cannot reject the null of long memory (H > ½) for all the return series.

Finally, in Tables 9 and 10, values of V(n,q) and estimates of $\widehat{H}$ according to the modified R/S statistic (Lo, 1991) are shown for evidence of short memory in return processes. Similarly to Table 8, in all cases $\widehat{H} < 1/2$. However, all these values are statistically insignificant at conventional levels 5% and 10%. A conclusion is that normalizing by $\hat{\sigma}_n(q)$, use of the modified R/S statistic weakens remarkably the findings from use of the traditional R/S statistic, a strong indication of consistency of the data with the short-memory null hypothesis. Regarding squared returns, there is strong evidence of existence of long memory in all cases but one (E.T.E.), as has been empirically well-argumented.

6. Conclusions

In this paper the existence of long memory and nonlinear dynamics in the Greek stock market is examined applying relatively new time series techniques. We found stationarity in return series as well as small autocorrelation in the single stock return processes (weak short memory), but noticeable serial correlation in the squared return time series (more evident short memory) and dependencies in remote time lags of many stocks in the sample (long memory indications). These results combined with strong evidence of overall autocorrelation in single and squared returns lead us to suggest a nonlinear path of stock returns and to apply an ARCH(1) (LM test) statistic in order to capture any conditional heteroscedasticity in the volatility of the returns. The results support strong conditional autoregressive evolution in the stock return path of time series suggesting the presence of short-range or local dependencies in the stock returns. Non-normality in the return time series was diagnosed and to this end we used the Rescaled Range (R/S) statistic as being more appropriate for determining long-range dependencies and more robust to other alternative methods or tools. The results indicate the presence of long memory in the single stock returns and more emphatically in the squares of the returns.

However, application of the modified R/S statistic supported evidence of short memory in all cases of the return series, but was in agreement with existence of long memory in the squared returns.

The general conclusion of the paper is evidence for short memory and nonlinearities in the stock returns to a considerable degree but stronger evidence of long memory was found in almost all cases of the squared returns for the sample of the stocks under consideration. The results are typically indicative of possibilities in the emerging Greek stock market either to the direction of short-horizon benefits or to the prospect of a long-term investment policy, a fact that probably does not reveal inefficiencies in the Greek capital markets but must be considered as a healthy symptom or status of a rapidly developing and strengthening market to the direction of an efficiently operating market as has happened historically with all current developed stock markets today.

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APPENDIX

Table 1: *Tests for stationarity of stock return series*

	ADF TEST		PP TEST	
STOCK	**SINGLE RETURNS**	**SQUARED RETURNS**	**SINGLE RETURNS**	**SQUARED RETURNS**
ALEASING	-23.32698	-18.77117	-47.07789	-41.35615
VIOTER	-24.25801	-19.78705	-43.91803	-43.54357
ELAIS	-22.67288	-17.16990	-47.86336	-45.39731
EMPORIKI	-22.20571	-16.82401	-45.00118	-41.95722
ERGASIAS	-23.84436	-16.80506	-45.97033	-40.37960
E.T.E.	-22.52657	-17.65693	-44.05696	-41.28086
IONIKI	-22.94094	-16.38543	-45.18465	-41.91275
IRAKLIS	-24.44613	-17.43162	-46.65224	-42.64883
MAKED-TRHACE	-25.44121	-14.14387	-47.42196	-50.67033
PETZETAKIS	-24.54888	-18.56030	-48.05626	-38.20937
PISTEOS	-23.60454	-18.37526	-47.04438	-43.13168
TITAN	-22.93873	-20.61206	-47.77279	-47.53180
GINDEX	-22.22777	-17.65373	-43.39218	-41.22177

Table 2: *Autocorrelations of stock return series*

	Number of significant autocorrelations (ACF)		Largest absolute ACF	
STOCK	**SINGLE RETURNS**	**SQUARED RETURNS**	**SINGLE RETURNS**	**SQUARED RETURNS**
ALEASING	44	111	0.091	0.256
VIOTER	21	40	0.153	0.252
ELAIS	15	116	0.076	0.196
EMPORIKI	40	159	0.137	0.264
ERGASIAS	28	75	0.113	0.289
E.T.E.	36	133	0.157	0.272
IONIKI	21	57	0.132	0.277
IRAKLIS	31	61	0.098	0.236
MAKED-TRHACE	21	19	0.095	0.333
PETZETAKIS	21	91	0.078	0.319
PISTEOS	34	127	0.091	0.225
TITAN	37	52	0.076	0.197
GINDEX	41	70	0.174	0.271

Table 3: *Partial autocorrelations of stock return series*

	Number of significant partial autocorrelations (PACF)		Largest absolute PACF	
STOCK	**SINGLE RETURNS**	**SQUARED RETURNS**	**SINGLE RETURNS**	**SQUARED RETURNS**
ALEASING	36	23	0.091	0.256
VIOTER	18	17	0.153	0.218
ELAIS	16	32	0.076	0.191
EMPORIKI	21	40	0.137	0.264
ERGASIAS	26	34	0.113	0.289
E.T.E.	32	56	0.157	0.272
IONIKI	20	20	0.132	0.277
IRAKLIS	21	22	0.098	0.236
MAKED-TRHACE	26	32	0.095	0.327
PETZETAKIS	21	42	0.076	0.319
PISTEOS	32	42	0.091	0.225
TITAN	32	33	0.076	0.197
GINDEX	31	27	0.174	0.271

Table 4: *Ljung-Box (Q) statistic for overall correlation*

STOCK	**SINGLE SQUARED**	**RETURNS RETURNS**
ALEASING	263.18 *	1851.1 *
VIOTER	348.36 *	934.2 *
ELAIS	264.45 *	2461.9 *
EMPORIKI	306.17 *	3646.8 *
ERGASIAS	304.86 *	1870.9 *
E.T.E.	378.56 *	3098.5 *
IONIKI	304.65 *	1863.0 *
IRAKLIS	293.69 *	1590.0 *
MAKED-TRHACE	328.39 *	1672.2 *
PETZETAKIS	289.41 *	2208.5 *
PISTEOS	316.14 *	2806.4 *
TITAN	330.69 *	1247.1 *
GINDEX	410.11 *	1895.5 *

* *Statistically significant at 5% significance level*

Table 5: *ARCH (1) effects on the stock return processes*

STOCK	LM VALUES
ALEASING	175.189 *
VIOTER	50.008 *
ELAIS	89.446 *
EMPORIKI	179.453 *
ERGASIAS	269.304 *
E.T.E.	170.595 *
IONIKI	171.311 *
IRAKLIS	161.500 *
MAKED-TRHACE	8.949 *
PETZETAKIS	290.186 *
PISTEOS	134.317 *
TITAN	30.275 *
GINDEX	204.042 *

* *Statistically significant at 5% significance level*

Table 6: *Descriptive statistics of the stock returns series*

STOCK	SKEWNES	KURTOSIS	JARQUE - BERA STATISTIC
ALEASING	0.2542 *	7.9275 *	2739.081 *
VIOTER	-0.6537	19.1967 *	29473.580 *
ELAIS	0.5540 *	8.4569 *	3460.929 *
EMPORIKI	0.4972 *	7.2851 *	2160.051 *
ERGASIAS	0.2188 *	7.0939 *	1892.213 *
E.T.E.	0.4916 *	8.3835 *	3343.049 *
IONIKI	0.2108 *	4.7426 *	358.832 *
IRAKLIS	0.1845 *	6.5469 *	1419.502 *
MAKED-TRHACE	0.6333 *	23.9350 *	49100.720 *
PETZETAKIS	0.2093 *	9.4273 *	4630.775 *
PISTEOS	0.5032 *	8.6407 *	3664.644 *
TITAN	-0.1165	15.2195 *	16673.570 *
GINDEX	0.1081	8.0399 *	2840.564 *

* *Statistically significant at 5% significance level*

Table 7: *Values of the variable* $V = Q\tilde{n} / \sqrt{\tilde{n}}$ *for the R/S statistic*

STOCK	SINGLE RETURNS	SQUARED RETURNS
ALEASING	1.868	5.826
VIOTER	2.188	3.163
ELAIS	1.512	6.291
EMPORIKI	2.205	5.990
ERGASIAS	1.746	5.205
E.T.E.	1.985	1.985
IONIKI	1.922	4.575
IRAKLIS	1.446	4.698
MAKED-TRHACE	1.854	3.241
PETZETAKIS	2.012	5.203
PISTEOS	1.809	5.559
TITAN	1.771	4.217
GINDEX	2.513	5.390

Table 8: *Hurts exponent (H) for the sample stock using the classical R/S statistic/ p-Values for the distribution of* $\tilde{V}$

SINGLE RETURNS			SQUARED RETURNS		
STOCK	H	p-VALUE	STOCK	H	p-VALUE
ALEASING	0.57917	0.005< P < 0.025*	ALEASING	0.72328	P< 0.005*
VIOTER	0.59917	P < 0.005*	VIOTER	0.64589	P< 0.005*
ELAIS	0.55240	0.10< P < 0.20	ELAIS	0.73300	P< 0.005*
EMPORIKI	0.60019	P < 0.005*	EMPORIKI	0.72679	P< 0.005*
ERGASIAS	0.57063	0.05< P< 0.10	ERGASIAS	0.70899	P< 0.005*
E.T.E.	0.58688	0.005<P< 0.025*	ETE	0.72290	0.005<P< 0.025
IONIKI	0.58275	0.005<P< 0.025*	IONIKI	0.69264	P< 0.005*
IRAKLIS	0.54671	0.20 <P< 0.30	IRAKLIS	0.69601	P< 0.005*
MAKED-THRACE	0.57824	0.025 <P< 0.05**	MAKED-THRACE	0.64898	P< 0.005*
PETZETAKIS	0.58857	0.005<P< 0.025*	PETZETAKIS	0.70895	P< 0.005*
PISTEOS	0.575074	0.025< P < 0.05	PISTEOS	0.71173	P< 0.005*
TITAN	0.57239	0.025 <P< 0.05	TITAN	0.68233	P< 0.005*
GINDEX	0.61673	P< 0.005*	GINDEX	0.71342	P< 0.005*

* *Statistically significant at 5% level and under a 2-tail test*

** *The value of V is 1.854, marginally equal to the critical p-Value 1.862.*

Table 9: *Values of the variable* $V = Qn/\sqrt{n}$ *for the modified R/S statistic*

STOCK	SINGLE RETURNS	SQUARED RETURNS
ALEASING	1.384	3.226
VIOTER	1.473	2.198
ELAIS	1.974	3.401
EMPORIKI	1.442	3.282
ERGASIAS	1.406	3.020
E.T.E.	1.456	1.459
IONIKI	1.481	2.306
IRAKLIS	1.404	2.423
MAKED-TRHACE	1.425	2.217
PETZETAKIS	1.469	3.008
PISTEOS	1.377	3.141
TITAN	1.362	2.285
GINDEX	1.596	3.107

Table 10: *Hurts exponent (H) for the sample stock using the modified R/S statistic/ p-Values for the distribution of* $\widetilde{V}$

SINGLE RETURNS			SQUARED RETURNS		
STOCK	H	p-VALUE	STOCK	H	p-VALUE
ALEASING	0.54120	0.20< P < 0.30	ALEASING	0.64838	P< 0.005 *
VIOTER	0.54906	P < 0.20	VIOTER	0.59977	P< 0.005 *
ELAIS	0.58615	0.005<P<0.25 *	ELAIS	0.65507	P< 0.005 *
EMPORIKI	0.54637	0.20 <P < 0.30	EMPORIKI	0.65056	P< 0.005 *
ERGASIAS	0.54316	0.20 <P < 0.30	ERGASIAS	0.64002	P< 0.005 *
E.T.E.	0.54759	0.20 <P < 0.30	ETE	0.54786	0.20<P <0.30
IONIKI	0.54976	0.20 <P <0.30	IONIKI	0.60585	P< 0.005 *
IRAKLIS	0.54299	0.20 <P < 0.30	IRAKLIS	0.61212	P< 0.005 *
MAKED-THRACE	0.54487	0.20 <P <0.30	MAKED-THRACE	0.60086	P< 0.005 *
PETZETAKIS	0.54871	0.20 <P <0.30	PETZETAKIS	0.63952	P< 0.005 *
PISTEOS	0.54052	0.20 <P < 0.30	PISTEOS	0.64499	P< 0.005 *
TITAN	0.53914	0.30 < P< 0.40	TITAN	0.60469	P< 0.005 *
GINDEX	0.55923	0.10 <P< 0.20	GINDEX	0.64362	P< 0.005 *