Time-Varying Risk Premia in the Single European Treasury Bill Market

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Abstract

This paper investigates the validity of the expectations hypothesis (EH) with time-varying, albeit stationary, term premia in the Ecu Treasury bill market. The analysis utilises the term premium factor representation proposed by Tzavalis and Wickens (1997) and the modified VAR approach by Cuthbertson et al. (1997). The findings indicate that once time-varying term premia are accounted for, estimated models cannot reject the predictions of the EH. However, these term premia do not exhibit strong persistence. The rejection of the spread restriction for \((n,m)=(26\text{-week},13\text{-week})\) may be due to a small \(I(1)\) term premium and/or a slight misalignment of investment horizons.

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1. Introduction

At its meeting on July 7, 1998, the Governing Council of the European Central Bank (ECB) emphasised the importance of stabilising money market interest rates in the European Monetary Union (EMU) area. This is considered to be a prerequisite for the successful implementation of a common monetary policy in the euro area, as the ECB will not need to frequently util-
ise open market operations (i.e., to intervene in the short term government money market) for fine tuning purposes. As such, the role of the ECB in the design of the European Union’s monetary policy is expected to become more transparent, and market participants will find it easier to distinguish between policy changes and technical adjustments.

One way to assess the stability of euro interest rates at the short end of the maturity spectrum is by testing the validity of the expectations hypothesis (EH) of the interest rate term structure, using euro money market rates. The EH relates the long term interest rate to anticipated future short term rates plus a term premium which may account either for risk considerations or for investors’ preferences about liquidity. Thus, according to the EH, the money market term structure at a given time should reflect the market’s current expectations for future short term rates. In a previous paper (Mylonidis (1999)), we investigate the validity of the EH with constant term premia in that segment of the international capital market acknowledged to be the precursor of the euro-denominated money market, namely the Ecu-denominated Treasury bill market. The empirical results give contradictory inferences. On the basis of the cointegration analysis, the EH is supported over the sample period (1989:01 – 1996:12). Nonetheless, the perfect foresight spread regressions and VAR procedures provide little or no support for the EH.

This paper attempts to illustrate why different tests give apparently conflicting inferences. In particular, we examine the possibility that the mixed evidence in support of the traditional EH in the UK Ecu Treasury bill market stems from the presence of time-varying, albeit stationary, term premia. In general, risk averse investors require extra compensation for holding risky assets during volatile periods. The history of the UK Ecu Treasury bill market was closely associated with the progress towards EMU. During periods of turmoil and scepticism concerning the future course of EMU, for example following the 1992 crisis in the Exchange Rate Mechanism (ERM), investors’ sentiment concerning prospects for the Ecu market was pessimistic. This possibly resulted in excessive premia. In the present study, we attempt to analyse the implications of modelling these time-varying risk premia in the term structure of the Ecu Treasury bill market. The theoretical insight this paper provides is the utilisation of term premia proxies that have single factor representations. By including these proxies in the perfect foresight regressions, any biases due to omitting the term premia should be removed.

There have been numerous attempts in the financial literature to model time-varying risk premia on bills and bonds. A number of earlier studies, including Fama (1976), Mishkin (1982), Jones and Roley (1983) and Shiller, Campbell and Schoenholtz (1983) use a measure of time-varying risk premia which is proportional to the change in the absolute value of the 3-month US Treasury bill rate. Nonetheless, although this proxy might constitute an im-
improvement on the risk neutrality assumption that underlies the traditional EH, it remains unfounded in any economic theory. Shiller et al. argue that this risk measure contrasts the finance theory which suggests that it is the covariance, rather than the volatility, of asset returns with other asset returns that determines their riskiness. Furthermore, the weighted average proxy does not make any distinction between anticipated and unanticipated changes in the interest rate, and hence it does not allow us to test for the rational expectations assumption.

Simon (1989) attempts to remedy the latter deficiency by introducing a proxy for time-varying risk premia which is proportional to the expectation of the square of the excess one-period holding period yield. Nevertheless, despite the explicit inclusion of optimal linear forecasts in the formulation of the risk premium, this proxy remains theoretically ad hoc. Furthermore, Simon’s empirical findings, based on US Treasury bill data, indicate that the proxy does not increase the predictive power of the yield curve in periods of high interest rate volatility (i.e., during periods of no or partial interest rate targeting).

The conceptual inadequacy of these measures of time variations in risk led to the development of other alternative approaches. In a pioneering study, Engle, Lilien and Robins (1987) utilise the Autoregressive Conditional Heteroskedasticity (ARCH) framework to model time-varying risk in the bill market. In particular, they assume that the expected excess yield of long bills over short bills is determined by the conditional variance of the returns. The intuitive economic argument that underlies this specification is that the larger the variance of the forecast errors, the higher the yield that investors require in order to hold long term bills rather than short term bills. The results of Engle et al. suggest that there are strong effects of the conditional variance on equilibrium returns, indicating the potential suitability of the model. The main advantage (and disadvantage at the same time) of the ARCH approach is its flexibility. That is, it allows researchers to model the time-varying variance in different ways, none of which provides the definite specification of the risk premium. Subsequently, it may prove difficult to obtain a tractable model of the term structure with time-varying variances.

Tzavalis and Wickens (1997) propose an alternative way of proxying the term premia, which is less restrictive to the approaches suggested by Jones and Roley (1983) and Simon (1989), but more specific than the ARCH model. Specifically, Tzavalis and Wickens utilise ex post excess holding period returns as proxies for the unobservable term premia, and (arbitrarily) assume that these term premia are related to each other through a single factor representation. The rationale behind this term premia interrelationship is that all excess holding period returns are subject to common economic forces, and hence they are jointly determined. Tzavalis and Wickens also
emphasise the importance of the stationarity of this term premium proxy in removing the bias in the estimates of the yield spread. Their empirical evidence provides a strong rationale for modelling a time-varying (yet stationary) term premium when investigating the EH. Further empirical support for the utilisation of the excess holding period return as a proxy for a possible time-varying term premium is provided by Cuthbertson, Galindo and Nitzsche (1997) who employ data on UK Treasury spot rates and show that a Vector Autoregression (VAR), including the term spread, the change in the short rate and the holding period return, provides incremental evidence on the validity of the EH.

The empirical analysis in this paper adopts the theoretical framework proposed by Tzavalis and Wickens (1997), as well as the modified VAR approach by Cuthbertson et al. (1997). The utilisation of excess holding period yields (or an approximation of them) as a proxy for time-varying (stationary) term premia allows us to incorporate in our analysis rational expectations and distinguish between expected and unexpected interest rate movements. In addition, the single factor representation relates term premia to each other, in accordance with Shiller et al. (1983) who argue that the riskiness of different asset returns should be jointly determined. Finally, the VAR approach provides the necessary mechanisms to evaluate the impact of (possible) time-varying term premia on future short rates, and hence to assess the predictive power of the expectations hypothesis.

The findings of the present empirical study indicate that the restrictions imposed by the rational expectations hypothesis of the term structure (RETS) seem to hold for the (3-month, 1-month), (6-month, 1-month) spreads. However, the impact of these time-varying term premia on one period excess returns is negligible. We argue that the rejection of the restriction for the (26-week, 13-week) spread is probably due to the presence of a small I(1) risk premium that distorts the estimated coefficients.

The remainder of the paper is organised as follows. Section 1 outlines the theoretical framework, whereas section 2 presents the methodological considerations. Section 3 reports and discusses the empirical results. Finally, Section 4 summarises and concludes the paper.

1. Theoretical considerations

The rational expectations of the term structure requires that at time $t$, the holding period return, $h(n,t+1)$, for investing in an $n$-period asset for one-period (i.e., to $t+1$) is equal to the one-period rate, $R(m,t)$, plus a term premium, $\lambda(n,t)$, that is,

$$E_t h(n, t+1) = R(m, t) + \lambda(n,t)$$

(1)
$E_t$ is the expectations operator conditional on information at time $t$, $h(n,t+1)$ is defined as $h(n,t+1) = \ln(c(n-1,t+1)) - \ln(c(n,t))$, and is the capital gain from holding the $n$-period bond for one-period, $c(n,t)$ denotes the price of a zero-coupon bond with a unitary face value, and finally, $\lambda(n,t)$ is the term premium perceived at time $t$ for the $n$-period bond. $\lambda(n,t)$ equals zero under the hypothesis of risk neutrality.

Assuming continuously compounded interest rates (spot yields), $\ln(c(n,t)) = -nR(n,t)$, and substituting $h(n,t+1) = nR(n,t) - (n-1)R(n-1,t+1)$ into equation (1) yields:

$$ER(n-1,t+1) - R(n,t) = \frac{1}{(n-1)}[R(n,t) - R(m,t)] \frac{1}{(n-1)} \lambda(n,t)$$

(2)

Solving equation (2) forward we obtain:

$$R(n,t) = \frac{1}{k} \sum_{i=0}^{k-1} E_t R_i + \sum_{i=1}^{k-1} E_t \Lambda(n,t)$$

(3)

where $k = n/m$ and should be an integer.

Subtracting $R(m,t)$ from both sides of equation (3) and re-arranging we have:

$$S_t(n,m) = E_t S^*_t(n,m) + E_t \Lambda(n,t)$$

where, $S_t(n,m) = R(n,t) - R(m,t) = \text{actual spread between the long and short rate}$

(4a)

$$S^*_t(n,m) = \sum_{i=1}^{k-1} (1 - i/k) \Delta R_i + \sum_{i=1}^{k-1} E_t \Lambda(n,t) = \text{perfect foresight spread}$$

(4b)

$$\Lambda(n,t) = \frac{1}{k} \sum_{i=0}^{k-1} \lambda(k - i, t + i) = \text{the rolling or average risk premium}$$

(4c)

Equation (4) is the spread equation and indicates that the actual spread is an optimal predictor of expected future changes in short rates plus future expected changes in the rolling term premium.

Tzavalis and Wickens (1997) assume that $\lambda(n,t)$ has a single factor representation, i.e., they are related to each other,

$$\lambda(n,t) = \alpha(n,x) \lambda(x,t) \text{ for all } n, x.$$  

(5)

If $\lambda(n,t)$ increases monotonically with maturity, then

$$\alpha(n,x) < 1 \text{ for all } x > n$$

(5a)

$$\alpha(n,x) > 1 \text{ for all } x < n$$

(5b)

If $\alpha(n,x)$ only depends on the number of periods between $x$ and $n$, then $\alpha(n,x)$ is constant for fixed $n-x$, and denoted $\alpha(n-x)$. 


Replacing expectations in equation (4) with realisations and ignoring the term premia produces equation (6) that can be used to test the rational expectations hypothesis of the term structure (RETS),

\[ S_t(n,m) = \alpha_0 + \alpha_1 S_t(n,m) + \varepsilon_{1t} \]  

(6)

where,

\[ \varepsilon_{1t} = \sum_{i=1}^{k-1} \left( 1 - \frac{i}{k} \right) \left[ \Delta R_t + \Delta R_{t+m} - E_t \Delta R_{t+m} \right] - \Lambda(n,t) \]  

(6a)

RETS predicts that \( \alpha_1 = 1 \). However, if \( \lambda(n,t) \) (and consequently, \( \Lambda(n,t) \)) is time-varying, then \( \varepsilon_{1t} \) is likely to display conditional heteroskedasticity, be correlated with the spread, and more importantly be serially correlated since its first component is a moving average process (of an \( (n-m-1) \) order) which reflects overlapping expectational errors in the changes in future short rates. As a result, the OLS estimator of \( \alpha_1 \) is likely to be inconsistent, and thus biased away from unity.

Now, taking into account equation (5), we can re-write equation (4c) as follows:

\[ \Lambda(n,t) = \frac{1}{k} \sum_{i=0}^{k-1} \alpha(k-i,k) \lambda(k,t+i) \]  

(7)

where \( \alpha(k,k)=1 \) when \( i=0 \).

Tzavalis and Wickens (1997) state that if \( \lambda(n,t) \) follows a first order autoregressive process, then equation (7) can be written as:

\[ \Lambda(n,t) = \frac{1}{k} \sum_{i=0}^{k-1} \alpha(k-i,k) \rho^i \lambda(k,t) \]  

(8)

where \( \rho \) is the autoregressive coefficient.

Equation (6) can be augmented by a proxy for the rolling term premium, \( \Lambda(n,t) \), as follows:

\[ S_t^*(n,m) = \beta_0 + \beta_1 S_t(n,m) + \beta_2 \Lambda(n,t) + \varepsilon_{1t} \]  

(9)

where \( \Lambda(n,t) \) is given by \( h(n,t+1) - R(m,t) \). Under the null of RETS with a time-varying term premium we have:

\( \beta_2 = 1 \)

\[ \beta_2 = -\left( \frac{1}{k} \right) \sum_{i=0}^{k-1} \alpha(k-i,k) \rho^i \]

\( \varepsilon_{1t} \) = zero mean innovation error.

Cuthbertson, Galindo and Nitzsche (1997) provide an alternative approach to test the expectations hypothesis with a time-varying term premium. Based on equation (4), they utilise the vector autoregression (VAR) approach.
by Campbell and Shiller (1987, 1991). Assuming that the term premium, \( \lambda(n,t) \) is stationary, they consider the matrix:

\[
X^* = [R(n,t), R(m,t), h(n,t+1)]
\]

where,
- \( R(n,t) \) is the yield of the \( n \)-period bond at time \( t \)
- \( R(m,t) \) is the short rate yield at time \( t \)
- \( h(n,t+1) \) is the excess one-period holding return of an \( n \)-period bond.

If all three variables which constitute \( X^* \) are integrated of order one (i.e., they are I(1) processes), then the three-equation system should be cointegrated with two cointegrating vectors. These vectors can be identified as the spread, \( R(n,t) - R(m,t) \), and the excess holding period yield, \( h(n,t+1) - R(m,t) \). If the above cointegration relationships hold, then the vector \( X = [S_t, \Delta R(m,t), h(n,t+1) - R(m,t)] \) should contain only stationary variables.

If this is the case, then vector \( X \) can be approximated by a VAR of order \( \rho \) which in companion form is:

\[
X_t = AX_{t-1} + u_t (10)
\]

\( A \) is the companion matrix of coefficients in \( X_t \). Next, we define the selection vectors, \( e_1, e_2 \) and \( e_3 \), such that \( e_1'X_t = S_t \), \( e_2'X_t = \Delta R(m,t) \), and \( e_3'X_t = h(n,t+1) - R(m,t) \). \( e_1, e_2 \), and \( e_3 \) are \((3 \times \rho)\) vectors with unity in the first, \((\rho+1)\)st, and \((2\rho+1)\)st rows, respectively, and zeros elsewhere. Multi-period forecasts of the VAR variables are computed from the chain rule of forecasting as:

\[
E_t[X_{t+i} | I_t] = E_t[X_{t+i} | X_t] = A^iX_t \quad (11)
\]

where \( I_t \) is the information set available to the econometrician, and is a subset of the full information set, \( F_t \).

We can use the VAR to test whether the term premium is time invariant. Equation (1) states that \( E_t(h(n,t+1) - R(m,t)) \) is a constant, only if the term premium does not change over time. Given the VAR representation in equation (10) and the formulation of the selection vectors, this implies:

\[
e_3'A=0 \quad (12)
\]

Violation of this linear restriction implies that the term premium is time variant.

Cuthbertson et al. (1997) also provide the means for assessing the importance of incorporating time-varying term premia in the EH. Specifically, they evaluate the impact of news about future interest rates, or news about future changes in the term premium, on the one-period holding period return, \( eh(t+1) = h(t+1) - E_t h(t+1) \). Returning back to equation (1), we note that an unexpected change in the holding period return must be due to either an unanticipated change in future short rates, \( eR(m,t+1) \), or to unanticipated changes in future one-period term premia, \( e\Lambda(n,t+1) \). For example, if there is an unexpected rise in the one-period holding return of an \( n \)-period bond, this implies an unexpected fall in the \( n \)-period bond yield, which in turn must be
due to an unexpected fall in current and/or future short rates. Alternatively, the *sudden* rise in the one-period return may result from an unanticipated fall in future term premia. That is,

\[ e_{h(t+1)} = -e R(m,t+1) - e \Lambda(n,t+1) \]  

where,

\[ e_{h(t+1)} = h(t+1) - E_{eh(t+1)} \]  

\[ e R(m,t+1) = \left( E_{t + 1} - E_{t} \right) \sum_{i=1}^{k-1} \left( 1 - \frac{i}{k} \right) \Delta R_{t + im,m} \]  

\[ e \Lambda(n,t+1) = \left( E_{t + 1} - E_{t} \right) \sum_{i=1}^{k-1} \lambda \left( k - i, t + 1 \right) \]  

To empirically assess the relative importance of *news* about future term premia, we utilise the residuals from the VAR system (equation (10)). The weighted sum of the residuals in future short rates,

\[ \left( E_{t + 1} - E_{t} \right) \sum_{i=1}^{k-1} \left( 1 - \frac{i}{k} \right) \Delta R_{t + im,m} \], corresponds to the surprise in future short rate changes. Correspondingly, the residuals of the excess return equation in the VAR (i.e., the third equation) indicate the *news* in the excess holding period return. Using equations (10) and (11) and the selection vectors \( e_{2}^{'}, \) and \( e_{3}^{'} \), the revisions to the future (one-period) term premia (equation (13)) can be written as:

\[ e \Lambda(n,t+1) = -e R(m,t+1) - e_{h(t+1)} \]

\[ = -e_{2}^{'} \left\{ (1/k)A_{1}((k-1)I+(k-2)A+(k-3)A^{2}+...+(k-(k-1))A^{k-2})u_{t+1} - e_{3}^{'}u_{t+1} \right\} \]  

where \( u_{t+1} \) are the one-period ahead residuals from the VAR (equation (10)).

Therefore, if *news* about future term premia are very small, then we expect the unanticipated change in the one-period return to fully capture the unexpected future short rate changes. From equation (13) it follows that the standard deviation ratio between \( eh \) and \( e R(m,t) \) should be equal to one, and their coefficient of correlation should be minus one. Formally, we write:

\[ \sigma[e R(m,t)] / \sigma(eh) = 1 \]  

\[ \rho[e R(m,t),eh] = -1 \]  

2. Methodological considerations: Stationarity of the term premium

An implicit assumption in the above theoretical analysis is that the term premium \( \Lambda(n,t) \) is stationary. Specifically, the success of equation (9) in removing the bias in the estimates of the slope coefficient \( \beta_{1} \) due to the time-varying term premium depends on the time series properties of that series. Given stationary changes in short rates and stationary yield spreads, it follows that the term premium should also be stationary. A non-stationary term
premium implies that the spread between the long and short rate and the term premium proxy are asymptotically uncorrelated, and thus the inclusion of \( h(n, t+1) - R(m, t) \) cannot eliminate the bias in the spread slope coefficient.

Evans and Lewis (1994) offer a contrary view on US data. They test whether the risk premia in excess US Treasury bill returns are stationary processes. The test is rejected for a wide range of maturities less than a year. The point estimates, however, of the cointegrating forward short rate are relatively close to unity indicating that the unit root component in the excess returns is empirically small. Evans and Lewis conclude that this finding is likely due to a small I(1) term premium (in the range of 5% - 10% of the variance of the stationary components), and that their results are consistent with the observation that excess returns appear stationary.

Tzavalis and Wickens (1997) suggest that the stationarity of the term premium proxy can be checked by the means of an ADF test. Alternatively, a cointegration analysis can be conducted between the sum of the future short term rates, \( \sum_{i=1}^{k-i} R_{t+im,m} \), and the forward rate \( kR(n,t)-R(m,t) \). To see this we re-arrange equation (3) as:

\[
[kR(n,t) - R(m,t)] - \sum_{i=1}^{k-i} R_{t+im,m} = \Lambda(n,t) + \varepsilon_{1t}
\]  (15)

Equation (15) states that if \( \Lambda(n,t) \) is stationary and \( \varepsilon_{1t} \) is a white noise error, then the left hand side of the equation should be cointegrated with cointegrating vector \((1, -1)\).

Having established that \( \Lambda(n,t) \) are stationary, we then proceed to estimate equation (9). Since the term premium proxy variable is defined as \( h(n, t+1) - s(t) \), which is contemporaneously correlated with the error term due to expectations errors, equation (9) requires an instrumental variable (IV) type of estimator. A generalised method of moments (GMM) estimator is also employed to correct the covariance matrix for the moving average error of order \((n-m-1)\) and possible heteroskedasticity (Hansen, 1982; Newey and West, 1987). The instrumental variable set consists of the constant, the deterministic time trend, the spread between the longest and shortest rate, the one-period lagged change of the short term rate, and the change in the long term rate. The selection of these variables can be justified on the grounds of their ability to forecast excess returns (Campbell (1987), Tzavalis and Wickens (1997)). Specifically, the change in the short and long rate captures the lagged innovation in the corresponding rates, and is a good measure of the current risk premium if premia are persistent through time. Furthermore, the use of first differences ensures that the time series processes are stationary. The spread between the long and short rate is also a powerful measure of the
risk premium on the long term bill, since by definition the spread is equal to the sum of the risk premium on the \( n \)-period bill and the expected \( n-m \) future changes in the \( m \)-period bill.\(^8\) Given that the number of instruments is bigger than the number of parameters to be estimated, i.e., the equations are over-identified, we can test the overidentifying restrictions, using Sargan’s (1964) statistic. This test is asymptotically distributed as a \( \chi^2 \) with \( s-k \) degrees of freedom, where \( s \) represents the number of instruments and \( k \) the number of estimated parameters.

3. Data and empirical results

3.1 Data description

The expectations hypothesis in the presence of a time-varying term premium is tested using the Ecu Treasury bill nominal yield to maturity data provided by the Bank of England. The maturities we consider are for 4-weeks (1-month), 13-weeks (3-months), and 26-weeks (6-months). This data set spans the period from January 1989 until December 1996, giving a total of 416 weekly observations (or alternatively, 96 monthly data points). The analysis is undertaken using both the 4-week (1-month) and the 13-week (3-month) rate as the representative short rate. Specifically, given the constraint that \( k (=n/m) \) should be an integer, we utilise the 13-week rate (sampled on a weekly basis) as a representative short rate for \( (n,m)=(26\text{-week},1\text{-month}) \), and the 1-month rate (sampled on a monthly basis) for \( (n,m)=(3\text{-month},1\text{-month}), (6\text{-month},1\text{-month}) \).

3.2 Unit roots and cointegration

Before proceeding to the estimation of equation (9), it is necessary to determine the time characteristics of the term premium proxy. In the previous section, we justify our theoretical expectation that the term premium should be stationary. This can be investigated by conducting unit root tests for the excess returns, i.e., \( h(n,t+1)-R(m,t) \). In Table 1 we report the results of the Dickey-Fuller (DF) test, with and without a deterministic trend. The inclusion of the deterministic trend in the auxiliary regression increases the statistical power of the DF test, and allows it to better distinguish between the null hypothesis of unit roots and the trend stationary alternative. The test results suggest the rejection of the null hypothesis of a unit root in the excess returns (term premia), for all \( n \) and \( m \). It is worth mentioning that in all instances, the slope coefficient of the trend, \( \delta \), is relatively small (\( \leq 0.020 \)), and in one case (26-weeks) statistically insignificant. The \( F^{\delta,\psi} \)-statistic tests the joint hypothesis \( H_0: \delta=\psi=0 \). The test results indicate that \( \delta \) is not significantly differ-
ent from zero. Therefore, its impact on the stationarity of the term premium, as indicated by the DF test statistics without a trend, is not significant.

[See Table 1]

In the previous section we argue that another way of investigating the stationarity of the term premium is to apply cointegration analysis between the sum of the future short rates, \( kR(n,t) - R(m,t) \), and the forward rate, \( kR(n,t) - R(m,t) \). The Pantula principle suggests that a constant should be included in the cointegration space and Schwartz’s criterion recommends that the maximum number of lags in the VEC model should be three. The results of the Johansen’s cointegration analysis are presented in Table 2, and strongly support the predictions of the RETS with a stationary rolling premium. In particular, both the \( \lambda_{\text{trace}} \) and \( \lambda_{\text{max}} \) test statistics determine that in all instances the rank of the cointegration space is one, and that the cointegrating vector is \((1, -1, *,*)\), where \((*,*)\) means that the constant is estimated without restrictions. These findings imply that the vector is driven by a unique common factor, and thus the term premium, \( \Lambda(n,t) \) is stationary. These results together with the DF test results provide strong evidence that the term premium is stationary.

[See Table 2]

We now turn to the estimation of equation (9) which involves the term premium proxy \( h(n,t+1) - R(m,t) \). GMM estimates of this equation are given in Table 3 - Panel A. The estimation results provide reasonable support for the expectations hypothesis with a time-varying term premium. Specifically, the theoretical value of the estimated term spread slope is not different from unity for \((n,m) = (3,1)\) and \((6,1)\) at the 5% significance level. Further support for the model is provided by Sargan’s test for overidentified restrictions. The test seems to validate the inclusion of the time-varying term premium proxy in the perfect foresight regression of the expectations hypothesis. Nevertheless, the values of \( \beta_2 \) are statistically insignificant in both cases. We comment on this finding in the following sub-section.

For \((n,m) = (26\text{-weeks}, 13\text{-weeks})\) the estimated term spread slope, even though it is statistically significant, is different from unity. This finding might be due to the starting assumption that term premia have a single factor representation instead of a multi-factor structure.\(^9\) Tzavalis and Wickens (1997) propose an alternative measure of \( \Lambda(n,t) \). Specifically, they assume that for fixed \( n-x, \alpha(n-x) \) remains constant, that is:
\[ \alpha(n,x) = \alpha(n-x) = \alpha(1) \]  
\[ (16) \]

Then, \( \Lambda(n,t) \) can be written as:

\[ \Lambda(n,t) = \frac{\alpha(1)}{k} \sum_{i=0}^{k-1} \lambda(k - 1 - i, t + i) \]  
\[ (17) \]

It follows that

\[ \frac{1}{k} \sum_{i=0}^{k-1} [h(n - 1 - i, t + i) - R(m, t + i)] \]

can be used as the rolling proxy premium.

Estimates of the revised equation (9) are reported in Table 3 - Panel B, and indicate no improvement in the test statistics. The slope coefficient of \( R(n,t) - R(m,t) \) is still different from unity, and the \( R^2_{adj} \) has fallen by approximately 30%. These results suggest that the proposed measures of term premia are not good proxies of \( \Lambda(n,t) \). Remember that the basic underlying assumption in the previous analysis is that \( \Lambda(n,t) \) is stationary. Evans and Lewis (1994), however, note that in the presence of a small I(1) term premium the estimated slope coefficient is biased towards zero. Therefore, the rejection of the null \( H_0: \beta_1=1 \) for S(26,13) may be due to the utilisation of weekly data in the present study. Specifically, the two 13-week investments may not have always fallen on the second Tuesday of the month (tender date), thirteen weeks hence.

[See Table 3]

3.3 VAR analysis

The lag length of the VAR is chosen according to the Schwartz criterion and is for all maturities equal to one. The restriction that the excess holding period return is not time-varying is rejected in all instances (Table 4-Panel A). This implies that the inclusion of \( h(n,t+1) - R(m,t) \) in the VAR provides a noisy proxy for a time-varying (yet stationary) term premium. However, results from equation (9) in Table 3 indicate that the estimates of the average future premia \( \Lambda(n,t) \) are relatively small and not statistically different from zero. These seemingly contradictory pieces of evidence are further investigated in Table 4-Panel B. Specifically, the time series behaviour of the unanticipated change in the holding period return, \( eh(n,t+1) \), is compared with surprises about future short rates, \( eR(m,t+1) \). For both maturities considered \( (n,m)=(3,1),(6,1)^{10} \) the coefficient of correlation is very close to minus one. The latter indicates that most of the variation in the Ecu one-period excess returns could be attributed to sudden changes in future short rates, and very little to unanticipated movements in future average risk premia. Finally, although the standard deviation ratios provide favourable evidence to the
above finding, in the sense that they are both positive, they suggest that the quantitative impact of \( eR(m,t+1) \) on \( eh(t+1) \) is relatively small in comparison with that required by the EH with time-varying term premia.

Conclusively, our results indicate that the inclusion of time-varying term premia removes the bias in the estimates of the slope coefficient \( \beta_1 \) in equation (9). Nevertheless, these term premia do not exhibit strong persistence. The latter finding implies that changes in short Ecu Treasury bill rates dominate changes in average risk premia.

[See Table 4]

4. Conclusions

In this paper we examine whether the inclusion of time-varying (yet stationary) term premia in perfect foresight spread regressions increases the ability of the expectations hypothesis to capture the dynamics of the UK Ecu Treasury bill term structure. The current empirical analysis is unique, in the sense that the Ecu Treasury bill data set has not been investigated before in the financial literature. Following Tzavalis and Wickens (1997), we opt for a single factor representation of the term premium and find that the predictions of the expectations hypothesis cannot be rejected for \((n,m) = (3\text{-}month, 1\text{-}month), (6\text{-}month, 1\text{-}month)\). However, the VAR analysis suggests that the importance of news in Ecu term premia may have been relatively small, indicating that most of the surprises in one-period excess returns are attributable to news about future short rates. The rejection of RETS restrictions for the \((26\text{-}week, 13\text{-}week)\) spread might arise from a small I(1) component in the term premium and/or a slight misalignment of investment horizons.

In general, our findings indicate that the inclusion of time-varying term premia can manifest a significant improvement in the predictive power of the expectations hypothesis. Our analysis has immediate implications for current policy discussions. The relative validity of the EH with time-varying, but stationary, term premia in the Ecu-denominated Treasury bill term structure indicates that there exists the necessary mechanism to facilitate the stabilisation of short run fluctuations in the euro-denominated money market. This, in turn, increases the ECB’s ability to operate efficiently in the euro money market and to implement a transparent common monetary policy in the euro area. The euro money market is directly linked to the ECB’s policy decisions, and hence it operates as a means for monetary policy diffusion to other segments of the euro capital market. As such, it is expected that in the long run, the existence of a stable money market will also benefit European public, private and equity markets, since their further development and integra-
tion are directly related to the credibility and accountability of the institution determining monetary policy.
## Table 1.
### Unit Root Tests on Excess Returns (at levels)

<table>
<thead>
<tr>
<th>Horizon (n)</th>
<th>3-months(^1)</th>
<th>6-months(^1)</th>
<th>26-weeks(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A (without trend)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DF</td>
<td>-6.286</td>
<td>-6.187</td>
<td>-22.332</td>
</tr>
<tr>
<td>LM(1)</td>
<td>0.760</td>
<td>0.056</td>
<td>2.776</td>
</tr>
<tr>
<td>(0.383)</td>
<td>(0.813)</td>
<td>(0.096)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B (with trend)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DF</td>
<td>-6.612</td>
<td>-6.502</td>
<td>-22.402</td>
</tr>
<tr>
<td>LM(1)</td>
<td>1.413</td>
<td>0.267</td>
<td>3.144</td>
</tr>
<tr>
<td>(0.235)</td>
<td>(0.605)</td>
<td>(0.076)</td>
<td></td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.008*</td>
<td>0.020*</td>
<td>0.003</td>
</tr>
<tr>
<td>(F^{\delta,\psi})</td>
<td>3.993</td>
<td>3.893</td>
<td>1.979</td>
</tr>
</tbody>
</table>

**Notes:** The estimated model is:
\[
\Delta y_t = c + \delta t + \psi y_{t-1} + u_t
\]
where \(y_t = h_{n,t+1} - R_{m,t}\); \(\delta=0\) in Panel A, whereas \(\delta\neq0\) in Panel B.

LM(1) denotes the Breusch-Godfrey Lagrange Multiplier test for the presence of first order serial correlation. The test is distributed as \(\chi^2\)-statistic with one degree of freedom. \(\rho\)-values are given in parentheses.

\(\delta\) is the estimated coefficient of the trend variable (Panel B only). \(F^{\delta,\psi}\) is the joint F-test that \(H_0: \delta = \psi = 0\). The 5% critical values are 6.49 (n=100 observations) and 6.30 (for n=500 observations).

\(^1\) For n=(3-month, 6-month), m is equal to the 1-month rate. 96 monthly data points were used to conduct the DF test. The 5% critical values of the test are -2.892 for the model without the trend and -3.422 for the model with trend.

\(^2\) For n=(26-weeks), m is equal to the 13-week rate. 414 weekly data points were used to conduct the DF test. The 5% critical values of the test are -2.869 for the model without the trend and -3.422 for the model with trend.

* indicates that the estimated coefficient is statistically significant at the 5% significance level.
### Table 2.
Cointegration Analysis for determining the stationarity of Excess Returns

<table>
<thead>
<tr>
<th>Horizon (n)</th>
<th>3-months&lt;sup&gt;1&lt;/sup&gt;</th>
<th>6-months&lt;sup&gt;1&lt;/sup&gt;</th>
<th>26-weeks&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_{\text{max}} )</td>
<td>( \lambda_{\text{trace}} )</td>
<td>( \lambda_{\text{max}} )</td>
</tr>
<tr>
<td>( r=0 )</td>
<td>34.81</td>
<td>36.00</td>
<td>25.56</td>
</tr>
<tr>
<td>( r=1 )</td>
<td>1.19</td>
<td>1.19</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Likelihood Ratio Tests of the Rank of \(\Pi\)-matrix

90% quantiles

Horizon (n)       | 3-months<sup>1</sup> | 6-months<sup>1</sup> | 26-weeks<sup>2</sup> |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_{\text{max}} )</td>
<td>( \lambda_{\text{trace}} )</td>
<td>( \lambda_{\text{max}} )</td>
</tr>
<tr>
<td>( r=0 )</td>
<td>34.81</td>
<td>36.00</td>
<td>24.09</td>
</tr>
<tr>
<td>( r=1 )</td>
<td>1.19</td>
<td>1.19</td>
<td>1.48</td>
</tr>
</tbody>
</table>

### Estimates of the Cointegrating Vector (normalised by the first element)

\[(1.00 \ -1.03 \ 0.31) \quad (1.00 \ -1.02 \ 0.26) \quad (1.00 \ -0.99 \ -0.07)\]

### Likelihood Ratio Test of the Cointegration Restriction \((1 \ -1 \ *.*)\)

\[
0.57 \\ 0.02 \\ 0.05 \\
(0.45) \quad (0.88) \quad (0.83)
\]

**Notes:**
The multivariate cointegration analysis is conducted on the vector:

\[
\left[ \sum_{i=1}^{k-1} R_{t+im,m}, kR_{n,t} - R_{m,t} \right]'
\]

<sup>1</sup> For \( n=(3\text{-month}, \ 6\text{-month}) \), \( m \) is equal to the 1-month rate. 92 monthly data points were used to conduct the cointegration analysis.

<sup>2</sup> For \( n=(26\text{-weeks}) \), \( m \) is equal to the 13-week rate. 412 weekly data points were used to conduct the cointegration analysis.

The LR test of the restriction \((1 \ -1 \ *.*)\) is distributed as a \( \chi^2(1) \). \( \rho \)-values are given in parentheses.
Table 3. Perfect Foresight Regressions with a Time-Varying Term Premium

<table>
<thead>
<tr>
<th>Horizon (n)</th>
<th>3-months$^1$</th>
<th>6-months$^1$</th>
<th>26-weeks$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>0.153</td>
<td>0.389</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>(0.151)</td>
<td>(0.296)</td>
<td>(0.109)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.004</td>
<td>-0.011</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.008)</td>
<td>(&lt;0.001)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.625</td>
<td>0.558</td>
<td>0.469</td>
</tr>
<tr>
<td></td>
<td>(0.276)</td>
<td>(0.331)</td>
<td>(0.204)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.104</td>
<td>0.004</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(0.387)</td>
<td>(0.183)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>$H_0$: $\beta_1$=1</td>
<td>-1.355</td>
<td>-1.336</td>
<td>-2.607*</td>
</tr>
<tr>
<td>$R^2_{adj}$</td>
<td>0.497</td>
<td>0.285</td>
<td>0.209</td>
</tr>
<tr>
<td>S(1)</td>
<td>0.013</td>
<td>0.014</td>
<td>1.167</td>
</tr>
</tbody>
</table>

Panel B

| $\beta_0$  | 0.066       |             |
|            | (0.031)     |             |
| $\delta$   | 0.001       |             |
|            | (<0.001)    |             |
| $\beta_1$  | 0.354       |             |
|            | (0.064)     |             |
| $\beta_2$  | -0.008      |             |
|            | (0.008)     |             |
| $H_0$: $\beta_1$=1 | -10.108* |  |
| $R^2_{adj}$| 0.130       |             |
| S(1)       | 0.033       |             |

Instruments: constant, trend, $S_{t,(n,m),}$, $\Delta R_{m,t-1}$ and $\Delta R_{n,t}$.

Notes: The estimated model in Panel A and B is equation (9) augmented by a deterministic time trend. In Panel A, $\Lambda_{n,t}$ is approximated by $(h_{n,t+1}-R_{m,t})$. In Panel B, $\Lambda_{n,t}$ is approximated by $\frac{1}{k} \sum_{i=0}^{k-1} (h_{n-1-i,t+i} - R_{m,t+i})$. The reported standard errors are corrected for heteroskedasticity and moving average errors of order (n-m-1), using Newey and West (1987) weights to guarantee positive semi-definiteness.
S(1) is Sargan’s test for overidentified restrictions. The test is distributed as a \( \chi^2 \)-statistic with one degree of freedom. The 5% critical value with one degree of freedom is 3.84.

\(^1\) For \( n=(3\text{-month, 6\text{-month})}, \ m \) is equal to the 1-month rate. 94 monthly data points are used in the estimation procedure.

\(^2\) For \( n=(26\text{-weeks}) \), \( m \) is equal to the 13-week rate. 412 weekly data points are used in the estimation procedure.

\(^*\) denotes that the null (\( H_0 : \beta_1=1 \)) can be rejected at the 5% significance level.

Table 4. Vector Autoregression (VAR) Analysis

<table>
<thead>
<tr>
<th>Horizon (n)</th>
<th>3-months</th>
<th>6-months</th>
<th>26-weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR Test for the Time-Variability of the Excess Holding Period Returns</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \chi^2(3) )</td>
<td>37.547</td>
<td>-196.626</td>
<td>-2,270.166</td>
</tr>
<tr>
<td>( \rho )-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Panel B</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>News about Future Short Rates and 1-period Excess Holding Returns</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p(eR_{m,t+1},eh_{t+1}) )</td>
<td>-0.998(^1)</td>
<td>-0.999(^1)</td>
<td>N/A</td>
</tr>
<tr>
<td>( \sigma(eR_{m,t+1})/\sigma(eh_{t+1}) )</td>
<td>0.470</td>
<td>0.630</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Notes: The variables in the VAR are \( X=(S_t, \Delta R_{m,t}, h_{n,t+1} - R_{m,t}) \). The lag length, chosen using the Schwartz criterion, is for all maturity bundles one. In Panel A, the null hypothesis for a time invarying (one-period) term premium is \( H_0 : e_{A}=0 \). In Panel B, the null hypothesis is that \( eh_{t+1} \) is solely due to surprises in \( eR_{m,t+1} \), implying that \( p(eR_{m,t+1}, eh_{t+1})=1 \), and \( \sigma(eR_{m,t+1})/\sigma(eh_{t+1})=1 \). The statistics of interest are not calculated for \( (n,m)=(26\text{-week, 13\text{-week})} \) due to near singularity of the corresponding series.

\(^1\) For \( n=(3\text{-month, 6\text{-month})}, \ m \) is equal to the 1-month rate.
References


**Endones**

1 Given the lack of long historical data on euro interest rates it is impossible to make reliable statistical inferences concerning the current structural money market situation.

2 For further discussion on the relation between the Ecu- and euro-denominated government bond markets see Bowe and Mylonidis (1999).

3 Bloem and Namor (1997) provide information on the 10-year Ecu-German Bund spread which indicates that the Ecu market was traded as high as 220 bp and as low as 30 bp during our sample period (1989-1996). The ERM crisis in September 1992, for example, resulted in Ecu-Germany spreads reaching a high of 220 bp. On the contrary, when January 1, 1999, was confirmed as the starting date for EMU at the Madrid summit (December 1995) the 10-year Ecu-Bund spread tightened to 60 bp. These examples provide some evidence against the risk neutrality assumption that the traditional EH asserts.

4 In particular, Fama (1976) attempts to measure variation in expected premia by taking the average of the absolute values of monthly changes in the 3-month US Treasury bill spot rate from one year before to one year after the current period. Mishkin (1982), Jones and Roley (1983) and Shiller et al. (1983) use simple averages of lagged absolute changes in the same spot rate over a period of eight quarters.

5 The implicit assumption here is that \( \lambda(n,t) \) (and consequently, \( \Lambda(n,t) \)) is stationary. A non-stationary term premium implies that the spread and \( E[R(n-1,t+1)-R(n,t)] \) in equation (2) are asymptotically uncorrelated, hence invalidating the ability of the RETS to be an equilibrium model.

6 Again \( \Lambda(n,t) \) is considered to be a stationary time series process.

7 For more information on the stationarity of the UK Ecu Treasury bill rates and yield spreads see Mylonidis (1999).

8 This identity holds under the assumption of continuously compounded interest rates.

9 The rapid decline of \( R^2_{adj} \) as \( n \) increases (Table 3) may justify this conjecture.

10 The standard deviation ratio and correlation coefficient are not computed for \( (n,m)=(26\text{-week, 13\text{-week)} because of near singularity of the corresponding series.}