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Long Memory in Volatility. An Investigation on the Central and Eastern European Exchange Rates

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Abstract:

Understanding the evolution of volatility on the financial markets is essential for the comprehension and for the analysis of risk. This paper regards the topic of persistence of volatility in the exchange rates for four Central and Eastern European countries: Czech Republic, Hungary, Poland, and Romania. Persistence in volatility shows how quickly financial markets forget large volatility shocks. The persistence of volatility is addressed as the presence of long-term memory in the second order moment of returns and in absolute returns. The main feature of a long-memory process is that its autocorrelation function decays slower than that of a short memory process, but faster than that of an integrated one. The paper also concerns the implications on risk assessment of detecting long-term memory in the volatility of the exchange rate.

Keywords: long memory, volatility, GARCH models.

JEL Classification: C14, D81, G17

1. Introduction

The human decision process design has a fundamental impact on the financial markets. The ways in which the human decision process deviates from perfect rationality generates phenomena intriguing the academic financial literature.

The affluence of empiric evidence regarding the asymmetric information perception and processing contradicts an efficient pricing framework congruent with a market composed integrally by identical traders concerning the inferring

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ability, simultaneous perception and identical resources. The human investors are optimal myopic strategist (Kauffman, 1994). Myopia in economic contexts is defined as the result of the heuristic decision process that incorporates implicitly a feedback mechanism allowing learning from historical experience. The heavy dependence on historical outcomes causes investors to underestimate the probability of future outcomes in making decisions.

The time and financial restrictions applied to the real decision process justifies heuristic and inexact techniques, providing an explanation for the auto-afine and the long term memory processes in the financial time series. The presence of long-memory components in the moments of asset returns has important implications for many of the paradigms used in modern financial economics (Lo, 1991).

The finding of long-memory in the volatility of the financial time series suggests the development of new methods of forecasting, portfolio optimization, risk assessing and aggregation.

Research on the long-memory processes arose from the examination of data in the physical sciences. Hurst (1951) found persistence in streamflow data, observing that long-memory processes can generate non-periodical cyclical pattern.

Persistence in volatility shows how quickly financial markets forget large volatility shocks. The main feature of a long-memory process is that its autocorrelation function decays slower than that of a short memory $I(0)$ s process, but faster than that of an integrated one.

To account for a discrete-time model that exhibits hyperbolic decay of its autocorrelation function, Granger and Joyeux (1980) and Hosking (1981) introduce independently the fractional integrated autoregressive moving average model (ARFIMA).

The importance of long-memory for assets returns was first discussed in Mandelbrot (1971). Mandelbrot shows that in the presence of long-memory perfect arbitraging is not possible, providing a table which relates the Hurst exponent values, the Sharpe Ratios, and the number of transactions needed to obtain profits under option strategies. Greene and Fielitz (1977) use the rescaled range statistic (R/S) to address the long-memory hypothesis for stock returns. Their analysis was conducted on the daily returns for 200 common stocks listed in the New York Stock Exchange and concluded that long-term dependence characterizes a significant percentage of the sample. Aydogan and Booth (1988) suggest that the Greene and Fielitz (1977) results might be corrupted by serial dependency and non-stationarities. Lo (1991) performed a refinement of the R/S method on the daily and monthly stock returns indexes of Center for Research in Security Prices over several time periods and, contrary to previous findings, found no evidence of long-range dependence in any of the indexes over any sample period or sub-period once short-range dependence was taken into account. Barkoulas, Labys, and Onochie (1997) used the classical R/S analysis to reevaluate the memory of future returns and found persistent long-memory in a significant group of future contracts.

The analysis of long-term memory in the second order moment of assets returns follows the analysis of conditional variance model seminal paper of Engle (1982), regarding autoregressive conditional heteroskedasticity (ARCH) models. The Engle's original work was extended by Boolerslev (1986) to generalized ARCH (GARCH) models. Engle and Boolerslev (1986) introduced integrated GARCH (IGARCH), allowing for high-frequency stock market data to display persistent volatility. Crato and de Lima (1994) applied a modified version of R/S statistic and GPH statistic to the squared residuals of various filtered U.S. stock returns indexes. Ding, Granger and Engle (1993) draw attention to the presence of long-memory in the higher moments of return series.

Two classes of models have been proposed to capture the slow decay of the autocorrelation function of volatility series. The natural extension of the ARCH, allowing a hyperbolic rate of decay for lagged squared innovations were introduced in the form of fractionally integrated GARCH (FIGARCH) and fractionally integrated exponential GARCH (FIEGARCH) models of Baillie, Bollerslev and Mikkelsen (1993) and Bollerslev and Mikkelsen (1993). In their application of the FIGARCH model to the exchange rate between U.S. dollars and the German mark, the hypothesis of IGARCH behavior against FIGARCH behavior is rejected. Similar results are obtained by Bollerslev and Mikkelsen (1993) in their application of the FIEGARCH model to daily returns on the Standard and Poor's 500 stock index. The second class of models that allows long-memory in volatilities is the stochastic volatility class of models of Breidt, Crato and de Lima (1994). Caporin (2002a) extends the analysis on the estimation and identification problems with a FIGARCH specification for the conditional variance. Caporin (2002b) employs a risk oriented approach.

Kirman and Teyssiere (2000) construct a microeconomic behavioral model with interacting agents that can replicate the empirical long memory properties of the two first conditional moments of financial time series. The essence of the model is that the second assumption of rationality – mutually consistence – no longer holds; the forecasts and the desired trades of agents are influenced by those of other participants, affecting the structure of the assets price dynamics. The series of squared returns display long-memory, while returns are uncorrelated. The basic foundation of the model developed in Kirman and Teyssiere (2000) is the existence of two groups of agents, who differ by their price forecast: fundamentalists and chartists. The differentiating feature of this model is that individuals change from a category to another. The variable size of the two groups has consequences for emergent market behavior. The model, although of a sequential nature, is an equilibrium one.

LeBaron (2007) shows that the long memory persistence in trading volume, volatility, and order signs can be a consequence of the imitation across trading behavior.

Understanding the evolution of volatility on the financial markets is essential for the comprehension and for the analysis of risk. Additional terms concerning risk and incertitude necessitate the development of econometric techniques to capture the empirical regularities, as well as the idiosyncrasies of financial markets.

We investigate the persistence of volatility in the exchange rates for four Central and Eastern European countries: Czech Republic, Hungary, Poland, and Romania. The persistence of volatility is addressed as the presence of long-term memory in the second order moment and in absolute returns. The paper also concerns the implications on risk assessment of detecting long-term memory in the volatility of the exchange rate.

The paper is organized in 4 sections. The 2nd section reviews the models of long-term dependence; the 3rd section presents the methods used to detect and to estimate long-memory characteristics; the final section outlines the empirical results and concludes.

2. Modelling long-memory in returns and volatility

The long-term dependence can be translated through the persistent influence of distant shocks on a series level. The standard characterization of long-term dependence comprises the autocorrelation function. A process (X_t) presents long-term memory if $\forall \tau \quad \lim_{T\to\infty}\sum_{\tau=1}^{T}|\rho(\tau)| = \infty$, where $\rho(\tau)$ is the autocorrelation function at rank τ. A time series is denominated as integrated of order d, $I(d)$, if $\lim_{T\to\infty}\sum_{\tau=1}^{T}|\rho_X(\tau)| = \infty$, $\forall \tau$ and d is the minimum positive number such as $\lim_{T\to\infty}\sum_{\tau=1}^{T}|\rho_Z(\tau)| < \infty$, $\forall \tau$, with $Z_t = (1-B)^d X_t$. The B operator is defined as $B^a X_t = X_{t-a}$. The d parameter quantifies the memory stock of the time series. The autocorrelation function for a long-memory process decays hyperbolically.

The ARFIMA specification represents a flexible and an efficient way to model the short- and long-term behavior of time series. The ARFIMA specification was introduced by Granger and Joyeux (1980), and Hosking (1981). These stochastic processes are not strong-mixing, and have autocorrelation functions that decay at much slower rates than those of weakly dependent processes (Lo, 1991).

A stochastic process (X_t) follows an ARFIMA process of p, d and q parameters, if

$$\phi(B)(1-B)^d X_t = \theta(B)\varepsilon_t . \tag{2.1}$$

The lag polynomials $\phi(B)$ and $\theta(B)$ are defined by

$$\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p , \tag{2.2}$$

$$\theta(B) = 1 - \theta_1 B - \ldots - \theta_q B^q , \tag{2.3}$$

and ε_t is a white noise process. Granger and Joyeux (1980) and Hosking (1981) show that when the quantity $(1-B)^d$ is extended to non-integer powers of d, the result is a well-defined time series.

The formulation of the fractional difference operator can be parameterized using the MacLaurin series.

$$(1-B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} B^k, \tag{2.4}$$

where $\Gamma(\cdot)$ is the Euler Gamma function. The Euler Gamma function is a method to generalize the usual factorial function to non-integer arguments.

The order of fractional integration governs the effect and the permanence of the shocks on the stationarity or the non-stationarity of the process. Hosking (1981) show that (X_t) is stationary and invertible for $d \in (-1/2, 1/2)$, and exhibits a unique kind of dependence that is positive or negative depending on the sign of d.

The ARCH class of models derives from Engle (1982). The ARCH models are capable of surprising stock market features as leptokurtic distributions, volatility clustering, leverage effects, non-trading periods, predictable events, volatility and serial correlation dependence, and common behavior in volatilities. The ARCH models segregate the second order conditioned and unconditioned moments; the conditional covariances depending non-trivially on previous states.

Similar to Engle (1982), an ARCH process is defined as

$$\varepsilon_t = \sigma_t Z_t, \tag{2.5}$$

where (Z_t) is an independent, identically distributed process, with $E[Z_t] = 0$ and $Var[Z_t] = 1$. The variable σ_t^2 is a positive, F_{t-1}-measurable function, where F_{t-1} is the sigma-algebra generated by $(Z_{t-1}, Z_{t-2}, ...)$; σ_t^2 is the conditional variance of the process.

Following Bollerslev (1986), in a GARCH (p, q) specification the variance is defined by

$$\sigma_t^2 = \omega + \alpha(B)\varepsilon_t^2 + \beta(B)\sigma_t^2, \tag{2.6}$$

where $\alpha(B)$ and $\beta(B)$ are lag polynomials similar to (2.2) and (2.3).

The GARCH process may be rewritten as the ARMA (m, p) process

$$[1-\alpha(B)-\beta(B)]\varepsilon_t^2 = \omega + [1-\beta(B)]\upsilon_t, \tag{2.6'}$$

where $\upsilon_t = \varepsilon_t^2 - \sigma_t^2$, and $m = \max\{p, q\}$.

In a stationary ARCH or GARCH model, memory decays exponentially. For example, if (ε_t) is ARCH (1), the autocorrelation function is $\rho(\tau) = ct^{\tau}$, implying an unrealistically fast decay.

To model a very slow decay in the sample autocorrelation function of squared returns measure of volatility, the GARCH model was extended to allow an approximate unit root in $1-\alpha(B)-\beta(B)$. Bollerslev and Engle (1986) defined the IGARCH (p, q) process as

$$(1-B)\varphi(B)\varepsilon_t^2 = \omega + [1-\beta(L)]\upsilon_t, \tag{2.7}$$

where the lag polynomial $\varphi(B)$ is of order $m-1$.

To model the long-memory in the volatility of financial instruments returns, Baillie, Bollerslev, and Mikkelsen (1993) extended the IGARCH specification by a natural parallel with ARFIMA processes. The FIGARCH (p, d, m) process is defined as

$$(1-B)^d \phi(B)\varepsilon_t^2 = \omega + [1-\beta(B)]\upsilon_t. \tag{2.8}$$

In a FIGARCH model the impact of the innovation lies between exponential decaying and infinite persistence. In stationary long memory models for volatility, the autocorrelations of (ε_t^2) obey a power law, $\rho(\tau) \approx \tau^{2d-1}$.

3. Detecting long-term memory and estimating the fractional difference parameter

We shall review in this section several methods employed in the detection of long-memory type of persistence and in estimating its parameters: the unit roots tests of Dickey-Fuller and Kwiatkowski, Phillips, Schmidt and Shin; the modified R/S statistic of Lo; the spectral regression method of Geweke and Porter-Hudak; and the local Whittle estimator. A gross categorization would divide the pre-mentioned techniques based on their appurtenance to time domain analysis and, respectively, to spectral domain analysis.

3.1 Unit roots method

Due to the fact that the standard Augmented Dickey-Fuller (ADF) tests tend to have low power against the alternative hypothesis of fractional integration, complementary unit root test ADF and Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS) can be used to detect long memory in the return series or in the volatility series. The combined result of ADF and KPSS can be synthesized as follows.

i. The rejection of ADF and the impossibility of rejecting KPSS provides the evidence of a wide stationary process; the series is $I(0)$.

ii. The impossibility of rejecting ADF and the rejection of KPSS provides the evidence of a process integrated of order 1.

iii. The joined impossibility of rejecting ADF and KPSS shows an informational insufficiency of the data generating process to the lower frequencies.

iv. The joined rejection of ADF and KPSS indicates the insufficiency of a representation either $I(1)$ or $I(0)$ and the necessity of a fractional alternative.

Charemza and Syczewska (1997) suggests that, where the ADF and KPSS statistics are jointly used, the conventional critical values for those tests should be replaced by symmetric critical power values, which corresponds to the probability of type 1 error for the ADF test and power of the KPSS test in the case both cumulative marginal distributions are equal.

3.2 R/S statistic

R/S is the short for range over standard deviation. The R/S was developed by Harold Edwin Hurst and is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. The classical rescaled range statistic $Q(T)$ is defined as

$$Q(T) = \frac{R(T)}{\hat{S}(T)}, \tag{3.1}$$

$$R(T) = \max_{0 \le j \le T}\left\{\sum_{j=1}^{T} X_j - E[X]\right\} - \min_{0 \le j \le T}\left\{\sum_{j=1}^{T} X_j - E[X]\right\}, \tag{3.2}$$

and $\hat{S}(T)$ is a standard deviation estimator.

The R/S statistic can detect long-range dependence in highly non-Gaussian, skewed and leptokurtic time series, is almost-sure convergence for stochastic processes with infinite variances, and can detect non-periodic cycles, cycles with period equal to or greater than the sample period. The distribution for the classic RS statistic is unknown and it can be contaminated by the short memory components.

The limit of the ratio $\log Q(T)/\log T$ is denoted by H and is called the "Hurst" coefficient. The fractionally difference parameter is $H - 1/2 = d$.

Lo (1991) modifies the R/S statistic such as its statistic behavior is invariant over a general class of short memory processes, but deviates in the case of long memory processes. The modified R/S statistic differs from the classical R/S by the denominator. Lo (1991) employes a long-memory consistent standard deviation estimator using Newey-West weights.

For ARMA processes $R(T)/\hat{S}(T,M)$ converges to T^H, with $H = 1/2$. A natural estimation for H, given a T length series is, therefore,

$$\hat{H} = \log\left[R(T)/\hat{S}(T,M)\right]/\log(T). \tag{3.4}$$

The critical values for the R/S statistic are provided in Lo (1991).

3.3 Spectral regression estimator

Geweke and Porter-Hudak (1983) introduced for the fractional integration order an estimate based on the spectral representation of a stationary stochastic process. The spectral density function of the $I(d)$ process satisfies the equation

$$f_X(\omega) = [2\sin(\omega/2)]^{-2d} f_Y(\omega), \tag{3.5}$$

where $f_Y(\omega)$ is the spectral density function for the process $Y_t = (1-B)^d X_t$.

GPH is a semiparametric procedure estimating the parameter d through the least squares applied to a transformation of (3.5) and assuming that $\ln[f_Y(\omega_j)/f_Y(0)]$ becomes negligible.

Geweke and Porter-Hudak (1983) argue that the resulting estimate of d could capture the long-memory behavior without being contaminated by the short-memory behavior of the process. This argument is asymptotically correct if, besides truncation of the higher periodograma frequencies, an additional truncation of the very first ordinates is performed. The usual t-test of the hypothesis that $d = 0$ against $d \neq 0$ is a test of the null hypothesis of short-memory against long-memory alternatives. The small sample properties of GPH test can be very sensitive to large autoregressive and moving average effects.

3.4 Spectral regression estimator

Sowell (1992) suggests to estimate the parameters of an ARFIMA (*p, d, q*) process with the method of maximum likelihood. The log-likelihood function is

$$L_T[X; E(X), \beta] = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\ln|\Omega(\beta)| - \frac{1}{2}[X - EX]'\Omega^{-1}(\beta)[X - EX], \tag{3.6}$$

where $X = (X_1, \ldots, X_T)'$, $\Omega = (\omega)_{ij} = \gamma_{|i-j|}$, and β is a vector of parameters including d, ARMA coefficients and unconditional variance. Based on the approximation proposed by Whittle, maximizing the log-likelihood function is equivalent to minimizing the (negative) spectral likelihood function

$$L_T^W(\beta) = \sum_{j=1}^{\left[\frac{T}{2}\right]} \left\{ \frac{I(\omega_j)}{f_X(\omega_j;\beta)} + \ln[f_X(\omega_j;\beta)] \right\}. \tag{3.7}$$

The resulting minimizer is known as the Whittle estimator. Although it is efficient to the exact MLE, it requires a priori specification of the spectral density function $f(\omega, \beta)$. To overcome this difficulty, H.R. Künsch and P.M. Robinson suggested a local Whittle estimator, which does not impose the gaussian assumption nor does it require a correct specification of the spectral density. The method is asymptotically more efficient than the Whittle estimator.

There are cases, when the tests concerned in detecting long-memory and estimating its parameters spuriously detect evidence on long-memory. Thus, a

pseudo-long-memory behavior can be generated by the sum of AR (1) processes, with coefficients drawn randomly from a suitable distribution and increasing number of terms, by the misspecification of conditional heteroskedasticity models, or by the presence of nonstationarities. The switching ARCH data can replicate the combined contrasting behavior: when applied to the levels, the tests indicate no evidence of long-memory, but when applied to the squares of the series the tests present evidence of long-memory.

4. Empirical results

This section is oriented towards detection of long-memory feature in the returns and in different specifications for volatility calculated for the exchange rates versus EUR of the Hungarian forint (HUF), Czech koruna (CZK), Polish Zloty (PLN) and Romanian Leu (RON). We analyzed daily data covering the interval January 1998 – December 2007.

Table 1 presents the results of the unit root tests. As one can observe, the null hypothesis of unit root is rejected by ADF for all the series, implying a degree of integration lower than one. The KPSS test provides more clear results, indicating a zero degree of integration for the returns of CZK, HUF, and PLN, for the squared returns of HUF and of RON. The rejection of the null hypothesis by the KPSS indicates a fractional degree of integration for all the series in absolute value, and for the squares for CZK and PLN.

The findings of the ADF and KPSS tests are supported by the estimated fractional difference parameter through the GPH method. The results are displayed in Table 2. The estimated values of d suggest that absolute returns show a degree of persistence greater than that of the squared returns.

To asses the ability of different conditional heteroskedasticity models to represent the studied financial series we estimated for each individual security GARCH (*1*, *1*), FIGARCH(*1*, *d*, *1*), and IGARCH(*1*, *1*) models. We implemented different conditional mean specifications to account for the low order correlation. Monte Carlo simulations show that AIC and SIC segregate effectively GARCH and FIGARCH alternatives. We exploited the opportunity offered by Ox Garch23 package to consider different error distribution: Gauss, Student-*t*, General Error Distribution, and Skewed Student-*t*.

The sum of GARCH α_1 and β_1 estimates are close to one for the majority of series, as a sign of persistence in volatility. The fractional differencing parameter is estimated as significantly different form zero; the estimated values being similar with the results presented in Table 2. The β_1 estimate falls considerably from GARCH to FIGARCH. The analysis of AIC and SIC implies that FIGARCH models fit the studied data series slightly better than GARCH, or IGARCH models.

Although the robustness of the usual long-memory procedures remains yet to be addressed, the jointly results of tests performed indicate evidence for the fact that volatility of analyzed exchange rates returns displays long-memory. The results also indicate that, with the exception of the RON exchange rate, the returns describe mainly short-term processes.

A crossing observation over the similar results of the performed tests induces the following incremental ranking, in terms of autocorrelation structure: returns, squared returns, absolute returns. The previous autocorrelation structure ranking suggests the suitability of the absolute returns volatility design.

A further development in the area would comprise fractional cointegration analysis to detect a usable formulation between return and the respective volatility specification.

The presence of long-memory in the volatility of financial assets is congruent with the persistence of shocks in returns, indicating that new models for risk assessment are required.

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APPENDIX

Table 1: Unit root tests

CZK			HUF			PLN			RON		
Ret.	Abs.	Sq.	Ret.	Abs.	Sq.	Ret.	Abs.	Sq.	Ret.	Abs.	Sq.
Augmented Dickey-Fuller											
-54.67	-2.92	-13.02	-51.30	-4.91	-15.38	-52.29	-5.02	-10.5	-39.31	-9.92	-15.8
Kwiatkowski, Phillips, Schmidt and Shin											
0.04	2.08	2.17	0.12	1.41	0.55	0.14	1.55	1.24	1.91	2.5	0.71

Source: Own calculations. Ret. stands for returns, Abs. for absolute value, and Sq. for conditional volatility. Shaded areas denote significance at 1%. Critical values for KPSS are 0,74 for 1%, 0,46 for 5% and 0,38 for 10%.

Table 2: GPH estimates for the fractional integration parameter

CZK			HUF			PLN			RON		
Ret.	Abs.	Sq.	Ret.	Abs.	Sq.	Ret.	Abs.	Sq.	Ret.	Abs.	Sq.
-0.13	0.59	0.56	-0.0	0.53	0.32	0.02	0.36	0.19	0.27	0.31	0.09
(0.13)	(0.09)	(0.09)	(0.11)	(0.1)	(0.1)	(0.11)	(0.1)	(0.09)	(0.15)	(0.09)	(0.04)

Source: Own calculations. Ret. stands for returns, Abs. for absolute value, and Sq. for conditional volatility. In parentheses are presented the standard deviations.

Table 3: Volatility models

FIGARCH (1,d,1)					
Series	α	β	d	Akaike	Schwarz
CZK	0.26	0.69	0.59	0.66	0.67
HUF	0.16	0.3	0.29	0.95	0.96
PLN	0.12	0.32	0.32	1.69	1.7
RON	0.28	0.67	0.6	1.65	1.66
IGARCH (1,1)					
CZK	0.09	0.91	-	0.66	0.67
HUF	0.03	0.97	-	0.98	0.99
PLN	0.14	0.86	-	1.7	1.71
RON	0.13	0.87	-	1.65	1.67
GARCH (1,1)					
CZK	0.08	0.9	-	0.66	0.67
HUF	0.09	0.87	-	0.97	0.98
PLN	0.12	0.84	-	1.7	1.71
RON	0.14	0.86	-	1.65	1.66

Source: Own calculations.