



Research Article

Time Variation, Asymmetry and Threshold Effects in Malta's Phillips Curve

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Abstract. This paper estimates a Phillips curve for Malta using data since the 1960s, using Bayesian methods to estimate a time-varying parameter model with stochastic volatility. It presents evidence that the curve has flattened over time. This implies that the link between inflation and economic activity has weakened, consistent with findings for other countries. This phenomenon is driven by downward price stickiness and threshold effects, where inflation is generally unresponsive to domestic economic conditions unless the economy is going through a strong boom. Meanwhile, this study finds an increasingly important role for import price shocks in driving inflation in the Maltese economy, owing to its increased openness and trade integration. The estimated variance of shocks to inflation was high in the 1980s, but has fallen greatly since then, rising somewhat in the run-up to the Great Recession.

Keywords: Inflation, NAIRU, time-varying parameters, Bayesian methods, Metropolis-Hastings, Gibbs sampling

1 Introduction

Understanding inflation dynamics has become particularly important in view of the low inflation regime now prevailing and because the traditional relationship between slack in the economy and inflation seems to have weakened significantly in some countries.

If confirmed, the flattening of the Phillips curve would be relevant for monetary policy because that relationship was the traditional linchpin of the transmission mechanism that gave central banks control of inflation. The subsequent focus on the role of expectations and their management in the toolkit of monetary policy reduced but did not elim-

inate the relevance of the traditional mechanism.

—Vítor Constâncio¹
Former ECB Vice-President

Central banks have striven to earn credibility in their quest to control consumer price inflation by, *inter alia*, improving their communication through the announcement of a preferred inflation rate. In the euro area, monetary policy is conducted with the primary objective of keeping inflation “below, but close to, 2% in the medium term” (ECB, 2001). When analysing economic developments, econometric models help shape views about the current and medium-term outlook for economic activity and inflationary pressures.

Since the financial crises of 2008 and the ensuing Great Recession, research is being directed at studying additional important channels through which shocks propagate. At the same time, economists observe that models which enjoyed a good track record at forecasting inflation tended to perform badly during the past ten years, predicting a more significant drop in inflation than what materialised. This was termed the period of the ‘missing deflation’ (Ball & Mazumder, 2011; Stock, 2011; Ball & Mazumder, 2015).

Economists believe that in the short run inflation moves in line with economic conditions. This relationship, known as the Phillips curve, traces its origins to an empirical exercise conducted in the late 1950s, showing a negative relationship between nominal wage growth and unemployment in the United Kingdom (Phillips, 1958). During times of strong demand, firms employ more workers, leading to a tighter labour market. This puts upward pressure on wage claims, and therefore also on

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firm operating costs, which are reflected in higher prices for goods and services, and so on. Low demand should generate the opposite effect. Thus, demand shocks boost economic activity, lowering unemployment and increasing inflation.

In the past, policymakers believed they could exploit this trade-off, reducing unemployment at the cost of higher inflation. However, advances in the theory behind the Phillips curve, in particular the incorporation of people's expectations in the late 1960s, as well as a better framework for firms' pricing behaviour in the 1970s and 1980s, showed that exploiting this trade-off did not really pay off in the medium to long run. Economists realised that as people come to expect higher inflation, unions would call for higher wage growth, which would increase unemployment back to an equilibrium level. The economy would return to the previous unemployment rate, living with a higher rate of price inflation. Thus, the Phillips curve is vertical in the long run, unrelated to economic activity.

Today the Phillips curve is a complex but important component of the New Keynesian DSGE model, which is the workhorse model in academia, central banks and other policy-making institutions. Despite the rich theory behind it, recent studies have shown that simple versions of the Phillips curve can capture inflation dynamics reasonably well. A more detailed review of the literature is given in Section 2.

The Phillips curve in Malta is relatively unexplored. It is embedded within the Central Bank of Malta's macro-econometric model (O. Grech & Micallef, 2014), where inflation responds to economic activity in the short run. A. Grech (2015a) plots annual data for the unemployment rate and inflation over the period 1960-2014 and finds a negative relationship, in line with theory. He argues however that the link is weak, implying that a tight labour market does not lead to strong upward pressure on prices, and vice versa.

This paper contributes to the literature by estimating the Phillips curve over a long time frame, using state-of-the-art techniques.² It also explores whether the relationship changed over time. The link between economic activity and inflation was high in the mid-1980s but fell progressively over the 1990s and was very weak thereafter. Further analysis shows that size and nature of economic disequilibrium also matters. Inflation tends to rise during strong economic booms, but does not fall during recessions. In recent years however, this relationship seems to have disappeared. Foreign price pressures play a strong and increasingly important role in driving inflation in Malta, since it is a small and very open economy.

²Studies using data with long time spans for the Maltese economy are relatively scarce.

Increased globalization and lower barriers to trade, mainly through EU accession, have led to an increase in competition, putting a lid on price pressures. In addition, increased participation in the labour market, especially female participation, together with a strong inflow of foreign workers boosted the labour supply, reducing labour market tightness. These factors can explain the observed 'flattening' of the Phillips curve.

2 The Phillips Curve

In this section I discuss the canonical specification of the Phillips curve, which has been rigorously developed over time, particularly with the incorporation of inflation expectations in the late 1960s, as well as micro-founded derivations of profit maximisation subject to nominal rigidities in the 1970s and 1980s.³ The hybrid version of the New Keynesian (NK) Phillips curve (Gali & Gertler, 1999; Gali, Gertler & Lopez-Salido, 2001; Gali, 2008) is given by

$$\pi_t = \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda \widehat{m}c_t, \quad (1)$$

where the parameters γ_f , γ_b and λ are functions of structural parameters, $\mathbb{E}_t \pi_{t+1}$ is expected future inflation, reflecting forward-looking behaviour, π_{t-1} is lagged inflation, capturing inflation inertia, and $\widehat{m}c_t$ is real marginal cost of production, which is the activity variable through which prices are affected. The latter term has been shown to be proportional to the output gap, under a number of assumptions (Gali & Gertler, 1999; Gertler & Leahy, 2008). For this reason, empirical studies proxy real marginal costs by a measure of the output gap (see Bermingham, Coates, Larkin, O'Brien & O'Reilly, 2012; Jordan & Vilmi, 2014).⁴

Other studies use the deviation of the unemployment rate from the Non-Accelerating Inflation Rate of Unemployment (NAIRU) as the activity variable, referred to as cyclical unemployment or the unemployment gap (see Ball & Mazumder, 2011; Peach, Rich & Cororaton, 2011; Bermingham et al., 2012; Kajuth, 2012; Simon, Matheson & Sandri, 2013; ECB, 2014; Speigner, 2014). Using cyclical unemployment as the activity variable is more reminiscent of the traditional Phillips curve.⁵

The definition of expected inflation varies across empirical studies. Expectations are proxied either by survey-based measures of expected inflation (Jordan & Vilmi, 2014), announced central bank targets (Simon

³See Kajuth (2012) for a list of the important contributions to this area.

⁴However it has been argued that for such specifications the proper proxy for marginal costs is the labour share of income; see Gali and Gertler (1999).

⁵Other studies use more complex specifications which take into account asymmetric/threshold effects and differences between the short term and long term unemployment; see *inter alia* Laxton, Rose and Tambakis (1999), Bermingham et al. (2012), Speigner (2014) and Ball and Mazumder (2015).

et al., 2013), or a long run average of realised inflation (Ball & Mazumder, 2011).⁶

In empirical studies aimed at determining the size and significance of the coefficients relating to the determinants of inflation, inference is based on a reduced-form Phillips curve. A number of authors have recently introduced time variation in the parameters, allowing the relationship between inflation and its determinants to change over time (Simon et al., 2013; Stevens, 2013; Álvarez & Urtasun, 2013; Oinonen, Paloviita & Vilmi, 2014; Riggi & Venditti, 2015). This was partly motivated by the poor forecasting performance for inflation during and after the financial crisis (ECB, 2015). While one reason behind the large forecast errors were incorrect real-time estimates of activity gaps, it has also been shown that the sensitivity of inflation to activity has changed recently. This topic has also been re-visited in a recent ECB conference (see Hartmann & McAdam, 2018). This highlights the importance of allowing for structural change in empirical models.

There are other factors which can explain apparent 'shifts' in the Phillips curve, for example, it may be subject to threshold and asymmetry effects (Laxton et al., 1999; Musso, Stracca & van Dijk, 2009; Bermingham et al., 2012; Speigner, 2014). Very strong booms or deep recessions may affect inflation differently than smaller, more typical booms and recessions. Owing to downward price rigidity, inflation may also not turn negative during recessions. These issues are explored in Section 4.3.

3 Data

This section describes the data that were used in this study. The main variables of interest are the Retail Price Index (RPI), registered unemployment, real GDP, and consumer price indices (CPI) of Malta's key trading partners. Further information on the data sources and workings can be found in Appendix Appendix A.

Fig. 1 shows yearly growth in the RPI and an index of foreign consumer prices since the mid-1960s.⁷ The long-run co-movement between these two price series has been high, which implies that both series were driven by

⁶Some studies adopt the hybrid approach, including both leads and lags of inflation. For instance, Simon et al. (2013), Blanchard, Cerutti and Summers (2015) and Blanchard (2016) list the following specification:

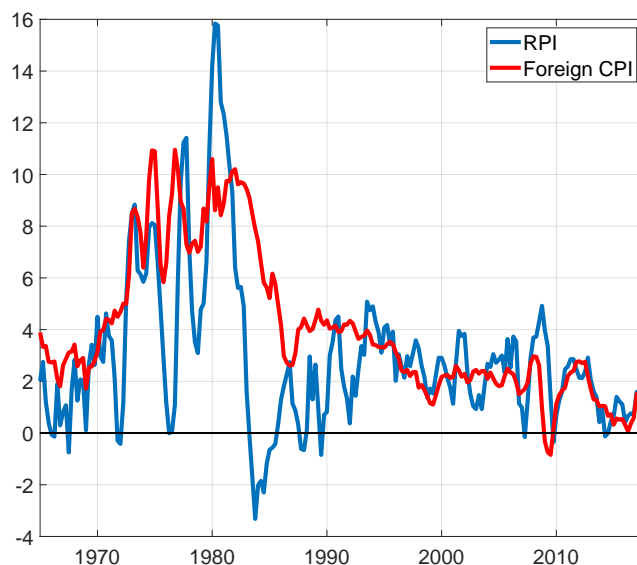
$$\pi_t = (1 - \vartheta)\pi_{t-1} + \vartheta\pi_t^e - \kappa\bar{u}_t + \gamma\pi_t^m + \varepsilon_t, \quad (2)$$

where ϑ measures the relative importance given to expectations of future inflation during wage and price setting, relative to information from past inflation, κ measures the slope of the Phillips curve on the activity variable (in this case cyclical unemployment \bar{u}_t) and γ measures the impact of imported inflation.

⁷The foreign CPI is an index based on CPI developments in France, Germany, Italy and the United Kingdom, which were historically the most important trading partners.

common factors, such as the oil price shocks in the 1970s and 1980s. The two series deviate somewhat in the mid-1980s, partly on account of the price controls that were enacted on some consumer goods at the time in Malta. Subsequently, both foreign and domestic inflation co-move and stabilise around lower levels.

Figure 1: Inflation indicators (yearly growth (%)).



Figures 2 and 3 show GDP growth and the unemployment rate and their underlying (and unobserved) trends, potential output and the Non-Accelerating Inflation Rate of Unemployment (NAIRU). The trends were estimated using an Unobserved Components Model (UCM) based on the Kalman filter; see Appendix Appendix B for details. The data match the stylised facts for a number of advanced economies, in which GDP growth becomes more stable after the 1980s (Summers, 2005) and the Maltese economy, being small and very open, enjoyed the spillovers of the so-called Great Moderation. The output gap and cyclical unemployment, defined as the difference between the data and estimated trends, are treated as indicators of excess demand. A positive output gap implies output growing at a faster rate than potential growth, and vice-versa. Similarly, negative cyclical unemployment indicates an excess demand for labour, and vice-versa.⁸

⁸The output gap and unemployment gap, as expected, are negatively correlated. Developments in GDP growth typically precede developments in the labour market, with a lag of about 2 to 3 quarters.

Figure 2: Real GDP and potential output growth (annual, %).

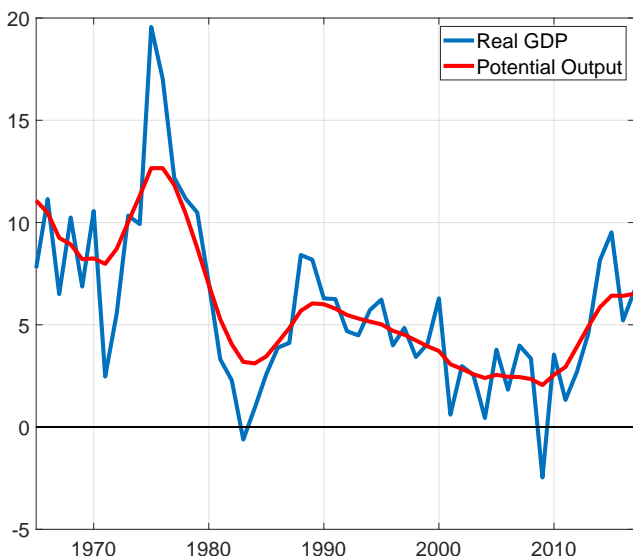
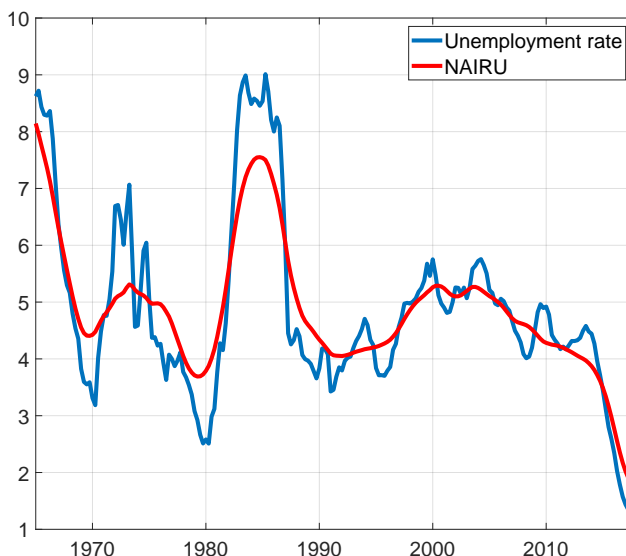


Figure 3: Registered unemployment rate and the NAIRU (%).



4 Empirical Results

In this section, I first present estimates of the Phillips curve under the assumption of constant parameters over the entire sample. I then relax this assumption and use a more sophisticated technique to uncover possible changes in the coefficients over time, and allow different inflation responses during booms and recessions.

4.1 A Standard Phillips Curve

The Phillips curve specification that is used in this paper is

$$\pi_t = c + \alpha X_{t-i} + \gamma \pi_{t-i}^f + \sum_i \rho_i \pi_{t-i} + \varepsilon_t, \quad (3)$$

where π is annual RPI inflation, X is the activity variable and π^f is relative import prices proxied by annual growth in foreign CPI less RPI growth. The appropriate lag length i for each variable is determined empirically. Note that this specification assumes that inflation expectations are purely backward-looking, or adaptive. Eq. (3) is estimated using both cyclical unemployment \tilde{u} and the output gap \tilde{y} as the activity variables ($X \in \{\tilde{u}, \tilde{y}\}$).⁹

Since \tilde{u} , \tilde{y} and π^f are all autocorrelated over time, including several lags of these variables introduces high

collinearity between regressors. Hence, after some testing, cyclical unemployment was only included in its third lag and imported inflation included only in its first lag. This choice was guided by the cross-correlogram for the dependant variable and the regressors, and the lag at which there was the highest correlation was chosen. Given the lagged co-movement between the output gap and cyclical unemployment, the output gap was lagged by 1 quarter. Since inflation is measured in annual percentage changes, the model includes the first four lags of inflation to control for residual serial correlation. Eq. (3) is estimated using Ordinary Least Squares (OLS), using quarterly data for the period 1965Q4–2017Q4.¹⁰ Inference is based on Newey-West standard errors (Newey & West, 1987).

The OLS estimates are shown in Table 1 below in column (1). The ‘slope’ of the Phillips curve, α , is statistically significant at conventional levels only when cyclical unemployment is used as the activity variable. This shows some link between economic activity and prices. Cyclical unemployment is arguably a more indicative measure of economic activity than the output gap, as transitory shocks to GDP, which affect the output gap, may be absorbed by firms and thus not reflected in employment through the extensive margin. Column (1) also confirms the important role of import price shocks on domestic inflation. The measure of fit of both models, assessed using the adjusted R-squared (\bar{R}^2) is very

⁹This specification is typically used in empirical studies; see *inter alia* Simon et al. (2013). It nests the hybrid NK Phillips curve since it is assumed that expected inflation is equal to inflation in the previous period and marginal costs are approximated by the slack variables $X \in \{\tilde{u}, \tilde{y}\}$. In open economy versions of the NK Phillips curve an additional variable is the (change in the) terms of trade Galí and Monacelli (2005). Other foreign variables appear in the marginal cost variable, but given that estimation is based on a reduced-form representation, foreign prices are explicitly included as a separate variable.

¹⁰Although inflation and unemployment are available in quarterly frequency, GDP, from which the output gap is derived, was only measured in annual frequency during roughly the first half of the sample. Appendix Appendix A explains how this data was interpolated to quarterly frequency.

high, although most of the fit can be attributed to inflation being explained by its history.

These estimates are based on a relatively long time series, during which the Maltese economy witnessed significant structural and socio-economic changes.¹¹ It is therefore likely that the relationship presented above might have changed over time. Two approaches are used to test the stability of the parameters. The first is to split the sample into two, an 'early' period spanning 1966–1995, and a more recent period over 1996–2017, and estimate the Phillips curve for each sub-sample.¹² The results are in columns (3) to (6) respectively in Table 1. By comparing the estimated coefficients over the two periods, we can assess any material shifts in the coefficients for economic slack and imported inflation over the two periods.

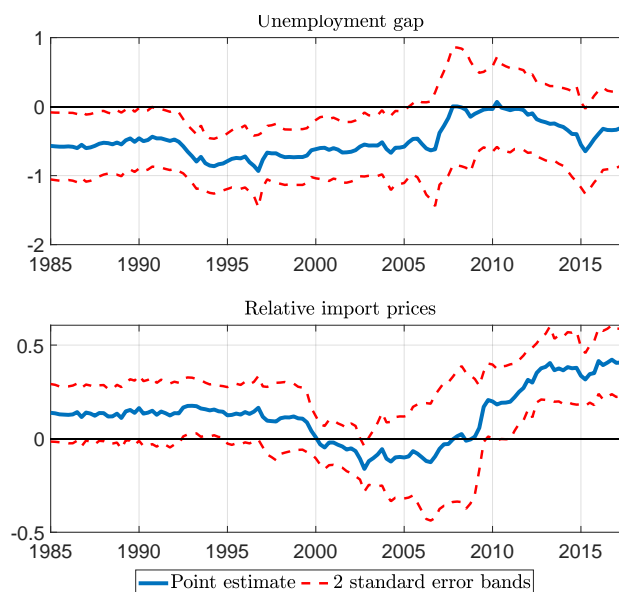
A second approach is to estimate rolling regressions for Eq. (3) to track the evolution, if any, of the parameters over time.¹³ Starting from 1965Q4, the first 80 observations, or 20 years worth of data (the 'window') are used to estimate the Phillips curve. The estimated coefficients are saved, and the sample is moved by one period forward in time, while keeping the same window length. I repeat the process until the end of the entire sample (2017Q4). The estimates from each recursion track 'smooth' changes in the parameters and report them as time series. I use the unemployment gap as the relevant activity variable, and show the results in Fig. 4.¹⁴

Both the sub-sample and rolling regression approaches provide evidence of a change in the slope of the unemployment gap version of the Phillips curve, α , over time. The parameter in the first sub-sample is -0.535 and statistically significant (column (3)), while in the second it is lower in absolute terms (-0.209) and not statistically different from zero (column (4)), indicating a significant change. This development is not easily seen in the output gap version, as in both sample periods the coefficient is negative and insignificant. As discussed above, this latter result could be due to the fact that the output gap is somewhat volatile, carrying less information about the cyclical position of the economy compared to the unemployment gap, which is more persistent.

These results are supported from the rolling regression estimates. The top panel of Fig. 4 shows that the slope of the Phillips curve was relatively stable for a long

time up to the late 2000s, after which it tended to zero. The same dynamics occur at a slightly earlier stage for the economy's sensitivity to import price shocks. It is interesting to note that the drop in the inflation sensitivity to economic slack and import price shocks, and wider error bands, coincide with the Great Recession.¹⁵ From 2010 onwards we observe a reversal, especially in the coefficient for imported inflation.

Figure 4: Rolling regression parameter estimates.



These results are interesting as they imply that significant changes, potentially structural, took place in the Maltese economy. Furthermore, and as noted above, volatility in inflation was high in the 1970s and 1980s, but then fell markedly since the 1990s. This phenomenon is also observed in many major economies as one of the characteristics of the Great Moderation. The fall in inflation and its volatility across advanced economies is argued to be primarily a consequence of credible inflation targeting central banks, which stabilized long-term inflation expectations (Simon et al., 2013; Carney, 2015).

4.2 Allowing for Stochastic Volatility: A TVP-SV Model

A model which allows the parameters to change over time but ignores the changing volatility in the dependent variable is likely to overestimate or lead to spurious variation in the coefficients, as these 'soak up' some of the variance of the residuals (Cogley & Sargent, 2005;

¹⁵Admittedly, the timing of these dynamics is somewhat sensitive to the window length used in estimation. A shorter window of 60 observations shows the same dynamics occurring earlier in time. However, the key point is the indication of instability in the relationship over time.

¹¹See A. Grech (2015a).

¹²The choice for splitting the sample around 1995 was made mainly to have roughly balanced sub-samples.

¹³This approach is frequently used in the literature, see Oinonen et al. (2014).

¹⁴The first parameter estimates are as at 1984Q4. The results using the output gap version of the model are not shown, but are qualitatively the same.

Table 1: OLS regression results.

	(1) 1965Q4–2017Q4	(2) 1965Q4–2017Q4	(3) 1965Q4–1995Q4	(4) 1996Q1–2017Q4	(5) 1965Q4–1995Q4	(6) 1996Q1–2017Q4
\tilde{u}_{t-3}	-0.450**		-0.535***	-0.209		
\tilde{y}_{t-1}		0.016			0.010	-0.046
π_{t-1}^f	0.130***	0.118**	0.144**	0.381***	0.137	0.392***
\overline{R}^2	0.863	0.862	0.871	0.668	0.865	0.668
Obs.	209	209	121	88	121	88

Notes: *** and ** imply statistically significant coefficients at the 1% and 5% level of significance respectively and are based on Newey-West standard errors. Obs. is the number of observations used in the estimation. The coefficients on the autoregressive terms are not shown, but throughout the regressions are jointly statistically significant and sum up to less than one, implying that RPI inflation is a stationary process.

Primiceri, 2005; Nakajima, 2011). For this reason, the Time-Varying Parameter model with Stochastic Volatility (TVP-SV) is explored next. This model allows both the parameters and the volatility of shocks to inflation to change over time.¹⁶ I use this technique on the unemployment gap version of the model.

The TVP-SV specification of the baseline Phillips curve is given by

$$\pi_t = c_t + \alpha_t \tilde{u}_{t-3} + \gamma_t \pi_{t-1}^f + \sum_i^4 \rho_{i,t} \pi_{t-i} + \varepsilon_t \sqrt{h_t}, \quad (4)$$

where coefficients now have a time subscript and shocks to inflation ε_t are augmented with a time-varying variance term h_t . The parameters of the model c , α , γ and ρ_i ($i \in [1, 4]$), and the logarithm of h , are assumed to follow random walks. When stacked into the vector $B_t = [c_t \ \alpha_t \ \gamma_t \ \rho_{1,t} \ \rho_{2,t} \ \rho_{3,t} \ \rho_{4,t}]'$, the evolution of the parameters can be represented as

$$B_t = B_{t-1} + \nu_t, \quad (5)$$

where ν_t is a vector of shocks. The evolution of the (log) variance of shocks is given by

$$\log h_t = \log h_{t-1} + \eta_t, \quad (6)$$

where η_t is a disturbance term. This setup constitutes a non-linear state-space model, as the state variable h_t is not linear in the observation equation (Eq. (4)).

The model is estimated using Bayesian methods, specifically a Metropolis-within-Gibbs sampler, using the algorithm of Carter and Kohn (1994) to extract the path for all the elements in B_t in every iteration.¹⁷ Follow-

¹⁶See Cogley and Sargent (2005) and Primiceri (2005) and the literature cited therein for a discussion of this model. An explanation of this setup for a univariate framework can be found in Nakajima (2011, p.109). More details are provided in Appendix C.

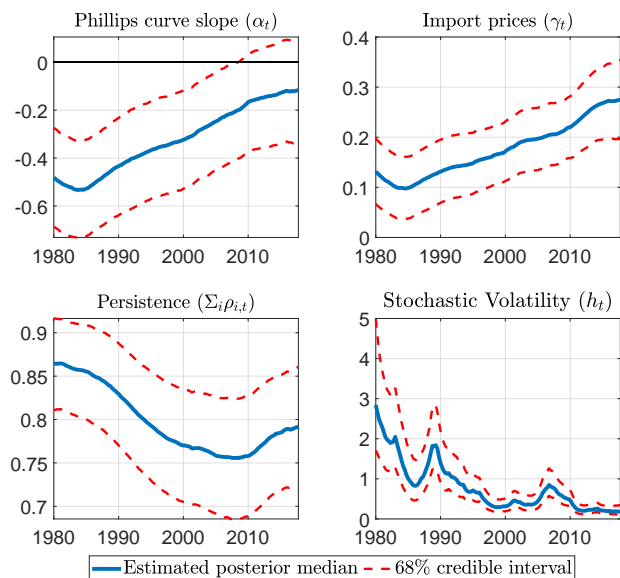
¹⁷See Jacquier, Polson and Rossi (1994) and Blake and Mumtaz (2012) for a discussion of Monte Carlo Markov Chain (MCMC) Bayesian inference in such models.

ing Primiceri (2005), a fraction of the data were used as a training sample to initialize the priors (1966Q1–1979Q4). Details on the estimation setup are available in Appendix Appendix C. The sample on which inference is based spans 38 years (1980Q1–2017Q4) and the estimation procedure is based on 20,000 iterations. The first 5,000 burn-in draws are discarded and the remaining draws are used to construct the posterior distributions of the parameters, shown in Fig. 5.

The time-varying Phillips curve slope, α_t , is estimated to have declined in absolute terms since the 1980s, implying a weakening in the relationship between economic activity and inflation. This confirms the findings of the previous section, and is a pattern that is observed in many advanced economies, in line with the findings in Simon et al. (2013) and Blanchard et al. (2015). Although there are no major changes during the period associated with the financial crisis, a slight but sudden change in the trend of the slope can be seen starting in 2010.

The estimates for γ_t show that the role of import prices was weakest during the period of the price controls in the early 1980s. Thereafter, imported price shocks played a progressively stronger role in explaining inflation in Malta. This trend is also in line with other studies; Stevens (2013) and Simon et al. (2013) find the same behaviour in the economies of the EU and of a number of OECD countries, respectively. The variance of shocks to inflation, h_t (bottom right panel), exhibited significant time variation, being high in the early 1980s but then falling significantly. Inflation volatility rose temporarily just before the 1990s and more recently in 2007, on account of the food price shocks that preceded the financial crises.

Inflation persistence fell significantly since the 1980s, implying that when everything else is kept constant, shocks to inflation used to die off much slower in the past. The anchoring of expectations in many major eco-

Figure 5: TVP-SV estimation results.

nomies and the increased synchronisation of the Maltese economy with such developments (through a rise in γ), are perhaps the key reasons for the decline in local inflation persistence. The uncertainty surrounding the time-varying parameter estimates is quite high, as the 68% credible intervals are relatively wide for α , γ and ρ_i . Nevertheless, these results highlight the possible changes that have occurred in the macroeconomy and are discussed further in Section 5. A model of the Maltese Phillips curve which assumes constant parameters and constant residual volatility, is therefore unable to capture all these interesting developments.

All the estimates presented above and in previous sections could be sensitive to how economic slack is measured. To this end the model is re-estimated using a measure of cyclical unemployment derived from the Hodrick-Prescott filter, which is another frequently used trend-cycle decomposition technique. The flattening of the Phillips curve is also observed on the basis of this activity measure. The sudden reversal in the slope starting in around 2010 is also confirmed in this set of estimates. Other robustness checks are discussed in a working paper version of this article (Gatt, 2016).

4.3 Asymmetry and Threshold Effects

The model specified above postulates that the economy behaves the same way irrespective of the state it is in, that is, irrespective of whether it is going through a boom or recession. This goes against the literature on downward nominal rigidities and Phillips curve convexity, which argues that one should not expect the same re-

lationship at different points along the Phillips curve.¹⁸ Empirical evidence shows that the response of inflation to slack may be state-dependent. For example, Demers (2003) and Barnes and Olivei (2003) find that the slope of the Phillips curve can change according to the state of the economy within a business cycle. Birmingham et al. (2012) also find the existence of threshold effects, whereby the link between economy and inflation is stronger during recessions compared to booms. Further discussion can be found in Musso et al. (2009).

To allow for the possibility of asymmetries in the Phillips curve, I modify the model in the previous section to allow different Phillips curve slopes conditional on booms ($\tilde{u} < 0$) and recessions ($\tilde{u} > 0$):

$$\pi_t = c_t + \alpha_t^b \tilde{u}_{t-3} \mathbb{1}_{(\tilde{u} < 0)} + \alpha_t^r \tilde{u}_{t-3} \mathbb{1}_{(\tilde{u} > 0)} + \gamma_t \pi_t^f + \sum_i^4 \rho_{i,t} \pi_{i,t-1} + \varepsilon_t \sqrt{h_t}, \quad (7)$$

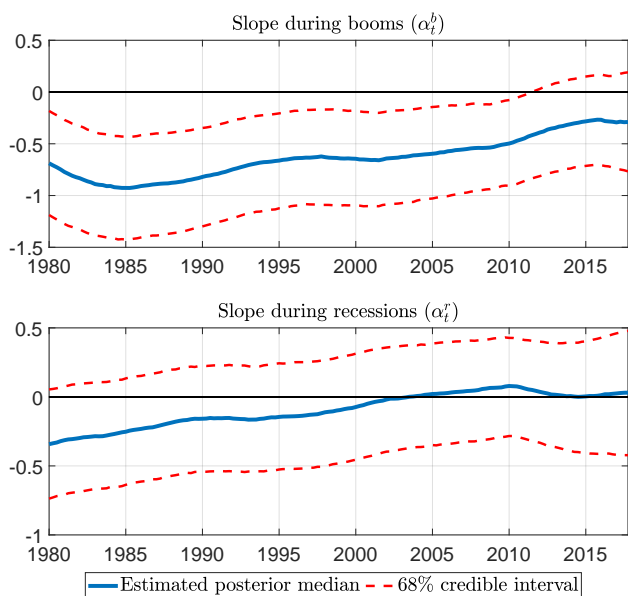
where $\mathbb{1}(\cdot)$ is the indicator function which takes a value of 1 when the condition within the brackets holds. As above, the parameter vector B_t and the log variance $\log(h_t)$ evolve as random walks. Estimation follows the same procedure as above and the results are summarized in Fig. 6. The drifts of the parameters γ and $\sum_i^4 \rho_i$ are similar to those in Fig. 5, so are not shown again.

Besides time variation, there is also evidence of asymmetry in the Maltese Phillips curve. The slopes associated with booms α_t^b and recessions α_t^r have both fallen in absolute terms since the 1980s. However, the relationship during a boom was much stronger throughout the sample period. Thus, the flattening phenomenon appears to be driven mainly by behaviour during economic booms, since the relationship during recessions was always weak.

The estimated unemployment gaps have varied in magnitude over time. Large deviations from the NAIRU occurred mostly during the early part of the sample. It may be the case that the Phillips curve relationship may also be sensitive to the *size* of labour market slack, as discussed in Barnes and Olivei (2003). The implication of this argument would be that the observed flattening may not reflect a change in the relationship, but merely the fact that prices are today reacting to much smaller shocks than in the past. Below a given threshold, inflation may respond very weakly (potentially in both directions), say due to menu costs.

¹⁸See Laxton et al. (1999) and Speigner (2014). A theoretical account of how rigidities affect the convexity of the Phillips curve is given by Daly and Hobijn (2014).

Figure 6: Asymmetric Phillips curve slopes.



To test this formally, while maintaining the separation between booms and recessions, I modify the Phillips curve to the following form

$$\begin{aligned} \pi_t = & \left[\alpha_t^b \tilde{u}_{t-3} \mathbb{1}(\tilde{u} < 0) + \alpha_t^r \tilde{u}_{t-3} \mathbb{1}(\tilde{u} > 0) \right] \mathbb{1}(|\tilde{u}| < \kappa) \\ & + \left[\bar{\alpha}_t^b \tilde{u}_{t-3} \mathbb{1}(\tilde{u} < 0) + \bar{\alpha}_t^r \tilde{u}_{t-3} \mathbb{1}(\tilde{u} > 0) \right] \mathbb{1}(|\tilde{u}| > \kappa) \\ & + \gamma_t \pi_t^f + \sum_i^4 \rho_{i,t} \pi_{i,t-1} + c_t + \varepsilon_t \sqrt{h_t}, \end{aligned} \quad (8)$$

where the indicator function outside square brackets switches on during periods of low ($|\tilde{u}| < \kappa$) or high ($|\tilde{u}| > \kappa$) labour market slack respectively. This specification nests that in equation (7), so that the threshold effect is tested along with the asymmetry effect discussed above.¹⁹ The threshold value κ was set at 1 standard deviation of the unemployment gap.²⁰ This specification effectively allows the economy to be in four distinct states and returns four slope parameters, which describe the relationship between inflation and slack during shallow and deep recessions (α^r and $\bar{\alpha}^r$ respectively) and small and large booms (α^b and $\bar{\alpha}^b$ respectively).

The results, shown in Fig. 7, shed further light into the degree of asymmetry and sensitivity of inflation to small and large shocks. The estimates are subject to a higher degree of uncertainty, given that few observations are available in each state, so these results should be

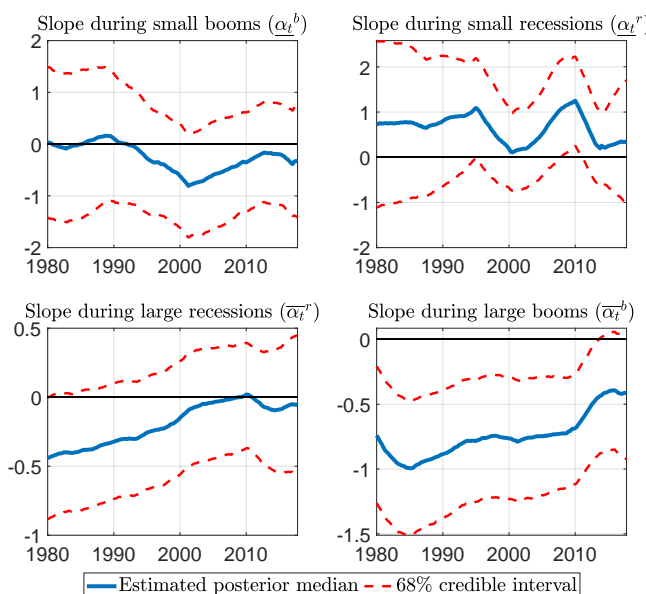
¹⁹Indeed, it also nests the fixed parameter model estimated using OLS.

²⁰Similar results were obtained at a lower (0.5 standard deviation) and higher (2 standard deviations) threshold.

interpreted with some caution.²¹

Nevertheless, the results related to the asymmetry of the Phillips curve discussed above remain valid. Shallow recessions have not been associated with a drop in inflation, and although there is evidence that deep recessions may have put downward pressure on inflation in the 1980s, the link has since then disappeared. Similarly, small expansions have not been associated with a rise in inflation. The asymmetry takes effect only during large expansions, and these estimates suggest that this relationship has also weakened somewhat since the 1980s.

Figure 7: Asymmetric-threshold Phillips curve slopes.



5 What Caused a Flat Phillips Curve?

I now discuss possible drivers of a falling Phillips curve slope. A number of theories have been fielded to explain this phenomenon, which as Simon et al. (2013) show, seems to be widespread across several advanced economies, with varying degrees of openness. The central argument raised in studies which report a flattening of the Phillips curve, is a general move towards ‘anchored inflation expectations’ (Simon et al., 2013; Ball & Mazumder, 2015; Blanchard et al., 2015). People’s belief of moderate and stable future inflation, brought about by successful central bank monetary policy, reduced pressure on wages by workers and unions seeking to maintain the purchasing power of income. Anchored inflation expectations were attributed to the so-called ‘missing deflation’ in OECD countries, in which economic activity

²¹There were 50 quarters of shallow recessions, 55 quarters of moderate booms, 25 quarters of deep recessions and 22 quarters of strong booms in the period 1980–2017.

dropped significantly but inflation did not turn negative.

Another key argument is the role of globalisation. Lower global inflation, in part due to increased openness to trade and cheaper imported goods – the so-called “China effect” (Lewis & Saleheen, 2014) – lowered domestic inflation. To this end, changes over both the general level of mark-ups and their relation to the economic cycle might have also changed pricing behaviour, and hence affected the Phillips curve slope (Carney, 2015). A theoretical account of how globalization drives a flat aggregate supply curve is given in Razin and Binyamini (2007). In fact, the empirical work of Borio and Filardo (2007) presents cross-country evidence of an increased role for global factors in explaining domestic price developments, especially since the 1990s. Furthermore Sbordone (2007) argues that globalisation may have led to a low inflation environment by moderating growth in marginal costs through increased competition.

Using a NK Phillips curve, as specified in Eq. (1) above, Kuttner and Robinson (2010) argue that changes in the persistence of marginal cost fluctuations can lead to a flattening of the slope, which is typically observed in reduced form estimates. However, they show that their estimate of the structural parameter, λ , linking developments in marginal costs and inflation in the United States, fell over time through an increase in the so-called Calvo parameter – the probability that firms in any point in time cannot revise prices.²²

Since the Maltese economy is very small and open, the globalization argument is considered the prime mechanism driving a flatter Phillips curve. Lower barriers to trade over time, brought about by EU accession and later the adoption of the euro, led to increased competition, which controlled price pressures. This was coupled with low and stable inflation in trading partner countries. A more recent phenomenon, the rise of online purchases from abroad, marks an additional development in product market competition. In fact, whereas only 34% of Maltese households with internet access had purchased goods online in 2005, this percentage rose to 66% by 2015.²³ All of these developments have led to a decline in trend inflation in Malta (Gatt, 2014).

While estimates of the asymmetric Phillips curve show that the flat slope relates mainly to periods of subdued economic activity, the expected impact of a strong boom on prices is uncertain. Based on the 68% credible

interval for estimated slope at the end of the sample, a 1 standard deviation downward shock to the unemployment gap is associated with an increase in inflation of between $[-0.09 - 0.93]$ percentage points on impact.

Developments in the labour market may have contributed to lower pressure on wage growth, through which we get a wage-price spiral. Trade unionisation rates have declined significantly from 33% in 1995 to 23% in 2013 (Micallef & Caruana, 2014). Labour participation rates, which were stable for decades, rose sharply after 1995, led by a near doubling of the female participation rate. This was also complemented by a significant inflow of foreign workers following EU accession (A. Grech, 2015b), and hence an overall increase in the labour supply may have dampened wage claims. These developments may be behind the stabilization in trend wage inflation, and thus explain the fall in the volatility of wage growth, as discussed in Gatt (2016). Therefore, while in Malta inflation tends to rise during an expansion, it does not fall during an economic slowdown.

6 Conclusion

This paper discusses and presents estimates of the Phillips curve in the Maltese economy using data starting from the mid-1960s. While OLS regression results show that the data fit the relationship over the full sample, sub-sample estimates point to a weakening of the relationship over time. Meanwhile, the same analysis shows an increase in the sensitivity of domestic inflation to import price shocks.

To analyse this further, I use a more flexible model which allows the Phillips curve parameters to change over time. Estimation of this model is based on a mix of Bayesian methods, and the results show significant changes in the parameters over time. The model is also able to track changes to the variance of shocks affecting inflation. The results show that, recently, shocks became smaller in magnitude on average compared to the 1980s, peaking only during the energy and food price shocks of 2007.

The decline in the slope of the Phillips curve is shown primarily to be due to an asymmetry in the relationship; the link between economic activity and inflation exists only during times of (strong) growth, implying downward price rigidity. The link between activity and nominal variables exists only when the shock to the economy is sizeable. However, increased openness and more stable economic growth are the key drivers for the observed flattening of the Phillips curve since the 1980s.

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²²This will have an effect on the transmission of monetary policy in a simple New Keynesian DSGE model. A flat Phillips curve reduces the effectiveness of monetary policy which follows a Taylor rule for the nominal interest rate. An exogenous positive shock to inflation causes the same central bank to lift interest rates more aggressively when the Phillips curve is relatively flat. See Appendix Appendix D for simulations.

²³See annual National Statistics Office reports titled ‘ICT usage by households’.

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Appendix A Data

Historic data is sourced primarily from A. Grech (2015a). All recent data are subject to revisions.

A.1 Inflation

Inflation is measured using the Retail Price Index, an index which was first estimated in 1936 (Micallef & Caruana, 1996) and has been the key indicator used to track inflation in Malta for many decades. This data was obtained in monthly frequency for the period 1950-2014 from the National Statistics Office. Since the index was re-based several times since 1950, the index was spliced into a consistent series and re-based.²⁴ Quarterly averages of the price index P_t were obtained and inflation is defined as $\pi_t = [\log(P_t) - \log(P_{t-4})] \times 100$.²⁵

A.2 Unemployment Rate

Data for registered unemployed and the labour supply were obtained from the Employment and Training Corporation (ETC; nowadays JobsPlus) from 1965 to 2014 in monthly frequency and spliced into a consistent series. The series was then seasonally adjusted using Census X12.

A.3 GDP

Annual data for GDP was obtained as a spliced series from different statistical methodologies over time.²⁶ The whole dataset was re-based in terms of millions of euros based on 2010 prices and was interpolated from annual to quarterly frequency using the Litterman interpolation technique (Litterman, 1983).^{27,28}

A.4 Foreign CPI

This index was constructed as a weighted average using the consumer price indices of four of Malta's major trading partners, namely the United Kingdom (UK), France, Germany and Italy. These countries accounted on average for more than 80% of all trade within the EU and just under 60% of all of Malta's trade between the period 1980-2014. CPI indices are those found in OECD (2010), and were extracted from the FRED© database. Relative weights were obtained from the historical

²⁴It should be noted that the coverage of the RPI basket was initially very limited; but was extended over time to include a wider range of goods and services.

²⁵Unless otherwise stated, all growth rates are calculated using this formula.

²⁶The data between 1954 and 2000 are those measured by the *System of National Accounts* (SNA) methodology, and two such series were available, one based on 1973 prices (which spans from 1954-1997) and another based on 1995 prices (data for which spans 1970-1995). GDP data from 2000 onwards is based on the *European System of Accounts 2010* (ESA2010) guidelines. All these series were spliced into a consistent and continuous series.

²⁷Litterman Match Sum technique in Eviews 9.

²⁸Although ESA2010 data is available in quarterly frequency, mixing this data with the interpolated quarterly frequency data would have introduced noise to a relatively smooth data series.

Direction of Trade statistics that can be found on the Central Bank of Malta's website. Data prior to 1980 was not available, and thus the earliest datapoint available was held constant to the past.

Appendix B Extracting Cyclical Indicators

This section describes the Unobserved Components Model (UCM) that is used to extract potential output growth, the output gap, the NAIRU and cyclical unemployment (as shown in Figs. 2 and 3) from the GDP and unemployment data. The trend-cycle decomposition is based on a state space representation of the system

$$\Delta GDP_t = \tau_t + \mu_t + \epsilon_t^{\Delta GDP}, \quad (B.1)$$

$$U_t = N_t + \lambda_t + \epsilon_t^U, \quad (B.2)$$

$$\tau_t = \tau_{t-1} + \epsilon_t^\tau, \quad (B.3)$$

$$N_t = N_{t-1} + \epsilon_t^N, \quad (B.4)$$

$$\mu_t = \rho_1 \mu_{t-1} + \rho_2 \mu_{t-2} + \epsilon_t^\mu, \quad (B.5)$$

$$\lambda_t = \theta_1 \lambda_{t-1} + \theta_2 \lambda_{t-2} + \theta_3 \mu_{t-4} + \epsilon_t^\lambda, \quad (B.6)$$

where ΔGDP_t is yearly GDP growth, U_t is the unemployment rate, τ is potential output, μ is the output gap, N is the NAIRU, λ is cyclical unemployment, $\epsilon_t^{\Delta GDP}$ and ϵ_t^U are measurement errors, and ϵ^i random shocks to $i \in \{\tau, N, \mu, \lambda\}$.

Eqs. (B.1) and (B.2) are the observation equations, which state that the left-hand side variable in each is the sum of a trend, a cyclical component and an irregular component which accounts for measurement errors. These sub-components are the unobserved state variables which the framework tries to identify. Therefore, τ and N represent potential output growth and the NAIRU respectively, and these are modelled in Eqs. (B.3) and (B.4) as random walks which are subject to white noise shocks $\epsilon_t^\tau \sim N(0, \sigma_\tau^2)$ and $\epsilon_t^N \sim N(0, \sigma_N^2)$.

The output gap and cyclical unemployment are modelled as μ and λ respectively in Eqs. (B.5) and (B.6). Owing to their cyclical nature they are modelled as stationary AR(2) processes, however the process generating cyclical unemployment is also a function of the output gap lagged by four quarters, in the spirit of Okun's law. This latter detail adds some economic structure to the decomposition implied by the system. Both of these processes are also subject to random shocks $\epsilon_t^\mu \sim N(0, \sigma_\mu^2)$ and $\epsilon_t^\lambda \sim N(0, \sigma_\lambda^2)$. The measurement errors follow white noise processes $\epsilon_t^{\Delta GDP} \sim N(0, \sigma_{\Delta GDP}^2)$ and $\epsilon_t^U \sim N(0, \sigma_U^2)$. All disturbances are uncorrelated with each other.

The model was parameterised as shown in the Table B.1 and run through the Kalman Filter. These parameters were chosen such that the resulting trend variables τ and N are not excessively volatile but evolve progressively over time. In Gatt (2016) I show that the resulting

estimate for potential output growth is consistent with production function-based estimates of potential growth for the Maltese economy.

Table B.1: UCM model parameters

Coefficients		Variances	
ρ_1	1.6	σ_τ^2	0.9
ρ_2	-0.8	σ_N^2	1.0
ϱ_1	1.6	σ_μ^2	1.8
ϱ_2	-1.1	σ_λ^2	2.1
θ	-0.1	$\sigma_{\Delta GDP}^2$	5.0
		σ_U^2	12.5

Appendix C The Time-Varying Parameter Model with Stochastic Volatility

The TVP-SV model is given by

$$\pi_t = c_t + \alpha_t \tilde{u}_{t-3} + \gamma_t \pi_{t-1}^f + \sum_i^4 \rho_{i,t} \pi_{t-i} + \varepsilon_t \sqrt{h_t}, \quad (\text{C.1})$$

$$c_t = c_{t-1} + \epsilon_t^c, \quad (\text{C.2})$$

$$\alpha_t = \alpha_{t-1} + \epsilon_t^\alpha, \quad (\text{C.3})$$

$$\gamma_t = \gamma_{t-1} + \epsilon_t^\gamma, \quad (\text{C.4})$$

$$\rho_{1,t} = \rho_{1,t-1} + \epsilon_t^{\rho_1}, \quad (\text{C.5})$$

$$\rho_{2,t} = \rho_{2,t-1} + \epsilon_t^{\rho_2}, \quad (\text{C.6})$$

$$\rho_{3,t} = \rho_{3,t-1} + \epsilon_t^{\rho_3}, \quad (\text{C.7})$$

$$\rho_{4,t} = \rho_{4,t-1} + \epsilon_t^{\rho_4}, \quad (\text{C.8})$$

$$\log h_t = \log h_{t-1} + \eta_t, \quad (\text{C.9})$$

where shocks to inflation ε_t are augmented with a time-varying variance term h_t and the parameters of the model α , γ , ρ and c and the logarithm of h are assumed to follow random walks. Stacking these parameters in the vector $B_t = [c_t \ \alpha_t \ \gamma_t \ \rho_{1,t} \ \rho_{4,t} \ \rho_{3,t} \ \rho_{4,t}]'$, we can express the evolution of these parameters as

$$B_t = B_{t-1} + \nu_t, \quad (\text{C.10})$$

where ν_t is a vector of shocks following normal distribution with mean 0 and variance Q

$$\nu_t \sim N(0, Q). \quad (\text{C.11})$$

Similarly the stochastic component in the log-volatility transition equation follows a Normal distribution with mean 0 and variance g

$$\eta_t \sim N(0, g). \quad (\text{C.12})$$

This setup constitutes a non-linear state-space model, as the state variable h_t is not linear in the observation

equation. The model is estimated using Bayesian methods, specifically a Metropolis-within-Gibbs sampler, using the algorithm of Carter and Kohn (1994) to extract the path for all the elements in B_t in every iteration.²⁹

Following Primiceri (2005), the prior for Q follows an inverse Wishart distribution ($Q \sim IW(Q_0, T_0)$) with scale matrix $Q_0 = (Q_{OLS} \times T_0 \times k)$, and T_0 degrees of freedom, where Q_{OLS} is the covariance matrix from an Ordinary Least Squares (OLS) regression of the Phillips curve on a training sample, T_0 is the number of observations in the training sample, and k is a scaling factor. The training sample spans 1966Q1–1979Q4 ($T_0 = 56$), and the value of k was set to 0.01, which is standard in the literature (see Primiceri, 2005; Cogley, 2005; Cogley & Sargent, 2005). A higher k reflects the prior belief of greater time-variation. Setting $k^* = 5k$ results in more changes in the parameters within B , while $k^* = \frac{k}{5}$ produces smoother dynamics, although in both cases the results remain qualitatively similar to those from the baseline settings.

Similarly, the prior for g , the variance of shocks to log volatility, follows the inverse Gamma distribution ($g \sim IG(\frac{V}{2}, \frac{S}{2})$), with prior degrees of freedom $V = 5$ and scale $S = 0.5$. This prior incorporates the belief that volatility shocks to inflation were historically large but places some uncertainty around this belief.

The estimation of this model proceeds in the following sequence:

1. Sample the process h_t

This procedure is derived in Jacquier et al. (1994) and Jacquier, Polson and Rossi (2004), which involves specifying the distribution for h_t conditional on h_{t-1}, h_{t+1} and the data Y_t as the product of Normal and log-Normal densities:

$$f(h_t | h_{t-1}, h_{t+1}, Y_t) = h_t^{-0.5} \exp\left(\frac{-\varepsilon_t^2}{2h_t}\right) \times h_t^{-1} \exp\left(\frac{-(\ln h_t - \mu)^2}{2\sigma_h}\right), \quad (\text{C.13})$$

where $\mu = \frac{\ln h_{t+1} + \ln h_{t-1}}{2}$ and $\sigma_h = \frac{g}{2}$. An independence Metropolis-Hastings algorithm was used to draw from the candidate density, which is the second term in (C.13). To sample the initial value of h_t , i.e. h_0 , the authors suggest assuming a prior for $\ln h_0$: $\ln h_0 \sim N(\bar{\mu}, \bar{\sigma})$ whose posterior density is given by:

$$f(h_0 | h_1) = h_0^{-1} \exp\left(\frac{-(\ln h_0 - \mu_0)^2}{2\sigma_0}\right), \quad (\text{C.14})$$

²⁹See Jacquier et al. (1994) and Blake and Mumtaz (2012) for a discussion of Bayesian inference in such models.

where $\sigma_0 = \frac{\bar{\sigma}g}{\bar{\sigma}+g}$ and $\mu_0 = \sigma_0 \left(\frac{\bar{\mu}}{\bar{\sigma}} + \frac{\ln h_1}{g} \right)$, which require a value for $\bar{\sigma}$, $\bar{\mu}$, h_1 and g . The hyperparameter $\bar{\mu}$ is estimated as the log of the variance of the residuals from an OLS regression, while $\bar{\sigma}$ is set to a high number to reflect the uncertainty around this estimate. In practice values for $\bar{\sigma}$ between 10 and 200 do not affect the results in a meaningful way. An estimate of the process h_t is obtained as the sequence of squared changes in the dependent variable in Eq. (C.2) (inflation), and the value for h_1 is simply the first number in this series. The value of g , the variance of the process $\ln h_t$, is initialised to 1. The process to sample the sequence $h_{t=1}$ to h_{T-1} (conditional on g and B_t) involves sampling from the density in (C.14) with $h_0 = h_t$, $\mu = \frac{\ln h_{t+1} + \ln h_{t-1}}{2}$ and $\sigma_h = \frac{g}{2}$. This draw is retained with probability

$$\chi = \min \left(\frac{h_{t,new}^{-0.5} \exp\left(\frac{-\varepsilon_t^2}{2h_{t,new}}\right)}{h_{t,old}^{-0.5} \exp\left(\frac{-\varepsilon_t^2}{2h_{t,old}}\right)}, 1 \right) > u \quad (C.15)$$

for $u \sim U(0,1)$. That is, if χ is greater than a draw between 0 and 1 from the uniform distribution, the new draw $h_{t,new}$ is accepted, otherwise the previous draw is retained. Finally, the value for h_T is sampled from the same density in (C.14) with $\mu = \ln h_{t-1}$ and $\sigma_h = g$ and the same acceptance probability is calculated.

2. **Sampling g**

For each full sequence h_t constructed above, the residuals η_t from the transition Eq. (C.9) are calculated and a value for g is drawn from the inverse Gamma distribution with degrees of freedom $T + V$ and scale $\Sigma\eta_t^2 + S$.

3. **Extracting B_t**

Conditional on h_t and Q , the processes for the time varying parameters are drawn using the Carter-Kohn algorithm (Carter & Kohn, 1994).

4. **Sampling Q**

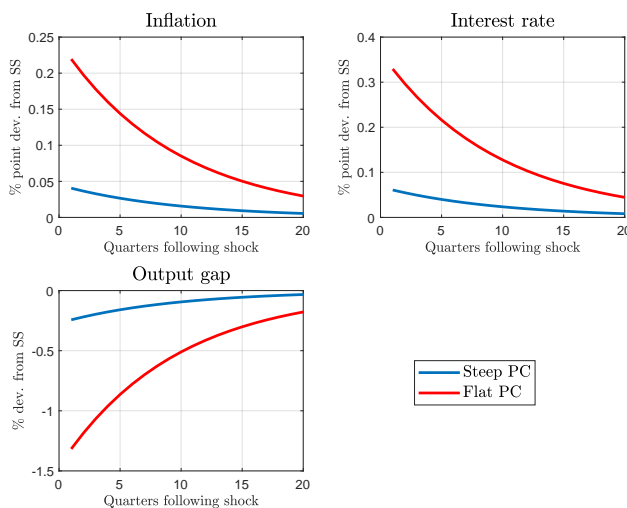
Conditional on B_t , Q is sampled from the inverse Wishart distribution with scale matrix $(B_t - B_{t-1})'(B_t - B_{t-1}) + Q_0$ and degrees of freedom $T + T_0$.

Estimation is based on 20,000 repetitions of steps 1 to 4 above, from which the first 5,000 draws are discarded as burn-in draws. The posterior distributions of the parameters are based on the retained draws.

Appendix D Phillips Curve Slope and Monetary Policy Effectiveness

In this appendix, I show simulations from a benchmark closed economy New Keynesian DSGE model as in Walsh (2017, Ch.8), to which I add an exogenous and persistent shock process to the log-linearized New Keynesian Phillips curve.³⁰ The central bank follows a simple Taylor rule for the interest rate, and is only concerned with inflation stabilization. Fig. D.1 shows the dynamics of two economies, which are the same, except the Phillips curve in one is flatter than in the other, and are both hit by the same exogenous inflation shock. Variables are shown in deviation from their steady state (SS) values.

Figure D.1: Simulations: An exogenous inflation shock.



The Phillips curve slope is affected by changing the Calvo probability that a firm in any period cannot change prices ($\omega \in \{0.5, 0.75\}$). The central bank in the economy with a flatter Phillips curve ($\omega = 0.75$) will need to raise rates by more, causing the real interest rate to rise, and the economy to experience a larger (negative) output gap.³¹ Inflation is nevertheless still higher, demonstrating that monetary policy in this economy is therefore, *ceteris paribus*, less effective.

³⁰The shock follows an AR(1) process with persistence $\rho = 0.9$. For more details on the model and calibration refer to the citation.

³¹The output gap is defined as the difference between supply and the level of output that is achieved under fully flexible prices.