

Einstein-Podolsky-Rosen correlation in Kerr-Newman spacetimeJackson Said¹ and Kristian Zarb Adami²¹*Physics Department, University of Malta, Msida, Malta*²*Physics Department, University of Oxford, Oxford, OX1 3RH, United Kingdom*

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The EPR correlation has become an integral part of quantum communications as has general relativity in classical communication theory, however when combined an apparent deterioration is observed for spin states. We consider appropriate changes in directions of measurement to exploit full EPR entanglement for a pair of particles and show that it can be deduced only up to the outer event horizon of a Kerr-Newman black hole, even in the case of a freely falling observer.

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I. INTRODUCTION

For some of the founders of quantum mechanics one of the troubling parts was the spooky action-at-a-distance. Originally this was thought up by Einstein, Podolsky, Rosen (EPR) in an attempt to challenge certain aspects of quantum theory at the time. Contrary to its original design, it is now the cornerstone of modern mainstream quantum physics, from cryptography to quantum computation; thus it is important to understand as many of the properties of quantum communications as possible. In particular, it is of importance to fully understand the effect of spacetime curvature on EPR states. This is completely different from classical information transport. In this scenario the space between observer and emitter does not have an effect on the transmission which means that only local spacetime effects matter when making measurements on transmissions.

In this paper, we apply the Terashima and Ueda [1] approach to the spacetime background of a Kerr-Newman black hole. In general relativity, the spin of a particle becomes deformed in all but the Minkowski spacetime. We present a method to extract the complete EPR correlation of two particles in a Bell state in Kerr-Newman geometry, ignoring helicity of infalling particles. These particles are defined locally and so suffer a precession of their spin component due an acceleration by an external force and the difference in the local inertial frame at different points about the given geometry. Taking these differences to arise from a continuous succession of local lorentz transformations (LLT), the spin component can be calculated since it precesses in accord with the Wigner rotation. It is therefore not a trivial task to describe the motion of a particle using quantum mechanics near a Kerr-Newman black hole because the Poincaré group does not act intuitively in this region.

This paper is organized as follows, in Sec. II we derive the spin precession in the Kerr-Newman background for an observer at infinity. Then in Sec. III we consider the EPR correlation for a pair of fully entangled particles. In an attempt to remove the coordinate singularities from the

derived angle we then calculate the spin precession for an infalling observer in Sec. IV. In Sec. V we discuss Bell's inequality for the observers at infinity and the infalling observer. Finally in Sec. VI we summarize our results.

II. KERR-NEWMAN DISTORTION

The most general vacuum solution of Einstein's field equations for black holes is the Kerr-Newman metric, any further complications requires one to consider hairy black holes. In this paper we take the Minkowski signature to be $\eta = \text{diag}(-, +, +, +)$ and use geometric units ($G = 1 = c$). Latin letters are run over the four inertial labels (0, 1, 2, 3) and Greek letters over the four general coordinate labels. Also repeated indices are to be summed. Then the metric for the Kerr-Newman spacetime in *Boyer-Lindquist coordinates* (t, r, θ, ϕ) for an observer at infinity is given by

$$\begin{aligned} ds^2 &= g_{\mu\nu}(x)dx^\mu dx^\nu \\ &= -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \\ &\quad + \frac{\sin^2\theta}{\Sigma}(adt - (r^2 + a^2)d\phi)^2 \end{aligned} \quad (1)$$

where

$$\Sigma(r) = r^2 + a^2\cos^2\theta \quad \Delta(r) = r^2 - 2Mr + a^2 + Q^2 \quad (2)$$

and Q , a and M the charge, angular momentum per unit mass and mass of the black hole, respectively. For the most part, the explicit statement of the dependence of Eq. (2) on r will be assumed and so it will be suppressed for brevity unless otherwise stated. Together these three parameters form a family containing a set of all classical black holes. The coordinate system breaks down twice for this metric, firstly for the radial part when $\frac{1}{g_{rr}} = 0$ then for the time part of the metric when $g_{tt} = 0$,

$$r_{\pm} = M \pm \sqrt{M^2 - a^2 - Q^2},$$

where $r_{+(-)}$ is the outer (inner) event horizon. In order to

relate local and global coordinates one must introduce a tetrad (or vierbein) formalism, which we have chosen to be

$$\begin{aligned} e_0^\mu(x) &= \left(\frac{a^2 + r^2}{\sqrt{\Delta\Sigma}}, 0, 0, -\frac{a \sin(\theta)}{\sqrt{\Sigma}} \right) \\ e_1^\mu(x) &= \left(0, \sqrt{\frac{\Delta}{\Sigma}}, 0, 0 \right) \\ e_2^\mu(x) &= \left(0, 0, \frac{1}{\sqrt{\Sigma}}, 0 \right) \\ e_3^\mu(x) &= \left(-\frac{a}{\sqrt{\Delta\Sigma}}, 0, 0, \frac{1}{\sin(\theta)\sqrt{\Sigma}} \right). \end{aligned} \quad (3)$$

This describes a local inertial frame for a particle, in this case they are rotating with respect to an observer at infinity. In this respect, Eq. (3) and its inverse are central to relating local and global coordinates. For example, the momentum in a local inertial frame $p^a(x)$ is $e^a_\mu(x)p^\mu(x)$ in relation to its global definition. It is imperative that properties can be related locally and globally on a manifold. Henceforth

$$\begin{aligned} u^t(x) &= N^{-1} \cosh(\zeta) = r \sqrt{\frac{(a^2 + r^2)^2 - a^2\Delta}{a^2(2Mr - Q^2)^2 + (\Delta - a^2)((a^2 + r^2)^2 - a^2\Delta)}} \cosh(\zeta) \\ u^\varphi(x) &= -N^{-1} N^\varphi \cosh(\zeta) + \frac{\sinh(\zeta)}{\sqrt{g_{\varphi\varphi}}} = \frac{r}{\sqrt{(a^2 + r^2)^2 - a^2\Delta}} \left[\frac{a(2Mr - Q^2) \cosh(\zeta)}{\sqrt{a^2(2Mr - Q^2)^2 + (\Delta - a^2)((a^2 + r^2)^2 - a^2\Delta)}} + \sinh(\zeta) \right] \end{aligned} \quad (5)$$

where ζ is the rapidity in the local inertial frame defined by

$$v = \tanh(\zeta) \quad (6)$$

and

$$N = \frac{1}{\sqrt{-g^{tt}}} \quad N^\varphi = \frac{g_{t\varphi}}{g_{\varphi\varphi}} \quad (7)$$

$$\begin{aligned} a^r(x) &= -\frac{1}{r^3(a^2(r(2M+r) - Q^2) + r^4)^2} [a\sqrt{\Delta} \sinh(2\zeta)(a^2(Mr - Q^2) + r^2(3Mr - 2Q^2))(a^2(2Mr - Q^2 + r^2) + r^4) \\ &\quad + \frac{1}{2}r(M-r)(a^2(2Mr - Q^2 + r^2) + r^4)^2 + \frac{1}{2} \cosh(2\zeta)(2a^4(Q^2 - Mr) + a^2(r^2(6M^2 - 3Mr + r^2) \\ &\quad + Q^2r(3r - 7M) + 2Q^4) + r^4(r(r - 3M) + 2Q^2))(a^2(2Mr - Q^2 + r^2) + r^4)]. \end{aligned} \quad (8)$$

The change in local inertial frame, $\chi^a_b(x) = -u^\nu(x)\omega_\nu^a_b(x) = u^\nu(x)e_b^\mu(x)\nabla_\nu e^a_\mu(x)$, between different points is now shown to be a boost along the 1-axis and a rotation about the 2-axis. In particular, $\chi^a_b(x)$ is a local quantity that relates the total change in local inertial frames along the path of the particle. Formally this quantity arises by the procedure that follows, in particular, one first notes that a particle with four momentum $p^a(x)$ will suffer a change $\delta p^a(x)$ in its four momentum when moving between points x^μ and $x^\mu + u^\mu d\tau$ on the curved space-

terms not shown are vanishing unless explicitly stated otherwise.

A straightforward, but tedious calculation yields the components of the connection one-form [2], $\omega_\mu^a_b(x) = e^a_\nu(x)\nabla_\mu e_b^\nu(x)$, these are a spin connection. These are very involved equations in the Kerr-Newman spacetime. One notes, however, that the nonzero Schwarzschild symmetry is preserved and extended to some other pairs of elements. The following are the nonvanishing one-forms restricted to the equatorial plane,

$$\begin{aligned} \omega_{t^1_1}(x) &= \omega_{t^1_0}(x) & \omega_{\theta^1_2}(x) &= -\omega_{\theta^2_1}(x) \\ \omega_{\varphi^1_3}(x) &= -\omega_{\varphi^3_1}(x) & \omega_{t^1_3}(x) &= -\omega_{t^3_1}(x) \\ \omega_{\varphi^0_1}(x) &= \omega_{\varphi^1_0}(x) & \omega_{r^0_3}(x) &= \omega_{r^3_0}(x) \\ \omega_{r^0_0}(x) &= \omega_{r^3_3}(x). \end{aligned} \quad (4)$$

A particle in a circular orbit on the equatorial plane ($\theta = \pi/2$) is now considered, with a radius $r(>r_+)$ and constant velocity. The four velocity of such a particle is given by

subject to the constraint $u^\nu u_\nu = -1$ as stated in [3], where N is the lapse function and N^φ is the nonvanishing component of the shift vector field.

This is not however a geodesic, so an external force must be applied to the particle to counter the gravitational field. The acceleration, $a^\mu(x) = u^\nu(x)\nabla_\nu u^\mu(x)$, of such a force will be

time, where $d\tau$ is the infinitesimal proper time between events. The local change in four momentum corresponds to a global change by

$$\begin{aligned} \delta p^a(x) &= \delta(p^\mu(x)e^a_\mu(x)) \\ &= \delta p^\mu(x)e^a_\mu(x) + p^\mu(x)\delta e^a_\mu(x). \end{aligned} \quad (9)$$

The change in four momentum is given by taking the Fermi-Walker derivative

$$\begin{aligned}\delta u^\mu(x) &= -[u^\mu(x)u_\mu(x)a^\nu(x) - u_\nu(x)u^\nu(x)a^\mu(x)]d\tau \\ &= -[u^\mu(x)a_\nu(x) - a^\mu(x)u_\mu(x)]u^\nu(x)d\tau\end{aligned}\quad (10)$$

Thus

$$\delta p^\mu = -\frac{1}{m}[p^\mu(x)a_\nu(x) - a^\mu(x)p_\mu(x)]p^\nu(x)d\tau\quad (11)$$

and the change in local inertial frame by

$$\begin{aligned}\delta e^a_\mu(x) &= u^\mu(x)d\tau\nabla_\nu e^a_\mu(x) \\ &= u^\mu(x)\omega_\nu^a_b(x)e^b_\mu(x)d\tau \\ &= \chi^a_b(x)e^b_\mu(x)d\tau\end{aligned}\quad (12)$$

where we have made use of the identity $e^a_\mu(x)e_b^\mu(x) = \delta^a_b$ and indeed $\chi^a_b = -u^\nu(x)\omega_\nu^a_b(x)$ with $\omega_\mu^a_b(x) = -e_b^\nu(x)\nabla_\mu e^a_\nu(x) = e^a_\nu(x)\nabla_\mu e_b^\nu(x)$. Combing the above into Eq. (9) gives

$$\delta p^a(x) = \lambda^a_b(x)p^b(x)d\tau\quad (13)$$

where

$$\lambda^a_b(x) = -\frac{1}{m}[a^a(x)p_b(x) - p^a(x)a_b(x)] + \chi^a_b(x)\quad (14)$$

The explicit values for $\chi^a_b(x)$ turn out to be

$$\begin{aligned}\chi^0_1(x) &= \chi^1_0(x) \\ &= \frac{\Delta}{a^4 r^2 (\Delta - a^2) + a^2 r^3 (4Mr(M - r) + 2r^3 - Q^2(a^2 + 2Mr - r^2) + \Delta(2M - r)) + r^6 (\Delta - a^2)} \\ &\quad \times \left[(2a^2 r^2 + r^4) \cosh(\zeta)(Q^2 - Mr) \sqrt{\frac{(a^2 + r^2)^2 - a^2 \Delta}{((a^2 + r^2)^2 - a^2 \Delta)(r(r - 2M) + Q^2) + a^2(Q^2 - 2Mr)^2}} \right. \\ &\quad - \frac{1}{((a^2 + r^2)^2 - a^2 \Delta)^{3/2}} \left[a(a^4(-r(r - 3M) + Q^2)) + a^2(Q^2(a^2 + 3Mr + Q^2 + r^2) \right. \\ &\quad \left. + r(\Delta(r - 3M) - r(6M^2 - 3Mr + r^2)) - Q^4) + r^4(r(r - 5M) + 3Q^2)) \right. \\ &\quad \left. \times \left(\frac{a \cosh(\zeta)((a^2 + r^2)^2 - a^2 \Delta)(2Mr - Q^2)}{\sqrt{((a^2 + r^2)^2 - a^2 \Delta)(r(r - 2M) + Q^2) + a^2(Q^2 - 2Mr)^2}} + \sinh(\zeta)(a^2(2Mr - Q^2 + r^2) + r^4) \right) \right] \quad (15)\end{aligned}$$

$$\begin{aligned}\chi^1_3(x) &= -\chi^3_1(x) \frac{\sqrt{\Delta}}{r^4} \left[\cosh(\zeta) \left(\frac{a(2Mr - Q^2)(2a^2(Q^2 - Mr) + r^4)}{\sqrt{(a^2 + r^2)^2 - a^2 \delta}} + 2a(Mr - Q^2) \right) \right. \\ &\quad \left. \times \sqrt{\frac{(a^2 + r^2)^2 - a^2 \Delta}{((a^2 + r^2)^2 - a^2 \delta)(-2Mr + Q^2 + r^2) + a^2(Q^2 - 2Mr)^2}} + \sinh(\zeta)(a^2(2Mr - Q^2 + r^2) + r^4) \right] \quad (16)\end{aligned}$$

The LLT is defined by Eq. (14), where m is taken to be the mass of the particle species under consideration. This is an infinitesimal LLT since $\lambda_{ab} = -\lambda_{ba}$. The LLT then turns out to have only four nonvanishing terms which can be separated in to two symmetries,

$$\lambda^0_1(x) = \lambda^1_0(x) \quad \lambda^1_3(x) = -\lambda^3_1(x). \quad (17)$$

The explicit expressions are lengthy and so are represented graphically in Fig. 1 for some average parameters of a Kerr-Newman black hole.

This turns out to also be a boost along the 1-axis and a rotation about the 2-axis, as in Schwarzschild black hole case. With constant momentum $p^a(x) = (m \cosh(\zeta), 0, 0, m \sinh(\zeta))$ pointing in the 3-axis, the change of the spin becomes a rotation as follows.

After an infinitesimal proper time $d\tau$, the particle moves in the 3-axis by an amount, $\delta\phi = u^\varphi d\tau$. Over this the momentum in the local inertial frame transforms under the LLT, $\Lambda^a_b(x) = \delta^a_b + \lambda^a_b(x)d\tau$, which corresponds to a

unitary operator that acts on the state of the particle. This operator changes the spin, in particular, it acts like the unitary matrix U in

$$U(\Lambda(x))|p^a(x), \sigma; x\rangle = \sum_{\sigma'} D_{\sigma'\sigma}^{(1/2)}(W(x))|\Lambda p^a(x), \sigma'; x\rangle, \quad (18)$$

as in [4], where $W(x) = W(\Lambda(x), p(x)) = L^{-1}(\Lambda p)\Lambda L(p)$ is a local Wigner rotation. It follows that $W^a_b(x) = W^a_b(\Lambda(x), p(x)) = [L^{-1}(\Lambda p)\Lambda L(p)]^a_b$, with a standard Lorentz transform (LT) $L^a_b(p)$ defined by

$$\begin{aligned}L^0_0(p) &= \gamma, & L^0_i(p) &= L^i_0(p) = \frac{p^i}{m}, \\ L^i_k(p) &= \delta_{ik} + (\gamma - 1) \frac{p^i p^k}{|\vec{p}|^2},\end{aligned}\quad (19)$$

where

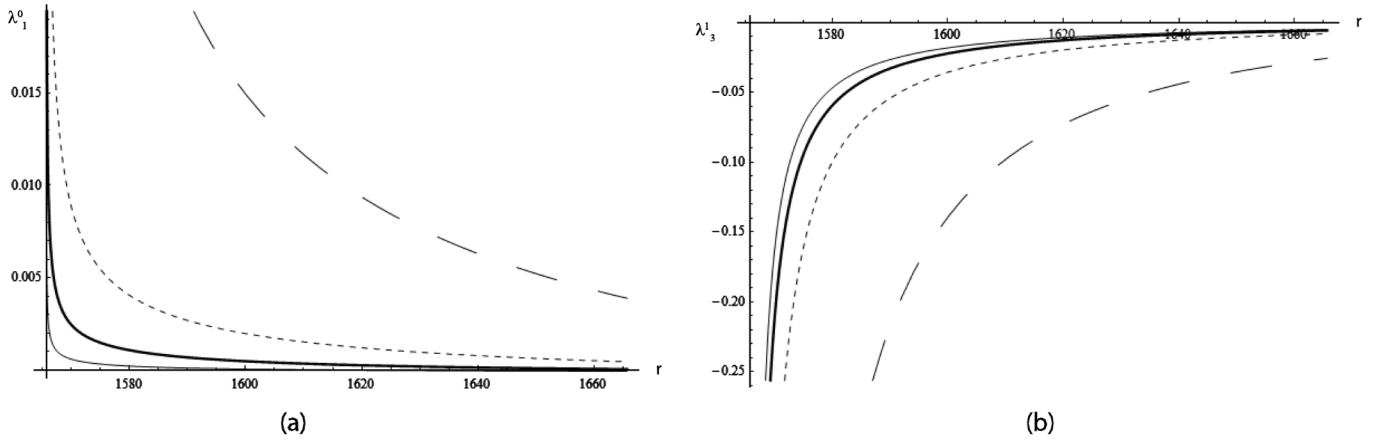


FIG. 1. The infinitesimal LLT for a Kerr-Newman black hole with parameters $M = 1000$, $a = 0.8M$, $Q = 0.2M$, and $v = 0.3, 0.5, 0.7$, and 0.9 for the black, thick, dashed, and thick dashed lines, respectively.

$$\gamma = \frac{\sqrt{|\vec{p}p|^2 + m^2}}{m} \quad \text{and} \quad i, k \in \{1, 2, 3\} \quad (20)$$

which turns out to be a rotation about the 2-axis though the angle $\vartheta^1_3(x) = -\vartheta^3_1(x)$, as will be shown in Eq. (30). This is yet another tediously long equation, so we employ graphical techniques in Fig. 2 to illustrate that a singularity is evident at the event horizon r_+ . This is expected since an observer at infinity can not make measurements at and beyond an event horizon.

It is important here to note that $\vartheta^a_b(x) \neq \lambda^a_b(x) \neq \chi^a_b(x) \neq \varphi^a_b(x)$, where

$$\varphi^1_3(x) = -\varphi^3_1(x) = u^\varphi(x), \quad (21)$$

which is a trivial change in the local inertial frame. The former set of inequalities results from the boost part of $\lambda^a_b(x)$, the acceleration and hence the force of the particle, and the curvature of the spacetime, respectively.

Considering now the special case when $M, a, Q \rightarrow 0$, the Minkowski spacetime is again recovered with

$$\chi^1_3(x) = -\chi^3_1(x) = \frac{\sinh(\zeta)}{r} \quad (22)$$

and

$$\vartheta^1_3(x) = -\vartheta^3_1(x) = \frac{\cosh(\zeta) \sinh(\zeta)}{r} \quad (23)$$

and most importantly for this situation, the Thomas precession of the spin, i.e. [Eqs. (23) and (22)] for $v \ll 1$, remains

$$[\vartheta^3_1(x) - \chi^3_1(x)]d\tau \sim -\frac{va}{2c^2}dt \quad (24)$$

where $a \equiv |a^r(x)| = c^2 \sinh^2(\frac{\zeta}{r})$. In this limit the change in the local inertial frame is just a rotation in the 2-axis through the angle $\chi^1_3(x)$ and the change in spin is also a rotation about the 2-axis given by $\vartheta^1_3(x)$. The difference between the latter two terms gives the spin precession in the low velocity limit per unit $dt = d\tau \cosh(\zeta)$

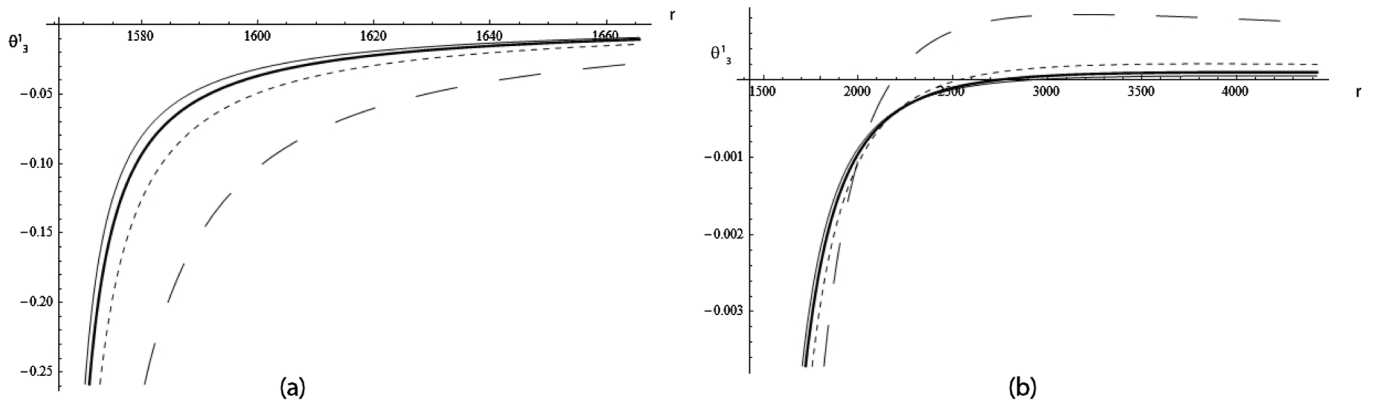


FIG. 2. The rotation angle for a Kerr-Newman black hole with parameters $M = 1000$, $a = 0.8M$, and $Q = 0.2M$ for the figure on the left and, $M = 1000$, $a = 0.9M$, and $Q = 0.1M$ for the one on the right, and both with $v = 0.3, 0.5, 0.7$, and 0.9 for the black, thick, dashed, and thick dashed lines, respectively.

III. EPR CORRELATION

We now move on to the actual gedanken experiment of this paper. Two observers are considered at azimuthal angles $\pm\Phi$ with $r > r_+$ and an EPR source generator at $\Phi = 0$ as seen in Fig. 3. The observers and EPR source are static in the local inertial frame Eq. (3), i.e. they are relatively static. The EPR source generates a pair of maximally entangled particles at $\Phi = 0$ which are sent in opposite directions with constant four momentum $p^\alpha_\pm = (m \cosh(\zeta), 0, 0, \pm m \sinh(\zeta))$. The pure state can be described by the Bell state,

$$\frac{1}{\sqrt{2}}[|p^a_+, \uparrow; 0\rangle|p^a_-, \downarrow; 0\rangle - |p^a_+, \downarrow; 0\rangle|p^a_-, \uparrow; 0\rangle], \quad (25)$$

where for notational simplicity we put the ϕ -coordinate in the state argument. After a proper time Φ/u^ϕ (since there is zero azimuthal acceleration) the particles reach their respective observers. An induced spin precession is observed because of the curved nature of the spacetime background, this is called the local Wigner rotation and in this case is given by

$$W^a_b(\pm\phi, 0) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\Theta) & 0 & \pm \sin(\Theta) \\ 0 & 0 & 1 & 0 \\ 0 & \mp \sin(\Theta) & 0 & \cos(\Theta) \end{pmatrix}, \quad (26)$$

where

$$\Theta = \Phi \frac{r}{\sinh(\zeta)} \vartheta^1_3, \quad (27)$$

which is similar but not identical to the case for when $a = 0 = Q$ since the Θ is different. The induced precession arises from Eq. (18), which moves in accord to the local Wigner rotation. For an infinitesimal LT

$$\Lambda^a_b(x) = \delta^a_b + \lambda^a_b(x)d\tau. \quad (28)$$

The infinitesimal Wigner rotation corresponds to

$$W^a_b(x) = \delta^a_b + \vartheta^a_b(x)d\tau, \quad (29)$$

where $\vartheta^0_0(x) = 0 = \vartheta^0_i(x) = \vartheta^i_0(x)$ and

$$\vartheta^i_k(x) = \lambda^i_k(x) + \frac{\lambda^i_0 p_k(x) - \lambda_{k0}(x) p^i(x)}{p^0(x) + m}. \quad (30)$$

In [1] it is shown that this is entirely equivalent to

$$\frac{1}{\sqrt{2}}[\cos(\Theta)(|p^a_+, \uparrow; \Phi\rangle|p^a_-, \downarrow; -\Phi\rangle - |p^a_+, \downarrow; \Phi\rangle|p^a_-, \uparrow; -\Phi\rangle) + \sin(\Theta)(|p^a_+, \uparrow; \Phi\rangle|p^a_-, \uparrow; -\Phi\rangle + |p^a_+, \downarrow; \Phi\rangle|p^a_-, \downarrow; -\Phi\rangle)] \quad (34)$$

as visible from Fig. 3. This is where the entanglement appears to breakdown because of the presence of the spin triplet state in the state description. First of all however we remove the trivial rotation that rotates the local inertial frames by $\pm\Phi$ as in Eq. (21). about the 2-axis at $\phi = \pm\Phi$. This is achieved through rotating the bases by $\mp\Phi$ about the 2-axis at $\phi = \pm\Phi$ respectively,

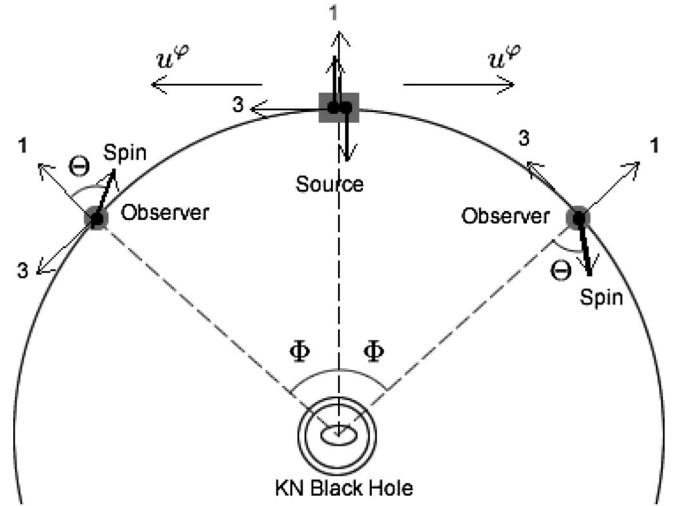


FIG. 3. The EPR setup in the Kerr-Newman spacetime. Gray circles are the observers and the gray square is the EPR source.

$$\begin{aligned} W^a_b(x_f, x_i) &= \lim_{N \rightarrow \infty} \prod_{k=0}^N \left[\delta^a_b + \vartheta^a_b(x_{(k)}) \frac{h}{N} \right] \\ &= T \exp \left[\int_{\tau_i}^{\tau_f} \vartheta^a_b(x(\tau)) d\tau \right], \end{aligned} \quad (31)$$

where T is a time ordering operator and the exponent is the Taylor series exponent of the whole matrix. In this situation however the time ordering term is not needed since $\vartheta^a_b(x)$ is constant during the motion of the particle (assuming that mass is not injected in to the system during this time).

The significant terms of the spin representation, i.e. the unity ones, can be condensed in to a Pauli representation as follows

$$\begin{aligned} D_{\sigma'\sigma}^{(1/2)}(W(x)) &= I + \frac{i}{2} [\vartheta_{23}(x)\sigma_x + \vartheta_{31}(x)\sigma_y \\ &\quad + \vartheta_{12}(x)\sigma_z] d\tau, \end{aligned} \quad (32)$$

where with the current parameters gives,

$$D_{\sigma'\sigma}^{(1/2)}(W(\pm\Phi, 0)) = \exp\left(\mp i \frac{\sigma_y}{2} \Theta\right). \quad (33)$$

Thus when the particle pair reach their respective observers the state is described by

$$|p_{\pm}^a, \uparrow; \pm\Phi\rangle' = \cos\left(\frac{\Phi}{2}\right)|p_{\pm}^a, \uparrow; \pm\Phi\rangle \pm \sin\left(\frac{\Phi}{2}\right)|p_{\pm}^a, \downarrow; \pm\Phi\rangle, \quad (35)$$

$$|p_{\pm}^a, \downarrow; \pm\Phi\rangle' = \mp \sin\left(\frac{\Phi}{2}\right)|p_{\pm}^a, \uparrow; \pm\Phi\rangle + \cos\left(\frac{\Phi}{2}\right)|p_{\pm}^a, \downarrow; \pm\Phi\rangle. \quad (36)$$

As in [1],

$$\frac{1}{\sqrt{2}}[\cos(\Delta)(|p_{+}^a, \uparrow; \Phi\rangle|p_{-}^a, \downarrow; -\Phi\rangle - |p_{+}^a, \downarrow; \Phi\rangle|p_{-}^a, \uparrow; -\Phi\rangle) + \sin(\Delta)(|p_{+}^a, \uparrow; \Phi\rangle|p_{-}^a, \uparrow; -\Phi\rangle + |p_{+}^a, \downarrow; \Phi\rangle|p_{-}^a, \downarrow; -\Phi\rangle)] \quad (37)$$

is found to describe the state, where

$$\Delta = \Theta - \Phi = \Phi \left[\frac{r}{\sinh(\zeta)} \vartheta^1{}_3 - 1 \right]. \quad (38)$$

Since the trivial rotation has been removed it is clear that a real deterioration of the perfect correlation between the spins is being observed, however only local unitary operations have been applied and the entanglement is invariant under unitary operations. Hence this must be an affect of the acceleration and gravity. If the pure state can be recovered then quantum computations may still be done while in the presence of a gravitational field. In particular, the respective observers at $\phi = \pm\Phi$ must take measurements at an angle $\mp\Theta$ in their local inertial frames. Since $\vartheta^a{}_b(x) \neq \chi^a{}_b(x)$, a parallel transport would not reproduce this angle. Hence by transforming in the appropriate direction, the full EPR correlation may still be recovered.

Now it was found that Δ is positive for $r \rightarrow \infty$ to a radius r_0 very close to the outer horizon r_+ . As r becomes smaller than r_0 and furthermore $r \downarrow r_+$, $\Delta \rightarrow -\infty$ and thus to extract the perfect EPR correlation each observer would require infinite accuracy in the measurement that even a small error would lead to a mixed state element.

IV. THE INFALLING OBSERVER

We adopt the Doran [5] metric to remove the coordinate singularities of the metric. The observer can now fall through the apparent singularities of the Kerr-Newman spacetime observed by an observer at infinity. For this observer the line element is given by

$$ds^2 = -dT^2 + \left[\frac{\Sigma}{\Omega} dR + b \frac{\Omega}{\Sigma} (dT - a \sin^2(\theta) d\phi) \right]^2 + \Sigma^2 d\theta^2 + \Omega^2 \sin^2(\theta) d\phi^2, \quad (40)$$

where

$$\Omega = (R^2 + a^2)^{1/2} \quad b = \frac{(2MR - Q^2)^{1/2}}{\Omega} \quad (41)$$

$$\Sigma = (R^2 + a^2 \cos^2(\theta))^{1/2}$$

and the time coordinate coincides with the proper time for the free fall observer. The vierbein is now chosen to be

$$\begin{aligned} \tilde{e}_0{}^\mu(x) &= (1, 0, 0, 0) \\ \tilde{e}_1{}^\mu(x) &= \left(b \frac{\Omega}{\Sigma}, \frac{\Sigma}{\Omega}, 0, -a b \sin^2(\theta) \frac{\Omega}{\Sigma} \right) \\ \tilde{e}_2{}^\mu(x) &= (0, 0, \Sigma, 0) \\ \tilde{e}_3{}^\mu(x) &= (0, 0, 0, \Omega \sin(\theta)). \end{aligned} \quad (42)$$

In the (t, r, θ, ϕ) coordinates the vierbein inherited the r_+ and r_- coordinate singularities, the above (T, R, θ, ϕ) also act as the metric does at those radii, and since the metric is singularity free there so is the vierbein. Now similarly to Eq. (5) we take a four velocity of the form,

$$\tilde{u}^T(x) = \tilde{N}^{-1} \cosh(\tilde{\zeta}) \quad (43)$$

$$\tilde{u}^\varphi(x) = \tilde{N}^{-1} \tilde{N}^\varphi \cosh(\tilde{\zeta}) + \frac{\sinh(\tilde{\zeta})}{\sqrt{g_{\phi\phi}}},$$

where $\tilde{\zeta}$ is the rapidity in the (T, R, θ, φ) local inertial frame. However, constraining the two free variables \tilde{N}^{-1} and \tilde{N}^φ by the normalization equation $u^\nu u_\nu = -1$ yields

$$\tilde{N} = \frac{a(Q^2 - 2MR)}{-2a^2MR + a^2Q^2 - a^2R^2 - R^4} \quad (44)$$

$$\tilde{N}^\varphi = \pm \frac{\sqrt{-2a^2RM + a^2Q^2 - a^2R^2 - R^4}}{R\sqrt{\Delta(R)}}. \quad (45)$$

The four velocity thus emerges out of the normalization condition. Now Eq. (45) clearly will be singular on both horizons since they are defined as the solution to $\Delta = 0$. In this way we find that even a freely falling observer can not extract the EPR correlation at and beyond the event horizon of a Kerr-Newman black hole. Thus no such observer can exist in this scenario which we attribute this to the strength of the frame dragging effects at the outer horizon since observers may be defined in the Schwarzschild black hole [1]. The result is a little surprising however it follows from the intrinsic nature of the Kerr-Newman black hole that no observer of this kind may be defined globally which is what would be required to make measurements on a bilocal property.

Furthermore, following the same method as with the observer at infinity, the local Wigner rotation also turns out not to be finite on and beyond the out horizon, which is

a clear consequence of the absence of observability and *not* the lost of the EPR correlation. Hence it is the inability of an observer, so defined, to make measurements that restrict the regions where the EPR correlation can be extracted successfully.

V. MAXIMUM VIOLATION OF BELL'S INEQUALITY

As in [1] we examined circularly moving particles in a local inertial frame. Similarly we measure the spin of one particle in the $(1, 0, 0)$ -direction (component Q) or in the $(0, 1, 0)$ -direction (component R) and the other particle in the $(-1, -1, 0)$ -direction (component S) or $(1, -1, 0)$ -direction (components T) in the local inertial frames $\phi = \Phi$ and $\phi = -\Phi$, respectively. In the Kerr-Newman geometry a decrease in the maximal violation of Bell's inequality is observed, in particular, as

$$\langle QS \rangle + \langle RS \rangle + \langle RT \rangle - \langle QT \rangle = 2\sqrt{2}\cos^2(\Theta). \quad (46)$$

This, however, still includes the trivial rotation Eq. (21) which could be causing the decrease in maximal violation of Bell's inequality. Rotating the components by $\mp\Phi$. The new spin components then become, $(\cos(\Phi), 0, -\sin(\Phi))$ -direction (component Q') or $(0, 1, 0)$ -direction (component R') for one particle and $\frac{1}{\sqrt{2}}(-\cos(\Phi), -1, -\sin(\Phi))$ -direction (component S') or $\frac{1}{\sqrt{2}}(\cos(\Phi), -1, \sin(\Phi))$ -direction (component T') for the other one. The violation of Bell's inequality still reduces from maximal as

$$\langle Q'S' \rangle + \langle R'S' \rangle + \langle R'T' \rangle - \langle Q'T' \rangle = 2\sqrt{2}\cos^2(\Delta). \quad (47)$$

This is due to the gravitational field and accelerations involved. Taking into account the general relativistic effect on spin measurements are thus take in the directions $(\cos(\Theta), 0, -\sin(\Theta))$ -direction or $(0, 1, 0)$ -direction for one particle and $\frac{1}{\sqrt{2}}(-\cos(\Theta), -1, -\sin(\Theta))$ -direction or $\frac{1}{\sqrt{2}}(\cos(\Theta), -1, \sin(\Theta))$ for the other one in the same respective local inertial frames. However, as the radius where the experiment takes place reduces to the outer horizon, a small error can build up which still causes maximal entanglement to be lost. This corresponds to a requirement of infinite accuracy in making measurements as the observer approaches infinitely close to the horizon.

Considering next the freely falling observer Eq. (42), the angle of precession is observed to become infinite on and beyond the outer event horizon r_+ since the observer loses the ability to take such measurements. Thus on the equatorial plane the EPR correlation cannot be extracted and so infers a region where information may be lost for such an observer making measurements locally.

VI. CONCLUSION

We considered the EPR correlation of two accelerated particles in a Kerr-Newman background and found that the correlation apparently decreases as seen in the directions of flat spacetime as does the degree of violation of Bell's inequality. We derived the Wigner rotation and showed that maximal violation of Bell's inequality can be achieved through appropriate coordinate transformations of the local inertial frames. In this new inertial frame, the EPR correlation can be extracted up to the outer event horizon, which is to be expected for an observer at infinity.

However, at the outer horizon r_+ and below, for both the observer at infinity and the free fall observer, the EPR correlation is unmeasurable. In particular, the rotation angle for an observer approaches negative infinity on both counts and so the correlation will not be extracted once both particles have gone over the outer event horizon. This occurs because the flow of spacetime itself does not allow the experimental setup required to extract the EPR correlation. Hence, due to frame dragging effects becoming so intense, one cannot achieve the relatively static condition required for extraction and so no further measurements may be made on the particles in question, which means that the information stored in their spin states will become irrecoverable. Despite the apparent loss of information as measured by such an observer, it is actually stored by the black hole up to the singularity where theory fails to predict what will happen.

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- [1] H. Terashima and M. Ueda, *J. Phys. A* **38**, 2029 (2005).
 [2] M. Nakahara, *Geometry, Topology and Physics* (Institute of Physics Publishing, Bristol, 1990).
 [3] D. Bini, C. Cherubini, A. Gerialico, and R. T. Jantzen, *Gen. Relativ. Gravit.* **40**, 1 (2008).

- [4] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 1995), Vol 1.
 [5] C.J.L. Doran, *Phys. Rev. D* **61**, 067503 (2000).