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Florenskij and Georg Cantor: Naming Infinity

It might seem surprising to talk about the relationship between a theologian and a mathematician. One deals with matters of faith while the other deals with hard, logical arguments — or not? The relationship might not seem so surprising if I could, in as non-technical terms as possible, explain Cantor's theory of infinite sets, the objections raised against it, and what an eminent defender of his theory said. I'll try to do this in these few minutes, without risking going out of point, because this is basically what Florenskij does in his 1904 paper *The symbols of the infinite (An essay on the ideas of G. Cantor)* (Italian translation),[1] and on which I was asked to comment for this session.

Cantor's cardinals

Cantor developed the theory of infinite cardinals and ordinals simultaneously, but I shall here deal briefly with cardinals since it is easier to give a nontechnical presentation. So, consider the totality of the positive integers,

1.2.3.. ..

and that of the even positive integers,

2.4.6.. ..

It should be clear (without going into a strict definition of what "infinite" means), that these are both infinite sets. It should also be clear that the second set is contained completely in the first set but there are many (in fact an infinity

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[1] Pavel A. Florenskij, "Simboli dell'infinito (Saggio sulle idee di G. Cantor)," in *Il Simbolo e la Forma: Scritti di Filosofia della Scienza*, ed. Natalino Valentini and Anatolij Gorelov (Torino: Bollati Boringhieri, 2007), 25-80.

of) integers in the first set not in the second. The first set should be larger than the second. In fact, it should be twice as large, should it not? But Cantor says that the two sets have the same size, or *cardinality*, because we can set up a one-one correspondence between the elements of the two sets: 1 in the first set corresponds to 2 in the second, 2 corresponds to 4, 3 corresponds to 6, and so on. Every number in the first set has one and only one mate in the second set under this correspondence, and every number in the second set has one and only one mate in the first set. Now this is strange: a set is in one-one correspondence with a proper subset (we shall see how this seeming contradiction can be turned on its head to give a definition). But this is only the beginning. In a slightly more complicated way one can show that there is a one-one correspondence between the positive integers and all the rational numbers (that is, integers and also fractions). The latter set seems to be so much larger than the first. The integers are discretely set apart whereas the rationals are dense, because between any two fractional numbers there is an infinity of other fractional numbers. But, rather surprisingly, Cantor tells us that their cardinalilities are equal, therefore one infinity is as large as the other. Even a little more technical to show, but equally true is that the set of all numbers (integers, rationals and irrationals — that is, those numbers that cannot be written as fractions or recurring decimals, such as the well-known 7r) between zero and one has the same cardinality as the set of all the numbers between zero and one hundred, or all the numbers without upper or lower limits!

A moment's thought might lead one to say that, of course, we are talking about infinity, and there is only one infinity, and it should make sense that all infinities are equally large. But here Cantor's theory took the first seemingly bizarre twist in 1874: he showed that there is no one-one correspondence between the set of integers, or the set of integers and rationals, and the set of all numbers. The cardinality of the latter set is a larger infinity than the cardinality of the former.

Several "infinities"

I think that this is where trouble might have started with Cantor's contemporaries. Yes, Cantor had the audacity to assign a symbol to infinity — to give it a name — but now this name has turned round to bite its inventor, because it has led to a multiplicity of infinities. Some theologians saw this as an argument in favour of pantheism. An eminent mathematician described his theories as "a grave disease" and another accused Cantor to be a "corrupter of youth." Years after Cantor's death in 1918, Wittgenstein was still raising difficult philosophical issues with the transfinite set theory.[2] But David Hilbert,[3] probably the foremost mathematician of his time, in 1926 used a religious metaphor to defend Cantor: *Out of the Paradise that Cantor has created for us, no one must be able to expel us.* So it is not that surprising that Floresnkij should be interested in Cantor's work both as a mathematician and a theologian; perhaps more surprising and to Florenskij's credit is that this happened in the early 1900s when there was still some dispute surrounding Cantor's work. But Cantor's work is particularly relevant to Florenskij's philosophy that naming a concept is akin to giving it life, as I shall try to explain.

Naming infinity

Let us, as Florenskij did in his paper, start with the Greek mathematicians' way of dealing with infinity. In their work we can distinguish between potential infinity and actual infinity. The Greeks always steered cautiously away from the latter. So, they said that given any straight line of finite length, it can always be extended to a longer (but still finite) length. Similarly, when Euclid shows that the number of primes is infinite what he does is to show that if you hypothesize a largest prime then he can always find a larger one. This is potential infinity. The Greeks never considered an infinite line, but only one that can be made as finitely long as required. And they never considered the totality of elements of an infinite set, but they used the fact that any finite subset of such a set can be made as large as required, but always remaining finite. I sometimes think that Zeno wanted to illustrate, through his famous paradoxes, the dangers of playing with actual infinity. For the Achilles and the Tortoise Paradox, I would be surprised if Zeno and his contemporaries did not possess the technical skill to sum what we call today a geometric progression and thereby "resolve" the paradox. But why should a sum of an infinite number of terms be finite? Maybe because the terms are becoming smaller? These would be reasonable questions which Zeno might have pondered. Again, I believe that from here it would have been an obvious next step to consider checking out this assertion by experimenting with the most obvious infinite sum whose terms become smaller, the harmonic series,

$1 + {}^{1}/_{2} + {}^{1}/_{3} + {}^{1}/_{4} + \dots$

And again, I am sure that the Greek mathematicians had the technical abilities to discover that assuming that this series has a sum leads one into trouble.

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[2] Morris Kline, *Mathematics: The Loss of Certainty* (London: Oxford University Press, 1982).

[3] David Hilbert, "Uber das unendliche," *Mathematische Annalen* 95 (1926): 161-190.

So, giving a name to something in mathematics is not an idle matter, especially where infinity is involved. If the listener wants a more prosaic example, then she could try this: find the area of a right-angled triangle whose sides are, respectively, 3cm, 4cm and 6cm long. If you let *A* be this area, then trying to manipulate *A* will give you absurd results, simply because a right-angled triangle with sides 3, 4 and 6 does not exist — in mathematics, giving names to unicorns does not bring them to life. But giving a name to something which works according to the rules of mathematics can turn magic into mathematical reality, even if counter-intuitive reality. (Here I am glossing over the issue of axiomatics raised by the relationship between naming and existence.)

And this is what Cantor did. He gave a name to actual infinity — and he was handed back with a hierarchy of infinities. And it seemed that he was sailing towards the rocks of mathematical contradiction. But it turned out that he was discovering a new continent, or a paradise, as Hilbert put it. That an infinite set is in one-one correspondence with a proper subset can be turned from a seemingly contradictory statement to a definition of what an infinite set is—in fact, after Cantor, it was realized that mathematics did not have an axiom to define an infinite set and one had to be devised. That there is a hierarchy of infinities gave mathematics the Continuum Hypothesis: that there is no set whose cardinality is between that of the integers and the real numbers. It turned out that either the truth or the falsity of this hypothesis can be assumed as a new axiom without disturbing the other accepted axioms of mathematics!

Cantor's "Paradise" in Florenskij's Words

The work of Cantor, especially carried out in the face of adversity for daring to give a name to actual infinity and opening what was then a Pandora's box, must have struck a strong chord with Florenskij, who was so close to Imiaslavie philosophy.[4] Hilbert might have used the word "Paradise" as a metaphor, intending no spiritual or metaphysical significance. But surely Florenskij was doing precisely that when he wrote, in his 1904 paper, "At what is he [Cantor] aiming? He is striving to create a "temple," to create the symbols for the Infinite. He wants to see the realization of the Divine forces, he wants to convince himself that this is possible, and he wants to do it as soon as possible. He must prove that the idea of the Transfinite is not intrinsically contradictory but it is legitimate and necessary."

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[4] Loren Graham and Jean-Michel Kantor, *Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity* (Cambridge, MA: Belknap Press, 2009).

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[5] Alexandre Borovik, "Being in Control," in *Understanding Emotions in Mathematical Thinking and Learning*, ed. Ulises Xolocotzin Eligio (New York: Academic Press, 2017), 77-96.

[6] S.S. Demidov and C.E. Ford, "On the Road to a Unified World View: Priest Pavel Florenskij — Theologian, Philosopher and Scientist," in *Mathematics and The Divine: A Historical Study*, ed. T. Koestner and L. Bergmans (Amsterdam: Elsevier, 2005), 595-612.