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Strong lensing as a test for conformal Weyl gravity

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ABSTRACT
Conformal Weyl gravity (CWG) predicts galactic rotation curves without invoking dark matter. This makes CWG an interesting candidate theory as an alternative to general relativity (GR). This removal of the necessity for dark matter arises because the exact exterior solution in CWG for a static, spherically symmetric source provides a linear potential γr, which can explain the observed galactic rotation curves with a value for γ of $\sim+10^{-26}\ \mathrm{m}^{-1}$. Previous work has also shown that CWG can explain lensing observations, but with $\gamma \sim -10^{-26}\ \mathrm{m}^{-1}$, in order to ensure converging rays of light rather than diverging ones. Even though different expressions for the angle of deflection have been derived in CWG in order to remove this inconsistency, this article shows that the γ required to fit the lensing data is several orders of magnitude higher than that required to fit galactic rotation curves.

Key words: gravitational lensing: strong.

1 INTRODUCTION

Ever since the first indications of an invisible mass present in the Universe, the existence of dark matter has remained an open question. Studies show that this matter could be composed of particles that do not emit light, but interact weakly with gravity. Despite the continuous search for particles that may make up dark matter, we still do not know whether such particles exist or whether another theory of gravity is needed to explain these phenomena. Gravitational lensing was predicted by Einstein's general theory of relativity (GR) and observed a few years later; nonetheless, it provides a useful method by which to test alternative theories of gravity.

Without invoking dark matter, but rather by modifying the expression for the gravitational potential, these theories aim to match the observed data with theoretical predictions. Thus, such theories might provide an explanation for phenomena such as gravitational lensing without the need for dark matter.

Using a modified gravitational potential, conformal Weyl gravity (CWG) shows remarkable predictions for galactic rotation curves without the necessity of an extra invisible mass. However, we cannot put aside dark matter and GR until we have more evidence that this theory proves the presence of this additional mass unnecessary. The objective of this article is to test whether this theory also gives the same predictions for strong lensing.

Section 2 introduces the principle behind CWG and in Section 3 we discuss its predictions, which fit observations for galactic rotation curves. Section 4 focuses on gravitational lensing by providing a schematic explanation of the effect and explaining how the lens equation was obtained by Einstein himself, using simple trigonometry. Before proceeding to the article's main contribution, we discuss previous studies on lensing and give an explanation for our use of different equations for the angle of deflection to test CWG in the strong lensing regime. We obtain an expression for Einstein's radius (R_{E}) in CWG by using the equation representing the deflection angle derived by Sultana & Kazanas (2010) and also that obtained by Cattani et al. (2013). A data-fitting algorithm was used to find the value of γ for which numerical predictions match observations. We follow this with a discussion on the results obtained in this study, before concluding in Section 5.

2 CONFORMAL WEYL GRAVITY

CWG uses the principle of local conformal invariance of the space–time manifold, in other words invariance under local conformal stretching (Kazanas & Mannheim 1989, 1991):

$$g_{\mu\nu}(x) \rightarrow \Omega^2(x) g_{\mu\nu}, \tag{1}$$

where $g_{\mu\nu}$ is the space–time metric. The unique action of CWG is represented by

$$I_w = -\alpha \int \mathrm{d}^4x\,(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}, \tag{2}$$

where $C_{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor and α is a dimensionless coefficient. Equation (2) leads to fourth-order equations of motion for the gravitational field, given as

$$-2\alpha W^{\mu\nu} = -2\alpha \left(C^{\lambda\mu\kappa\nu}{}_{;\lambda\kappa} - \frac{1}{2} R_{\lambda\kappa} C^{\lambda\mu\kappa\nu} \right) = \frac{1}{2} T^{\mu\nu}, \tag{3}$$

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Table 1. A sample of clusters.

Cluster	z_L	M_{gas} (10^{13} $m_\odot$)	H_0 (km s^{-1} Mpc^{-1})	θ_{obs} (arcsec)	β_{gas}[a] (10^{15} m)	M_{lens} (10^{13} $m_\odot$)	z_{arc}	$\gamma_{E\&P}$ (10^{-26} m^{-1})	$\gamma_{S\&K}$ (10^{-14} m^{-1})	$\gamma_{Cat\,et\,al.}$ (10^{-15} m^{-1})
A 370	0.375	$0.73^{+0.07}_{-0.06}$	70	39.0	10.78	29.0	0.725	−5.65	−3.81	1.23
A1689	0.183	$1.56^{+0.03}_{-0.03}$	70	45.0	23.04	36.0	1.000	−5.50	−0.16	0.12
A2163	0.201	$0.24^{+0.02}_{-0.02}$	50	15.6	3.54	4.1	0.728	−4.18	−2.15	1.09
A2218	0.175	$0.18^{+0.01}_{-0.01}$	50	20.8	2.66	6.2	0.702	−4.69	−8.79	3.09

[a]Geometric mass.

for a source $T_{\mu\nu}$. Hence, any vacuum solution for Einstein's field equation, i.e. when $R_{\mu\nu}$ is zero, leads to a vacuum solution for Weyl gravity, since $W_{\mu\nu}$ vanishes. In fact, the exact exterior static and spherically symmetric vacuum solution for CWG is given by the metric (Kazanas & Mannheim 1989)

$$ds^2 = -B(r)\,dt^2 + \frac{dr^2}{B(r)} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{4}$$

where

$$B(r) = 1 - \frac{\beta(2-3\beta\gamma)}{r} - 3\beta\gamma + \gamma r - kr^2 \tag{5}$$

and β, γ and k are constants of integration. Outside the source, k is related to the cosmological constant ($k \sim \Lambda/3$, where $\Lambda \sim 10^{-52}$ m^{-2}) in a Schwarzschild–de-Sitter background and $\beta = (GM)/c^2$ (i.e. the geometric mass). Thus, when γ and k tend to zero, equation (5) recovers the Schwarzschild solution in vacuum.

The key addition of CWG is γ, since this describes the gravitational effect otherwise attributed to dark matter. As explained in Section 3, γ has succeeded in explaining this effect for galactic rotation curves, where Solar system constraints require $|\beta\gamma| \ll 1$.

3 CWG AND GALACTIC ROTATION CURVES

While studying the galactic rotation curve, a Swiss astronomer, Zwicky (1937), proposed that more mass must be present than is observed. Newtonian gravity failed to explain the observed rotational curves, specifically for stars further away from the galactic centres. According to Newtonian mechanics, the rotational speeds should first increase with radius and then drop due to the absence of visible mass. Instead, observations show that the curve reaches a limit, which remains constant for higher radius. This could only be explained by invoking an additional invisible mass that was not detected (Zwicky 1937).

Such observations were explained by CWG without dark matter by using the modified gravitational potential described by equation (5). As shown in Kazanas & Mannheim (1989), if γr is comparable to the Newtonian $1/r$ for a regular galactic scale (10 kpc), the equation would represent an increasing potential for $r > 10$ kpc, a constant for $r \sim 10$ kpc and becomes Newtonian for $r < 10$ kpc. CWG fits rotation curves (Mannheim 1993, 1997) with the following relation:

$$\gamma = \gamma_0 + \left(\frac{M}{M_\odot}\right)\gamma^\star, \tag{6}$$

where $\gamma_0 = 3.05 \times 10^{-30}$ cm^{-1} and $\gamma^\star = 5.42 \times 10^{-41}$ cm^{-1}. This implies that, for galactic rotation curves, dark matter is not essential and a modified gravitational potential may explain the observed phenomena. If equation (6) is applied to the clusters used in this analysis, where $M \sim 10^{13}$ M$_\odot$, then γ is found to be 10^{-26} m^{-1}, i.e. the inverse order of the Hubble length. Using a sample of galaxy clusters given in Tables 1 and 2, we test the lensing predictions for CWG.

4 CWG AND GRAVITATIONAL LENSING

Strong gravitational lensing describes the bending of light in the presence of a gravitational field. Light is deflected as it passes through the gravitational potential of an intervening mass, a galaxy or a cluster of galaxies, between the source and the observer. The effect is similar to that caused by a lens. Fig. 1 shows the lensing geometry where D_S, D_L and D_{LS} are the angular diameter distances between the source and the observer, the lens and the observer and the source as seen by the lens, respectively. The Einstein angle, θ_E, is related to the Einstein radius shown in equation (7) (Wambsganss 2001):

$$\xi_E = \theta_E D_L. \tag{7}$$

Here, $\hat{\alpha}$ is the angle of deflection and ξ represents the impact parameter. Assuming the lens is spherically symmetric, then

$$\theta D_S = \beta D_S + \alpha(\theta) D_S. \tag{8}$$

Thus, when placing the source on-axis behind the lens, i.e. $\beta = 0$ in equation (8), the lens equation becomes

$$\theta D_S = \alpha(\theta) D_S, \tag{9}$$

where

$$\alpha(\theta) = \frac{D_{LS}}{D_S}\hat{\alpha}. \tag{10}$$

Substituting for $\alpha(\theta)$, we obtain

$$\theta_E = \frac{D_{LS}}{D_S}\hat{\alpha}. \tag{11}$$

Equation (11) was derived using trigonometry and does not depend on a particular gravitational potential. Thus the same equation can be used for different theories of gravity, where $\hat{\alpha}$ is replaced accordingly and the distances should be computed with the respective expressions. Previous works on the angle of deflection in CWG will be discussed in the next section, together with an explanation as to why we apply the equations derived by Sultana & Kazanas (2010) and Cattani et al. (2013) in this study.

4.1 Previous work

Previous studies of lensing in CWG used the gravitational potential in equation (5) to find the deflection angle (Edery & Paranjape 1998). The expression they obtained is

$$\hat{\alpha}_{E\&P} = \frac{4\beta}{\xi} - \gamma\xi. \tag{12}$$

Table 2. A sample of clusters (Wu & Xue 2000) showing constraints on γ.

Cluster	z_{L}	M_{gas} (10^{13} m$_{\odot}$)	R_{obs} (kpc)	$\beta_{\mathrm{gas}}{}^{a}$ (10^{15} m)	M_{lens} (10^{13} m$_{\odot}$)	z_{arc}	$\gamma_{\mathrm{S\&K}}$ (10^{-14} m^{-1})	$\gamma_{\mathrm{Cat\,et\,al.}}$ (10^{-15} m^{-1})
A 370	0.373	$2.81^{+0.25}_{-0.23}$	350.0	41.50	130.0	1.3	−0.57	0.12
A 963	0.206	$0.27^{+0.02}_{-0.02}$	80.0	1.77	6.0	0.711	−2.30	1.08
		$0.12^{+0.01}_{-0.01}$	51.7	3.99	2.5	–	−4.50	2.16
A1689	0.181	$1.56^{+0.03}_{-0.03}$	183.0	23.04	36.0	–	− 0.34	0.17
A2218	0.171	$0.21^{+0.01}_{-0.01}$	84.8	3.10	5.7	0.515	−5.80	2.55
		$1.61^{+0.04}_{-0.04}$	260.0	23.78	27.0	1.034	−1.00	0.33
A2219	0.228	$0.35^{+0.02}_{-0.02}$	79.0	5.17	5.6	–	−1.10	0.56
		$0.66^{+0.04}_{-0.04}$	110.0	9.75	16.0	–	−0.59	0.30
A2390	0.228	$1.43^{+0.20}_{-0.21}$	177.0	21.12	25.4	0.913	−0.36	0.16
A2744	0.308	$0.51^{+0.06}_{-0.07}$	119.6	7.53	11.4	–	−1.50	0.57
C10500	0.327	$1.21^{+0.14}_{-0.15}$	150.0	17.87	19.0	–	−0.31	0.15
MS0440	0.197	$0.23^{+0.03}_{-0.03}$	89.0	3.40	8.9	0.530	−5.30	2.20
MS0451	0.539	$1.51^{+0.05}_{-0.05}$	190.0	22.30	52.0	–	−0.42	0.21
MS1008	0.306	$1.18^{+0.15}_{-0.16}$	260.0	17.43	61.0	–	−2.30	0.52
MS1358	0.329	$1.02^{+0.11}_{-0.12}$	121.0	15.06	8.3	4.92	−0.20	0.08
MS1455	0.257	$0.52^{+0.02}_{-0.02}$	98.0	7.68	8.6	–	−0.74	0.37
MS2137	0.313	$0.44^{+0.05}_{-0.04}$	87.4	6.50	7.1	–	−0.77	0.39
PKS0745	0.103	$0.17^{+0.01}_{-0.01}$	45.9	2.51	3.0	0.433	−1.80	1.51
RXJ1347	0.451	$6.15^{+0.47}_{-0.48}$	240.0	90.82	42.0	–	−0.02	0.02

^{a} Geometric mass.

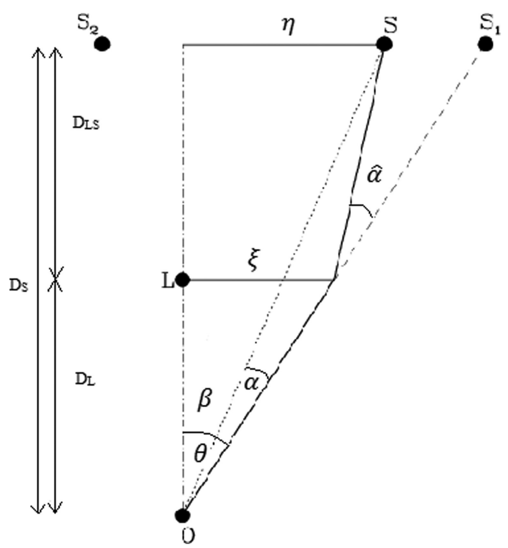

Figure 1. Gravitational lensing by a point mass (Wambsganss 2001).

Pireaux (2004) derived a similar result, where the two middle terms obtained by Pireaux were neglected by Edery & Paranjape, using the Solar system constraint $|\beta\gamma| \ll 1$. This leaves two main contributions to the angle of deflection: the first term, which is the same as that in GR, together with the last term $-\gamma\xi$. As highlighted by both Edery & Paranjape (1998) and Pireaux (2004), for the last term to bend light towards the source, $\gamma < 0$ is required.

A negative γ in equation (12) would lead to a 'paradoxical result', as described in Sultana & Kazanas (2010). The linear relation between $\hat{\alpha}_{\mathrm{E\&P}}$ and ξ implies that 'the larger the light ray's impact parameter with respect to the lens, the larger the deflection angle' (Sultana & Kazanas 2010). The deflection is an effect caused by an object's gravitational field. Since this effect decreases further away from the object, so does the deflection. In other words, the closer to an object, the stronger the gravitational effect, i.e. light should bend more as it travels close to an object than it would further away.

To address this problem, Sultana & Kazanas derived another expression for the bending of light from the same metric (4) using the gravitational potential in equation (5). In contrast to previous studies, Sultana & Kazanas do not obtain a '$+\gamma\xi$' term in the expression for the angle of deflection. Instead, the term is inversely proportional to the impact parameter. Such a result is expected and moreover contributes to the GR term, which is also inversely proportional to the impact parameter.

Sultana & Kazanas (2010) followed the approach used by Rindler & Ishak (2008), where the cosmological constant affects the bending of light. From the metric (4), the null geodesic equation is given as

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi} + u = 3\beta\gamma u + \frac{3}{2}(2 - 3\beta\gamma)\beta u^2 - \frac{\gamma}{2}, \tag{13}$$

where $u = 1/r$. One notes that the cosmological term vanishes in equation (13) but is introduced at a later stage. The final expression for the angle of deflection in CWG is (Sultana & Kazanas 2010)[1]

$$\hat{\alpha}_{\mathrm{S\&K}} = \frac{4\beta}{\xi} - \frac{2\beta^2\gamma}{\xi} - \frac{k\xi^3}{2\beta}. \tag{14}$$

When replacing $\hat{\alpha}$ in equation (10) by (14) and using the following substitution $\xi = \theta D_{\mathrm{L}}$, we obtain the quadratic equation

$$\frac{k D_{\mathrm{L}}^4}{2\beta D}\theta^4 + \theta^2 - \frac{2\beta}{D}(2 - \beta\gamma) = 0, \tag{15}$$

[1] S&K is used to refer to the expression obtained by Sultana & Kazanas.

where $D \equiv D_\mathrm{L} D_\mathrm{S}/D_\mathrm{LS}$. Solving for θ and taking the positive real solution, the Weyl angle ($\theta_{\mathrm{S\&K}}$) in CWG is expressed by

$$\theta^2_{\mathrm{S\&K}} = -\frac{D\beta}{D_\mathrm{L}^4 k} + \frac{\beta}{D_\mathrm{L}^4 k}\sqrt{D^2 + 4D_\mathrm{L}^4 k(2-\beta\gamma)}. \tag{16}$$

Recent work by Cattani et al. (2013) has shown that a negative γ is not necessary for lensing. They derived an expression for the deflection angle comparable with that obtained by Sultana & Kazanas (equation 14) where, in the former, the middle term has a positive contribution rather than negative. In comparison to the gravitational potential used by Sultana & Kazanas, Cattani et al. use

$$B(r) = \alpha - \frac{2\,M}{r} + \gamma r - kr^2, \tag{17}$$

where M is the luminous mass and $\alpha = (1 - 6M\gamma)^{1/2}$. Cattani et al. follow the same approach as Sultana & Kazanas. They explain that if $M = [\beta(2 - 3\beta\gamma)]/2$ and $\alpha = 1 - 3\beta\gamma$ (Cattani et al. 2013) were replaced in their gravitational potential (equation 17), their equation for $\hat{\alpha}$ would be equivalent to equation (14). This approach led to a positive γ term and a coefficient of 15 instead of -2:[2]

$$\hat{\alpha}_{\text{Cat et al.}} = \frac{4\,M}{\xi} + \frac{15\,M^2\gamma}{\xi}. \tag{18}$$

The Weyl angle is thus expressed as (Cattani et al. 2013)

$$\theta^2_{\text{Cat et al.}} = \frac{4M + 15\,M^2\gamma}{D}, \tag{19}$$

where $D \equiv D_\mathrm{L} D_\mathrm{S}/D_\mathrm{LS}$ as before.

4.2 This study

In order to have an idea of the order of magnitude of γ, we represent $\hat{\alpha}_{\mathrm{CWG}}$ as

$$\hat{\alpha}_{\mathrm{CWG}} = \frac{4\beta}{\xi} + \frac{\varepsilon\beta^2\gamma}{\xi} - \frac{k\xi^3}{2\beta}, \tag{20}$$

where $\varepsilon = -2$ for the S&K bend angle (equation 14) and $\varepsilon = +15$ for the Cattani et al. bend angle (equation 18). Comparing the above equation with that derived for GR (Rindler & Ishak 2008) in a Schwarzschild–de-Sitter background, the second term alters the effective Newtonian potential. As mentioned in the previous section, this term is inversely proportional to the impact parameter, i.e. the light ray bends less as the ray travels at large distances from the object. The change in sign of γ depends on the equation used for the analysis, such that equation (16) requires $\gamma < 0$, while $\gamma > 0$ is required for equation (19).

Using $k \approx \Lambda/3$, the k term of equation (20) is the same as the Λ term in Rindler & Ishak (2008) and Ishak et al. (2008); therefore, we can equate the first two terms of equation (20) with the expression for $\hat{\alpha}$ in GR (when $\Lambda = 0$), i.e.

$$\frac{4\beta}{\xi} + \frac{\varepsilon\beta^2\gamma}{\xi} = \frac{4\,M}{R}. \tag{21}$$

Thus, the second term of equation (21) is added to the first, where M on the right-hand side (hereafter RHS) is the total geometric mass. In order to show how γ should behave for lensing, the RHS of equation (21) is re-written as follows:

$$\frac{4M}{R} \rightarrow \frac{4M_{\text{baryonic matter}}}{R} + \frac{4M_{\text{dark matter}}}{R}. \tag{22}$$

[2] Cat et al. is used to refer to the expression obtained by Cattani et al.

In CWG, β is only related to the baryonic matter, so that the second terms should be equivalent, leading to

$$\frac{\varepsilon\beta^2\gamma}{\xi} = \frac{4M_{\text{dark matter}}}{R}. \tag{23}$$

Now $\xi \simeq R$, which leaves the most important relation for this analysis,

$$\beta^2\gamma \approx \varepsilon' M_{\text{dark matter}}. \tag{24}$$

From the above relation, one notes that the Solar system constraint $|\beta\gamma| \ll 1$ does not hold for gravitational lensing, unless ε' is of the order of $\approx 10^{-12}$. In fact, equation (24) predicts that γ should have the negative order of β or a few orders less, so that the left-hand side (hereafter LHS) has the same order of magnitude as the RHS, i.e. γ is inversely proportional to β (the baryonic mass). This observation is clear, since the dark matter mass in galaxies and clusters of galaxies is either of the same order of magnitude as that of the baryonic matter mass or a few orders higher. In other words,

$$\gamma \approx \varepsilon' \frac{M_{\text{dark matter}}}{\beta^2}. \tag{25}$$

As pointed out earlier in this section, the change in sign of γ depends on the equation used and we proceed to confront this analysis with observations using the sample of galaxy clusters mentioned earlier. Equations (16) and (19) were used to fit for the γ that best explains lensing observations. The angular diameter distances were obtained using

$$d_\mathrm{A} = \frac{d_\mathrm{L}}{(1+z)^2}$$

(Varieschi 2011), where the respective expression for the luminosity distance (d_L) in CWG (Mannheim 2003b; Speirits, Hendry & Gonzales 2007; Diaferio, Ostorero & Cardone 2011) is given as

$$d_\mathrm{L} = \frac{c(1+z)^2}{q_0 H_0}\left[\left(1 + q_0 - \frac{q_0}{(1+z)^2}\right)^{1/2} - 1\right]. \tag{26}$$

Here, q_0 represents the deceleration parameter, for which a value of ~ -0.37 was found (Mannheim 2003a; Diaferio et al. 2011), and H_0 is the Hubble constant. The redshifts of the background sources (z_{arc}) were also included in Tables 1 and 2. The equations derived by Edery & Paranjape (1998), Sultana & Kazanas (2010) and Cattani et al. (2013) were used for clusters A370 (Richard et al. 2011), A1689 (Morandi, Pedersen & Limousin 2011), A2163 and A2218 (Makino & Asano 1999), shown in Table 1, so as to highlight the different behaviour of γ arising from different expressions of the deflection angle.

In the analysis for $\theta_{\mathrm{S\&K}}$, k was taken to be constant and of the same order as the cosmological constant $\Lambda \sim 10^{-52}\,\mathrm{m}^{-2}$. Ishak et al. (2008) and Rindler & Ishak (2008) explain that the cosmological contribution of Λ is small; nonetheless, the k term of equation (14) was not discarded but kept for the expression of the angle in $\theta_{\mathrm{S\&K}}$ (equation 16).

Our fitting procedure for each arc computes β ($=GM/c^2$) for each respective enclosed gas mass (M_{gas}) and then adjusts γ until the Weyl angle (equations 16 and 19) matches the observed arc size. The resulting γ values for a sample of clusters taken from Wu & Xue (2000) (with $H_0 = 50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$) are given for each arc in Table 2. Fig. 2 shows the resulting log $|\gamma|$ values plotted against the enclosed gas mass, log M_{gas}, as obtained from equations (16) and (19). Two series were plotted for equation (16) (Sultana & Kazanas 2010), to show that the k term has a small effect on log $|\gamma|$.

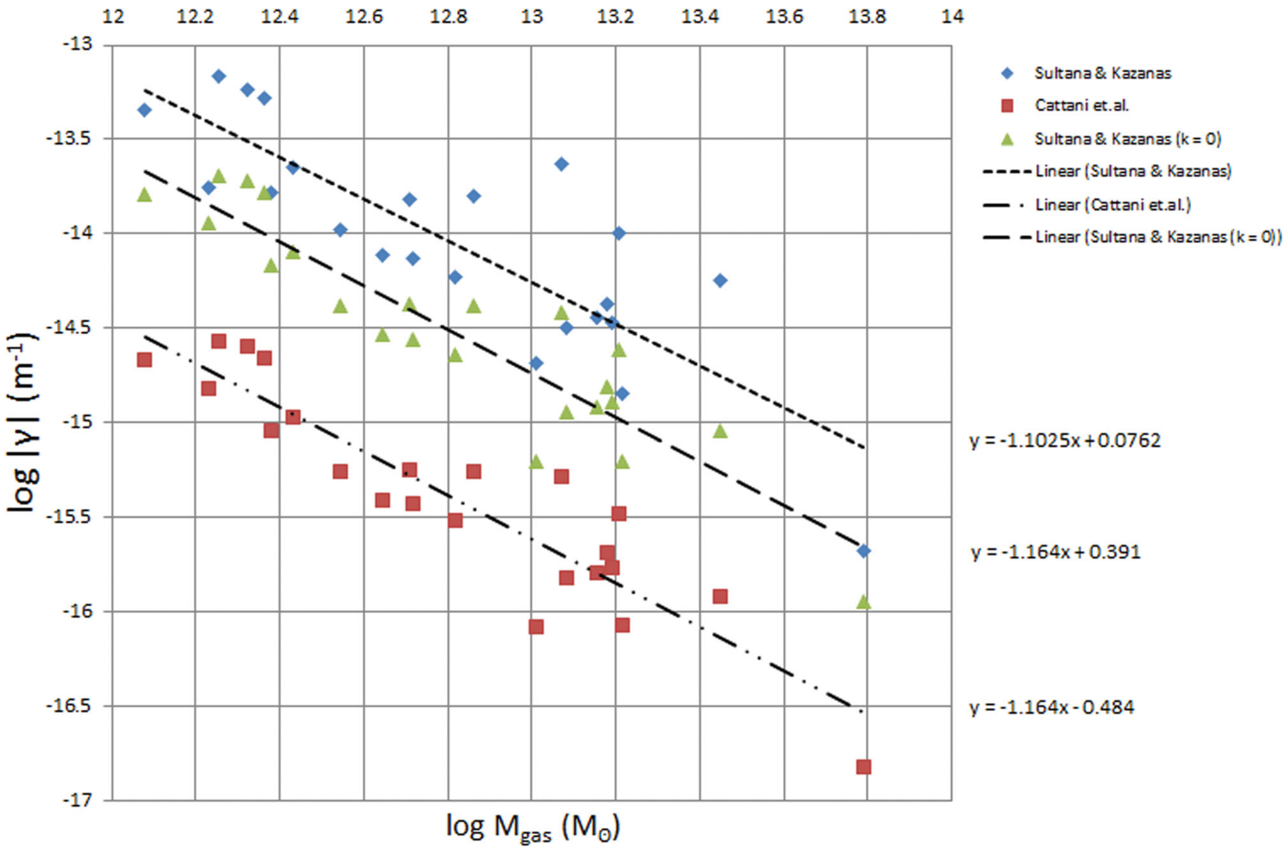

Figure 2. A plot of log γ (m^{-1}) against log M_{gas} (M$_{\odot}$).

In fact, Fig. 2 shows how small the contribution of the k term to the bending of light is; this was pointed out by Rindler & Ishak (2008) and Ishak et al. (2008), who discuss the small contribution of Λ ($k \sim \Lambda/3$). If this were not the case, then the ratio $\gamma_{\mathrm{S\&K}}/\gamma_{\mathrm{Cat\ et\ al.}}$ would be greater than 10.

The expected inverse relation between γ and M_{gas} is also evident in Fig. 2. For each series, we add a trend line to the points and their respective equations are shown next to the end of the line. In fact, all three equations show a negative gradient, which supports our expectations. The small contribution of the k term discussed above is shown again from the first data series, where γ and M_{gas} are still inversely related, however with a different slope from the other two lines. The y-intercept of the S&K equations shows that including the k term would result in a higher value of γ than that obtained in the other series. Therefore, higher fits for k (Mannheim & O'Brien 2012) would result in a larger negative term in equation (20) and hence a larger positive γ term than was obtained in this article is necessary to account for its contribution.

5 CONCLUSION

Previous studies showed that a γ of different sign from that found in the study for the rotational velocities of galaxies, is required for gravitational lensing in CWG (Edery & Paranjape 1998; Pireaux 2004), but it is of the same order of magnitude. However, the deflection angle that led to such conclusions increased as $\gamma\xi$, rather than decreasing with impact parameter ξ. This problem was addressed by Sultana & Kazanas (2010), who derived another expression for the deflection angle in CWG, and recently also by Cattani et al. (2013). In this study, we used the latter two equations to understand how γ behaves when the angle of deflection is inversely proportional to the impact parameter.

For CWG to fit galaxy rotation curves, we find that γ is positive. However, in the expression for the angle of deflection (equation 14), as noted by previous studies (Edery & Paranjape 1998; Pireaux 2004), $\gamma < 0$ is required to fit lensing observations. Despite agreement on the sign, this work disagrees on the order of magnitude for γ. Using equation (14), γ turns out to be approximately 10^{12} orders higher than the value obtained by Edery & Paranjape (1998). A similar behaviour of γ was obtained using Cattani et al.'s (2013) expression for the angle of deflection with $\gamma > 0$.

The gravitational potential (equation 5) used to derive the equations for the angle of deflection represents the exact exterior solution for a static, spherically symmetric source. One could argue that the analysis should include the interior and the exterior solution in CWG (Mannheim & Kazanas 1994). Hence, light travelling in the vicinity of an intermediate mass between the source and an observer necessarily passes through an exterior mass distribution and not through a vacuum. Such scenarios would have two types of deflection to be considered: the bending of light caused by the interior mass acting as a lens bending light towards it and the divergence of the light ray away from the lens caused by the exterior distribution. Thus, γ should account for the effect otherwise attributed to dark matter and also for the divergence caused by the exterior mass distribution. If the effect of the exterior distribution is small, then one would obtain similar results for γ to those obtained in this work. This further confirms the result of this article in showing that, for lensing, γ has to be several orders of magnitude higher than the value obtained from the galactic rotation curve.

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