Helping Children with Mathematics Learning Difficulties:

An Intervention Programme carried out with Children with Mathematics Learning Difficulties only and Children with both Mathematics Learning Difficulties and Reading Difficulties

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Declaration

I, the undersigned, declare that this thesis is my original work, and has not been presented in fulfillment of other course requirements at the University of Malta or any other University.

Esmeralda Zerafa

Abstract

Although Mathematics Learning Difficulties (MLD) are known to affect 5%-8% of any population (Geary, 2004), when compared to Reading Difficulties (RD), MLD are yet limitedly understood. Furthermore, recent emerging studies within this field focus mainly on the causes of MLD, mainly neurobiological ones (Nieder & Dehaene, 2009) with little emphasis on what intervention 'works' with learners with MLD. The main research aim of this study was to look at two groups of learners: those with only MLD and others with both MLD and RD. The study explored which strategies were effective with both groups. A subsidiary research question investigated whether these two groups of learners had the same mathematical profile pre-intervention whilst another explored how similar the profile of the learners assessed with dyscalculia was to that of the other participants. The study looked into each participants' nature and degree of MLD. The last subsidiary research question investigated whether mathematics anxiety was another difficulty experienced by the participants. Vygotsky's theories concerning the notions of 'internalisation', 'cultural tools', the 'zone of proximal development (ZPD)' and the role of the 'more knowledgeable other (MKO)' underpinned the study. The study was mainly qualitative but had two phases. Phase 1 involved finding local norms for Grade 5 (9 - 10-year-old) boys attending Catholic Church schools in Malta because the participants of Phase 2 had these demographic characteristics. Since no numeracy assessment had yet been standardised locally, Phase 1 ensured that the right participants were chosen for Phase 2. Phase 2 consisted of multiple-case intervention studies. An intervention programme was carried out with six learners on a one-to-one basis. The programme used the structure provided by the intervention programme Catch Up® Numeracy and was carried out for 15 minutes, twice weekly over a period of six months. Class observations and interviews with the pupils' teachers and parents were also carried out. The data was analysed by looking at both pre-determined themes, including Tharp's (1993) seven modes of facilitating learning, and other emerging ones. The findings showed that the intervention programme was beneficial in supporting the learners to internalise the targeted numeracy components and in increasing confidence levels. The programme's impact was not determined by whether the learner was in the MLD only or MLD and RD group, but possibly by other domain-general abilities. The analysis gave rise to a pedagogical model that shows which strategies seemed to work with the participants. The model illustrates how MKO-driven strategies, Learner-driven strategies and Tools-assisted strategies come together in a symbiotic relationship to facilitate the internalisation of numeracy components.

Key words: Mathematics Learning Difficulties (MLD); primary mathematics education; intervention programme; teaching-learning model; internalisation; numeracy skills.

Dedication

To all children struggling with mathematics…

May you find the support to improve in this subject and nurture a love for it.

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Although I am the author of this thesis, it could not have materialised without the support of my family, supervisors and friends. Primarily, infinite thanks must go to my husband Kurt for his patience and support throughout these recent years in which this study took much of my time and attention. As important has been the support of my parents who believed in me and my capabilities, putting me before themselves on many occasions. I thank them for all the help they gave me along the way. They taught me to always fulfil my dreams and to be ambitious enough to reach my full potential. Special thanks also go to my two sons, Alessandro and Francesco: too young to give their support but they had to give up plenty of their precious quality time with their mother so that this thesis could be completed. Thanks also go to my sisters and my 'in laws' for their encouragement and help.

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Table of Contents

List of Figures

List of Tables

Chapter 1
Introduction

Chapter 1: Introduction

1.1 Introduction

The journey of my teaching career began when I started teaching mathematics in a school for girls. Although I graduated as a teacher with a specialization in primary education, my first post was focused mostly on teaching mathematics to girls in Grade 6 (aged 10 to 11). This was because the school in which I worked used a subject teaching system. Today I feel fortunate to have had this experience because it encouraged me to develop a passion towards helping learners struggling with mathematics. I remember the difficulty I encountered when a percentage of my cohort each year could not cope with the mathematics being taught. This situation prompted me to ask questions about the nature of the mathematics learning difficulties (MLD) my pupils were encountering and to reflect on how I was going to help them to gain at least a minimum level of success in the subject. As a result, I pursued a Master's degree in education focusing on helping children identified with dyscalculia to overcome their difficulties in diverse components of mathematics. This post-graduate degree was an eye opener and the springboard for me to further research a field which is to date under-researched, especially locally.

1.2 My Research Journey

The research I carried out as part fulfillment of my Master's degree was crucial in my professional development and helped me develop insights I later brought to this present study. Thus, I feel that a brief outline of what the former study entailed is crucial in allowing the reader to understand the baggage of knowledge and experience I brought with me to this study.

My Master's dissertation focused on dyscalculia. I identified three pupils with a profile of dyscalculia using the *Dyscalculia Screener* (DS) (Butterworth, 2003). Two of the pupils were in Grade 6 (aged 10) and one was in Grade 3 (aged 8). I then worked with the children using an intervention programme. Cognitive and affective gains were recorded for all three learners and some characteristics of effective intervention for learners with dyscalculia were outlined. However, I must admit that some stones were left unturned due to the limitations of the study. For example, the difference, if any, between mathematics learning difficulties (MLD) and dyscalculia and the relationship between mathematics learning difficulties (MLD) and reading difficulties (RD) were two issues which I explored in a very limited manner. Moreover, although I had outlined some characteristics of what seemed to work with children with dyscalculia, I had not expanded much on these and had not explored how these may be useful with other children also struggling with mathematics. I used these unresolved issues to develop a framework for the current study including my present research aims. One of the main impacts of my first research on this area was the difficulty of identifying pupils with 'pure' dyscalculia. This difficulty shaped the objectives of the present research study. I had started off my Master's study with the belief that most of the pupils who were underachieving in mathematics had dyscalculia. However, the fact that only two out of the 15 children assessed were shown to be dyscalculic brought up numerous questions. A major one was whether there was therefore a difference between MLD and dyscalculia since although the *Dyscalculia Screener* (DS) (Butterworth, 2003) had not assessed my case-study children with a profile of dyscalculia, they were still experiencing MLD of some sort of nature and degree.

Therefore, when it came to choose my focus for my doctoral research, I decided to work with children having MLD rather than dyscalculia. An outline of how I perceive dyscalculia to be different to MLD is found in the literature review (Chapter 2). My previous experience made me realise that I wanted to help more learners who were failing at mathematics and this prompted me to broaden the spectrum of learners I was going to target through the present study. This is also evident in my choice to focus on children with both MLD and reading difficulties (RD) and children with MLD alone. Through the present study I wanted to investigate whether these two categories of children have a different nature and degree of MLD and whether the strategies which work with one group are as effective with the other group.

My previous study also shaped the research methods used for collecting the data for the present study. In particular, it influenced my choice of assessments for identifying students with MLD and the intervention programme chosen. To a certain extent the research carried out for my Master's served as a pilot study for this thesis, as many lessons had been learnt to ensure greater reliability and validity of the data collected as part of this new research endeavour.

After I finished my second degree, I was given the opportunity to take the role of *Complementary* teacher at a Church[1] school for boys. This role involved supporting learners struggling with literacy through small groups or one to one intervention sessions across the primary grades (Grade 1 to 6 - children aged 5 to 11). One of the reasons why I took up this role was that

[1] Locally there are three types of schools – State schools are run by the State, Church Schools are run by the Catholic Church through the Episcopal Curia, and Independent schools, which are privately owned schools.

I hoped I would be able to intervene for mathematics too in the future using the skills acquired from the master level study. This shift in my career also permitted the carrying out of the data collection of the present study, namely carrying out multiple-case intervention studies (explained in Chapter 4) with a group of students struggling with mathematics. It gave me the chance to understand better the nature of MLD and to get to know the students' characteristics and learning styles in detail. It allowed me to study their mathematics anxiety levels as well as their strengths and weaknesses in mathematics. This provided a deeper perspective on the characteristics of these learners and on the strategies that could be of support.

1.3 The Local Education Scene

Unfortunately, mathematics learning difficulties have been given very limited attention in Malta. Although international researchers have shown greater interest in MLD during the last decade, this is not yet being reflected in the local scenario. To date only a couple of local undergraduate dissertations and two postgraduate dissertations (Master level – one being my own) have focused on MLD and/or dyscalculia (Asciak, 1987; Farrell, 2018; Said, 2010; Zerafa, 2011). Moreover, whereas policies and intervention for literacy and literacy difficulties, including dyslexia, have been in place for a number of years, similar policies and interventions are still non-existent for mathematics, MLD and dyscalculia. The only intervention programme being used in Malta at this time is the *Numicon* programme (Wing, Tacon, Campling, & Atkinson, 2001). This is being used in two non-State schools for children in the early school years. It is also being used by a non-profit organization with children with Down Syndrome and MLD. The latter recently carried out a pilot study using Numicon with children underachieving in mathematics to explore its effectiveness (Micallef & Gatt, 2015). No other intervention programme has yet been adopted or researched. Moreover, no studies in Malta have so far explored the difference in degree and nature of MLD between learners with MLD and reading difficulties (RD) and those with only MLD; thus, my research is pioneering within the local context. I do hope that this research will serve as a ray of hope to all those children and adults who have either struggled with mathematics, or still do so, and for whom failing mathematics had a negative impact on their life path.

1.4 Rationale and Aims of the Research Project

Although research has estimated a prevalence of MLD ranging between 4% to 7% (Fuchs, & Fuchs, 2002; Geary, 2004; Gross-Tsur, Manor, & Shalev, 1996; Lewis, Hitch, & Walker, 1994) studies about MLD have been far less numerous than those focusing on RD (Desoete, Roeyers, & De Clercq, 2004; Fuchs et al., 2004). Noel (2000) illustrates how for the same period (1974 to 1997) only 28 articles about MLD were available in PsycInfo[2] as compared with 747 articles found about literacy difficulties. Moeller, Fischer, Mag, Cress and Nuerk (2012) also highlight the discrepancy in the number of publications for dyslexia as compared with dyscalculia. This is shown in Figure 1.1.

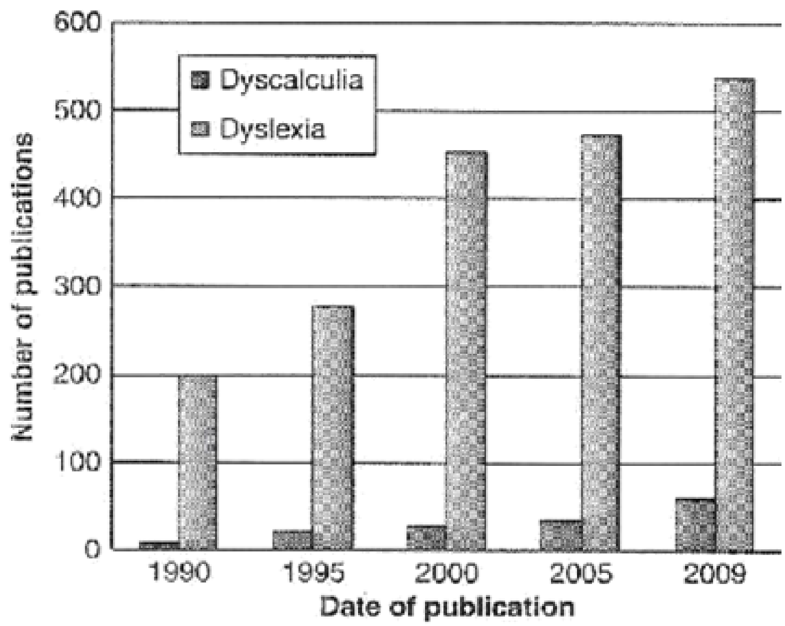


Figure 1.1: Overview of the number of publications in the last 20 years as found on the ISI Wed of Knowledge for entering the topics of 'Dyscalculia' and 'Dyslexia' (Moeller et al., 2012, p. 234).

The limited attention given to MLD is of concern when one considers that skills related to mathematics are not only needed during school life but also in adulthood (Rivera-Batiz, 1992). The long-term consequences of not tackling mathematics difficulties in children are presented in the study carried out by Bynner and Parsons (2000). In this study some literacy and numeracy assessments provided by the Basic Skills Agency were given to a group of 37-year-olds. All the chosen subjects had participated in the National Child Development Study (NCDS). NCDS is a longitudinal study of 17,000 people born in either England, Scotland or Wales in a particular week during 1958. Its aim was to collect information, throughout the participants' lives, about various aspects including physical and educational development. In this study it was concluded that almost a quarter of the tested population had 'very low' numeracy skills. Having such lack of skills was

[2] Psycinfo is an expansive database with more than 3 million records devoted to peer-reviewed literature in the behavioural sciences.

hindering them from performing everyday tasks successfully. In addition, Bynner and Parsons (2000) showed how 17% of the male cohort and 74% of the female population having only MLD (as opposed to having both MLD and literacy difficulties) were not in full-time employment. This contrasted with the 10% of males and the 58% of females who were identified as only having 'very low' literacy difficulties and who were not in full-time employment.

These results showed that not only are numerical skills extremely important in one's adulthood but also that numerical skills are at least as important as literacy skills in an individual's daily life. Hence, they should be given due importance through an increase in studies about MLD. Furthermore, it is essential that policies are developed and that intervention programmes are provided for educational settings to help individuals grasp at least the basic numerical skills needed to lead a good quality life. Through this research I hope to contribute to the body of knowledge currently available about MLD. I also wish to shed light on two areas within this field that are understudied. Primarily I wish to explore which strategies are useful with children having MLD and therefore which kind of intervention, if any, would help the learners to improve. In particular I will be focusing on aspects related to the *number* strand of mathematics. Secondly, I would like to delve deeper in the relationship between MLD and RD and to understand better whether the numeracy difficulties encountered by children with solely MLD are similar to, or diverse from, those experienced by children having both MLD and RD. To achieve my aims, I decided to use a specific intervention programme called Catch Up® Numeracy (CUN[3]). CUN provides a formative assessment which assesses ten components of numeracy. The ten components include *counting verbally* and *counting objects*; a full list will be given in Section 2.3. CUN then provides a specific structure for intervention sessions and some activities which can be used during sessions with children. Further details will be given about the programme and its characteristics in subsequent Chapters.

1.5 Research Questions

The following main research question was developed:

i. Is an intervention programme carried out with children having MLD only and with children having both MLD and RD beneficial to these learners? Which

[3] *Catch Up® Numeracy,* abbreviated as CUN, will be used to refer to the actual intervention programme.

characteristics of the intervention programme are effective with each group of learners?

Moreover, four other subsidiary questions were posed, namely:

ii. What are the teachers' and parents' perspectives on how the children's MLD affects their daily lives at school and at home?

iii. Do children with solely MLD and those with both MLD and RD have similar mathematical profiles? Are both groups of children strong/weak in the same areas?

iv. Do the children assessed with MLD or both MLD and RD and also with a profile of Dyscalculia as identified by the Screener (Butterworth, 2003) have difficulties in all numeracy components or in just a few?

v. Is mathematics anxiety one of the difficulties experienced by the learners?

The study is divided into two phases. Phase 1 includes the choice of suitable assessments, including standardised tests, used to identify the appropriate participants for the second phase. Phase 1 will also involve finding local norms for the standardised tests chosen. Phase 2 is dedicated to the intervention case studies of six children – three with MLD and three with both MLD and RD. During Phase 2, these children will be assessed formatively and given intervention sessions with the hope of helping them to overcome some of their difficulties. Moreover, classroom observations of the participants will be carried out and interviews conducted with their parents and teachers.

1.6 Conclusion

The Maltese National Curriculum Framework (NCF) (Ministry of Education and Employment, 2012) states that, "All children need to experience mathematics as a rewarding and enjoyable experience" (p. 53). I hope that my research will help this statement to become reality for more learners. I hope that the participants of this study nurture a more positive view of mathematics and that they achieve success in the subject. I also wish that my research endeavour raises greater awareness about MLD amongst professionals and provides them with further insight on how to support pupils encountering MLD more effectively so that other learners struggling with grasping mathematics may also benefit from the findings of this study.

The rest of the thesis shall be structured in the following manner. Chapter 2 will present a critical literature review which will illustrate different perspectives in the field of MLD. Chapter 3 will deal with the methodology, explaining my conceptual framework and how this underpinned the planning and implementation of the intervention programme as well as the analysis of data. Chapter 4 will then present the research methods adopted. Chapters 5 will present the quantitative data collected in Phase 1 of the study, whilst Chapter 6 will provide an analysis of the qualitative data gathered in Phase 2, including a discussion of themes that emerged from the analysis of data. The concluding chapter, Chapter 7, will provide a discussion of the main points that have allowed me to answer the research questions. In this final chapter, I will also present implications for local education and indicate areas which could be given importance in future research projects.

Chapter 2
Literature Review

Chapter 2: Literature Review

2.1 Introduction

This Chapter will present various theoretical perspectives and literature about Mathematics Learning Difficulties (MLD). Moreover, it will illustrate how this knowledge has shaped my own perspectives about the subject and has therefore influenced my own research project. It will tackle literature about learners who have both mathematics learning difficulties and reading difficulties. It will give an overview of the literature currently available about reading difficulties as well as existing studies of mathematics intervention for children with both mathematics learning difficulties and reading difficulties. This Chapter will also shed light on the research findings of other researchers to which I will be able to compare my own research findings.

2.2 Becoming Numerate – The Origins of an Innate Ability

Like the innate ability to acquire language, human beings appear to be born with the ability "to respond to the numerical properties of their visual world, without benefit of language, abstract reasoning, or much opportunity to manipulate their world" (Butterworth, 2005, p.5). Research has shed light on how infants are equipped with several domain-specific mechanisms such as that of 'number sense' which allow them to perceive and understand quantities in an intuitive manner (Dehaene, 2009). Other studies have also demonstrated the existence of some numerical abilities from infancy; (Antell & Keating, 1983; Izard, Sann, Spelke, & Streri, 2009; Starkey, Spelman, & Gelman, 1990). An early study carried out by Starkey and Cooper (1980) with infants aged 4-6 months old, revealed that even young infants are already able to differentiate small quantities based on the numerical properties of a display. In this research they made use of the phenomena of 'habituation' and 'dishabituation'. The former signifies losing interest in a stimulus after repeated presentation whilst the latter means re-gaining interest after a change has occurred. Thus, dishabituation indicates that the change has been registered. The researchers showed the participants a few black dots. The participants turned out to be sensitive to the quantity (numerosity) of the sets presented, because they dishabituated when shown sets with new quantities. This was only the case with regard to numerosities of 3 or less.

Brannon (2002) carried out a study of 11-month old infants. The researcher showed the infants a series of displays each representing an increasing number of dots and then a test display illustrating a decreasing number of dots. The researcher noticed that the children looked at least twice as long when the test display (with the decreasing number of dots) was shown. Infants who were two months younger than those in this sample did not, however, show a similar reaction. This suggests that the ability to distinguish addition from subtraction develops by the end of the first year, but may not be innate, unlike the ability to distinguish small quantities.

The studies just mentioned have given rise to questions about the characteristics of typically developing infants that will allow them to go on to develop mathematical skills and concepts. In recent literature (Butterworth, 2010; Leibovich & Ansari, 2017) one finds frequent reference to the notion of numerosity and how this appears to be crucial for human beings to become numerate. A longitudinal study carried out by Geary, Hamson, and Hoard, (2009) illustrated that understanding the numerosity of a set is one of two of the main predictors of low achievement in mathematics; the other predictor is the ability to roughly position a number on the number line. Studies (Piazza, 2010; Piazza, Pinel, Le Bihan, & Dehaene, 2007) have shown that the inability to perceive and interpret the numerosity of a set of objects may derive from a deficit in the Approximate Number System (ANS) – a cognitive system that is used by humans to estimate numerical quantities. The Approximate Number System (ANS) is described by Mazzocco, Feigenson, and Halberda (2011) as,

> a mental system of approximate number representations that is activated and used during both nonsymbolic approximations (e.g., judging which array of items is more numerous, irrespective of item size) and symbolic number tasks (e.g. judging whether a series of Arabic numerals refers to increasing or decreasing quantities (p. 1224).

These studies, as well as other research in this field (Nieder & Dehaene, 2009; Piazza et al., 2007; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004), seem to conjecture that numerosity is fundamental for humans' development of mathematical ability and competence. We seem to have inherited this ability from evolution (Feigenson, Dehaene, & Spelke, 2004), and hence this is part of our biological development (Starkey & Cooper, 1980).

The concept of numerosity and associated ones are a crucial part of what is often referred to as *number sense* (Bird, 2009; Emerson & Babtie, 2010). As Emerson and Babtie (2010) point out, "number sense underlies the ability to make sense of number relationships, patterns within and between numbers, and the way numbers are built from other numbers" (p.4) and is thus crucial to mathematics learning (Van de Rijt & Van Luit, 1998). Another important aspect of number sense is the ability to subitize (perceive without counting) a set of objects. This ability is also of fundamental importance in the acquisition of more complex mathematical skills and concepts (Bird, 2009).

2.3 Defining Numeracy, Arithmetic and Mathematics

Throughout this thesis the terms *numeracy*, *arithmetic* and *mathematics* will be used. Various authors and researchers seem to use the terms *numeracy*, *arithmetic* and *mathematics* interchangeably and synonymously (Chinn, 2012) as will be explained in the subsequent section. However as suggested by Aldrich and Crook (2000), "'arithmetic', 'mathematics' and 'numeracy' are not simply interchangeable terms. They have had separate identities and separate historical trajectories" (p.44). This is a statement that I support and thus in this thesis each will be used intentionally and specifically as will now be outlined.

2.3.1 Numeracy and Arithmetic

The terms *numeracy* and *mathematics* were used synonymously in the National Numeracy Strategy presented in 1999 in the United Kingdom (UK) (Haylock, 2010). The shift in the way the term numeracy was used in the United Kingdom (UK) was radical because previously the term was used to refer to the basic skills one needs for everyday life including the four operations. Once the National Numeracy Strategy was in place, the term numeracy was used interchangeably with mathematics to the extent that the mathematics lesson was also called the numeracy hour. The new mathematics curriculum in the United Kingdom (UK) (Department for Children, Schools and Families/Qualifications and Curriculum Development Agency [DCSF/QCDA], 2010) re-established some balance by clearly suggesting that numeracy is specifically about making use of, and applying, the aspects of mathematics learnt. It also indicated that apart from being confident at applying mathematics in their learning and everyday lives, pupils need to use numerical skills to solve problems and interpret data.

Similar to this definition, Dowker (2004 in Catch Up®, 2009[4]) sub-categorises the term numeracy into 10 components namely;

i. Counting verbally

ii. Counting objects

[4] Catch Up (2009) will be used to refer to the file with resources provided by the programme itself.

iii. Reading and writing

iv. Hundreds, tens and units

v. Ordinal numbers

vi. Word Problems

vii. Translation (changing objects to numbers, numbers to objects and number words to objects)

viii. Derived Facts

ix. Estimation

x. Remembered Facts

Each of these terms will be explained in Chapter 4 (see Section 4.7) since the components are the basis of the intervention programme used in this study. Dowker argues that these components are the foundations of mathematics, although she states that the notion of mathematics is more complex and involves other areas of development such as data handling, measurement and geometry. This is similar to what the Department for Education and Skills (DfES; 1999) suggests. However, authors like Sousa (2008), refer to mathematics similarly to the way numeracy has been defined by Dowker (2004 in Catch Up®, 2009) and the DfES (1999).

The scenario becomes more complex because the term *arithmetic* is also used frequently even though for a while this term was not used much in Mathematics Education. It is controversial whether this is similar to, or the same as, the notions of mathematics and/or numeracy. According to Chinn (2012), arithmetic is "that part of mathematics that focuses on numbers and the four operations: addition, subtraction, multiplication and division" (p.3). Similarly, Dowker (2005a; 2005b) regards the term arithmetic as different from that of mathematics. She (Dowker, 2005b) explains that in relation to arithmetic, "mathematics includes many other topics such as geometry, measurement, and algebra" (p.324). She suggests that "it is well known that individual differences in arithmetical performance are very marked in both children and adults" (Dowker, 2005a, p.5). Dowker also highlights that arithmetic is not unitary but is made up of different components and sub-components. The British organization 'National Numeracy' states that "being numerate means being able to reason with numbers and other mathematical concepts and to apply these in a range of contexts and to solve a variety of problems. Being numerate is as much about thinking and reasoning logically as about 'doing sums'" (taken from https://www.nationalnumeracy.org.uk/what-numeracy). Dowker (5th November 2018) indicated that, "arithmetic usually refers to the subject of numbers and numerical operations and does not carry the same implications about how much people can understand, reason about or apply these topics" (personal communication).

Dowker (2005b) highlights that "most studies of young children's MD [Mathematics Difficulties] have dealt mainly with number and arithmetic" (p.324) and she therefore explains that although much research speaks about *mathematics learning difficulties* it would be more appropriate to speak of *arithmetical difficulties*. Making such a distinction shows that the terms *mathematics* and *arithmetic* evidently imply two different constructs. Moreover, Butterworth (2010) makes use of the terms: *arithmetic ability*, *arithmetic difficulties, low arithmetic attainment*, *arithmetic learning* and *arithmetic disabilities*. It is unclear whether he uses these terms to refer to a similar construct as Dowker (2005a, 2005b) does when using the term *arithmetic*. However, from the examples Butterworth (2010) provides, I can conjecture that he refers to *arithmetic* in a broader sense, and therefore similar to the way *mathematics* has been defined as will be discussed in the next section (Section 2.3.2). After viewing the different perspectives presented by the mentioned authors (Butterworth, 2010; Chinn, 2012; Dowker, 2005a; 2005b), my view is similar to that of Dowker (5th November 2018, personal communication). Although arithmetic and numeracy are closely related, they do not refer to exactly the same construct. Arithmetic seems to be the basis of numeracy as the latter does not only involve numbers and numerical operations but also the skills to apply these as well as understand and reason them out.

2.3.2 Defining *Mathematics*

Mathematics has a central role in school programmes and has been emphasized in educational documents over the years. For example, it was discussed in some detail in the Cockcroft Report (1982). In its opening paragraph, this Report states that, "there can be no doubt that there is a general agreement that every child should study mathematics at school" (p.1). Moreover, the Report highlights that, "mathematics is in some way thought to be of especial importance" (p.1), when compared to other subjects taught in schools such as Geography and History. The Report gives mathematics the same status as English in the UK saying that both subjects are deemed by most people as undeniably essential. Bramall (2000) argues that the underlying argument within the Cockcroft Report (1982) "in favour of granting special status to mathematics in the curriculum is the idea that mathematics is something like a special language with which we can describe, and communicate about the world" (Bramall, 2000, p.50). Most literature agrees that the ability to develop mathematical skills and be able to apply mathematical thinking in the daily situations we face is crucial and inevitable to life in modern society (Bynner & Parsons, 1997; Dowker, 2005a; Ernest, 2011; Haylock, 2010; Ministry for Education and Employment, 2012). Apart from this utilitarian aim, Haylock (2010) also identifies other reasons why the teaching of mathematics in schools is fundamental. These include: the importance of the subject to other curricular areas; its importance in the learner's intellectual development; its importance in enhancing the learner's enjoyment toward learning as well as its epistemological aim which highlights that we teach mathematics because "it is a significant and distinctive form of human knowledge with its own concepts and principles and its own ways of making assertions, formulating arguments and justifying conclusions" (p.16).

Although there seems to be a consensus among researchers about the importance of teaching mathematics in schools, a stance with which I also agree, debates arise as to defining mathematics and as to what should be taught as part of mathematics curricula in schools. Defining mathematics is somewhat problematic as no universal consensus about the constituents of mathematics has been reached. Hence, I thought it important to look at how different researchers have defined mathematics and what constitutes it. Lerner and Johns (2009) state that "mathematics is a symbolic language that enables human beings to think about, record, and communicate ideas" (p. 478) about the properties and relationships of quantity. Additionally, Van De Walle, Karp, and Bay-Williams (2015) identify that mathematics involves the use of numbers, form, chance, algorithms and change. In the available literature, most authors do not explicitly define mathematics, but some give accounts of what it means to be successful in mathematics. Haylock (2010), for example, suggests that being a successful mathematics learner involves "constructing understanding through exploration, problem solving discussion and practical experience – and through interaction with a teacher who has a clear grasp of the underlying structure of the mathematics being learnt" (p.3). The role of the teacher as mentioned in this quotation will be discussed at a later stage. The key point at present is that authors measure success in mathematics in various ways and express diverse views of what one should consider mathematics and success at this subject.

What knowledge, skills and dispositions are considered to be mathematical is debatable because as outlined by Watson and Winbourne (2010) "in the practice of teachers and learners, 'knowledge' adequately stands for mathematics which might be seen as a tool for problem-solving, or as rules of participation in mathematical activity, or as the outcomes, through learning, of activity" (p.4). Thus, the authors argue that different forms of knowledge in mathematics may be recognized even when one is rigorous and follows conventions. For example, the United States (US) National Council of Teachers of Mathematics (NCTM; 2000) indicate that there are ten standards which "are descriptions of what mathematics instruction should enable students to know and do. They specify the understanding, knowledge, and skills that students should acquire from prekindergarten through grade 12" (p. 29). The ten 'standards' are divided into the 'Content Standards' and the 'Process Standards'. The 'Content Standards' are made up of *number and operations*, *algebra*, *geometry*, *measurement*, and *data analysis*. The 'Process Standards' involve *problem solving*, *reasoning and proof*, *communication*, *connections*, and *representation*.

Ernest (2000) argues that, "school mathematics is neither uniquely defined nor value-free and culture-free" (p.1). Although some of the mathematical content focused upon in schools are drawn from history and popular practice, other aspects have been selected based upon what is considered important in different cultures and contexts. Hence the nature of school mathematics is rather diverse and open (Ernest, 2000) and there is no universal consensus as to which concepts should be taught. Although in the first instance, it is generally agreed that mathematics is a crucial subject mainly due to its usefulness to everyday life, researchers such as Ainley (2000) suggest that when creating a mathematics curriculum for primary schools, it is rather problematic to justify the choice of concepts to be tackled according to their degree of usefulness. In particular, most situations for problem solving presented in textbooks involve adult activities such as shopping and cooking, rather than those most typically relevant to children. Secondly, it seems that children are constantly given the message that topics tackled in primary school curricula are important as the basis of more complex mathematics to be learned during the secondary years. This may again reduce the subject's relevance to primary school children.

Hoyles and Noss (2000) indicate that "for most, the study of mathematics now comprises elementary number work, a little algebra, shape and space (a euphemism for geometry) and some 'data handling' as statistics is quaintly called" (p. 157). In the National Curriculum for England (DCSF/QCDA, 2010) the level descriptions for both primary and secondary schools, are categorized under four similar titles to those mentioned by Hoyles and Noss (2000). These are: Using and Applying Mathematics; Number and Algebra; Shape, Space and Measures; and Statistics. Since the present study was carried out in Maltese primary classrooms, I investigated the knowledge we consider as being of a mathematical nature in Malta to see whether the Maltese mathematics curriculum and syllabus include similar areas of study. In our local National Curriculum Framework (NCF) (Ministry for Education and Employment, 2012), being successful at mathematics is taken to mean that "learners acquire a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers" (p. 35). In this document an emphasis is placed on the actual knowledge to be learnt for success to be reached. The recently revised mathematics syllabus for the primary years (Department of Curriculum Management, 2014), clearly places the knowledge to be developed in this subject into four strands. These are: number and algebra, measurement, space and shapes and data handling. These are identical to those found in the National Curriculum of England (DCSF/QCDA, 2010), possibly because of the great influence that the English curricula have always had on the local ones. The Primary Syllabus (Department of Curriculum Management, 2014) does not justify why its focus is based on the four strands mentioned earlier. However, one might assume that these have been included because of their utilitarian purpose which seems a predominant means of selecting mathematical content for school curricula (Ainley, 2000). The four strands which the Primary Mathematics Syllabus (Department of Curriculum Management, 2014) focuses on are the same as those used in the Learning Outcomes Framework (LOF) (Ministry for Education and Employment, 2015) which is the new framework that will replace the 2014 syllabus (Department of Curriculum Management, 2014). However, in this latter framework, the four strands have been placed into even smaller components so that there are seven strands. 'Number and Algebra' has been split into: Strand 1 – The Number System, Strand 2 – Numerical Calculations and Strand 3 – Fundamentals of Algebra; 'Shape, Space and Measure' have been taken together and divided into: Strand 4 – Measures; Strand 5 – Euclidean Geometry and Strand 6 – Transformation Geometry. 'Statistics and Data Handling' are tackled through Strand 7 under the heading 'Data Handling & Chance – Statistics'.

Although, as stated earlier, there is a general agreement about the importance of mathematics in schools, some researchers have questioned whether mathematics should act as a 'critical filter', implying that equal opportunities are denied to pupils who fail at the subject for life-long learning (Sells, 1973). Moreover, *social theory* has brought to light "questions on who fails in mathematics and how such failure is connected to the very same pedagogy that apparently claims to be "inclusive"" have been raised (Pais & Valero, 2014). Pais and Valero (2014) argue that, "failure in school mathematics is less a problem of "deficit" – individual or collective – and more a result of the way schooling is structured in today's society" (p.243). Hence, when looking at mathematics education through a social perspective, emphasis is placed on changing the structures which underpin mathematics teaching and learning rather than the learner himself. Although in this thesis I focus on changing and improving the individual participants' arithmetical abilities, on a much wider national scale, it would be appropriate to evaluate whether our structures for mathematics learning are failing our pupils and how these may be changed for more learners to succeed. As a result, one of the aspects that we would have to evaluate is the content that is part of the local school curricula and whether this is indeed being inclusive or whether it is excluding some categories of learners from the outset. This idea will be explored further in Section 2.8.1 which will tackle socio-economic disadvantages. However, in conclusion, I must highlight Pais and Valero's (2012) argument that "fields of research such as mathematics education emerged with the task of finding new ways of ensuring that the subject of mathematics in the curriculum reaches all students (p.10)". Hence, this thesis clearly falls within this research category.

2.3.3 My Working Definitions for Mathematics, Numeracy and Arithmetic

I now come to the working definitions I shall attribute to each of the terms for this study. Although at various points throughout this thesis I cite other authors who might have different definitions for the terms discussed so far, I will make use of each term intentionally and specifically. I will take *mathematics* to have a broader meaning than the terms *numeracy* and *arithmetic*. When using the term *mathematics*, I will refer to the wider sense of the term. This will involve the four strands outlined in the revised Maltese primary syllabus for mathematics (Department of Curriculum Management, 2014) as well as the seven strands these have been split into as part of the LOF (Ministry for Education and Employment, 2015).

Numeracy will be taken to be similar to the term *arithmetic*. However, when the term arithmetic is used, I will refer solely to the carrying out of numerical operations. When the term *numeracy* is used it will be taken to mean explicitly the understanding of the ten components of numeracy (Dowker, 2004 in Catch Up®, 2009) and their application. *Arithmetic* and *numeracy* will therefore be taken to be the foundation of mathematics. To conclude, I view the relationship between each term presented so far in the following manner: The two domain-specific mechanisms of n*umber sense* (numerosity and other related abilities such as subitizing which are innate or develop through experience) and more specifically *numerosity* (the innate ability to perceive quantities of sets of objects) are the foundation to developing the skills and concepts related to *arithmetic* abilities and to master the ten *numeracy* components (Dowker in Catch Up®, 2009). Once achievement is reached in most, if not all, of these components, the learners can engage in *some* crucial mathematical activities involving algebra, geometry and data handling as will be explained in the following section. I use the term 'some' here since although some complex mathematics tasks such as fraction and decimal activities depend on numeracy and arithmetic, others are less related to these.

2.4 Defining *Mathematics Learning Difficulties (MLD)*

Although research about MLD is now growing rapidly, as shown through the new scientific research in this field (Dowker, 2008; Mazzocco, 2007), a universal definition of MLD has still not been established. In various studies, a variety of terms is used instead of, or together with, the term MLD. These terms include *Mathematics Disorder* (American Psychiatric Association, 2000); *Number Fact Disorder* (Temple & Sherwood, 2002) and *Psychological Difficulties in Mathematics* (Allardice & Ginsburg, 1983). A list of the different terms used in literature is shown in Table 2.1.

Table 2.1: Terms used to refer to MLD and their authors.

Developmental Dyscalculia (DD)	Temple (1991) Shalev & Gross-Tsur (1993) Sharma (2003) Butterworth (2003) Rubinsten & Henik (2009) Ashkenazi & Henik (2010) Rubinsten & Sury (2011)
Dyscalculia	Chinn (2004; 2012) Emerson & Babtie (2010) Ernest (2011)
Mathematical Learning Difficulty (MLD)	Hopkins & Egeberg (2009)
Mathematics Difficulty (MD)	Powell, Fuchs, Fuchs, Cirino, & Fletcher (2009)
Mathematics Learning Difficulty (MLD)	Van Steenbrugge et al. (2010)
Mathematical Disability (MD)	Geary (1993) Mazzocco, Myers, Lewis, Hanich, & Murphy (2013)
Mathematics Learning Disability	Rousselle & Noël (2007)
Mathematic Disorder	American Psychiatric Association (2000)
Arithmetic Learning Disability (AD, ARITHD or ALD)	Siegel & Ryan (1989) Koontz & Berch (1996) Geary & Hoard (2001)
Number Fact Disorder (NF)	Temple & Sherwood (2002)
Psychological Difficulties in Mathematics	Allardice & Ginsburg (1983)
Arithmetical difficulties	Dowker (2005a)

Some researchers such as Geary (1993) and Geary and Hoard (2001) emphasise that the different terms found in most literature and research seem to be referring to a common difficulty. This is a difficulty to conceptualize and apply the necessary number concepts and skills in order to understand and engage in mathematical tasks. Some researchers (Bartelet, Ansari, Vaessen, & Blomert, 2014; Mazzocco et al., 2011) explicitly state that they use the terms *mathematics learning difficulties* and *dyscalculia* synonymously.

As Kaufmann et al. (2013) suggest, “what is treated as DD [Developmental Dyscalculia] in one study may be conceptualized as another form of mathematical impairment in another study” (p. 1). For example, Mazzocco (2007) use the term *Mathematical Learning Difficulty* abbreviated as MLD to refer to the same condition other researchers define as *dyscalculia* (Butterworth, 2003; Chinn, 2012) or even *developmental dyscalculia* (Ashkenazi, Mark-Zigdon, & Henik, 2009; Chan, Au, & Tang, 2013; Devine, Soltész, Nobes, Goswami, & Szűcs, 2013) – which is a 'specific' learning difficulty in mathematics. Thus, the term *specific* refers to a difficulty in mathematics that derives from a domain (number) specific difficulty – either a difficulty with number sense or even more explicitly numerosity – and includes a smaller portion of the population (5 – 8%). On the other hand, Mazzocco (2007) then uses the term *Mathematical Difficulties* to refer to general difficulties in mathematics and therefore to include more of the population struggling with mathematics learning. The term *general* refers to all those children who are performing way below expected number age in relation to their actual age and whose difficulties may derive from several factors including difficulties with working memory. However, these general difficulties are referred to in most of the other literature as Mathematics Learning Difficulties (Hopkins & Egeberg, 2009; Van Steenbrugge, Valcke, & Desoete, 2010) – which abbreviated becomes MLD like Mazzocco's (2007) abbreviation of mathematics learning disability.

In their study, Mazzocco et al. (2013) seem to amalgamate the definitions above by using *Mathematics Learning Disability* and putting the term *dyscalculia* in brackets rather than using another term, such as *Low Mathematics Achievement*. In addition, Dowker (2005a) states that terms like *Difficulties with Arithmetic*, *Mathematics and Numeracy* have been used in a more generic sense to refer "to children or adults who struggle or fail to cope with some of the aspects of arithmetic that are necessary or desirable for educational or practical purposes" (p.11). As Geary, Hamson, and Hoard (2000) suggest it is therefore essential that until a working definition is accomplished for MLD, individual researchers define their own understanding of the terms used and provide a clear rationale as to why the particular terms were chosen. Hence, although most literature does not distinguish clearly between whether the terminology being used is referring to a specific learning difficulty in mathematics or to general mathematical learning difficulties, I consider that making such a distinction will be crucial in my own thesis.

2.4.1 Working Definition for *MLD* and Identification of MLD

In my view the term *Mathematics Learning Difficulties* (MLD) has a broader construct than the terms *Dyscalculia* or *Mathematics Disability*. The term *Mathematics Learning Difficulties* (MLD) refers to all the individuals who underachieve in mathematics no matter what the underlying cause may be. I opt to make use of the term *Mathematics Learning Difficulties* (MLD), as opposed to terms like *Dyscalculia* and *Mathematics Disability* because, in my thesis, poor achievement serves as a primary criterion for classifying the main participants in this study. Additionally, since I acknowledge that arithmetic is a complex ability made up of a wide spectrum of skills (Dowker, 2005a), I have also chosen this term because various researchers have suggested that this group of people, namely those having MLD, is likely to constitute a heterogeneous group of subjects (Bartelet et al., 2014; Geary, 2010; Kaufmann & Nuerk, 2005), which is the case if MLD refers to all those learners who underachieve in mathematics.

Despite a working definition, identifying learners with MLD is still not an easy task. As Moeller et al. (2012) indicate, the cut-off test scores used, and the way they are used, in different studies to define the term *Mathematical Learning Difficulties* (MLD) vary noticeably. However, in this study, the term MLD will be taken to indicate all those pupils who fall below a cut-off point of the 30th percentile, since various studies (Geary, Hoard, & Hamson, 2001; Passolunghi & Siegel, 2001; Passolunghi & Siegel, 2004) and numeracy standardised tests (Gillham & Hesse, 2001) use this cut-off point to identify pupils who are underachieving in mathematics due to diverse potential causes. These potential causes do not necessarily include having a biological inherited weakness in mathematical cognition (Gersten, Jordan, & Flojo, 2005; Hanich et al., 2001; Jordan, Kaplan, Olah, & Locuniak, 2006). The choice of this cut-off point will allow me to study a larger population of pupils who are struggling with mathematics than if I had chosen to set more stringent criteria. Since the rationale of this research project does not involve setting identification criteria for both groups of learners – those with MLD only and those with MLD and RD - but revolves around finding effective strategies to remediate individual weaknesses in mathematics learning, even if these are not of a biological nature, taking this broader perspective is feasible. Moreover, since there is yet no clear distinction in the literature between MLD and Dyscalculia/Mathematics Disability, as will be discussed in the next section (Section 2.5), it seems appropriate to take this wider construct rather than to focus on a fine line which has not yet been well defined.

2.5 Defining *Mathematical Disability* or *Dyscalculia*

Mathematical Disability, Specific Disorder of Arithmetical Skills, Developmental Dyscalculia or *Dyscalculia*, are commonly used to indicate a specific learning disability in mathematics. Since research about mathematical disabilities is still very limited it is difficult to find a common definition for this term. Defining a learning disability is "among the most problematic classifications because of vagaries and antagonisms surrounding its definitions" (Mather & Roberts, 1994, p. 49). However, the great similarity between the meanings indicated by the terms presented in Table 2.1 (see p. 28) is also pointed out by Butterworth (2010). He explains that the terms *developmental dyscalculia* and *mathematical learning disability* have the same constructs and adds that they refer to a "modal prevalence of approximately 6.5%" (p. 534). Additionally, Fuchs, Fuchs, and Prentice (2004), point out that the term *Mathematics Disability* (MD) has sometimes been used to describe different but specific categories of children. These include pupils who have been identified by the school as being two grade levels below their actual grade (Parmar, Cawley, & Frazita, 1996), pupils who were one grade level below their age-appropriate level (Russell & Ginsburg, 1984) and pupils scoring below the 30th percentile on standardised tests (Hanich et al., 2001). The latter however contradicts the fact that in most of the research currently available, children who score below the 30^{th} percentile are referred to as having Mathematics Learning Difficulties (MLD) and those who score below the 10^{th} percentile are referred to as having Mathematics Disability (Mazzocco, 2007).

The Diagnostic and Statistical Manual of Mental Disorders (DSM IV-TR) (American Psychiatric Association, 2000) recommends three criteria to identify whether a learner has Mathematics Learning Disabilities (MLD). These include:

i. The learner must perform lower in mathematics when compared to his or her attainment in other school subjects and general intelligence (IQ)[5];
ii. The pupil scores 2 or more *standard deviations* (SD) below the norm established by any mathematics standardised test;
iii. The pupil is considered by his/her teacher as not making expected improvement even after appropriate classroom instruction.

Dyscalculia, a term which is being used in most contemporary research, has been defined by the UK Department for Education and Skills (DfES, 2001) as,

> a perseverant condition that affects the ability to acquire mathematical skills despite appropriate instruction. Dyscalculic learners may have difficulty understanding simple

[5] An Intelligent Quotient (IQ) is a score derived from one of many standarised tests made-up to assess intelligence. The IQ score defines one's intelligence in relation to the mean score of individuals on the same test.

> number concepts, lack an intuitive grasp of numbers, and have problems learning, retrieving and using quickly number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence (p.14).

Research has reported different figures for the prevalence of dyscalculia. However, Shalev, Auerbach, Manor, and Gross-Tsur (2000) conclude that a realistic figure is that 5% of school aged children have dyscalculia. More recently, Bird (2013) stated that dyscalculia "affects roughly 4-6% of the population. This equates to at least one child in any average classroom" (p. 4). Differences in the reported figure have arisen from the fact that to date no universal criteria have been established to diagnose this condition. If, as Dowker (1998) suggests, dyscalculia implies having a difficulty in all aspects of mathematics, and only in this subject, then it is probably very rare and represents far less than 6% of the population. However, as she indicates (Dowker), if dyscalculia is taken to indicate difficulties with certain aspects of mathematics, which are sufficient to cause significant practical and educational problems for the child, then they are probably more prevalent than the 6% found in literature and may in fact indicate 15% to 20% of the population. I have opted to use the term dyscalculia to define the former population mentioned in Dowker's definition: i.e. the bottom 6% or fewer because this figure is closer to that used by other researchers (Butterworth, 2010; Geary, 2004). Moreover, my teaching experience prompts me to use this prevalence rate. As explained in the Introduction Chapter, my Master's dissertation allowed me to conjecture that only a small number of those struggling with mathematics do so because of dyscalculia.

2.5.1 Working Definition for *Dyscalculia*

I take dyscalculia to mean a specific learning disability which impinges on most, if not all, aspects of mathematics. In this thesis I therefore take *dyscalculia* to be an umbrella term for the different conditions that may cause specific difficulties with mathematics including the terms and conditions referred to by *mathematical learning disability*, *mathematical disability*, *developmental dyscalculia*, *arithmetic disability* and *number fact disorder* as mentioned in Table 2.1 (see p. 28). This corroborates with Emerson's and Babtie's (2010) work in their publication of *The Dyscalculia Assessment*. Additionally, contemporary research is indicating that some of the causes of dyscalculia include an inability to understand the numerosity of a set (Geary et al., 2009). This does not allow dyscalculics to perform well in approximate number tasks due to a deficit in the Approximate Number System (ANS) (Piazza et al., 2007; Piazza, 2010). These studies therefore indicate that dyscalculia may be caused neuro-biologically and that therefore it is possibly a genetic inherited condition as indicated by studies such as that by Ansari and Karmiloff - Smith (2002).

When using the term *dyscalculia*, I will therefore be referring to: those individuals who have a specific learning difficulty in mathematics; who are at the lowest end of the continuum for mathematics learning difficulties; who normally perform within the 10th percentile in a given mathematics standardised test - a cut-off point suggested in most tests (Gillham & Hesse, 2001) and used in other studies (Mazzocco, 2007); and whose difficulties stem from their lack of number sense and inability to perceive numerosities. Further causes of dyscalculia and characteristics of subjects with this specific learning disability will be described in subsequent sections.

To conclude, *Mathematics Learning Difficulties*, abbreviated as MLD, will be the expression used most often in this thesis. This will include all those learners underachieving in mathematics for one cause of another. The word will be used as an umbrella term to also include those learners with dyscalculia who would be evidently underachieving in mathematics. The term *dyscalculia* will be used to indicate a specific learning disability which describes a fewer percentage of the population and is therefore only one possible cause for MLD.

2.6 Characteristics of Learners with MLD

It has been shown that learners with mathematics learning difficulties are a heterogeneous group of learners (Bartelet et al, 2014; Dowker, 2005b; Geary, 2010; Kaufmann & Nuerk, 2005; Rubinsten & Henik, 2009). As a result, Dowker (2004) suggests that a learner should not be described as simply ‘good’ or ‘bad’ at mathematics but that one should understand better the components of mathematics in which the child is struggling. An individual may be good at one component and weak at another, suggesting that there is no such thing as *arithmetical ability* but only *arithmetical abilities* (Dowker, 2005a). It is usually due to a weakness in one or more of these components that a learner underachieves in numeracy and is thus hindered from grasping mathematics (Chinn, 2004; Dowker, 2004; Dowker, 2005a). Dowker (2005a) suggests that such learners may have difficulties with essential numerical skills for mathematics learning including counting, reading and writing numbers and estimation.

Various studies and literature have highlighted the key characteristics for individuals with MLD including learners with dyscalculia. These have been: a poor sense of number (Bird, 2009; Emerson & Babtie, 2010); a marked early delay in understanding some of the principles of counting (Geary et al., 2000; Geary, Hoard & Hamson, 1999); a delay in using counting techniques for addition (Geary et al., 2000; Jordan & Montani, 1997); an over reliance on finger counting strategies (Geary, 2004; Jordan & Montani, 1997; Ostad & Sorensen, 2007); a difficulty with sequencing (Emerson & Babtie, 2010); a deficit in various components of working memory (Geary, 2004; Roselli, Matute, Pinto & Ardila, 2006); an impairment in central executive tasks (D'Amico & Guarnera, 2005; Dehaene, Piazza, Pinel, & Cohen, 2003) and a deficit in spatial working memory (D'Amico & Guarnera, 2005; Gathercole & Pickering, 2000). Additionally, some studies have indicated that these learners have a deficit in verbal working memory (Wilson & Swanson, 2001). However, since others (Passolunghi & Siegel, 2004) have not included the latter, whether this should be considered as one of the key characteristics is yet debatable.

Although a learner with MLD may have a deficit in either one or more of the areas mentioned above, recent studies indicate that the main difference in characteristics between an individual with dyscalculia and any other learner with MLD is that the former will, above all, have a poor sense of number and a deficit for understanding numerosities (Butterworth, 2010; Emerson & Babtie, 2010; Halberda, Mazzocco & Feigenson, 2008; Piazza et al., 2010). The phenomenon of having poor number sense has been studied widely but, as Berch (2005) points out, different studies seem to refer to different abilities when using the term *number sense*. As he suggests, this notion "reputedly constitutes an awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line" (p.333). Having number sense is key to achieve everything from understanding what numbers represent to solving problems (Berch).

Contemporary studies about dyscalculia have also focused on the basic numerical processing of these learners. These studies have indicated that within this area of processing, learners with dyscalculia specifically encounter difficulties with size congruity (comparing two numerical quantities presented in different physical sizes) (Rubinsten & Henik, 2005), subitizing (Dehaene, 2001) and comparing magnitudes (the quantities of a set of objects) (Ashkenazi et al., 2009; Geary et al., 2000). These deficits in learners with dyscalculia are possibly the result of abnormalities or the activity of a specific section of the brain which is mainly involved in mathematical activities – the parietal lobes. This argument will be dealt with in more detail in the section dedicated to the causes of dyscalculia (Section 2.9.3).

Some studies into the nature of MLD have prompted researchers to divide learners with MLD into different groups. Bartelet et al. (2014), for example, suggest that there are six cognitive subtypes of mathematics learning difficulties in the primary educational setting. They consider MLD and dyscalculia to be synonymous. Their study was carried out with 226 children between Grade 3 and Grade 6 who were taken from a clinical and non-clinical sample. An arithmetic fluency test was given to all the classes, following which eleven cognitive processing tasks were carried out individually. These tasks included a dot comparison task, matching objects task and dot enumeration task. A full list of these tasks and examples is found in Appendix A. Their results indicated that there are six subtypes of MLD. The names and definitions of these six subtypes are presented in Table 2.2.

Table 2.2: Six subtypes of MLD and their definitions (adapted from Bartelet et al., 2014, pp. 664-666).

	Name of Subtype	**Definition**
i.	A weak number line group;	The group of children who performed within average or above average range in all tasks except for number line performance.
ii.	A weak Approximate Number System (ANS) group;	Children who had strong spatial short-term working memory skills, next to high-average nonverbal IQ [6] and counting abilities but low performance on the approximate numerical knowledge and number line tasks, which are both tasks relying on the ANS.
iii.	Spatial Difficulties group;	Children with weak approximate numerical knowledge and spatial short-term working memory proficiency, and next to average-to-low average performance on the other cognitive measures.
iv.	Access Deficit group;	Children who had deficits with counting skills, despite having relatively strong general cognitive capabilities.
v.	No numerical cognitive deficit group;	This was the group with the fewest MLD children. No number processing strength or weaknesses could be distinguished on most tasks. However, they showed very strong verbal short-term memory [7] skills.
vi.	Garden-variety group.	Children who had a weak nonverbal IQ, but strong performance on the number line task. In the other cognitive processing measures, children's score was average-to-low average.

[6] Nonverbal IQ refers to the ability to analyse information and to solve problems using reasoning skills of a visual and hands-on nature.
[7] Verbal memory refers to the ability to remember some information and to use it to complete a given task.

Other researchers have proposed different sub-types for MLD. Rourke and Conway (1997) presented two sub-types of mathematical disabilities: a verbal type and a spatial type. Conversely, Geary (1993; 2004) suggests three sub-types including: a procedural sub-type, a semantic memory subtype and a visuo-spatial sub-type.

Several lists have been created to outline the key characteristics of children with MLD and/or dyscalculia (Bird, 2009; Chinn, 2012). These act as a guide for educators to identify learners with MLD as early as possible. For example, Bird's (2009) list of indicators for MLD can be found in Table 2.3.

Table 2.3: Characteristics of Children with MLD and/or dyscalculia (Bird, 2009, p.2).

Characteristics of Children with MLD and/or Dyscalculia
i. An inability to subitize (perceive without counting) even very small quantities; ii. An inability to estimate whether a numerical answer is reasonable; iii. Weaknesses in both short-term and long-term memory; iv. A weakness in visual and spatial orientation; v. Directional (left/right) confusion; vi. Slow processing speed when engaged in mathematical vii. activities; viii. Trouble with sequencing; ix. A tendency not to notice patterns; x. A problem with all aspects of money; xi. A marked delay in learning to read a clock to the time; and xii. An inability to manage time in their daily lives.

Geary (2004) and Bird (2009) showed that children with MLD have slow number processing. Similarly, Hannell (2005) highlights the difficulty children with MLD find with reading and understanding the concept of time. Moreover, Poustie (2000) points out that as adults, these individuals tend to write down appointments in their diaries in the wrong manner which in turn has its own repercussions. Bird (2009) and others (Chinn & Ashcroft, 2007; Dowker, 2005a; Ott, 1997) suggest that many learners with dyscalculia find difficulty with reading and writing numbers. For example, learners may write the numbers in the wrong direction, change the position of digits in a multi-digit number, omit numerals, or replace one numeral with another. Difficulties with sequencing and carrying out multiple processes have also been noted in these learners (Poustie, 2000; Townend & Turner, 2000). Bryant, Bryant, and Hammill (2000) carried out a study with 870 with MLD whom they compared to 854 students without MLD. They discovered that the sample with MLD had many difficulties in many areas, compared with those who did not. The most common were a persistent difficulty to solve word problems and a difficulty with carrying out *multi-step arithmetic*. Nonetheless, Bryant at al. demonstrate that not all individuals with MLD had difficulties in all areas of mathematics or even the same specific areas. My teaching experience allows me to agree to such a conclusion. Studies have shown *dissociations* and *double dissociations*. The term *dissociation* refers a single individual who does well in one component of arithmetic and not in another. *Double dissociation* refers to situations in which individuals with MLD are compared and found to have different mathematical profiles. This study will hopefully contribute to the knowledge currently available in this aspect as this area is explored through one of the subsidiary questions.

In this section I have intentionally left out another four characteristics of children with MLD. These are: a difficulty with number fact recalling; a weakness in memory functioning; a difficulty with linguistic skills including a deficit in acquiring and using mathematics language and; high mathematics anxiety. These four characteristics have been given increased importance in diverse studies and have been relatively widely researched. I will thus consider them in subsequent sections.

2.6.1 Working Memory

A definition of working memory, or short-term memory, proposed by Roselli et al. (2006) states that "working memory refers to the mental capacity responsible for the temporary processing and storage of information" (p. 804). The original working memory model was put forward by Baddeley and Hitch (1974). Working memory is believed to be made up of three main components. These are the *central executive,* the *phonological loop* (or *verbal memory composite),* and the *visual sketch pad* (Raghubar, Barnes, & Hecht, 2010; Van der Sluis, Van der Leij, & De Jong, 2005). Baddeley and Hitch's (1974) model has been adopted in various works about numerical processing (Raghubar et al., 2010; Van der Sluis et al., 2005). In their model, Baddeley and Hitch (1974) explained how working memory has three components which serve as storage spaces for data which is to be stored in working memory. Each of these components has a different function. Details of the function of each of these can be found in Figure 2.1.

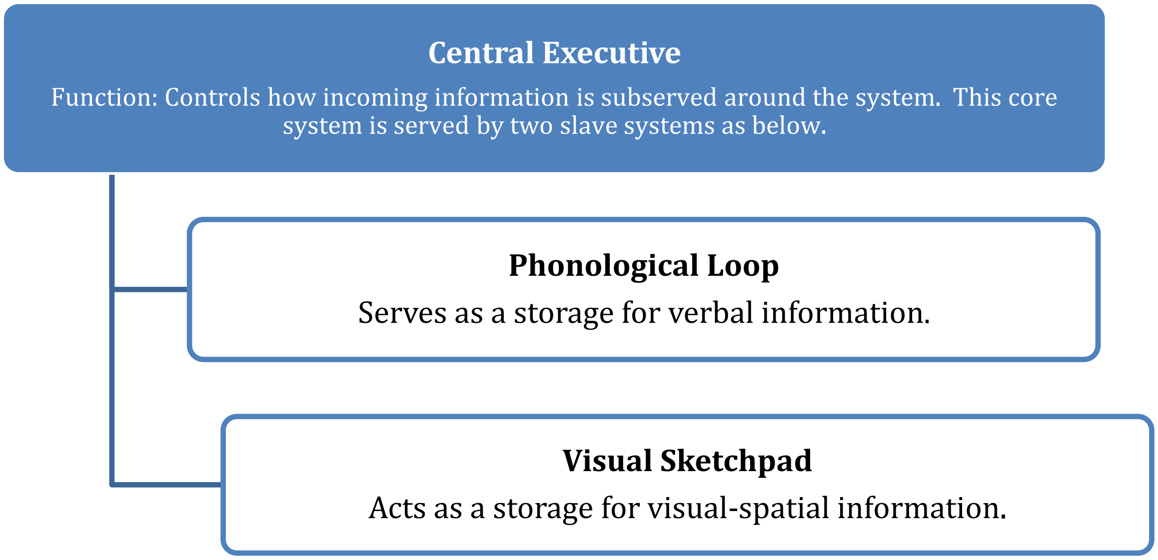


Figure 2.1: The components of working memory.

It has often been argued that a connection exists between working memory and mathematical abilities. In a review presented by LeFevre, DeStefano, Coleman, & Shanahan (2005) about this correlation, the authors propose that although such propositions have been made repeatedly, the evidence to support this argument is still quite limited. In this regard many controversies still remain as to whether domain-general abilities such as working memory are involved in mathematical development, abilities and performance. However recent studies have indicated that working memory is indeed related to, and an important component of, mathematical abilities and therefore for optimal performance on mathematical tasks.

Two different schools of thought have provided theories about the importance of domain-general abilities. One school of thought illustrates how domain-general abilities, like working memory and language, significantly predict performance in mathematics (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). For example, in a study by Gullick, Sprute, and Temple (2011), 17 adults were recruited to explore which domain-general factors were the foundations of individual differences in mathematics performance. Their research looked into working memory, nonverbal IQ, and other brain mechanisms linked to symbolic and non-symbolic number processing. After exposing the participants to diverse stimuli and functional Magnetic Resonance Imaging (fMRI) scanning, they found that the working memory of the individual participants (when compared to nonverbal IQ and mathematics achievement) showed the strongest effect on mathematics performance and conclude that “individual differences in very basic number processing tasks are associated with working memory ability” (p. 652).

Another school of thought illustrates that domain-specific mechanisms like numerosity and subitizing (Butterworth, 1999; Butterworth & Reigosa, 2007) are better predictors of mathematics abilities and performance than domain-general mechanisms such as working memory. Nonetheless the latter school does not underestimate the importance of the role played by domain-general cognitive abilities for mathematics performance. Instead it indicates that these domain-specific mechanisms are as important and can act as early indicators of mathematics ability. This because earlier studies (Starkey et al., 1990; Starkey & Cooper, 1980;) have indicated that infants react to different numerosities and thus may exhibit their ability in domain-specific tasks from an early age.

Raghubar et al. (2010) review four different forms of studies that have explored whether working memory has any impact on mathematical performance and ability. The studies involve:

i. Experimental dual task studies – experimental investigations of working memory and mathematics processing e.g. Mc Kenzie, Bull and Gray, (2003).

ii. Individual difference studies – working memory in children with MLD e.g. Peng and Fuchs (2016);

iii. Studies that tackle if, and in what ways, working memory is linked to specific mathematical outcomes and processes e.g. Halberda et al. (2008);

iv. Longitudinal studies e.g. Geary et al. (2009).

Following their review of the numerous studies, some of which will be described further in the coming paragraphs, Raghubar et al. (2010) state, “working memory is indeed related to mathematical performance in adults and in typically developing children and in children with difficulties in mathematics” (p. 119). However, they also explain that the relationship between mathematics and working memory is a complex one which is possibly affected by numerous factors including age, language of instruction, skills level and the way in which a mathematical task is presented. They thus underscore that studies need to cater for further precision in their description of mathematics outcomes and measures taken for working memory.

A study conducted by McKenzie et al. (2003) compared the process in which a sample of children (6 to 7 years old) performed in arithmetic problems under three different conditions to the process undertaken by an older group of children (8 to 9 years old) in the same tasks. They concluded that the younger children seemed to rely more heavily on visual-spatial strategies, making use of the visual-spatial sketchpad component of working memory more than the older children. On the contrary, through the strategies they used, the older children seemed to use a mixture of both the phonological loop and the visual-spatial sketchpad components of working memory. In a separate study by Imbo and Vandierendonck (2007) it was illustrated how the participants needed central executive working memory components to solve lengthy addition problems. This was especially evident when the subjects were also required to retrieve the answer from long-term memory on their own. Both these studies show that all parts of the working memory domain may be crucial in mathematics performance. This has been supported by different studies which have also indicated working memory deficits as having a main role in children's poor mathematics attainment (Keeler & Swanson, 2001).

A study that presents a convincing argument that working memory is related to mathematical achievement, is a longitudinal study carried out by Geary et al. (2009). The researchers followed their participants from kindergarten to the third grade. They used a specific test battery to measure the central executive, phonological loop and the visuo-spatial sketchpad (see Figure 2.1). The study illustrated how the different ways in which the working memory mechanism worked could discriminate between children's different levels of impairment and their different mathematical abilities. The central executive seemed to have a very high importance in the correct retrieval of addition facts.

As indicated by Gullick et al. (2011) "digit span tests are a well-validated measure of working memory thought to involve both the executive and phonological working memory systems" (p.645). Digit span tests are made up of two sub-tests: the forward counting span and the backward digit span. Both of these are tests for working memory. Whereas counting span is when the participant is asked to repeat a sequence of numbers forward, backward digit span is when they are asked to repeat the numbers starting from the last number read out. Different studies (Andersson & Lyxell, 2007; Passolunghi & Siegel, 2004; Wu et al., 2008) seem to agree that difficulties in mathematics are predicted more clearly by counting span rather than backward digit span. Counting span seems to differentiate children with MLD from children with other difficulties more evidently. The effectiveness of using backward digit span tests to identify children with mathematics learning difficulties is still debatable because although some studies have found this effective (D'Amico & Guarnera, 2005; Fuchs et al., 2008; Roselli et al., 2006), others have not found this relation (Landerl, Fussenegger, Moll, & Willburger, 2009; Van der Sluis et al., 2005).

2.6.2 Language Skills and the Knowledge of Mathematics

Language is a crucial part of mathematics learning. Research has repeatedly underscored that learning mathematics and engaging in mathematical tasks are both closely interwoven into written and oral language (Adams & Lowery, 2007; Schleppegrell, 2007). As stated by Melanese, Chung, and Forbes (2011) "every part of learning [mathematics] is dependent upon language, from the arousal of curiosity, to the teacher's explanation of a concept, to the formation of an understanding and the verbalization or written expression of that understanding" (p. 4). The fundamental role that language plays in mathematics learning is also highlighted by the National Council for Teaching Mathematics (NCTM, 2000). Furthermore, in the diagram presented by Haylock and Cockburn (2013) (see Figure 2.2), language is also seen as a major link in making connections and understanding mathematical ideas.

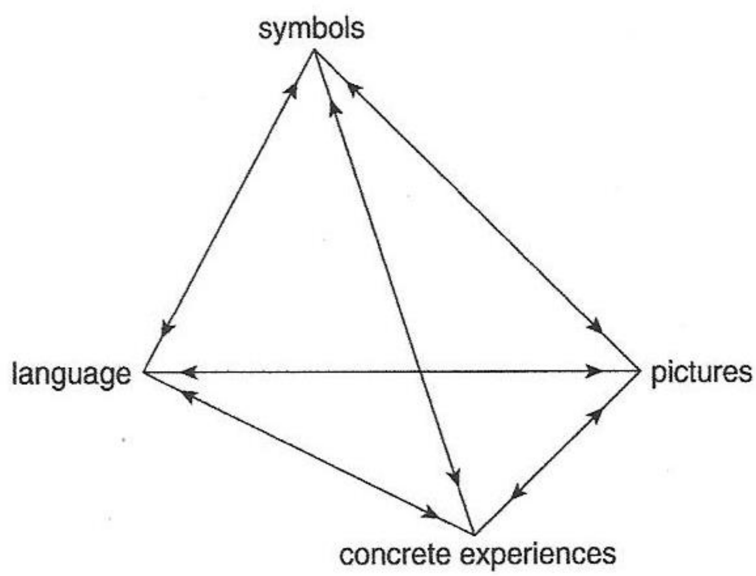


Figure 2.2: Significant connections in understanding number and number operations (Haylock & Cockburn, 2013, p.10).

Some research has indeed indicated that mathematics difficulties may stem from deficient linguistic processes rather than quantitative processes which are commonly associated to numerical processing (Vukovic, 2012). Such suppositions originate from research (Dehaene et al., 2003) which has illustrated that more than one part of the brain is responsible for manipulating numbers in a verbal form (processing numbers lexically, phonologically, and syntactically, like any other word). This research has shown how for this manipulation to take place, a linguistic circuit which seems to reside within a part of the brain called the left angular gyrus needs to support the specialized quantitative domain present in the part called the Inferior Parietal Lobule (or parietal lobe) (see Figure 2.3).

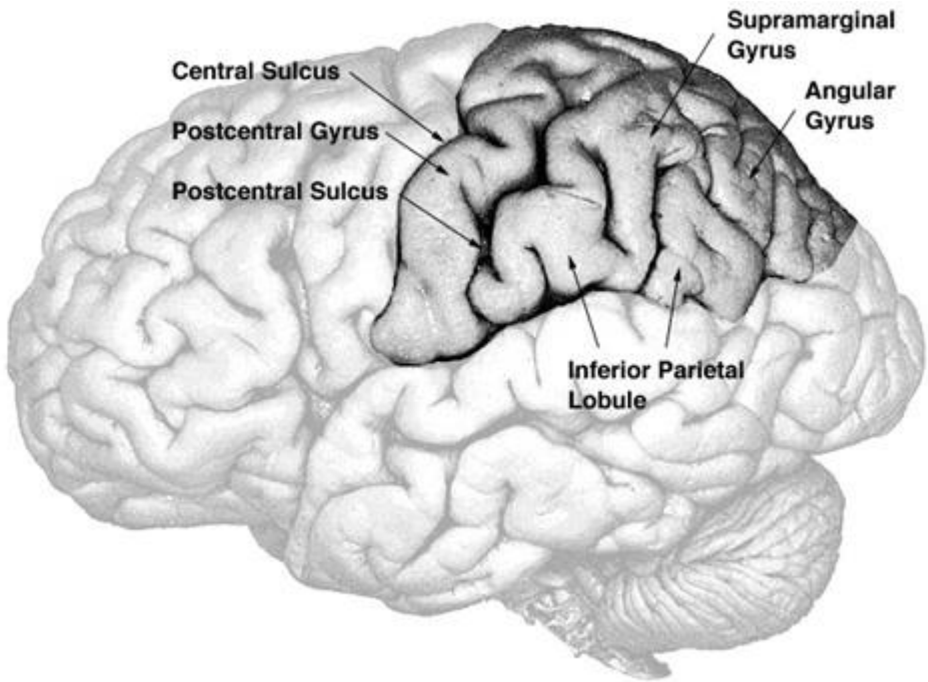


Figure 2.3: Sections of the brain highlighting the angular gyrus and parietal lobe (National Academy of Neuropsychology, 2000).

Moreover, various studies have demonstrated how phonological processing may be critical for mathematics learning (Jordan, Kaplan & Hanich, 2002; Simmons & Singleton, 2008). However, some children who have mathematics difficulties are good word readers and therefore would have probably mastered their phonological skills, thus indicating that phonological awareness (phonological awareness – blending sounds and segmenting them amongst others) is not the only influential factor (Jordan & Hanich, 2003; Landerl et al., 2009). Albeit these controversies and the acknowledgement that linguistic skills may have a prominent role in mathematics acquisition, few studies have explored how these may impinge on children's performance in mathematics.

The findings of a study carried out by Vukovic and Lesaux (2013) with 287 third graders illustrate how a child's general verbal ability[8] is directly involved in the process of numerical reasoning whereas skills related to phonology[9] are related to the execution of arithmetic problems. Their research was built upon a model proposed by LeFevre et al. (2010), known as the *Pathways to Mathematics* model (Figure 2.4). This model is important because it highlights the cognitive skills and early numeracy knowledge which seem to provide 'pathways' to mathematical outcomes.

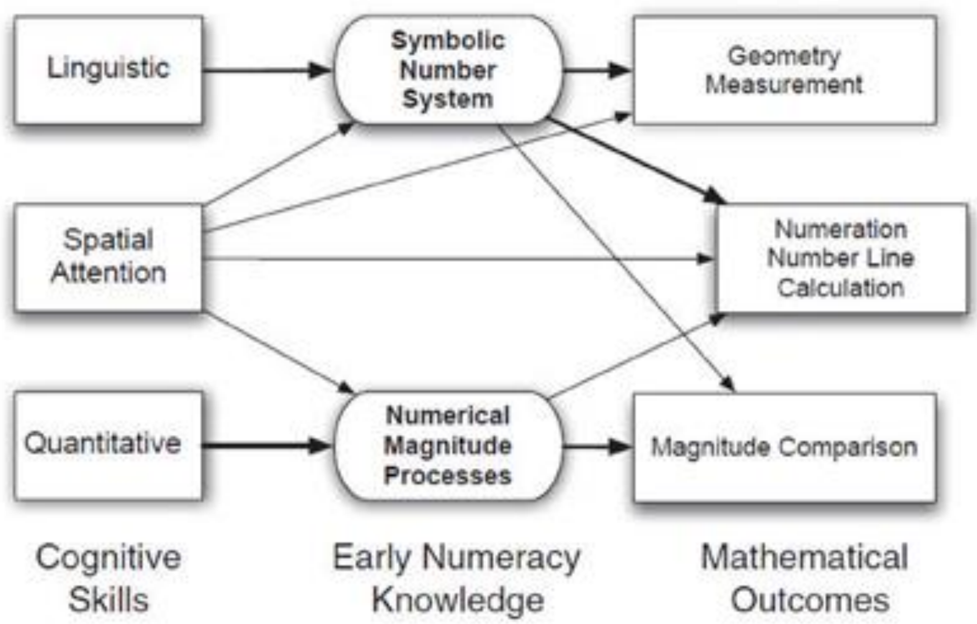


Figure 2.4: Pathways to Mathematics model (LeFevre et al., 2010, p. 1755)

Vukovic and Lesaux (2013) adapt this model and focus on the relationships among linguistic skills, symbolic number skill (which include number identification and numerical reasoning), and arithmetic knowledge. Symbolic number skill impacts on arithmetic knowledge which Vukovic and Lesaux classify in two forms: *procedural arithmetic* (the ability to add, subtract and solve whole number arithmetic computations) and *arithmetic word problems* (solving problems read out to them). On the other hand, linguistic skills have both a direct impact on arithmetic knowledge and an indirect one through their influence on symbolic number skill. Hence, linguistic skills have a dual impact on arithmetic knowledge, accentuating their importance. These connections can be seen in Figure 2.5.

[8] The authors (Vukovic & Lesaux) refer to general verbal ability as the ability to acquire "knowledge and skills associated to language and its everyday use" (p.88).
[9] The skills required to encode and manipulate numerical symbols in the same way humans encode and manipulate lexical ones (Vukovic & Lesaux)

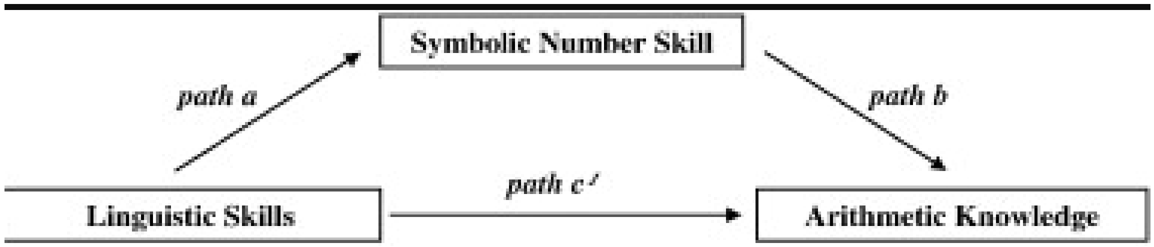


Figure 2.5: Predicted relationships among linguistic skills, symbolic number skill, and arithmetic knowledge (Vukovic & Lesaux, 2013, p.88).

Vukovic and Lesaux (2013) assessed the participants in six areas, namely: verbal ability, phonological skills, symbolic number skill, procedural arithmetic, arithmetical word problems, working memory and visual-spatial ability[10]. In their conclusions Vukovic and Lesaux (2013) highlight that, although mathematical thought can be seen as separate to language, "children most often need language to express, understand, and learn mathematics, which begs a nuanced understanding of its role in children's ability to express, understand, and learn mathematics" (p. 90).

Zhang and Lin (2015) also carried out a study basing their theories on the model presented by LeFevre et al. (2010) (see Figure 2.4), focusing on the linguistic cognitive aspect of the model. Their study was conducted with 88 Chinese 4-year-olds who were first tested at kindergarten second grade (K2). The participants were then also assessed at kindergarten third grade (K3) at age 5. The K2 assessments encompassed a spectrum of language skills including phonological skills, morphological skills (understanding and using word parts), and visual-orthographic skills (ability to recognize whether letters and numerals are correctly written) as well as visual-spatial skills. The subjects were then assessed in K3 for arithmetic skills including symbolic arithmetic (Arabic digits), word problems and non-symbolic arithmetic (subitizing arrays of objects). The results indicated that visual-spatial skills in K2 predicted the children's performance in all three arithmetic components assessed in K3. Additionally, the researchers noted that morphological skills seemed to predict word problems whereas phonological skills were good indicators for written arithmetic. Lastly, visual-orthographic skills seemed to influence both written and non-symbolic arithmetic. Their conclusions illustrated that general cognitive skills, particularly spoken and written language skills and visual-spatial skills, were related to the participants' competence in the three forms of arithmetic as assessed.

[10] Visual-spatial ability or skills refer to the ability to manipulate spatial relations among objects or space.

Emerson and Babtie (2010) recommend that "if a child has difficulty with the language of mathematics, it must then be considered whether they have a general language difficulty or a specific difficulty with the language of mathematics and mathematics-related words" (p.7). If the learner experiencing difficulties in mathematics does not have linguistic difficulties as those which have been just outlined, than it is important to gauge whether their difficulties stem from a difficulty with the subject-specific, mathematics language and mathematics-related words. Duffy (2003) describes two kinds of words used in mathematics classrooms; the *content* type and the *function* type. The content language describes the concept we are teaching and is linked to a mental picture, for example, *fraction*. The function type involves language that needs to be used to express ideas, for example, *you can solve this problem by…*. Acquiring mathematics language is rather complex especially since some words in mathematics have a different meaning to their use in everyday life (Adams, 2003). This language is specific to mathematics and is a component of the *mathematics register* (Halliday, 1978). Halliday (1978) defines a register as, "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p.195).

As stated by Lee (2006), "the mathematics register is a way of using symbols, specialist vocabulary, precision in expression, grammatical structures, formality and impersonality that results in ways of expression that are recognizably mathematical" (p.12). A key aspect of the *mathematics register* used by the teacher and the learner is subject-specific terminology such as *take away*, *product of* and *more than*. The kind of vocabulary which needs to be learnt is described in three categories (Halliday & Martin, 1993): words that have the same meaning in mathematics as they do in natural language e.g. *altogether*, words which are subject-specific and thus only have a meaning in mathematics language e.g. *numerator*; and words that have a different meaning in mathematics in relation to everyday language e.g. *take-away*. Moroever, Bobis, Malligan, & Lowrie (2004) argue that symbols (e.g. £), abbreviations (e.g. km) and conventions (e.g. dashes on the lines of a shape to show that they have the same length) are very widely used in mathematics so that mathematical ideas can be expressed concisely and thus this symbolic language is also part of the mathematics register. It has been noted that this symbolic language may increase "the difficulties that learners may experience because of this unique and complex part of the mathematics register" (Quinell & Carter, 2013, p.8). Hence, it is imperative that learners acquire both the vocabulary related to mathematics and its symbolic language too because as highlighted by Lee (2006) "using mathematical language can be a barrier to pupils' learning because of particular requirements and conventions in expressing mathematical ideas" (p.2). Similarly, Henderson, Came, and Borough (2003) also accentuate this reality when suggesting that "most of the difficulties seen in mathematics result from the underdevelopment of the language of mathematics" (p.20).

It is imperative that pupils learn to use the correct mathematical language to be able to express their mathematical ideas. This is essential because as underscored by Perry and Dockett (2002) "without the sufficient language to communicate the ideas being developed, children will be at a loss to interact with their peers and their teachers and therefore will have their mathematical development seriously curtailed" (p. 101). Difficulties with acquiring and making use of mathematical language is especially evident in students with MLD and dyscalculia. Poustie (2000), for example, stresses that dyscalculic learners tend to confuse language related to time with that related to money. As a result, different authors (Townend & Turner, 2000; Pimm, 1987) state that it is crucial that educators focus on the mathematical register to be acquired and that they provide different situations in which they model the use of such language whilst allowing learners to express themselves with similar language.

O'Connell (2007), recommended that specialized mathematics vocabulary should be taught explicitly. Every mathematics learner should be considered as a *mathematics language learner* (Thompson, Kersaint, Richards, Hunsader, & Rubinsten, 2008). Marzano (2004) lists eight research-based features of teaching direct vocabulary effectively. Two of these are: providing opportunities for students to discuss the terms being learnt and teaching students the meaning of word parts e.g. *kilo*- and *milli*-. These are indeed two strategies which I found effective when teaching mathematical language during my professional experience.

A particular area of mathematics learning, in which language may have a great impact, is word problems. As outlined by Zhang and Lin (2015) numbers can be represented in at least three modes: *Arabic digits* (1, 2, 3 etc.), *words* (one, two, three etc.) and *non-symbolic quantities* (arrays of objects). A combination of these three modes is necessary for word-problem solving, a crucial component of mathematical acquisition and achievement. Consequently Geary, Hoard, Nugent, and Bailey (2013) have illustrated that mastering competence in these different modes of mathematical representations is fundamental for more advanced mathematical learning to take place, including solving word problems. Solving arithmetic word problems is an important component of mathematics learning (Castro, 2008). However, word problems also have a linguistic component. As Bernardo (2005) suggests, "word problems have a clear linguistic component because the problem elements are embedded within a text" (p. 415). The language component within such problems takes mathematics to a higher difficulty level than non-verbal computations because such tasks are highly dependent on an individual's ability to read and understand the problem at hand (O'Connell, 2000; Rothman & Cohen, 1989).

When considering the influence of language on solving word problems one must also consider the difficulty incorporated in learning mathematics through a second language. This is the case in Malta where mathematics is taught in English which is a second language to most of our learners. Although Maltese children are brought up in a bilingual community, most of them still speak mainly Maltese at home and at school (National Statistics Office, Malta, 2011). A study carried out by Bernardo (2002) focuses on the language component in mathematical word problems specifically among bilinguals. Bernardo illustrated that the results indicated "first-language advantage in both understanding the word problem texts and in solving the word problems" (p. 295).

A similar local study carried out by a colleague and myself (Baldacchino & Cassar, 2008) drew similar conclusions to that of Bernardo (2002). Thirty participants were asked to solve word problems in both Maltese and English. Subsequently, they were asked to solve the same computations needed to solve the word problems in a non-verbal form. The findings of this study indicated that the subjects generally performed better in the non-verbal computation sheet than in the word problems regardless of the language in which they were presented showing that language did influence the performance of the subjects in solving arithmetic word problems. Moreover, the children generally performed better in the word problems which were presented in their first language and seemed more able to solve the more difficult problems when these were presented in their first language rather than their second. The researchers also noted that when the problems were presented in their second language the children seemed to use 'tricks' to solve them, for example, always associating the term *altogether* with addition. As a result, the children sometimes added instead of subtracted, to find the correct answer, because they had seen this term.

This section has outlined the complex connection between mathematical competence and linguistic skills. The importance of mathematical language in the acquisition of mathematics has been highlighted and the influence of learning mathematics in a second language has also been explained. The next section will delineate the lifelong consequences related to MLD and how the characteristics of MLD mentioned so far may impinge on an individual's quality of life.

2.6.3 MLD and Lifelong Consequences

MLD may have a major influence on one's adulthood if it is not tackled through the right intervention at an earlier stage. Despite the growing importance that employers place on numeracy skills, many still believe that being able to deal with numbers and graphics is not as significant as being able to read and write proficiently (Bynner & Parsons, 2005). The longitudinal study carried out by Bynner and Parsons (2000), presented in Section 1.3, illustrates the substantial influence which numeracy skills may have on one's opportunity for employment. Further data related to this same study was collected in 2005. This data produced similar findings. Some of the effects found for men and women who lacked basic numeracy skills can be seen in Table 2.4:

Table 2.4: Conclusions drawn for men and women who lacked basic numeracy skills (Bynner & Parsons, 2005, p. 34 - 35).

Effects for Men	**Effects for Women**
Less likely to find a full-time job and more likely to be unemployed and in semi-skilled or unskilled employment; Less likely to use a computer at work and to have had training related to their work and to have had a promotion in the current employment; Less likely to have access to an employer pension; Less likely to be home owners and more likely to form part of a non-working household; More likely to be arrested (this may also be due to a general low educational achievement independent of numeracy alone); A greater risk of developing depression having a lack of control over their lives	To be out of the labor market; To live in a non-working household; Not to vote; Not to have any political interest; To have poor physical health; To be depressed; To feel they lacked control over their lives.

The findings are thus significant ones that emphasise the importance of gaining at least the basic skills in numeracy to live a better-quality life since the absence of these skills causes the disadvantages mentioned in Table 2.4. Butterworth and Yeo (2004) also illustrate the importance that this should be given by highlighting that adults who do not have the basic numeracy skills are more likely to be unemployed, ill, depressed and arrested.

Other research has drawn similar conclusions. The National Mathematics Advisory Panel (2007) of the United States (U.S.) has suggested that pupils who would have mastered their numeracy skills by the end of their middle school are more likely to graduate from college. Sadler and Tai (2007) have also illustrated that the mastery of advanced mathematics skills is closely related to the success one may have at college-level courses and vocations in the sciences. As a result, Jordan, Glutting, & Ramineni (2008) underscore that "poor achievement in mathematics can have serious educational and vocational consequences" (p.45).

Further research has focused upon the conspicuous impact that poor numeracy skills may have on a dyscalculic's life. Poustie (2000) suggests that dyscalculic learners may "exist in a world of total mathematical confusion" (p.147). Moreover, Bevan and Butterworth (2007) have explained how these difficulties may impinge on the individual's affective domain and lower their confidence in mathematics as well as develop various negative feelings towards the subject.

2.7 The Affective Domain

Bloom (1956) as well as Anderson and Krathwohl (2001) illustrate how learning takes place in three domains: the cognitive, affective and psycho-motor domain. The cognitive domain is related to a person's acquisition of knowledge, reasoning and understanding. The psychomotor domain encompasses the acquisition of gross and fine motor skills whilst the affective domain is responsible for feelings, emotions and beliefs. Similarly, Johnston (1998) segments an individual's learning process into three sectors: cognitive, conative (performing an action using gross and/or fine motor skills) and affective. As early as 1965, Evans had indicated the great influence that the affective domain has on mathematics learning. However only lately has there been an increased interest in the role played by the affective domain. This has given rise to research about this crucial sphere in one's learning (Grootenboer & Hemmings, 2007; Schuck & Grootenboer, 2004). A study carried out by Grootenboer (2003a) indicates that positive mathematical beliefs, feelings and attitudes have a constructive impact on mathematics achievement.

In a separate study, Grootenboer (2003b) provides a model for the conceptualization of the affective domain (see Figure 2.6).

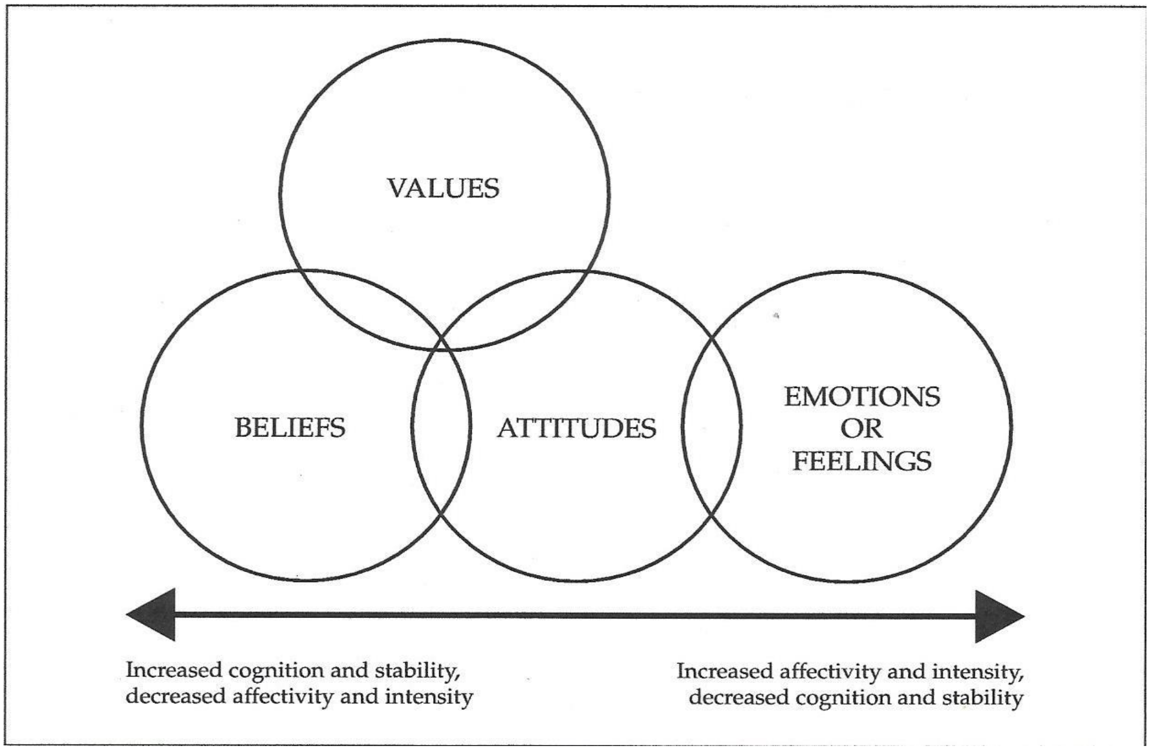


Figure 2.6: A model representing conceptions of the affective domain as presented by Grootenboer (2003b).

The model suggests that positive values, beliefs, attitudes and emotions lead to an increase in *cognition* and *stability* and a decrease in *affectivity* and *intensity* which is optimal for learning. On the contrary, negative values, beliefs, attitudes and emotions towards a subject, negatively impinge on learning as they result in decreased cognition and stability.

In contrast with Grootenboer's (2003b) model (see Figure 2.6), in which the affective domain has four main components, Sriraman (2003) proposes that there are three sectors to the affective domain: *beliefs*, *attitudes* and *emotions* excluding *values*. Sriraman excludes *values* saying that they are usually seen as having the same construct as *beliefs*. On the other hand, however, Clarkson, FitzSimons and Seah (1999) suggest that "values are demonstrated in the actions carried out by a person, whereas beliefs can be verbally assented to, but do not necessarily lead to observable behavior in public" (p.3). I agree that values and beliefs are similar but different sectors of the affective domain.

More recently, Ernest (2011) suggested that the affective domain is made up of the following factors: *attitudes to mathematics, beliefs about mathematics, appreciation of mathematics, perception of mathematics, classroom climate and other aspects like feelings, values about the subject* (p.100). This seems to provide a more comprehensive list of components which impinge on the affective domain and which can be useful to educators so that they can provide an optimal learning environment for mathematics learning to take place. As Gresalfi and Cobb (2006) recommend, "it is not sufficient to focus exclusively on the ideas and skills that we want students to learn" (p.55). The experience which individuals have of mathematics inside the classroom and outside has an impact on all the aspects of the affective domain in relation to mathematics learning.

Values and beliefs about mathematics develop through experiences and are very difficult to change. On the other hand, attitudes seem more malleable (Hannula, 2002). Albeit the segment of 'attitudes' has been mostly widely researched, when compared to the other segments of the affective domain, the term *attitude* is yet considered to have an ambiguous construct (Zan & Di Martino, 2007). However, despite different definitions for the term 'attitudes' having been put forward, all these definitions "generally include the idea that attitudes are learnt, manifest themselves in one's response to the object or situation concerned, and can be evaluated as either being positive or negative" (Way & Relich, 1993, p.581). Attitudes are closely related to emotions and feelings. Hogg and Vaughan (2005) describe them as, "a relatively enduring organization of beliefs, feelings, and behavioural tendencies towards socially significant objects, groups, events or symbols" (p. 150). Three of the components of the 'attitudes' domain which may impinge on the performance of an individual in mathematics are motivation (Silver, 1985), confidence in mathematics (Wilkins, 2000) and mathematics anxiety (Ashcraft & Kirk, 2001). Much research has shown how mathematics anxiety has a high impact on pupil performance and achievement. Hence, this aspect will be focused upon in the next section.

2.7.1 Mathematics Anxiety

One of the aspects which Ernest (2011) identifies as a key factor within the 'attitudes' realm is the fear of mathematics – more commonly known as *mathematics anxiety*. Richardson and Suinn (1972), who designed one of the earlier assessments for mathematics anxiety – Mathematics Anxiety Rating Scale (MARS), defined this fear of mathematics as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of ordinary life and academic situations" (p.551). Faust, Ashcraft and Fleck (1996) have indeed shown that mathematical tasks often cause a high level of anxiety, to a much greater extent than other difficult activities. Burns (1998) obtained results suggesting that more than two thirds of the adult population feel anxious about performing mathematical tasks. Similarly, Chinn (2012) has indicated that mathematics anxiety can influence the acquisition of mathematics.

The biological composition of the human body allows mathematics anxiety to act as a disability in itself. Mathematics anxiety has the potential of filling the area within the brain responsible for the working memory, thus interfering with the effective processing of numerical tasks (Adler, 2001). Sousa (2008) explains how anxiety triggers off the release of a hormone called cortisol in the bloodstream. Cortisol forces the brain to refocus its attention on the cause of anxiety rather than the processing of numerical tasks. As a result, it has been evidenced that mathematics anxiety may also have other negative effects on the body's performance. These may include increased heart rate, sweating and other physical features that indicate worrying. An illustration of how this occurs can be found in Figure 2.7.

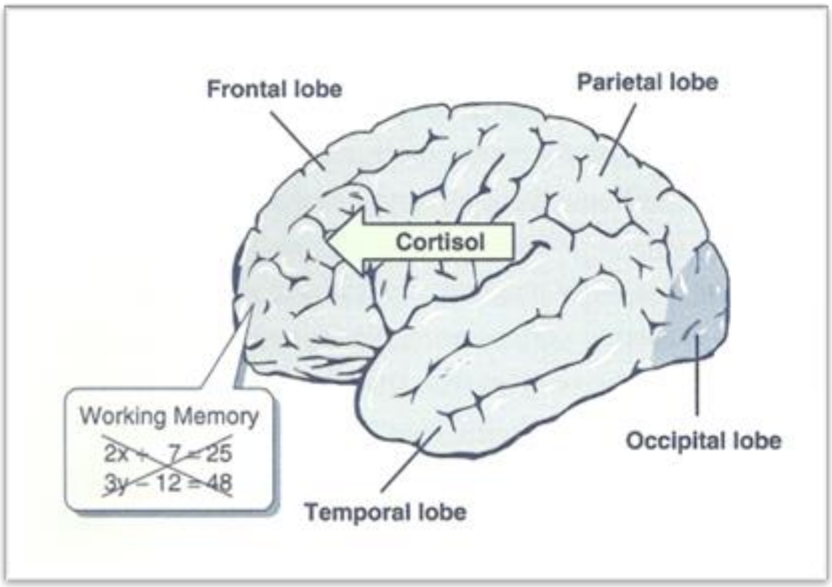


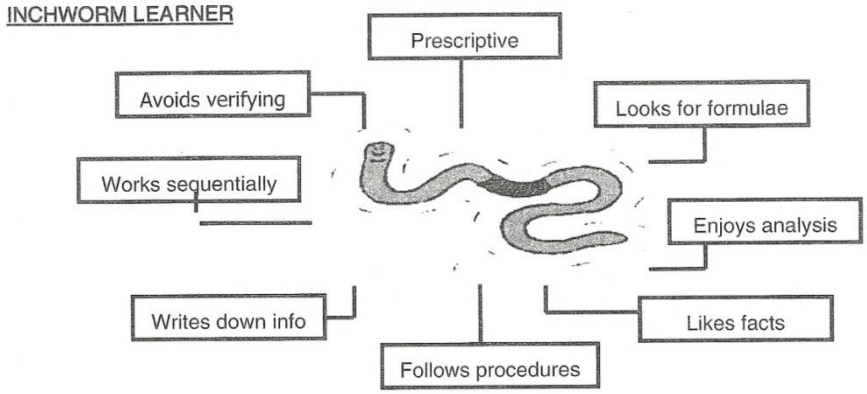
Figure 2.7: The release of cortisol in the blood due to anxiety caused by mathematics (in Sousa, 2008, p.172).

2.5.1.1 Causes of Mathematics Anxiety

Research about mathematics anxiety has shown that anxiety can stem from different causes. Tobias (1978), who is considered as a pioneer in this field of research, suggests that this anxiety can derive from either one or more of: *the language of mathematics, an inadequate preparation for the subject, repeated absenteeism (since mathematics is cumulative in nature) and the nature of mathematics as being as 'exact science'*. More recently, Hadfield and McNeil (1994) offered a model to illustrate that mathematics anxiety can derive from three factors: *environmental, intellectual and personal*. A similar tripartite model was also proposed by Strawderman (2017) who presented mathematics anxiety as originating from a social/motivational, an educational and an emotional dimension. Albeit different terms, Hadfield and McNeil (1994) and Strawderman (2017) seem to be referring to essentially the same construct.

One of the environmental/social causes of mathematics anxiety is the childhood experiences related to mathematics. Providing positive experiences for children is thus crucial as these experiences may influence their future attitudes and achievement in the subject. As Bishop and Nickson (1983) suggest, "as a result of the kind of mathematics they [pupils] had at primary level and more particularly, their achievement or lack of it with respect to the subject, attitudes to it are likely to be entrenched by the time they enter secondary school" (p.20). Moreover, research has shown that teachers who have higher levels of anxiety unintentionally pass this anxiety to their students (Department for Education and Skills, DfES, UK, 2002).   Consequently, teachers should be made aware of this anxiety rather than suppress it. Another issue that teachers should be aware of is that gender differences and stereotypes influence the mathematics anxiety levels of pupils, impacting their performance in the subject. This will be discussed at greater length in Section 2.8.

Some of the intellectual/educational factors which increase mathematics anxiety include difficulties with mathematics learning especially when these are related to dyscalculia. It has been noted that learners with dyscalculia seem to have an increased measure of anxiety when engaging in mathematical tasks. In a study carried out by Rubinsten and Tannock (2010) with 23 participants (12 with dyscalculia and 11 control), they observed that there was a direct link between mathematics anxiety and achievement. They illustrated that all the children with dyscalculia permitted their mathematics anxiety to have an impact on their performance, to a much greater degree than typically developing children. Additionally, teaching and learning approaches to mathematics also seem to have an impact on the learner's affective learning domain. While there are many perspectives as to how far one can identify different learning styles for mathematics, some researchers have proposed that learners engage in mathematics using one of two styles: the 'inchworm' and 'grasshopper' styles or a mix of both. Figure 2.8 illustrates the characteristics of each learning style as presented by Henderson et al. (2003).



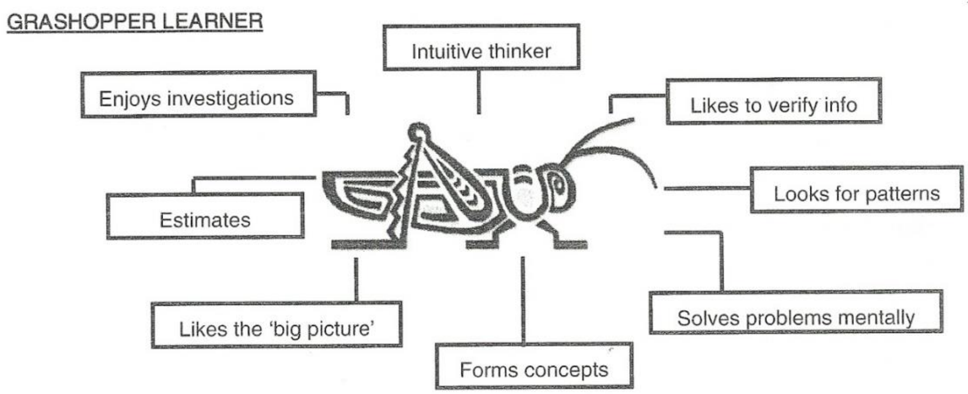


Figure 2.8: Characteristics of inchworm and grasshopper type learners (Henderson et al., 2003, p. 25).

Henderson et al. (2003) propose that if the teaching styles used by the teacher in the classroom match the learning styles of the learners, this may positively affect their attitudes towards mathematics (Henderson et al., 2003).

The personal/emotional causes of mathematics anxiety are numerous. Primarily, feelings of avoidance and frustration towards mathematics may stem from low self confidence in the subject resulting from frequent failures. As Adler (2001) outlines, "to be able to maintain motivation, a genuine feeling of success is essential" (p.7). Feelings of failure thus seem to play a key role in the levels of mathematics anxiety. The direction of causation is complex, as failures lead to anxiety and anxiety may lead to further failures. In fact, Emerson and Babtie (2010) show that anxiety levels "can actually rise to the point where it paralyses their [dyscalculic learners] ability to perform even simple mathematics operations" (p.9). Moreover, Ashcraft and Kirk (2001) have illustrated that a part of poor achievement in mathematics, which highly anxious learners normally have, originates from the depletion of the cognitive functions which are needed to carry out mathematics tasks. Thus, although mathematics anxiety may be a problem for both high and low achievers, low achievers may begin to experience failure from a tender age and develop negative feelings, such as fear towards mathematics. It seems that there is a vicious cycle in which fear instigates low performance and vice versa.

2.7.2 Gender Differences and Mathematics Anxiety: The Case for Boys

Gender differences in the long-term consequences of acquiring basic numeracy skills show that the lack of acquiring these skills has a greater impact on the life of women (Bynner & Parsons, 2005). However, gender differences in mathematics achievement are now diminishing or even non-existent in countries which promote gender equality (Devine, Fawcett, Szucs, & Dowker, 2012; Else-Quest, Hyde & Linn, 2010; Guiso, Monte & Sapienza, 2008). Albeit a gender stereotype still persists in many cultures that treats girls as performing worse than boys, this myth has been seriously challenged. The Trends in International Mathematics and Science Study (TIMSS) results reported by Sturman et al. (2012) have illustrated that the country that performed best within the UK, Northern Ireland, showed no significant gender difference in the mathematics attainment of male and female participants. Moreover, of the 50 countries of the world, which participated in this assessment, just under half (24 countries) showed a significant gender difference. In most countries this gender difference favoured boys, but the reverse was true in four. However, various studies have shown gender differences apparent in the affective domain rather than the cognitive domain of learning mathematics. They have shown that females have higher mathematics anxiety which in turn may influence their mathematics performance negatively.

Beilock, Gunderson, Ramirez, and Levine (2010) carried out a research study with 17 first and second grade female teachers and their 117 students. Their teachers were asked about their mathematics anxiety levels and their respective students were also given tests of mathematics performance as well as asked about their beliefs about gender differences in mathematics. The results of this study showed that "by the school year's end, female teachers' mathematics anxiety negatively relate[d] to girls' gender ability beliefs" (p.1861). The researchers speculate that mathematics-anxious female teachers influence girls into believing the stereotype that boys are better than girls at mathematics thus negatively impacting girls' achievement causing difficulties with their learning of mathematics. They illustrated that the more anxious the teachers were about mathematics "the more likely girls (but not boys) were to endorse the commonly held stereotype that 'boys are good at mathematics, and girls are good at reading' and the lower these girls' mathematics achievement" (p.1860).

In a separate study conducted by Devine et al. (2012) similar findings emerged. This study once again focused on finding out more about the effect of gender difference in the affective domain of mathematics learning. The researchers carried out their study with 433 British secondary school children in Years 7, 8, and 10 (ages 11 to 14). The students were asked to complete mental mathematics tests and both 'mathematics anxiety' and 'test anxiety' questionnaires. Their findings indicated that males normally have low mathematics achievement which leads to high mathematics anxiety at all Year group levels. However, in the case of girls, the prior low achievement in the subject seems to only predict high mathematics anxiety at critical transition points like when they are to move from the first phase of secondary education to the second phase. The authors offer a possible explanation for this suggesting that "girls tend to experience MA [mathematics anxiety] whether or not they have any intrinsic difficulties in mathematics, whereas MA in boys is more likely to reflect initial problems in the subject" (Devine et al., 2012, p. 3). Hence, since the participants of this study are boys who are underachieving in the subject, their anxiety level may be higher than that of other boys not experiencing such difficulties. Devine et al. list several reasons why some girls may be more anxious than boys. Primarily they show that the difference in experience offered to boys and girls in their childhood may lead to sex-role socialization which pushes girls to view mathematics as a male domain rather than as a female one. It may however also be that females may be more honest about the mathematics anxiety they feel in relation to the learning of mathematics, leading to an exaggeration of the actual differences in anxiety. Moreover, other studies have indicated that there seems to be a gender difference in self-confidence displayed in relation to mathematics learning. Boys seem to be more confident at the subject (Guiso et al., 2008) and seem to have higher mathematics self-efficacy levels (Pajares, 2005).

2.7.3 Increasing Positive Affect

The affective domain has a high impact on one's learning of mathematics. It is thus essential that this domain is given prominence when implementing any form of intervention programme with children having MLD. Children with MLD usually need plenty of support in raising their general self-esteem, enjoying mathematical tasks more (Silver, 1985), increasing their confidence in mathematics (Stevens, Olivarez, Lan, & Tallent-Runnels, 2004) and nurturing intrinsic motivation (Sternberg, 1983).

A study carried out by Supekar, Iuculano, Chen, and Menon (2015) with 46 children in third grade suggested that an intensive 8-week one-to-one cognitive tutoring programme which was designed to improve arithmetical fluency, had a positive impact on the children's attitudes towards mathematics. Following the programme, the participants showed reduced mathematics anxiety on the Scale of Early Mathematics Anxiety (SEMA) and this was also reflected in alterations to the brain circuits related to mathematics anxiety. Supekar et al. (2015) described this as 'exposure treatment' – a technique in behaviour therapy which is known to alleviate anxiety disorders. These results could, however, also be due to an increase in arithmetical fluency because of the training, contributing to reversal of the vicious circle in which anxiety causes poor performance which in turn increases anxiety.

Goldin (2000) suggests that *meta-affect* is an essential aspect of the affective domain. As explained by DeBellis & Goldin (2006) "meta-affect refers to affect about affect, affect about - and within - cognition about affect, and the individual's monitoring of affect through cognition" (p.136). Skills of meta-affect involve thinking about one's own progress and recording one's achievements to remind the self about one's successes. In my view, at school learners are seldom given the opportunity to reflect about what they have learnt, and to highlight their successes, because most of the time this is done by an external person like the class teacher. However, teaching the learner to carry out such an exercise may help them to change their attitudes towards mathematics to more positive ones and therefore to perform better in mathematics.

2.8 Other factors related to MLD

Different studies have proposed several other factors which impinge on MLD. Literature (Duncan et al., 2007; Jordan & Levine, 2009; Sammons et al., 2002) indicates that children coming from low-socioeconomic backgrounds are disadvantaged in the learning of mathematics and thus tend to struggle with mathematics learning throughout their years at school. The home environment which children come from may indeed accentuate MLD as will be discussed later (Crosnoe & Cooper, 2010). Additionally, MLD seem to be hereditary and therefore this may be yet another factor which influences MLD. Finally, many studies have been dedicated to exploring the neurobiological causes of MLD especially when these are specific, as in the case of children having dyscalculia. These factors will be discussed in the following sections.

2.8.1 The Socio-economic Disadvantage & Home Background

"Poverty, rurality, ethnicity, gender, language, culture, race, among others, have been defined as the variables that constitute socioeconomic influences of mathematical achievement" (Valero & Meaney, 2014, p.984). Some research within the field of MLD has explored whether the socio-economic background of children has an impact on their acquisition of mathematics. The outcomes of these studies have shown that children who come from low socio-economic backgrounds seem to be disadvantaged in relation to their peers who come from higher socio-economic backgrounds (Burr, 2008). Jordan and Levine (2009) suggest that "low-income children are four times more likely than their middle-income counterparts to start school at a low level and to show flat growth between kindergarten and first grade in key areas of number competence" (p. 64). It has been illustrated that this is the result of the lack of acquiring basic numerical skills which provide the necessary readiness to engage in the academic learning that takes place at school (Duncan et al., 2007; Hertzman & Power, 2006; Lee & Burkham, 2002; National Research Council, 2009). Moreover, it has been indicated that this disadvantage can be documented even before children enter formal schooling (Burchinal et al., 2011). As a result, recent studies (Dyson, Jordan, & Glutting, 2013; Geoffroy et al., 2010) have discussed the influence which formal childcare may have on empowering these children to develop the necessary numeracy skills which other children may develop from home since these skills affect a learner's readiness to acquire more complex ones.

The gap between children from different backgrounds may be the result of numerous factors. Although many parents engage in 'number talk' (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006) with their children, others may not appreciate the importance of such talk at an early age. Additionally, some parents may be unaware of the importance which counting and comparing quantities have in a child's development of numeracy. Some children coming from low socio-economic backgrounds may have limited number sense when they enter formal schooling (Malofeeva, Day, Saco, Young, & Ciancio 2004). Number sense in Malofeeva et al.'s (2004) study is taken to be based on three important pillars of numeracy acquisition including the understanding of whole numbers, number operations and number relationships.

Ramaa (2015) conducted a research project in India to explore which mathematical competencies are difficult for children from socially disadvantaged backgrounds. The research was carried out with 138 participants in Grade 5 (ages 9 to 10) from low socio-economic backgrounds. The participants included learners who were extremely socially and materially deprived due to numerous factors encompassing poverty, discrimination or other unfavourable circumstances. Participants were excluded unless they were performing at an average level in reading and writing, to eliminate other factors which could impinge on the acquisition of mathematics such as sensory difficulties and/or a below average IQ. Ramaa assessed the participants' performance in all four operations (addition, subtraction, multiplication and division). 47.11% of the learners were found to struggle with addition, 52.83% had not mastered subtraction whilst 78.71% and 65.92% of the participants had not mastered multiplication and division respectively. Following these findings Ramaa suggested that the socio-economic background of the students has a strong influence on their mathematical performance, even in the case of those who are performing at an average level in literacy.

A study carried out by Ginsburg, Lee, and Boyd (2008) indicated that children come to school with a baggage of informal experiences related to mathematics. This baggage of experiences seems to be highly predictive of children's attainment in mathematics as they start formal schooling. Longitudinal studies (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008) have confirmed that specific number competencies in kindergarten, namely counting, number knowledge and number operations, are highly predictive of later mathematics achievement in one's school trajectory at least till Grade 3 (7 to 8 years old). It appears that the baggage carried by children coming from low socio-economic backgrounds is not as rich as that gained by their counterparts coming from middle class backgrounds. As a result, researchers and policy-makers (Baroody, Eiland, & Thompson, 2009; Chard et al., 2008) are exploring the possibility of using childcare to minimize gaps between different groups of learners since these competencies have been shown to be malleable at such a tender age.

Another influencing factor of MLD related to the learner's social and economic background is the home environment. Studies (Crosnoe et al., 2010; Crosnoe & Cooper, 2010; Hannell, 2005) have indicated that growing up in an environment that stimulates cognitive development predicts the learner's immediate development and longer-term academic achievement. Moreover, a study carried out by Young-Loveridge (1989) conjectured that children whose mothers were not confident with mathematics were also not confident themselves.

A study carried out by Sonnenschein and Galindo (2014) in the United States (U.S.), with children coming from different race and ethnic backgrounds, examined the impact of the home and classroom environment on the mathematical achievement of kindergarteners. The results of this study illustrated that no matter what background in relation to race/ethnicity, children who started kindergarten with pre-school mathematics skills achieved scores which were much higher than those not having such skills. The researchers found that the home and classroom environment of participants belonging to the various backgrounds - Black, Latino and White - were significantly different. Nonetheless reading at home was one of the factors which seemed influential on mathematics achievement. If the children began kindergarten with some proficiency in mathematics (like being able to count till 10), the mathematics gaps present for Latino-White pupils was no longer significant after controlling for home and classroom factors. On the contrary, the mathematics gaps between the Black-White remained significant. When the learners would not have started kindergarten with the required proficiency in mathematics both the Latino-White and Black-White mathematics gaps remained significant. The proficiency in mathematics gained from the informal experiences of mathematics which children normally acquire from their home environments and from their parents/guardians is highly significant (Sonnenschein & Galindo, 2014) and can impinge on individuals' academic achievement in mathematics.

The studies presented in this Section illustrate that mathematics education is not equally accessible to all learners. The OECD (2014) reports that "across OECD countries, a more socioeconomically advantaged student scores 39 points higher in mathematics – the equivalent of nearly 1 year of schooling – than a less-advantaged student" (p.12). This urges me to refer back to the *social theory* presented in Section 2.3.2 that problematizes some of the approaches generally taken to the teaching and learning of mathematics and the content in school curricula which should be accessible to all learners but in fact may not be (Pais & Valero, 2014). The participants of this study will have different home and socioeconomic backgrounds which may have influenced their mathematics learning trajectory. Although the content in the school curriculum and its relevance to these learners' lives may be questionable, I will aim to improve the children's experiences within it and to support them to acquire the basic skills needed to close their gaps in their learning of mathematics.

2.8.2 Genetic Influences on MLD

As early as 1950, Hallgren (1950) indicated that genetics influence the development of mathematical skills. Other research has also since drawn similar conclusions (De Fries & Gillis, 1991; Wadsworth, DeFries, Fulker, & Plomin, 1995). Barakat (1951) carried out a review which indicated that when Cyril Burt studied children having difficulties with mathematics, it was common to find that there were other family members who struggled with mathematics learning. Barakat (1951) shows that the number of family members found to have difficulties with mathematics for those learners who were also struggling with mathematics was nearly three times more than it was for other learners who were not having such difficulties. It should be noted, however, that Burt's studies have since come under question. Kosc (1974) found that specific learning difficulties in mathematics tended to run in families.

A more recent study was conducted by Davidse, de Jong, Shaul, and Bus (2014) with a pair of 9-year old identical twins with mathematical difficulties. Both twins performed similarly in the tasks given. Their performance was far below that obtained by the control group on a non-symbolic comparison task and on a subitizing task. Both were found to have impairments in visual-spatial processing and working memory and this seemed to be hindering the development of number sense which is fundamental for the acquisition of mathematics. Although the main aim of this study was to determine whether number sense was the core impairment hindering mathematics learning, Davidse et al. also deduced that both twins performed in similar ways. Although this was a study of just one set of twins, it suggests that genetics may influence the acquisition of mathematics.

A larger-scale twin study (Alarcón, DeFries, Light, & Pennington, 1997) indicated that when one of the twins was assessed with dyscalculia there was an increased possibility that the other twin would be assessed with the same profile. In this study, Alarcón et al. established that 58% of the identical twins participating in their study and 39% of fraternal twins were concordant for dyscalculia. A separate study carried out by Shalev and Gross-Tsur (2001) also revealed similar findings. The researchers found that half of the siblings of children assessed with dyscalculia were also dyscalculic themselves. The results of this study indicated that the risk of being assessed with dyscalculia is 5 to 10 times more than for those children whose siblings have never displayed traits of this specific learning difficulty. Albeit the studies mentioned in this section have contributed to an understanding of the etiology of MLD, this aspect of MLD is still under researched and further research is necessary.

2.8.3 Neurobiological Perceptions of MLD

Many studies within the field of MLD and dyscalculia have investigated the hypothesis that brain function differs in children with MLD or dyscalculia from that of typically developing children. A study produced by Rubinsten and Henik (2009) sought to outline what research from the field of cognitive psychology has shown so far. Their study is fundamental because it provides links between the biological level – the part or parts of the brain which may be functioning in unusual ways – and the cognitive level as well as the behavioural level for each. Primarily, Rubinsten and Henik identified the areas of the brain which show reduced activation in children with MLD and dyscalculia. These are the Intraparietal Sulcus (IPS), Angular Gyrus, Fusiform Gyrus and the frontal brain areas. A representation of the brain is given in Figure 2.9. The parts Rubinsten and Henik identify have been highlighted.

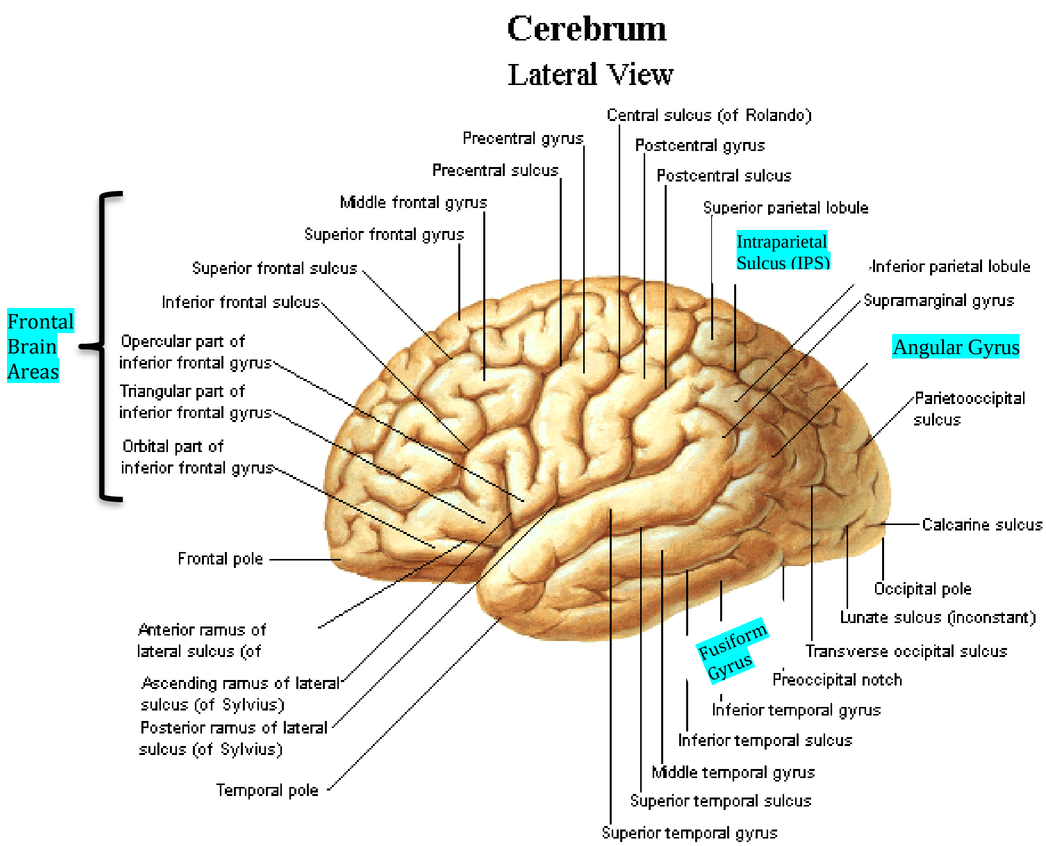


Figure 2.9: Diagram of the brain showing all the components of the brain mentioned in Rubinsten and Henik's (2009) framework (adapted from Khan, 2014).

Rubinsten and Henik provide a new theoretical framework as to the neurobiological deficits which learners with dyscalculia may have, versus to those with Mathematics Learning Disabilities, who do not meet their criteria for Developmental Dyscalculia (DD). They advocate that Developmental Dyscalculia (DD) is used in several studies to refer to a deficit in core numerical abilities, and thus indicate a single core deficit hypothesis for dyscalculia (Piazza & Izard, 2009). On the contrary, Mathematics Learning Disabilities are caused by "several cognitive deficits such as deficient working memory, visual-spatial processing or attention" (Rubinsten & Henik, 2009, p.92). Figure 2.10 is a visual explanation present in their work illustrating this theoretical framework. The authors indicate how each biological difference is related to the cognitive and the behavioural level of an individual.

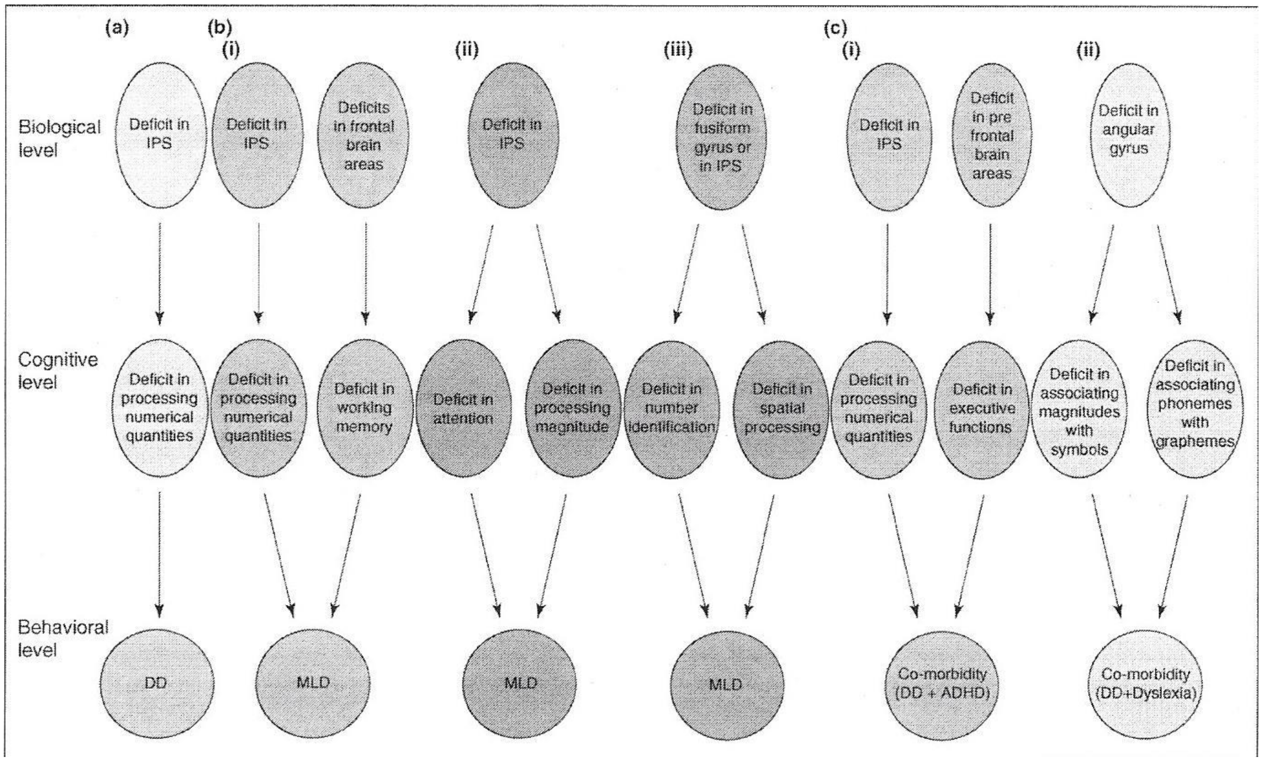


Figure 2.10: Three alternative frameworks for the origin of mathematics deficits and their underlying neurocognitive deficits (Rubinsten & Henik, 2009, p.95).

As can be seen in Figure 2.10, Rubinsten and Henik propose that Developmental Dyscalculia (DD) and Mathematics Learning Disabilities originate from different biological functions. DD is proposed to stem from a deficit, resulting from reduced activity in the Intraparietal Sulcus (IPS), and manifests itself as a deficit in processing numerical quantities. This is compatible with some other findings (Butterworth, 2005; Landerl, Bevan & Butterworth 2004; Pinel, Piazza, Le Bihan & Dehaene, 2004) which will be discussed shortly. On the other hand, Mathematics Learning Disabilities may be associated with multiple biological levels including deficits (from reduced activity) in the IPS, in the frontal brain areas and in the fusiform gyrus. Each may represent a separate characteristic at cognitive level. It should, however, be noted that not all studies show identical results, and that the level of activation of different brain areas may cause differences in cognition. Rubinsten and Henik summarize evidence that DD is frequently co-morbid with other specific learning difficulties like dyslexia and Attention Deficit Hyperactivity Disorder (ADHD). They discuss possible biological correlates of each of these and how these might influence cognitive functioning.

Other studies explore the function of different parts of the brain in numerical development (Cantlon et al., 2011; Soltész, Goswami, White, & Szűcs, 2011) and the differences in the brain functions of children with dyscalculia as opposed to typically developing children (Cohen Kadosh, Cohen Kadosh, Kaas, Henik, & Goebel, 2007; Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007). Early studies which used Positron Emission Tomography (PET) imaging (Dehaene et al., 1996; Pesenti, Thioux, Seron, & Volder, 2000) illustrated that the parietal cortex is the area which is mostly activated during numerical tasks. Later similar studies (Dehaene et al., 2003) using fMRI scanning, which is more precise, identified three sections in this parietal area of the brain which seem to interact differently whilst representing and processing numerical content. The areas identified were the left angular gyrus, the posterior superior parietal lobe (PSPL) and the horizontal intra-parietal sulcus (HIPS). Each of these areas seems to be mainly responsible for different functions as shown in Table 2.5.

Table 2.5: The responsibilities of each part of the parietal lobe which is responsible for numerical processing as per different studies.

Left Angular Gyrus • Seems to be most responsible for numerical tasks that require verbal memory like exact addition and multiplication (as opposed to approximate addition and subtraction) (Lee, 2000).
Posterior Superior Parietal Lobe (PSPL) • Appears to be activated when numerical tasks which involve estimation, number comparison and subtraction activities (Pinel et al., 2001).
Horizontal intra-parietal sulcus (HIPS) • HIPS seems to be needed for quantity manipulation or tasks involving number sense which include estimating, comparing numbers as per size and magnitude, and subtracting (Dehaene et al., 1999).

Differences have been noted in the activation patterns of these parts of the parietal area between children with dyscalculia and typically developing children. The findings of these studies do not necessarily indicate that neurological deficits are causing arithmetical difficulties but may indicate that brain activation patterns are showing different arithmetical strategies employed by individuals. Evidence that this might be the case, emerges from studies that have shown that training which seeks to improve the arithmetic of children with MLD also seems to change patterns of brain activation making it like that of typically developing children (Iuculano et al., 2015).

Since studies have agreed that the intraparietal sulcus (IPS) within the parietal lobe is the fundamental part of the brain which is active during any number processing, it also plays an active key role when the Approximate Number System (ANS) is employed to estimate magnitudes and fundamental to developing number sense. As a result, some studies conclude that the difficulties of learners with dyscalculia stem from a single core deficit (Butterworth, 1999, 2005; Gersten & Chard, 1999; Robinson et al., 2002). They conclude that the ANS is deficient in these children (Mazzocco et al., 2011; Piazza et al., 2010). Mazzocco et al. (2011) conducted a study with 71 pupils in 9th grade (14 - 15 years old) by taking participants from a prospective longitudinal study (Mazzocco & Myers, 2003). They indicated that in children with dyscalculia the ANS is much poorer in precision than all other categories of learners including low achievers in mathematics who do not have dyscalculia.

Piazza et al.'s (2010) study also suggests that children with dyscalculia seem to have a single core deficit - a deficit in the ANS. Their research was carried out with children having dyscalculia and with children without dyscalculia. Additionally, they also assessed kindergarteners and adults. Following the tasks given to the participants, Piazza et al. (2010) concluded that the number acuity of children with dyscalculia is severely impaired. They concluded that their participants with dyscalculia who were aged 10 scored at the level of a 5-year-old of a child without dyscalculia.

Other studies (Butterworth, 2005; Kucian et al., 2006; Landerl et al., 2004) have also supported the idea of a single core deficit. They have reported that there is a distinction between areas of the brain which are activated when a learner manipulates the magnitude of numbers and when rote memory tasks are undertaken. A study conducted by Kucian et al. (2006) with 18 children with dyscalculia and 20 typically developing children from 3rd Grade and 6th Grade clearly indicated this. The subjects were exposed to fMRI scans whilst performing different numerical tasks. As can be seen in Figure 2.11A, the brain activation of children with dyscalculia is far less than that of typically developing children during approximate calculations.

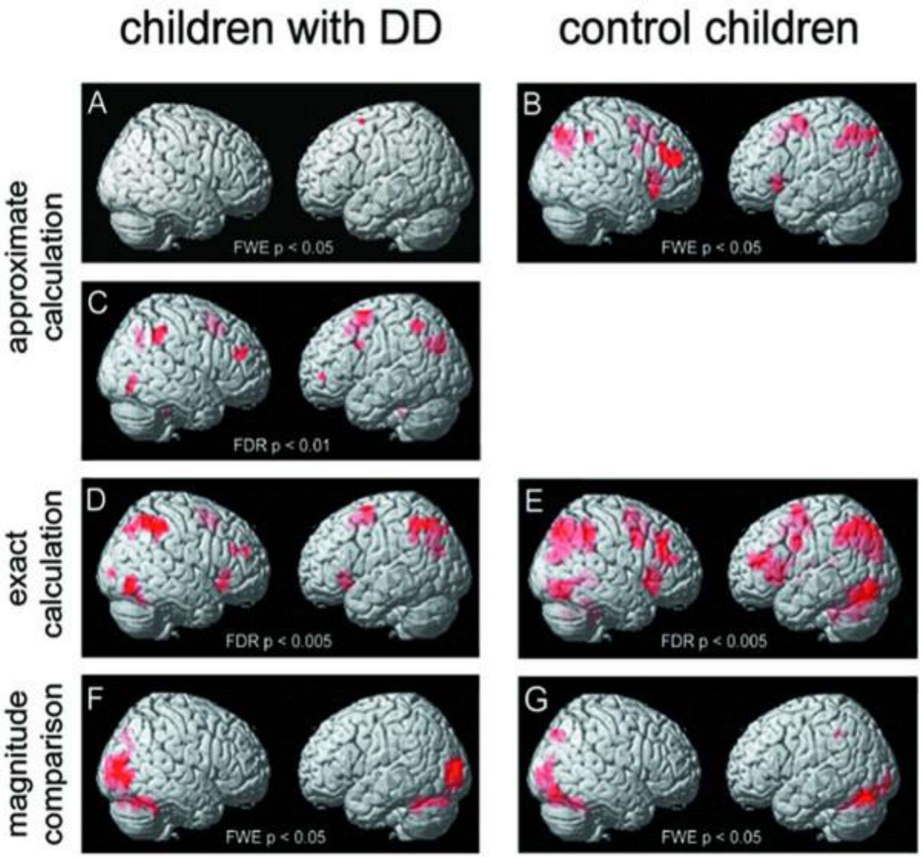


Figure 2.11A - G: Brain activation for children with dyscalculia and typically developing children in three different tasks or numerical processing (Kucian et al., 2006, p. 8).

Figure 2.11B-G demonstrates the areas of the brain that the fMRI scans found activated during different tasks for both the children with dyscalculia and those who were typically developing. These images (Figure 2.11 B-G) show that the areas of activation during the different tasks for both groups of learners were similar. Kucian et al. conclude that, "no differences could be found in brain activation between both groups during exact calculation and magnitude comparison, only during approximate calculation did control children exhibit stronger activation in two small cluster" (p.9). This agrees with the arguments posed by research (Butterworth, 2005; Landerl et al., 2004) indicating that number sense, the ability to make sense of numerosity, number relationships and patterns, is the main core deficit in children with dyscalculia. In my view, this study therefore indicates that there is one core deficit causing MLD – that of a difficulty with number sense, more specifically manipulating numerosities.

The findings presented by Kucian et al. (2006) seem to contrast a review carried out by Wilson and Dehaene (2007) which challenges the notion of having a single core deficit. They propose that subtypes of dyscalculia exist and that they result from different deficits within the areas responsible for numerical processing and representation within the parietal cortex. They argue that other subtypes of dyscalculia which may exist are: poor verbal symbolic representations which is evident when dyscalculic learners find difficulty with retrieving number facts; deficient executive functions which impinge on the execution of more complex tasks; and a weakness in spatial attention which influences the ability to recognize small numerosities (subitize) and thus hinders the learner from manipulating symbolic and non-symbolic numbers.

The findings of a study carried out by Iuculano, Tang, Hall, and Butterworth (2008) also contradict the notion of a single core deficit. Iuculano et al. carried out a study with 36 children in 4th Grade (8- to 9-year-olds) from three different state schools. The participants were divided into three sub-groups, namely; the dyscalculic group, low numeracy group and normal achievement group. The children were assessed using a battery of assessments including the Dyscalculia Screener (Butterworth, 2003). The authors concluded that there was no relationship between exact tasks and approximate tasks in all groups of learners. They conjectured that the main difference in performance between the typically achieving children and the other two groups of participants was not that of a poor grasp of numerosities but the result of a poor understanding of symbolic numerals. This corroborated the findings of another study carried out by Rousselle and Noel (2007).

Mussolin, Mejias, and Noel (2010) support the argument against a single core deficit being the cause of MLD, more specifically dyscalculia. They illustrated that their dyscalculic participants had a deficit in the processing of both symbolic and non-symbolic quantities. Additionally, within the same domain of mathematical acquisition, individuals exhibit different characteristics of deficits (Dowker, 2005a). These arguments sustain the notion that individuals have individual characteristics in the learning and execution of mathematics. Research studies on dyscalculia have drawn similar conclusions.

Although it is interesting to consider research about neuroscience and specific learning difficulties in mathematics "a frequent criticism of brain imaging studies involving learning is their restricted applicability to education and classroom interventions" (Kaufmann, 2008, p.167). Although there is hope that studies of such a nature may shed light on the complex way in which the brain works when engaging in mathematical activities, I believe it would be ideal if emerging studies would focus on linking more closely neuroscience and education by suggesting implications of these studies to education and intervention.

2.9 Assessing for MLD

Forms of assessment for MLD are still limited in relation to assessments for other learning difficulties such as reading difficulties. However, the Diagnostic and Statistical Manual of Mental Disorders (DSM IV-TR) (American Psychiatric Association (APA), 2000) offers some guidelines for assessing for Mathematics Learning Disabilities. These have been explained in Section 2.5. However, the more recent DSM-V (APA, 2013) takes a different stance to considering mathematics learning disability. In this new version of the manual, there is a section which is dedicated to specific learning disorders (SLD). It indicates that specific learning disorders (SLD) are:

> A neurodevelopmental disorder of biological origin manifested in learning difficulties and problems in acquiring academic skills markedly below age level and manifested in the early school years, lasting for at least 6 months; not attributed to intellectual disabilities, developmental disorders, or neurological or motor disorders (APA, 2013).

Added to this definition are three specific areas through which the learning disorder may manifest itself. One of them is that of having a specific learning disorder with mathematical processing. It is expected that clinicians and assessors determine the severity of the diagnosed specific learning disorders (SLD) as mild, moderate or severe. As explained by Tannock (2012) since the new version of the DSM has shifted its definition to a more comprehensive one, rather than one focusing mostly on IQ, new criteria for identification have been set. These are:

i. Having one of six symptoms specified by the same manual which last at least 6 months;
ii. Having an apparent discrepancy between actual age and achievement in the specific area;
iii. That the learning difficulty would have become to be most visible when the individual would have started formal learning;
iv. That the learning difficulty is specific and not related to an intellectual disability.

Additionally, when explaining which forms of assessment are suitable for assessing dyscalculia, Michaelson (2007) suggests that these include: standardised tests, direct observation of the student and the Dyscalculia Screener (DS) (Butterworth, 2003). However, the following assessment tools are also valid tools for assessing for MLD: the compilation of relevant checklists, the use of a mathematical interviews, formative assessments like those proposed by Emerson and

Babtie (2010), as well as Dyscalculium (Trott & Beacham, 2010) (a screener for dyscalculia suitable for students 16 and over). In subsequent sections I will give further details of each of these modes of assessments. However, it is crucial to underscore that arguments have been presented in favour and against all the various modes of assessments, as will shortly be discussed. Therefore, the use of diverse methods of assessment would provide a more complete description of the strengths and weaknesses of a child in mathematics. Assessments should also consider factors such as social inequalities and absenteeism (Ginsburg, 1997).

2.9.1 Norm-referenced Testing

Norm-referenced tests (NRT), also known as standardised tests, are probably the mode of assessment which are most commonly used to assess for MLD. Most norm-referenced tests (NRTs) focus on place-value, writing the numbers before and after a given number, the four operations (+, -, x, ÷), fractions of a shape and continuing a sequence of numbers which follow a pattern. Nonetheless different norm-referenced tests (NRTs) will have different tasks which are purposely put together to start from the easiest and move on to more difficult tasks. Each NRT is usually accompanied by a grid to aid the assessor in the analysis of data. The raw score achievement by an individual in such a test may be converted to a standardised score or a number age, a percentile and a quotient. As a result, NRT normally assess the learner's mathematical knowledge with respect to their number age (Shalev & Gross-Tsur, 2001). Although the commonest practice is for NRT to be carried out by educational psychologists, these tests can be carried out in the classroom by the class teacher. Most NRT can be administered on an individual basis or in groups. A variety of these tests have been developed. These include the Basic Number Screening Test (BNST) (Gillham & Hesse, 2001) and Progress in Mathematics (PIM) (Clausen-May, Vappula, Ruddock, & NfER, 2009). A new NRT, which is a profiler, is that proposed by Karagiannakis and Cooreman (2015). This profiler aims at assessing four important domains for mathematics learning: core-number, visual-spatial, memory and reasoning. The outcomes of the profiler can then be used to inform intervention.

Every NRT will indicate a cut-off point showing whether the assessed learners would fall into the category for children with MLD. Some even indicate whether the score obtained by the individual is within the category dedicated for learners with a special educational need in mathematics. This latter category of learner is usually indicated for those learners who obtain a score which is within the 30th percentile. However, cut-off scores may be different for different tests due to the inexistence of a universal definition for MLD. There seems to be a general agreement that a specific learning difficulty in mathematics like dyscalculia is evident when the individual performs much lower than expected in relation to their age. When this difference is at least that of two years, an assessment of dyscalculia is very likely (Semrud-Clikeman et al., 1992).

Most research studies about MLD have made use of NRT to identify children with MLD or even dyscalculia. However, there is yet no universal understanding of one cut-off point which truly indicates MLD. This is also evident in the DSM-V (APA, 2013) in which no standardised scores are indicated as a cut-off point for SLD. The DSM-V (APA) in fact reminds clinicians that there is no natural cut-off point to identify learners with SLD from other typically developing learners mainly because academic skills are continually distributed in the population. Thus, it asks clinicians to consider different factors before suggesting whether an individual has an SLD or otherwise and prior to determining the severity of the SLD when present.

Since there is no natural cut-off point for SLD, different research adopts different cut-off scores, and hence, the prevalence of MLD and dyscalculia is highly subjective. Thus, there is yet no consensus on how many learners are likely to have MLD or dyscalculia in a given population. Consequently Devine, Soltész, Nobes, Goswami, and Szűcs (2013) highlight that “it is important to have clear diagnostic criteria in order to understand the prevalence of DD and also to assess likely genetic origins” (p.31). Devine et al. present a Table and a Figure to show primarily how different studies have adopted different criteria (cut-off points) to assess for MLD and how these criteria have impinged on the outcomes of the prevalence of MLD and dyscalculia in the relevant studies. These are represented as Table 2.6 and Figure 2.12 respectively. The criteria adopted for each study indicate whether, for example, the authors have taken the bottom 10%, 25% or 2 Standard Deviations (SD) below the mean of the group as the population with MLD.

Table 2.6: A summary of the cut-off points used by different researchers as presented by Devine et al. (2013, p. 32).

Table 1

Summary of DD prevalence studies.

First author	Country	Sample	Prevalence	Criteria
Kosc (1974)	Slovakia	375	6.4%	≤10% + control
Badian (1983)	US	1476	3.6%	≤20%
Klauer (1992)	Germany	546	4.4%	<2 *SD*
Lewis et al. (1994)	UK	1056	1.3%	<16% + control
Gross-Tsur et al. (1996)	Israel	3029	6.5%	2 year performance lag + control
Badian (1999)	US	1075	3.9%/2.3%[a]	<20%/<25% [a]
Hein et al. (2000)	Germany	181/182	6.6%	<17%/<25% + control
Ramaa and Gowramma (2002)	India	251/1408	5.98%/5.54%[b]	Exclusionary criteria/2 year performance lag
Mazzocco & Myers, 2003	US	210	9.6%[a]	≤1 *SD*/<10% + control
Desoete et al. (2004)	Belgium	3978	2.27%/7.7%/6.59%[c]	≤2 *SD* + control + RTI
Koumoula et al. (2004)	Greece	240	6.3%	<1.5 *SD* + control
Barbaresi et al. (2005)	US	5718	5.9%/9.8%/13.8% [b]	Regression formula; discrepancy formula; <25% + control
Barahmand (2008)	Iran	1171	3.8%	≤2 *SD* + control
Dirks et al. (2008)	Netherlands	799	10.3%/5.6% [b]	<25%/<10% + control
Geary (2010)	US	238	5.4%	≤15% + control
Reigosa-Crespo et al. (2011)	Cuba	11,652/1966[d]	3.4%	<15%/<2 *SD*[d]

Note. Where possible, reported prevalence estimates are for mathematics disability only. RTI = resistance to intervention.

[a] Persistent DD.

[b] Prevalence estimates when using the different criteria.

[c] Prevalence estimates for the Second, Third and Fourth grades respectively.

[d] Two stage diagnosis.

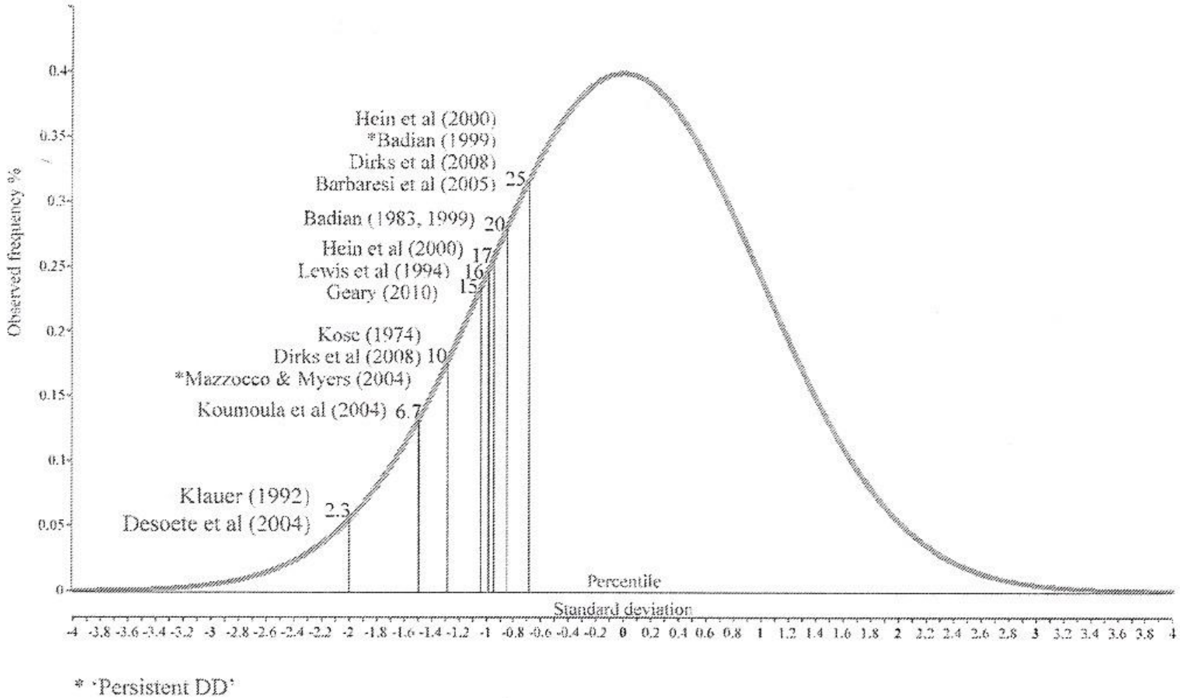


Figure 2.12: The cut-off points used to outline the prevalence of MLD in different studies. Percentile values are shown on top on the normal distribution curve (Devine et al., 2013, p.32).

Hence, it is crucial that assessors use a triangulation of modes of assessment not only to gain a holistic picture of the individual's mathematics abilities and needs, but also to outline as precisely as possible whether that same individual has MLD or dyscalculia and to what degree. Furthermore, since these standardised tests are limited in providing information which will help to formulate the appropriate intervention for the concerned individual, using more formative assessments (see Section 2.10.2) may provide further information for the persons involved in intervening upon the difficulties being experienced by the learner.

Unfortunately, no numeracy assessment has yet been standardised locally and therefore the scores of the Maltese individuals assessed using these tests are compared to the norms of learners in the UK which might differ to those of the Maltese population. Two factors which might vary the norms are the language in which the test is presented (because these NRTs are in English and therefore one must seek to identify whether this places a barrier for some learners in Malta), and the actual mathematics curriculum adopted by the different countries.

2.9.2 Formative Assessments

Detailed formative assessments provide an account of the nature and degree of the MLD experienced by the learner; they are the best means of identifying which mathematical skills and concepts the learner is experiencing most difficulty with. As Ashlock (2015) suggests,

> diagnosing should be refreshing – like getting water from a deep well – because it permits us to help each student move ahead successfully. Diagnosing can be deep like a well; we have to probe deeply to find what the difficulties are (p.234).

Consequently, most numeracy intervention programmes are accompanied by a thorough formative assessment to gauge the abilities and difficulties of each learner in different components of mathematics. However, apart from the formative assessments which accompany and are aligned to specific intervention programmes, one of the few formative assessments which are available to highlight the nature of the MLD being experienced is that produced by Emerson and Babtie (2010) called *The Dyscalculia Assessment*. As indicated in the book itself it "is a detailed investigation of basic numeracy….to find out what the child can do and how they reach their answers" (Emerson & Babtie, p.16). This assessment targets the following areas:

i. Number sense and counting;

ii. Calculation;

iii. Place value;

iv. Multiplication and division;

v. Word problems;

vi. Formal written numeracy (Emerson & Babtie).

Although the criteria and topics set here are rather similar to those found in NRTs, a main advantage is that this assessment is carried out on a one-to-one basis. Hence, particular skills, like number sense and counting, may be dealt with much better through this test. Furthermore, the assessor is encouraged to observe the learner's behaviour closely during each task given. This assessment can be used as the basis for any intervention which may be directed at supporting an individual learner to overcome at least some of his/her difficulties in mathematics learning.

Another form of formative assessment is that of a *mathematics interview*. The idea is that one not only observes the results of the tests as with NRT but also how the learner would have arrived at the answer since this can be very revealing. One example is that one can determine whether the learner is over-reliant on finger counting as a strategy for addition and subtraction. This might be an eye opener since finger counting is a strategy that many dyscalculic learners and learners with MLD rely on.

An informal method of assessing formatively can be achieved by classroom observations. Classroom observations are very important because teachers have a crucial role in identifying potential MLD, as early as possible. To guide these observations, some checklists like the one presented by Henderson et al. (2003) have been drawn up. The checklist includes the following criteria: *has very high levels of fear and anxiety when it comes to Mathematics* and *finds remembering mathematics rules and formulae difficult*. A full list of the criteria can be found in Appendix B. These checklists give a clear direction as to what a teacher should observe. According to the number of ticks a learner gets in the checklist, the teacher can understand whether that child has MLD or possible dyscalculia and can then refer the child to other professionals accordingly.

2.9.3 Screeners for Dyscalculia

To date only two screeners have been produced for dyscalculia. These are the Dyscalculia Screener (DS[11]) (Butterworth, 2003) and the Dyscalculium[12] (Trott & Beacham, 2010). The main difference between the two screeners is that the former has been developed for learners who are between 6 and 14 years of age, whilst the other is intended to be used with adults over 16 years of age. Both screeners are computer based standardised tests however a paper version also exists for Dyscalculium. According to Voutsina and Ismail (2007), the DS has been designed to diagnose dyscalculia as a separate condition to other factors which may impinge on the learning of mathematics like difficulties in communication and interaction as well as behavioural, social and emotional development. Since the DS has been on the market far longer than the Dyscalculium, more research has been done on the former, resulting in contrasting views about the efficacy of this screener to assess for dyscalculia.

The DS has been used in numerous international studies (Gifford, 2006; Messenger, Emerson, & Bird, 2007). It focuses on assessing for an individual's number sense, more specifically a learner's ability to manipulate numerosities and number relationships, since the

[11] Throughout this section 'DS' will stand for the Dyscalculia Screener developed by Butterworth (2003).
[12] Throughout this section 'Dyscalculium' will refer to the screener developed by Trott and Beacham (2010).

notion which inspired its development is that of the single core deficit. One of its advantages is that it assesses for mathematics learning difficulties independently of other influences like reading, as all its instructions are read out. Instructions are clear and very easy to follow. Since the DS is computer-based its results are objective and eliminate human error. Additionally, it is not as time consuming as formative assessments. It also allows you to assess the learner without getting to know him/her well, as would be necessary when completing a checklist or a formative assessment.

Some research has outlined disadvantages of the DS. Voutsina and Ismail (2007) indicate that the tasks proposed by the DS are lengthy and therefore some learners may get bored and this might impinge on their performance. Moreover, a study carried out by Messenger et al. (2007) has proposed that some learners who performed well in mathematics were assessed by the DS with a profile of dyscalculia, whilst others who performed poorly were not. This might derive from the fact that other functions of the brain come into play when engaging in mathematical activity like language, attention, spatial and sequential ordering (Messenger et al.). Additionally, my own Master level study (Zerafa, 2011) showed that there were discrepancies in the assessment of pupils in the sense that some pupils who were very poor at mathematics were not identified by the DS as having a profile of dyscalculia. Furthermore, sometimes the DS report did not indicate clearly whether the learner had dyscalculia or not, suggesting that the test be repeated. These findings once again indicate that it is unwise to use one form of assessment since knowledge of MLD and dyscalculia is still limited, and numerous inconsistencies seem to prevail.

The Dyscalculium has not yet been used much for research purposes so very little data is available about this tool. However as outlined by Beacham and Trott (2006) the Dyscalculium has already been piloted several times. This screener has been developed based on a cognitive model of dyscalculia. The first phase of piloting was carried out with 19 adults in higher education. It seemed to identify correctly whether the adults had dyscalculia or not (Beacham & Trott). The pilot was carried out with adults having dyscalculia only, dyslexia and no Specific Learning Difficulties (SpLD). The screener assessed all the adults having dyscalculia with a profile of being ‘at risk’ of having a specific learning difficulty with mathematical and numerical understanding. On the contrary it showed that all adults who did not have a Specific Learning Difficulties (SpLD) as not having any difficulties with mathematical understanding, thus giving a realistic assessment. Following this first phase, some changes were made to the first screener including modifications to the background colour and layout (Beacham & Trott, 2005). The second phase of the study was carried out with 109 students in higher education who were asked to complete the screener and a mathematics competency test (Beacham & Trott, 2006). This second phase was also carried out with dyscalculic adults, dyslexic adults and adults with no Specific Learning Difficulties (SpLD). This phase had similar positive results however for some of the cases, questions arose about the language barrier and how other conditions like Attention Deficit Hyperactivity Disorder (ADHD) may impinge on the results of the screener.

Another more recent assessment for dyscalculia is the Dynamo Profiler (Jelly James Publishing Ltd., 2014) which is an online assessment, suitable for learners aged 6 to 13. This Profiler is not a screener per se, however it does illustrate the areas of mathematics learning which the learner is still struggling with and indicates whether these areas are related to 'dyscalculia' or only 'mathematics developmental delay'. It is based on an MMR Framework (Dowker, 2016) through which the children's understanding of number Magnitude, Meaning and Relationships are assessed. The Dynamo Profiler (Jelly James Publishing Ltd., 2014) was published after my collection of data, hence I could not consider it for use in my study. Having said this, the Profiler (Jelly James Publishing Ltd., 2014) also provides an intervention programme which is individualized according to the needs identified by it. The company has also very recently published the Dynamo Profiler Pro (Jelly James Publishing Ltd., 2018). The Dynamo Profiler Pro is a similar assessment to the one for children but has been developed to identify traits of dyscalculia and mathematics developmental delays in young adolescents and adults (ages 13+). Research is yet to be carried out to identify the efficacy of the Dynamo Profiler Pro (Jelly James Publishing Ltd., 2018).

2.10 Intervention for MLD

There is evidence that learners with MLD and dyscalculia can make significant progress when they are provided with the right intervention to support them in overcoming their difficulties with mathematics (Dowker & Holmes, 2013; Kaufmann, Handl, & Thöny, 2003). For example, Kaufmann et al. (2003) conducted a qualitative study with six children having dyscalculia. They not only show that children with dyscalculia can make great progress when given the right intervention but also indicate that the positive effect of the programme is mainly due to the explicit teaching of specific numerical components which are usually not given importance in school mathematics such as 'counting backwards'. These conclusions are similar to those drawn by other authors (Dowker & Sigley, 2010). As a result of this evidence, different intervention programmes (see Section 2.11.1) have been proposed and evaluated. No matter which programme is adopted, there are several strategies which have been presented as being effective with children having MLD and dyscalculia. Primarily, it is essential to bear in mind that "MLD students show greatest progress when provided with direct, explicit, and multisensory instructions adapts to their individual learning profile, strengths and weaknesses" (Karagiannakis & Cooreman, 2015, p. 269). Similarly, Emerson (2015) points out that, "a multi-sensory approach to arithmetic is recommended for pupils with poor number sense" (p. 223). Emerson (2015) suggests that a multi-sensory approach often "finds starting points based at the proximal level of development of each pupil so that teaching starts at a point where the pupil can succeed at the edge of their competence before moving on in small steps" (p.226). This corroborates other research that also implies that intervention should be individualized (Dowker, 2004) and that therefore all intervention needs to be supported by a thorough assessment (Ashlock, 2015; Moeller et al., 2012). The intervention cycle presented by Henderson et al. (2003) indicates that appropriate intervention should be preceded by detailed assessment and followed by evaluation which instigates new practice.

Henderson et al. (2003) provide a guideline regarding which areas of mathematics learning one needs to focus on when intervening on MLD. These include mathematics language and mathematics anxiety. The authors also mention the importance of selecting teaching strategies which are appropriately linked to the learning styles of the pupils. To date, a few publications (Bird, 2007; 2009; 2011; Chinn, 2004; 2012; Emerson & Babtie, 2014; Henderson, 2012) have outlined different strategies targeting MLD and practical activities to support learners with such difficulties. Two important reports in the UK, The William Report (William, 2008) and The Rose Review (Rose, 2009) underscored the importance of having appropriate intervention strategies and programmes in place which help learners as early as ages 5 to 7 to grasp the basic numerical skills and concepts which are the foundations of mathematics learning. According to William (2008), closing the gap between children with difficulties and typically developing children is important. Nonetheless, one must consider that "there is no evidence that any one programme is best for most of our children…different programmes would be suitable for different groups of children" (National Strategies/Primary, 2009 cited in Henderson, 2012, p.29).

2.10.1 The Local Context

Although in recent years there has been a growing awareness of the importance of assessing and intervening for MLD and dyscalculia as early as possible throughout formal schooling, as yet, this is still limited. Moreover, the importance that has been given to this area of mathematics learning in Malta has been far less than that given to literacy difficulties and intervention programmes. Presently, in state schools, teachers are encouraged to refer students struggling with mathematics in the Upper Primary Grades (Grade 4 to 6) (8 to 11 years of age) to the Mathematics Support Team (MST). The Mathematics Support Team (MST) have developed checklists to assess for gaps in mathematics learning and to classify the level of mathematics of children with MLD, identifying their abilities as being within Grade 1, 2 or 3 level. These children are then given the possibility to sit for an alternative mathematics examination paper. Through the Checklist one can see which skills, concepts or competences an individual learner would have mastered, partially mastered or not mastered at all. The Checklists being used differ from standardised tests as they are more formative in nature. Unlike in norm-referenced tests (NRTs), no comparison can be made between the individual and other children their age with the use of a standardised score. To date no screening for dyscalculia is being carried out and although teachers may informally identify individuals with this SpLD, they cannot refer them for any further assessment. Moreover, assessments and screening are rarely done in the secondary or post-secondary education settings.

In view of the fact that so many interventions are now available to support students with MLD, it is unfortunate that locally no specific intervention programme is yet in place for children with MLD. While children with literacy difficulties are given the needed support through complementary lessons, children with MLD are not given similar support. To date, only the programme *Numicon* has been used and mainly with children having Down's Syndrome by the Non-Governmental Organisation (NGO) *Inspire* and by two independent schools who are using this programme through a whole school approach in the Early Years. In other mainstream settings, children with MLD are not being given any form of specialized intervention apart from the support provided by the class teacher or Learning Support Educator (LSE). When one considers the number of students on our Core Competence Programme (CCP) (a programme for students in Grade 7 (11 to 12 years old) who have not yet reached Grade 4 level), it is evident that timely interventions which are specific and individualised need to be carried out. This need is accentuated by the number of students failing SEC mathematics (a national examination which one needs to pass to go to tertiary education) every year. As outlined by Dowker (2005a) "ignoring the existence of individual differences [in mathematics achievement and learning] (whatever their sources) is not going to make them disappear" (p. 261).

To date only my own Master level research (Zerafa, 2011) has investigated the effectiveness of a specific intervention for mathematics locally. More research is thus fundamental not only to identify the prevalence of MLD and dyscalculia on the Maltese Islands but also to find out what kind of teaching and learning strategies are effective with children with MLD.

Section 2.10.2 The Impact and Important Characteristics of Effective Intervention Programmes

The importance of intervention programmes for children with MLD goes back to the early 1920s, when such programmes began to be developed in the US (Brownell, 1929; Buswell & John, 1926). However, as reported by Dowker (2009), it is only lately that the number of intervention programmes being offered for MLD have increased. Some interventions proposed are very intensive, like *Mathematics Recovery*, originally developed in Australia by Wright, Cowper, Stafford, Stanger and Stewart (1994). This intervention suggests that a learner has 30 minutes of individualized intervention every day. However, these interventions are normally seen as impracticable with overcrowded classrooms and considering the lack of human resources in schools. As a result, other interventions were proposed which were less intensive such as *Catch Up® Numeracy* (Catch Up®, 2009) which is based on the *Numeracy Recovery* (Dowker, 2001) intervention and which suggests two weekly individualized sessions lasting 15 minutes each.

Different intervention programmes have worked around a wide range of principles. Intervention programmes like *Numeracy Recovery* (Dowker, 2001), *Mathematics Recovery* (Wright, Martland & Stafford, 2000; Wright, Martland, Stafford, & Stanger, 2002), *Numeracy Intervention* (Kaufmann et al., 2003), the intervention presented by Karagiannakis and Cooreman (2015) and *Number Worlds Curriculum* (Griffin, 2003; Griffin, Case, & Siegler, 1994) are planned to target different components of numeracy (see Section 2.3) believing that these different components make up mathematical knowledge and allowing participants to work on those areas with which they are finding difficulties. However, other programmes take a different view and focus all their training on one specific component with the believe that progress in this specific component will lead to development in other areas. An example of such an intervention is Fischer, Moeller, Bientzle, Cress, and Nuerk's (2011) attempt at developing children's mental number line precision with the hope that this will allow them to perform better on other mathematical tasks. Moeller et al. (2012) compiled a list of intervention programmes providing details about each. Their tabulation is reproduced here (Table 2.7).

Table 2.7: Information about the intervention approached evaluated by Moeller et al. (2012) (Moeller et al., 2012, p.258).

Authors	Title	Target children	Age group	Type of approach	Range of tasks	Domain of knowledge	Intervention material	Treatment setting	Evaluation
Butterworth and Laurillard (2010)	Basic numeracy games	Children with special educational needs	?	Intervention	Multi-componential	Basic numerical competencies	Electronic	Individual	In progress
Sarama and Clements (2002, 2004)	Building Blocks software	Children without dyscalculia	Pre-kindergarten to grade 6	Prevention/training	Multi-componential	Basic numerical competencies	Electronic	Individual	Whole program
Dowker (2001)	Numeracy Recovery Programme	Children with dyscalculia	Age 6–7 years	Intervention	Multi-componential	Basic numerical competencies	Tangible	Individual	Whole program, no control group
Fischer et al. (2011)	Dance mat training	Children without dyscalculia	Kindergarten	Training	Magnitude comparison task	Basic numerical competencies	Electronic	Individual	By task
Griffin et al. (1994), Griffin (2003)	Number Worlds Curriculum	Children with/without dyscalculia	Pre-kindergarten to grade 6	Prevention and Intervention	Multi-componential	Basic numerical competencies and higher arithmetic	Electronic and tangible	Classroom, small-group or individual	Whole program
Kaufmann et al. (2003)	Numeracy Intervention Programme	Children with dyscalculia	Third grade	Intervention	Multi-componential	Basic numerical competencies	Tangible	Individual	Whole program
Metzenleitner (2007)	Multi-Componential Training Approach	Children with dyscalculia	Third grade	Intervention	Multi-componential	Basic numerical competencies	Tangible	Small-group	Whole program
Siegler and Ramani (2009), Booth and Siegler (2008)	Linear number board games	Children from low-income background	Kindergarten	Intervention	Linear board game	Basic numerical competencies	Tangible	Individual	By task
Wilson et al. (2006a, b)	Number race game	Children with dyscalculia	Age 5–8 years	Intervention	Comparison tasks	Basic numerical competencies	Electronic	Individual	By task
Wright et al. (2000, 2002)	Mathematics Recovery Programme	Children with dyscalculia	Age 6–7 years	Intervention	Multi-componential	Higher arithmetic	Tangible	Individual	Whole program

2.10.2 Computer-assisted intervention

Computer-assisted intervention for MLD is a new means of supporting learners with MLD to overcome their difficulties. This mode of intervention seems to be promising since intervention through computer-assisted programmes has not only been shown to be very effective (Räsänen, Salminen, Wilson, Aunio & Dehaene, 2009) but has also managed to increase levels of motivation for these students when engaging with digital mathematical tasks (Mayer, 2001). In a report by Laurillard and Baajour (2009) for the British Educational Communications and Technology Agency (BECTA), the authors suggest four fundamental elements for interventions. These are: *start early, personalize learning, use technology to help personalized learning; design technology-based learning* (p.4). Examples of digital intervention programmes offered are *Basic Numeracy Games* (Butterworth & Laurillard, 2010), *Building Block Software* (Sarama & Clements, 2004) and the *Number Race* (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). In a study using *The Number Race* (Wilson et al., 2006), the authors have shown that children increased their performance on core number sense activities following this intervention. The computer game was tested with a sample of nine pupils aged 7 – 9 years. The subjects performed more accurately on subtraction tasks and subitizing after following the programme for half an hour a day for four days per week over a period of five weeks. However, their performance on addition and base-10 comprehension did not improve.

Dowker (2005a) has discussed ways in which computer-assisted training has similar advantages and disadvantages to self-teaching systems. One of the advantages is their adaptability to the unique learning patterns presented by different individuals. This is also proposed by Moeller et al. (2012). Dowker (2005a) also suggests that the lack of social pressure which accompanies such digital technologies is also another of their benefits. Emerson (2015) suggests that yet another advantage is that they can help students to build visual representations of number concepts which they would not have internalized otherwise.

However, these computer-assisted interventions also have some disadvantages. Laurillard and Baajour (2009) point out that commercial software that is currently available for dyscalculia and low numeracy attainment has several problems. These include *screens that are too busy, tasks that rehearse known facts rather than support learning* and *learner input that promote guessing rather prediction or reflection* (p.9). As indicated by Dowker (2009) these programmes often do not give learners the opportunity to interact and communicate with others. She also points out that these interventions may place too much emphasis on the response of the learner rather than on the process undertaken to get to that response.

Although computer-assisted interventions seem to have benefits, there seems to be a general agreement that no computer-assisted intervention can completely replace the importance of the social interaction happening between a learner and a more knowledgeable other during intervention. Moreover, digital manipulatives should not replace tangible materials as children need to touch and manipulate concrete objects to develop a better understanding of the concepts being taught. Therefore, as stated by Emerson (2015) digital manipulatives "are an adjunct to, not a replacement for concrete materials, allowing pupils to work independently after they have worked with a teacher" (p.226). Similarly, Moeller et al. (2012) conclude that "the full potential of computer-assisted instruction is only unfolded when combined with direct instructions from a teacher" (p.264).

There has been an attempt to combine computer-assisted intervention and the intervention by a more knowledgeable adult through some intervention programmes. One example of such a programme is that offered by *Dynamo Mathematics* (Jelly James Publishing Ltd., 2014). The intervention set by this programme involves on-line digital games, a lesson carried out by a more knowledgeable other and a worksheet.

2.10.3 *Catch Up® Numeracy*

Catch Up® Numeracy (CUN) is one of the intervention programmes which have been developed to support learners with MLD. This intervention programme is being used in many schools across the UK as well as in other countries such as New Zealand. In my study I have chosen to use the framework provided by CUN as the basis of the intervention programme provided to the main participants. My reasons for choosing this programme will be explained in Chapter 3 however in this section I provide some background information about this programme.

CUN was developed in collaboration with the Catch Up® Trust, a not-for-profit charity, based on the pilot programme *Numeracy Recovery* (Dowker, 2001). *Numeracy Recovery* was first piloted with 169 children aged 6 and 7. Following some changes, such as the number of numeracy components, methods of delivery, and the introduction of training programmes for teachers and teaching assistants (TAs), it was was transformed into the CUN programme (Dowker & Sigley, 2010). The programme is delivered on a one-to-one basis and is tailor-made to suit the needs of individual learners. Its foundations include a thorough assessment that provides the more knowledgeable adult with a detailed outline of the abilities of the learner in each of the ten numeracy components outlined in Section 2.3. The importance of having such an assessment is in line with arguments presented by different researchers (Emerson, 2015; Karagiannakis & Cooreman, 2015; Moeller et al., 2012). The intervention session suggested by the programme is highly structured. CUN recommends that two sessions carried out twice a week are sufficient to register progress when working with children struggling with mathematics making it far less intensive than other interventions. Another difference between CUN and other interventions is that they illustrate that the intervention programme can be done by any adult, teacher or Teacher Assistant, who is trained to deliver the programme and therefore it does not specify that a specialized teacher must implement it. A detailed explanation of how the assessment is carried out and how each session should be structured will be given in Chapter 4.

CUN was piloted between 2007 and 2008. As a result, two reports then presented the outcomes of each phase (Evans 2007; 2008). The first phase was carried out with 62 children with MLD who attended 40 schools across six authorities in the UK and Wales and another four schools in the Oxford area. Following this first phase, the report concluded that the "intervention was a powerful adjunct to pupil learning in numeracy" (Evans, 2007, p.8). Through the 16 one-to-one interviews and two group interviews held with headteachers, teachers, teaching assistants (TAs) participating in this pilot project, the following positive feedback was collected:

i. The one-to-one structure of the sessions was a crucial contribution to the children's learning;

ii. The way the programme promoted individualized and targeted intervention specifically on the numeracy components allowed the children to develop particular numerical skills and concepts which they had not yet grasped;

iii. CUN presented new tools and methodologies (teaching strategies);

iv. The programme also had a positive effect on the affective domain of the children's learning since the negative attitudes towards mathematics the participants had at the beginning of the programme changed to more positive ones by the end;

v. The participants gained confidence in numeracy and engaged better in classroom activities; and

vi. Students were self-aware that they were progressing in the area (Evans, 2007).

In addition to these positive comments, a standardised test which was administered prior and post intervention showed that the participants had gained an average of 7.4 months vis-à-vis their number age between March and July. On the other hand, a control group which was given the same test, registered a number gain of 2.9 months within the same period not having been exposed to the same intervention (Evans, 2007). The second phase of piloting CUN had similar positive conclusions. This phase was carried out with six pupils in each of three schools in the UK. Whereas four of these participants were given the intervention programme, the other two from each school acted as the control group. The teachers participating in this project reported that the learners receiving the intervention managed to grasp better the mathematical language and strategies taught. Also, they indicated that the formative assessment accompanying the programme was very detailed and well-set to allow for the appropriate identification of weak numerical competencies which needed to be intervened upon. In addition, after standardised tests were administered to the group of 33 pupils receiving the intervention, Evans (2008) deduced that these children had made an average number age gain equivalent to 11.6 months after approximately 5.75 hours of support. Contrastingly the control group only gained 6.75 months in number age over the same time span (Evans, 2008).

To date the most extensive study carried out using CUN is that by Holmes and Dowker (2013). This was a quantitative research conducted with 440 pupils from 15 local authorities in England and Wales. The research project was divided into Study 1 and a sub-study which they refer to as Study 2. Study 1 compared the performance of the participants who had received the intervention programme with the control group which was made up of learners who had either not followed a targeted intervention programme or any kind of intervention. Study 2 analysed the data provided by the schools of 141 of the participants regarding the time spent on the intervention programme per week. This was done to explore whether the time dedicated to intervention has any effect on the number age gain of the pupils.

The main findings of the research (Holmes & Dowker, 2013) showed that the children receiving the intervention programme managed to attain average number age gains which are more than twice that expected of typically attaining children. Additionally, they illustrated that children receiving the intervention attained an average gain which was more than twice that attained by children receiving support in mathematics which was not targeted. Both findings not only indicate the effectiveness of this intervention as a targeted component-based approach (based on the numeracy components mentioned in Section 2.3), but also highlight that children's numerical knowledge and understanding is highly malleable and if intervened upon - even if not intensively and/or by a specialist teacher - children with MLD can make important improvement (Holmes & Dowker).

My own previously-mentioned research (Zerafa, 2011) also involved the use of CUN. The research was of a qualitative nature and was carried out with 3 students identified as having a profile of dyscalculia. The outcomes of this study also showed benefits for the children who were given the intervention. Two out of three of the participants managed to make a significant number age gain. One student gained 18 months of number age within three months of intervention, whereas another gained six months. In addition, the intervention had other positive effects on all three participants. Primarily, their attitude towards the learning of mathematics turned into a more positive one and by the end of the intervention they admitted liking the subject more. Moreover, they seemed less reliant on their fingers for counting and thus became more efficient at working out simple addition and subtraction computations. Finally, the learners developed metacognitive techniques that helped them to think about their learning, to reflect on what they had previously done wrong and to think about more effective strategies where appropriate (Zerafa).

2.11 Reading Difficulties

Since my main research question seeks to explore which strategies are effective with learners having both MLD and RD, it was important that literature about RD was also presented and discussed in this review. Learning to read is the most fundamental pursuit in the first years of formal schooling and is the foundation for future academic achievement (Chall, 1983). The importance of reading is highlighted when one considers that it is not only tightly connected to the cognitive development of an individual but also to their behaviour and socio-emotional development (Hinshaw, 1992). Consequently, there has been a global quest to understand literacy issues since "the ability to read, to communicate, and to develop one's personal and economic independence is a basic fundamental right" (Firman, 2009, p.28). Hulme and Snowling (2009) underscore "of all the cognitive deficits that occur in children, reading disorders are the most studied and best understood" (p. 37). Early work carried out by a French neurologist Dejerine in 1896 shed light on these difficulties and allowed further research to understand and explore better the field of reading difficulties (Wolf & Bowers, 1999). Dejerine (1892) introduced the term *alexia* to define the inability to read and indicated that reading difficulties seem to occur due to a damage to the brain's temporal lobe.

It is well known that some pupils struggle with learning this skill and thus develop Reading Difficulties (RD). Difficulties with reading single words accurately and fluently can persist and affect up to 6% of the population – based on samples taken from the UK (Yule et al., 1974). Other international research (mainly in the US and the UK) (Compton, Fuchs, Fuchs, Elleman, & Gilbert, 2008; Vaughn et al., 2010) has shown that RD persist in the secondary levels of schooling and sometimes even beyond and can either start in the early years or later at Grade 4 or beyond. Additionally, a phenomenon was identified in 1983 as the 'fourth-grade slump' (Chall & Jacobs, 1983) and more recent literature has supported this phenomenon (Willingham, 2009). These researchers noted that when children do not become proficient readers by Grade 3 this impinges very negatively on their academic achievement and they perform far lower than expected in Grade 4, probably due to the increased demands on reading in many areas of the curriculum that emerge at this point. It is not unusual that students in Grade 4 to 12 manifest a wide spectrum of RD (Wanzek et al., 2013). Some pupils exhibit difficulties related to the comprehension of text and need support in vocabulary (Torgesen et al., 2007), whilst others demonstrate more severe difficulties and are unable to read texts which are normally read by typically developing children. The gap between their reading ability and that of others is sometimes more than two years apart. These learners will normally need more intensive support in word reading and fluency (Cirino et al., 2012).

Other studies (Badian, 2005; Mortimore & Dupree, 2008) have indicated four focal areas of deficit which hinder a learner from developing the necessary reading skills. These are: *phonological awareness, rapid automatic naming, visual-orthography and the double deficit hypothesis*. Phonological Awareness, as suggested by Allor (2002), is "the ability to understand and use the sound system of our language" (p. 48). This is done by segmenting and blending phonemes to be able to read words. Phonological awareness seems to be the strongest factor in predicting reading difficulties in pupils (Badian, 2005). Rapid Automatic Naming (RAN) refers to the importance of recalling letters and words at a quick speed and therefore of developing a bank of words which an individual is able to retrieve automatically. Although some researchers have placed Rapid Automatic Naming (RAN) within the Phonological Awareness category (Savage & Frederickson, 2006), others believe that the two causes are independent from one another (Wolf & Bowers, 1993). Visual-orthography is the ability to identify whether numerals or letters are correctly oriented (Badian, 2005). A deficit in such skills may also contribute to the development of RD in learners. The double deficit hypothesis suggests that when a learner has a nemesis in more than one of the above mentioned factors, s/he is more at risk of becoming a poor reader than a learner that has a deficit in only one factor.

Difficulties with other general cognitive domains like working memory are also evident in learners with RD. A study carried out by Wang and Gathercole (2013) explored the cause of the impact which working memory may have on reading. In this research an initial number of 689 children aged 8 to 10 years of age were screened for RD and out of these, 46 children were identified as having RD after being assessed using several tests. The researchers (Wang & Gathercole) deduced that the children with RD as opposed to the control group had pervasive difficulties in the simple and complex span tasks. These tasks involved reading several unconnected sentences aloud, remembering the last word on each sentence after the sentence is read out and recalling the last word of every sentence after the whole set is read out. Moreover, they experienced more difficulties when trying to coordinate two cognitive demanding tasks.

On the local scene, RD are somewhat compounded since our pupils are brought up in a bilingual, sometimes even multilingual, educational setting. Pupils are expected to become competent in using both Maltese and English and in accessing the curriculum in both languages (Ministry of Education and Employment, 2012). Due to this, Firman (2009) reports that "bilingualism, and indeed, multilingualism, though in themselves a necessity and a great asset in today's fast-moving world, nonetheless pose problems through the juxtaposition of two or more languages" (p.28). It is thus fundamental that the language skills pertaining to both languages are well-taught and developed.

However, it is important to highlight that the conclusion drawn by Firman refers to children having reading difficulties since research has suggested that learning in a bilingual setting can have numerous advantages. These advantages include that bilingual learners understand concepts related to mathematics more easily and solve word problems with greater ease (Zelasko & Antunez, 2000); that they find it easier to ignore irrelevant information (Kovács & Mehler, 2009); that bilingual individuals find it easier to think about language and to use logic (Castro, Ayankoya, & Kasprzak, 2011); and that their ability to think in a second language allows them to move away from biases and stereotypes as they are open to an additional culture (Keysar, Hayakawa, & An et al., 2011).

2.11.1 Dyslexia

RD are defined as the key feature of the specific learning difficulty known as dyslexia. The World Federation of Neurology (1968) has definied dyslexia as "a disorder manifested by a difficulty in learning to read, despite conventional instruction, adequate intelligence and socio-cultural opportunity (p. 21)." Similarly, Peterson and Pennington (2012) refer to dyslexia as "a neurodevelopmental disorder that is characterized by slow and inaccurate word recognition" (p. 1997). A recent publication of the DSM-V (American Psychiatric Association (APA), 2013) also refers to this specific learning difficulty as a disorder similarly to the way it presents the meaning of dyscalculia (see Section 2.5). However, when referring to dyslexia, it mentions that it is manifested through an impairment in reading and an impairment with writing expression. Albeit these two aspects of literacy are tightly linked, an individual can have one characteristic and not the other. Additionally, a more recent definition established by the International Dyslexia Association (IDA) (2002) has suggested that dyslexia "is characterised by difficulties with accurate and/or fluent word recognition and by poor spelling and decoding abilities" (www.interdys.org/FAQWhatIs.htm). It also highlights that these difficulties are typically the result of a deficit in the phonological aspect of language.

Longitudinal studies (Scarborough & Dobrich, 1990; Snowling, Gallagher, & Frith, 2003) have shown that phonological deficits present in children with dyslexia are present before the child begins to read. Snowling et al. (2003) followed a group of 56 children who were at high risk of developing RD (due to other members of the family having it) from before their fourth birthday until they were 8 years of age. The outcomes of the research showed that the children who were later assessed with a profile of dyslexia had difficulties with early language development when first assessed by the researchers especially in object naming, letter knowledge and non-word repetition

(Snowling et al.). When they were 6 years old, these same subjects had persistent oral language deficits and their phonological awareness was very poorly developed.

Research about dyslexia has been far more numerous than that for dyscalculia. In fact, Peterson and Pennington (2012) report that "of all the neurodevelopmental disorders, dyslexia has been the most studied and is the best understood" (p. 2004). There seems to be a general agreement that dyscalculia is where dyslexia was roughly 30 years ago (Bird, 2009). This is certainly true when one considers the local situation. Whereas the Dyslexia Association was established in Malta in 1986 mainly to raise awareness about this SpLD, no association for dyscalculia exists as yet. Additionally, although locally children with RD are given support through complementary teaching and the State-run SpLD unit, children with MLD and dyscalculia are still left to struggle with mathematics with no such systematic support.

Research about dyslexia has illustrated a prevalence of this SpLD which is similar to that of dyscalculia: around 3 – 6% of children (Rutter et al., 2004). Additionally, similarly to dyscalculia, it seems that the brain activation of individuals with dyslexia is often atypical when carrying specific functions such as motor-related tasks, language tasks or orthographic processing of words (Feng et al., 2017). Brain activation for reading is different for children with dyslexia as opposed to typically developing children (Rumsey et al., 1994). Dyslexia is considered as a specific learning difficulty, with a significant genetic contribution (DeFries & Light, 1996; DeFries, Vogler, & LaBuda, 1986; Willcutt et al., 2002). Dyslexia has a spectrum of severity. As described by Voeller (2004) "some children are very severely involved, whereas others appear to be functioning well" (p. 741). Research on dyslexia has shown that individuals with dyslexia usually also have difficulties with working memory (Swanson & Ashbaker, 2000; Swanson & Siegel, 2001).

Studies have shown that dyslexia can be co-morbid not only with Attention Deficit Hyperactivity Disorder (ADHD) but also with language impairment and speech disorders (Catts, Bridges, Little, & Tomblin, 2008; Peterson, Pennington, Shriberg, & Boada 2009). This point will be discussed further in Section 2.13.

2.11.2 Assessment for Reading Difficulties

Assessing for RD is similar to assessing for MLD albeit many more assessments for reading are available in contrast to assessment for MLD. One may assess for RD and dyslexia using standardised tests, formative assessments, a screener or even a paid-for or free online assessment. Pearson, Valencia, and Wixson (2014) state that for the past 30 years there have been numerous collective and individual battles with reading assessment internationally. They highlight that these battles involved "efforts to release the profession from the stranglehold of standardised tests of minute subskills and superficial comprehension, to championing authentic performance assessment, to building state assessments that support challenging curriculum and teaching" (Person et al., p. 236). There are therefore numerous ongoing debates about how reading should be assessed and whether standardised tests are sufficient to assess for RD or whether more formative assessment should be used. Nonetheless, a vast range of standardised tests for literacy have emerged and reading normally forms part of such assessments. An example of such as assessment is the Dynamic Indicators of Basic Early Literacy Skills (DIBELS) (University of Oregon Center on Teaching and Learning, 2016). However, fewer assessments focus on reading alone. An example of such as assessment is the Single Word Reading Test (SWRT, Foster, 2007).

In the local scenario, language assessments for Maltese and English writing and comprehension skills have been written and standardised (Firman, Martinelli, Camilleri, & Ventura, 2010). A Maltese single word reading test has also been developed (Bartolo, 1988). Moreover, in local schools, children who seem to be struggling with literacy are asked to sit for a formative assessment, that of the Checklist (Directorate for Quality and Standards in Education, (DQSE), 2012). A checklist exists for Grades 1, 2 and 3 for both English and Maltese. The Checklists assess for blending (putting letter sounds together) and segmenting (separating letter sounds) skills, phonological awareness, reading fluency, reading single phonetic words and high frequency words, spelling and writing skills. The checklists are official and State school teachers are expected to use them to identify children struggling with literacy (DQSE, 2012). The checklists help for planning the intervention carried out by the classroom teacher and through complementary sessions with another adult. Hence intervention is individualized according to the child's strengths and needs.

Locally, RD are usually assessed using more than one assessment. Our children are expected to develop both Maltese and English (Ministry for Education and Employment, 2012) but it is well-known that the learners will always have a first language preference (Baker, 2011; Martiniello, 2008). As a result, children with RD are defined as those who perform poorly in both the Maltese and English assessments, showing that their difficulties go beyond limitations in one of the languages.

2.12 Co-morbidity

The notion of co-morbidity has been extensively studied and it is well known that developmental disorders like dyscalculia may be co-morbid with other such cognitive or even behavioural disorders. Co-morbidity is the notion commonly used to refer to the fact that some learners experience difficulties related to two separate SpLD and/or disorders and therefore these developmental disorders tend to co-occur in the same individual. Research has been carried out to understand better the co-occurrence between dyscalculia and Attention Deficit Disorder such as ADHD, ADD and Asperger Syndrome (Badian, 1983; Monuteaux, Faraone, Herzig, Navsaria, & Biederman, 2005; Shalev & Gross-Tsur, 2001; Shalev, Manor, & Gross-Tsur, 1997), poor eye-hand coordination (Siegel & Feldman, 1983; Siegel & Ryan, 1989), poor memory for experiences which are non-verbal (Fletcher, 1985) and Gerstmann's syndrome (Grigsby, Kemper, & Hagerman, 1987). However, Landerl et al. (2004) have highlighted that apart from showing that there is a correlation between dyscalculia and the different developmental disorders, it is still far from illustrating whether these same disorders have a causal effect on individuals with dyscalculia.

Research has been carried out to explore the co-occurrence of dyscalculia and ADHD. Lindsay, Tomazic, Levine, and Accardo (2001) noted that the participants in their study having dyscalculia made more errors related to commission and omission in the test given (Conners' Computerized Continuous Performance Test (CPT)) than participants without dyscalculia. Additionally, they suggest that many of the difficulties experienced by dyscalculics can be connected to problems with recruiting attention. Similarly, Shalev, Auerbach and Gross-Tsur (1995) showed that children having dyscalculia had higher attention deficits than typically developing children. Another study by Ashkenazi and Henik (2010) was carried out with university students. Similarly, they stated that the individuals with dyscalculia had an attention deficit which was accentuated when numerical processing took place.

A study carried out by Monuteaux et al. (2005) sought to investigate the familial relationship between dyscalculia and ADHD. The report produced explained the outcomes of two separate studies (Biederman et al., 1992; Biederman et al., 1999). The first study (Biederman et al., 1992) was carried out with 140 male participants with ADHD being 6 to 17 years old (with 174 siblings and 280 parents) and 120 participants without ADHD (with 129 siblings and 239 parents). The following study (Biederman et al., 1999) was carried out with females also aged 6 to 17 years old. This study was also carried out with 140 subjects with ADHD (with 143 siblings and 274 parents) and 122 controls without ADHD (with 131 siblings and 238 parents). All the participants were assessed through a diagnostic interview and a battery of cognitive assessments. The results illustrated that high rates of ADHD were seen in relatives of participants with ADHD, irrelevant of their dyscalculia status, and elevated rates of dyscalculia were found in relatives with dyscalculia, irrespective of their ADHD status. This supports the notion that though these conditions may be co-morbid, they are separate and should be identified and treated as such.

Literature such as the work by Yeo (2003) has clearly shown that other SpLDs like dyspraxia can also co-occur with MLD. As defined by the Dyspraxia Foundation (2016) "Dyspraxia, a form of developmental coordination disorder (DCD) is a common disorder affecting fine and/or gross motor coordination in children and adults. It may also affect speech" (on https://www.dyspraxiafoundation.org.uk/about-dyspraxia/). He illustrates how the deficits experienced by dyspraxia learners in relation to poor long-term and working memory, sequencing problems, directional confusion and other such difficulties impinge on the children's acquisition of mathematics skills and thus then develop a co-occurrence between dyspraxia and MLD. Yeo (2003) underscores that these difficulties also characterise the co-occurrence of dyslexia and MLD. Most research about co-morbid disorders has in fact been carried out on MLD and RD and/or dyscalculia and dyslexia. As indicated by Vukovic (2012), MLD and RD co-occur much more often than they occur alone. This corroborates other studies that drew similar conclusions (Dirks, Spyer, van Lieshout, & de Sonneville, 2008; Rubinsten, 2009). The strong link between these learning difficulties has thus been explored extensively as will be outlined in the next Section.

2.12.1 Co-morbid MLD and RD

Much research has investigated the co-morbidity between MLD and RD and between their related SpLD dyscalculia and dyslexia respectively (Andersson & Lyxell, 2007; Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007; Geary et al., 1999; Jordan, Hanich, & Uberti, 2003; Powell et al., 2009). Butterworth and Yeo (2004) suggest that 20% to 60% of dyscalculic learners also have difficulties with reading and writing. Albeit this, it is crucial to remember that not all students with MLD have difficulties learning to read. Chinn (2004) states that he has encountered children who struggle with mathematics but were good at the languages. On the other hand, Attwood (n.d.) explains that 10% of dyslexics excel at mathematics, 30% of them perform at expected level, 10% of dyslexics appear to be below average due to short term memory difficulties whilst another 25% perform at below average level in mathematics because of reading difficulties. As with ADHD, although it is common that these two difficulties co-occur, they are separate, and each can exist without the other.

Several studies have indicated similar difficulties for children having MLD only and those having MLD and RD. Amongst these Geary et al. (1999) have shown that both groups of learners show more procedural errors than children without MLD. Moreover Hanich et al. (2001) have suggested that both groups perform worse than their peers on word problems. Others have also indicated that both learners with solely MLD and those having both MLD and RD have lower mastery of number facts when compared to typically developing children (Powell et al., 2009). Wang, Tsai, and Yang (2012) suggest that children with the combination of dyslexia and dyscalculia have difficulties in a spectrum of mental processing which cause them to experience problems with both reading and acquiring mathematical skills. In a study carried out by Landerl et al. (2004), the authors compared 8- and 9-year-old pupils with dyscalculia and RD. They too found no qualitative difference in the numerical competences of children in both groups.

However, others have pointed out that children who have both MLD and RD tend to have more profound deficits (Cirino et al., 2007; Jordan & Hanich, 2000; Jordan & Montani, 1997). They have thus suggested that although difficulties presented by both group of learners are similar, the difficulties of children with both MLD and RD are more intense in nature and more profound (Powell et al., 2009). Nonetheless as outlined by Vukovic (2012) studies have not yet explored and given a clear indication of whether the mathematics disorder which children in the former group experience are the same as those in the latter group. For example, Jordan et al. (2003) suggest that the main difference between learners with solely MLD and those with MLD and RD is that the latter group has a more language-based disorder because of the reading difficulties. As a result, the fundamental need emerges to explore which MLD are independent of RD (Vukovic, 2012). Indeed, some studies have indicated that predicting traits for RD are enough to detect children who are at risk of developing MLD (Fletcher, 2005). However, Vukovic (2012) challenges this idea. In her longitudinal study, 203 children were followed from kindergarten level up to Grade 3. The researcher's main interest was to investigate MLD and five crucial areas of mathematics learning including: *working memory, short-term memory, cognitive processing speed, early numerical skills, and phonological processing* (p. 284). These were considered independent of reading ability. Vukovic drew two conclusions. First, that a deficit in early numerical skills is one of the fundamental features that outline MLD with or without RD. This illustrates that numerical skills are independent to reading skills and that therefore without detecting deficiencies in numerical skills it would be difficult to predict later MLD. Additionally, she suggests that these early numerical skills, added to phonological processing, influence the acquisition of mathematics from kindergarten to third grade.

Other studies (Hart, Petrill, & Thompson, 2010; Kovas, Harlaar, Petrill, & Plomin, 2005; Plomin & Kovas, 2005) have also indicated similar conclusions. All suggest that there are unique genetic factors involved in cognitive numerical processing which are independent of those involved in reading. Following their study with twins having MLD and RD, Hart et al. (2010) underscored that there may indeed be independent factors that impinge on different functions like mathematics and the fluency and decoding of reading. Thus, one must bear in mind that although there seems to be a significant overlap between the genetic factors involved in reading and mathematics (Petrill & Plomin, 2007) some specific characteristics seem to be unique to the separate domains.

As a result of her study and the literature currently available, Vukovic (2012) recommends that,

> more diagnostic research is necessary to uncover the gaps in mathematical thinking that characterize children with MD, to ascertain whether these gaps reflect deficits in underlying processes, and to learn how to best address these gaps. Without such research, addressing the fundamental issue of whether MD is a learning difficulty separate from RD is difficult (p. 295).

This concurs with other researchers' (Ansari, 2010) suggestion that more longitudinal studies are essential to explore the degree to which some skills underlie a general atypical development such as that when both RD and MLD are present versus the degree to which deficits in more specific skills underlie a precise disorder such as that of MLD. Qualitative research which investigates more deeply the nature and degree of MLD is also very limited. Additionally, such research has rarely dealt with intervention and effective strategies for both groups of learners.

One of the few qualitative research studies in the field is that carried out by Tressoldi, Rosati, and Lucangeli (2007). In this study seven multiple single-cases were studied. The participants were two children with dyslexia only, two with dyscalculia only, and three children with both dyslexia and dyscalculia. Each subject was assessed using a spectrum of tests for mathematics, reading and other cognitive tests. The outcomes of their study showed that the characteristics of dyscalculia are somewhat different and independent from reading ability. This was brought to the fore after the participants with dyscalculia showed specific impairments in mental and written calculations, retrieving number facts, comparing numbers, number alignment[13] and the identification of mathematical signs.

One area of mathematics learning in which both MLD and RD overlap are word problems. Some studies have researched how children with MLD only and those with MLD and RD perform when completing word problems. A study carried out by Powell et al. (2009) investigated the difficulties which children with no MLD, those with MLD and those with MLD and RD had when solving word problems. The subjects were in third grade when assessed using 14 different word problems made up of *total*, *difference* and *change* problem types. They conjectured that mathematical cognition is different for subtypes of MLD groups i.e. those with solely MLD and others with MLD and RD. The types of problems children with MLD and those with both MLD and RD found difficulties with, were different. The researchers (Powell et al., 2009) concluded that the position in which the missing information was found in the word problem (i.e. whether the problem was of the type '4 + __ = 7', '__ + 3 = 7' or '4 + 3 = __') did not have an impact on all of the pupils' performance.

Other studies have concluded that children with MLD only will struggle particularly with solving word problems. For example, Jordan and Hanich (2000) suggest that pupils with MD "may be at particular risk for having difficulties in mathematics in later elementary school as higher order problem-solving skills receive more emphasis in the mathematics curriculum" (p.576). Moreover,

[13] This task involved vertically aligning eight series of numbers according to the number's value. The numbers ranged from one to four digits and were presented horizontally on a sheet of paper (Tressoldi et al., 2007).

a separate study investigated the responsiveness that children from different subgroups of MLD had towards a mathematical problem-solving intervention programme (Fuchs et al., 2004). The study was conducted with 301 students who were present for a pre-test, followed a 16-week mathematical problem-solving intervention programme and a post-test. The intervention focused on developing the ability of pupils to solve different types of problems. After the given intervention, it was evident that the group of students who were not at risk of MLD improved more than all the other groups (MLD only, MLD and RD and RD only) on computations and labelling. The pupils with both MLD and RD improved least when assessed for conceptual underpinnings. The findings seem to indicate that the mathematical deficits of children with both MLD and RD impinged much more on their responsiveness to the intervention than RD alone (Fuchs et al.). Studies in this field are yet very limited and hence, it is necessary that the differences between these two sub-groups of learners (MLD only; MLD and RD) on more complicated forms of problems solving (Fuchs et al.) is investigated further.

2.13 Conclusion

In this chapter I have presented the existing knowledge on which the questions of this research project are based. As has been shown throughout this Chapter, several questions still prevail regarding this relatively new area of research. Through my study I shall seek to close further some of the gaps, mainly regarding effective intervention strategies for supporting learners with MLD and both MLD and RD. Hence, I will investigate which strategies support these learners to overcome at least some of their difficulties with the acquisition of basic mathematics concepts and skills. This reflects my belief that every individual pupil deserves to receive appropriate support to progress in their mathematics learning.

Chapter 3
Learning through an Intervention Programme: Theoretical Underpinnings

Chapter 3: Learning through an Intervention Programme: Theoretical Underpinnings

3.1 Introduction

Phase 2 of this research project was made up of multiple-case intervention studies. I conducted an intervention programme with six learners – three with MLD only and three with both MLD and RD. The programme was carried out on a one-to-one basis and tackled the ten numeracy components (Dowker, 2004 in Catch Up®, 2009). Before choosing this method of data collection and designing the individual intervention programmes for each of the learners, it was fundamental to select a conceptual framework which would underpin these processes. The conceptual framework needed to be most suited towards my own beliefs as a teacher and researcher. It also had to ensure that all the practices undertaken followed these same beliefs so that an inclusive approach (Bartolo et al., 2007; Ministerial Committee for Inclusive Education (MCIE), 2000; Peterson & Hittie, 2003) was always maintained.

I decided to focus on *social constructivism* because my own beliefs are in line with the epistemological assumptions indicated by this school of thought. My choice of *constructivism* derives from the notion that as stated by Davar (2012), “Constructivism is a theory which proposes that people create their own meaning by combining their existing knowledge with new experiences” (p. 156). My role in this study was that of a teacher-researcher. I believe that as a teacher and researcher my beliefs of how learning takes place were the foundations of all the choices I have made – including my choice of having an individualized intervention programme for each learner and what I choose to place in the programme itself. As a result, I feel it is crucial for me to highlight my own beliefs and explain why these are linked mostly to constructivism.

Primarily, as a teacher, I believe that learning should be an active process which allows the learner to construct their own knowledge through inquiry and interactions with others. Secondly, I believe that all learners come to school with a prior baggage of knowledge and that all learning should seek to be connected to the learner’s prior knowledge. Third, in line with the epistemological assumptions of constructivism, I believe that reflecting on the learners’ lived experiences during the programme, is important to facilitate the creation of new knowledge.

3.2 Psychological or Social Constructivist Theories?

Learning theories such as those originating from Piaget (1896-1980), known as *psychological constructivism*, and Vygotsky (1896-1934), *social constructivism*, have sought to answer questions related to the science of learning like 'What is learning?' and 'How can effective learning occur?' Donovan, Bransford, and Pellegrino (1999) argue that although each constructivist school of thought has its own 'blueprint' there are three common essential learning principles. Firstly, that children come to the classroom with a baggage of previous knowledge about how the world around them evolves; Secondly, that to acquire competence in a specific field of inquiry children must have a deep understanding of its underlying factual knowledge, understand these facts within a conceptual framework and be able to form knowledge in a way which facilitates its retrieval. Lastly that a 'metacognitive' approach to teaching can give students the tools to define learning objectives and understand their progress in achieving them, thus taking further control of their learning.

The theories proposed by both Piaget (1896-1980) and Vygotsky (1896-1934) had such a great impact on the development of psychology and education that their contributions to these two areas of studies cannot go unnoticed (Daniels, Cole, & Wertsch, 2007; Kirschner & Martin, 2010). Lourenço (2012) rightly suggests that, "it if were asked who are the two main geniuses in the field of developmental psychology, many, if not all, developmentalists would certainly point to Jean Piaget (1896-1980) and Lev Vygotsky (1896-1934) in either order" (p. 281). Hence, what I now discuss are the main theories proposed by these two theorists in developmental psychology, how they have contributed to education and why I chose Vygotsky's (1896-1934) to be at the heart of my conceptual framework.

Piaget's (1896-1980) theories are central to psychological constructivism. Psychological constructivists believe "that an individual can construct new understanding using his own knowledge and beliefs…they focus on the knowledge, beliefs and self-concepts (inner psychological life) of an individual" (Davar, 2012, p. 156). These beliefs originate from Kant's ideology that an individual's ideas are already embedded in the mind waiting to be discovered. As indicated by Piaget (1980) the learner is an active participant in the construction of knowledge which is based on existing schemata of thoughts. The learner then steadily modifies and expands these schemata. Knowledge is thus constructed by the self, based on previous knowledge through what Piaget (1952, 1980, 1985) labels as *accommodation* and *assimilation*. A main characteristic of Piaget's (1896-1980) works is the sense of autonomy of a learner (Bruner, 1997). Within a Piagetian perspective, the learner usually constructs his/her knowledge through his/her own exploration.

Piaget's theories emphasise the importance of developmental stages in an individual's life. Thus, Piaget proposes that the current state of the child's development will determine the learning taking place. Furthermore, according to Piaget (Inhelder & Piaget, 1969; Piaget, 1942), scientific concepts such as number and classification naturally emerge in a learner's thoughts without the need of teaching or any other form of direct intervention. However, in this study I assessed the participants for what they knew prior to the intervention programme and guided them to mastering the numeracy component which they should have grasped in their earlier stages of development. In this case, learning led to development rather than the other way around. Moreover, since the numeracy components at hand had not 'naturally' been mastered, direct intervention was needed to facilitate their mastery. Furthermore, in my study, the focus was social interaction, guided by myself, which would facilitate the learners' mastery of the numeracy components being tackled. A social constructivist perspective, specifically Vygotsky's (1896-1934), was thus deemed more closely aligned with my own beliefs.

Social constructivism states that all forms of learning are socially constructed. This school of thought derives from the works of Vygotsky (1896-1934). Vygotsky argued that all higher psychological processes begin as social processes which have been shared between people, particularly between adults and children. As a result, Ernest (1998) suggests that social constructivist theorists believe that the mind is socially formed, and that knowledge is constructed through socially situated interactions or conversations. Thus, social constructivist theories focus mostly on the impact that culture has on one's construction of the world around him/her. This idea is what mainly contrasts Vygotskian thought with Piaget's (1896-1980) theories, since the notion of heteronomy, i.e. the need for the learner to interact with adults and more competent peers for development to take place, is highly present in Vygotsky's works (Lourenço, 2012). Social constructivism proposes that the learner learns most if guided by a peer or adult rather than on his own accord. From a social constructivist perspective, learning precedes development, as it is learning that facilitates a child's development. Lastly, Vygotsky's (1896-1934) view of the notion of how scientific concepts emerge naturally in a child's thought contrasted that of Piaget (1896-1980). Vygotsky (1962) highlighted that, "verbal intercourse with adults thus becomes a powerful factor in the development of the child's [scientific] concepts" (p.69). This is indeed highly compatible with my own beliefs of how children's scientific and mathematical concepts develop, that is through rich conversations with their teachers, within the classroom and other learning situations.

Thus, the fundamental concern which geared me towards which learning theory suited my beliefs as a teacher and researcher was the applicability of the learning theory to the classroom setting. As a result, I concluded that social constructivism would serve as an appropriate underlying theory for my study. In summary the reasons why, I chose to underpin my study with social constructivism are my beliefs that:

i. the teacher has a central role in guiding a learner through the learning process;
ii. the learning process is effective only when learning is geared towards the current state of the learner's development;
iii. rich social interactions can enhance learning especially through the rich tool of social conversation and other forms of language; and
iv. resources and language can be used to facilitate learning and to support the learner's development.

As summarized by Wertsch (1985a & 1985b), Vygotsky's theories can be understood through three main themes. These are: a dependence on a genetic/developmental method; the assertion that social life can impact on the development of an individual's higher mental functioning; and the claim that a major part of human mental functioning can only be understood by looking closely into the tools and signs that underpin it. Taking Vygotsky's theories into account whilst developing and implementing the programme and whilst evaluating it allowed me to provide deeper insights into what strategies were effective for the participants studied. In subsequent sections I will give further detail about the main tenets of Vygotsky's theories as each will be discussed in much detail in the forthcoming sections. However, I find that at this stage, a working definition of each of the principles he highlights is helpful. The following list introduces key terms which have either been used directly by Vygotsky in his works or derive from his works (*Scaffolding*). The list also provides a brief definition which I have written up for each term:

- **Zone of Proximal Development (ZPD):** The Zone of Proximal Development (ZPD) is the potential learning which a learner is able construct through guidance;

- **More Knowledgeable Other (MKO):** The More Knowledgeable Other (MKO) is the person, who knows more than the learner, whether a parent, a peer or a significant other like me, and can thus provide suitable guidance for the learner to move from the actual state of knowledge to the potential one;
- **Tools**: Tools support the learning and the More Knowledgeable Other (MKO) in guiding the learner to the potential zone of development. Tools may be psychological or technical as will be explained in Section 3.4;
- **Scaffolding** *(term not used by Vygotsky himself):* This describes how the MKO ensures that tasks move from one level of difficulty to another in a gradual manner to support the learning process.
- **Internalisation**: When the learning process is effective, the learner should 'internalise' the learning, that is to understand and retain it, and show this by being able to use it on other occasions.
- **Semiotic Mediation**: Semiotic Mediation refers to social interaction in which language is used by the MKO to facilitate internalisation.

All these terms will be used in the various sections of this Chapter and in subsequent Chapters, as I explain the importance of each one, and demonstrate how they have underpinned the analysis of data and interpretation of findings.

3.3 *Cultural Mediation* – Social Interaction and Internalisation

Vygotsky (1978) emphasises that a child's development is highly influenced by their culture and social interaction. Unlike Piaget, Vygotsky illustrated that a child develops through a transformative collaborative practice which involves cultural tools, cultural influences, and other significant adults (Vianna & Stetsenko, 2006). Sociocultural theories, of which Vygotsky is the forefather, propose that all learning is a social enterprise. As Smith, Teemant, and Pinnegar (2004) point out, "Individuals actively construct personal understandings and abilities by way of cooperative interaction and negotiation of shared meanings in social contexts" (p.2). I shall therefore be using the powerful means of social interaction by providing struggling learners with meaningful interactions with the intention of enhancing development and therefore learning. Moreover, I will seek to analyse my data in a similar way to that in which Vygotsky's (1978) research team analysed his own. Vygotsky (1978) clearly indicates that the analysis of his team "accords symbolic activity a specific organizing function that penetrates the process of tool use and produces fundamentally new forms of behaviour" (p.24). Thus, when analysing my data I shall seek examples and situations in which the interactions provided by my learning programme, including my use of tools (as will be explained shortly) would have altered the child's cognitive ability or behaviour to such an extent that they now *know* more vis-à-vis the numeracy component being focused on before the actual interaction itself.

Vygotsky's emphasis on internalisation is also a crucial perspective which will be strongly related to my research. As highlighted by Mariotti (2009), "Vygotsky's approach to learning is not separable from his approach to teaching, and the central role played by internalisation constitutes the unifying element" (Vygotsky, 1981, p. 162). Vygotsky's internalization model proposes that development happens when learning proceeds from the social plane to the individual one. Such internalisation becomes the basis for an individual's thoughts. Matusov (1998) highlights that the concept of internalization was defined by other sociocultural theorists as "a transformation of intermental (interpsychological) external functions distributed among participants of joint sociocultural activity into intra-mental (intrapsychological) internal functions of individual skills, or as Vygotsky put it, '*the social plane*' transforms into '*the psychological plane*'" (p.328). Vygotsky (1981) illustrated his idea by giving the example of a child pointing his finger to show his mother an object he would like to have. Vygotsky explains that primarily this happens casually but that when, following repetition of this same process, the child internalizes that pointing a figure to an object demonstrates to others that he would like it, the child starts using this action with the actual intention to get grasp of the object. As a result, Vygotsky (1978) argues that "what a child can do with assistance today she will be able to do by herself tomorrow" (p.81).

Vygotsky (1978) indicates that it is the uniting of perception, speech and action that ultimately results in the construction of new knowledge. Thus, Dunphy and Dunphy (2003) conclude that an individual's lifelong learning is characterized by numerous cycles, within the learner's Zone of Proximal Development (ZPD), "from other-assistance to self-assistance, recurring over and over again for the development of new capacities" (p. 50). One of the main tenets of the internalization theory presented by Vygotsky (1978) is that an individual's development should be studied by comparing the person's skills and functions before, during and after a specifically planned programme that aims at facilitating learning and therefore internalization. This is known as Vygotsky's 'formative experiment' methodology (van der Veer & Valsiner, 1991). I myself have used this methodology for this study.

Analysing situations where internalization will take place will be fundamental since "internalisation transforms the process itself and changes its structure and functions" (Vygotsky, 1981, p. 163) and it is these transformations which I will be looking at closely. However, as argued by Wertsch (1985b) the core question about Vygotsky's concept of internalization remains; '*How does the social plane become the individual?*' Barbara Rogoff (1995) sought to find answers to this question by observing sociocultural activity on three dimensions: participatory appropriation, guided participation and apprenticeship. The aspect which Rogoff (1995) proposes which is most closely related to Vygotsky's theory of internalization is the 'participatory appropriation' theory (Matusov, 1998). Rogoff (1995) argues that the "basic idea of appropriation is that, through participation, people change and, in the process, become prepared to engage in subsequent similar activities" (p. 148). Although developed from the model of internalization, Rogoff (1995) uses the terms 'appropriation' and 'participatory appropriation' in contrast to the term 'internalisation' because she focuses on discussing gain from the learners' involvement in social interaction. Rogoff (1995) explains that "rather than viewing the process as one of internalization in which something static is taken across a boundary from the external to the internal, I see children's active participation itself as being the process by which they gain facility in an activity" (p. 148). This idea is similar to my own since in order to analyse instances of internalization, I intend to explore the process through which this takes place including the relationship between myself as the More Knowledgeable Other (MKO), the learner and the use of cultural tools. This is in line with Rogoff's participatory appropriation model which highlights that development and learning are the result of:

- the interdependence of the learner and his/her social partner – both having active and dynamically changing roles; and
- the processes by which they communicate and take decisions together (Rogoff, 1995).

However, throughout this study I will still use the term 'internalisation' to demonstrate the children's mastery of a numeracy component, mathematical language, skill or competence. I wish to stick to Vygotsky's original theories so that at the end of my analysis I can propose a holistic view of how his theories may be applied to mathematics education, specifically to MLD.

The main question resulting from Vygotsky's internalization model, *'how does the social plane become individual?'* (Wertsch, 1985b) links very well to my main research question which seeks to investigate which strategies are effective in supporting the internalization process. Rogoff's (1995) 'participatory appropriation' theory demonstrates way in which the MKO and the learner are *interdependent*. However, I will seek to develop this interpretation further by looking deeper at the interdependence of the MKO and the learner during intervention sessions which aim to support learners to internalize the numeracy components; exploring the applicability of this theory to the field of MLD. Moreover, I also wish to look at the way in which cultural tools (discussed in the next Section) play a role in making this interdependence a success. In my data analysis I will thus not only seek to highlight the learner's internalisation both of the knowledge as well as of the mastery of using the tools introduced but will also shed light on which strategies supported this process. In this way, I hope that my study will contribute to answering Wertsch's (1985b) question about the internalization process and to develop Rogoff's (1995) theory further; namely, to gain a better understanding of what contributes effectively to shifting knowledge and skills at the social plane to the individual one. For my participants, this shift would not have happened in the classroom setting since they would not have yet internalized the numeracy components. Hence, I will seek to identify how my intervention might support the shift in planes more effectively than in the classroom setting.

3.4 Cultural Tools: Technical and Psychological

Internalization will only take place through the use of what Vygotsky refers to as *cultural tools*. Vygotsky suggests (1978) that "the child's system of activity is determined at each specific stage both by the child's degree of organic development and by his or her degree of mastery in the use of tools" (p.21). In more recent studies, *cultural tools* are also referred to as *artefacts* (for example, Bartolini Bussi, 2011). I will, however, mainly use the term *cultural tools*, unless the term 'artefacts' is used by the author/s being cited, to remain faithful to the terminology used by Vygotsky. The term *cultural tools* encompasses both technical tools and psychological/cognitive tools. These tools support the process of signification, or process of attaching meaning to symbols, which becomes more and more complex as the child develops. Technical tools differ from psychological tools which are also referred to as *signs*. In Vygotsky's (1981) terms,

> The most essential feature distinguishing the psychological tool from the technical tool, is that it directs the mind and behaviour whereas the technical tool, which is also inserted as an intermediate link between human activity and the external object, is directed toward producing one or other set of changes in the object itself (p. 140).

Technical tools "serve as the conductor of human influence on the object of activity; it is externally oriented" (Vygotsky, 1978, p.55). In the field of mathematics technical tools would include, for example, the use of a ruler or a protractor.

On the other hand, psychological tools are those tools which are directed inward, and which gear the mind and change the process of thinking. The psychological tools act as a means of changing one's own mental process and include mathematical symbols, numerical systems and language. As highlighted by Knox and Stevens (1993),

> What needs to be stressed here is his [Vygotsky's] position that it is not the tools or signs, in and of themselves, which are important for thought development but the *meaning* encoded in them. Theoretically, then, the *type* of symbolic system should not matter, as long as meaning is retained...They [tools embedded in action] allow a child to internalize language and develop those higher mental functions for which language serves as a basis (p. 15).

One of the uses of psychological tools, as explained by Vygotsky (1978), is for learners to "personally create a temporary link through an artificial combination of stimuli" (p. 51).

Vygotsky (1981) also suggests that a tool can have both a technical function and a psychological purpose. He explicitly mentions that *language*, *various systems for counting*, *diagrams* and *mechanical drawings*, amongst others are examples of these. This idea of one tool having a dual purpose is also highlighted by Bartolini Bussi, Corni, Mariani, and Falcade (2012) who argue that "In some cases, the same "tool" may be conceived as technical and psychological" (p. 3). This idea is one which will be explored further in my analysis of data.

Vygotsky did not apply his notion of cultural tools specifically to mathematics education. However, the application of this theory to mathematics education has been explored by various researchers (Abtahi, 2014, 2017 & 2018; Graven & Lerman, 2014; Mariotti, 2009). Mathematical tools do not carry any mathematical meaning with them, however learners build their mathematical ideas through their interaction with the tools and by thinking about their actions (Clements & McMillen, 1996). Nonetheless, some children have difficulties perceiving the relationship between a specific mathematical tool and the concept that it represents (McNeil & Uttal, 2009). As a result, this has prompted researchers such as Abtahi (2018) to explore the role played by mathematical tools as learners interact with them when engaging in a mathematical task. Abtahi (2018) specifies that in her most recent research study her focus is "on both the tools and the role the tools play in the learning/knowing that happens in the interaction, within the Zone of Proximal Development (ZPD)" (p.1). This article was published after I analysed my data however it is important to note that it is very similar to the focus I took when analyzing the use of tools. In this most recent publication, Abtahi (2018) proposes that interactions within the ZPD may be sign- and/or tool-mediated and that therefore the mathematical tools may sometimes take the role of the MKO in supporting internalization. This idea is one which I also look at through the analysis of the use of tools during the intervention sessions provided.

Graven and Lerman (2014) suggest that tools can take the role of the MKO. They describe the situation of a child named Lila interacting with her mother, Mellony, as Lila began to use the remote control to count in threes. The buttons on the remote control were in threes and she counted the buttons in rows. In this situation, Graven and Lerman argued that whereas the mother was primarily the MKO, Lila later took up the role of MKO. They stated that,

> Initially, Lila is the activator of the emergence of the ZPD. In this respect and given that she is holding the artefact which mediated her discovery, Lila is the more knowledgeable other of the discovery and her mother is the learner connecting the relationship between the structuring of the numbered buttons on the artefact and Lila's counting in three (p.30).

Following this, Graven and Lerman demonstrated that the two parties changed roles again. They argued that when Lila had shown and explained her discovery to her mother, Mellony took up the role of MKO again as she had to affirm Lila's discovery and endorse the correctness of naming this pattern. When writing about this study, Abtahi (2014) confirmed that the mother-child interaction was making use of role inversion since she suggested that in this situation, "the role of being the more knowledgeable other is alternating between the mother and the child" (p.14). Moreover, Abtahi also insinuated that the artefact, in this case the remote control, could have also played the role of the MKO at one point since she posed the following question, "if the ZPD emerged when Lila's attention was caught by the remote…who/what was the more knowledgeable other?" (p.14) and later asked, "is it possible that the more knowledgeable other in Lila's interaction with the remote is the remote control itself?" (p.15). The role of the MKO is to guide the learner from their current zone of development to their potential one. One can ask: if a tool has the potential to do this, is it possible that it can take up the role of the MKO? In a real classroom situation, if a child cannot add '1' to a given number, the teacher might give the learner a number grid (as a tool) to help the child complete the given task. In this case, one may argue that the number grid is assisting the child instead of an adult or more knowledgeable peer. Hence, in my analysis I will explore this shifting role between the cultural tool and the MKO and how possible it is for tools to take the role of an MKO. A main question I will seek to answer is whether using a tool, such as a digital tool, instead of the MKO is an effective strategy in facilitating internalisation or whether the MKO still needs to maintain his/her role before and after the use of the tool itself. I will thus explore the relationship between the MKO and the use of tools since, as suggested by Wertsch (1993), “On the one hand, cultural tools cannot play any role in human action if they are not appropriated by concrete individuals acting in unique contexts. On the other hand, we cannot act as humans without invoking cultural tools” (p. 170).

Another relevant idea that has been explored in mathematics education is that of viewing digital games as tools. Although, of course, Vygotsky did not explore digital games himself, the idea of digital games may be considered as serving as tools within a neo-Vygotskian perspective. Studies (Borba & Bartolini Bussi, 2008; Mariotti, 2009) in this field refer to them as ‘technological tools’. When referring to Vygotsky’s theories, Mariotti (2009) suggested that, “in the past 20 years, a different theoretical framework has become relevant for the issue of integrating technological tools into mathematics education” (p. 427). Researchers (Borba & Bartolini Bussi, 2008; Mariotti, 2009) who have explored this aspect of mathematics education have indicated that technological tools may be important for effective semiotic mediation. Mariotti (2009) highlighted that the relationship between knowledge and artefacts is complex and that careful analysis of this connection is needed “to avoid useless oversimplification and allow to fully exploit the potential that the use of technology (and in particular new technology) offers to mathematical education” (p. 428). When analyzing the intervention sessions, I will seek to determine whether digital tools may have been one of the reasons why the children managed to master mathematical concepts and how these tools contributed to effective semiotic mediation.

As highlighted by Caven (1952), “The mathematics teacher should not use any aid unless he can reasonably assure himself that he can, by the use of this aid, create a better learning situation with regard to some definite area of his subject” (p.334). Throughout my intervention sessions I will ensure that each tool is used well to support the learning of the specific numeracy component at hand. During the intervention sessions I will make use of several technical tools to facilitate internalisation through effective scaffolding. These include the ruler, interlocking cubes (used as objects for counting tasks), paper clips, number cards, mini whiteboards and dice. These items will have a utilitarian purpose that is, they will be used for an exercise or an activity to be carried out successfully. Hence, they will be considered as technical tools. For example, the paper clip will be used to mark a number on the number line. Any other means of marking a number on the number line could have been used hence the role of the paper clip is to serve as a technical tool rather than to alter the learner's cognitive functioning or behaviour. Similarly, number cards will be used as a means of getting a random number. Dice will also be used for this purpose, showing the utilitarian purpose of using these tools. The use of these technical tools and some examples of how they were used will be discussed and illustrated in Section 6.11.1. Technical tools will not serve the same purpose as psychological tools throughout the intervention sessions as they will not "direct the mind and behaviour" (Vygotsky, 1981, p.14) as the latter would. However, they will be indispensable for the MKO to carry out a specific task or too provide meaningful social interaction through activities.

When I planned what would serve as psychological tools in my sessions, I identified that the psychological tools used would be those in which the meaning encoded in them was more important that the tools themselves (Vygotsky, 1978). Hence, in this study, psychological tools were the tools and artefacts used to support the internalisation of the numeracy components directly. The main psychological tool used throughout the sessions will be language, both general and subject-specific. This tool will be discussed shortly (Section 3.4.2). The other psychological tools that will be used are manipulatives, visual aids and digital games.

Using both technical and psychological tools I shall seek to support the children in using external objects to support learners in using basic mental functions like *attention*, *memory* and *perception* to develop numerical concepts. Vygotsky (1978) labels attention as the major function which is crucial for making use of cultural tools. The learner's attention needs to be caught and maintained for the effective use of tools. Vygotsky mentions two forms of memory; natural memory and mediated memory. Natural memory relies on perception and is the outcome of external stimuli whilst mediated memory is that which is mediated by signs and which results from social development. In his writing, Vygotsky explains how a human may tie a knot in a handkerchief to remind them of something else. In my study, natural memory features since I shall use tools like mnemonics to help the children to transform remembering into an external activity by helping the students to "personally create a temporary link through an artificial combination of stimuli" (Vygotsky, 1978, p.51). Memory will thus also be catered for using concrete materials which will support the development of visual mental representations. Mediated memory will also play a key role. Vygotsky underscores that "through verbal formulations of past situations and activities, the child frees himself from the limitations of direct recall; he succeeds in synthesizing the past and present to suit his purpose" (p. 36). The children will be encouraged to explain concepts and ideas to me because this helps them to remember more. The children will also be encouraged to make links between the past sessions and the present one. A specific way in which this will be done is by asking the children to reflect about the miscues carried out during previous sessions so that these are not repeated.

Another idea which I will consider is Vygotsky's (1978) explanation of perception; more specifically 'the perception of real objects'. In his writing Vygotsky explains that humans do not simply see colour and shape in objects but attribute meanings to different objects. As a result, for example, a clock is not normally seen as a round object with two hands but as a *clock*, which it is. An individual may understand what it is used for and, for example, distinguish between the use of both hands. One of the ways in which children's behaviour should change through my programme will thus be by changing their perception of particular tools. For example, when using the Cuisenaire rods, they might primarily have a perception of the rods based on their shapes and colours. However, I will help the children to develop this perception and use these tools as a means of building mental representations of the number system. Perception is also highly linked to language from the very early stages of an individual's life since the latter is needed to give meaning to real objects. Vygotsky (1978) explains that language plays an indirect role even in non-verbal tasks since inner language is always present. This fundamental system of signs will be tackled in the next section.

3.4.1 Representations

Boaler, Chen, Williams, and Cordero (2016) suggest that good mathematics teachers typically use visuals and manipulatives to support learners' understanding of mathematical concepts. Boaler et al. (2016) also argue that these two tools are significant for providing learners with multiple representations of the same mathematical idea. The notion of representations is an important one in mathematics education. As outlined by Ennis and Witeck (2007),

> representations take many forms: an algorithm that represents a problem situation, a graph that represents data collected in the classroom, an array that shows a multiplication process, or a diagram that illustrates area or perimeter, numbers, pictures, diagrams, equations, graphs, and models are all forms of mathematical representations (p.2).

Representations support learners to develop what Skemp (1978) referred to as *relational* understanding in mathematics learning. This means that the children gain a deeper understanding of the concept at hand rather than an *instrumental* one. Representations are also a means of communicating mathematical ideas, "as well as powerful tools for thinking" (National Council for the Teaching of Mathematics, NCTM, 2000, p. 136). Thus, learners should also be encouraged to make use of them as an alternative form of communication and thought process. In mathematics there are multiple representations of the same concept and "it is up to the teacher to help students make the connection between the models and the mathematics" (Ennis & Witeck, 2007). Bruner (1966) grouped representations into three categories: the concrete, pictorial and abstract and described how learning should be a journey from one representation to another. All three are essential in mathematics learning.

The CPA Approach (Bruner, 1966) proposes that learning should start with concrete experiences through which the learner uses real objects to gain a relational understanding of the concept at hand. As indicated by Marzano, Pickering, and Pollock (2001), "the very act of generating a concrete representation establishes an image of the knowledge in students' minds" (p. 78). These concrete experiences lay the foundations for the use of symbols later on (National Council for the Teaching of Mathematics, NCTM, 2000). In this study, concrete experience will mainly be provided through the use of manipulatives. The concrete representation is to be followed by a pictorial representation. Hence, the activities which will follow the introductory hands-on task will be of a pictorial nature and will seek to promote visual images and mathematical representations pictorially to support internalisation. As suggested by Ennis and Witeck (2007), "students needing alternate ways of expressing their thinking often times find that pictorial representations open a much-needed path for communication" (p. 7). Pictorial representations will thus be provided through visual aids and drawings (for example of arrays of objects to show different operations). Abstract (also known as symbolic) representations, including number sentences (with no images) will only be given to the learner at the end of the session through a task or the linked recording section. This approach will hopefully contribute to facilitating the internalisation of the concepts being covered. What I will thus present here are some studies that have explored visuals and manipulatives which are indispensable for offering learners a wide array of representations in mathematics.

As indicated earlier, manipulatives serve as tools for providing concrete mathematical representations. "Manipulatives are physical objects that students and teachers can use to illustrate and discover mathematical concepts, whether made specifically for mathematics (e.g., connecting cubes) or for other purposes (e.g., buttons)" (Van de Walle, Karp, & Bay-Williams, 2015, p.46).

The importance of the use of manipulatives as essential aids for mathematics learning has been recommended by many textbook schemes and publishing companies (Nührenbörger & Steinbring, 2008). Such a hands-on approach has been shown to support the learner's understanding of mathematical concepts and language as well as to provide opportunities to express mathematical ideas. As suggested by Coggins, Kravin, Coates, and Carroll (2007) "it is important for students to be *actively* using materials to investigate mathematical ideas" (p. 49). As indicated by the same authors, it is not enough for students to use manipulatives (concrete materials) in lessons which are led by the teacher's talking and thinking. The use of concrete materials is fundamental. They should be used throughout the lesson starting from the teacher's explanation. Concrete materials should also be used by the students themselves to explain their mathematical ideas. When used effectively, manipulatives can inspire pupils to think more deeply about the mathematics concepts being studied, as well as to make connections with related concepts. Hence, most intervention sessions will start with the use of manipulatives. This also supports Bruner's Enactive – Iconic – Symbolic approach, more commonly now known as the Concrete – Pictorial – Abstract (CPA) approach, which I also kept in mind when planning the sessions.

Visual aids will support the development of mental representations which the learners will be able to use to solve future mathematical tasks. As highlighted by Shabiralyani, Hasan, Hamad, and Iqbal (2015), the use of visual aids has several advantages including proper use of visual aids helps to retain concepts longer and they help to increase the vocabulary of the pupils (p.226). During the intervention programme, the visual aids used will include: a number line, a number grid, flashcards with number names and important mathematics vocabulary (see Section 2.6.2), as well as dot patterns to enhance learners' subitizing skills. Visual aids will play a crucial role in providing the learners with the pictorial representation of the concrete representation experienced using the manipulatives.

Visual aids are also important because the average human brain needs them to understand quantities. Menon (2014) suggested that a widely distributed area of the brain supports the mental processing of mathematics knowledge. One of the fundamental areas of the brain in this process is the dorsal visual pathway that plays a key role when learners look at visual or spatial representations of quantity. As a result, a few studies have found out that representations such as the number line and the number grid that present number quantity are important for the development of numerical knowledge (Kucian et al., 2006; Schneider, Grabner, & Paetsch, 2009).

Similarly, literature (Chinn, 2012; Emerson & Babtie, 2014) has also indicated that repeatedly showing children visual representations of number patterns helps them to develop their number sense and ability of subitizing better also indicating the validity of using such visual tools.

Both manipulatives and visual aids are also referred to as 'artefacts' (for example, Bartolini Bussi, 2011). Artefacts have been shown to serve as tools for semiotic mediation in previous studies (Bartolini Bussi & Mariotti, 2008). In their study, Bartolini Bussi and Mariotti suggest that, "any artefact will be referred to as tool of semiotic mediation as long as it is (or conceived to be) intentionally used by the teacher to mediate a mathematical content through a designed didactical intervention" (p. 754). Hence, my belief is that cultural tools are needed to provide learners with multiple representations. Although some authors (for example, Van de Walle, Karp & Bay-Williams, 2015) seem to attach the same meaning to 'tools' as to 'representations', similarly to Bartolini Bussi and Mariotti, I argue that a tool only becomes a representation when used with intent and meaning. This is similar to Vygotsky's (1978) idea of psychological tools, indicating that when used with the right intent, psychological tools will become representations, however technical tools merely have the potential to become so.

3.4.2 Language

Language is given prime importance in Vygotsky's work. In *Thought and Language* (1962) Vygotsky explains that language is the most important sign-using behaviour in children's behaviour. As highlighted by Walshaw (2017), "In Vygotskian thinking, much of joint intellectual activity and meaning making is derived from language" (p.295). Vygotsky believed that language is the most important means by which individuals may express themselves and thus share their thoughts. They also use language to prepare themselves for the future and learn to plan, organise and regulate their own behaviour and that of others. In his later publication *Mind and Society* (1978) he maintains the same importance he had previously given to language. Vygotsky (1978) underscores that,

> the specifically human capacity for language enables children to provide for auxiliary tools in the solution of difficult tasks, to overcome impulsive action, to plan a solution to a problem prior to its execution, and to master their own behaviour. Signs and words serve children first and foremost as a means of social contact with other people. The cognitive and communicative functions of language become the basis of a new and superior form of activity in children, distinguishing them from animals (p.28 - 29).

The importance Vygotsky gave to language was also supported by other researchers who followed him. Bakhtin (1986), for example, indicates that "all the diverse areas of human activity involve the use of language…" (p. 60). Due to the importance given to language in a learner's cognitive development, I shall analyse individual situations in which language serves as a psychological tool to guide the learner. Vygotsky (1978) explained that psychological tools "allow a child to internalize language and develop those higher mental functions for which language serves as a basis" (p.15). Hence, language is not only a psychological tool in itself but is also the basis of other psychological tools that need to be used. Vygotsky refers to the instances in which language is used for this purpose as *semiotic mediation*. Semiotic mediation will be used throughout the sessions and will be critical to providing social interaction between the MKO and the learner that will support the child's development within the ZPD. As indicated by Walshaw (2017), "if the teacher's talk fails to keep the student's mind attuned to the teacher's, scaffolding loses its impact and the development of shared understanding is minimized" (p.295).

The language which will be used throughout the sessions will be both general (for example, when giving instructions or feedback) and subject-specific. The latter form of language will be used when completing the mathematical tasks and therefore the *mathematics register* (Halliday, 1978) will be emphasized (see Section 2.6.2). During each session I shall focus on specific language and/or mathematics terms and symbols. In my data analysis, I shall explore, for example, how the general language used supported internalisation through the use of instructions. I will also discuss how mathematical terms and expressions were used by the MKO to help the learners express themselves mathematically. Whilst implementing my intervention programme I will also use language to help the children reflect about their process of learning through metacognitive questioning techniques. I shall also use language to communicate effectively with the learners and scaffold their process of development. In my analysis I will focus on how language is used to support the learners in their internalisation process. Analysing situations in which semiotic mediation takes place will allow me to understand better how language can be used to support the learner whilst moving within the Zone of Proximal Development (ZPD) (see Section 3.5). Through such an analysis I will be able to observe whether the learners internalise the language emphasized and how, if in any way, they make use of this language.

3.4.2.1 Metacognition

One essential use of language is when it is used for the process of metacognition. There is general agreement amongst researchers that metacognition refers to "the act of thinking about one's own thought processes" (Kustritz & Clarkson, 2017, p. 338). However, a more specific definition is that provided by Flavell (1979 in Silver, 2013) in which he indicates that metacognition refers to knowledge concerning one's own cognitive process and products, and the cognition of others. The process of metacognition involves engaging the learner in higher order thinking through questioning techniques, self-monitoring, regulation and evaluation (Montague, Warger, & Morgan, 2000). In this way, language thus becomes the "teacher's instrument for building models of the child's or the student's thinking" (Sierpinska, 1998, p.32). Although Vygotsky does not use the term himself, he does discuss what may be called metacognitive behaviour in his writings and describes it as a higher form of the use of language. In his text *Mind and Society* Vygotsky (1978) suggests that,

> the greatest change in children's capacity to use language as a problem-solving tool takes place somewhat later in their development, when socialized speech (which has previously been used to address an adult) is turned inward. Instead of appealing to the adult, children appeal to themselves; language thus takes on an intrapersonal function in addition to its interpersonal use (Vygotsky, 1978, p.27 – 28).

Research (Borkowski, 1992; Carr & Biddlecomb, 1998; De Corte, Verschaffel, & Op 't Eynde, 2000) has described metacognition as being fundamental in the learning of mathematics. As a result, the use and teaching of metacognitive skills will be embedded in my intervention programme most evidently through the questioning techniques which will be used to support the participants in engaging in thinking about their learning process. This will also involve helping students to identify their own miscues, highlight them and develop them to learn from their own misconceptions. This aspect of the learning process will thus also be analysed to understand better the level of effectiveness which this strategy has for the learners being studied and the degree to which learners learn from those same miscues.

3.5 Scaffolding and the Zone of Proximal Development (ZPD)

As described by Vygotsky (1978) himself, the ZPD is, "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (p. 86). As highlighted by Daniels (2005) "The ZPD provides the setting in which the social and individual are brought together. It is in the ZPD that so-called 'psychological tools' (particularly speech) and signs have a meditational function" (Daniels, 2005, p. 7). Dunphy and Dunphy (2003) identified four stages within the ZPD. These are:

i. Stage one - Performance is assisted by the more knowledgeable other – in this stage the "learner's responses are acquiescent or imitative" (p. 49);
ii. Stage two – the learner carries out a task without any assistance;
iii. Stage three – the performance is 'automatized' [used with automaticity] and internalized – the learner is self-regulated to carry out the task and does so in a smooth and integrated manner;
iv. Stage four – this is where the performance of the learner is de-automatized and leads back to the ZPD (pp. 49-50).

Although the notion of the ZPD has been widely used, the popularity of the term has had its consequences. As Wertsch (1984) explains, unless this notion is developed further "it will be used loosely and indiscriminately, thereby becoming so amorphous that it loses all explanatory power" (p. 7). In a recent publication, Stott (2018) still feels that the term ZPD is one of the most used and least understood constructs. Stott (2018) argues that in Vygotsky's perspective, the ZPD only takes place if the child's learning leads to the development of higher psychological functions. She argues that "it can be difficult to identify development of higher-level psychological functions in his [Vygotsky's] terms" and that thus "it is often easier to observe and possibly measure learning" (p.22). This supports the argument, presented by Chaiklin (2003) much earlier, suggesting that in the educational sphere it would be better to consider the ZPD as a zone of potential learning (ZPL) rather than a zone of development. As a teacher-researcher this is an argument which I favour since learning is measurable, but development is more elusive. Hence, throughout this study I feel that it would be more practical and precise to demonstrate the internalization of numeracy components by comparing the children's zone of learning prior to the programme to that after the programme. Since what the pre- and post- tests will measure is the *learning* that took place and not the *development*. Nonetheless, learning does lead to development. Hence, although it is learning that will specifically be measured, I will still refer to the ZPD instead of the ZPL considering that, as also in Vygotsky's view, learning does lead to the development of the self.

As suggested by Daniels (2005), Vygotsky himself rarely indicates which interactions – whether instructional or collaborative - are appropriate for guiding learners through their ZPD. However, authors who have looked closely at Vygotsky's work such as Wood, Bruner and Ross (1976) and Tharp (1993) have developed Vygotsky's idea of the ZPD and summarized the kind of teaching instruction that not only guides the learner's performance through the ZPD but also acts as a way of supporting the social interaction between the more knowledgeable other and the learner to develop meaningful dialogue. One of the ways in which this theory was extended was by the introduction of the term scaffolding. McLeod (2012) suggests, "The ZPD has become synonymous in the literature with the term scaffolding" (p. 2). Although not a term introduced by Vygotsky himself, the term 'scaffolding' has been defined by its originators (Wood et al., 1976) as a process "that enables a child or novice to solve a task or achieve a goal that would be beyond his unassisted efforts" (p. 90). Hence, the terms 'scaffolding' and 'ZPD' are closely related since the former is the support offered by the MKO to assist the learner in moving from the zone of actual development to the proximal one. In Anghileri's (2006) terms, "the notion of 'scaffolding' has been used to reflect the way adult support is adjusted as the child learns and is ultimately removed when the learner can 'stand alone'" (p. 33). Wood et al. (1976) indicate that scaffolds require the MKO to control "those elements of the task that are initially beyond the learner's capability, thus permitting him to concentrate upon and complete only those elements that are within his range of competence (p. 90). Through their study, which explored the role of tutoring in problem solving, Wood et al. originally identified six key elements that would support scaffolding. These were:

- Recruitment – obtaining the child's interest and refocusing their attention to the actual problem which needs to be solved;
- Reduction in degrees of freedom – Simplifying the given task by providing step-by-step feedback on how to get to a solution;
- Direction maintenance – Keeping the learner focused on what it is they are trying to solve;
- Marking critical features – using a variety of means to highlight or accentuate specific parts of the task which are most relevant;
- Frustration control – offering support which allows the learner to feel more at ease since he needs to complete the task with help rather than without;
- Demonstration – modelling or demonstrating how the task is to be solved by imitating the ideal form of solution a learner should come up with. (p. 98)

Tharp and Gallimore (1988) took up this idea and proposed the term 'assisted learning' that has the same meaning which has been attributed to the term 'scaffolding'. They propose six interdependent strategies that are crucial for assisting learning. However, in a later writing, Tharp (1993) proposes a seventh tool to assist learning – that of *task structuring*. These seven tools are presented in Table 3.1.

Table 3.1: The seven means of assisting and facilitating learning presented by Tharp (1993, p. 271-272).

Modelling	Offering behaviour for imitation. Modelling assists by giving the learner information and a remembered image that can serve as a performance standard.
Feedback	The process of providing information on a performance as it compares to a standard. Feedback is essential in assisting performance because it allows the performance to be compared to the standard and thus allows self-correction. Feedback assists performance in every domain from tennis to nuclear physics. Ensuring feedback is the most common and single most effective form of self-assistance.
Contingency Management	Application of the principles of reinforcement and punishment to behaviour – this is when the More Knowledgeable Other uses rewards and sanctions to support the learner in completing the task by providing the encouragement for the child to replicate the desired behaviour.
Instructing	Requesting specific action. It assists by selecting the correct response and by providing clarity, information and decision making. It is most useful when the learner can perform some segments of the task but cannot yet analyse the entire performance or make judgements about the elements to choose.
Questioning	A request for a verbal response that assists by producing mental operation the learner cannot or would not produce alone. This interaction assists further by giving the assistor information about the learner's developing understanding.
Cognitive Structuring: "explanations"	Cognitive structuring assists by providing explanatory and belief structures that organize and justify new learning and perceptions. Another term for cognitive structuring is 'explanation'.
Task Structuring	Chunking, segregating, sequencing, or otherwise structuring a task into or from components. It assists learners by modifying the task itself, so the units presented to the learner fit into the ZPD when the entire structured task is beyond that zone.

The "scaffolding functions" proposed by Wood et al. (1976) and the seven means of assisting learning put forward by Tharp and Gallimore (1988) and Tharp (1993) have multiple similarities. For example, both give importance to the strategy of modelling – in the former it is described as *demonstration*, whilst in the latter it is labelled as 'modelling'. Moreover, feedback is a feature of what Wood et al. (1976) mention as *reduction in degree of freedom*. I chose to take Tharp's seven means of assisting the learner to facilitate internalization because they are, to my knowledge, the most holistic list of strategies which can be applied to the education context. Tharp's (1993) seven means of assistance can also serve as a structured list of guidelines for providing intervention within the ZPD, something my intervention sessions aim to do. Hence, Tharp's (1993) seven means of assisting and facilitating learning will be a main part of my conceptual framework.

Hobsbaum, Peters, and Sylva (1996) introduced the notions of *strategic scaffolding* and *incidental scaffolding*. These derived from their study of reading instruction. *Strategic scaffolding* is when the MKO makes use of specific scaffolds to support learning. *Incidental scaffolding* refers to any additional scaffolds which the MKO might use to develop the child's learning without thinking of the strategy used specifically as a scaffold - as many times happens when a parent supports their child in activities and tasks completed in their company. In this study, Tharp's (1993) seven means of assistance will be used for strategic scaffolding, however, whilst I analyse the data from the audio recordings, I will be open to any strategies that may have serve the purpose of incidental scaffolds.

The use of tools was not mentioned in this list provided by Tharp (1993) although it was highly accentuated in Vygotsky's works. In my study, apart from focusing on Tharp's (1993) seven means of facilitating the internalization process, I will also ensure that I use the right tools, whether these are language, visual aids and/or manipulatives because I believe that they are important too in helping the child reach his potential stage of development. Moreover, as I design the intervention sessions, I will take into account the learners' profiles since the mathematical profile they carry along will influence the session's outcome. As accentuated by Anghileri (2006), "it is crucial to consider the role of the learner, as sociocultural factors cannot be ignored" (p.35). How this will be considered throughout the study will be explained further in Section 3.6.

The guidance provided by the MKO, whether a parent, a peer or a significant other like me, needs to support learners in their journey to acquire the basic mathematics skills which they have the potential to acquire. I thus acknowledge that my role shall be central in this because Vygotsky (1978) as well as Tharp (1993) highlight the impact and influence the guidance provided by a MKO has on the child's achievement. This is one of the main reasons why I chose a teacher guided intervention programme rather than an intervention programme in which the computer plays the key role. The role of the MKO, in this case that of the teacher, in facilitating the learning process within the ZPD is fundamental. As Tharp and Gallimore (1988) highlight, the teacher plays a fundamental role in the learning process. They indicate that an effective teacher must not only be an expert in the skills and concepts being taught but also an expert in pedagogy. Moreover, as

Dunphy and Dunphy state, "the amount and type of assistance will vary (referring to the ZPD) with the experience and level of performance of the teacher" (Dunphy & Dunphy, 2003, p. 52). Thus, the MKO is seen as a crucial figure in the learning process and his/her knowledge, expertise and skills may impact the effectiveness of the learning process. The importance given to the role of teacher is in line with my own beliefs as expressed earlier.

An idea that has more recently been given importance, is that the role of the MKO can, in some situations, be inverted with that of the learner. For example, Roth and Radford (2010) suggested that *alternation* in role has been used to examine children's mathematical learning. In my study, this idea will also be taken up throughout the intervention sessions. It will not be used because it is suggested by the framework I will be following, mainly Tharp's (1993) seven mode of assistance, but rather because through my experience, I feel this strategy may be effective in supporting some learning situations. In my analysis I will thus also present this idea and explain whether and how this seems to be compatible with the framework I will be following when planning the intervention sessions. As discussed earlier in Section 3.4, other studies have also explored the idea of the cultural tool taking up the role of the MKO. This latter idea is related to the role inversion explored between the MKO and the learner. However, they have been treated separately in studies, and to my knowledge, researchers who have explored one of these aspects, have not explored the other. Hence, in my analysis, these ideas will be treated separately, however I will seek to demonstrate how they may be related.

3.6 Analysing the data steered by a Vygotskian perspective

Vygotsky's theories will thus steer all the thoughts, actions and choices carried out in this study. As suggested by Maschietto and Trouche (2010), "Based on Vygotsky's work, the theoretical construct of *semiotic mediation* has been elaborated and applied to mathematics education" (p.38). Moreover, Bartolini Bussi, and Falcade are two authors who "have been conducting research in the field of mathematics education for years and have contributed to the development of a theoretical framework for semiotic mediation in the mathematics classroom" (Bartolini Bussi et al., 2012). My study will seek to contribute to this epistemology focusing on MLD, an area which, to my knowledge, has been limitedly explored in this respect.

Vygotsky (1978) himself declared that,

> children confronted with a problem that is slightly too complicated for them exhibit a complex variety of responses including direct attempts at attaining the goal, the use of tools,

> speech that simply accompanies the action, and direct, verbal appeals to the object of attention itself. If analyzed dynamically, this alloy of speech and action has a very specific function in the history of the child's development (p. 30).

It is these responses which will be scrutinized to determine their effectiveness vis-à-vis supporting the children to develop their thoughts and grasp a better understanding of the numeracy components (in Section 2.3) at hand. Thus, I will seek to analyse which strategies lead to the internalization of important mathematical concepts and skills. I will aim at identifying:

- how my intervention as the More Knowledgeable Other (MKO) has an impact on the internalization process and whether the strategies used, including the seven modes of facilitating internalization, facilitated the learning of the numeracy components focused upon;
- the role played by cultural tools in the internalization process;
- how the learners' mathematical profiles, brought to the social interaction, influence the process of internalization.

These are represented in Figure 3.1 below.

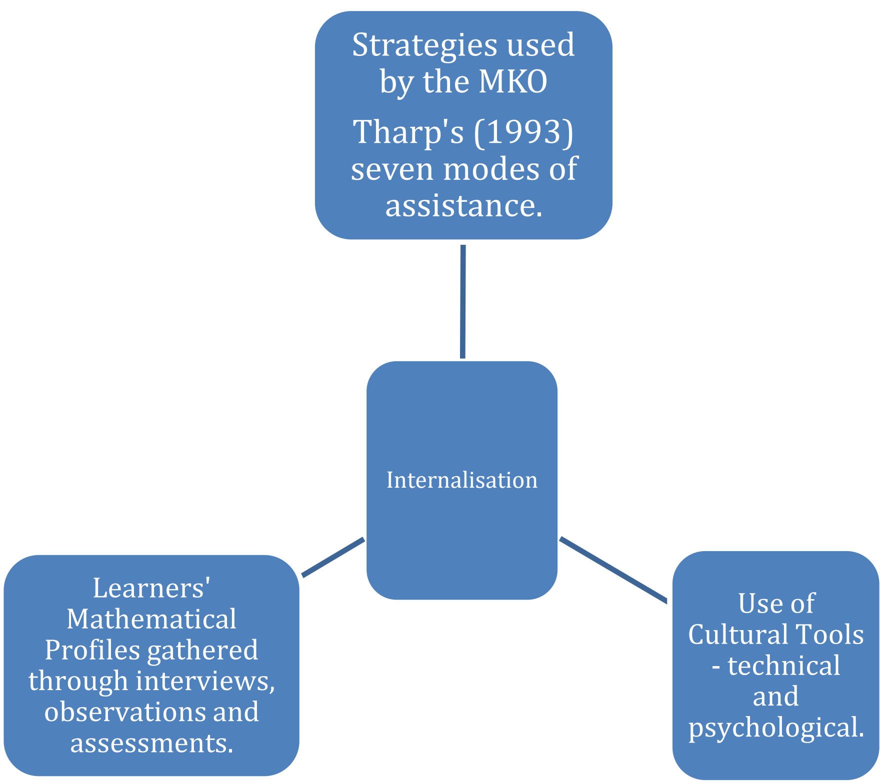


Figure 3.1: Model followed when analysing the intervention sessions.

Vygotsky suggested that a child develops through a transformative collaborative practice (referred to in Section 3.3). Hence, whilst analyzing the data collected through the audio-recordings of the sessions, I will seek to identify how the three factors identified above came together in a symbiotic relationship to support the internalization process, developing a new pedagogical model based on those already available. Moreover, I will apply this pedagogical model to mathematics education, more specifically in relation to providing intervention to learners struggling with mathematics, and to understand better what transformative collaborative practice is involved in facilitating the internalization of the numeracy components. This may allow me to contribute to the development of Rogoff's (1995) participatory appropriation model and to use this in relation to the teaching of mathematics for learners struggling in this field. I will look at the data by explicitly keeping in mind what would serve as evidence of internalization, to explore which strategies had an impact on the internalization process and how these were interlinked.

As illustrated in Section 3.3, Vygotsky's approach to learning was not separable from his approach to teaching (Mariotti, 2009). Hence, when searching for instances of internalisation, I will look at both the teaching and learning of the numeracy components. Thus, the eventual pedagogical model developed would hopefully also serve as a teaching-learning model so that internalisation could be explored by researchers and teachers through a holistic view when evaluating intervention programme for children with MLD.

To draw conclusions of what would serve as evidence of internalization, I will look back at what meaning Vygotsky gave to the notion of internalization. Vygotsky (1978) stated that "what a child can do with assistance today, she will be able to do by herself tomorrow" (p.81, referred to in Section 3.3). Vygotsky (1978) explained evidence of internalization when he gave the finger pointing example and mentioned that the MKO models 'finger pointing' and the child then uses this strategy without being told. Therefore, latching onto this meaning, evidence of internalization will include situations that illustrate how the learner was first unable to do something without assistance, both from the MKO or the use of tools e.g. the number line, and was then able to do it in a following situation with no help at all. I thus determined that the following six situations would serve as valid evidence of internalization:

- The pre-intervention and post-intervention scores obtained by each of the learner in the numeracy components and the standardised numeracy assessments;
- The linked recording exercise completed at the end of each session with no help;

- The use of the correct mathematical language which had been focused upon (in the sessions themselves);
- Situations in which the learner was able to take up the role of the teacher and explain an exercise well or use the correct instructions and imitating the right behaviour;
- Situations in which the learner was previously making a mistake (e.g. missing a number when counting) and later managed to do this correctly – at times also remembering the miscues made in previous sessions;
- Situations in which the learner chose to make use of a particular 'tool' e.g. blocks, number line etc. out of his initiative and could use it appropriately to work out a sum following modelling and intervention by the MKO.

With these in mind, I will analyze the data manually looking for pre-determined themes, namely Tharp's (1993) seven means of assistance (see Section 3.5). I will also be open to any other emerging themes that would make the idea of a 'transformative collaborative practice' put forward by Vygotsky applicable to providing effective intervention that leads to the internalization of the numeracy components. Being open to new themes will ensure that I can identify any other strategies which may not have been mentioned by Vygotsky himself, but which would have also seemed to contribute to the 'transformative collaborative practice' needed for the learners to grasp the numeracy components that are much needed for mathematics learning.

3.7 Conclusion

My beliefs as a teacher have inevitably influenced my beliefs as a researcher. As a researcher I favoured qualitative research methods since through a qualitative approach one can gain a deeper understanding of the phenomenon at hand. As a researcher I believe that the knowledge researchers construct depends on the experiences and situations they encounter during the data collection and analysis processes and that these realities are thus subjective. This point will be explained in more detail in Chapter 4.

My intervention sessions will be based on a thorough and formative assessment provided by the CUN programme itself and by adding additional assessments to get a holistic profile of the child. This will be represented through individual detailed reports which will provide a detailed profile of each child being studied. Doing this will allow me to assess all the children's unique current state of development to use what they know in order to be able to bring to their "mind and exploit those aspects of their past experience that we (as experts) but not they (as novices) know to be relevant to what they are currently trying to do" (Wood, 1998, p. 97). Additionally, carrying out sessions on an individual basis will ensure that unique rich interactions are provided to each learner, facilitating their learning, and thus, progress from their current zone of development. To adhere to Vygotsky's view of how development occurs, I shall provide challenging activities which will allow the students to move on further in their areas of mathematics development. Such activities will allow me to enhance each child's development through my social interaction and guidance within the ZPD.

Chapter 4
Research Design

Chapter 4: Research Design

4.1 Introduction

In this Chapter I outline the research questions which this study aims to answer. I also present the research paradigm chosen to underpin my study. I will explain the research methods adopted for the collection of data, including sample choice and ethical considerations. Finally, I will describe the intervention programme used.

The first step of research endeavours is that of identifying a suitable arena for the research, to specify the aims and purposes of the study, how these will be operationalised, and to generate research questions (Robson, 2004). Hence, after identifying the research issues (Bassey, 1995) which I wanted to concentrate on and outlining the aims of the research, I drew up the following research questions:

i. Is an intervention programme carried out with children having MLD only and with children having both MLD and RD beneficial to these learners? Which characteristics of the intervention programme are effective with each group of learners?

   Moreover, four other subsidiary questions were posed, namely:

ii. What are the teachers' and parents' perspectives on how the children's MLD affects their daily lives at school and at home?

iii. Do children with solely MLD and those with both MLD and RD have similar mathematical profiles? Are both groups of children strong/weak in the same areas?

iv. Do the children assessed with MLD or both MLD and RD and also with a profile of Dyscalculia as identified by the Screener (Butterworth, 2003) have difficulties in all numeracy components or in just a few?

v. Is mathematics anxiety one of the difficulties experienced by the learners?

These research questions have steered my choice of methodology, research design and research methods. The data collection process was divided into two phases. In Phase 1, I collected norms for some numeracy and reading standardised tests to be able to select the participants for Phase 2 of the study, which is the Main Phase. Phase 2 involved a triangulation of research methods including classroom observations, interviews with the parents, assessment with the participants as well as an intervention programme carried out on a one-to-one basis with each participant. In the coming sections I will explain my choices.

4.2 Selecting a Research Paradigm

Understanding theoretical paradigms as well as the philosophical assumptions that underpin them is one of the initial steps to conduct research. As indicated by Denzin and Lincoln (2000) “the net that contains the researcher’s epistemological, ontological, and methodological premises may be termed a paradigm” (p.19). I primarily explored different paradigms to identify which one was most closely related to my own epistemological assumptions and ontological beliefs. Whereas epistemology is a theory about what counts as knowledge, ontology relates to views of reality, and thus the meaning of reality generated by the children and myself. I concluded that I would need to explore two different paradigms due to the diverse nature of Phase 1 and Phase 2 of this study.

To select the right participants for Phase 2 of the study, it was necessary to use reliable and valid modes of assessments. Since no assessments have ever been standardised locally for numeracy and reading, I had to assess a large group of pupils to develop tentative local and specific norms for these assessments. Consequently, Phase 1 was underpinned by a positivist paradigm. Positivism is based on the premise that “the social world can be studied in the same way as the natural world, that there is a method for studying the social world that is value-free, and that explanations of a causal nature can be provided” (Mertens, 1998, p. 7). Additionally, positivism “claims that science provides us with the clearest possible ideal of knowledge” (Cohen, Manion, & Morrison, 2007, p. 11). Within this paradigm, there is only one reality and that, within probability, this reality is knowable. Moreover, the epistemological assumptions accompanying this paradigm indicate that objectivity is prominent in the process of collecting and analyzing data. Quantitative research methods are thus considered as important as qualitative ones and are fundamental to acquire knowledge.

The use of standardised tests (justified and problematized in Section 4.2.1), to identify the main participants of the study, supports the assumptions of this paradigm. The process of finding indicative norms specific to my needs and of statistically analyzing them was objective in that tests were distributed to many pupils and corrected using a 'right or wrong' approach. Test scores were analysed using the Statistical Package for the Social Sciences (SPSS) (Version 21.0). The interpretation of the results was also objective in that norms obtained were taken *prime facie* to identify pupils for Phase 2 of the study.

Following the quantitative phase, my study took a qualitative approach. As suggested by Cohen et al. (2007), positivism is less suitable "in its application to the study of human behaviour where the immense complexity of human nature and the elusive and intangible quality of social phenomena contrast strikingly with the order and regularity of the natural world" (p. 11). As a result, a paradigm shift was necessary within this research project since Phase 2 of this study is based within the interpretive paradigm. Throughout this writing I will refer to the chosen paradigm as interpretive because, as will be explained later, interpretation and subjectivity is a main feature of this paradigm and therefore of my main research method.

The interpretive paradigm contrasts with the underlying assumptions of the positivist paradigm. The epistemological assumptions and ontological views of this paradigm derive from grounded theory (Glaser & Strauss, 1967). With a growing understanding that epistemology may be approached through other perspectives other than a positivist view, a new school of thought emerged. As indicated by Charmaz (2000), "throughout the research process, grounded theorists develop analytic interpretations of their data to focus on further data collection, which they use in turn to inform and refine their developing theoretical analyses" (p. 509). Grounded theory methods, which are mainly of a qualitative nature, have been criticized as will be explained in Section 4.3. This new way of thinking gave rise to new paradigms for conducting research, one of which is the interpretive paradigm. I did not use Grounded Theory because among others I did not revisit the data as is normally done in this paradigm and as indicated in the cycle produced by Glaser and Strauss (1967). Rather, I found the interpretive paradigm to be more appropriate for the reasons explained hereon.

One of the epistemological assumptions of the interpretivist paradigm is that there is an interaction between the researcher and the participants in the data collection phase and this tallies with my choice for providing one-to-one sessions with the main participants. As indicated by Cohen et al. (2007) "the central endeavor in the context of the interpretive paradigm is to understand the subjective world of human experience" (p.21). This paradigm was primarily deemed most suitable since "MLD" is socially constructed; researchers may perceive it in different ways (see Section 2.4). Furthermore, the epistemological assumptions underlying this paradigm are also consistent with my belief of how children learn as explained in Section 3.1.

The epistemology underlying the interpretivist paradigm underpinned my choice of research methods for Phase 2 of the study. Patton (1990) suggests that the research aims, and the chosen paradigm are valid reasons for choosing qualitative research methods. Hence Phase 2 of this study was inevitably qualitative in nature. Since I wished to provide an in-depth description and to focus on "words rather than quantification in the collection and analysis of data" (Bryman, 2008, p.264), qualitative methods were considered as indispensable for Phase 2. The interpretive paradigm assumes that the researcher and the participants are in a constant interpersonal relationship with each other and that they have an impact on one another. In fact, the intervention programme provided to every learner was uniquely designed for his needs and the progress made by the learner influenced the strategies employed. Moreover, this paradigm makes the study a very personal one since as outlined by Charmaz (2000) researchers can "still study empirical worlds without presupposing narrow objectivist methods and without assuming the truth of their subsequent analyses" (p. 511). The personal aspect of maintaining an interpretive paradigm as the backdrop to my study is also evident in the fact that I use personal pronouns throughout the writing of this thesis. This is a common characteristic of interpretivism (Bassey, 1995, p.13).

The ontological views underlying the interpretivist paradigm were also deemed suitable since interpretivism assumes that there is no such thing as *one* reality since realities are socially constructed. The reality lived through the social interaction which took place in the one-to-one sessions is multi-faceted and may be interpreted in multiple ways. Thus, I acknowledge that the 'reality' which is presented in Chapter 6, is one which the participants and I have constructed. This interpretation of reality also fits the ontological views which constructivism promotes, namely that researchers "do not find or discover knowledge so much as construct and make it" (Schwandt, 1994, p.125). As a researcher I am aware that my interpretation of the social interactions with the participants will be subjective and that the baggage of knowledge I currently hold will impinge on the way I construct the reality at hand. Just like interpretivists, I too believe that "the descriptions of human actions are based on social meanings, and people living together interpret meanings of each other, and these meanings change through social intercourse" (Bassey, 1995, p. 13).

Phase 2 of this study fits well within this paradigm since within this view of reality the researcher "opts for a more personal, interactive mode of data collection" (Mertens, 1998, p.13). As indicated by Bassey (1995), research carried out within the interpretive paradigm is normally richer in language than the data normally offered by positivists. I opted to carry out multiple – case intervention studies (discussed further in Section 4.3.2) to record all the language interaction between myself, as the researcher, and the participants and to make this data the main source of my analysis process. Furthermore, my use of the interpretivist paradigm is fundamental in that through this study I "search for deep perspectives on particular events and for theoretical insights" (Bassey, 1995, p. 14) in relation to mathematics learning and mathematics learning difficulties. I also acknowledge that my analysis of the data will "offer possibilities, but no certainties, as to the outcome of future events" (Bassey) since the observations which will be presented will be related to the specific situation with an individual participant. It should also be highlighted that consequently my analysis will only give rise to theoretical and analytical generalisations since as suggested by Yin (2014), "case studies, like experiments, are generalisable to theoretical propositions and not to populations or universes" (p. 21). This form of generalisation will be discussed further in Section 4.3.2. when I explain my choice of using case studies as a main instrument for this study.

Finally, the interpretive paradigm "is characterized by a concern for the individual" (Cohen et al., 2007, p.21). This concern is reflected in the nature of this research study in which my main concern is the well-being of children who are struggling with mathematics, whether or not they are also experiencing reading difficulties. The focus of this research was thus based on the process and development of its main participants and on emphasizing individualized outcomes, thus seeking to find effective strategies to provide these children with a high-quality education (NCF, Ministry of Education and Employment, 2012) which will allow them to develop numeracy skills which are needed for everyday life.

4.3 Research Methods

As suggested by Cohen, Manion, and Morrison (2000) "the purposes of the research determine the methodology and design of the research" (p.73). Moreover, as Mertens (1998) underscores, "the nature of the research question itself can lead a researcher to choose qualitative methods" (p. 162). It is essential that the research methods adopted are appropriately related to the conceptual framework the study is underpinned to. All three factors (the research aim, research questions and methodology) form the research design which "is the "blueprint" that enables the investigator to come up with solutions to these problems and guides him or her in the various stages of research" (Frankfort-Nachmias & Nachmias, 2008, p.89).

Although the main aims and questions of the research required qualitative methods to be used, quantitative methods were also needed (see Section 4.3.1) leading to the use of a mixed methods approach. One important aspect of mixed methods is to understand that this approach is not merely about combining or mixing methods, but that the use of any methods or set of methods used is also intertwined to particular epistemologies, methodologies, and ontological assumptions and connected to the researcher's own perspectives (Mertens & Hesse-Biber, 2013). Hence research methods gain their meaning from the methodologies that form and lead them; therefore, any discussions of mixed methods "must be discussions of mixing methodologies, and thus of the complex epistemological and value-based issues that such an idea invokes" (Mertens & Hesse-Biber, p.6). The way in which I have mixed methodologies and methods in this study is presented in Table 4.1.

Table 4.1: An illustration of the way methodology and methods were combined in this research based on work presented by Guba and Lincoln (1994) and Mertens (1998).

	Phase 1	**Phase 2**
Paradigm	Positivist	Interpretivist
Ontology	One reality which can be discovered within probability.	Multiple realities which are socially constructed.
Epistemology	Objectivity is paramount; the researcher collects data in a dispassionate, objective manner.	A relationship is built between the researcher and the subjects; values are explicit, and the findings are created.
Research Methods	Standardised Tests for selecting children with MLD and both MLD and RD. Re-assessment of all the children in the classes of the main participants.	Multiple-case intervention studies; Classroom observations; Interviews with the parents and teachers.

Phase 1 was mainly viewed through a utilitarian aim and solely made use of as an inevitable part of selecting the participants for Phase 2 of the study. To conduct my study, I had to identify the most suitable candidates to receive the intervention program and to be studied. One way of making this selection could have been by asking for the class teachers' opinion about the learners' abilities and identifying three children with solely MLD and three with MLD and RD. However, this was not deemed to be rigorous enough. Consequently, I deemed it necessary to assess all the children currently in Grade 5 at the school where I taught using assessments for numeracy and reading to be able to choose the 'right' participants. However, the fact that no numeracy and reading assessments had been standardised locally meant that I had to first find indicative local norms specific to the group of learners to be studied. Hence, in Phase 1 of the project, my focus was breadth rather than depth, an approach which is typically associated with quantitative research methods (Wimmer & Dominick, 1994, p.140). Larger scale assessment was also used in the final step (as all the class was re-assessed).

The utilitarian nature of Phase 1 of the study is very evident in the research design since whereas the main quantitative method used was that of conducting assessments, the qualitative methods used involved multiple-case intervention studies including various formative assessments, one-to-one sessions with the subjects, interviews and observations. The planned research design involved multiple steps. The collection of data would commence with the trialing of the numeracy assessments. Once these were selected, a pilot study would be carried out and eventually the numeracy and reading assessments would be administered in all Church schools for boys. When local norms for this cohort were found, the tests would be administered to the Grade 5 boys at the school where I taught. The results would allow me to identify three pupils with MLD only and three with both MLD and RD. Further tests would be carried out with these six children and an intervention programme would follow. The pupils' parents and class teachers would also be interviewed, and class observations would be carried out. Following the intervention programme, the six learners would be re-assessed. The rest of the class would also be re-assessed. A plan of the research design is found in Figure 4.1.

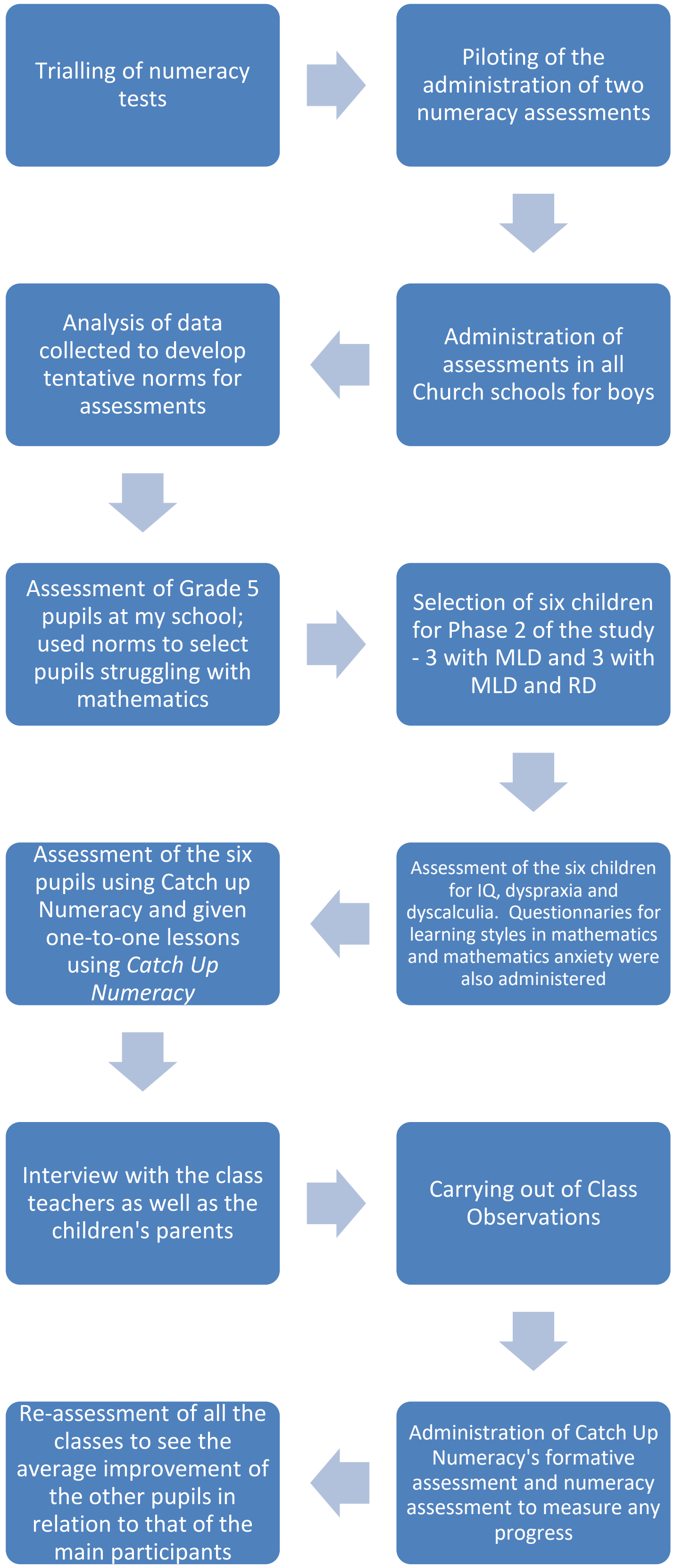


Figure 4.1: Outline of Steps carried out.

4.3.1 Standardised Tests

A long history of controversy exists about standardised testing (Higgins, 2009). Researchers (Gladwell, 2001; Phelps, 2003; Zwick, 2002) have debated the advantages and disadvantages of using standardised testing and have questioned the validity of results resulting from these tests.

Standardised tests are advantageous because they provide information about a learner's achievement that is more objective than that given through a teacher-created assessment. They give parents, teachers and the learners a clear picture of where the latter stands vis-à-vis their peers and the baseline score for children or adolescents of their age. A study carried out by Marlow et al. (2014) compared students' performance on teacher assessment (referred to as APP) to that on a standardised test (Weschler Individual Achievement Test-II; WIAT-II) for both literacy and numeracy. They concluded "there was a strong correlation between the APP and WIAT [II] for literacy but not for numeracy" (p.412). Marlow et al. provide possible explanations for this; they explain that this result may be due to a longer tradition of administering literacy achievement and to thus the establishment of more suitable assessment standards for this area. Another reason provided is that the WIAT-II was normed in 2004 in the UK and therefore the teaching practices may have changed by the time the test was administered.

Numerous studies have indicated ways in which teacher assessments may not be as accurate and valid as standardised tests. Allal (2013) and Wyatt-Smith and Klenowski (2013) explain how teachers develop judgements about their pupils that may impinge on assessment. Moreover, Harlen (2004) suggested that teachers might have biases regarding gender, special educational needs and others that can impact assessment. These studies suggest that standardised testing may provide a more objective perspective towards measuring a learner's achievement. As a result, standardised tests are usually seen as more valid and reliable (Marlow et al., 2014).

On the other hand, some disadvantages of the use of standardised tests have also been highlighted. These include the possibility that they create additional stress for teachers and learners, increase competition between students and schools, and may be used to discriminate between groups of learners (Miller, 2003). A study conducted by Chu, Guo and Leighton (2013) explored the relationship between student interpersonal trust and attitudes towards standardised tests. They argue that the affective domain is an important aspect of student learning and that it should be accounted for in assessment. A main finding of the study was that the students' attitudes towards the tests varied according to their interpersonal trust and attitudes towards the value they gave to the test itself. Thus, they concluded that, "affective variables need to be more fully considered when considering practice and generating policy to improve student test performance" (Man-Wai et al., p. 167). Since doubts and fears about the test may impinge on test performance, addressing these may indeed support students in becoming more proactive test-takers (Gal & Ginsburg, 1994).

Inspite of the arguments against standardised testing, standardised tests remain an important way of measuring a learner's achievement. As underscored by Higgins (2009) "standardised tests are not a perfect tool, but they are the best tool we have to measure student achievement" (p.1). Keeping in mind that standardised tests are not perfect I opted for the use of a triangulation of assessment methods that would ensure the results were as valid and reliable as possible. This triangulation included the use of multiple standardised tests, and a comparison with teacher assessment and parents' feedback. Moreover, I ensured that the standardised tests chosen had 'content validity' (the content was what I wished to test) and 'construct validity' (constructed in a way which gives valid results). For example, I eliminated one of the tests on the basis that it had a lot of written language thus threatening its construct validity. The test required the pupils to read and thus might have given a distorted picture of who had MLD or who struggled due to an inability to read the text. Precautions were also taken whilst administering the standardised tests. All tests were administered by myself, and I made sure to introduce myself, inform the learners what the test was going to be used for and reassured them so that they did not feel stressed by the test itself. Additionally, norms for local pupils were sought thus making the test fairer. Moreover, the children's achievement was compared to that of other pupils learning in similar educational settings i.e. Church schools for boys.

4.3.2 The Use of Case Studies

According to Yin (2014) a case study is "an empirical study that investigates a contemporary phenomenon (the "case") in depth and within its real-world context, especially when the boundaries between phenomenon and context may not be clearly evident" (p.16). A case study can be the single study of one case. However, a researcher can also opt to "study *multiple cases*: a number of single parents; several schools; two different professions" (Gillham, 2000, p. 1) as in this case. Yin (2014) explains the features of a case study. He suggests that case studies:

i. rely on more than one source of evidence;
ii. benefit from the development of theoretical proposition prior to the commencement of the data collection and analysis process; and
iii. cope with the situation of having more variables of interest than data points.

In this thesis each child was considered as a case study of its own. The phenomenon investigated was that of finding effective strategies that work for children with MLD – these were explored through the intervention programme provided. Each case study served to highlight which intervention strategies were effective with the different 'cases'. The context considered was the struggle the children face.

Stake (1995) identifies three functions of case studies. He describes them as *intrinsic*, *instrumental* and *collective*. *Intrinsic* refers to case studies that are carried out as the result of an intrinsic interest to get to know more about a particular case. *Instrumental* is when a case study is used to understand something else rather than the case itself. Thus, in this study, the case serves the purpose of providing information about a specific phenomenon. Lastly, the term *collective* refers to when multiple case studies are carried out to investigate a specific phenomenon. The case studies used in this thesis, have served the *instrumental* and *collective* function. The instrumental function is evident. The cases were not only studied due to my genuine concern for the children but were also carried out to provide more knowledge about the phenomenon of MLD and what teaching and learning strategies are effective with these learners. The collective function is evident through the use of multiple case studies as will now be discussed.

Faced with the choice of conducting a single case study or multiple case studies, I decided on the latter, since I could select children with similar profiles. As highlighted by Yin (2014) "multiple-case designs may be preferred over single-case design…the analytical benefits from having two (or more) cases may be substantial" (p. 64). Conducting multiple case studies is a more robust way of collecting data (Herriott & Firestone, 1983). In choosing the number of participants for the multiple cases to be studied, I bore in mind that case study research is different to sampling research (Stake, 1995). The idea of having more than one case study is not to sample since "we do not study a case primarily to understand other cases. Our first obligation is to understand this one case" (Stake, p.4). Thus, I decided that three pupils with MLD only and three with both MLD and RD would provide a good basis for both literal and theoretical replications

(Yin, 2014) whilst ensuring that the time spent on collecting the data would be manageable for one researcher.

Stake (1995) argues that "the representation of a small sample is difficult to defend" (p.5) but as stated by Yin (2014) "in doing case study research, your goal will be to expand and generalize theories (analytical generalisations) and not to extrapolate probabilities (statistical generalisations)" (p. 21). My approach towards analyzing the case studies will thus be one in which the single cases are presented. These will then be followed by a discussion of different 'themes' that may arise from a cross-case analysis of the case studies.

As indicated by Gillham (2000) "Case study is a *main* method. Within it, different sub-methods are used: interviews, observations, document and record analysis, work sample and so on" (p. 13). In fact, the case studies carried out as part of this study involved all the research methods mentioned by Gillham (2000). As indicated by Yin (2014) "the case study's unique strength is its ability to deal with a full variety of evidence - documents, artifacts, interviews, and observations" (p. 12). Every one of these research methods contributes to providing further evidence that will give a deeper insight into the subject at hand (Gillham, 2000). In addition, to the mentioned methods, data was mainly collected through the intervention sessions that were part of the case studies themselves. The intervention programme itself has many characteristics of action research, defined as "any systematic inquiry conducted by teachers…for the purpose of gathering information about how their students learn" (Mertler, 2009). However, in my situation I was not working with children who I would have taught anyway had it not been for this study. Moreover, the intervention programme was only one of the research methods used for collecting the data, hence viewed as part of a wider aim which I sought to reach through case studies. Case studies like mine, which incorporate an element of intervention, have been used in educational psychology under the term 'single case experimental design'. As highlighted by Wilson (2000) "single case experiments are scientific investigations in which the effects of a series of experimental manipulations on a single participant are examined" (p. 60). Wilson (2000) suggests that an example of how this design could be used is to assess the effect of intervention on one individual. This is in line with what I sought to achieve through this study. However, it is interesting to note that more recently, this means of gathering data within an educational setting seems to have become more popular (e.g. Koponen, Aro, & Ahonen, 2009) and in a paper published after my own collection of data, Koponen et al. (2018) refer to this research method as "single-case intervention studies" (p.3). Since I feel that the name is more appropriate to my research method as it shows that the case studies carried out also incorporated an element of intervention, I will refer to my research method as 'multiple-case intervention studies'. Each of the different methods used to collect data for the multiple-case intervention studies (e.g. interviews and the intervention programme itself), and how they have been used, will be discussed in Section 4.7.

4. 4 Gaining Access

In any research project, it is crucial that the researcher gains access to the field through the gatekeepers of the specific field. As highlighted by Mertens (1998) "before data are collected, the researcher must follow appropriate procedures to gain permission from the gatekeepers (typically defined as those with power in the organization or agency) of the organization of community" (p. 177). As a researcher I had to bear in mind that access is not a right. Researchers "have to demonstrate that they are worthy, as researchers…of being accorded the facilities needed to carry out their investigations" (Cohen et al., 2007, p. 55).

Feldman, Bell, and Berger (2003) stress that scholars should design research whilst considering access. Access should thus be practicable, "there is, after all, little point in designing research that can never be fulfilled" (Feldman et al., p.4). Since I was working as the Complementary teacher, at a Church school for boys, I could not leave my school to conduct Phase 2 of the study in a different setting. As a result, the most practical solution was for me to carry out Phase 2 (assessment and intervention programme) of this project at the school where I taught. Phase 1 consisted of finding norms to be used to identify the participants for Phase 2 of the study. Hence, after thorough consideration, I decided to collect norms by administering the tests to a cohort that was as similar to that of the school I taught in as possible. Consequently, it was decided that Phase 1 would be conducted in other Church schools catering for boys only since this cohort would reflect the participants that would later take part in Phase 2. This corroborates the procedures for sampling identified by Frankfort-Nachmias and Nachmias (1996) who indicate that "once the researchers have defined the population, they draw a sample that adequately represents that population" (p. 181) and who also highlight that a high degree of correspondence is fundamental between the sampling frame and the sampling population. It also supports arguments put forward by Henry (1990) who underscores that the external validity of research within a positivist paradigm needs to be analysed regarding its generalizability vis-à-vis the group of participants for which its findings are to be generalized. Although I wanted scores from a finite population (Frankfort-Nachmias & Nachmias, 1996), sampling was still deemed necessary since it would not have been "feasible to collect data from every individual in a setting or population" (p. 253). After using the set formula to find an appropriate sample size which would be practicable, I concluded that it would be best to take half the population of boys in Grade 5 attending Church schools - details of how this was worked out will be explained in subsequent Chapters.

Permission was thus asked to carry out both Phase 1 and Phase 2 in the aforementioned schools through the Secretariat for Catholic Education, as a sub-division of the Maltese Episcopal Curia (See Appendix C). When written consent was granted from this central entity (see Appendix D), every Church school catering for boys was contacted to ask for permission to conduct Phase 1 of the study at their school. During that scholastic year, there were 704 boys attending Church schools. Access was thus obtained to conduct the tests with half this population at the seven Church schools catering only for boys.

The tests to be administered were the Basic Number Screening Test (BNST[14]) (Gillham & Hesse, 2001), parts of Chinn's (2012) assessment and the Single Word Reading Test (SWRT[15]) (Foster, 2007) (further details about each test will be given in Section 4.5.1). Parental consent was gained through the ethical procedures that will be explained in Section 4.9. Once schools received clearance from the parents, they helped me to plan a suitable date with the class teachers to disrupt children's learning as little as possible.

Meanwhile my Head of School and I discussed the logistics and the need to carry out a piloting phase of some tests with a small group of pupils in Grade 5. This had to be done because I was still undecided about which tests were best for my study. I also explained that I would need to administer the confirmed tests with half the cohort of Grade 5 pupils during that scholastic year (without including the pupils who would have taken part in the pilot study). This because I needed scores from pupils at our school too so that the norms collected would be a true reflection of the whole population of Grade 5 pupils in Church schools for boys. We also discussed how Phase 2 of the study would unfold. Several practical measures were taken so that access for this phase of

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[14] BNST will refer to the Basic Number Screening Test developed by Gillham and Hesse (2001) throughout this and subsequent Chapters.

[15] SWRT will refer to the Single Word Reading Test developed by Foster (2007) throughout this and subsequent Chapters.

the study would be granted, since gaining access to a research field is fundamentally about persuading gatekeepers to let you in (Robson, 2004). As suggested by Mertens (1998), "administrators usually look for some assurances about the amount of control they will have over the research process and the use of the findings" (p. 177). I promised the Head of School that sessions would be held when the children would not miss any crucial lessons. Since the intervention sessions were short (15 minutes), this was not a difficult task. It was also important that I commit myself to honour all my other commitments and to carry out the intervention programme during my own free lessons. Following this meeting, my Head of School gave me access (see Appendix E) to carry out Phase 2.

4.5 Selecting the Participants for Phase 2

The process of identifying the main subjects was complex. I wanted to ensure that as a researcher working within the constructivist/interpretivist paradigm I "select my samples with the goal of identifying information rich cases that allow [me] to study a case in-depth" (Mertens, 1998, p. 261). Within this paradigm, researchers tend to start their research by identifying "groups, settings and individuals where (and for whom) the processes being studied are most likely to occur" (Denzin & Lincoln, 1994, p.202). The sample for Phase 2 of the study was chosen purposively. I wanted three participants with MLD and another three who had both MLD and RD. I managed to find the desired participants in the same cohort.

I opted to have six participants for the main research phase because this sample size was deemed manageable. Having a larger number of pupils would have made it difficult for me to follow during school hours with the desired depth. On the other hand, an even smaller number of pupils would not have allowed comparison to be made between the two groups of learners, i.e. those with MLD only and those with MLD and RD as well as between the different learners themselves. The analysis chapter will thus reflect this approach. I intend to present both the rich data of all the individual pupils separately as well as try to find similarities and differences between the groups of participants or within the groups themselves which would allow me to discuss important themes to find answers to my research questions.

After deciding on the sample size, other important factors were also considered regarding student characteristics. One was that the RD of the children would not be severe and that thus they did not have dyslexia. Although the children had RD it was still ensured that Checklist 3 (Directorate for Quality and Standards in Education (DQSE), Malta, 2012)[16] was attained and thus that the basic reading skills - up to a grade 3 level (8 to 9 years old) – had been mastered. Nonetheless they would have obtained a low score in the reading test administered indicating that even though they may have reached a basic level they were still not reading at an age-appropriate level.

Another characteristic which was considered was performance on an Intelligence Quotient (IQ) test. It was important for the participants to have at least an average level of performance on an IQ test since there seems to be a general agreement in the literature that children considered to have MLD should have at least an average IQ. The main reason for this agreement is that a learner with at least average IQ should have the cognitive ability to be able to achieve in mathematics and therefore it is expected that such children do not struggle with mathematics. Hence when children with at least an average IQ struggle in this area of learning, it should be taken as a strong indication that the learner has a specific difficulty in mathematics. Furthermore, since I wanted my sample to have learning difficulties specifically with either mathematics or both mathematics and reading, I had to ensure that they did not have an IQ so low that it impeded them from achieving in all the areas of the curriculum rather than solely in mathematics and/or reading. The six children were first identified using the standardised tests. The learners identified as having MLD and/or RD achieved a score which was at the 30th percentile or lower, since the tests administered indicated the 30th percentile score as a cut-off point for children with MLD.

The participants were then given the British Ability Scales II (BAS II, Elliott et al., 1996). The Scales were administered by a qualified psychologist on a one-to-one basis and took about two hours each. The exercise indicated that all the participants were within the average range of IQ apart from one pupil who had been identified as having MLD only. This pupil’s IQ was much higher than average. After consulting with the Senior Management Team (SMT) and the class teacher, I learnt that the child had severe socio-economic difficulties, and this was probably why he was performing poorly. As discussed in Section 2.8.1, children coming from low-income backgrounds are four times more likely to struggle with mathematics than their middle-class

[16] In Malta children are assessed in literacy through Checklists. Three official Checklists are available. Checklist 1 are the skills and competencies to be reached by the end of Grade 1 (5 to 6 years), Checklist 2 by the end of Grade 2 (6 to 7) and Checklist 3 by the end of Grade 3 (7 to 8). Pupils who master the skills and competencies needed for Checklist 3 are considered to have obtained at least a standard basic skills level in literacy.

counterparts (Jordan & Levine, 2009). Moreover, it has been shown that the difficulties in mathematics experienced by this group of children stem from not having the basic numerical skills which are needed to engage in the academic learning that takes place at school (Duncan et al., 2007; Hertzman & Power, 2006; Lee & Burkham, 2002; National Research Council, 2009). As a result, I thought that this learner would not be a good candidate for the main sample since his difficulties with mathematics learning were probably not of the same origin as those experienced by the other participants. Since I wanted to reduce variables as much as possible, I decided to search for another participant (whilst offering my lessons to the specific pupil anyway as will be explained in Section 4.9).

After speaking to the class teachers and re-examining the scores obtained by all the 50 pupils currently in Grade 5, I chose another candidate for my study. This pupil was also struggling with mathematics only and needed some support sessions. The educational psychologist administed the BAS to this child. The child was within the average range for IQ and thus I asked his own and his parents' consent to participate in Phase 2 of the study. Ultimately, I decided that I should assess all six pupils for dyspraxia too. Dyspraxia can sometimes be confused with MLD and to ensure that the chosen group really had MLD, and not a problem with gross and fine motor skills, I asked their teachers to complete a checklist provided by the National Health Service, UK (n.d.) to see if they had characteristics of dyspraxia. The full checklist can be found in Appendix F. I chose this checklist because it was the most detailed that I could find.

Once these assessments were done, I also used further assessment tools (Firman et al., 2010), with the six children, to confirm my classification of children having only MLD and those having both MLD and RD. These further assessments involved two reading comprehension assessments which allowed me to confirm whether the children had MLD only or both MLD and RD. When the sample for Phase 2 of the study was confirmed, intervention programme could then begin.

4.5.1 Tests administered for Sample Selection

Prior to choosing specific tests, I looked into most of the standardised tests available on the market for both mathematics and reading that could be administered by class teachers. Choosing a reading assessment was quite straightforward. Not many reading assessments are available and after consultation with my supervisors, I decided to choose the Single Word Reading Test (SWRT)

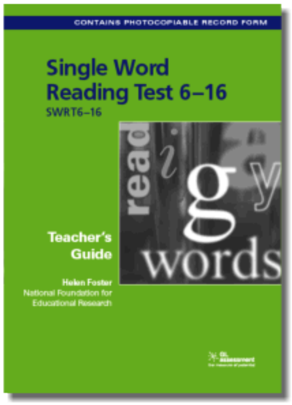


Figure 4.2: Cover Page of SWRT Manual (Foster, 2007)

(Foster, 2007) (see Figure 4.2). The SWRT is appropriate for pupils aged 6 to 16. It comprises a series of single words that the pupils should be asked to read on a one-to-one basis. The words vary in difficulty from very easy to very difficult. The main target of the assessment is assessing the children's ability to decode the presented words either through sight, in the case of sight words, or by blending sounds in the case of phonetic words. The main reason why I chose this test was that out of the tests considered, this provided the most words with which Maltese children are familiar. This decision was not only based on my own personal experience but was also discussed with other professionals, including an educational psychologist, who believed it was the best choice due to this characteristic. This ensured that the children were not misdiagnosed with a profile of reading difficulties because of their unfamiliarity to the words. This test is scored through counting all the words read out correctly. The raw score is then used to obtain a reading age. However, this assessment had not been standardised locally and therefore the norms were UK based. I thus decided to establish indicative norms for the group I would be working with.

Choosing a mathematics test was more complex. I looked into the mathematics assessments available; none of these had been standardised locally. After scrutiny of the assessments I decided that three were most appropriate: the Basic Number Screening Test (BNST) (Gillham & Hesse, 2001), Progress in Mathematics (PIM[17]) (Clausen-May et al., 2009) and Chinn's (2012) assessment. This decision was based on the facts that:

i. the assessments were in line with our curriculum;
ii. the assessments focused more on number operations and algebra than measures, data handling, shape and space – this was important since most children with MLD have difficulties with the former areas rather than the latter.

I thought that using all three tests would be too much for the children. Although it was essential that I had a triangulation of results to ensure validity of the instruments, I decided that two would suffice if the scores obtained in both agreed and if both would have indicated a learning difficulty in mathematics. Being undecided about which two to select, I decided to trial all three tests to decide which two were most appropriate and then carry out a pilot study using the two chosen tests.

[17] PIM will refer to Progress in Mathematics 9 developed by Clausen-May et al. (2009) in this and subsequent Chapters.

As indicated by Cohen et al. (2000), “a pilot has several functions, principally to increase the reliability, validity and practicability” (p.260) of the research instrument and therefore is an important step in the research process.

In the trial phase, ten pupils were selected who were currently in Grade 5 at the school where I taught. I asked the teachers to help me select pupils with mixed ability for this exercise. I wanted to compare how the different pupils would fare in the three assessments. All ten pupils were asked to sit for the BNST. Then five out of the same sample were asked to sit for Chinn’s (2012) assessment whilst the other five sat for PIM. After analyzing the data obtained from the tests, it was evident that all three tests were reliable as similar results were obtained as will be seen in the analysis chapter. However, I decided to choose the BNST and Chinn’s (2012) assessment because PIM had a lot of written instructions and children with reading difficulties found it hard to complete. Thus, using this test might run the risk of pupils being identified as having MLD because of their weak reading ability rather than their mathematical abilities. Also, it took the children a long time to complete the PIM assessment, approximately between 45 minutes and an hour. This contrasted with the 20 – 25 minutes taken to administer both Chinn (2012) tests and the BNST. Thus, I concluded that using PIM would not have been feasible because it would have taken up too much learning time in schools. In addition, since it took participants a long time to complete, some got bored and began to guess some answers. Therefore, using this test might have increased the risk of obtaining invalid results.

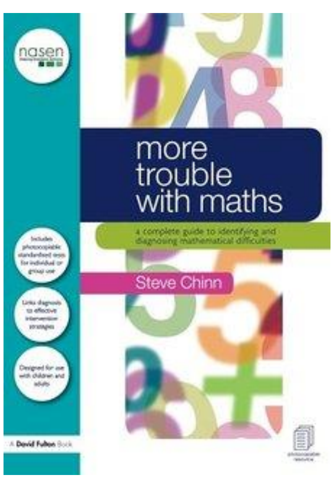


Figure 4.3: Front cover of Chinn's (2012) assessment

Through the trialling phase, Chinn’s (2012) assessment (see Figure 4.3) was thus deemed effective for identifying the learner’s MLD without reading ability becoming a confounding factor. This assessment can be carried out with both children and adults. The test has been standardised in the UK. It has no written instructions. One of its parts is made up of an assessment of the four operations (+, -, x and ÷). The pupils are given one minute to complete as many addition and subtraction facts as possible. They are then given two minutes to complete the multiplication and division facts. This specific component of the test has been standardised with 2058 pupils from over 40 schools in the UK aged between 7 and 15 (Chinn, 2012) and thus standardised scores are available for this age group of pupils. The test is then followed by a

15-minute assessment. This latter component of the test was standardised with over 2500 pupils and adults, from over 50 schools, colleges, organisations and some of the author's friends; 1783 of the population were from schools for pupils and students aged from 7 to 15 whilst the rest were those aged 16 to 59 (Chinn, 2012). In this test component, no written instructions are involved, and the children need to complete the operations given. Computations vary in difficulty and are graded starting with computations which are appropriate for children aged six to computations which are suitable for secondary students (aged 16). Some sheets from the test have been included in Appendix G – due to copyright issues, the whole test could not be included. Information about the reliability and validity of the test, that sometimes is included in such standardised tests, does not appear to be present for this assessment and thus could not be included.

The children were encouraged to complete as many items as they could. The assessment instrument has other sections to it to assess for mathematics anxiety, mathematics learning styles and other basic skills. However, these are to be done on a one-to-one basis and are not standardised. Consequently, I did not use these for the sample selection process but made use of the 'mathematics anxiety' and the 'learning style' sections of the assessment when the six main participants were identified, to get a deeper understanding of the learners. The mathematics anxiety questionnaire is accompanied by standardised scores for ages 7 to 15. These latter two components of the assessment allow the administrator to learn more about the pupil to be able to cater further for individual needs.

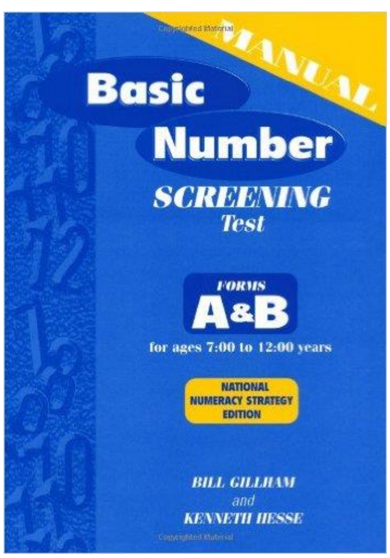


Figure 4.4: Front Cover of the BNST manual (Gillham & Hesse, 2001)

The BNST (see Figure 4.4) was selected because it takes roughly 25 minutes to complete and has no written instructions. The test was developed by Hodder Education (2001). It was originally standardised with a total of 3,042 children in the UK and is suitable for learners aged 7 to 12. The test is composed of different exercises that assess basic numeracy components such as addition, subtraction, multiplication, division and fractions. One sheet of the test has been included in Appendix H to illustrate what sort of exercises are included. The raw score allows the assessor to find out the number age of the child and a percentile and quotient for that child's performance. The assessor reads out the instructions and the children have to immediately complete the task so that if the children have difficulties with memory this does not influence the mark. The reliability and validity of this test has been found to be high. The Pearson product-moment correlation coefficient was +0.93 illustrating a high reliability. On the other hand, Spearman's Rho used to measure the validity of the test indicated an average of +0.82 (Gillham & Hesse, 2001). This information does not appear to be present for Chinn's (2012) assessment and thus could not be included.

Instructions for the BNST are read out twice. In order that the children's understanding of the English language would not influence the mathematical score obtained, I translated all the instructions to Maltese and when collecting the data, I read out the instructions in both Maltese and English. Translating the instructions included multiple steps. I primarily translated the instructions myself. One of my supervisors checked the translation together with a professional in linguistics. Changes were then made. A professional translator back translated the instructions into English. This allowed me to check whether words or phrases could be misinterpreted. The back translation was a fruitful exercise and other minor changes were then carried out to the instructions so that the translation was as precise and valid as possible.

The results from the pilot study illustrated that the tests were reliable and valid as the results obtained in both tests were similar. Following the pilot study, I decided to change the order in which I administered the tests. During the pilot study I administered Chinn's (2012) test first. However, I then decided that it would be better to do it second. During the pilot I realised that the children found Chinn's (2012) test rather fun to do due to fact that it is timed, and they had to finish off as much of it as possible in the time given. By doing it second, I hoped that the children remained motivated to do both tests and not begin to guess answers due to boredom.

Following the intervention phase, I made use of these same two tests to assess any quantitative progress that the children may have made. I thus wanted to identify whether their raw score, and therefore number age, had improved or regressed in any way following intervention. It was deemed suitable to use the same tests for pre- and post- intervention since this would give a comparable measure of any changes in the children's mathematical ability. Since the time frame between the pre- and post- test was long, roughly seven months, I believed it would be impossible for the children to remember answers to the questions. Measures taken when administering the tests to safeguard validity and reliability will be discussed in Section 4.10.

4.5.2 The Dyscalculia Screener (DS)

Prior to commencing the intervention phase, I administered the Dyscalculia Screener (DS) (Butterworth, 2003) with each of the six chosen participants. The DS (Figure 4.5) is,

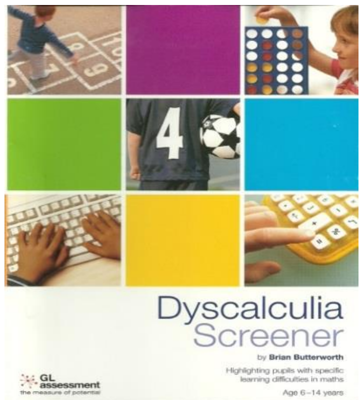


Figure 4.5: Front Cover of Dyscalculia Screener (Butterworth, 2003)

> "a computer based, standardised test designed to diagnose dyscalculia in children aged 6 to 14 years and to distinguish this condition from other issues that can affect performance in mathematics such as difficulties in communication and interaction, behavioural, emotional and social development" (Voutsina & Ismail, 2007, p.85).

I was interested in seeing whether any of the participants would be identified by the screener as having dyscalculia. This would give me further insight into the abilities of the participants I would be working with. Moreover, it would give me the possibility to investigate the similar and diverse abilities, if any, in the numeracy components, between the learners diagnosed with a profile of dyscalculia (if applicable) by the DS and the other participants. Furthermore, it would allow me to see whether these children were affected by my intervention or otherwise.

There is a general agreement, between international (Gifford, 2006; Messenger et al., 2007; Voutsina & Ismail, 2007) and local studies (Zerafa, 2011; 2015) that the DS is not a totally reliable measure and there are discrepancies between the learners' abilities and the results of the DS. For example, Voutsina & Ismail (2007) suggest that the tasks are too lengthy and that therefore some children might get bored thus underachieving in a task due to boredom, carelessness and disinterest. They also indicate that the children might guess the answers and simply press 'yes' or 'no' on the keyboard without knowing the real answer. Furthermore, Messenger et al. (2007) explained how they assessed children who were high achievers in mathematics but were assessed by the DS as being dyscalculic and others who were had poor mathematical abilities but were not assessed by the Screener as having dyscalculia. They indicate that this may result from the fact that other brain processes and functions are needed for numerical processing. These include: attention, higher-order thinking, language, sequential ordering and spatial ordering (Messenger et al.) (see Sections 2.6 and 2.9.3).

Although I was fully aware of its limitations, I decided to use the DS anyway since it is the only available screener for dyscalculia to date. Moroever, the DS is user-friendly and gives the participants three or four (depending on the pupil's chronological age) numerical tasks. It is recommended that it is administered on an individual basis. Each test takes about thirty minutes per child. The first task measures the reaction time of the individual taking the test. The other tasks are dot enumeration (comparing several dots of the number figure and saying whether they match); number comparison (comparing the value of different numbers and selecting the largest; addition and multiplication sums (checking whether each given sum is correct). Screenshots of some of the tasks have been included in Appendix I. The Screener's diagnosis of dyscalculia is based on the results of the different numerical tasks given and the time taken to complete the said tasks. The latter is compared to the learner's reaction time to conclude whether the time taken on the numerical tasks is similar to the child's reaction time overall. A detailed report is then provided (example in Figure 4.6) indicating whether the learner's abilities are typically of learners of the same age or whether dyscalculia has been detected.

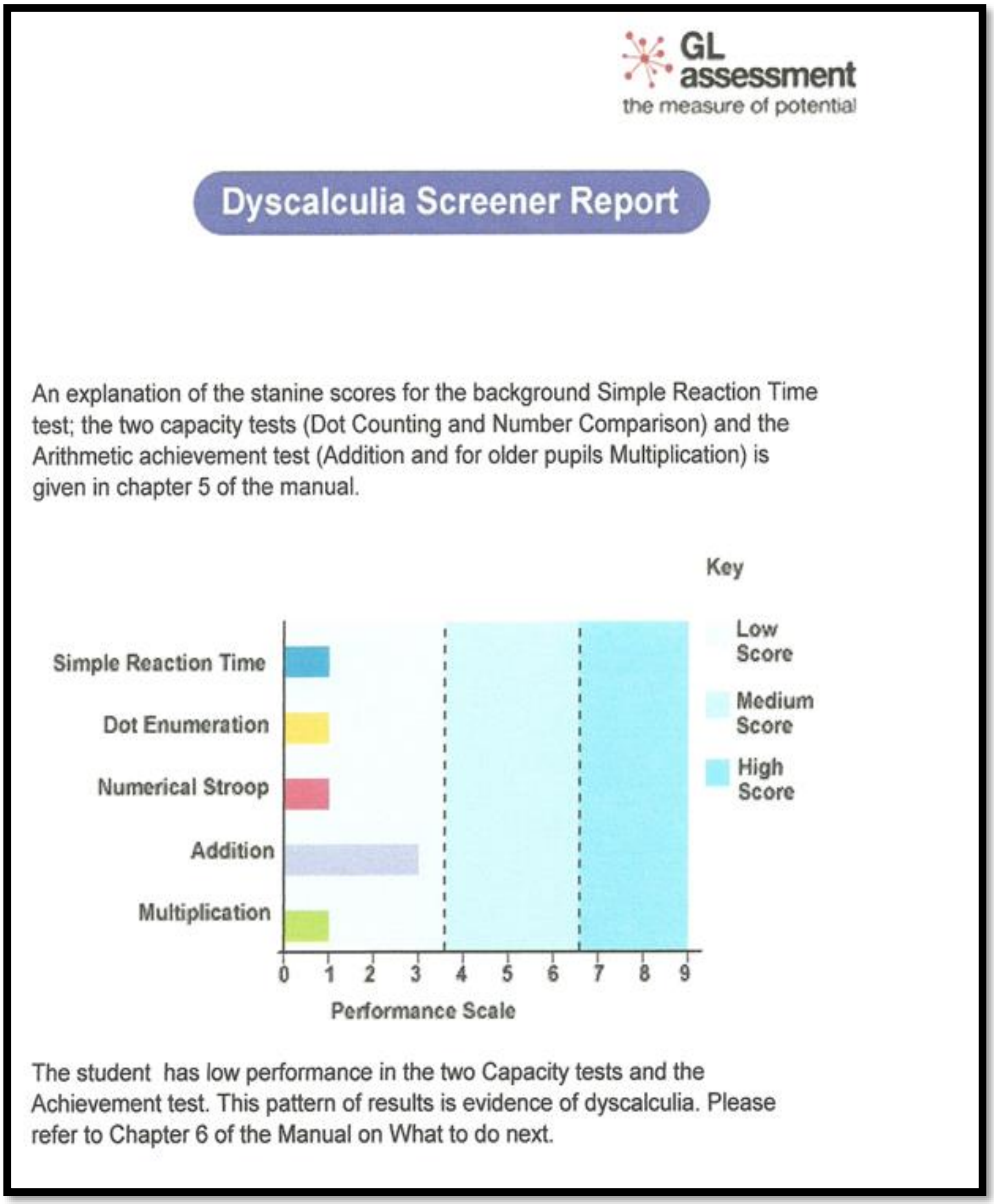


Figure 4.6: An example of a report provided by the DS indicating dyscalculia.

4.6 The Intervention Phase: Catch Up® Numeracy

This section provides an outline of the Catch Up® Numeracy (CUN) programme, which was used as the framework for the intervention. I had learnt about the earlier programme some years earlier while attending a conference in the UK. The organization that developed it had offered to support my Master's level research by offering me training in making use of the programme for free. Although they had given me this sponsorship, at no point did the organization dictate how I was to conduct the research, what findings I should have or what to report, nor did they expect me to 'conceal' the sponsor. As Cohen et al. (2007) suggest, these characteristics ensure that conflict of interest does not threaten the ethical aspect of the study. Our agreement was solely that at the end of my research I was to provide the organization with details of my results. At the time, the organisation's gain was multi-fold. Through my Master's research, Catch Up® was supporting a study that focused on children with dyscalculia and the results could have helped them to keep improving upon the programme to make it more accessible to dyscalculics. Moreover, the research was conducted outside the UK allowing the organization to determine its effectiveness in another country. Furthermore, Catch Up® could use my results to advertise the programme if these satisfied their criteria. Thus, although I was obliged to give Catch Up® a detailed transcript of my results, the actual research was in no way compromised. Similarly, for this research I asked Catch Up®'s Deputy Director, Dr Graham Sigley, for consent to be able to use their numeracy intervention programme. However, this time our agreement was different since Catch Up® gave me their consent to use the programme without asking for anything "in return". I plan to forward to Catch Up® any future publications based on this research, for their interest.

Before deciding on this programme, however, I did look into different programmes and also some training in other intervention programmes. This allowed me to make an informed decision as to which intervention programme was most suitable for my study. The reasons why I preferred to use CUN for this study are the following:

i. Although CUN sessions have a fixed structure, the person doing the intervention is still left free to create his/her own activities to suit the specific components;
ii. Different manipulatives can be used, at the discretion of the teachers, as part of the programme itself. This is something which is not normally allowed in other intervention programmes;
iii. It comes with a formative assessment that allows the professional to identify the child's strengths and weaknesses in the numeracy components;
iv. It is based on only 30 minutes a week but has been seen to have great impact on the children's learning;
v. It targets the cognitive, affective and psychomotor domains of learning to provide a rich mathematical experience to the learners;
vi. It provides a file with suggested activities and ways in which to carry out the intervention;

All the other programmes considered, lacked one or more of these criteria, steering my choice towards the CUN programme.

The CUN programme focuses on intervening upon ten numeracy components (see Section 2.3.1). Much like the developers of the programme, I decided to focus on the ten numeracy components because of two main reasons. Primarily, these ten components are found in many school mathematics programmes, including the local ones. As explained in Section 2.3.2, most mathematics curricula have a ‘number and algebra’ component which is given much importance because it is the foundation of mathematics learning. This is also in line with the early numerical competencies identified by Powell and Fuchs (2012), from the findings of various studies, which are important to young learners. Figure 4.7 provides an image which shows these competencies.

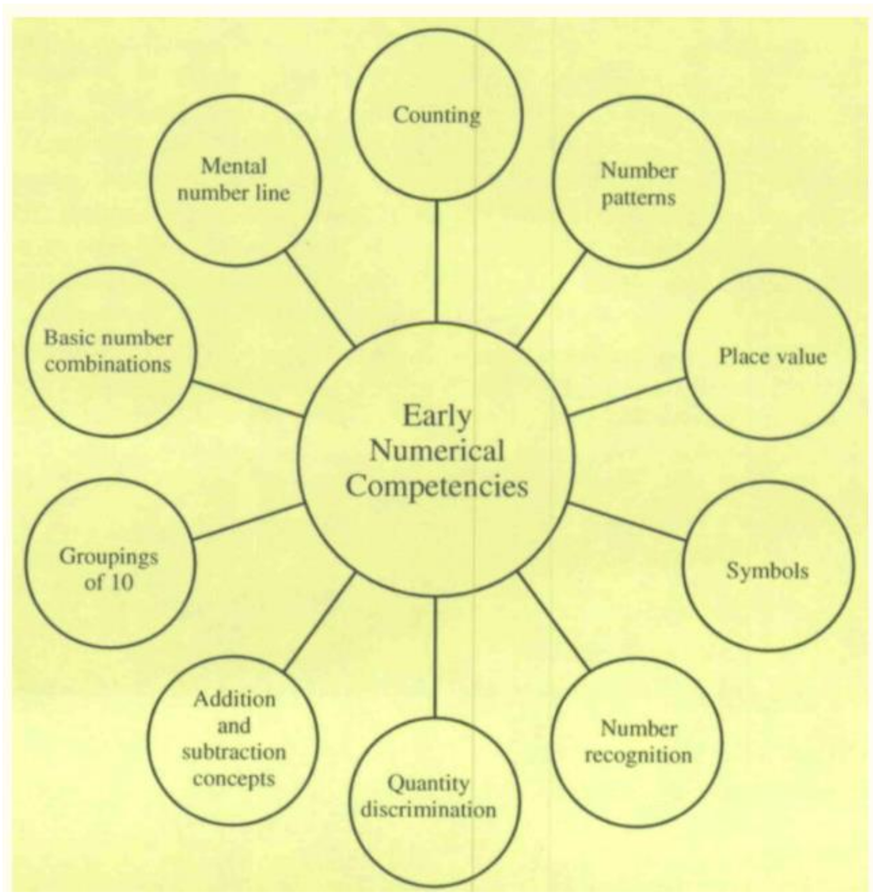


Figure 4.7: The early numeracy competencies important for young learners as per findings from various studies (Powell & Fuchs, 2012).

These early numerical competencies are present in the ten numeracy components too. Mastering the ten numeracy components is thus essential for the learning of more complex mathematics. Secondly, research in the field of MLD has shown that the most accentuated differences in arithmetical ability lie within these components. However, although as suggested by Girelli, Bartha, and Delazer (2002) mathematics knowledge may be factual, conceptual or procedural, “it is still true to say that most educational interventions thus far have targeted just one component, most commonly factual knowledge, trained by drilling" (Koponen et al., 2018, p.3). A similar conclusion has also been made by Jordan et al. (2009). Hence, through this study I hope to provide a more holistic view of the nature of difficulties experienced by learners having MLD and to be able to share other effective strategies that can support the learning of the different components rather than simply drilling.

In the CUN intervention programme, each numeracy component is split into sub-components. All the areas are first assessed through the programme's assessment so that together they provide a detailed overview of the learner's ability and needs in each of the areas. Table 4.2 depicts how each numeracy component is divided into other components. These have been extracted directly from the resource file provided by Catch Up® (2009). Nonetheless, I have added my own explanation of each sub-component to indicate what each one refers to.

Table 4.2: Numeracy component (adapted from Catch Up®, 2009).

Numeracy Main Component	Sub-Components	Example/Explanation
Counting Verbally	Counting Verbally Counting On Counting Back	To count verbally from 0 to a number (5, 8 or 10). To count on from a number to another. To count back from a number to another.
Counting Objects	Counting Objects Order Irrelevance Adding Objects Subtracting Objects	To count a number of objects. To understand the order irrelevant of counting objects. To add given objects. To subtract given objects.
Reading and Writing	Reading Numbers Reading Number Words Writing Numbers	To read numbers e.g. 1, 2, 3, etc. To read number words e.g. one, two three, etc. To write numbers e.g. 1, 2, 3, etc.
Hundreds, Tens and Units	Number Comparison Adding Tens and Units Subtracting Tens and Units	To compare two numbers and say which is greatest. To add tens and units e.g. 10 + 3 = 13. To subtract tens and units e.g. 12 – 4 = 8.
Ordinal Numbers	No Sub-components	To use ordinal numbers e.g. first, second, etc. correctly.
Word Problems	No Sub-components	To solve word problems.
Translation	Objects to Numbers Numbers to Objects Number Words to Objects Number Words to Numbers	To represent objects with number sentences. To represent number sentences with objects. To represent word problems with objects. To represent word problems with numbers.
Derived Facts	Identical Commutative N + N -	e.g. If 4 + 3 = 7, then 4 + 3 = 7. e.g. If 4 + 3 = 7, then 3 + 4 = 7. e.g. If 4 + 3 = 7, then 4 + 4 = 8. e.g. If 4 + 3 = 7, then 4 - 2 = 6.
Estimation	No Sub-components	Estimating the answer to sums.
Remembered Facts	No Sub-components	Recalling number facts without counting.

As part of the data collection process, assessments were administered as indicated in Catch Up®'s (2009) file and on a one-to-one basis. Each assessment lasted approximately one hour and a half. The assessments were carried out during my free periods and when the children were not having lessons for Core Subjects (Mathematics, Maltese and English) in class. For each sub-component, the learner was asked to complete a specific task. For example, for the sub-component of 'counting back', the learner was asked to count back from a given number to another (e.g. count back from 5 to 0). For the 'word problems' sub-component, the learner was asked to work out different types of word problems. Some problems were 'join' type, while others were 'separate', 'part-part whole' and 'comparison' types. Moreover, he was then asked why he had solved the problem in the way he did, and I would take note of whether the explanation given was reasonable or otherwise. When working through the 'translation' component, I first read the problem and asked the learner to translate this into a computation. Each learner was encouraged to draw when they got stuck in a translation, so that this might help them to make the translation. Figure 4.8 illustrates some of the working/drawing that one of the learners used to work out the given word problem. The support I offered him when he got stuck, in the form of prompting, is in pink.

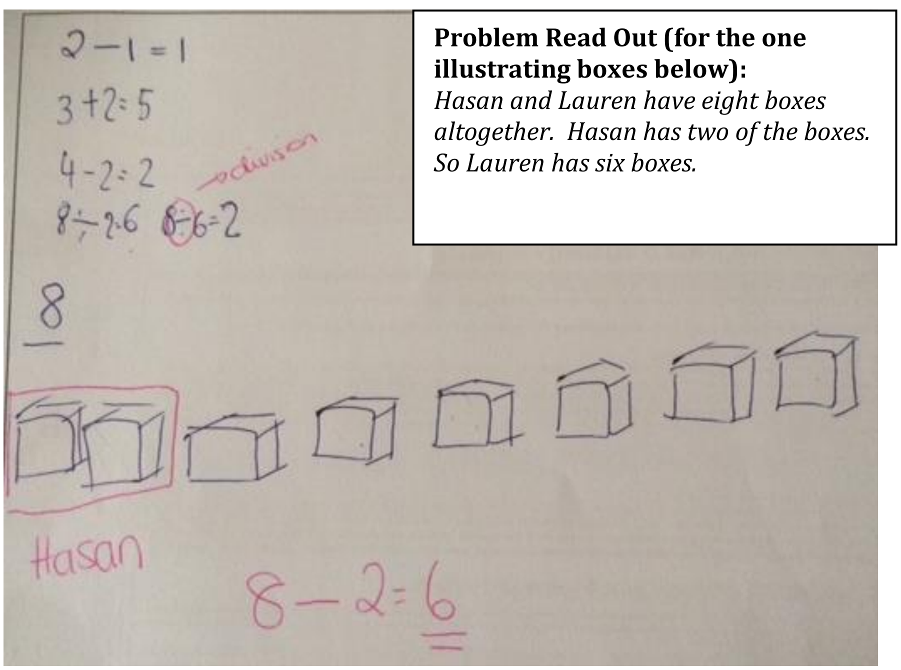


Figure 4.8: Working/Drawing done by one of the pupils when solving word problems.

The assessment of each sub-component began at the most basic number range (range 1-5). The exercises then increased in number range. Each sub-component was assessed in the following number ranges (0-8, 0-10, 0-15, 0-18 and 0-20). As soon as a pupil was seen to be struggling with a number range, the assessment was stopped, and intervention would then be planned to begin with that specific component and number range. When all assessments for the different components were carried out, a pupil's profile could then be drawn up to determine the strengths of the learner and the areas which still needed development. The pupils' profile was crucial for me to identify which numeracy components necessitated intervention as well as prioritisation. Components were also given priority especially if they were needed as a basis to master other components. CUN's assessment was the last assessment carried out before intervention began so that the information gathered would be a realistic image of the learner's difficulties at that point in time.

4.6.1 Structure of Each Session

As prescribed by the programme, intervention sessions were 15 minutes long and were carried out twice weekly. The programme's idea of keeping intervention sessions short is mainly based on providing a short intervention which would maintain the learner's attention throughout, thus making it more effective. All the sessions were carried out at school. I liaised with the class teachers to ensure that the children missed as few lessons as possible. The teachers were normally very cooperative and helped me to identify time slots in which no new topic was being introduced and to select lessons that were either revision lessons or deemed as less important than others. I did however refrain from taking the pupils for intervention when they had lessons such as Physical Education or Art because when I did try this once the learner came to my session upset and not much was achieved.

All sessions were carried out in English. This choice of language was based on two factors. Primarily, two of the participants' first language was English and the other pupils were very fluent in English. Secondly, at the boys' school mathematics lessons always took place in English, since this was the school's practice throughout the primary, as indicated by the school's language policy. As in the daily lessons, code switching during the intervention sessions took place only when the child was unclear about something or did not understand the instructions given. In fact, code switching was rarely needed and the children whose first language was Maltese only switched to this when commenting about experiences other than those strictly related to mathematics. For example, during one of the sessions when a ball was mentioned, the participant seemed to recall something associated to this and said, '*Taf x'se nagħmel il-lejla? Sejjer nilgħab futbol!*' [Do you know what I will be doing this evening? I will be playing football!] When transcribing the data, any utterances in Maltese were translated to English as faithfully as possible, indicating that this was originally stated in Maltese.

Each participant was exposed to 20 sessions spread over 14 weeks. My intention was to carry out more sessions, however I realised that due to school holidays, outings and other activities this was going to be impossible. As indicated by the programme, each session was focused on one particular component, with which the child was struggling. Each 15-minute session was made up of three parts and each part had a stipulated number of minutes it. The process is outlined in Figure 4.9.

Figure 4.9: The three parts of each CUN intervention session (adapted from Catch Up®, 2009).

These parts will be explained in detail in Sections 4.6.2 to 4.6.4. Before each session, I would plan my own outline of activities to be carried out with each learner to target the specific numeracy component. I generally created my own 'review and introduction' and 'linked recording'. However, these were sometimes based on the numeracy activity suggested by Catch Up® (2009). For the main 'numeracy activity' I normally turned to Catch Up®'s (2009) section of suggested activities and adapted a suitable activity as necessary. I sometimes drew on activities from Catch Up®'s online resources, in other sources or online. My choice of activity was usually based on how appropriate it was to introduce or review a specific component and on how compatible it was to the child's learning style (refer to Figure 2.8 in Section 2.5.1.1), which I established through one of Chinn's (2012) test components. As a result, for the same component, I sometimes made use of different activities with different learners because I sought to find the task which would be most successful with each specific learner. Also, since some of my learners also had RD, I took this into account when preparing my resources. One of the ways in which this was done was by using large and clear print. Additionally, I made use of pastel coloured paper to reduce the contrast between the colour of the paper and the print and thus minimize scotopic sensitivity (Ott, 1997) which can cause children to perceive words as moving on the page.

For each session I took note of the following important information:

- The numeracy component and the number range focused upon;
- Any miscues which the child had displayed during the session and any misconceptions which I observed. This indicated areas which still needed to be addressed;
- Any other components which were targeted indirectly through the specific session; for example, when targeting the remembered facts component, the reading and writing numbers component was inevitably also invoked;
- The open-ended questions posed with regard to *prediction*, *process* and *reflection*.
- Any comments about the learner's performance during the particular session and comments by the learner himself about his own progress;
- Any follow-up tasks to be carried out in subsequent sessions.

4.6.2 The 'Review and Introduction' Phase

The 'review and introduction' section of each session had two main objectives. It aimed at reviewing what the learner would have covered during the previous session so that this could be used as the starting point to the new number range of component being covered during the day's session. Furthermore, it allowed me to share the teaching objectives for the session with the learner and therefore to specify the numeracy component and number range which would be the focus of the particular session. Moreover during this initial phase of the session I would decide what related mathematical language would be focused upon and introduce it to the learner. In Table 4.3 I give examples, taken from my notes, of how the 'review and introduction' unfolded during one of the sessions.

Table 4.3: Sample ‘review and introduction’ phase taken from my planning notes.

Previous Session	Numeracy Component: Counting Back Number Range: 0 - 15
Actual Session	Numeracy Component: Counting Back Number Range: 0 – 18
Review	The child will be given the number cards from 0 to 15. He is to put them in descending order starting from the largest number. Once the sequence of numbers is ready, ask the child to count back starting from the largest number. Emphasise the mathematics language for counting back using the flashcards seen below (Figure 4.10).  Figure 4.10: Sample flashcards to rehearse mathematics terms.
Introduction	Introduce the new number range which will be 0 – 18 and explain the objective of today’s session which is that of counting back verbally using the numbers 0 to 18.

4.6.3 The ‘Numeracy Activity’

The ‘Numeracy Activity’ section was the main part of each session. Here an activity was carried out in relation to targeted teaching objective of the particular session. Its main aim was that of allowing the child to practise the skill being introduced or focused upon.

The CUN programme suggests that during this phase of the session the more knowledgable other supports the child in engaging in a metacognitive process. This is done by asking questions relating to prediction, process and reflection. The examples of questions for all the questioning domains, as provided by Catch Up® (2009), can be seen in Figure 4.11.

Response and understanding

Prediction e.g.,

What do you think the answer will be?

Why do you think that?

Process e.g.,

How are you going to work it out?

What will you do first?

Reflection e.g.,

Was your answer accurate?

What will you do next time?

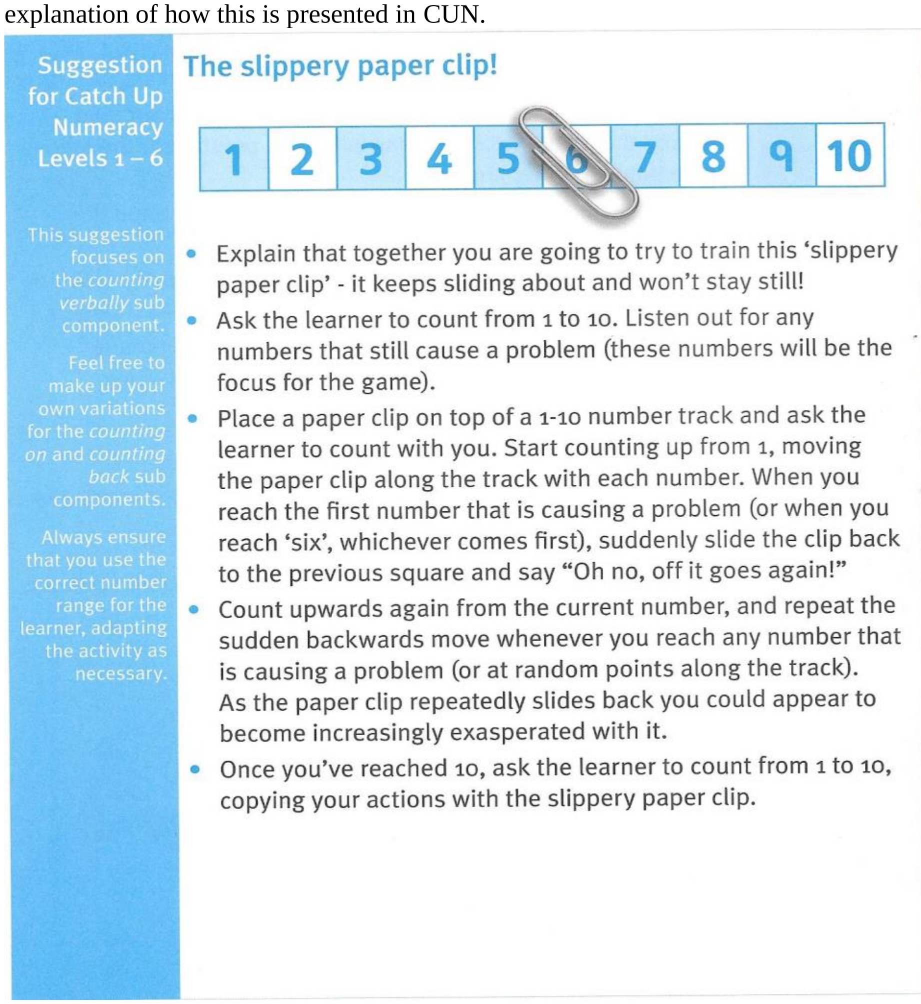
If you had a friend who couldn't do this, what would you tell them to do?

Figure 4.11: Sample questions provided by Catch Up® Numeracy (2009).

Metacognitive thought, as highlighted in Section 3.4.2.1, is fundamental as it allows the child to engage in the process of thinking about the way s/he is processing the information being learnt. In addition, it allows the educator to provide guidance within the child's Zone of Proximal Development (ZPD) (see Section 3.5). Therefore in the following example of the 'numeracy activity' (Table 4.4) the metacognitive questions are planned.

4.6.4 ‘Linked Recording’

Table 4.4: A sample ‘numeracy activity’ (Counting Forward, 0 - 15).

Numeracy Component	Counting Verbally: Counting forward
Number Range	0 – 15
Activity	Move a paper clip along a number line with numbers from 0 to 15. The child in to count forward from the number indicated by the paper clip to the number 15. This activity has been adapted from those suggested by Catch Up® (2009), more specifically ‘The Slippery Paper Clip!’ activity. Figure 4.12 illustrates a detailed explanation of how this is presented in CUN.  Suggestion for Catch Up Numeracy Levels 1 – 6 This suggestion focuses on the *counting verbally* sub component. Feel free to make up your own variations for the *counting on* and *counting back* sub components. Always ensure that you use the correct number range for the learner, adapting the activity as necessary. The slippery paper clip! 1 2 3 4 5 6 7 8 9 10 • Explain that together you are going to try to train this ‘slippery paper clip’ - it keeps sliding about and won’t stay still! • Ask the learner to count from 1 to 10. Listen out for any numbers that still cause a problem (these numbers will be the focus for the game). • Place a paper clip on top of a 1-10 number track and ask the learner to count with you. Start counting up from 1, moving the paper clip along the track with each number. When you reach the first number that is causing a problem (or when you reach ‘six’, whichever comes first), suddenly slide the clip back to the previous square and say “Oh no, off it goes again!” • Count upwards again from the current number, and repeat the sudden backwards move whenever you reach any number that is causing a problem (or at random points along the track). As the paper clip repeatedly slides back you could appear to become increasingly exasperated with it. • Once you’ve reached 10, ask the learner to count from 1 to 10, copying your actions with the slippery paper clip. *Figure 4.12: The Slippery Paper Clip activity as presented in Catch Up® Numeracy (2009).*
Metacognitive Questions to be Asked	• What is happening to each number as you count along the number line? Is there a pattern? What is it? • How do you remember the number sequence? • Have you been making the same mistake over again? What should you remember to be able to count better?

The ‘linked recording’ section is the final part of every session. Here the child is asked to complete a task or written exercise related to the numeracy activity conducted during that particular session. The task or exercise is recorded on the sheet that is used to log the details for every session. The ‘linked recording’ section allows the child and the educator to assess whether the learning outcomes of the session have been achieved or whether more practice is needed in subsequent sessions. Since it is recorded it is evidence of the child’s progress and can be referred to at any time. Figure 4.13 below provides a sample of the work completed for counting forward (0 – 20).

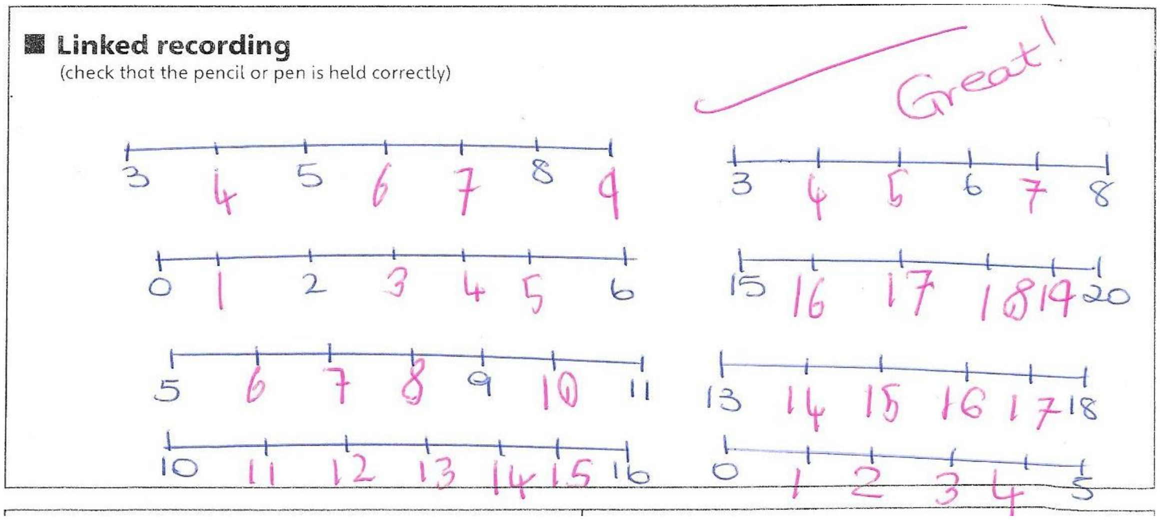


Figure 4.13: Sample ‘linked recording’ (Subtracting Objects, 0 -10).

4.7 Interviews

Since the main research phase involved six single-case studies, I thought that it would be beneficial to interview the parents and teachers of the participants. The aims of the interviews were to get to know more about the children’s development of numerical skills, whether, and to what degree, the children had difficulties in specific components of numeracy and to learn more about how the parents and teachers had been supporting the learner regarding the development of these skills. Moreover, I felt that it would serve to get a clearer profile of each of my main participants. It was important for me to get to know specific information about the child such as when they had begun to struggle with mathematics and which areas of mathematics they found most difficult. The interviews provided richer data in line with the interpretivist paradigm underpinning Phase 2 of this study.

Kvale (1996) highlights that an interview is “an interchange of views between two or more people on a topic of mutual interest, sees the centrality of human interaction for knowledge production, and emphasizes the social situatedness or research data" (p.14). Notwithstanding the fact that the interview is not merely a neutral means of data collection (Barker & Johnson, 1998) and that it is also a social, interpersonal encounter (Cohen et al., 2000) I felt that it was the best tool to collect the data I needed from the parents and the teachers. I was aware that as the researcher I was also the research instrument and that not only was my knowledge of the child going to come in play but also my expertise in communication and interaction (Kvale, 1996). Moreover, I knew that the structure of the interview and the questions posed would be crucial in obtaining the desired data. Although an interview inevitably has bias, I tried to recognize this and control it. I made myself aware of what I knew about each child and prepared a set of questions that would guide both the interviews with the parents as well as those with the class teachers.

Since the structure of the interview would influence the data collected, I began by looking at the different types of interviews to see which would suit the aims of my study. LeCompte and Preissle (1993), indicate six types of interviews: *standardised interviews, in-depth interviews; ethnographic interviews; elite interviews; life history interviews and focus groups*. Furthermore, Bogdan and Biklen (1992) add another two types; group interviews and semi-structured interviews. As Kvale (1996) suggests, interviews normally differ with regard to openness of their purpose or the degree of the structure. Since I was working within an interpretive paradigm, I discarded structured interviews because they might have provided a very limited response on the interviewee's part. I decided to make use of semi-structured interviews (Bogdan & Biklen, 1992) since this would allow me to avoid the rigidity present in structured interviews whilst allowing me to present a set of guiding questions to have some form of uniformity in the participants' responses to allow comparison. Although during semi-structured interviews I would be able to ask further questions about anything which the participants would bring up, I had to prepare a "handful of main questions with which to begin and guide the conversation" (Rubin & Rubin, 1995, p. 145).

Several questioning techniques have been proposed (Charmaz, 2002; Kvale, 1996; Stringer, 2004). When planning the questions, I tried to keep them as open-ended as possible. Open-ended questions are one form of questioning technique which can be used in an interview schedule. This type of questioning strategy has numerous advantages: it is flexible, allows the interviewer to probe to be able to go into more depth; helps establish a rapport between the researcher and participant; and allows for the arising of unexpected answers (Cohen et al., 2000, p. 275). When preparing the question guide, I also tried to make use of the 'funnel' technique (Kvale, 2007) by beginning my interview with broader questions that gradually became more specific. Interview questions for both the parents and the teachers were written in English since all participants could understand the language well. However, when the clarification of a question was requested, I tended to clarify the question in English and then repeat this in Maltese.

The following are some of the questions which I prepared as part of my interview schedule for the parents:

- *Has your child always had difficulties with mathematics?*
- *When did you first notice that your son was struggling with mathematics?*
- *Does your child do his mathematics homework alone at home?*
- *If not, what do you observe him doing whilst completing given tasks?*

A full list with the questions can be found in Appendix J. On the other hand, some examples of the questions which I asked the teachers are:

- *How does (NAME) perform in Mathematics – class participation, school work, homework, exams, test etc.?*
- *Are there particular topics in which he performs well?*
- *Are there particular topics in which he needs support?*

The entire list of questions can be found in Appendix K. I prompted the parents and teachers to elaborate their responses, discuss further and to give me more but without leading them.

Interviews were held in my classroom, at school. I closed the door so that the participants would feel as comfortable as possible. I also closed the windows of the classroom and pulled the curtains as well as hung a note on my classroom door which asked people to come back later. This was done to minimize interruptions and disruptions (Field & Morse, 1989). The interviews were audio recorded. As indicated by Bryman (2008) "with approaches that entail detailed attention to language…the recording of conversations and interviews is to all intents and purposes mandatory" (p.451). Audio recording allowed me to focus all my attention on the participants rather than having to take detailed notes of what was being said. It also gave me the possibility of listening to the same interviews repeatedly and analyse the data collected. I seemed to notice new details every time I listened to an interview. Parts of the interviews were transcribed and will be presented in the Analysis Chapter. It should be noted that in circumstances where the participants replied in Maltese, the text was translated to English as faithfully as possible. Moreover, I believed that it would be important to take 'jotted notes' (Lofland & Lofland, 1995; Sanjek, 1990). This was done to take note of any non-verbal actions, gestures and body language since these details are fundamental to collect richer data. Note taking was done as discretely as possible and did not seem to interfere with the interview in any way.

4.8 Observations

As highlighted by Cohen et al. (2000), "observation methods are powerful tools for gaining insight into situations" (p. 315). Since I wanted to gain as much insight as possible about each of the participants on my study, I deemed it beneficial for me to observe them *in situ* (Patton, 1990). Thus, I decided to conduct three classroom observations in each of the Grade 5 classes in which the children learnt mathematics. Ethan[18], Mike and Nathan were in the same class, hence classroom observations happened on the same occasion. Similarly, Andrea, Thomaz and Seb were observed in the other class at the same time. Two of the classroom observations took place at the beginning of the intervention programme to help me to get to know the learners better and to understand each participant's level of participation and engagement in the classroom. These initial observations also allowed me to gauge the child's understanding of the topic at hand and his transfer of this knowledge to applying it in individual work set in the classroom. The last observation took place mid-way through the programme aiming at providing further insight as per the first two observations but also to be able to note any differences in learners. Since the aims of conducting these observations were not related to observing the children as they tackled one or more specific mathematical concepts, lessons to be observed were chosen based on convenience. Table 4.5 illustrates the schedule followed for carrying out the observations.

Table 4.5 Schedule for Classroom Observations for each learner.

Schedule of Sessions (Session Numbers marked 1 to 20) and Class Observations (marked as CO)																							
E	CO	1	CO	2	3	4	5	6	CO	7	8	9	10	11	12	13	14	15	16	17	18	19	20
M	CO	1	CO	2	3	4	5	6	CO	7	8	9	10	11	12	13	14	15	16	17	18	19	20
N	CO	1	CO	2	3	4	5	6	CO	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	CO	1	2	CO	3	4	5	6	7	CO	8	9	10	11	12	13	14	15	16	17	18	19	20
T	CO	1	2	CO	3	4	5	6	7	CO	8	9	10	11	12	13	14	15	16	17	18	19	20
S	CO	1	2	CO	3	4	5	6	7	CO	8	9	10	11	12	13	14	15	16	17	18	19	20
Legend: E – Ethan, M – Mike, N – Nathan, A – Andrea, T – Thomaz and S - Seb																							

[18] All the names given here are pseudonyms.

The data collected through my observations was mostly related to the *interactional setting* (e.g. the interactions that are taking place) and the *programme setting* (e.g. pedadogic styles and curricula and their organisation) (Cohen, Manion & Morrison, 2011). The observations permitted me to obtain first-hand experience about the interactions that took place between the teacher and each of the learners that might have been formal, informal, verbal, non-verbal, planned or even unintentional. Moreover, they gave me the chance to view the pedagogical styles being used in the classrooms where these children were learning as well as the resources used. These observations will be discussed in Chapter 6.

The degree of structure in observations may vary. Observations may be *structured*, *semi-structured* or even *unstructured* (Morrison, 1993). Structured observations are those in which the researcher is very clear about what s/he is looking for and takes note of just that. On the other hand, unstructured observations are those in which the research enters the field with no guidance as to what to observe but simply takes note of anything that may happen. Whereas in the former, the research hypothesis is set, and observations are used to illustrate that the hypothesis is correct or otherwise, in the latter the observations tend to formulate the hypothesis. A balance between these two forms of observations can be found through semi-structured observations. As indicated by Cohen et al. (2000) "semi-structured observation will have an agenda of issues but will gather data to illuminate these issues in a far less pre-determined or systematic manner" (p. 305). I decided to use this form of observation because it allowed me to observe specific issues whilst providing some form of flexibility knowing that specific observations might give rise to new hypothesis. Consequently, my observations were carried out after drawing up a set of guidelines that steered what I was to focus on whilst carrying out the classroom observations. These guidelines can be found in Appendix L.

When entering the field, I also had to decide what kind of observer I wanted to be. Cargan (2007) identifies three main types of observers: nonparticipant observers, complete participant observers and, something in between, which he names as either the observers-as-participant or participant-as-observer. I could not opt to be a nonparticipant observer because the children knew me as the school's complementary teacher, and so I could in no way act as a complete observer. Furthermore, one must also bear in mind that the researcher cannot observe a situation without being part of it (Adler & Adler, 1994) and thus without being participant at least to a minimal extent. Thus, I decided to consider myself as an observer-as-participant. During the observation sessions my main aim was to observe what was happening, but I acknowledge that I was also a part of the situation myself and that although I did not intentionally seek to alter the situation myself, since the children knew me well, my presence might have provoked this unintentionally.

It is well known that field notes can have varying forms and lengths (Bogdan & Biklen, 1992; LeCompte & Preissle, 1993). Short field notes were taken during the observations and more lengthy ones were written after each observation to jot down any details which I would have observed but did not have the time to elaborate upon during the observation session itself. This was crucial in providing details that were relevant to gaining a deeper insight of each of the participants of my research and thus to have a more holistic picture of each participant and their interaction throughout the mathematics lessons observed. An example of two short notes taken during the observation itself are *'learner was able to explain why the clock hand turns 180° from 12 o'clock to 6 o'clock'* and *'learner was able to use right angles and degrees'*. Immediately after the session I elaborated on this detail so that I had richer data to use during analysis. Thus, the short notes were transformed to:

> *The learner was able to explain why the clock hand turns 180° from 12 o'clock to 6 o'clock. During the lesson he explained that there is one right angle from 12 o'clock to 3 o'clock and another from 3 o'clock to 6 o'clock so 180° in all. He also added that this means that there are two right angles. When the teacher asked him to explain why the answer was 180° the learner seemed to get excited and hesitated however after the teacher prompted him he plucked up courage and explained his reasoning using the correct mathematical language – 'right angles' and 'degrees'* (Sample of my notes from the class observations).

4.9 Ethical Considerations

After obtaining clearance from the relevant research ethic committees at the University of Malta, the process of gaining access from gatekeepers began. As suggested by the British Educational Research (BERA) (2004), after gaining access to the research field, consent is to be obtained from "those who act in guardianship" (p. 6) as well as from the children themselves for both phases of the study. As Gregory (2003) underscores, "every code of ethics designed to guide research involving human subjects gives primacy to the requirements of fully informed voluntary consent on the part of the individuals concerned" (p. 35). I thus ensured that both the participants and the parents of each phase of the study were informed about what my research would involve. Additionally, throughout the process of data collection and analysis I reflected on the various issues that came up to ensure that my duties as a researcher were sustained while safeguarding the participants' rights (Hammersley & Traianou, 2012).

In my email to the schools I explained the nature of the assessments. I offered to send them a consent form for parents and some of the schools did ask for one (see Appendix M). Other schools thought this was not necessary since they collect forms at the beginning of each scholastic year to ask for permission to give the children periodic assessments. The form was written in English since all the schools had a policy of sending forms to parents in English. The consent form mentioned that the results would be kept confidential and would only be used to find norms for the larger population of boys in Grade 5 attending Church schools for boys. As highlighted by Mertens (1998) confidentiality "means that the privacy of individuals will be protected in that the data they provide will be handled and reported in such a way that it cannot be associated with them personally" (p. 279). The School Administration were very helpful in sending out a circular regarding the assessments which would be carried out with the Grade 5 cohort together with the consent forms. Parents who did not wish their child to participate were asked to communicate with the school by a given date. When I analysed the data collected during Phase 1 the issue of confidentiality was kept in mind so that no data would be used to compare schools. Furthermore, during this phase of data collection, anonymity could also be safeguarded. As Mertens (1998) underscores this "means that no uniquely identifying information is attached to the data, and thus, no one, not even the researcher, can trace the data back to the individual providing it." (p. 279). The students were asked to write a number instead of their names on the actual test papers. Since my aim was that of collecting test results to be able to find norms, there was no need for me to trace the data to the individual student.

At the beginning of the following scholastic year (2013/2014) I sent an information letter (see Appendix N) to the parents of all the pupils who were in Grade 5 during that year. As indicated by Cohen et al. (2000) "whatever the specific nature of their work, social researchers must consider the effects of the research on participants, and act in such a way as to preserve their dignity as human beings" (p. 56). The information letter explained my research aims and methods and informed the parents that their children would be sitting for some standardised assessments so that I could select learners who would be struggling with mathematics. Since I needed to trace the data to the individual who would have taken the test to identify the six main participants, in the letter I could not promise anonymity during the data collection of Phase 2. However, I did promise to anonymise their identities in the text. Privacy was nonetheless always safeguarded. The letter also clearly stated that once such learners would be identified, I would inform the parents to provide them with more information about the research and a plan of action.

I telephoned each parent to tell them that their son had been chosen and gave them further details about the study. I also offered to meet them should they have wished so. Since Phase 2 of the study would entail more complex ethical considerations, I wanted to ensure that all parents understood my intentions, duties as well as their rights as participants. As indicated by the British Sociological Association (BSA) (Bryman, 2008) it is the researcher's responsibility to "explain as fully as possible, and in terms meaningful to participants, what the research is about" (p. 481). I also gave the parents my contact details and invited the parents to contact me if they wished further clarification. After the telephone call, a consent form (see Appendix O) was attached to the information letter asking the parents for their consent to conduct the mathematics intervention programme with their children. As indicated by Frankfort-Nachmias and Nachmias (1996), "consent must be *voluntary* and *informed*" (p. 84). Thus, I ensured that whilst asking for consent, these two important criteria were taken into account. I promised to keep all the data collected during the sessions confidential and to present the data in full anonymity. Since it would have been easy to trace the school in which this research project was carried out, as it is one of the schools in which I taught, I ensured that pseudonyms would be used. I also ensured that no photos would show the pupils' faces, their uniforms or any other characteristic. The consent form also asked for the parents' consent to audio record the sessions with the children. It also specified that only myself, and my supervisors if needed, would listen to recordings, and that all data would be treated with strict confidentiality and anonymity.

At a later stage, I also asked the participant's parents and teachers to participate in an interview; both confidentiality and anonymity were promised. Consent was asked for the interview to be audio recorded. I also asked for consent from the parents to carry out additional assessments with their children. I felt it was important to inform them that the British Ability Scales II (BAS II) would not be carried out by myself but by a qualified educational psychologist since this assessment can only be administered by such a professional. I explained that all assessments would be carried out at school during school hours so that the children's daily routines were minimally altered.

An information sheet (see Appendix P) was also handed out to the 6 chosen children using simpler terms to explain to them what my research was about. I felt it was essential to give the children a voice since this would ensure that they came to sessions out of their own free will and therefore took further benefits from the intervention programme. The children were also then asked to sign a assent form (Appendix Q).

One of the main ethical issues centered on the fact that I, as the Complementary teacher, could have had a conflict of interest. I reflected about the power issues which concerned my role since parents and children may have felt obliged to participate in the study. A number of measures were considered and communicated to the parents and children to ensure that the parents' and children's consent was truly voluntary. Primarily, I informed the parents that they had the right not to give their consent for their children to participate in this study. Moreover, I also made it clear, that if they decided not to allow their child to participate in this study, they would still be entitled to complementary lessons in numeracy which would not be based on the *Catch Up*® *Numeracy* framework but would still be tailor-made to the learner's needs to support the child in the mathematics areas he would be struggling with. Another measure that was emphasized with both the parents and the children was that they had the right to withdraw from the study at any point without giving any reason for their withdrawal. Additionally, they would still be entitled to the complementary lessons without these being compromised.

No deception was involved and a trusting relationship with both the parents and the children was developed and maintained. I felt that my relationship with the parents was an open one as when they had queries, they communicated them to me immediately and we worked on them. For example, after a few sessions, one parent had asked me whether I had seen any progress in her son's mathematical skills and competences. I had showed her my record sheets to show her that her son had already mastered some of the numeracy components that I had covered with him.

I would like to declare that my external co-supervisor has herself worked extensively on the CUN programme. She has supported its development and has led various studies that use it as a main research instrument (Dowker & Sigley, 2010; Holmes & Dowker, 2013). When I decided to use the CUN programme for this study, I searched for an external supervisor who knew the programme well and who could supervise my work at doctoral level. Dr Ann Dowker, from the University of Oxford, agreed to be nominated for this role. Dr Dowker has not only been actively involved in researching the CUN programme but has also been involved in researching other intervention programmes, namely *Numbers Count* (Torgerson et al., 2011), *Dynamo Mathematics* (Dowker, 2016), *SELKIS* (Koponen et al., 2018) and another pilot programme with Kindergarten children (Kaufmann et al., 2005). It was clear that Dr Dowker could support this piece of work through her expertise and experience in the field of dyscalculia and MLD. Hence, Dr Dowker's 'double role' served to support this study and did not cause any ethical issues such as conflict of interest since all the data was analysed by myself and no pressure was imposed by either Dr Dowker or CUN on the findings which were discussed in this study.

4.10 Validity and Reliability

One of the fundamental criteria in any research is that the data collection and analysis process are both externally and internally valid and reliable (LeCompte & Goetz, 1982). Although I acknowledge that threats to validity and reliability cannot be eliminated, I am aware that "the effects of these threats can be attenuated by attention to validity and reliability throughout a piece of research" (Cohen et al., 2007, p. 133). *Validity* implies the credibility of a specific research including its data collection and analysis process. Whilst *reliability* is used to refer to the repeatability of findings and thus the consistency of a specific measure. Reliability is necessary but not sufficient for validity whilst validity can be sufficient but not indispensable for reliability.

The terms *validity* and *reliability,* stem from quantitative research and have in fact been widely criticized by qualitative researchers (Lincoln & Guba, 1985; Winter, 2000). The approach towards validity and reliability taken by qualitative and quantitative research methodologies differs since both approaches embrace these concepts in their own way. Since I have made use of a mixed method approach in this research, I will explain how validity and reliability issues were considered during both phases of this project.

In the quantitative phase of the research, a high degree of research validity was attained by ensuring the norms were found with the right representative sample. Moreover, validity was safeguarded by piloting different possible standardised tests which could be used with the sample before actually selecting the instruments that would be most suitable in allowing me to answer my research questions. This ensured that a high level of content validity was obtained. The validity of this part of the research was also tackled whilst analyzing the data collected using SPSS. All data was inputted in the programme – that is test scores obtained from half the population at each school as explained in subsequent sections. This ensured that the norms obtained would truly mirror the ability of the cohort being analysed and therefore increase the external validity of the results obtained.

In the process of finding norms for suitable assessments to be used to identify the children with MLD and MLD and RD, it was deemed necessary to use more than one test so that results could then be compared and the validity of the choice of participants would increase. Since "each of these data collection methods has certain advantages as well as some inherent limitations" (Frankfort-Nachmias & Nachmias, 1996, p.205), it was important to make use of more than one research instrument in Phase 1 of the study. Children were then selected based on whether they attained a low achievement in all tests and not just one. Although all these aspects of validity were looked into, nonetheless I acknowledge that this quantitative stage still had a degree of *margin of error* that was inevitable. This refers to the random sampling error which is inevitable when one takes samples, as I did, rather than the whole population. Although the large sample taken makes the test valid, as the norms collected should be similar to those collected if the whole population was tested, certainty cannot be assured.

A high degree of the reliability of the quantitative phase of the study was maintained by considering the main three principles of reliability that are those of stability, equivalence and internal consistency (Hartas, 2010). Stability, which indicates a consistency over time and over similar samples, was obtained by administering the assessment with a large cohort of children that would represent the population being targeted. Reliability of equivalence was also safeguarded. When collecting the norms, I ensured that all the tests were carried out within during the same period of time so that the pupils had covered roughly the same material in class. This period was also maintained when the main participants were then assessed the following year. Additionally, reliability of equivalence was also tackled by using the same tests with all the students as much as possible, I carried out the tests at the same time of the day, during the morning, so that the children were not restless. Internal reliability was secured by administering the tests myself, in the school settings of the children and in the same order and participants were asked to cover their work as well as to separate desks where this was possible to ensure that they did not copy from each other. All these minimized possible variables that could have lowered the degree of reliability of this first part of the study.

With regard to the qualitative nature of Phase 2 of the project, Maxwell (1992) and other researchers such as Lincoln and Guba (1985) argue in favour of looking into the authenticity of a qualitative study rather than its validity and/or reliability. In fact, the latter authors state that qualitative research can also be examined if its *trustworthiness* is examined. This includes looking into four main issues: *credibility, transferability, dependability* and *confirmability*.

Credibility deals with answering whether the findings of the study are congruent with reality (Merriam, 1998) and is usually used instead of the term *internal validity* used in positivism. Guba and Lincoln (1994) argue that credibility is the most important factor to secure the trustworthiness of a study. In this research credibility was facilitated by:

- Taking notes of my observations and thoughts in the form of an investigative learning journey;
- Audio recorded the intervention sessions and the interviews with the participants' parent/s and teachers.
- Taking photos of different activities that took place, so that I could later provide extensive descriptions about the phenomenon under study;
- Transcribing parts of the recordings to understand and examine better the situation at hand;
- Collecting and analyzing the data with honesty and thus as truthfully as possible;
- Adopting commonly used qualitative research methods and sharing the data collected with supervisors and colleagues through various conferences to scrutinize the project;
- Choosing to carry out the study at the school where I taught to ensure familiarity with the context;
- Selecting the participants of Phase 2 of the study by making use of a triangulation of assessments so that my own bias could not influence the process of sample selection;
- Using *methodological triangulation* in Phase 2 of the study.

Methodological triangulation also helped to safeguard the study's confirmability. As advocated by interpretivism, in qualitative research it may be difficult to ensure real objectivity (confirmability). Hence, it is fundamental that methods such as triangulation are used to ensure that the study's findings are truly the result of the informant's experiences rather than the researcher's ideas. Since the use of multiple methods "contrasts with the ubiquitous but generally more vulnerable single-method approach that characterizes so much of research in the social sciences" (Cohen et al., 2007, p. 141) it seemed best to use this method to add rigour to the data collected whilst adding to its validity and reliability. Multiple perspectives of the learners and their difficulties with mathematics were gathered through the one-to-one sessions, interviews and observations. Moreover, the progress, if any, which they were making through the support of the intervention programme could also be recorded through the various ways.

Confirmability was maintained by clearly stating my conceptual framework, beliefs and assumptions throughout this write-up. It was secured by stating the parameters of this study and how these could impinge on the process of the study. Moreover, an audit trail was created for the whole process of the data collection stage. An audit trail "allows any observer to trace the course of the research step-by-step via the decisions made and procedures described" (Shenton, 2004, p.72). Table 4.6 provides a detailed account of all the steps undertaken together with a timeline and other relevant details which usually characterize 'theoretical' audit trails (Shenton).

Table 4.6: Detailed Outline of the Research Design in the form of an Audit Trail.

November 2012

Trial phase using Chinn's (2012) assessment, Basic Number Screening Test (Gillham & Hesse, 2001) and Progress in Mathematics (Clausen-May et al., 2009). The trial phase was carried out with 10 male students in Grade 5. This phase was carried out to choose two out of the three research instruments identified as most appropriate.

↓

January 2013

Pilot Study using Chinn's (2012) assessment and the Basic Number Screening Test (Gillham & Hesse, 2001). Pilot study carried out with 10 students male studens in Grade 5. Pilot study was carried out to re-confirm choice of research instruments, to pilot the sequence of how the tests would be administered and to pilot the translation of the test's (BNST) to Maltese.

↓

February 2013

Administered Chinn's (2012) assessment, the Basic Number Screening Test (Gillham & Hesse, 2001) and the Single word reading test (Foster, 2007) in all Church schools for boys to find local norms for these assessments. Assessments were carried out with half the cohort of Grade 5 pupils in Grade 5 at each school.

↓

March to August 2013

Analysed data collected from the standardised tests to find norms for pupils in Church schools for boys at Grade 5 level. These norms would be used to identify the main subjects of the research project.

↓

October/November 2013

Administered tests to Grade 5 cohort at the school where I taught and where the main study would be carried out. Tests were corrected and six children were identified who according to the norms found previously had MLD. The pupils' scores were checked to see whether the children would be in the MLD only group or the group for MLD and RD.

↓

November/December 2013

The six children identified for the main study were assessed for IQ using British Ability Scales II (BAS II, Elliott et al., 1996) and a checklist for dyspraxia. Following these assessments, it was decided that one of the children was not suitable for the main study because he had a high IQ and therefore his difficulties with mathematics were mostly associated to his socio-economic background. The child was still offered intervention but not as part of the study. Another student was identified to replace him.

↓

January 2014

The main participants were assessed using the Dyscalculia Screener (Butterworth, 2003). They were also assessed using local Maltese and English comprehension tests (Firman et al., 2010) to confirm whether the pupils also had RD or otherwise. They were also assessed using Catch up's (2009) formative assessment. To identify the children's learning styles towards mathematics, Chinn's questionnaire for learning styles was carried out. Moreover a questionnaire for mathematics anxiety was carried out (Chinn, 2012) to determine each child's anxiety level towards mathematics.

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February to May 2014

The Intervention programme was carried out with the six main participants based on their needs and learning style. The programme followed the Catch Up Numeracy framework. Sessions were held on a one-to-one basis and were tape recorded.

February 2014

Interviews were held with the class teachers about each of the main participants and their mathematics learning. This was done to gain insight of the child's behaviour and interaction in mathematics lessons within the classroom context and to learn more about each individual learner.

March 2014

Interviews were held with each of the parent/s of the six participants of the main study. Interviews were used to collect information from the parents mainly with regard to their children's development of numerical skills.

February to May 2014

During this period classroom observations were made in each of the Grade 5 classrooms to see the main participants working within the classroom setting**.** This allowed me to see the pupils during the daily mathematics lessons and thus in their usual learning environment and not on a one-to-one basis.

May/June 2014

The formative assessment from Catch Up Numeracy and the standardised tests for numeracy were carried out again to identify any improvements in the areas intervened upon and to identify whether the pupils showed any improvement in their number age. An informal conversation was also carried out with each of the participants to establish what they liked best about the programme and areas which they would have changed.

June 2014

All the cohort of Grade 5 were re-assessed to identify whether their score had improved so as to compare this to that of the main participants of the study. This was done to be able to compare the improvement in score of the children following intervention with that of the rest of the pupils in class who did not.

It was problematic to maintain a high degree of transferability and dependability in this research. Transferability refers to the study's applicability to other situations (Merriam, 1998). Dependability implies that if the study had to be repeated using the exact same context and methods, findings would be similar. Since my main concern was that of understanding how each of the main participants was effectively supported to internalise the specific numeracy components, I made use of multiple-case intervention studies. As a result, since each social situation is unique and cannot be replicated, it is problematic to ensure transferability and dependability in any qualitative research. However, to ensure transferability is maintained, it is the researcher's responsibility to provide enough information about the context of the specific fieldwork sites to allow other researchers to develop a full understanding of it. This is necessary to be able to compare "the instances of the phenomenon described in the research report with those that they have seen emerge in their situations" (Shenton, 2004, p. 70). I have sought to provide thick descriptions which would permit other researchers to make such comparisons. On the other hand, for dependability to be ensured, Guba and Lincoln (1994) argue that a high level of credibility is maintained. Moreover, they state that the research report should include sections which provide extensive details about the research design, implementation, data gathering and an appraisal of the effectiveness of the process of study undertaken. This has been taken into account in this research report.

4.11 Overview of the Analysis Process

Following the collection of both the quantitative and qualitative data, it was crucial to analyse each type of data through valid and reliable means. The methods employed in analysing the quantitative data were different to those which were used to analyse the qualitative data. Details of how the quantitative and qualitative data was analysed will be given in Section 5.3 and Section 6.8 respectively. However, in this section I provide an overview of the analysis phase.

In Phase 1 of the study, after administering the tests to half the cohort of boys attending Church schools for boys (352 in total), all the scripts were corrected by myself. All raw scores were coded and entered on Microsoft Excel (2016). All scores were then transferred to SPSS 23 so that this data could be analysed using this software. A z-score (standardised score) was computed for every raw score. These z-scores were saved as variables and used to find norms that would be used for the identification of the pupils for the Main Phase of the study.

As outlined in Section 2.4.1, in this study, the term Mathematics Learning Difficulties (MLD) refers to learners who fall below a cut-off point of the 30th percentile. Since various studies, for example that carried out by Geary, Hoard, & Hamson, 2001), and several numeracy standardised tests (such as the Basic Number Screening Test (BNST) (Gillham & Hesse, 2001) suggest this cut-off point to identify children with MLD, this cut-off point was deemed most appropriate. Following this decision of using the 30th percentile as a cut-off point to identify children with MLD and both MLD and RD, the 30th percentile was extrapolated using SPSS 23. The raw scores for each assessment which fell within this percentile were highlighted to be used in identifying the participants of Phase 2 of the study. Since I wanted to know the raw scores that fell within the 20th and 10th percentile so that I could understand better the severity of the main participants' mathematics learning difficulties, these were extrapolated as well.

On the other hand, the qualitative data collected mainly from: the audio recordings of the intervention sessions, as well as the interview and classroom observations were analysed using a method that is more appropriate for this type of data. As highlighted by Maguire and Delahunt (2017), "data analysis is central to credible qualitative research" (p.1). Analysing qualitative data is probably more complex than analysing data that is of a quantitative nature. As a result, Thorne (2000) argues that in the case of qualitative research, data analysis is the most complex phase. Thus, it was important to search for a data analysis method that would be trustworthy and rigorous. Thematic Analysis (Braun & Clarke, 2006) was deemed suitable to provide a structure that would ensure that the data was analysed in a valid way whilst providing insightful findings. Thematic analysis is the process of identifying themes within the qualitative data collected (Braun & Clarke). As Braun & Clarke themselves suggest, "thematic analysis is a method for identifying, analysing, and reporting patterns (themes) within data" (p. 6). This method of analysing qualitative data has become "increasingly recognised and valued" (Nowell, Norris, White, & Moules, 2017). Thematic analysis has been deemed by some (Holloway & Todres, 2003; Ryan & Bernard, 2000), as an essential tool to assist in the analysis of qualitative data. Others (Braun & Clarke, 2006; Nowell et al., 2017) consider it is a separate research method "that can be widely used across a range of epistemologies and research questions" (Nowell et al., p.2).

Braun & Clarke (2006) provide a six-step framework which can serve as a structure for thematic analysis. This framework has been given much importance in recent years. Consequently, Maguire and Delahunt (2017) suggest that the framework put forward by Braun &

Clarke (2006), "is arguably the most influential approach, in the social sciences at least, because it offers such a clear and useable framework" (p. 3). The six-step framework involves the following steps:

i. Become familiar with the data;
ii. Generate initial codes;
iii. Search for themes;
iv. Review themes;
v. Define themes; and
vi. Write-up (Braun & Clarke, 2006).

When analysing my own data, these steps were followed. First, I analysed the interviews and classroom observations by transcribing them and identifying the pre-determined themes related to the learners' mathematical profiles including their home and class experiences of mathematics. I also transcribed all the conversations that took place during the intervention sessions. I listened to the recordings several times to ensure that I became so familiar with the data that anything which could contribute to providing more insightful findings was highlighted. Second, initial codes were attributed to the transcribed data. The initial codes were primarily related to the pre-determined themes identified and presented in Section 3.6. These included Tharp's (1993) seven means of supporting scaffolding: Modelling, Feedback, Contingency Management, Instructing, Questioning, Cognitive Structuring and Task Structuring and the use of both psychological and technical tools. However, as I analysed the data, other themes emerged, and some patterns became very evident. These allowed me to find other common threads between the data collected to find richer answers to the research questions of this study. After reflection on the initial codes, I reviewed the themes. Reviewing the themes was an ongoing process as throughout the process of transcribing and coding the data, some themes remained apparent, others were noted, whilst some themes lost their importance as they did not emerge in other intervention sessions, interviews or observations. The two final steps, as per Braun & Clarke's (2006) six-step framework, were to define the themes and write (report) them. I defined the themes by categorising them and seeking to find ways in which they are related, in order to be able to link one theme to another. Finally, the process culminated itself in the writing and reporting of the themes that will be presented in Chapter 6 and Chapter 7 of this thesis. In these final Chapters all the themes, both pre-determined and emerging, will be discussed, and a model of how they are related will be proposed.

The main advantage of thematic analysis is that it is a flexible approach that can be applied to different qualitative studies. More importantly, although flexible, it still acts as a data analysis tool, which allows the researcher to analyse the data in a detailed way ensuring that the account of data is a rich and rigorous one. As argued by Braun and Clarke (2006) and King (2004), thematic analysis can be used for:

- Looking at the perspectives of the individual participants of the study;
- Identifying similarities and differences between the different views; and
- Generating unanticipated insights.

The flexibility provided using thematic analysis may lead to inconsistencies when themes are developed using the data collected. However, as suggested by Holloway and Todres (2003), consistency and cohesion can be maintained if an explicit epistemological position is applied to the whole data collection and analysis process to coherently underpin the research's empirical claims. This has been done in this study: Vygotsky's theories as well as those of neo-Vygotskian researchers, have been woven into the whole process of data collection and analysis. These theories will be a major influence on the way the themes identified during the thematic analysis process come together at the end of this thesis.

4.12 Conclusion

Chapter 4 has outlined how the research aims, questions and choice of paradigm steered my choice of research methods. A detailed account of how access and consent to the research fields were obtained and the ethical considerations taken into account. Chapter 4 has provided a detailed explanation of the sample selection process and the tests/assessments used to identify the main participants of the study with scrutiny to provide rich data about the phenomenon being studied. Chapter 4 has also provided information about the intervention programme itself, *Catch Up® Numeracy*, and has presented both its different phases and the parts that make up each intervention session. Moreover, it has explained the nature of the interviews carried out with the parents of the main participants and the classroom observations carried out.

The next Chapter will provide an analysis of the data obtained from the quantitative phase of the study, namely the trial phase, pilot study and the exercise of collecting norms of the chosen tests. This will be followed by a further two Chapters that will focus on the qualitative phase which is the focal point of this research project. The first part of the qualitative analysis Chapter will provide a detailed profile of each of the six main participants, together with the observed cognitive affective development, if any, for each learner. Furthermore, it will present themes which emerged from the observations, interviews and intervention sessions that provide insight into effective strategies which can help learners with MLD or MLD and RD overcome at least part of their barrier to the learning of mathematics.

Chapter 5
An Analysis of the Norm Collection Process

Chapter 5: An Analysis of the Norm Collection Process

It was essential to establish local norms for numeracy tests to ensure the validity of the identification of the main participants of the study and of the evaluation of their progress through the intervention programme. Choosing the right tests that would serve to assess children with either MLD or both MLD and RD required the development both of a trial phase and a pilot test, and collecting norms for these. In this Chapter, I describe the various steps carried out to ensure that the norms finally collected for all the numeracy tests and the reading assessment were reliable and valid. The first part of the chapter gives an account of how the numeracy assessments were chosen, based on the conclusions drawn from the trial phase and pilot study. It also reports all the scores obtained during these phases of the numeracy assessments and reports an analysis of these scores. The second part of the chapter explains how the results obtained when the tests were administered to the larger sample were then used to identify the participants for the Main study.

5.1 Analysing the data acquired from the Trial Phase

The trial phase was carried out with 10 pupils who were in Grade 5 (9 – 10 years old). The cohort was made up of 10 boys of mixed ability. Three tests were administered: the Basic Number Screening Test (BNST) (Gillham & Hesse, 2001), Chinn's (2012) assessment and Progress in Mathematics (PIM) (Clausen-May et al., 2009) (see Section 4.5.1). In the subsequent sections I present the analysis of the trial phase and the scores obtained in the various tests by the participants. I also outline how this phase led to the next phase: collecting the norms.

5.1.1 Scores obtained from the Basic Number Screening Test (BNST) (Gillham & Hesse, 2001)

As outlined in Section 4.5.1, the main advantages of the BNST test are that it can be completed in a short period of time and that it does not include any written instructions, therefore removing the influence of language skills as much as possible. The test was also deemed suitable for the collection of norms as questions are graded and it is possible to extrapolate a standardised score, number age, percentile and quotient from the raw score. Table 5.1 gives the age of the pupils, raw score, number age, quotient and percentile mark retrieved from the data collected through the BNST.

Table 5.1: Results from the Basic Number Screening Test (Gillham & Hesse, 2001).

	Age of Pupils	**Raw Score**	**Number Age**	**Quotient**	**Percentile**
P1	9 yrs 3 mts	6	7 yrs 3 mts	75	5th
P2	9 yrs 6 mts	17	9 yrs 9 mts	96	40th
P3	9 yrs 3 mts	20	10 yrs 3mts	113	80th
P4	9 yrs 7 mts	24	11 yrs 3 mts	119+	90th+
P5	9 yrs 6 mts	27	12 yrs 0 mts	119+	90th+
P6	9 yrs 4 mts	22	10 yrs 9 mts	119+	90th+
P7	9 yrs 5 mts	23	11 yrs 0 mts	119+	90th+
P8	9 yrs 7 mts	19	10 yrs 0 mts	104	60th
P9	9 yrs 8 mts	22	10 yrs 9 mts	108	70th
P10	10 yrs 3 mts	13	8 yrs 9 mts	87	20th

The administration of this test ran smoothly. Most of the children understood the few instructions given and were encouraged to ask me to translate the instructions into Maltese when they did not understand. Very little translation had to be done. One of the translations a child asked for was of a situation in which they were asked to shade in *one quarter* of a chocolate bar (drawn on their test paper). The question said,

> *"Next you have the drawing of a bar of chocoloate. Suppose your mother says that you can break off a quarter of it to eat it. Shade in a quarter of the chocolate bar, to show what you've eaten."*

This was one of the questions in which the instructions involved most wording.

I will now analyse some of the children's scores individually as part of validating this test. P1 was the child who found most difficulty with the test as a whole; he scored at the 5th percentile. He had difficulty in understanding some of the instructions and a lot of difficulties computing basic sums of the type 'x+y=?'. His class teacher had identified him as the weakest of the 10 children with whom the test was carried out and the test indicated that P1 was struggling with rather severe mathematics learning difficulties (MLD). Pupils P1 and P10 both exhibited mathematics learning difficulties as their raw scores were at the 5th and 20th percentile respectively (see Section 2.4.1 for an explanation of how cut-off points were identified). According to the test's manual, these children have special needs in the learning of mathematics. On the other hand, four pupils scored very highly even though two of them were considered to be just average achievers. This was a preliminary indicator that the local percentile scores and mean scores found for boys in Grade 5 attending Church schools mismatched those in the UK.

Most of the children got the first few sums correct and then gradually started to lose marks as the test questions became harder. This indicated that the test items were graded effectively. This analysis of data led me to conclude that the test was valid, as most of the children scored according to their ability as perceived by the class teacher.

5.1.2 Analysis of Chinn's (2012) Test Scores

Chinn's test (2012) is made up of a variety of items and can be administered either as a group test or individually. The main two components of this assessment are a 15-minute test made up solely of computations (therefore no verbal instructions are involved) and four sheets addressing addition, subtraction, mutiplication and division. Further details about this test, including its standardisation, can be found in Section 4.5.1. The 15-minute test and the four sheets testing fluency in working out computations with the four operations (+, -, x and ÷) are a good indication of whether a child has MLD. Another two components are also provided and should be carried out on a one-to-one basis. These involve a questionnaire for mathematics anxiety and another one for learning styles. These questionaires can be used to get to know more about the learner once they are identified as having MLD using the aforementioned tests. Chinn's (2012) test serves to assess learners for:

1. an inability to retrieve basic number facts quickly;
2. an inability to complete simple number computations; and
3. an inability to complete a sequence of number by identifying and following a pattern.

These are all areas in which learners with MLD usually experience difficulties.

The two test components of Chinn's (2012) assessment were administered to five of the children who had already done the BNST (P1 – P5). These children were selected according to their mathematical ability so that low achievers, average achievers and high achievers were part of the sample. I also kept in mind the fact that P1 was having difficulties understanding the instructions given during the BNST. Since no verbal instructions are involved in Chinn's (2012) test I asked him to take this latter test for two reasons. First I wanted to know whether he would score badly in Chinn's (2012) test too, which would indicate that he did indeed have MLD. Secondly, I considered that giving him the PIM, in which instructions may be too complex for him, would increase the variable of possible language difficulties. Pupil P2 was also given Chinn's (2012) test, rather than the PIM test. P2 had a short attention span and, since PIM is a 55-minute test, he would have been likely to lose concentration well before completing it, possibly influencing his performance on the test. Moreover, P2 also had reading difficulties so Chinn's (2012) test seemed more appropriate for him. The relatively short duration of Chinn's (2012) test may have increased its reliability and validity. This also made it more practical to use when testing large sample or pupils. The test results are found in Table 5.2.

Table 5.2: Scores of Chinn's (2012) Test.

	Results of 15-minute test Average Score for the Age group given by Chinn (2012): 17.2		**Results for Addition, Subtraction, Multiplication and Division Sheets**							
	Raw Score	**Percentile**	**Addition Score Percentile**		**Subtraction Score Percentile**		**Multiplication Score Percentile**		**Division Score Percentile**	
P1	5	Below 5th	12	10th	11	10th	10	5th	5	10th
P2	9	10th 5th	22	Average	18	Average	15	10th	22	Above Avg,
P3	13	25th	21	Average	21	Average	13	10th	13	Avg. – 25th
P4	22	80th-75th	11	10th-5th	12	25th-10th	14	5th	22	Abov Avg.
P5	16	50th-40th	36	Above Average	28	Above Average	22	Avg.-25th	26	Above Avg.

In agreement with the scores from the BNST, P1 scored below the 5th percentile, indicating that he had MLD. P2, who in the BNST scored 2nd best from the group of pupils P1 – P5, also scored second best in Chinn's assessment. Therefore the test results did complement one other. As for P3 and P5, it seemed surprising that they had done so well in the BNST but significantly less well in Chinn's (2012) test. Possible reasons for this may be that the latter test includes questions of a greater level of difficulty and that speed is a feature of Chinn's (2012) test since all parts of the tests are timed. Chinn's (2012) test did however, succeed at:

1. providing a test that reduced the role of verbal instructions.;
2. identifying P1 as a student with MLD and P2 as a student with low achievement in mathematics, who might also be considered as having mild to moderate MLD.

5.2.3: Analysis of Progress in Mathematics 9 (PIM) (Clausen-May et al., 2009) scores

The expected total time to be taken for the PIM is 55 minutes. I read out all the questions to the children in order to reduce the potential difficulty of verbal instructions,encouraged them to ask me to translate instructions into Maltese if need be. Pupils P6 to P10 were given this test. When choosing the pupils I kept in mind the important of including learners with diverse mathematical learning profiles, in order to have an adequate range of ability to be able to analyze the test results adequately. I also chose to include only children with a reasonable command of the English language, to ensure that they would be able to understand the instructions and questions. Table 5.3 gives the scores obtained.

Table 5.3: Scores from Progress in Mathematics 9 (Clausen-May et al., 2009).

Pupil	Raw Score	Standardised Score	Stanine	Comment as indicated by test
P6	36	113	7	Above Average
P7	29	101	5	Average
P8	33	106	6	Average
P9	20	87	3	Below Average
P10	7	69	1	Very Low

The scores on this test were again similar to those obtained in the BNST. Again, P10 was identified as having MLD and P6, P7 and P8 were identified as being above average and/or average students. P9 was the only child who scored in the 'average' range in the BNST but in the 'below average' range for this test. Possible reasons for this could be the fact that he did not understand some of the instructions or that this test targets mathematical concepts rather than simply numerical ones and includes some data handling, shapes and measure.

5.2.4: Conclusions drawn from the findings of the Trial Phase

After examining the results, I concluded that all three tests were appropriate for identifying MLD as the scores converged in indicating that P1 and P10 had severe MLD whilst P2 had mild to moderate MLD. Moroever, both the teachers' perceptions and scores on the tests were similar. However I anticipated two main difficulties with using PIM with the main sample. First, it contains written instructions in English, which would affect performance by pupils with English limitations or reading difficulties. Reading the questions out myself would obviously help, but there was also the problem that the first language of most of the sample was Maltese and therefore a translation of this text might become necessary. The other main difficulty was the fact that the test takes 55 minutes to complete, which could create problems for children with a short attention span. When evaluating Chinn's (2012) test, I concluded that it would be impractical to give all the parts of the test to the whole sample. However, it appeared appropriate to give all the children the components of the test during the trial phase (the 15-minute and four operations components) and then to give the remaining components of the test with those children identified as having MLD only.

After the analysis of these data, I concluded that it would be most appropriate to standardise the BNST and Chinn's (2012) test in local Church schools for boys and to use these as my main tool for identifying the children with MLD. PIM could also be used, if needed, as an additional test for the children identified with MLD together with the rest of the sections for Chinn's (2012) test. Moreover I decided that the instructions to the BNST should be translated intoto Maltese so that language limitations did not affect the results.

5.2 The Piloting Phase

Following the conclusions drawn from the trial phase, I began by translating the instructions of the BNST. I first translated the instructions and had these checked by a my principal supervisor, Dr Marie Therese Farrugia. Some minor changes were made. After this I asked a professional translator to do a back translation of the instructions from Maltese to English, so that I could evaluate the accuracy of the translation into Maltese. Following the back translation, some changes were made. For example, instances throughout the script in which the phrase *'Ħarsu sewwa lejn dak li għandkom tagħmlu u iktbu t-tweġiba' ['Look carefully at what you have to do and write the answer']* had been used were changed to the following *'Ħarsu sewwa lejn is-sinjali li jurukom dak li għandkom taħdmu u iktbu t-tweġiba' ['Look carefully at the sign that is showing you what you need to work out and write the answer'].* Since the original instruction in the manual said *'Look carefully at the sign, work out the answer and write it down'*, the second version of the instructions made them more faithful to the original English text. The final set of instructions used in Maltese can be found in Appendix R. Once changes were made, the final instructions were again reviewed by the professional translator and my supervisor. Both confirmed that the translations was now as accurate as possible.

When the translation process was complete, I conducted a pilot study in which I administered the BNST with both sets of instructions (English and Maltese) and Chinn's (2012) assessment. The pilot study was carried out with an additional 15 pupils in Grade 5 at the same school in which the trial phase took place. This phase was deemed necessary to re-evaluate the accuracy of the instructions given in Maltese for the BNST and to ensure that nothing else needed to be changed before the tests were administered to the larger cohort of Grade 5 pupils as part of the norm collection phase.

During the pilot none of the pupils asked for clarifications and I concluded that the pupils seemed to have no difficulty with understanding the instructions in either English or Maltese. I did observe that some pupils who had not understood the English version of the instructions waited for the Maltese version, and then managed to complete the task. This indicated that the translation of the instructions to Maltese served its purpose of ensuring that the instructions were made accessible to all and therefore reducing the language variable. The accuracy of the translation ensured that the final norms obtained would have a high degree of validity and reliabilty. The results obtained by the participants of the pilot study are illustrated in Table 5.4.

Table 5.4: Scores obtained in the Pilot study by the 15 participants.

Pupil Code	BNST Score (/30)	Chinn's 15-min. Assessment Scores (/40)	Chinn's Addition (/36), Subtraction (/36), Multiplication (/36) and Division Scores (/33)
P1	27	23	Addition 25 Subtraction 24 Multiplication 32 Division 27
P2	19	12	Addition 20 Subtraction 15 Multiplication 19 Division 18
P3	22	20	Addition 23 Subtraction 29 Multiplication 31 Division 22
P4	22	19	Addition 21 Subtraction 20 Multiplication 22 Division 23
P5	25	22	Addition 24 Subtraction 29 Multiplication 32 Division 30
P6	26	21	Addition 23 Subtraction 27 Multiplication 25 Division 11
P7	23	21	Addition 26 Subtraction 24 Multiplication 25 Division 16
P8	21	12	Addition17 Subtraction17 Multiplication 21 Division18
P9	18	12	Addition 10 Subtraction 20 Multiplication 25 Division 25
P10	25	24	Addition 26 Subtraction 33 Multiplication 26 Division 31

P11	24	23	Addition 31 Subtraction 33 Multiplication 31 Division 33
P12	25	21	Addition 24 Subtraction 26 Multiplication 32 Division 31
P13	17	14	Addition 17 Subtraction 16 Multiplication 22 Division 17
P14	20	17	Addition 19 Subtraction 21 Multiplication 22 Division 14
P15	21	18	Addition 19 Subtraction19 Multiplication 16 Division 24

The scores obtained by the different children were generally internally consistent as they were similar for the two tests. This indicated that both tests could be used to complement one another and that the choice to use a triangulation of instruments would be effective in safeguarding the validity and reliability of the norms collected.

The thorough analysis of the scores allowed me to identify those pupils whose scores were similar on both tests, as well as any participants who seemed to obtain scores that indicated a contradiction to their teacher-rated mathematical achievements. Pupils P1, P10, P11 and P12 obtained a score that was above average in all tests showing that the learners were not struggling with mathematics. On the other hand, P2, P13 and P14 obtained low scores on both tests, indicating the pupils did have difficulties in mathematics. I scrutinized the scores of those pupils who performed differently in the two measures. For example, P3 obtained a rather high score for all the simple addition, subtraction, multiplication and division computations from Chinn's (2012) test. However, he did not score well on either Chinn's (2012) 15-minute test or the BNST, both of which contain more complex mathematical tasks. I thus decided to speak to the pupil's teacher to determine whether P3 was struggling with mathematics. The class teacher confirmed that he was not achieving as expected in the subject. Therefore, since this difficulty had been identified by the

BNST as well as the main part of Chinn's test (2012) it was decided that the results were reliable and valid.

One main observation made during the piloting phase that could have impinged on the consistency of the scores obtained in the various tests, was that the children enjoyed completing Chinn's (2012) assessment more than the BNST. During the piloting phase Chinn's (2012) assessment was administered first and therefore some learners became restless by the time they completed the BNST. The participants seemed to be more motivated to complete the former assessment because Chinn's (2012) assessment is timed and therefore they took it up as a challenge to complete as many of the tasks as possible in the given time frame. This reflection led to a further change to the test administration procedure since it was deemed beneficial to change the sequence of the tests. Therefore, when norms were collected with the larger sample, the BNST was administered before Chinn's (2012) assessment. This was done so that the children's motivation was maintained throughout all tests and to avoid the situation in which some learners would become demotivated by the tasks and give up or start guessing the answers to the tasks out of boredom.

5.3 The Establishment of Norms

Since as explained in Section 4.5.1, selecting a reading assessment was rather straightforward, it was not deemed necessary to pilot the reading assessment. Hence, the collection of scores to establish norms began soon after the numeracy tests to be used had been confirmed. The chosen numeracy tests were administered to half the cohort of Grade 5 boys attending Church schools for boys – 352 boys. The Single Word Reading Test (SWRT) (Foster, 2007) was also administered alongside the numeracy assessments. All scripts were given a code so that the school name could be traced in case there were any clarifications needed when the tests were corrected. The raw scores were primarily entered into an excel sheet. Scores were first classified according to which school they had been collected from. This was done so that only half the scores collected from schools that had three Grade 5 classes would be taken at random. Scores were also classified according to the schools so that records could be kept about their provenance.

All the scripts were scored by me. The raw scores were entered on SPSS 23 and a z-score (standardised score) was computed for every raw score. These z-scores were saved as variables and used to find norms that would be used for the identification of the pupils for the Main Phase of the study. The quotient for the scores was first calculated through Microsoft Excel (2016) by using the formula 'z-score * 15 +100'. On transformation, the data satisfied the requirements of an interval scale. In accordance with various studies (Geary, Hoard, & Hamson, 2001; Passolunghi & Siegel, 2001; Passolunghi & Siegel, 2004), and the recommendations of the standardised reading and numeracy tests used, I selected the cut-off point of 30th percentile to identify children with MLD and both MLD and RD. Moreover, when scores were below the 30th percentile, it was also useful to idenfity how low they were. As a result, scores for the 20th and 10th percentiles were also used to gain a better understanding of the severity of the difficulties. The 30th, 20th and 10th percentile scores extrapolated following the analysis of data are represented in Table 5.5.

Table 5.5: Scores extrapolated for the 30th, 20th and 10th percentile following the collection of data.

Assessment	30th Percentile Score	20th Percentile Score	10th Percentile Score
Chinn's Addition	18 and below	17 and below	14 and below
Chinn's Subtraction	16 and below	14 and below	12 and below
Chinn's Multiplication	20 and below	18 and below	15 and below
Chinn's Division	16 and below	13 and below	9 and below
Chinn's 15-minute assessment	15 and below	14 and below	11 and below
Basic Number Screening Test	22 and below	21 and below	19 and below
Single Word Reading Test	38 and below	35 and below	32 and below

When norms were established for Grade 5 boys attending Church schools, these were compared to the norms extrapolated from the UK score for all tests. The scores obtained locally and in the UK are compared in Table 5.6.

Table 5.6: A comparison of the 30th percentile local score to that of the UK.

Assessment	30th Percentile Local Score	30th Percentile UK Score
Chinn's Addition	18 and below	18 and below
Chinn's Subtraction	16 and below	16 and below
Chinn's Multiplication	20 and below	19 and below
Chinn's Division	16 and below	12 and below
Chinn's 15-minute assessment	15 and below	13 and below
Basic Number Screening Test	22 and below	14 and below
Single Word Screening Test	38 and below	No percentile scores available for this test.

Some observations were carried out when comparing the local norms with those found in the UK. These are listed hereunder:

- The scores for Chinn's assessment were very similar. The UK and local norms for both the Addition and Subtraction components were identical;
- The local norms found for this cohort of learners for the division and multiplication components were slightly higher than those found in the UK;
- The local norms found for the BNST were much higher than the UK norms. This finding correlated with the findings from both the trial phase and pilot study in which most learners did much better in this assessment than in Chinn's (2012) 15-minute assessment. The outcome seems to indicate that the local population that participated in this collection of norms performed generally better in the mathematics components assessed by this test than the UK population. However, one must use caution in drawing conclusions, since the sample included only Church school boys and different results might have been obtained for Maltese girls and/or children attending other types of schools.
- It was interesting to note the inconsistency between the scores obtained for the BNST and Chinn's assessment (2012). Although both assessments seem to test similar mathematical skills and concepts, most of the learners performed better in the former than the latter. This discrepancy is also evident in the scores that were collected in the UK for both tests.

5.4 Using norms in the identification of the main participants

Once norms were established, the assessments were administered to the cohort of 50 pupils at the school where I taught at the start of the scholastic year when the intervention programme would be carried out. All scripts were corrected, and the pupils' raw scores and percentiles were analysed and compared to the established norms. Based on these norms six pupils were identified for further study and intervention: three with MLD only and three with both MLD and RD. However, following the assessment carried out by the educational psychologist using the British Ability Scales II (BAS II, Elliott et al., 1996), one of the pupils scored an above average IQ, that of 115 points. Looking into this specific case, it was discovered that the pupil had many social difficulties and I conjectured that the child was underperforming in mathematics due to these. Following discussions with my supervisors, it was decided best to exclude this pupil to render the group of six pupils as homogeneous as possible. Another boy, who also had similar scores to the other five children, replaced the pupil. The final six pupils who participated in the intervention programme were thus identified. These were: Ethan, Seb, Thomaz, Nathan, Andrea and Mike. These names are pseudonyms which will be used throughout the rest of this thesis. Their percentile scores in the numeracy and reading assessments are found in Table 5.7.

Table 5.7: Percentile scores (based on the local norms collected) in the numeracy and reading assessments of the 6 pupils chosen for the intervention programme.

	Percentile Score BNST	**Percentile Scores in Chinn's 15-min Assessment**	**Percentile Scores in Chinn's Addition Task**	**Percentile Scores in Chinn's Subtraction Task**	**Percentile Scores in Chinn's Multiplication Task**	**Percentile Scores in Chinn's Division Task**	**Percentile Scores for Single Word Reading Test Score**	**Type of Learner**
Ethan	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	MLD + RD
Seb	Below 10^{th} percentile	Below 10^{th} percentile	10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 30^{th} percentile	MLD + RD
Thomaz	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	MLD + RD
Nathan	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	Above 30^{th} percentile	MLD Only
Andrea	Below 10^{th} percentile	Below 10^{th} percentile	20^{th} percentile	Below 10^{th} percentile	Below 10^{th} percentile	30^{th} percentile	Above 30^{th} percentile	MLD Only
Mike	10^{th} percentile	Below 10^{th} percentile	20^{th} percentile	30^{th} percentile	Below 10^{th} percentile	20^{th} percentile	Above 30^{th} percentile	MLD Only

After identifying the six participants for the intervention case studies, other assessments were carried out with the children to ensure that they were appropriately selected to participate in the intervention programme. The children were separated in two groups – learners having only MLD and others having MLD & RD. The results of the tests for each learner are presented in Table 5.8. The table also illustrates the group to which each learner belonged at the bottom of it.

Table 5.8: Scores obtained by the six participants of the intervention programme.

Name of Pupil	**Ethan**	**Seb**	**Thomaz**	**Nathan**	**Andrea**	**Mike**
Reading Assessments						
Single Word Reading Test (/60)	27	57	12	40	40	40
Suffolk Reading Scale 2 (/75)	50	57	29	51	50	43
Naqra u Nifhem (/62)	40	27	9	49	38	60
Suffolk Reading Scale List (/75)	38	38	33	61	57	59
Mathematics Assessments						
Chinn's Addition Task (/36)	10	14	7	12	17	17
Chinn's Subtraction Task (/36)	9	4	4	8	15	9
Chinn's Multiplication Task (/33)	11	0	1	9	9	12
Chinn's Division Task (/33)	7	0	0	4	13	18
Chinn's 15-minute test (/44)	8	2	2	8	7	9
Basic Number Screening Test (/30)	17	11	1	17	15	19
Aston Auditory Sequential Memory Assessment						
Digits Forward (/5)	3.5	5	5	4.5	4.5	5
Digits Reverse (/5)	2.5	3.5	1	2.5	3	3.5
IQ – British Ability Scales II (BAS II)						
Non-verbal Reasoning (percentile)	55th	63rd	27th	47th	53rd	53rd
Mathematics Anxiety						
Chinn's (2012) Mathematics Anxiety Scale (/80)	54	40	55	48	38	55
Group of Learners	**MLD & RD**	**MLD & RD**	**MLD & RD**	**MLD**	**MLD**	**MLD**

Chapter 6
What ‘works’ with children having MLD or both MLD and RD?
A Qualitative Analysis

Chapter 6: What 'works' with children having MLD or both MLD and RD? A Qualitative Analysis

6.1 Profiles – from interviews to intervention sessions: Ethan

In this Chapter I present the qualitative analysis of the data collected through the delivery of the intervention programme. I start by providing a profile of each of the six children who participated in the programme. First, I give the profiles of the learners with MLD and RD, then I provide those of the learners with only MLD. These profiles serve to outline all the information gathered both from the interviews with the children's parents and the classroom observations. The profiles also allow me to provide answers to my first subsidiary research question, 'What are the teachers' and parents' perspective on how the children's MLD affects their daily lives at school and at home?'

In the transcripts from the interviews, some phrases were originally in Maltese however these were translated, by myself, as faithfully as possible into English. These phrases have been marked in a **bold** font. The information I have chosen to present here is data that allowed me to understand each child as a learner of mathematics. This information was important so that I could address any issues that might have had an impact on the effectiveness of the sessions. It also gave me a better picture of the learners' mathematical abilities so that I would know the areas with which they were struggling. In the first part of this Chapter I will seek to show how this information was made use of when planning the intervention sessions. Following the profiles, I will then focus on the intervention programmes and discuss what appeared to be effective strategies that support children with either MLD alone or both MLD and RD. These strategies will be discussed in an attempt to provide answers to my main research question.

The first learner I will describe is Ethan. Ethan's mother explained to me that he had always had difficulties with mathematics and that his kindergarten teacher had already indicated this. His mother had in fact asked Ethan's kindergarten teacher to assess him for difficulties, but the pediatrician had told her Ethan was still too young. When asked about what sort of activities he struggled with, Ethan's mother told me that even counting several blocks was difficult for him. This indicates that he had difficulties with numerosity and number sense which is a typical characteristic of learners with MLD (Butterworth, 2005; Emerson & Babtie, 2010). As highlighted in Section 2.2, understanding the numerosity of a set is one of two of the main predictors of low achievement in mathematics (Geary, Hamson & Hoard, 2009), hence these early childhood difficulties may have been the first evidence for Ethan's future difficulties with mathematics. Ethan's mother recalled instances when he was young when she tried to engage him in mathematical activities, but he failed at completing the tasks. She said,

> **I used to tell him to get 2 onions, for example. Alright…he would go to the cupboard but then just go somewhere else without getting me the number of onions I would have asked for** (Ethan's Mother).

Ethan's mother also added "he never showed any interest towards mathematical activities". She explained that even learning nursery rhymes with numbers was a struggle.

Ethan's mother explained to me that by the time Ethan got to Year 4 (aged 8), his difficulties were very evident, especially with the start of examinations. She mentioned that Ethan was very visual and that if he actually saw something, he could memorise it more easily. When I asked whether Ethan suffered from mathematics anxiety, she said that he feared mathematics and that he was already worried about the upcoming mathematics examinations. This is very typical of children with MLD (Ashcraft & Kirk, 2001; Chinn, 2012) (Section 2.7.1). She mentioned that his self-esteem in the subject was very low and this is also a characteristic of children with MLD (Wilkins, 2000). Ethan's mother also highlighted other characteristics that are common amongst children with MLD (see Section 2.6), namely, that Ethan found great difficulty with:

i. Doing tasks related to money and time;
ii. Solving word problems;
iii. Finishing off his H.W. in a reasonable time;
iv. Understanding new mathematical topics – he seems to have a mechanical understanding of the topics learnt and forgot them after some time;
v. Estimating that an answer is unreasonable as sometimes he subtracted a large number from a smaller one without realizing he would not be able to work this out since they had not yet covered negative numbers;
vi. Sequencing – finds great difficulty with continuing a sequence even in other areas which are not related to mathematics such as when learning a sequence during Karate sessions; and
vii. Concentrating for a long period of time on mathematical task.

Ethan's teacher, Mr. Desira, highlighted the same difficulties that his mother had mentioned. He mentioned that Ethan did not participate much during lessons and only very rarely asked questions. Mr. Desira mentioned that Ethan got distracted very easily whilst doing schoolwork. He said,

> Ethan starts his work and gains a steady pace, but then I realise he has stopped, and I need to draw his attention because he would be completely lost (Mr. Desira).

Mr. Desira confirmed that Ethan had great difficulties with number work especially the four operations. He also confirmed that Ethan found work related to shapes, data and measure much easier. These two characteristics are very typical of children with MLD (Bird, 2009). Mr. Desira explained that Ethan's mother was very concerned because Ethan was constantly asking for help in class and his self-esteem was now becoming lower and lower since he was conscious that he was struggling in the subject much more than his peers. Regarding mathematics language, Mr. Desira explained that Ethan tried to use the correct language but still did not use this language to express himself correctly. He mentioned that Ethan was still struggling to acquire the mathematics language related to the more complex concepts such as multiplication and fractions.

As the intervention programme unfolded, I observed Ethan in class on three occasions (for a schedule of the observations, see Section 4.8). The transfer from MKO directed learning to independent self-completion of tasks seemed problematic in all lessons observed. This was mainly apparent when Ethan had to apply knowledge and skills related to the four operations e.g. applying division in a context of angles and applying multiplication to find area. Most times Ethan did not seem to have difficulty with the actual operation but with making the connection between the two concepts.

During the first lesson, the topic being covered was 'Area'. The second and third lessons covered multiplication strategies and angles respectively. I present here some of the main observations made. Ethan answered the teacher when he was asked about how many turns an object had made to the left. He did get the answer correct after the teacher went on to drawing a visual representation of the problem on the board indicating that this visual representation was of great support to the learner. Ethan was also able to explain why the clock hand turns 180° from 12 to 6. He said that there is one right angle from 12 to 3 and another from 3 to 6 so 180° in total – which he correctly explained as making 2 right angles. When the teacher asked him to explain why it was 180°, Ethan hesitated at first to reply. However, following the teacher's praise and encouragement he explained his reasoning. Ethan was able to use the mathematical terms 'right angles' and 'degrees' correctly.

The lesson then tackled dividing the degrees to find out how many degrees there are in every 5-minute turn. The teacher used a visual representation once again. Ethan was able to come to the front of the class and work out 90 ÷ 3. He did this with confidence. Although Ethan was able to do this and had seemed to understand, he was unable to do his schoolwork. One possible reason could have been that although he could answer the separate parts of the sum with the teacher's prompting, he found it difficult to answer a complete task without the teacher.

From the interviews and observations, I got to know Ethan better and I took note of things to keep in mind when designing and delivering the intervention sessions. These are listed hereunder:

- Ethan was a very visual learner and could memorise things more easily when they are presented visually;
- Ethan suffered from mathematics anxiety and needed to be praised and encouraged to raise his low self-esteem in mathematics;
- He found difficulty with number work, solving problems, estimation and sequencing (all components would be targeted through the intervention programme);
- Ethan seemed to understand concepts mechanically hence a deeper understanding of the concepts is needed;
- He was not very participative in class;
- Ethan needed to learn the mathematical language related to the areas of development;
- Ethan needed to be supported better when moving from guided work to independent work.

During the intervention programme, I took these into account mainly by ensuring that concepts were covered as visually as possible, by constantly praising and encouraging him and putting him at ease, by emphasizing the mathematics terminology for every component, and by ensuring that his areas of difficulties were targeted.

6.2. Seb

Seb's mother said that her son's main difficulties with the subject started during the previous year when he first joined the school (Grade 4). His mother mentioned that at first, he had found the subjects somewhat easy because at his previous school they had worked on more 'advanced' mathematics. However, he suddenly started to struggle and seemed to be completely disengaged in class. She mentioned that at home he did not do any revision unless he had to. Seb did his homework at an afternoon club with a teacher but could be very moody. His mother mentioned that at times, when she picked him up from the Club, his teacher would tell her that he would not have done anything. According to Seb's mother, Seb had a very short attention span, especially in class. This was confirmed by his class teacher during the interview I had with her. Seb's mother was very concerned about his mathematics because mathematics is needed for life. She reported that Seb still found difficulty with the concept of time but that he had grasped the idea of money and could use it in everyday life situations. Struggling with time is a common characteristic of children with MLD (Section 2.7). Seb's mother felt that he was not showing his real potential when it came to summative assessments, including the end of year exams. She explained to me that during the last exam he had left out most of the paper and just drew question marks against every sum he did not know. This is also typical for children with MLD since they tend to need plenty of support in raising their general self-esteem, enjoying mathematical tasks more (Silver, 1985), increasing their confidence in mathematics (Stevens et al., 2004) and nurturing intrinsic motivation (Sternberg, 1983) (see Section 2.7.2).

Seb's teacher, Ms. Camilleri, gave a similar account of what Seb's areas of strength and difficulties are. She explained that Seb was distracted very easily and also found ways of distracting others. Ms. Camilleri reported that Seb found difficulties with mental mathematics. She recalled that Seb's mother had told her that at his previous school, they learnt many mechanical procedures by heart without really gaining an understanding of the concept at hand. Seb's teacher also mentioned that Seb seemed to find most difficulty with number work and that he generally participated more when the lesson was related to shapes, data and measures. Ms. Camilleri also mentioned that Seb had grasped some of the more basic mathematics language such as 'add' or 'take-away'. However, Ms. Camilleri pointed out that he still struggled when using subject-specific language to express himself, for example using terms like 'numerator' within a context.

The classroom observations allowed me to experience first-hand what Seb's mother and teacher had told me. Seb was very distracted during the first two lessons observed (multiplication and rounding). Not only did he seem not to be paying attention, but he also disrupted others. For example, at one point he started to click his pen constantly and to drop things on the floor purposely. During independent work he did not know what he was meant to do, and the teacher had to sit down next to him and re-explain. Following the re-explanation, he managed to complete the exercises but needed the multiplication grid to be able to complete the multiplication sums given. Seb seemed more engaged during the lesson about perimeter. He could explain to the teacher why we write 'cm^2' in our answers for area and only 'cm' for perimeter answers. He was able to do the independent task in this lesson and only needed re-directing once when he got distracted, arguing with the child sitting next to him because the latter's stationery was on his desk. His independent task was mostly correct but when it came to count the sides of irregular shapes, he kept forgetting to add some of the sides.

From the data collected, the information most relevant for the programme was:

- Seb got distracted very easily during mathematics and could get disengaged quickly;
- Seb had difficulty with mental mathematics;
- He had a mechanical understanding of concepts and needed to develop a deeper understanding of them;
- Seb had difficulty with mathematics language and needed to grasp this to express himself mathematically.

This information helped me to ensure that the intervention sessions were tailor-made for Seb and to engage him in the sessions as much as possible. Furthermore, it allowed me to gain a better insight into his areas of difficulties to target them during the programme. When planning the sessions, I took this information into account by: keeping tasks short, trying to use activities in which Seb could move around the classroom, using concrete materials to support Seb in developing a relational understanding of the concept at hand (Skemp, 1978), and by giving Seb the opportunity to develop his mathematics language which is fundamental to the learning of mathematics as discussed in Section 2.6.2. The latter would be done through visual aids and situations in which he could make use of specific mathematical terms verbally and in writing.

6.3 Thomaz

Thomaz's mother highlighted several difficulties that Thomaz had in relation to mathematics. She explained that Thomaz found difficulty with:

- Sequencing – he was still not confident with counting and saying the number sequence correctly. For example, he sometimes said '20' instead of '12';
- The concepts of time and money;
- Developing a relational understanding of the concepts at hand. Thomaz seemed to learn concepts mechanically and had difficulty with applying concepts and skills to different situations; and
- Understanding the magnitude of numbers.

Thomaz's mother stated,

> "It could be that he did not have a solid foundation to the subject…I noticed immediately that something was wrong…when he started Grade 4, he seemed to confuse the sequence of the numbers and the number system".

When asked whether she recalled identifying other difficulties in mathematics before Grade 4, Thomaz's mother replied,

> The alarm bells went off when he started level 4 and concepts got harder…I might have been focusing mostly on literacy and that's why I didn't realise [that he had] his mathematics difficulties (Thomaz's mother).

Thomaz seemed to enjoy topics related to data, shapes and measures. His mother reported that he liked finding the fraction of a shape, for example. Thomaz seemed to have mathematics anxiety. According to his mother, "he doesn't enjoy mathematics and is scared of the subject". She also explained to me that on the day of the mathematics exam, he woke up and said that he did not feel like going to school. His mother said that his confidence in the subject was very low. This characteristic is typical for children with MLD (Silver, 1985; Stevens et al., 2004).

Thomaz's teacher, Ms. Camilleri, reported similar difficulties to those mentioned by his mother. Ms. Camilleri reported that Thomaz had yet not mastered 'basic skills' in mathematics and that he was thus finding it very difficult to cope with Grade 5 work. She mentioned that Thomaz got distracted very easily but she was not sure if this distraction was caused by his inability to manage the tasks at hand. Ms. Camilleri explained to me that the class Learning Support Educator (LSE) usually supported Thomaz during mathematics lessons. Thomaz and the Learning Support Educator (LSE) sometimes worked together out of class as she gave him adapted work with the hope that he would feel better at the subject and was able to do some of the given tasks. Thomaz's teacher also explained that Thomaz's achievement in mathematics was very poor and that he was performing worst in mathematics out of all her pupils.

When I observed Thomaz in class during the 'long multiplication' lesson, he appeared to follow the teachers' explanation. He seemed to be paying attention to the lesson and when the teacher asked him to work out '43 x 5', he was able to. His adapted worksheets included only the multiplication tables of 2, 3, 4, 5 and 10, whilst those of the other learners also had the other tables. However, when the other children were completing the classwork, he carried out some adapted work instead. When doing the topic of rounding, the teacher gave him smaller numbers and he was also allowed to use the number grid. During independent work he was able to copy the teacher's example by looking for the closest ten to the number given on the number grid. However, he had not really understood what this meant. Such mechanical working is typical of children with MLD (DfES, 2001). The lesson on perimeter was followed better, but still with difficulty. Thomaz had the ability to add 1-digit numbers and was able to work out the perimeter of regular shapes. His difficulty was evident when he had to find the perimeter of irregular shapes. He needed prompting and the shared LSE in class had to explain to him how he had to ensure that he added all the lines because he kept forgetting some.

The gathered data prompted me to keep the following characteristics in mind:

- Thomaz still struggled with most of the numeracy components that were fundamental to be able to follow the more complex mathematics being covered at his grade level;
- He had not yet developed an understanding of the magnitude of numbers;
- Thomaz had high anxiety levels especially when he had to sit for the mathematics summative assessments;
- Thomaz needed to learn how to apply what he had learned to new situations.

The intervention sessions targeted his mathematics anxiety by bringing together the use of the computer, which he loved, and mathematics. The other areas were targeted through a multisensory approach with the intention of helping him develop a better relational understanding of the various concepts at hand.

6.4 Nathan

Both Nathan's mother and father attended the initial interview. At the start of the interview, Nathan's father said, **"you need to push him to complete any mathematics tasks he may have".** Nathan's father explained that Nathan did not seem to have any difficulties with numerical concepts when he was young. He mentioned that teachers up to Grade 3 had never pointed out that his child was struggling with mathematics. Both Nathan's mother and father revealed that Nathan's self-confidence with mathematics was very low. This profile of low confidence in mathematics is common to children with MLD (Wilkins, 2000). They explained that as soon as they asked him to complete a sum, he usually said that he did not know how to solve it. They also mentioned that without someone giving him constant support and encouragement, he usually gave up on mathematical tasks. When asked how Nathan reacted when the mathematics exam was around the corner, they said that they would be able to tell that he was not his 'usual self' and that he would be worrying about it. Taking exams and tests is one of the causes of mathematics anxiety (Chinn, 2012).

When I asked Nathan's parents about the topics that Nathan likes, his father said,

> **His private lessons teacher said that the [class] teacher gets annoyed when Nathan doesn't perform well because he seems to have grasped time and other more complex concepts, like degrees, but then gets stuck in multiplication and division which he seems unable to grasp** (Nathan's father).

Nathan was attending tuition for a 2-hour session once a week. During this session the teacher usually covered the core subjects especially English and Mathematics, which were the subjects Nathan seemed to be struggling with most. His father also said,

> **I can see he hasn't grasped the basic concepts yet and I understand that no teacher in class has the time to go back to concepts that have been covered in Year 1 or Year 2** (Nathan's father).

At home it seemed that Nathan's dad helped him most when it came to mathematics because his mother said that she did not feel too confident with the subject either and admitted struggling with mathematics too. This is not surprising since as discussed in Section 2.8.2, Barakat (1951) found that the number of family members of learners struggling with mathematics was nearly three times more than it was for other learners who were more able in the subject. When Nathan's parents were asked if they were concerned about Nathan's learning difficulties in mathematics, his dad said that he was very concerned especially because the following year Nathan would need to sit for his end of primary school exams. He concluded by saying that "even if he gets a 50, I would be happy" which showed their deep concern for their child's achievement in mathematics.

Nathan's teacher, Mr. Desira, was also concerned about Nathan's achievement in mathematics. Mr. Desira explained that Nathan **"rarely participates in mathematics lessons…he participates very little, I think he feels insecure and is afraid to do something wrong"**. Mr. Desira mentioned that he often had to give Nathan one-to-one support during the mathematics lesson. He recalled how he sometimes went around the classroom during independent classwork and found Nathan staring at his work. Mr. Desira said that he then re-explained, and Nathan was able to do a few other sums. Nathan's teacher mentioned that Nathan finds great difficulty with using mathematical language. This is a common characteristic of learners with MLD (Section 2.6). As concluded by Vukovic and Lesaux (2013) [in Section 2.6.2] "children most often need language to express, understand, and learn mathematics, which begs a nuanced understanding of its role in children's ability to express, understand, and learn mathematics" (p. 90). Mr. Desira explained that, "for example in fractions, he [Nathan] would say 'the one on top' or 'the one at the bottom' instead of using 'numerator' or 'denominator'". Mr. Desira also went on to explain that,

> for example, we have done square numbers, someone else would tell me, please re-explain square numbers again…but Nathan would say, those numbers that are the same, that we times together etc. without actually being able to use the term square numbers…(Mr. Desira)

This illustrates that Nathan was not able to recall the specific language that was needed to express himself precisely in the mathematics topics at hand.

Nathan was very reserved during all three lessons observed. He seemed to be listening to the teacher's explanation however, when during the first lesson observed, the teacher asked him a question related to the explanation, he was unable to reply, meaning he might not have really been understanding. Thus, I kept an eye open during the subsequent lessons observed. Nathan found difficulty with understanding the lesson related to angles. He seemed to have difficulty with the mathematical vocabulary because he had not understood what was meant by '3 right angle turns' and '90 degrees'. During the independent task, the teacher in fact had to sit next to him to explain the exercise once again. I observed the teacher writing the terms on separate sheets and drawing a picture to show Nathan what right angle turns were. The teacher then asked Nathan to stand up and make the turns with his own body. When I asked the teacher why he had decided to do this, he explained to me that Nathan seemed to understand better when visual aids and multisensory experiences were used. Nathan also had difficulty with recalling the multiplication tables during the multiplication lesson. Due to this, he took longer than his peers to complete a task since he continuously reverted to his fingers to count on from one number to another to get to the product of two numbers. For Nathan, this difficulty was also an issue when it came to the topic of area. Even though for this concept, the multiplication tables used in the textbook were rather easy (as only the 2, 3, 4, and 5 times tables were used), he still struggled to recall them. Nathan seemed not to ask for help even when he needed it.

The following characteristics were kept in mind when planning Nathan's sessions:

- Nathan still had difficulties with the numeracy components that are the foundations of the mathematics being covered in class as well as with recalling important mathematics facts like the multiplication tables;
- Nathan had high levels of mathematics anxiety;
- Nathan was still struggling to use mathematics language correctly;
- Nathan did not participate much during mathematics lessons;
- Visual aids and multisensory strategies helped Nathan to learn;
- Nathan would not ask for help even when he needed it.

The fact that I got to know that he does not generally ask for help even when he needs it, was an eye opener as during my sessions with him, I checked whether he had truly internalized the numeracy component by gaining written or oral evidence rather than by simply asking him about it.

6.5 Andrea

When I spoke to Andrea's mother during the interview, she explained that Andrea had not seemed to have difficulties with mathematics when he was very young, and at school when attending nursery. Moreover, at this young age Andrea also participated in mathematical activities at home. Andrea's mother explained that Andrea's difficulties with mathematics became evident in Grade 3 when the mathematical concepts and skills covered became harder for him to grasp. She stated that he struggled a lot when working out operations mentally. This conforms to common difficulties highlighted for children with MLD (Bird, 2009; Henderson et al., 2003). Moreover, Andrea found difficulty with problem solving, recalling his times tables, long division and long multiplication. On the contrary, Andrea was able to complete tasks related to shapes, data and measure such as angles and graphs. This characteristic is also typical of children having MLD as they seem to struggle more with number related concepts (Butterworth, 2010; Leibovich & Ansari, 2017). The concepts of time and money were also reported to be challenging for Andrea.

Andrea's mother described her son to be rather careless in his work and very easily distracted especially when it comes to completing mathematical tasks. She stated, **"he distracts himself with everything".** She also admitted that she was not confident with mathematics herself and that sometimes she could not help him with his homework because **"I am weak at mathematics myself too".** As outlined in Section 2.9.1, Andrea's mother's confidence in the subject might have impinged on his confidence in it, although I am not in a position to state this with any certainty. Andrea's mother said that he attended private sessions once weekly in which he does one or more of the core subjects (Mathematics, English and Maltese). Therefore, mathematics is not targeted on a weekly basis as sometimes he is supported in English and Maltese instead.

Andrea's teacher, Ms. Camilleri, had similar concerns about Andrea. She mentioned that he does not participate much during mathematics lessons and that he is sometimes caught daydreaming. She explained that the class LSE supported him to understand the topics being covered in class and gave him attention especially during classwork. Ms. Camilleri mentioned that he was still struggling with the basics in mathematics learning. She added that Andrea still had difficulty completing his homework even when this was related to topics like subtraction of both 1- and 2-digit numbers which the children were meant to have mastered in previous years.

During classroom observations, Andrea was rather inattentive. He seemed to be lost as the teacher explained 'long' multiplication (multiplication of a 2-digit number by another 2-digit number). As the teacher was explaining how to work out long multiplication operations, she asked him to come to the board to do the first part of the operation, however he was unable to. The teacher explained it again to him but when they were given independent schoolwork, he still struggled. He also struggled to recall the basic tables. Andrea also struggled during the lesson about rounding.

On the other hand, when I observed Andrea during a lesson on perimeter, he was very attentive and participative and was able to work out the independent task given. Moreover, when I asked him what he was doing, he could explain to me without difficulty the concept which in his terms was 'I am adding up all the sides to find the total length of the lines' (referring to the shape). Even if at times he added the measurements of the sides incorrectly, he had understood what the perimeter of a shape was and could find it. The observations reflected what Andrea's mother and his teacher had said during their interviews – that Andrea seems to find great difficulty with numbers and number work but performs better in topics related to shapes, data and measures.

The details about Andrea that were most useful for me when planning for his sessions were the following:

- Andrea struggled with mental work;
- He had difficulties with number work;
- Andrea was sometimes careless in his work; thus an error may be the result of carelessness rather than lack of knowledge;
- Andrea did not engage in the mathematics lessons in class.

This information prompted me to design short, engaging tasks so that Andrea would remain focused. It also led me to ensure that Andrea was reminded to check and edit his work when completing a task. Knowing that Andrea still struggled with number work confirmed that supporting him by focusing on the numeracy components should be beneficial.

6.6 Mike

Both Mike's mother and his father attended the interview and were able to highlight Mike's difficulties with mathematics. His mother said that "sometimes he doesn't manage to finish off in time" (referring to mathematics tasks) whilst his father added that "he spends a lot of time thinking [about] which method he is going to make use of (to solve a given operation)".

Both parents agreed that Mike struggled with the concept of time and they explained that he had difficulty with reading time from a clock, converting units of time and reading ‘quarter to’ and ‘quarter past’ times. Mike’s mother said that Mike was very fearful of mathematics and had low self-esteem when it came to this subject. These are two common characteristics of learners with MLD (Section 2.6). She mentioned that before the mathematics exam he usually told her “**don’t expect me to do well**”. As highlighted in Section 2.7.3, increasing the confidence of learners with MLD is usually a priority (Stevens et al., 2004). Mike also needed help with mathematics homework and his mum usually helped him out by working out a few examples, showing him the procedure to be followed. Mike had difficulty with working out long multiplication and division and converting different units of measurement. Mike attended private lessons for both Maltese and English. His mother explained that the lesson was not on a one-to-one basis but that there were other children with him and that sometimes other subjects were covered. When asked about Mike’s mathematics exam, Mike’s father commented that Mike had left out some questions completely from his mental paper (a summative assessment in which the child must work out different operations quickly). He added that he thought Mike was unable to work it out because this paper is timed and he said, **“he knows the answers but at that point he just blanks out”**. This implied that Mike’s mathematics anxiety was rather high.

Mike’s teacher, Mr. Desira said that Mike tried very hard and was not afraid of taking up a challenge. Mr. Desira mentioned that Mike could not concentrate for long periods of time. He stated that Mike could get higher marks in all other subjects but seems unable to make progress in mathematics. Mr. Desira mentioned that often Mike participated in class and asked for help when this is needed. He also explained that Mike could usually finish off his classwork alone and that even though the operations would not all be correct, he would have still tried his best. Mr. Desira mentioned that sometimes Mike copied from his neighbour, which was a coping strategy that he had developed. Mike also suffered a lot from mathematics anxiety and got over excited; word problems seemed to make him most anxious.

When observed in class, Mike was very participative. During the lesson about area, Mike seemed confident with using the formula ‘Length x Breadth’ to find his answers. Following the teacher’s explanation, I observed whether Mike was completing the task correctly and indeed he was. However, when I then asked him what area was, he kept repeating that it was ‘Length x Breadth’ showing me he had not really developed a relational understanding of what area was, apart from mechanically grasping the formula and replicating it in his exercise. Moreover, when asked to solve a problem related to area, Mike struggled.

He seemed unable to apply what he had learnt to the given context. I helped him to create a visual to help him solve the problem. The drawing seemed to help him create a better understanding and he used this tool to figure out what he had to do.

In the lessons on Multiplication and Angles, Mike seemed to be following the explanation and appeared to have understood what he was meant to do. However, when he was later asked to do the task on his own, he needed prompting and re-explaining. In the lesson tackling multiplication strategies, Mike seemed attentive and participative and seemed able to mechanically complete tasks in which he had to multiply a 2-digit number by a 1-digit number. However, he struggled to recall the multiplication tables, which led him to take long to complete the tasks, and also to make mistakes in the computations.

The information gathered from the interviews and the observations about Mike that would influence my sessions was the following:

- Mike usually took a lot of time to finish his mathematics tasks;
- He had low self-esteem when it came to mathematics;
- He could not concentrate for long periods of time when doing mathematical tasks;
- Mike had high levels of mathematics anxiety;
- Mike asked for help when he needed it.

This data prompted me to ensure that tasks were short and engaging, that I encouraged and praised Mike during the sessions and that I allowed Mike to have sufficient time to finish his tasks to reduce frustration and anxiety.

After providing a detailed profile for each of my main participants, I will now move on to analyse the qualitative data obtained from the intervention sessions with each learner. I will first present the data from the assessments of the ten numeracy components carried out pre- and post- intervention sessions. The identified strategies that seemed to be effective with both groups of learners will then be analysed and discussed to formulate theories about which of these strategies were effective in supporting the learners in making the much-desired cognitive and affective development in mathematics learning.

6.7 Comparison of Pre- and Post- Test Results: Similarities and Differences between Pupil Profiles

After answering the first subsidiary research question; 'What are the teachers' and parents' perspective on how the children's MLD affects their daily lives at school and at home?' I now move on to answering the other subsidiary research questions of this study, which are:

ii. Do children with solely MLD and those with both MLD and RD have similar mathematical profiles? Are both groups of children strong/weak in the same areas?

iii. Do the children assessed with MLD or both MLD and RD and also with a profile of Dyscalculia as identified by the Screener (Butterworth, 2003) have difficulties in all numeracy components or in just a few?

iv. Is mathematics anxiety one of the difficulties experienced by the learners?

In this section I seek to answer these subsidiary research questions. Primarily, I present the results from the pre-intervention and the post-intervention numeracy assessments for each individual child. I then compare the results obtained by the different learners to bring out any similarities and differences between learners with different profiles (i.e. MLD only, MLD and RD and dyscalculia).

The initial results of the individual learners include information about the number ranges achieved in all the numeracy components before and after the intervention programme for each learner. The specific components that were intervened upon during the programme are highlighted (in blue). When deciding which components to prioritise, I decided to start by targeting those that are important for other components. These were: counting verbally, order irrelevance, adding and subtracting objects, place value, estimation and remembered facts. Once these were tackled and mastered, I then continued with the other components. Tables 6.1 - 6.6 present the scores obtained by Ethan, Seb, Thomaz, Nathan, Andrea and Mike respectively.

Table 6.1: Pre- and post-intervention scores in numeracy assessments for Ethan.

Standardised Tests		
	Pre-Test	**Post-Test**
Chinn's Addition Task	10 out of 36	11 out of 36
Chinn's Subtraction Task	9 out of 36	11 out of 36
Chinn's Multiplication Task	11 out of 33	17 out of 33
Chinn's Division Task	7 out of 33	13 out of 33
Chinn's 15-minute test	8 out of 44	11 out of 44
Basic Number Screening Test	17 out of 30	23 out of 30
Catch Up® Formative Assessment		
Numeracy Component	**Pre-Assessment**	**Post-Assessment**
Counting Verbally	0-20	0-20
Counting On	0-20	0–20
Counting Back	0-10	0–20
Counting Objects	0-20	0-20
Adding Objects	0-20	0-20
Order Irrelevance	None	0-20
Subtracting Objects	0-10	0-20
Reading Numbers	0-20	0-20
Reading Number Words	0-20	0-20
Writing Numbers	0-20	0-20
H, T, U: Number Comparison	0-20	0-20
H, T, U: Adding Tens and Units	0-15	0-20
H, T, U: Subtracting Tens and Units	0-15	0-20
Ordinal Numbers	1-5	0-20
Word Problems	0-8	0–10
Translation: Objects to numbers	0-15	1-5
Translation: numbers to objects	0-20	1-5
Translation: number words to objects	1–5	0–15
Translation: number words to numbers	1-5	0-8
Estimation	0-15	0–15
Derived Facts	None	None
Remembered Facts	1-5	0-8

Table 6.2: Pre- and post-intervention scores in numeracy assessments for Seb.

Standardised Tests		
	Pre-Test	**Post-Test**
Chinn's Addition Task	14 out of 36	23 out of 36
Chinn's Subtraction Task	4 out of 36	12 out of 36
Chinn's Multiplication Task	0 out of 33	12 out of 33
Chinn's Division Task	0 out of 33	5 out of 33
Chinn's 15-minute test	2 out of 44	11 out of 44
Basic Number Screening Test	11 out of 30	24 out of 30
Catch Up® Formative Assessment		
Numeracy Component	**Pre-Assessment Level**	**Post-Assessment Level**
Counting Verbally	0-20	0-20
Counting On	0-20	0-20
Counting Back	0-15	0-20
Counting Objects	0-10	0-20
Adding Objects	0-10	0-20
Order Irrelevance	0-20	0-20
Subtracting Objects	0-10	0-20
Reading Numbers	0-20	0-20
Reading Number Words	0-20	0-20
Writing Numbers	0-20	0-20
H, T, U: Number Comparison	0-20	0-20
H, T, U: Adding Tens and Units	0-20	0-20
H, T, U: Subtracting Tens and Units	0-18	0-20
Ordinal Numbers	0-20	0-20
Word Problems	0-10	0-18
Translation: Objects to numbers	0-15	0-20
Translation: numbers to objects	0-20	0-20
Translation: number words to objects	0-20	0-20
Translation: number words to numbers	1-5	1-5
Estimation	None	0-18
Derived Facts	None	None
Remembered Facts	1-5	0-20

Table 6.3: Pre- and post-intervention scores in numeracy assessments for Thomaz.

Standardised Tests		
	Pre-Test	**Post-Test**
Chinn's Addition Task	7 out of 36	11 out of 36
Chinn's Subtraction Task	4 out of 36	8 out of 36
Chinn's Multiplication Task	1 out of 33	5 out of 33
Chinn's Division Task	0 out of 33	3 out of 33
Chinn's 15-minute test	2 out of 44	3 out of 44
Basic Number Screening Test	1 out of 30	4 out of 30
Catch Up® Formative Assessment		
Numeracy Component	**Pre-Assessment Level**	**Post-Assessment Level**
Counting Verbally	0-10	0-20
Counting On	0-8	0-20
Counting Back	none	0-10
Counting Objects	0-20	0-20
Adding Objects	0-10	0-18
Order Irrelevance	none	0-20
Subtracting Objects	1-5	0-10
Reading Numbers	0-18	0-18
Reading Number Words	0-18	0-15
Writing Numbers	0-20	0-18
H, T, U: Number Comparison	0-20	0-20
H, T, U: Adding Tens and Units	0-10	0-15
H, T, U: Subtracting Tens and Units	1-5	0-10
Ordinal Numbers	none	1-5
Word Problems	1-5	0-10
Translation: Objects to numbers	0-8	0-8
Translation: numbers to objects	none	1-5
Translation: number words to objects	none	1-5
Translation: number words to numbers	none	1-5
Estimation	1-5	1-5
Derived Facts	None	None
Remembered Facts	None	None

Table 6.4: Pre- and post-intervention scores in numeracy assessments for Nathan.

Standardised Tests		
	Pre-Test	**Post-Test**
Chinn's Addition Task	12 out of 36	19 out of 36
Chinn's Subtraction Task	8 out of 36	18 out of 36
Chinn's Multiplication Task	9 out of 33	16 out of 33
Chinn's Division Task	4 out of 33	12 out of 33
Chinn's 15-minute test	8 out of 44	17 out of 44
Basic Number Screening Test	17 out of 30	22 out of 30
Catch Up® Formative Assessment		
Numeracy Component	**Pre-Assessment Level**	**Post-Assessment Level**
Counting Verbally	0-20	0-20
Counting On	0-15	0–20
Counting Back	0-8	0–20
Counting Objects	0-20	0-20
Adding Objects	0-20	0-20
Order Irrelevance	0-20	0-20
Subtracting Objects	0-20	0-20
Reading Numbers	0-20	0-20
Reading Number Words	0-20	0-20
Writing Numbers	0-20	0-20
H, T, U: Number Comparison	0-20	0-20
H, T, U: Adding Tens and Units	0-15	0-18
H, T, U: Subtracting Tens and Units	0-15	0-20
Ordinal Numbers	0-20	0-15
Word Problems	0 - 18	0–20
Translation: Objects to numbers	0-8	0-20
Translation: numbers to objects	0-20	0-20
Translation: number words to objects	0-18	1-5
Translation: number words to numbers	0-15	1-5
Estimation	0-8	0 – 15
Derived Facts	None	None
Remembered Facts	0-20	0-20

Table 6.5: Pre- and post-intervention scores in numeracy assessments for Andrea.

Standardised Tests		
	Pre-Test	**Post-Test**
Chinn's Addition Task	17 out of 36	18 out of 36
Chinn's Subtraction Task	15 out of 36	19 out of 36
Chinn's Multiplication Task	9 out of 33	13 out of 33
Chinn's Division Task	13 out of 33	14 out of 33
Chinn's 15-minute test	7 out of 44	13 out of 44
Basic Number Screening Test	15 out of 30	21 out of 30
Catch Up® Formative Assessment		
Numeracy Component	**Pre-Assessment Level**	**Post-Assessment Level**
Counting Verbally	0-20	0-20
Counting On	0-18	0–20
Counting Back	0-20	0–20
Counting Objects	0-20	0-20
Adding Objects	0-20	0-20
Order Irrelevance	0-20	0-20
Subtracting Objects	0-15	0-20
Reading Numbers	0-20	0-20
Reading Number Words	0-15	0-20
Writing Numbers	0-20	0-20
H, T, U: Number Comparison	0-20	0-20
H, T, U: Adding Tens and Units	0-18	0-20
H, T, U: Subtracting Tens and Units	0-18	0-20
Ordinal Numbers	0-10	0-15
Word Problems	0-10	0-18
Translation: Objects to numbers	None	0-8
Translation: numbers to objects	0-20	0-18
Translation: number words to objects	0-18	0–18
Translation: number words to numbers	1-5	0-8
Estimation	0-15	0-20
Derived Facts	None	1-5
Remembered Facts	0-15	0-20

Table 6.6: Pre- and post-intervention scores in numeracy assessments for Mike.

Standardised Tests		
	Pre-Test	**Post-Test**
Chinn's Addition Task	17 out of 36	22 out of 36
Chinn's Subtraction Task	9 out of 36	17 out of 36
Chinn's Multiplication Task	12 out of 33	20 out of 33
Chinn's Division Task	18 out of 33	23 out of 33
Chinn's 15-minute test	9 out of 44	17 out of 44
Basic Number Screening Test	19 out of 30	26 out of 30
Catch Up® Formative Assessment		
Numeracy Component	**Pre-Assessment Level**	**Post-Assessment**
Counting Verballly	0-20	0-20
Counting On	0-20	0–20
Counting Back	0-20	0–20
Counting Objects	0-20	0-20
Adding Objects	0-10	0-20
Order Irrelevance	0-20	0-20
Subtracting Objects	0-10	0-20
Reading Numbers	0-20	0-20
Reading Number Words	0-20	0-20
Writing Numbers	0-20	0-20
H, T, U: Number Comparison	0-20	0-20
H, T, U: Adding Tens and Units	0-20	0-20
H, T, U: Subtracting Tens and Units	1-5	0-20
Ordinal Numbers	1-5	0-20
Word Problems	0 - 10	0–18
Translation: Objects to numbers	1-5	0-10
Translation: numbers to objects	0-20	0-20
Translation: number words to objects	1-5	0–10
Translation: number words to numbers	1-5	1-5
Estimation	1-5	0–20
Derived Facts	0-10	0–20
Remembered Facts	0-8	0-20

The analysis of the data collected from the pre- and post- intervention assessment, allowed me to answer three of my subsidiary research questions. The second subsidiary research question was, 'How do children with solely MLD and those with both MLD and RD perform in the ten numeracy components? Are both groups of children strong/weak in the same components?' The mathematical profiles of the learners (presented in Tables 6.1 – 6.6) confirmed what other researchers have suggested regarding this matter i.e. that learners with mathematics learning difficulties are a heterogeneous group of learners (Bartelet et al, 2014; Dowker, 2005b; Geary, 2010; Kaufmann & Nuerk, 2005; Rubinsten & Henik, 2009). As explained in Sections 2.4.1 and 2.6, it is best to speak not of mathematical ability but of mathematical abilities, since learners with MLD will have difficulties with a variety of numerical components (Dowker 2005a). Therefore, although MLD is used as an umbrella term, the characteristics of one learner with MLD may be completely different to that of another. To be able to compare the results obtained by each learner on the numeracy components, I collated all the data collected regarding the individual mathematical abilities, specific to the components in one Table. Table 6.7. thus, illustrates each of the ten components and the areas in which the children had difficulties classified by whether the child belonged to the group with solely MLD or both MLD and RD.

Table 6.7: Areas of Difficulties as per the pre-intervention assessment for children with MLD and those with MLD and RD.

Catch Up® Formative Assessment						
Numeracy Component	**Areas of Difficulty – Children with MLD and RD**			**Areas of Difficulty – Children with MLD**		
	Ethan	**Seb**	**Thomaz**	**Nathan**	**Andrea**	**Mike**
Counting Verbally			■			
Counting On			■	■	■	
Counting Back	■	■	■	■		
Counting Objects		■				
Adding Objects		■	■			■
Order Irrelevance	■		■			
Subtracting Objects	■	■	■		■	■
Reading Numbers			■			
Reading Number Words			■		■	
Writing Numbers						
H, T, U: Number Comparison					■	
H, T, U: Adding Tens and Units	■		■	■	■	
H, T, U: Subtracting Tens and Units	■	■	■	■	■	■
Ordinal Numbers	■		■		■	■
Word Problems	■	■	■	■	■	■
Translation: Objects to numbers	■	■	■	■	■	■
Translation: numbers to objects			■			
Translation: number words to objects	■		■	■	■	■
Translation: number words to numbers	■	■	■	■	■	■
Estimation	■	■	■	■	■	■
Derived Facts	■	■	■	■	■	■
Remembered Facts	■	■	■		■	■
Total Areas of Difficulties	**13**	**11**	**18**	**10**	**14**	**11**

The pre-intervention assessment of the numeracy components illustrated that all learners had a different profile of mathematical abilities. As evident in Table 6.7, no two learners had the same areas of difficulties in the ten numeracy components. It is interesting to note that some of the components presented a difficulty for all learners. These were: 'H, T, U: subtracting Tens and Units', 'Word Problems', 'Translations: Objects to Numbers', 'Translation: Number Words to Numbers', 'Estimation' and 'Derived Facts'. It may be the case that these components have a higher level of difficulty when compared to the other components, hence this difficulty. This would corroborate other research (referred to in Section 2.6.2) which, indicates that 'word problems', for example, have a higher level of difficulty than simple computations. However, one would have to show this generality by carrying out the assessment with a larger number of children. Another observation made is that, although one may conjecture that learners with MLD and RD may have greater difficulties with solving word problem due to the language component, both groups of learners struggled with this component. As illustrated through the last row in Table 6.7, it is evident that for this group of children, the number of areas a learner has difficulties in is not related to whether he was in the MLD only group or the MLD and RD group. The findings show that the number of difficulties was not more for learners with both MLD and RD and that each profile presented a different number of areas needing support depending on the individual needs of the learner. Hence, this suggests that assessing learners individually and creating a profile of strengths and needs is fundamental for any intervention to be effective. It also denotes that one-to-one intervention may indeed be more effective than small group intervention, especially if the group of learners have a different profile of needs.

Through Table 6.7, I could also see whether the children with dyscalculia, in this set of six children, had difficulties in all the numeracy components. This allowed me to answer the third subsidiary research question of this study which was, 'Do the children assessed with MLD or both MLD and RD and also with a profile of Dyscalculia as identified by the Screener (Butterworth, 2003) have difficulties in all numeracy components or in just a few?' The only child who had been assessed by the Dyscalculia Screener (Butterworth, 2003) as actually having dyscalculia was Thomaz. Table 6.7 shows that Thomaz did not have difficulties with all the numeracy components indicating that learners with dyscalculia also have strengths, which can be tapped upon. Having said this, Thomaz did present the greatest number of areas of difficulty – a total of '18' when the average areas of difficulties was '12.83'. This may show that learners with dyscalculia have a wider spread of difficulties hence leading to the more severe nature of their struggles. Moreover, since it has been suggested that dyscalculia might stem from a different deficit to other MLDs (Rubinsten and Henik, 2009), such as a deficit in the ANS (Piazza et al., 2010), the difficulties of children with dyscalculia may be indeed more profound than those of learners with MLD. This finding corroborates those of various studies (Kucian et al., 2006; Mazzocco et al., 2011; Piazza et al., 2010) mentioned in Section 2.8.3. As concluded by Mazzocco et al. (2011), in dyscalculic learners the ANS is much poorer in precision than all other categories of learners, including low achievers in mathematics who do not have dyscalculia. Similarly, Piazza et al. (2010) indicated that the number acuity children with dyscalculia is severely impaired since participants with dyscalculia who were aged 10 scored at the level of a 5-year-old of a child without dyscalculia in a given test for number acuity. Although the data presented here for Thomaz is showing similar characteristics to those of other studies, one must bear in mind that the sample was very small and further research would be needed to make more general conclusions.

The fourth subsidiary research question for this study was, ‘Is mathematics anxiety one of the difficulties experienced by the learners?’ By administering Chinn’s (2012) anxiety questionnaire I could determine whether the children were experiencing mathematics anxiety or not. I have collated the scores obtained in Chinn’s (2012) mathematics anxiety questionnaire in Table 6.8.

Table 6.8: Scores for Mathematics Anxiety Questionnaire (Chinn, 2012) obtained by each learner.

Name of Learner	Score obtained (out of 80)
Ethan	54
Seb	40
Thomaz	55
Nathan	48
Andrea	38
Mike	55

The average score given by Chinn (2012) for male 9-year-olds in the UK is that of ‘36.7’. This score is based on a sample size of 251 participants. The standard deviation for this age and gender group is that of ‘11.0’. When compared to the average score for 9-year-old males in the UK, all six learners did exhibit higher than average mathematics anxiety since all the scores obtained were higher than ‘36.7’. Moreover, Nathan’s score (48) was one Standard Deviation (SD) (47.7) higher than that of the norm. Whereas Ethan (54), Thomaz (55) and Mike (55) scored almost two Standard Deviations (SD) (58.7) higher than average. This indicates that all four boys were experiencing relatively high mathematics anxiety levels when compared to other boys their age. Only two of the pupils had anxiety levels that were within average range, namely, Andrea (38) and Seb (40). This finding supports other research studies that have illustrated that learners with MLD usually have higher than average anxiety levels (Ashcraft & Kirk, 2001). The intervention programme would thus hopefully also reduce the learners’ anxiety levels and allow them to fear mathematics less as was the case in other studies such as that carried out by Superkar et al (2015) (see Section 2.7.1).

As previously concluded when looking at the areas of difficulties, it does not seem that there is a pattern to these scores because the total number of areas of difficulty does not seem to be related to the score obtained in this questionnaire. Ethan, Thomaz and Mike had higher scores than the other learners, however, their total number of difficulty areas were 13, 18 and 11 respectively. Thomaz and Mike scored the same on the questionnaire however whereas Thomaz's score in the areas of difficulty is the highest, Mike's is one of the lowest. Although all the learners scored a higher than average score in this questionnaire indicating that mathematics anxiety is a difficulty they may be experiencing, Chinn's (2012) cut-off score for high anxiety levels for 9-year-old males is 59. Only 3.1% of 9-year-old males are in this category. The learners participating in this study would not classify as part of such a group as all their scores were below 59. This pattern of results may stem from the fact that the participants of this study were male. As discussed in Section 2.7.2, the levels of mathematics anxiety are higher for females than for males (Beilock et al., 2010). As highlighted by Devine et al. (2012) boys are more likely to experience mathematics anxiety when they encounter initial difficulties in the subject. Hence, although the learners' scores in Chinn's (2012) scale were not deemed as too high, their mathematics anxiety may increase if their difficulties are not intervened upon.

After answering the subsidiary research questions set for this study, I will now discuss findings for the main research question which was,

> Is an intervention programme carried out with children having MLD only and with children having both MLD and RD beneficial to these learners? Which characteristics of the intervention programme are effective with each group of learners?

Although I will first look at the scores the children achieved in the standardised assessments before and after the intervention programme, this analysis will only serve as an indication of whether the children found the intervention programme beneficial on a cognitive level. Since I used these tests before the intervention programme to identify the main participants of the study, I found it appropriate to revisit these scores and investigate whether these had improved following the programme. Nonetheless, since I am taking a social constructivist stance, of more significance are the various situations which will be presented as evidence of the internalisation of the numeracy components. Moreover, I shall also seek to identify whether the programme was also beneficial to the children on the affective domain. Hence, although progress will be measured using the scores obtained from the standardised scores before and after the programme, this progress will be only one aspect of the benefits of the programme which will be discussed in this Chapter, as I seek to illustrate how the learners developed holistically through their participation in the programme.

The scores the learners obtained in the standardised numeracy tests before and after the intervention programme show that progress was recorded. The pupils generally achieved a higher score after the intervention in all the different numeracy tests taken, irrespective of whether they had MLD only or both MLD and RD. This suggests that the intervention programme was effective with both groups of learners. As explained in Section 4.10, all the cohort of Grade 5 learners attending the same school were also re-assessed at the end of the year. This was done so that the impact of the intervention programme could also be compared to progress made by the other learners who were following the normal mathematics programme in class. As indicated by Aunio (2019), "in general, it is possible to say that an intervention program is effective if the children with low performance or learning difficulties progress better than their performance control peers" (p. 710).

Much like the results of the six participants of the intervention programme, most of the learners' scores had improved. Some made little progress whilst others made good improvement. In Table 6.9 I illustrate the mean additional raw score (showing progress) obtained for the general cohort attending Grade 5 during that year. I also compare the individual progress scores of the learners to this mean score to show the progress each learner following the intervention programme made in relation to the other learners. The fields in blue are the tests in which the individual learner scored higher than the mean raw score gained in the test by the class.

Table 6.9: Comparison of cohort average gain in the tests to the individual progress scores.

Name of Test	Cohort Average Gain (Raw Score)	Ethan	Seb	Thomaz	Nathan	Andrea	Mike
Chinn's Addition	7.11	1	9	4	7	1	5
Chinn's Subtraction	3.78	2	8	4	10	4	8
Chinn's Multiplication	11.67	6	12	4	7	4	8
Chinn's Division	11.72	6	5	3	8	1	5
Chinn's 15-min. test	4.44	3	9	1	9	6	8
Basic Number Screening Test (BNST)	3.77	6	13	3	5	6	7

This exercise allowed me to look for patterns in the results obtained by the learners. One observation is that all the learners, apart from Thomaz made a larger gain in the raw score obtained in the BNST than that of the class. Although Thomaz made a slightly below average gain in this test, his progress was still significant. Thomaz was the learner who was struggling with the largest number of numeracy components (see Table 6.3) and who was also identified by the Dyscalculia Screener (DS) (Butterworth, 2003) as having dyscalculia. In my view, both aspects contribute to making his progress a significant one. The overall results obtained by all six learners in the BNST indicate that the intervention programme was indeed fruitful with both groups of learners (MLD only and MLD and RD).

Four of the six participants also got a higher than average progress score on Chinn's (2012) 15-minute test. Moreover, 5 out of the 6 learners also made an 'above average' gain in the subtraction component of Chinn's (2012) tasks. This again indicates that the intervention programme was impactful since possibly they would not have made such a gain had it not been for the programme. However, it is interesting to note that the pattern of results obtained clearly shows that the participants of the intervention programme made a much better gain in the BNST than in Chinn's (2012) different tasks. One possible reason for this is that the former test is not timed, whereas the latter is. This result may indicate that although the six participants did internalize some of the components they were struggling with, as they have shown in the BNST, they were still rather slow, compared to the class, when engaging in these mathematical tasks.

This pattern of results is especially true for Ethan and Thomaz, the learners having most areas of difficulties at the start of the programme. Since both Ethan and Thomaz obtained 'above average' or near above average progress in the BNST, their lower achievement in Chinn's (2012) assessment may originate from them still using ineffective strategies for adding and subtraction, such as finger counting. Due to the timed nature of the task, such strategies may have been an obstacle for them to obtain an even higher gain in the test itself. This indicates that although the intervention programme itself was impactful, a non-timed test may be a more appropriate way of gauging impact with similar mathematical profilers to those of Ethan and Thomaz. Generally, this may also imply that if learners with MLD are not given extra time during assessments, these learners would not be able to show what they really know.

As can be seen in Table 6.9, Seb was the learner who made most progress. He obtained an 'above average' progress score in all tests apart from the division task (Chinn, 2012). However, Seb had originally obtained a '0' in this task showing he did not know how to complete it at all. Gaining a '5' in the post-test thus shows that his progress was significant. Seb's non-verbal reasoning percentile score on the British Ability Scales II (BAS II, Elliott et al., 1996) (Table 5.4) was also the highest (63^{rd}) out of the six participants showing that his ability to grasp mathematics was the best when compared to the other participants. The participants who, although showed progress in all the tests, obtained a progress score that was below the 'average' score in most tasks were Ethan and Thomaz. Although Ethan and Thomaz, like Seb, were part of the MLD and RD group, their results were different to his. It is interesting to note that whereas Seb had the highest non-verbal reasoning score on the British Ability Scales II (BAS II, Elliott et al., 1996), Thomaz had the lowest (27^{th} percentile), whilst Ethan's was also low (third lowest - 55^{th} percentile), hence their disposition to learning mathematics could have also been determined by this ability. These results emphasise that the progress made by each individual learner does not depend on whether he belongs to the MLD only or MLD and RD group but that it might be attributed to the individual's characteristics, including their non-verbal reasoning ability. Hence, these findings indicate that progress and achievement in mathematics is also dependent on domain-general abilities such as non-verbal reasoning ability. This conclusion is similar to that drawn by Lefevre et al. (2005) and Geary et al. (2007) (Section 2.6.1). Moreover, as already mentioned, Ethan and Thomaz were the learners with most areas of difficulties. This may indicate that it takes longer for learners like Ethan and Thomaz to make 'above average' progress and hence it may be the case that a longer intervention programme would have shown the results desired.

After looking closely at the scores obtained in the BNST and Chinn's (2012) tasks, I also examined the pre- and post- scores obtained vis-à-vis the numeracy components. These scores demonstrate that every learner made significant progress in the numeracy components assessed through the CUN assessment. The learners improved in all the components included in the intervention and managed to tackle the component with a higher number range than that achieved before the intervention. Indeed, most of the components included in the intervention were completely mastered after the intervention programme, so that the children could now complete tasks within the full number range of 0 to 20. It was also evident that the children had made progress in some of the other numeracy components, which were not included in the intervention. It may be the case that intervening on some numeracy components may have a positive effect on other components that are related to them. It might also imply that children with MLD can make progress only when the foundations of mathematics are covered (Kaufmann et al., 2003). However, bearing in mind that this was a relatively small sample, and that there was no formal control group being assessed in the CUN numeracy components, I cannot rule out the possibility that the children might have improved simply through maturation or due to their regular schoolwork.

One finding that does suggest that the intervention itself was important in the children's improvement, is that in a very few instances, regression was recorded for numeracy components which had not been intervened upon. An example is the 'translation: numbers to objects' component as part of Andrea's assessment in which he scored a number range of '0-20' prior to the intervention programme and '0-18' post intervention. Since in this case the discrepancy is minimal, this may have happened because Andrea would have not been focusing enough and might have made a careless mistake. However, other instances of regression have also been recorded. These are more accentuated. For example, in Ethan's assessment of the numeracy component of translation, he scored '0-20' prior to the programme on the 'translating objects to numbers' task, and only '1-5' after the programme. Moreover, Ethan regressed in the 'translating numbers to objects' component too. It is interesting to note that none of the scores of the components intervened upon regressed. This may indicate that, while children do make progress in the components intervened upon through the programme, they make less progress, or even regress, in components that do not undergo intervention. One of the reasons why this may happen is that learners with MLD generally have a nemesis with working memory. Working memory is especially important when learning mathematics and this has been discussed in Section 2.6.1. This prompted me to look at the regressions of each learner and compare these to their scores in the Digit Span Tests (forward and backward) the children completed as part of the first set of assessments (see Section 5.4, Table 5.8). When doing this exercise, it was clear that the learners with the lowest scores for working memory, Ethan, Thomaz and Nathan respectively, were the learners with the greatest number of regressions recorded. On the contrary, Seb and Andrea, who obtained a high score in the memory test had no regressions. This supports the literature presented earlier (Section 2.6.1): As highlighted by Raghubar et al. (2010) “working memory is indeed related to mathematical performance in adults and in typically developing children and in children with difficulties in math” (p. 119). Hence, this regression may have resulted from this deficit with working memory. This finding indicates that when numeracy components are not intervened upon, slow progress or even regression may be recorded. Thus, appropriate intervention may not only be important in enabling adequate progress in mathematics learning but may also prevent regression. This may be one of the reasons why, if no intervention is provided to children struggling with mathematics, their difficulties may become greater and the gap between their abilities and those of their peers may widen. However, further research is needed if we are to have a full understanding of the risk of regression, and the role of intervention in preventing this.

6.8 Strategies for Effective Intervention: An Overview

The scores that the learners obtained in the numeracy assessments showed that progress was recorded. Moreover, results indicated that the numeracy components intervened upon were generally mastered. I now present a qualitative analysis of which strategies seemed to be effective for supporting learners with only MLD or both MLD and RD focusing on interactive strategies. Since both groups of learners made substantial progress in the numeracy components, I will assume that the strategies that will be explored are effective with both groups of learners. Moreover, as has been discussed in the previous section (Section 6.7), the effectiveness of the intervention programme did not seem to be dependent on whether the learners had MLD and RD or MLD alone, but rather on other domain-general abilities such as working memory. In this light, the strategies presented were deemed effective with both groups of learners and therefore examples to support my arguments will be taken from various intervention sessions carried out with both groups of learners. Examples will include snapshots from the children’s work, screen shots of the digital games played as well as photos of the resources used. Examples and illustrations will also include transcripts from the intervention sessions. The text presented in **bold** shows instances in which the learner or I used Maltese. These phrases have been translated to English as faithfully as possible. Any non-verbal gestures and other comments will be mentioned using brackets ([ ]) whilst pauses have been indicated using the word pause in paranthesis [pause]. Finally, at times part of the transcript was deemed unnecessary to reproduce for the argument being made. Hence, some text was left out. This has been represented with three dots (…).

As explained in Section 4.11, the themes identified when analyzing the data were aligned to the conceptual framework presented in Chapter 3 and therefore underpinned by Vygotskian perspectives. Upon analyzing the data, emerging themes, mainly other effective strategies than the ones used intentionally, became evident. A main finding was that effective strategies, which appear to support learners in their acquisition of mathematics, should be tri-fold for achievement to be gained. Three main kinds of strategies were identified as follows:

i. Strategies made use of by the More Knowledgeable Other (MKO) - these will be referred to as MKO-driven Strategies;
ii. Strategies that can be led by the learner himself - referred to as Learner-driven Strategies;
iii. Strategies implemented by the MKO through the use of tools – both psychological and technical ones – these will be referred to as Tool-assisted strategies.

Each of these three 'protagonists': the MKO, the psychological and technical tools and the learner, appeared to contribute to the impact of the intervention programme.

Previous studies have used Vygotsky's theories to explore the influence of the MKO in facilitating the internalisation of mathematical concepts (Abtahi, 2014; Abtahi, 2017; Mariotti, 2009; Walshaw, 2017). Moreover, preceding research has also explored the role of cultural tools, including artefacts, in learning mathematical concepts (Bartolini Bussi et al. 2012; Bartolini Bussi & Mariotti, 2008; Clements & McMillen, 1996; McNeil & Uttal, 2009). The idea of learner-driven strategies, however, have not, to my knowledge, been explored in the same way as the other two forms of strategies. In this Chapter I will explain how each strategy seemed to contribute to the internalisation process throughout the intervention programme with each of the six learners. As will be discussed, some of these strategies sometimes overlap, however in this Chapter, I have separated them to focus on each one specifically through the different sections.

What will serve as evidence of internalisation has been explained in Section 3.6. The six mentioned situations will allow me to demonstrate that internalisation had taken place. Multiple strategies driven respectively by the MKO, the tools used, and the learner will be identified and discussed in the hope of later proposing a pedagogical model to demonstrate how these strategies came together to ensure that intervention was successful in assisting the learner to internalize the numeracy components. I hope that the latter part of my discussion can contribute to epistemology since, although past researchers (Sections 3.4, 3.5 and 3.6) have explored the impact of some of the different strategies individually, none, to my knowledge, have investigated what connects these strategies together. Thus, I intend to present a pedagogical model which can be useful in supporting children with MLD to internalise the numeracy components.

6.9 MKO – driven Strategies

The MKO played a significant role in the process of supporting the learner to internalize the numeracy components being focused upon. Several strategies employed by the MKO seemed to work together to ensure the learner was supported effectively to move within his Zone of Proximal Development (ZPD). Some of these, the seven modes of assistance presented in Section 3.5, were made use of intentionally to explore whether these would be effective in supporting learners with MLD. These seven modes brought forward by the neo-Vygotskian, Tharp (1993), were modelling, feedback, contingency management, instructing, questioning, cognitive structuring and task structuring (p.271-272, see Table 3.2). These seven strategies will now be presented together with instances from the intervention programmes in which they were used.

6.9.1 Modelling

Modelling was one of the strategies used most often throughout the sessions; it was used at least once in almost every session with each of the boys. "Modelling is the process of offering behaviour for imitation…different forms of modelling may be more effective depending upon the conceptual and verbal skills of the learner" (Dunphy & Dunphy, 2003, p. 51). Vygotsky (1978) himself referred to this form of support as *imitation*. When citing work from Shapiro and Gerke, Vygotsky (1978) underscores the importance which *imitation* plays in helping the child to develop. As indicated by Vygotsky, in the view of these two researchers (Shapiro & Gerke) "social experience exerts its effects through imitation; when the child imitates the way adults use tools and objects, she masters the very principle involved in a particular activity" (p. 22). The effect of *imitation* is usually influenced by the response the child receives, since if the MKO praises the child for what is imitated, the child will probably repeat this in other situations. A negative response would have the adverse effect.

Modelling was used in several ways. One way in which it was used was in situations when a new task was presented. This was done not only when introducing an activity during the session but also when presenting the linked recording exercise. An example of the modelling of an activity is that recorded during the third session with Seb. The following is a transcript of this modelling.

EZ — I am going to be counting back from 20 to 0. At first, I am going to use the number line. Look at what I am going to do and then you are going to do the same. [I started counting backwards whilst clapping as I said each of the numbers]. 20, 19, 18, …, 3, 2, 1, 0.

Seb — 20, 19, 18, …, 3, 2, 1, 0 [Seb clapped as he said each number].

(Seb, Session 3)

Similarly, modelling was also used when introducing a digital game. In Section 2.10.2 several arguments were presented in favour of, and against, digital games. A conclusion I drew from this analysis of data is that no computer-assisted intervention can replace the importance of the social interaction that takes place between the MKO and the learner during intervention. Modelling is one evident way in which the MKO is instrumental in the intervention process even when digital games are used. Very few games have a verbal explanation of how they are to be played. Moreover, digital tasks are rarely modelled. In Session 6 with Thomaz, I showed him a digital game that he was going to play but he did not know what he had to do because it was a new game and no instructions were provided by the computer. The task involved ordering the numbers from least to greatest by dragging the numbers and placing them in the correct order on a branch. A screenshot of the task is shown in Figure 6.1.

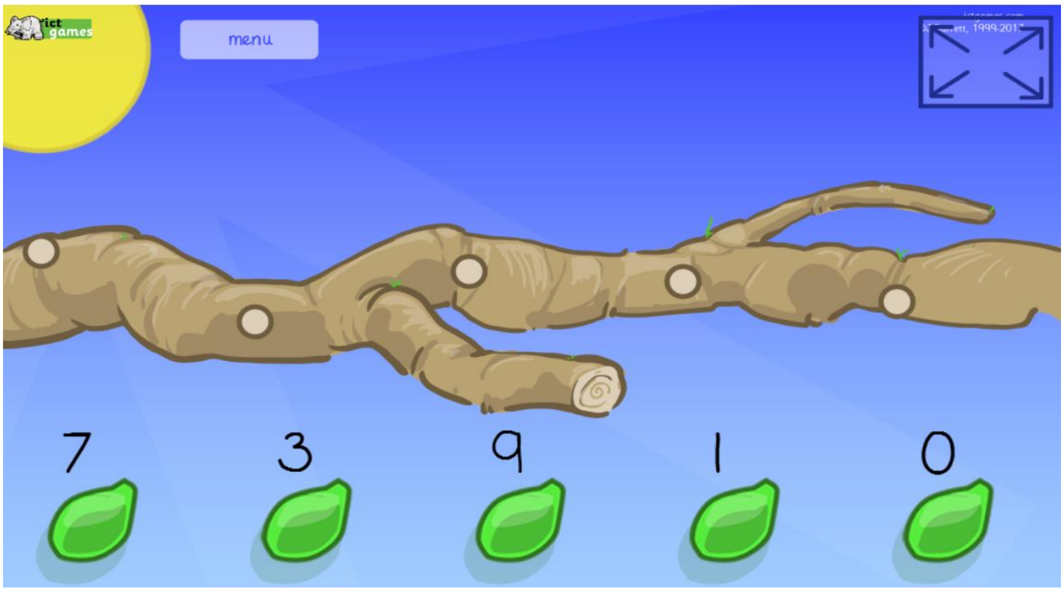

Figure 6.1: 'Counting Caterpillar' task which Thomaz had to complete during his 6th session.

Since at first, he was lost, I explained to him how to play the game and modelled playing it myself. I showed Thomaz how he had to drag the numbers and place them on the branch in the correct order. Moreover, since Thomaz was still struggling with this numeracy component, I also modelled counting forward by saying the numbers from 0 to 9 in sequence whilst clicking and dragging the numbers on the screen to the log as I said each one. This was done to show Thomaz how he could do the same to complete the game. Following this, Thomaz was then able to complete the task successfully. He had understood how to 'click' and 'drag' the numbers and was doing so correctly. More importantly, he was able to count forward, as I had done, to complete the task when he got stuck. This kind of modelling behaviour and interaction was also used in all other sessions in which digital games were used (a total of 32 sessions out of the 120).

Modelling was also used when assigning a task for the linked recording section given during each session. As the MKO I went through the process the child was expected to engage in when working out the tasks so that he could then imitate this process facilitating internalization. For example, in session 12 with Mike, he had to write the position of the yellow box using the ordinal numbers in digits and words. I went through the process of counting, using ordinal numbers and wrote down the answer to the first question as a model of what he had to write. Figure 6.2 illustrates this.

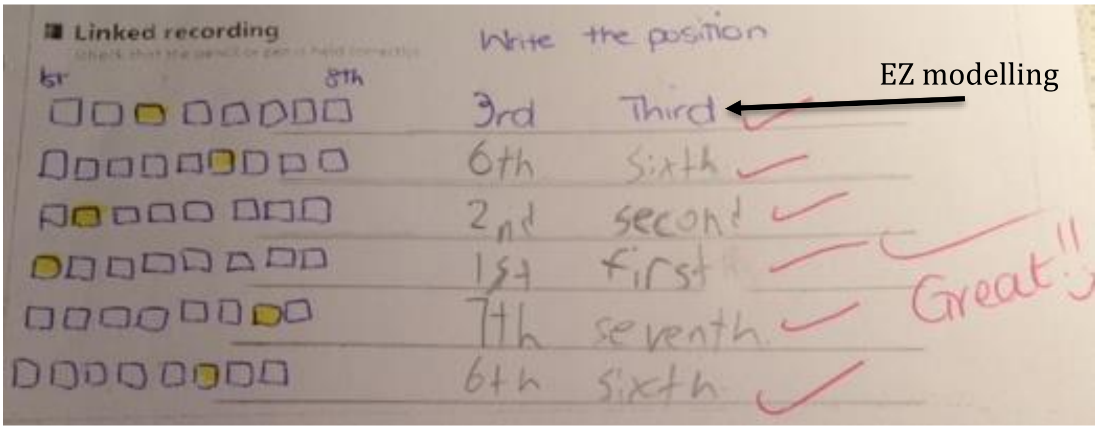

Figure 6.2: The first question completed by myself as a model.

Modelling was also used when the learners made a mistake. An example of such an occasion was recorded during the fifteenth session with Mike, in which we were doing an activity to reinforce ordinal numbers. A row of cubes was placed on the table. Mike had to say the position in which a cube with a specific colour was. Mike identified the wrong position since when counting in ordinal numbers he missed one of the numbers, showing he had not fully mastered this skill. I thus modelled counting using the ordinal numbers to show him how this was to be done. Following my modelling, Mike was able to count using ordinal numbers correctly and to identify the position of the specified cube.

In all the examples mentioned it was evident that modelling had facilitated the internalization process because the children had either imitated the behaviour correctly or managed to work out the tasks on their own after following the modelled example. However, modelling could not always be used on its own and therefore it had to be used in tandem with the other strategies.

6.9.2 Feedback

Feedback given by the MKO was another fundamental strategy that supported the internalization process. Feedback, both oral and written, was used in all the sessions with all the learners. During the sessions, oral feedback was generally given in instances when the child needed encouragement to complete a task, or when praise and/or guidance appeared necessary to keep the learner on the right track. Oral feedback was usually either in the form of a comment or as a question. An example of the former is the following commentary that I made during Ethan's third session,

EZ	Excellent. You did not miss the number 13 this time because you are really thinking hard about it.

(Ethan, Session 3)

Feedback provided was usually not merely a comment indicating whether the learner had done something correctly or otherwise. It was also used as an opportunity to highlight any miscues the child had made and to remind the child how the miscue had been tackled. This seemed to be an important aspect of the feedback provided because this strategy helped to increase the effectiveness of feedback in supporting the internalization of the specific numeracy component.

Feedback was sometimes given as a question. Questioning techniques will be explored separately as another means for the MKO to effectively support learners struggling with mathematics. However, in this section I will illustrate how questions were used specifically as a means of providing feedback. Feedback questions included ones in which the child's thinking was re-routed and others in which the child was asked whether they thought their answer was correct to constantly elicit learner reflection. An example of how questions were used to give feedback is taken from Nathan's 11th session that tackled estimation. Nathan was asked to estimate the answer of '7 take away 3'. At first, he said '12' but then realised it was a 'silly' guess (he had to say whether it was a 'good' (close to the real count) or 'silly' (far to the real count) guess as per the terms used by CUN).

EZ	Why is it a silly guess?
Nathan	It is far away [pause] because minus we need to remove

EZ	Exactly. So, should the answer be more or less?
Nathan	It should be less

(Nathan, Session 11)

In this session, Nathan then applied this new understanding of minus as ‘having less’ and adding as ‘having more’ to the subsequent number facts and he managed to get the rest of the estimations correct. This demonstrated that my question prompted him to make a sensible guess that would not only be useful for this one fact, but also when working out other addition and subtraction computations.

Another example of how questions were used as a form of feedback is that which took place with Seb during the third session, also about estimation. Seb was shown a few objects and had to estimate how many there were.

EZ	How many do you think there are now?
Seb	11. I don’t know! [Seb counts the objects] There are 17.
EZ	Is that a good or silly guess?
Seb	Silly
EZ	Yes, quite far away. What could you do next time? Could you have done something different to get a better guess?
Seb	Look at them and think three seconds about it.

(Seb, Session 3)

I concluded that Seb realised that he was making guesses without really thinking about them and that was why he suggested that he thinks three seconds before giving a reply. During the rest of the session, Seb used the strategy he had come up with during our conversation and managed to get much better estimates. This is evidence that he had internalized how to make a reasonable estimation as per the evidence of internalisation presented in Section 3.6 and Section 6.8. In this example, the questions also served the purpose of prompting the learner to reflect about his miscue and thus supported metacognition (see also Section 3.4.2.1).

Feedback was also given in writing when correcting the linked recording section and at the end of the session in which both the learner and the teacher had to give their feedback. When all the exercise was correct, and the child seemed to have internalized the numeracy component being focused upon, feedback was generally one of praise. An example of this type of feedback was given when Seb had mastered estimation and managed to get all the exercise correct. In this exercise, Seb had to estimate how many cubes there were of a particular colour, count the cubes and decide whether his guess was ‘good’ or ‘silly’. The exercise including feedback is seen in Figure 6.3.

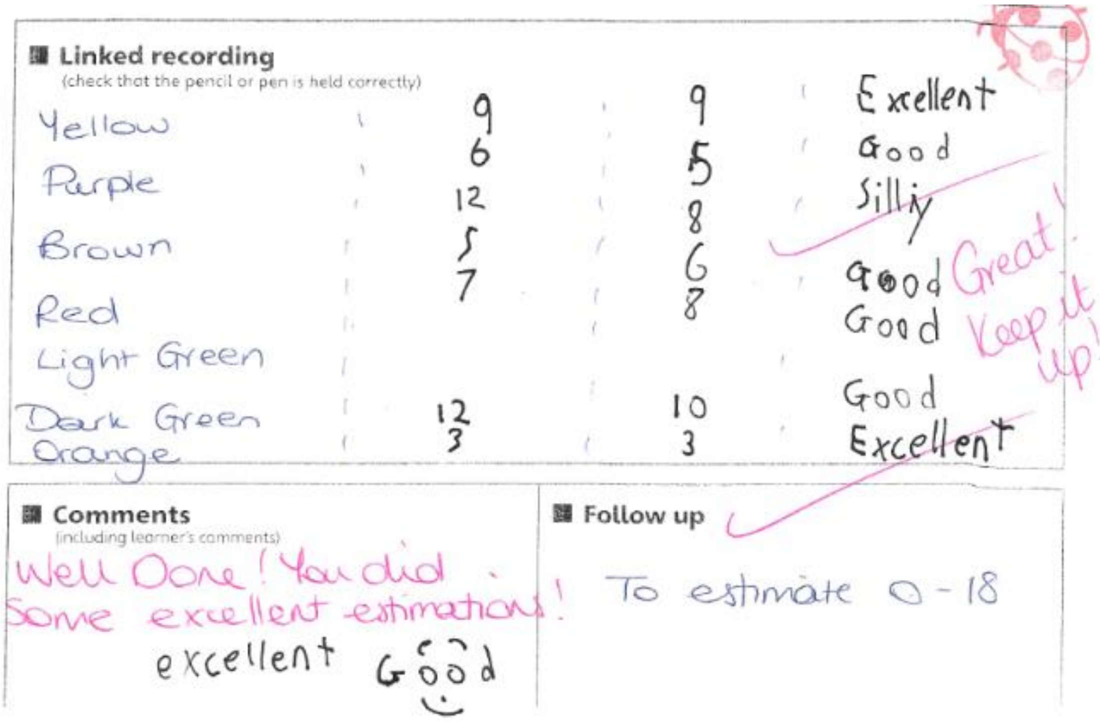

Figure 6.3: Linked recording and comments section showing feedback given when component was internalized.

He decided to write 'excellent' when the count and estimate were a perfect match.

However, when the learner seemed not to have completely mastered a component, written feedback was used in the same constructive way as oral feedback, rather than to simply point out whether answers were correct or not. As a result, the feedback given not only gave praise and encouragement but also highlighted the boys' area for development. An example of this is shown in Figures 6.4 in which Mike seemed to still be struggling with an aspect of the component tackled.

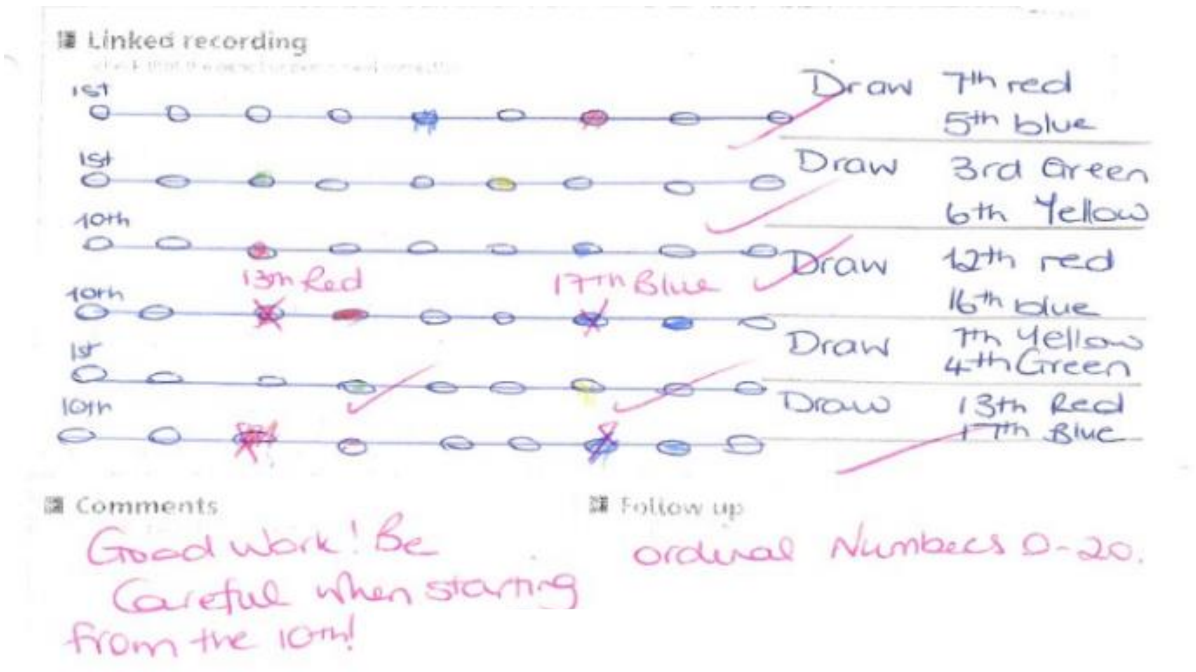

Figure 6.4: Linked Recording and comment section completed with Mike during his 15th session that tackled ordinal numbers.

As can be seen from Figure 6.4, Mike seemed to be still struggling when it came to write ordinal numbers for positions that did not start from the first position but from the 10th. Thus, not only did I give oral feedback as I went through the exercise and re-explained why his answers were incorrect (as shown through the 'X' in pink) but I also commented on this in the comments section (writing in pink pen at the bottom) in the hope that this would facilitate the internalization process. During the following session with Mike, I focused on ordinal numbers again but with a larger number range (0-20). Mike managed to get all the exercise (shown in Figure 6.5) - which included similar questions to those given during the previous session - correct, indicating that the feedback given during the previous session was effective in facilitating learning.

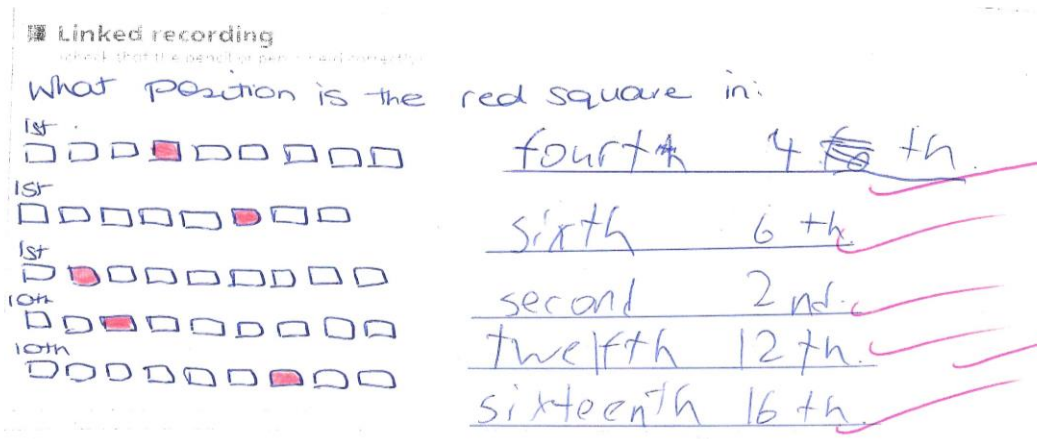

Figure 6.5: Mike's linked recording section for his 16th session showing he could complete the exercise correctly.

Another example of how this form of feedback was given was that provided in the 16th session with Andrea in which ordinal numbers were also being focused upon. Andrea's difficulty was not to count and colour the correct position but to write correctly the ordinal number in words. This was mentioned both in linked recording section as well as the one for comments to support the internalization of the 'ordinal numbers' component. This can be seen in Figure 6.6.

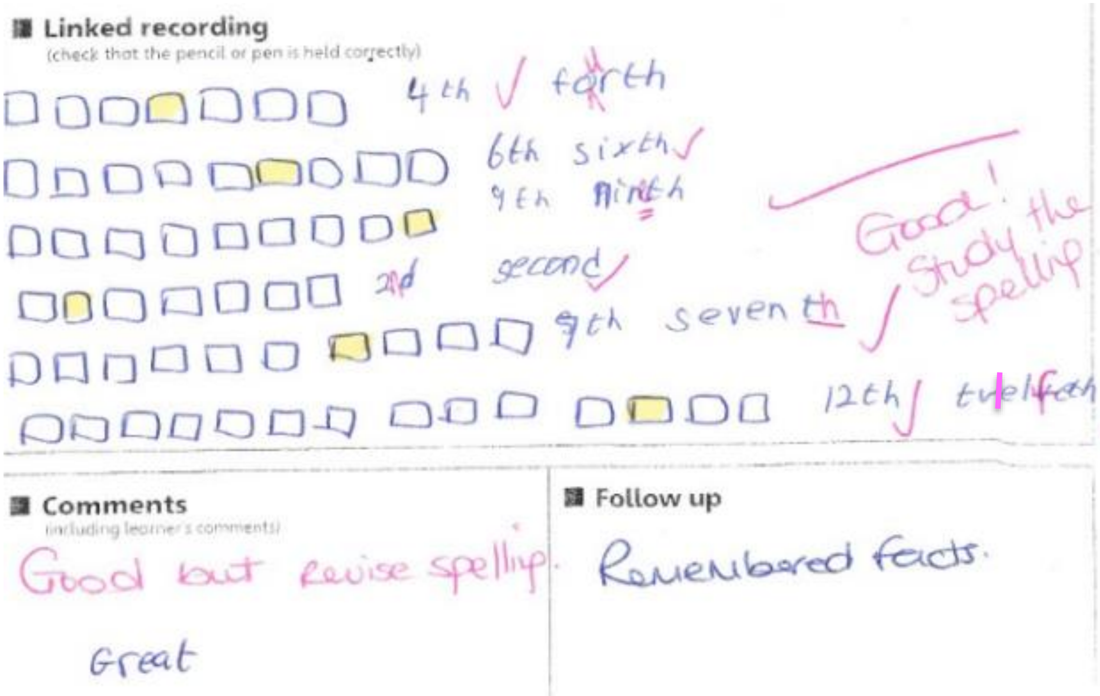

Figure 6.6: Linked Recording and comment section completed with Andrea during his 16th session that tackled ordinal numbers.

I gave Andrea flashcards with the terms he had not yet grasped (see Section 6.12.2.2 about visual aids) and asked him to revise them at home. At the start of session 17th, as a 'review' activity, I asked Andrea to write some of the ordinal numbers in words on his mini whiteboard. Andrea was able to write them correctly except for the word 'nineth' evidencing internalization, as outlined in Section 3.6. The words were revised during subsequent sessions, both orally and in writing, to ensure they were fully mastered.

6.9.3 Contingency Management

Contingency Management was another strategy which I intentionally used to encourage desired actions and language, and to support the child to get back on the right track when these were incorrect. Contingency management is rather behaviourist as an approach. However, I can understand why Tharp (1993) included it as one of the seven modes of facilitating internalization. In Vygotsky's own example (see Section 3.3), which suggests that a child learns to point to an object he wants by observing and imitating his mother, contingency management is what was being used. Contingency management advocates that the MKO should praise and encourage the learner when he makes the right gestures or imitates action correctly e.g. when counting on a number line or uses the right language e.g. when expressing himself mathematically. This with the hope that the learner internalizes the desired actions and language, showing this by repeating the actions and/or language unaided and applying it to other similar situations. Moreover, contingency management should also be used to show the learner that an action or language used is incorrect and therefore should not be used again. Hence, contingency management was deemed necessary to provide the right social interaction to facilitate the internalisation of the relevant actions and language related to each numeracy component.

Contingency management was used in most sessions with all learners. Contingency management took two forms – the verbal form and non-verbal one. Using these two forms ensured that I provided situations of rich social interaction which would include both intrinsic (coming from the learner) and extrinsic (deriving from external factors) motivation and conditions for the learner to imitate the desired actions and language to facilitate internalisation. In line with the literature and research presented in Sections 2.7.1.1 and 2.7.2, the affective domain is highly involved in the learning process. As I highlighted in Section 2.7.1.1 when referring to Adler (2001), "to be able to maintain motivation, a genuine feeling of success is essential" (p.7). Thus, contingency management was used intentionally to maintain a high level of positive affect and intrinsic motivation (Sternberg, 1983) which have been shown to reduce mathematics anxiety (see Section 2.7.1.1). Contingency management was also used to provide a positive disposition that helped to support learners struggling with mathematics to achieve in the various mathematical components. Research suggests that intrinsic motivation is more effective than extrinsic motivation (Jensen, 1995; Kohn, 1993; Sullo, 2007). Kohn (1993) highlights that educators tend to agree that, "at any age, rewards [referring to extrinsic motivation] are less effective than intrinsic motivation for promoting effective learning" (p. 144). Hence, although extrinsic motivation, like giving the learner a sticker when they used the correct action or language repeatedly, was used throughout the programme as a form of contingency management, maintaining a high level of intrinsic motivation was given priority.

Intrinsic motivation was essential for contingency management to be used successfully. It provided the right environment for encouraging the learners to relate to the MKO well enough to feel secure about imitating her actions and language and not shutting down when they were corrected for doing something 'wrong'. Intrinsic motivation was maintained by providing enjoyable sessions which involved the use of manipulatives and digital games, both of which the children loved using. It was also maintained by keeping tasks short and by using examples related to things the children liked. For example, I knew that Andrea loved football, so I tried to use examples from the world of football when possible.

Verbal cues included praising the child when a desired action or language was repeated or applied to a situation correctly. An example of this is taken from the transcription of Seb's third session.

EZ	I am going to be counting back from 20 to 0. [pause] First I am going to use the number line. Look at what I am going to do and then you are going to do the same. 20, 19, 18, 17 … [I clapped to each number]
Seb	20, 19, 18, 17, …, 3, 2, 1, 0 [Seb clapped to each number and counted correctly back to 0]
EZ	Good job! Well Done!

(Seb, Session 3)

In this episode, this verbal encouragement was given by me as the MKO to provide the intrinsic motivation for the child to keep repeating the correctly imitated actions. In fact, the activity was then repeated, and the degree of difficulty was constantly intensified but the verbal cues ensured that Seb was clearly shown that he was doing the task correctly. Verbal promises were also sometimes made to the learners as 'rewards'. For example, since I knew that Thomaz loved playing the online computer games, at times I promised that he would use the computer if he completed the task or exercises given correctly. This provided the extrinsic motivation to maintain focus on the task and to imitate the desired actions or language as many times as necessary. This form of contingency management would allow me to ensure that internalization took place because without the extrinsic motivation, at times the children would get bored of repeating the same actions and get distracted.

Verbal cues were also used when something was not being done correctly. Although Tharp (1993) refers to this as punishment, in my sessions, contingency management was used to re-direct the learner back to the right track rather than to 'punish' him. For example, in Seb's second session, Seb was given a computer game to complete. He had to insert the correct numbers in a sequence in descending order. Although he was instructed to count backwards to reinforce his ability to count back, he completed the sequence by counting forward. Hence, I verbally asked Seb to do them by counting backwards. I repeated what he needed to do and showed him in a playful way that I had caught him "cheating" in the task. Following this contingency management, Seb started to complete the tasks correctly.

Non-verbal strategies were also used for contingency management. These included rewards such as stamps or stickers. Sometimes, at the end of the session, if any of the learners had done the task correctly and appeared to have internalized the actions and language related to the numeracy component being tackled, they were also given a stamp apart from the positive feedback in the feedback or linked recording box. During the third session with Mike, I was supporting Mike in internalizing the use of a number line to find out what needs to be added to a number to produce another given number. Since Mike managed to make use of the number line, by imitating my actions correctly (using his finger to point to one number at a time as he added the given number), I rewarded him with a sticker. This can be seen in Figure 6.7.

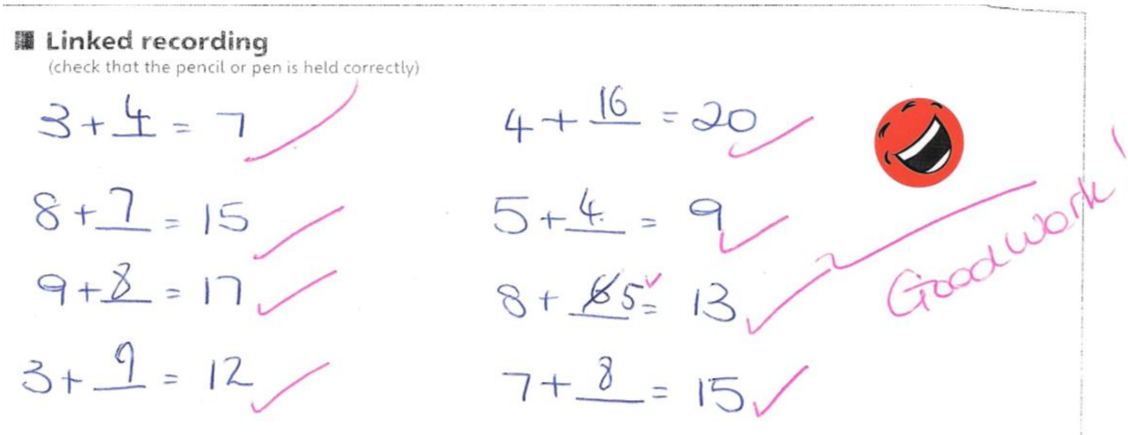

Figure 6.7: The linked recording section from Mike's third session showing the sticker rewarded as contingency management.

At times, I also gave the children the opportunity to choose a sticker to take home if they had successfully learnt the action, task or language that I would have introduced them to during that session. An example of this was when, during his second session, Andrea used different terms for 'counting forward' like 'counting on' (explained further in Section 6.10.2). I allowed him to choose a sticker as a reward for using the correct mathematics language.

Since the learners responded positively to the ways in which contingency management was used, I conjecture that the chosen forms of contingency management may have had a positive impact on the internalization process. Contingency management allowed me to interact with the learner positively both when showing him that the actions or language used were appropriate as well as when these were incorrect and had to be adjusted. This allowed me to reduce mathematics anxiety which, as outlined by previous studies in Sections 2.7.1.1 and Section 2.7.2, not only has a positive impact on the learner's self-confidence in mathematics but also has a constructive and direct influence on the internalisation process. Contingency management also permitted me to encourage the learners to use and apply actions and language imitated correctly and to refrain from re-using these when they were incorrect.

6.9.4 Instructing

Another mode of facilitating the learner's journey within the ZPD is instructing (Tharp, 1993). This strategy was used very often with all learners, in one or two ways. Instructing was made use of in two main ways. Primarily, instructing was used in a general sense to ask the learner to do something e.g. 'pick a number card' or 'come and sit at the computer'. Secondly, instructing was used to provide clear directions to complete a task. Moreover, instructions were usually accompanied by modelling, to ensure that the learner was clear about what had to be done in the task given (hence about the procedure itself). Most instructions were given orally during the different parts of the sessions. For example, during the tenth session with Andrea I instructed him to read my number word cards as follows.

EZ I have number words and non-number words [pause] digits, you need to read out the numbers as I show them to you [pause] this is number 4 [showing him the digit four on the card]…you just need to say four.

(Andrea, Session 10)

Andrea completed the task correctly showing that he had understood my instructions.

Instructions were also used in the digital games. Some games included on-screen instructions that explained how the individual had to play the game. Other games had no instructions. When no instructions were given, these had to be provided by myself. When the game did give them instructions, I always had to re-explain the digital game, in simpler terms, after the children had listened to the instructions given by the game itself. In Nathan's nineth session, Nathan was asked to play the game entitled 'Island Chase'. A screenshot of the first screen can be seen in Figure 6.8.

Figure 6.8: Screenshot of the online game 'Island Chase' which Nathan played during his ninth session (taken from https://www.arcademics.com/games/island-chase).

No instructions were provided by the digital game itself and so I gave Nathan instructions myself. This is the extract from the session in which I explained the game.

EZ **Look at what you have to do** [pause] **You have a speedboat…You need to do** 3 minus 2 **and be as fast as you can,** OK? Equals 1 for example [EZ clicks the answer on the screen]. 8 minus 3, five [EZ clicks the answer on the screen]. 6 minus 5, one. You are the blue boat. Every time you get a correct answer you move forward a bit further. Ok? You need to try and beat the other boats. Can you see it is moving? We are first in fact. [Nathan starts to try it out]. If you do it wrong, you move a bit back. **Let me do one wrong on purpose so you understand** [did one wrong] OK? Did you understand? Who finishes first is the winner. Ok, we came second. Ok, Ready?

Nathan, Session 9

Following my instructing, Nathan was able to complete the game with success. Primarily, as on this occasion, I also modeled the digital task to support the child in internalizing how the game should be played and therefore the steps of playing the game. Secondly, I used modelling to remind the child of how to complete the mathematical aspect of the task, in this case answering subtraction sums. The role of the MKO in acting as a facilitator when digital games were used will be discussed at a later stage. In Section 6.12.2.3 I will argue that, although digital games can be effective in supporting the internalisation of the numeracy components they deal with, they should never take the role of the MKO since social interaction is important in ensuring learning is facilitated in the best possible ways.

During written tasks I sometimes wrote instructions myself. In the linked recording section, I generally gave the learners written instructions to complete the task so that they could refer to these as often as required so as to be reminded about the aim of the task. An example of such instructions can be seen in Figure 6.9 that shows the written instructions "Write the position" given to Mike to complete the linked recording task (12th session).

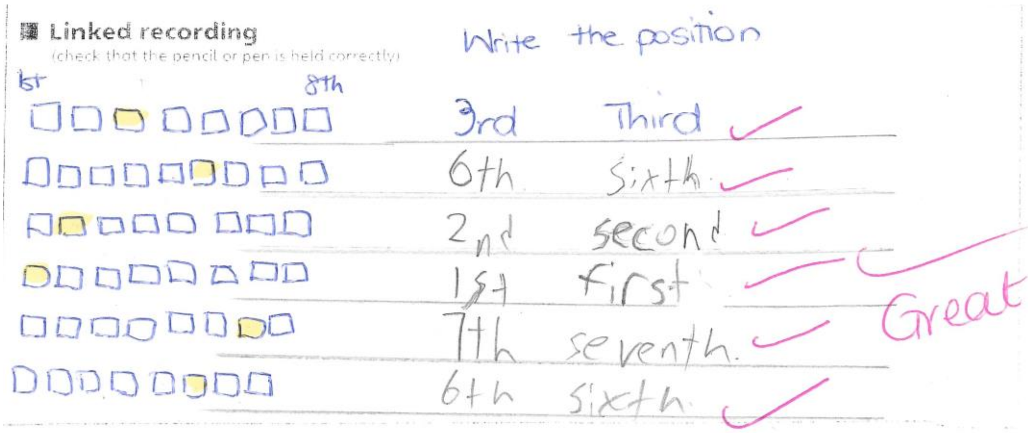

Figure 6.9: Written instructions as part of the linked recording task from Mike's 12th session.

When, during the linked recording part of the sessions, written instructions were not used, verbal instructions were used instead. Verbal instructions became another important strategy to support the learning of the numeracy component being targeted. For example, during the 11th session with Ethan, I gave Ethan oral instructions to complete the task shown in Figure 6.10.

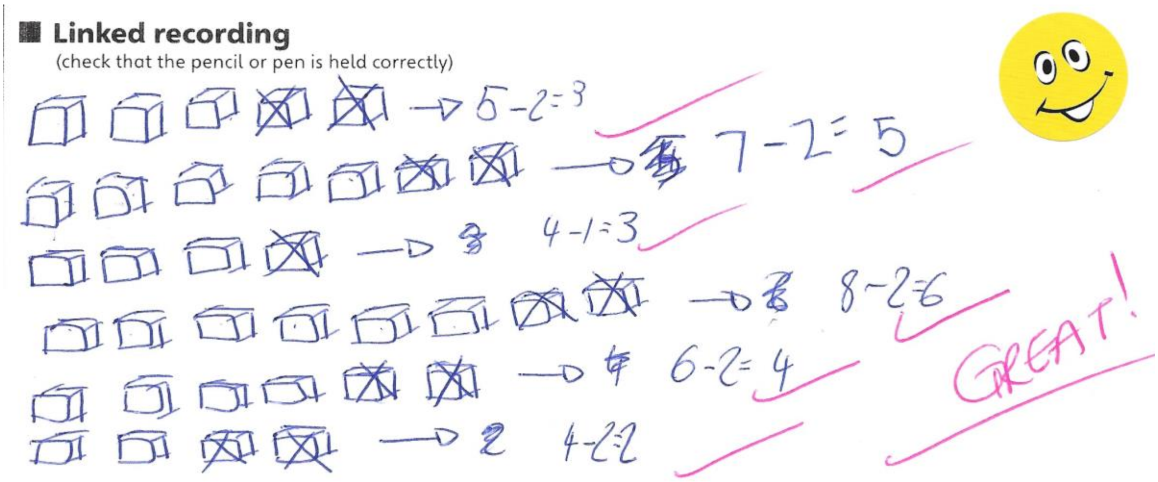

Figure 6.10: Task given to Ethan during his 11th session.

In this task Ethan had to translate from objects to symbols by writing a computation that showed what was being represented in the images. Written instructions were given on the mini whiteboard saying, *'Write a sum to show what is happening with the cubes below'*. However, Ethan had not understood how he had to complete the task. He looked at me puzzled and I re-explained verbally. This is my re-explanation:

EZ	Let me show you what you have to do [pause] [indicated the first example] [pause] For example, what is it showing you here?
Ethan	[Ethan looked at the cubes to interpret them] 5 minus 2
EZ	5 minus 2 equals [pause]You have to write the whole subtraction sum including the answer, not like this [I wrote 5 – 2] like this [I wrote 5 – 2 = 3]

(Ethan, Session 11)

Ethan was then able to write the sums as I had modeled them. Ethan could read what I had written. However, it seemed that he had difficulties with interpreting my written instructions. Had I not been working with him on a one-to-one basis, I probably would not have been able to provide the right scaffolds for him to understand how to work out the operations. The social interaction used when I, as the MKO, gave him verbal instructions about how the task had to be completed, was much more effective than the written tasks themselves.

6.9.5 Questioning

Different questioning techniques were used during every session with each of the learners. My analysis of the data allowed me to conclude that throughout the intervention programme, questioning techniques had three fundamental functions in facilitating the learning process:

i. Questions were important in assessing the learner's understanding to gain insight into where the child was at, vis-à-vis their zone of proximal development – hence to get to know the previous learning and experiences they have brought to the session and at which level my intervention should be pitched. This form of formative assessment has been discussed in Section 2.9.2;
ii. Questions were instrumental in engaging the learners in a metacognitive process that encouraged them to think about their learning process (see Section 3.4.2.1);
iii. Questioning techniques were also very helpful in shedding light on any misconceptions the learner had developed such as when, through questioning, I found out that one of the learners did not know the difference between estimation and counting.

When planning for the intervention sessions, I made sure that several questioning techniques were made use of as mentioned in Sections 3.4.2.1, 3.5 and 4.7.3. Questions asked were either open-ended or close-ended. Some of the questions were subject-specific and therefore were specifically asking for mathematics knowledge. Two examples of these kind of questions were "What is 13 take away 11?" (Nathan, Session 9) and "How can you work out

10 minus 8 using a number line?" (Nathan, Session 10). Other questions asked were more general. For example, on many occasions, following instructions given, I asked whether the learner had understood the task at hand. These questions would then also guide subsequent questions, actions or verbal communication. For example, in the situation mentioned earlier (see Section 6.9.4) when Nathan did not understand how to play the 'Island Chase' game on the computer, it was only because Nathan said that he had not understood the game that I explained it and modeled the task. I also kept asking him whether he had understood and kept checking his understanding whilst completing the task. This allowed me to assess the effectiveness of my explanation and modelling of the task in facilitating learning. Thus, questioning techniques allowed me to know when I needed to facilitate the learning process.

As pointed out in Section 3.4.2.1, one of the crucial strategies of engaging the learner in metacognitive thought processes through higher order thinking is questioning (Montague, Warger, & Morgan, 2000). Questions thus also served this purpose. *Catch Up® Numeracy* (2009) provides questions (see Figure 4.9) that should be used frequently in the intervention sessions. These were used throughout the session. For example, in the 11th session with Ethan I noticed that he was counting backwards. Just before this point, he had been counting backwards incorrectly and an exercise was completed after his miscues, in which he had to count back but could use the number line to help him. The same exercise was then repeated without the number line and Ethan now managed to complete the task. At that point I asked:

EZ	How are you remembering the numbers now?
Ethan	Like this [pause] first I say 11 then I put a zero instead of 1 then they just go down…

(Ethan, Session 11)

What I believe Ethan had done was to develop a pattern that he kept in mind regarding how the numbers change as they become smaller. His main difficulty was counting back from the number 11 to 10 but he could now do so by remembering that the 1 in 11 'became' a 0. Through the question I posed, I supported Ethan in thinking about how he had internalized the number sequence like a pattern. Other questions asked to support this metacognitive process included: "How did you work it out?" (Nathan, Session 10), "Why is it wrong?" (Seb, Session 2) and "How else could you have done it?" (Andrea, Session 14).

Questioning techniques seem to have been effective in contributing to the internalization process since they allowed me to assess any miscues the children had and to address these. Various situations in which questions were used allowed the learner to master the knowledge and skills at hand. One of these is evidenced through the following dialogue that took place between Nathan and me during Session 11:

EZ	What is to estimate? [pause] **What is to estimate?** What does it mean to estimate? [Nathan didn't reply]
EZ	Do you know what it means? [Nathan didn't reply] Do you know the difference between to count and to estimate?
Nathan	Times?
EZ	Times? If for example I say look at these cubes and count them, what are you going to do?
Nathan	**Count them**
EZ	Ok, count them, so can you count them please?
Nathan	1, 2, 3, 4, 5, …, 8
EZ	Good. Then if I tell you look at these cubes and estimate how many there are, what would you do?
Nathan	**I guess**
EZ	You try to guess how many there are [pause] so the answer does not have to be exact as long as your estimate is close to the number of objects [pause] you need to look at the number of objects and tell me how many there are [Nathan then completed the whole exercise correctly].

(Nathan, Session 11)

In this example the questioning techniques used and my explanation of what "estimate" means appeared to help him to recall something he already knew from his previous experiences. As discussed in Sections 2.6 and 2.6.1, most learners with MLD have a deficit in short-term and long-term memory. Nathan had the second lowest percentile score in the non-verbal reasoning test from the BAS II (47^{th}) hence this sort of questioning was important for him to recall important concepts or terms covered earlier. In this situation, questioning seemed to have facilitated Nathan's remembering and understanding of the term 'estimate'.

6.9.6 Cognitive Structuring

The penultimate MKO-driven strategy presented by Tharp (1993) is cognitive structuring. Cognitive structuring was used in most sessions with all the boys. According to Tharp (1993) 'Cognitive structuring' is a means through which the MKO provides the explanation of a task or newly taught concept that supports the internalization process by providing the right cognitive structure for the learner to be able to internalize the new learning. In my study, cognitive structuring was made use of mostly when the learner was exposed to a new concept, when he seemed to have misconceptions about a concept that he had already been taught earlier, and when he was unsure of how a tool (refer to Sections 3.4 and 6.12) could be used effectively. For example, in the third session with Ethan, it was clear that Ethan did not know the difference between a number line and a number chart – two of the technical tools (see Section 3.4) we used during the sessions. Thus, I explained what each of them was:

This is a number line [I pointed to a number line]. It is a line and it has the numbers from 0 to 20. A number chart is square and has many more numbers than this [I pointed to a number chart] (EZ).

This explanation is a clear example of cognitive structuring since it was intended to help the learner distinguish between a number line and a number chart. It served as an explanatory structure that could organize and justify this new learning (see Table 3.2 in Section 3.5). Indeed, when I referred to the number line in the rest of the activities Ethan knew to what I referred, showing that internalisation had taken place.

On another occasion, during Seb's first session, Seb was counting backwards incorrectly as he was leaving out some of the numbers and was very hesitant to count back especially from '18 to 10'. Thus, I modeled counting backwards and asked him to identify what his miscues were and together we took note of the fact that he kept forgetting to write the '0' (when completing the linked recording task) in the 'miscue' section. This note can be seen in Figure 6.11.

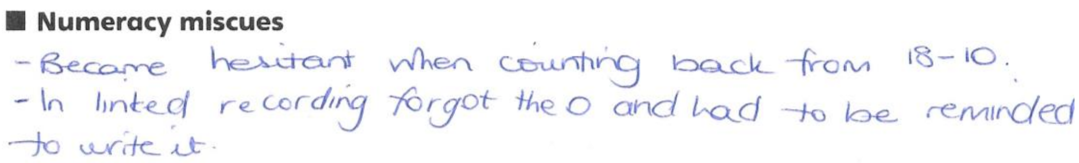

Figure 6.11: Numeracy Miscues section taken from Seb's first session showing Seb's difficulties.

This seemed to help him to remember to say the numbers he was leaving out and also to continue counting until he got to '0'. In fact, when he was given a similar task in his second session, he did not forget any numbers but counted backwards correctly and remembered to count back to '0' as instructed. Moreover, he was able to complete the linked recording section of his second session correctly without help. Figure 6.12 presents the linked recording section completed by Seb during the second session which demonstrates his internalisation of counting back.

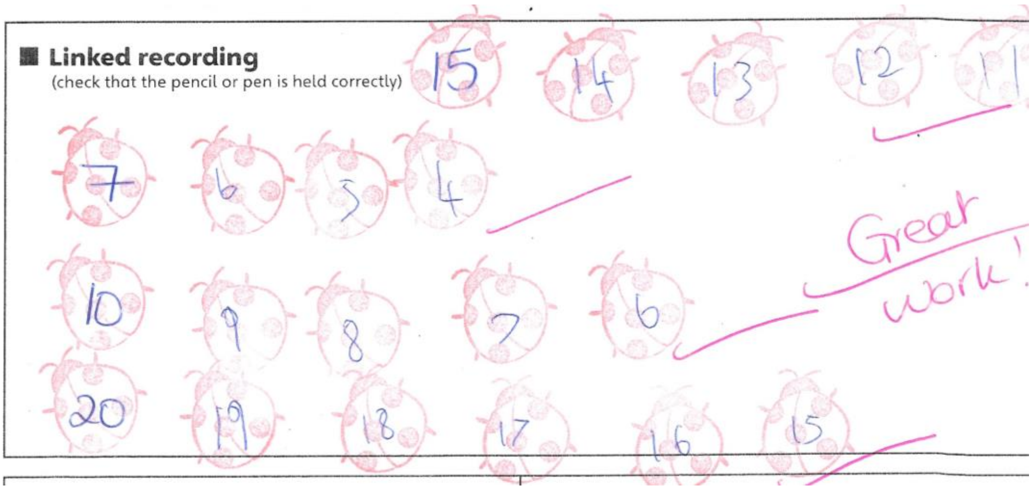

Figure 6.12: Linked Recording section taken from Seb's second session showing he had internalized counting back.

Cognitive Structuring took various forms. On most occasions it was an oral explanation given by the MKO however at times Cognitive Structuring was carried out through modelling (as seen in the example just given – taken from Seb's second session), by giving an example when the child was stuck or by using questioning. Questioning was explored in the section about modelling (see Section 6.9.1) and in Section 6.9.5 when it was mentioned that questioning techniques permitted the MKO to provide the right support for internalization to take place. Indeed, the seven modes of assistance overlap since social interaction is complex in its nature.

6.9.7 Task Structuring

Tharp's (1993) last means of assisting and facilitating internalization is Task Structuring (in Section 3.5). This is when the MKO modifies a task, or part of it, to fit the actual level of the child development when the task is beyond the child's ZPD. Thus, Task Structuring took place every time the learner could not complete a specific task because it was beyond his zone of proximal development for that particular concept or skill. This was usually the case when the learner did not know how to complete a task, could not give an answer to a mathematical question posed or seemed unable to make use of a provided tool. Task Structuring was sometimes carried out in the following situations:

i. To re-explain using simpler terms any instructions that the learners had not understood including those given by computer games;
ii. To change a given task to a simpler one by for example giving the learner smaller numbers to work with;
iii. To provide oral instructions when these were written and could not be read or understood;

iv. To give the learner different tasks with the same aim which provide opportunities to gain more practice in newly mastered concepts;

v. To provide clear and easier to follow instructions.

Task Structuring did not take place very often since I knew the children very well and was aware of their actual zone of development, so the tasks provided were mostly tasks that they would be able to complete. Moreover, the *Catch Up® Numeracy* (2009) programme is structured and scaffolded, as it requires that tasks are mastered in a lower number range before moving to a higher one. However, at times, even though I would have thought that the child would be able to complete a given task and work within an appropriate number range, they were not able to and hence I would intervene. For example, in one situation during my tenth session with Ethan I asked him to estimate several cubes (there were 15) and Ethan seemed unable to. The following interaction took place.

EZ	Do you think I have more or less than 9? [Ethan seemed to be counting.] **Come on**, just say how many you think there are
Ethan	15

(Ethan, Session 10)

Ethan's reply showed that he had counted the cubes. Thus, I went back to estimating smaller amounts of cubes and gradually increased the amount.

Task Structuring was sometimes carried out by offering the child alternative strategies for how he could complete a task in which he got stuck. For example, when Nathan got stuck in his 15th session and could not work out the operation '19 – 7', I offered him alternative strategies and said, "you can use the number line, you can use any strategy you like, even the cubes, if you can do it with the cubes" (EZ). In this same session, Nathan then managed to work out the operation using the number line. This meant that Task Structuring was successful in supporting the learner to complete the task in this case.

6.10 Additional Strategies Emerging from the Data

Up to now, I have discussed how each of Tharp's (1993) strategies was made use of intentionally as a *strategic scaffold* (Hobsbaum et al., 1996) (see Section 3.5) to facilitate the internalization process. Following my intervention using one or more of these strategies the learner was successful in mastering the components and this success undoubtedly contributed to the progress the children made as shown by their end results. Tharp's (1993) seven means of assisting scaffolding were pre-determined themes when analyzing the data using thematic analysis (see Section 4.11). However, as I analysed the recordings from the sessions I noticed that these seven strategies may not have been the only factors and that I seemed to have used other strategies repeatedly which may also have contributed to the internalization process. I used these latter set of strategies as they are ones that I have used throughout my teaching experience and which I found to be effective in facilitating the learning process. However, they were not originally part of the set of strategies (presented in Section 3.6) that I was intentionally planning to use during the intervention sessions. Hence, they can be labelled as *incidental scaffolds* (Hobsbaum et al., 1996). These strategies, that emerged as new themes during the analysis of data, shall now be discussed.

6.10.1 Role Inversion

What I am referring to as 'role inversion', were situations in which the learner was invited to take the role of the MKO. When I started planning the intervention sessions, the concept of 'role inversion' was one that I had come across (Roth & Radford, 2010) (Section 3.5) however, I had not actually planned to use it as a strategic scaffold since it had not been applied to mathematics education. Later, after exploring this aspect further through my analysis, I found that other recent neo-Vygotskian researchers (Abtahi, 2014; Graven & Lerman, 2014) also investigated this shift in role, specifically within mathematics education. Hence, the concept of 'role inversion' is now not new in mathematics education. In fact, a study by Abtahi (2017), published after my design and delivery of the programme, proposes some similar findings to my own was. In her study Abtahi (2017) highlighted that after Vygotsky many researchers,

> have studied children's mathematical learning within the ZPD, viewing the more-knowledgeable others as agents and viewing the role of the more knowledgeable as alternating – in other words, pointing to an 'other' who is the more knowledgeable, regardless of whether he or she is a child or an adult (p.36).

Albeit the participants in my study were never truly the MKO due to their difficulties with mathematics, when role inversion was used, I gave them the possibility of being the MKO. This was done by acting as if I did not know the numeracy component being tackled. In fact, I continuously asked them to explain any related tasks to me, empowering them to take up this new role.

Cultural tools have also been explored for their potential of taking up the role of the MKO. Abtahi (2017) argued that in the situation presented by Graven and Lerman (2014) (see Section 3.4), not only could the child take up the role of the MKO, but that a cultural tool, such as a digital one (discussed further in Section 6.12.2.3), could also take up this important role. Although these preceding studies presented a similar idea to what I label here as *role inversion*, the way in which this idea has been applied to social interactions which facilitate mathematics learning specifically for children with MLD is, to my knowledge, a new contribution in mathematics education.

Throughout the sessions, it was obvious that I repeatedly made use of role inversion between the teacher and the learner. This alternating in roles will be discussed in this section however I will also discuss the possibility of cultural tool, including artefacts, replacing the MKO in some circumstances. Inverting roles between the MKO and the learners seemed to have had multiple positive outcomes. Primarily, role inversion seemed to give the learner the opportunity to verbalise his thoughts and to feel empowered to take hold of his learning process. This undoubtedly supported the internalization process since the child was also given the chance to make use of that new learning within a context that in itself seemed to help them to make sense of what they had learnt. This strategy was effective with the learners, as they appeared to recall more readily the situations in which they acted as the MKO and even referred to these instances in later sessions. I must admit that this was also one of the strategies that the children preferred because they would eagerly take up my role and pretend that they were the adult figure. Secondly, role inversion allowed me to see whether the learner had really internalized the numeracy component, and the related mathematical language, at hand. What follows are now some examples of situations when role inversion was used.

In Andrea's second session, we were doing the paper clip activity in which a paper clip is placed somewhere on the number line and the learner is to count on from that number up to 20. Usually it is the MKO who leads the activity and places the paper clip somewhere and then asks the learner to count. However, the forthcoming excerpt shows how I asked Andrea to lead the task instead.

EZ	Now you put the paper clip somewhere and tell me what to do
Andrea	[Andrea put the paper clip on the number 3] Count up from 3 to 20
EZ	3, 4, 5, …, 20 [pause] Now, one last one: you put it where you want
Andrea	Count forwards from 12 to 20

(Andrea, Session 2)

In this role inversion situation not only did I allow Andrea to practise asking me to count up using the language introduced during the previous session (count up, count forward and count on) but the role inversion also acted as evidence of internalization as Andrea was able to use them appropriately. Hence, role inversion supported the learner to internalize the learning by thinking about what has been learnt and communicating it to the MKO. Moreover, I also argue that in this circumstance, role inversion was evidence of the internalisation that took place since if the child could take up this role, it showed that he had truly internalized the learning.

Another example of how role inversion was used will be presented in the Miscue Analysis section (Section 6.10.5) in which both strategies were used together. Through this analysis, my findings are similar to other studies, which have shown that the role inversion between the learner and MKO is valuable in the analysis of mathematical learning (Lerman & Meira, 2001; Roth & Radford, 2010). However, one must treat these conclusions with caution since as indicated in other studies (Abtahi, 2017) it is complex to identify who the MKO is in specific interactions.

6.10.2 Recapturing

Another strategy that seemed effective in supporting the learning process was what I chose to label as 'recapturing'. When analyzing the data, I noticed that on many occasions I reviewed or 'recaptured' the learning that took place to summarise the main learning points tackled in a previous learning circumstance or intervention session. At the start of each session it had become a standard question to ask the learner what we had done during the previous session. This appeared to be an effective strategy to use at the start of the session because it helped to frame and link the previous learning to the learning that would then take place during that particular session. Moreover, it would give me the opportunity to assess the child's capability (actual zone of development) to undertake the new learning that I had planned to take place during the forthcoming session. This happened since when asking about the previous session I could elicit important information from the learner which included whether the child knew and could remember the numeracy component, number range and mathematical language covered in the previous session. An excerpt taken from Andrea's second session illustrates the use of recapturing.

EZ	From what number to what number were we counting on?
Andrea	0 to 20
EZ	And I gave you the cards to take home, right? [Andrea nods.] Yes, good, what was written on the cards?
Andrea	Counting up, counting forward and counting on
EZ	Excellent, so I want to hear you make use of these new phrases you have learnt.

(Andrea, Session 2)

One of the strategies used for recapturing was that of asking the child about whether he remembered what his miscues were in the previous session. This allowed me to ensure that the learner had grasped the numeracy component focused upon in the previous session to be ready for the new learning now taking place. For example, in his third session, Seb remembered that he was forgetting to say the zero when counting back. In Ethan's third session, he remembered that he kept forgetting the number 13 when counting back. Recapturing their mistakes demonstrated that the Miscue Analysis strategy (discussed further in Section 6.10.5) used during the sessions was effective in supporting internalisation. Reminding the child of the miscues done during the previous session, through recapturing, also helped the child to ensure that these miscues were not repeated during the current session.

'Recapturing' also took place after a task or at the end of a session. I used this strategy to recapitulate the main learning points. This summary allowed the learner to focus on the specific learning of the numeracy component at hand. Frequently, repeating the focus of learning was noted to support the learner to master that learning. Hence, at the end of the session I felt that 'recapturing' the learning that had taken place during that time facilitated the process of remembering and also internalizing that learning.

6.10.3 Prompting

Prompting was yet another strategy, that although used as an *incidental* scaffold (Hobsbaum et al., 1996) (Section 3.5), appeared to facilitate the learning process. What I labeled as 'prompting' were situations in which verbal and non-verbal cues were given to help the child remember a concept, skill or language that had previously been learnt. Prompting was given in various ways. One way was by re-explaining a task. This allowed me to assist the learners in completing a specific task and therefore in that case prompting also served as a strategy to provide Cognitive Structuring (Tharp, 1993, refer to Section 6.9.6). Another way of prompting was that of asking a question that supported the learner to complete a task. For example, in his second session Ethan had to complete a number sequence in writing. However, he got stuck on the number that comes before the number 15. So, I prompted him by asking the following question "Which number comes before 15?" which was an echo of the written task that Ethan could not complete. The verbal cue assisted Ethan in getting to the right answer quicker. Questions such as these were not only used as questioning techniques but also served as prompts.

At times, prompting also served as a tool to provide the right scaffolding for the learner to be able to move from the current zone of development to the potential one. This because for some tasks prompting was initially used but once the learner could complete the task on his own, prompting would be removed. For example, in Seb's first session I explicitly said, "For the first couple of times I am going to leave the number line [on the table] and then I will remove it" (EZ). Hence the artefact in this example was used as a prompt. However, artefacts had other purposes, and these will be referred to in subsequent sections. Other ways in which prompting was carried out included: helping the learner to choose the correct strategy or tool to complete a task, as well as starting a task myself and asking the learner to complete it.

6.10.4 Putting the Learner at Ease

Another evident finding that was drawn from the transcriptions was that I, as the MKO, had made the learners feel at ease. This was evidenced in the numerous times in which the different learners started to talk about things that related to their everyday life rather than those related to mathematics learning. For example, in Ethan's tenth session he started talking about karate, "It is the first time I am going to karate today…I am going to karate lessons with my cousin". Another example is the following taken from Nathan's ninth session.

EZ	Does you mum bake cakes? [Nathan nods] Do you eat any of her cakes?
Nathan	When they are hers, yes, but not when they are for others, for example, when they are for someone of our family. She sells them as well!
EZ	Great! Then when someday I will need a cake I will tell her!
	(Nathan, Session 9)

Nathan was a rather reserved child who did not speak to me much. However, in this situation, it was obvious that he was completely at ease as he was talking about something personal. Allowing the children the time to talk briefly about personal aspects of their lives served to build a good rapport with them.

The self-evaluation sections completed at the end of each session were also evidence that the children were enjoying the sessions and feeling at ease. These sections allowed the children to engage in meta-affect (DeBellis & Goldin, 2006) and thus may have also in themselves been a main contributor to this shift in attitude. During Thomaz's fourth session, when I asked him to complete the self-evaluation section and to reflect about how he felt the session had gone, he wrote the comment 'I love it!'. This can be seen in Figure 6.13.

Figure 6.13: Thomaz's self-evaluation section – taken from his 4th session.

Thomaz did not like mathematics when he first started the sessions and yet, as the sessions unfolded, he constantly asked me to continue with the mathematics session for longer. This illustrated that his attitude towards the subject had shifted because as suggested in Section 2.7, although values and beliefs about mathematics are not malleable, attitudes are (Hannula, 2002). This shift in attitude is important since, as highlighted by Grootenboer's (2003b) model (Figure 2.6), positive attitudes lead to *stability* and *cognition* which are optimal for learning. As indicated by Gresalfi and Cobb (2006), "it is not sufficient to focus exclusively on the ideas and skills that we want students to learn" (p.55), but it is as important to focus on the affective domain of learning.

These situations allowed me to conclude that making the learner feel at ease is an important requirement of any intervention programme. As a result, it appears that when the participants of this study were at ease with the MKO and the situation, they were more appropriately disposed for the learning to take place. This disposition may have resulted in an increased positive outcome of the intervention programme. What my finding seems to indicate is in line with previous studies (Chinn, 2012; Sousa, 2008; see Sections 2.7.1 and 2.7.2) and shows that unless the affective domain is catered for, learners with MLD may not reap the maximum benefits possible of the intervention provided. This finding also leads me to conjecture that the one-to-one nature of the sessions was also a successful factor in the internalisation process as this circumstance allowed the learner to be given the individual attention required to feel at ease with the MKO and thus to focus on the mathematical knowledge and skills at hand. To conclude, I believe that another important MKO-driven strategy in the facilitation of the learning process is ensuring that a good rapport is built between the learner and the MKO. It is crucial that the MKO assists the learner in feeling at ease throughout the learning process.

6.10.5 Miscue Analysis

Miscue analysis - analyzing the learner's mistakes through questioning and reflection - was another effective MKO-driven strategy which served as an *incidental* scaffold (Hobsbaum et al., 1996) (Section 3.5) and had not been planned for as a part of the model (Figure 3.1) I was following. When analysing the data, it was evident that miscue analysis was a powerful way for the MKO to gain insight into the children's misconceptions. Moreover, this strategy was also powerful because it seemed that when the learners were asked to reflect about their miscues, they were more aware of their mistakes. In fact, when in subsequent sessions they were asked to recapture the previous learning, most of them could also mention their miscues. Furthermore, they consciously tried not to repeat these mistakes in that same session or subsequent ones. For example, in the second session with Ethan we were working on counting back; he kept forgetting the number '13' and this was pointed out to him. In the following session, I gave Ethan an exercise which he completed well ensuring that he included the number '13'. Furthermore, when I asked Ethan whether he remembered what mistake he was doing during the previous session, he immediately said that he was leaving out the number '13'. This is evidence that miscue analysis did support the internalization process because Ethan was now aware of his previous mistake and tried not to repeat it. In another session with Seb (3rd session), Seb was asked to count from a given number back to zero. However, he kept forgetting to say the zero at the end of the sequence. So first I asked him to tell me what mistake he was making, and he immediately replied that he was forgetting the zero. Following this I used the role inversion and miscue analysis strategies together and asked him to ask *me* to count back. When he did, I did not mention the zero purposely to see if he would notice. He immediately told me that I had not said 'zero' illustrating that miscue analysis may thus be yet another effective strategy for facilitating the learning process. This mistake was referred to over again during the session. Even when giving feedback, I made sure to mention his miscue and said, "Very good…and I can see that you are remembering to write the zero". Such repetition appeared to support the learners to remember their miscues.

In the 16th session with Mike, he had been writing the ordinal numbers incorrectly. I had filled in the 'miscue analysis' section as seen in Figure 6.14.

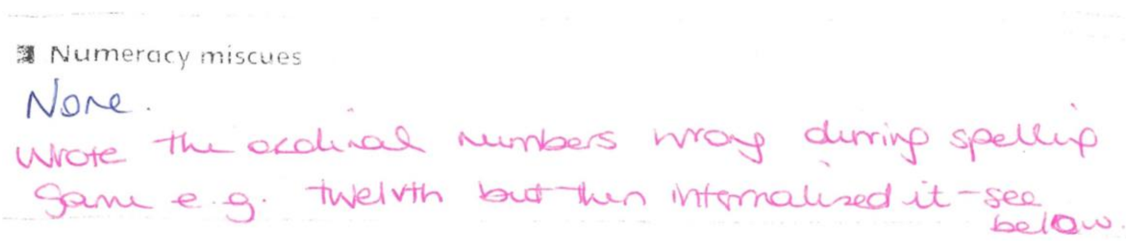

Figure 6.14: Numeracy miscues identified during Mike's 16th session.

The spelling of the ordinal numbers was emphasized through miscue analysis, by looking at the number he was writing incorrectly and discussing why they are incorrect. This seemed to work because Mike then managed to complete the 'Linked Recording' section on his own using the correct spelling. This can be seen in Figure 6.15.

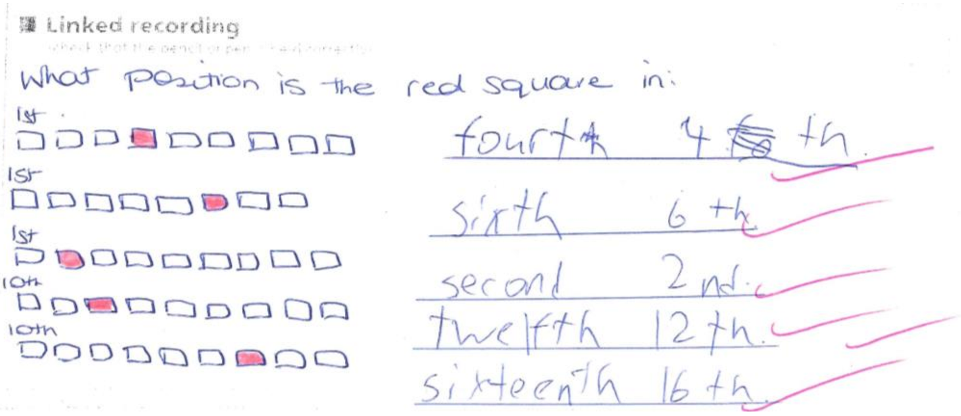

Figure 6.15: Linked recording section showing how Mike wrote the ordinal numbers correctly following miscue analysis.

6.10.6 Reviewing MKO-driven strategies

Section 6.9 to Section 6.10.5 have allowed me to explore different MKO-driven strategies that appeared to facilitate the learner's internalization process. Some of these strategies had been put forward by Tharp (1993) and had been used intentionally throughout the intervention programme since they are inspired by Vygotsky's works and have been demonstrated to assist the learner with the internalisation of the numeracy components. However, my analysis of data brought to light an additional five strategies, acting as incidental scaffolds (Hobsbaum et al., 1996). These incidental scaffolds appeared to be effective in supporting the learner to make move within his Zone of Proximal Development (ZPD) and thus to master the mathematical concepts, skills or language at hand. These strategies were not planned as part of the pedagogical model I was following in which Tharp's seven modes of assistance were included. However, they were used because my teaching experience had previously shown me that the children enjoyed them and that they seemed to be effective. Although some of the emerging themes discussed have been identified by previous studies, my finding contributes to the epistemology available in the field of research about mathematics education (as outlined in the different sections presented – see Section 6.10.1 to Section 6.10.5) by providing a holistic picture of all the MKO-driven strategies that seem to facilitate learning. All these strategies derive from Vygotsky's theories and are the result of the social interaction provided throughout each session. All the strategies place the learner at the centre of learning and allow the MKO to facilitate internalisation by guiding the learner from the current zone of development to the potential one. Figure 6.16 illustrates all the MKO-driven strategies that I believe to be effective in assisting children with MLD in the internalization process. It shows how MKO-driven strategies that were presented by Tharp (1993) were merged with others to provide an intervention programme that was successful at supporting the learners in mastering the numeracy components that one would have expected they mastered in earlier years.

Figure 6.16: All the MKO-driven strategies that have been found to be effective in supporting learners with MLD and both MLD and RD to internalise the numeracy components.

Apart from these MKO-driven strategies, other effective strategies were also identified. These were related to the learner and how he was empowered to steer specific strategies himself. As a result, I labelled this second set of strategies as 'Learner-driven' strategies. The Learner-driven strategies discussed in the next Sections (Sections 6.11 – 6.11.4) were themes that emerged during the 'thematic analysis' process (see Section 4.11). I originally set out to look at themes concerning the children's experience of mathematics at home and at school, however, as I analysed the data, I found that the learner also steered some strategies which possibly contributed to their improvement in the ten numeracy components. This second set of strategies will now be discussed.

6.11 Learner-driven Strategies

According to Vygotsky, the learner is at the center of the learning process. Hence, it was essential that the learner was placed at the center of the intervention programme through the choice of strategies employed by the MKO and the use of tools as assistance. This was done through the deep understanding of the learners' profile as presented in the first part if this Chapter. The learner featured in my conceptual framework when I set out to gather data about each individual participant through the observations, interviews as well as the mathematics anxiety and learning style questionnaires. This allowed me to provide intervention which was learner-centered. The children's profiles per se could not be altered, however, as I analysed the data, I noted that some strategies derived from the learner himself. Hence, I defined another type of strategies that were used throughout the intervention programme and had not originally been planned for as part of my initial model (Section 3.6; Figure 3.1). These were Learner-driven strategies that not only seemed to support the internalization process but also seemed to empower the learners to take control of future learning processes. The data revealed three Learner-driven strategies which seemed to support internalization and that I had included because my teaching experience had shown them to be useful. These additional *incidental* scaffolds (Hobsbaum et al., 1996) (Section 3.5) were:

- encouraging the learner to reflect upon their own internalization process and achievement in the form of self-evaluation;
- asking the learner to reflect about their miscues through Miscue Analysis; and
- asking the learner to take the role of the MKO.

These strategies will now be presented and discussed.

6.11.1 Self-Evaluation

As suggested by CUN, at the end of each session, the learners were encouraged to reflect about their performance and any new learning that had taken place during that specific session. This was done for two main reasons. Primarily, it aimed at empowering the learners to engage in metacognitive and meta affective thought processes that would facilitate internalisation. The importance of these have been discussed in Section 3.4.2.1 and Section 2.7.3 respectively. The "act of thinking about one's own thought process" (Kusritz & Clarkson, 2017, p.338) has been identified as a strategic means of facilitating learning. Moreover, encouraging the learner to think about his own achievements through meta-affect has been found as yet another effective strategy to support the internalisation process (Debellis & Goldin, 2006). Secondly, this process of self-evaluation served the purpose of boosting the children's self-esteem in the subject by allowing them to reflect and acknowledge their own achievements. The importance of this has been highlighted in Section 2.7.3 which underscores the need of supporting learners with MLD to enjoy mathematics tasks more while raising their self-confidence in the subject (Silver, 1985; Stevens et al., 2004).

Throughout the sessions, the children wrote a variety of comments in the 'comments' section – sometimes even drew images that helped them to express how they felt throughout that session. An example can be seen in Figure 6.17 completed by Ethan, during his 19th session, in which he drew a smiley Dracula face and another smiley following his comment 'good work'.

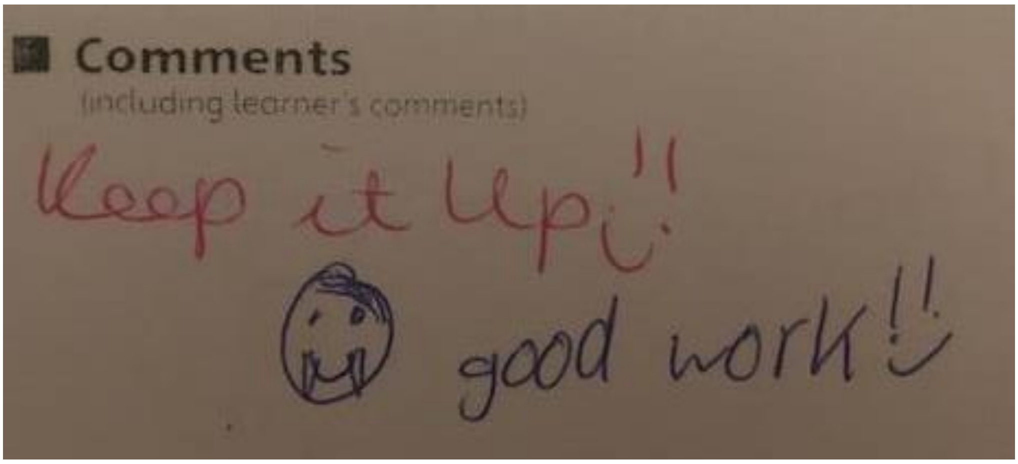

Figure 6.17: Ethan's drawing in the 'comments' section during Session 19.

By the end of the sessions the children did not need me to remind them about filling in this section because they would do so willingly and out of their own initiative.

The children came up with their own, unique comments to capture their achievements. To follow are some examples of the way the children completed this section. For example, in the 7th session with Mike, in which he had done very well I wrote 'well done!!' in the 'Comments' section. When I asked him to add his comment, he wrote 'moving forward' showing that he was feeling and acknowledging that he was improving in this numeracy component. This section can be seen in Figure 6.18.

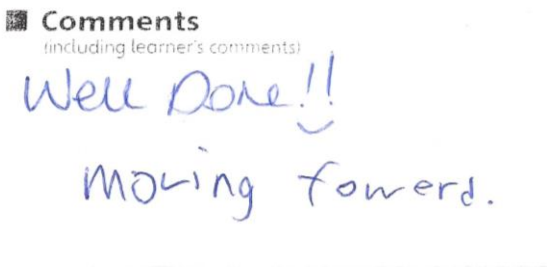

Figure 6.18: Self-evaluation section taken from Mike's 7th session.

Another example is taken from Ethan's 10th session. Ethan had completed his tasks well and after I indicated this, he wrote that he felt he had done very well, and he highlighted this to emphasise it. This can be seen in Figure 6.19.

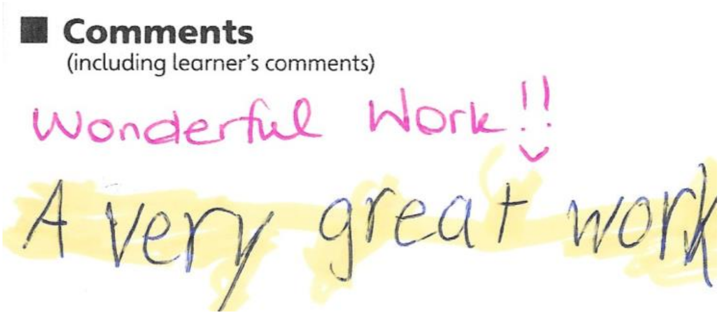

Figure 6.19: The self-evaluation section taken from Ethan's 10th session.

Both these images, as well as many others, show that for the learners, completing the self-evaluation section was beneficial because it gave them the time to reflect on their learning and to acknowledge that they had done well and were improving.

6.11.2 Miscue Analysis

Another strategy that seemed highly effective in supporting internalisation was that of Miscue Analysis. I have already discussed Miscue Analysis and how this was directed by the MKO in Section 6.10.5. Miscues were listed in the 'Miscue Analysis' section that is part of the sheet completed after each intervention session. However apart from filling in this section, I also encouraged the learners to think about their mistakes during that session and to reflect on those same mistakes in subsequent sessions. In fact, on some occasions it was evident that the learners had internalized the learning because they had made a mistake during a previous session and when they were about to repeat the mistake in subsequent sessions, they remembered by themselves or were prompted by the MKO to remember the previous mistake made. This led them not to repeat the same mistake. Making this observation prompted me to conjecture that such skills of meta-affect (discussed in Section 2.7.3) are important for facilitating learners' internatlisation process. In agreement with theories about meta-affect (Debellis & Goldin, 2006), the focus placed on reflecting on the child's performance during the session, and thus on their miscues, seemed to have a positive impact on the learners' internalisation of the numeracy component at hand. An example of this was seen in the excerpt taken from Ethan's 2nd session that was presented in Section 6.10.5. In this excerpt I showed how Ethan kept forgetting the number '13' when counting backwards and that he remembered this mistake during subsequent sessions.

Another example, also taken from Ethan's sessions, was when we started the order irrelevance component during his fourth session. Ethan could not realise that no matter from where he started counting a group of objects, the total number remained the same. During the task, I gave him a group of different coloured cubes. I asked him to start counting from a specific colour and he would give me the total number of cubes there were. Then I repeated the task asking him how many there were if he had to start counting from a different colour. He counted again every time without realizing that the total number would remain the same. By the end of the session, after I prompted him to reflect about his misconception, he had understood this since he started telling me the total amount without counting the objects again if he had already done so. Moreover, every time I asked him why he could tell me the total without counting the objects again, he was able to give me the correct explanation. During the following session when I asked him about what he could not grasp during the previous session he immediately said that he now knew that the total number of objects remain the same, no matter which one he started counting from. This example clearly shows that the miscue analysis carried out during the previous session was effective in facilitating the learner's internalisation process. Ethan had internalized the concept, and in this particular example, the child then told me the correct total number of objects (for the same group of objects) without having to count again after the first round.

After asking them to remember their previous mistakes at the start of each session, the children seemed empowered to think of their previous mistakes without being prompted. For example, in the 4th session with Nathan, he immediately told me what mistake he had been making during the previous session. He then did not repeat the same mistake during that session. There were various instances for each learner in which he thought of his miscues without being asked and which he then kept in mind in order not to repeat them during subsequent sessions. This showed that this strategy was not only effective, but also that the children were empowered to analyse their own miscues in future learning situations making miscue analysis an evidently effective Learner-driven strategy.

6.11.3 Taking Teacher's Role

As I analysed the data, I noticed that there was yet another Learner-driven strategy which had not been pre-planned. This was asking the children to take up the role of the MKO. In Section 6.10.1, I argued that role inversion was an MKO-driven strategy which facilitated internalisation. In that section, I gave examples of how this strategy was used from the perspective of the MKO and how this had an impact on the social interaction with the child and provided an opportunity to understand the learners' ZPD better. In this section, I now focus on role inversion from the learner's point of view. I discuss how this strategy impacted the learner themselves and how the learner was central to the social interaction when they took up the role of MKO. Hence, I argue that role inversion had an impact on both the MKO and the learner and that during situations of such interaction, the MKO had an impact on the learner and their ZPD, whilst the learner also influenced their own achievements and their interaction with the MKO. This is in line with the argument presented by Abtahi (2017) when she suggested that in the example given by Graven and Lerman (2014), the child, Lila, took up the role of the MKO, leading to her better understanding of counting in threes (see Section 3.4).

By changing their role from that of being the learner to that of becoming the MKO, the learners were empowered to take hold of their learning process and to direct the session themselves. Taking the role of the MKO seemed to have multiple benefits; it allowed the learner to:

- make use of the mathematical language he had been exposed to;
- express his thought processes – think aloud about the process involved in completing a task thus helping the child to internalize the process;
- boost his self-confidence in the subject;
- drive the use of tools and select which tools to make use of when explaining the task;
- allow the MKO to gain an understanding of where the child stood in the ZPD.

Another example of role inversion, that I can add to the ones given in Section 6.10.1, is taken from Ethan's 10th Session. I gave him a group of cubes, asking him to estimate the amount and then count them. I then asked him how many there would be if I had to remove one or two of the cubes. After three modeled examples, I asked Ethan to pick a number of cubes. I specifically told him that he could not pick up more than 15. He picked up several cubes and asked me to estimate how many there were. There were 13 so he had picked correctly. He then asked me: "If I remove 2, how many do you think there would be?" I asked him to answer his own question and he did correctly. He repeated this until he could not remove another group of 2 from the cubes in hand and his answers were all correct. The fact that he could express himself correctly, using the correct mathematical terms, did show that he had internalized the language to express himself in relation to the mastered numeracy component – an insight I could get from this role inversion activity. The fact that in the post-assessment he managed to complete all the tasks successfully also illustrates that he had internalized the concept of subtracting two objects from a given number of objects.

6.11.4 Concluding on Learner-driven Strategies

The Learner-driven strategies presented in these sections are far less numerous than the MKO-driven strategies presented earlier. Nonetheless, I believe they were important contributors to the successful internalisation of the numeracy components. In this account, I have given various examples from the intervention sessions which show how the MKO-driven strategies and the Learner-driven ones facilitated the internalisation process, applying them specifically to mathematics. Nonetheless, these pedagogic strategies may also be applied to other areas of learning, possibly also having such a positive impact.

As I selected the pre-determined themes for the analysis of data, I decided that I would also investigate the use of tools based on Vygotsky's perspectives. Hence, Tools-assisted strategies will be discussed next. Although the strategies which will be discussed in the coming sections are driven by either the learner or the MKO, they are also assisted by tools, which do not have agency on their own. Without the use of such tools, the strategies would be rather general pedagogic ones, as the ones identified for the MKO-driven and Learner-driven strategies. Using tools, especially ones that are specifically related to mathematics, seemed to contribute to the internalisation process of the numeracy components and may thus be an important feature of intervening for MLD.

6.12 Tools-assisted Strategies

One of the ways in which Vygotsky himself analysed his data was by analyzing "tool use" (Vygotsky, 1978). Similarly, I had to explore what tools, specific to mathematics education, were made use of intentionally in the intervention programme and how these were of assistance in the internalization process. Since Vygotsky (1978) suggested that, "the child's system of activity is determined by his or her degree of mastery in the use of tools" (p.21), I also looked at whether and how the learners mastered the use of the tools introduced throughout the programme. In fact, this was one of the situations that would serve as evidence of internalization (see Section 3.6).

The use of tools was essential to make the intervention programme successful. As highlighted by Wertsch (1993), human beings need tools to create meaningful social interaction that facilitates internalization. Throughout the intervention programme I thus made use of several psychological and technical tools that would support the internalization process and allow the children to use these ‘external objects’ as stimuli to extend their basic mental functions of memory, attention and perception as Vygotsky suggests (see Section 3.3). In this Section I will thus discuss which cultural tools, specific to mathematics education, were made use of and I present situations in which the use of these tools seemed to assist or lead to internalization. I will also explain how these tools had an important role in semiotic mediation which, as explained by Mariotti (2009), is the “knowledge-construction as a consequence of instruments activity where signs emerge and evolve within social interaction” (p. 428). The focus will remain on how this social mediation facilitated the internalisation of the numeracy components.

When Vygotsky (1978) put forward his idea of cultural tools, this was not specific to mathematics education. He proposed that cultural tools were formed from technical and psychological tools. Thus, when planning the intervention sessions, I intentionally made use of both technical and psychological tools that would support the learners in internalizing the numeracy components. Moreover, Vygotsky implies that the connection between tools and signs produces artefacts hence psychological tools may also be labeled as such. All the tools made use of as part of my conceptual framework are presented in Figure 6.20.

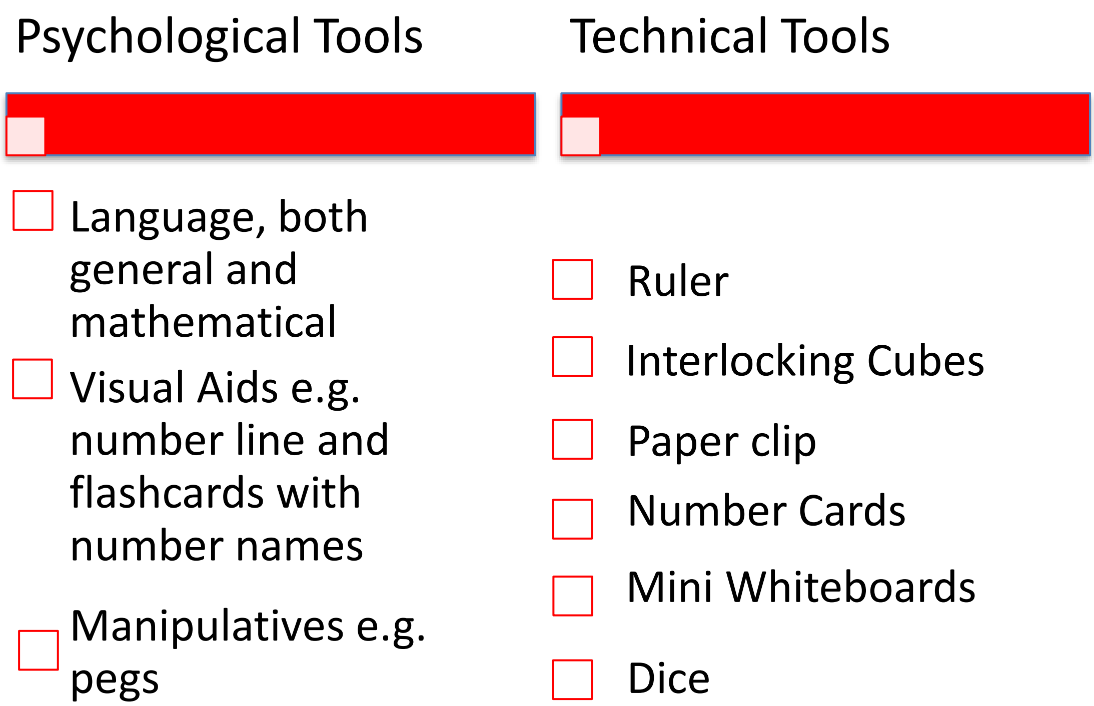

Figure 6.20: Tools intentionally made use of in the intervention programme.

All these tools are specific to mathematics education itself and therefore have not directly been referred to by Vygotsky. Nonetheless, their purpose was technical or psychological in nature as per the meaning Vygotsky gave to each of these terms. The way in which I have categorized these tools as psychological and technical has been discussed in Section 3.4. The way in which Vygotsky's theory on cultural tools has been applied to mathematics resources in this way is, to my knowledge novel. As discussed in Section 3.4, other studies (Bartolini Bussi & Mariotti, 2008; Borba & Bartolini Bussi, 2008) have explored the use of specific mathematics resources such as manipulatives and digital games. However, none, to my knowledge, have explored a more comprehensive list such as the one proposed here (Figure 6.20) or have applied Vygotsky's theory of cultural tools to suggest how mathematics resources can be used to provide effective scaffolding for children having MLD. In the subsequent sections I will discuss how the technical tools supported the internalization process and how each of the psychological tools used played a fundamental role in the 'transformative collaborative practice' leading to internalization.

6.12.1 Technical Tools

As highlighted in Section 3.4, a technical tool "serves as the conductor of human influence on the object of activity; it is externally oriented" (Vygotsky, 1978, p.55). When planning the intervention sessions, the use of some technical tools was planned for, as highlighted in my conceptual framework (Section 3.4). Throughout the programme, the technical tools used were a ruler, dice, number cards, paper clip, mini whiteboards and objects such as counters and interlocking cubes. These items had a functional purpose in that they were used for the purpose of an exercise or an activity to be carried out successfully. As previously outlined in Section 3.4, technical tools did not serve the purpose of psychological tools in that the situations in which they were used did not "direct the mind and behaviour" (Vygotsky, 1981, p.140) of the learner. Nonetheless, without them the MKO would not have been able to carry out the task.

For example, dice were used very often with the learners. In many activities, dice were used simply to generate a number. For example, in the 4th session with Andrea, he had to roll two dice and represent the total number on the dice using interlocking cubes. The numeracy component being focused upon during this session was that of subtracting objects. After representing the number of objects according to the number on the dice, Andrea had to subtract one or two cubes each time I instructed him to. Thus, in this situation, any other means of generating a number, for example, number cards could have been used. Similarly, in the 1st

session with Ethan, he had to throw three dice and count back from the total number on the dice to 0. Again, the dice were used for number generation here, and any other form of getting a number as a starting point could have been used. Paper clips were also sometimes used in some sessions to mark a number. One of the activities in which this was used was the ‘paper clip activity’. A number was marked on the number line using a paper clip and the child had to count back from that number to ‘0’. The number line would serve as the psychological tool in assisting the internalization of the number sequence directly (see Section 3.4), however the paper clip served the purpose of generating a number that indirectly led to the successful completion of the task.

Interlocking cubes were used in a number of activities. Their main purpose was to serve as objects, usually to be counted. For example, in the 5th session with Ethan, he was asked to take a few cubes in one hand. He then had to estimate how many cubes he had picked. Hence, any other object such as counters or beans could have replaced the cubes. In the 12th session with Mike, interlocking cubes were also used, however this time they were used in relation to the component of ordinal numbers. A row of different coloured cubes was fixed together and shown to Mike. Mike had to say the position of different coloured cubes – for example, in which position the red cube was. In this situation the cubes served as a means for completing the task, thus indirectly supported internalization. Any other object apart from the cubes could have been used for this purpose.

Technical tools did not directly impinge on the internalization process because they were not being used as a mathematical representation in themselves (as also explained in Section 3.4). However, they had an indirect role in ensuring that the activities planned and administered were successfully completed to facilitate the internalization process. It was indispensable for the MKO to use technical tools to ensure that the social interaction situations created were fruitful and led to internalization. Psychological tools, on the other hand, had a direct impact on facilitating the learners’ internalisation of the numeracy components being focused upon.

6.12.2 Psychological Tools: Language

As outlined in Section 3.4, to analyze what served as psychological tools in my sessions, I identified the tools used in which the meaning encoded in them was more important that the tools themselves (Vygotsky, 1978). As highlighted in Section 3.4, one of the most important psychological tools made use of was language, which allowed me to provide effective semiotic mediation. Semiotic mediation was used throughout the sessions and was critical to providing social interaction between the MKO and the learner that would support the child's development within the ZPD.

In the sections about MKO-driven strategies, I have already provided examples of how internalization was successful thanks to semiotic mediation. Language was the basis of all the strategies made use of by the MKO to support the internalization of the numeracy components (see Figure 6.16). For all the strategies mentioned, language was the main mode of communication between the MKO and the learner. Language manifested itself in verbal and written forms, as well as non-verbal forms such as gestures e.g. clapping. What I will give in this section is a further example of how language also played a crucial role in the MKO's capacity to assess the learner's ZPD and at which phase the learner was in the internalisation process.

In Session 11 with Thomaz, semiotic mediation was crucial in understanding that Thomaz had not internalized counting backwards and that therefore further intervention had to be provided for internalisation to take place. When Thomaz was completing the 'linked recording' section, I realised that he was counting forward rather than backwards, and I asked him to say the number out loud whilst counting backwards. Thomaz got completely stuck and could not count backwards. This is an excerpt of what happened after.

EZ	I saw that to see what comes before 8 you had to say all the numbers first
Thomaz	[Thomaz laughed] You heard me in my mind.
EZ	Yes, 7 [pause] what comes before? [Thomaz got completely stuck again] Can you use the number line to help you for now?
Thomaz	Ok. The answer is 6.

(Thomaz, Session 11)

This interaction allowed me to understand that Thomaz was still not competent at counting backwards and that although during previous sessions, when completing the same task, it had seemed as though he had internalized the number sequence, he still had difficulties with this. Thomaz was the participant with the lowest score on the Aston Auditory Sequential Memory assessment (Table 5.8) and had only scored '1' out of '5' on the 'Digit Reverse' task indicating a severe deficit in working memory which may have been a reason why he needed constant reminding to be able to complete the tasks given.

6.12.2.1 Manipulatives

Manipulatives were also used as psychological tools. One of the uses of psychological tools, as explained by Vygotsky (1978), is for learners to "personally create a temporary link through an artificial combination of stimuli" (p. 51) (refer to Section 3.4). As explained in Section 3.4, manipulatives are also referred to as 'artefacts' in some studies (Bartolini Bussi, 2011) which serve as tools for semiotic mediation (Bartolini Bussi & Mariotti, 2008). These artefacts, referred to in my study as manipulatives, assisted the learners to internalize the numeracy component focused upon since the multisensory experience provided seemed to be highly beneficial for the learners to develop a deeper understanding of the concepts and skills presented. Most sessions started with the use of manipulatives to provide the learners with a concrete (real) mathematical representation of the concept at hand (see Section 3.4 which refers to the CPA approach). Several manipulatives were used during the sessions, including cubes, pegs, hangers, beans and dice. Each of these were sometimes not merely used as an object needed for the activity to be completed, thus a technical tool (see Section 6.12.1), but also to support the development of a mental mathematical representation deriving from the concrete one. Examples of how some of these manipulatives served as psychological tools, seeking to alter the child's mental processes, will follow. In some of the sessions I made use of a hanger and pegs to illustrate different number sentences. Figure 6.21 illustrates this.

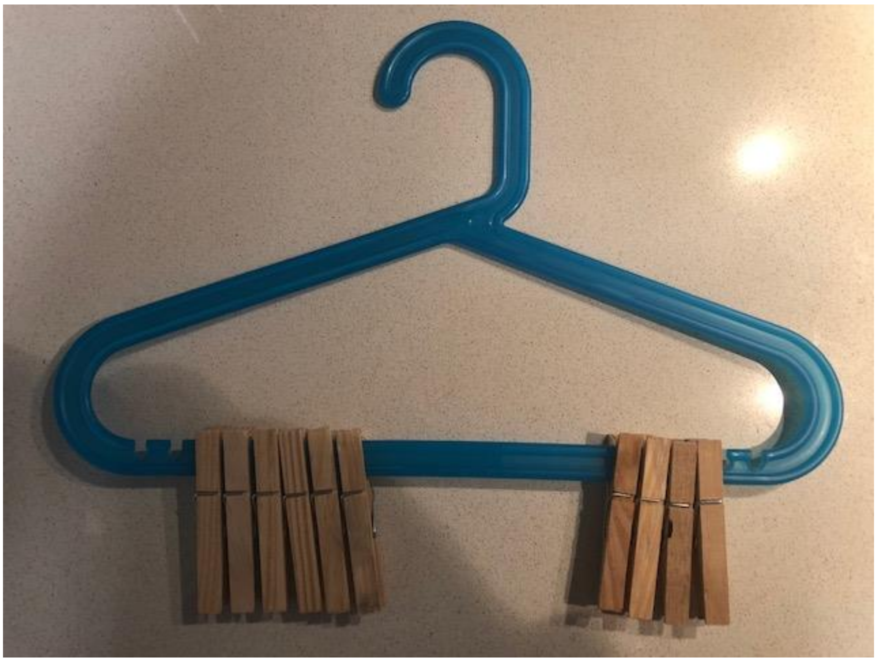

Figure 6.21: The pegs on the hanger represent the sum '6 + 4 make 10'.

Moving the pegs around and seeing how different numbers add up to 10 helped the children not only to recall the number sentences but also to see patterns, such as if you add one peg on one side, you would have to remove it from the other to keep the same total. This activity was done during Nathan's 7th session since before the intervention session, he was unable to add two numbers (smaller than 18) correctly without counting on his fingers. After the activity Nathan was able to use other strategies, including mental ones to add two numbers. Hence, the concrete representation of the number sentences supported him in internalizing a mental representation for the same sums. Evidence of this is that after the activity we moved on to adding using other strategies including working a sum out mentally and looking at number patterns. Nathan was able to use other strategies of addition other than finger counting at the end of the sessions. Evidence of this can be seen through the linked recording section that he was able to complete alone (Figure 6.22).

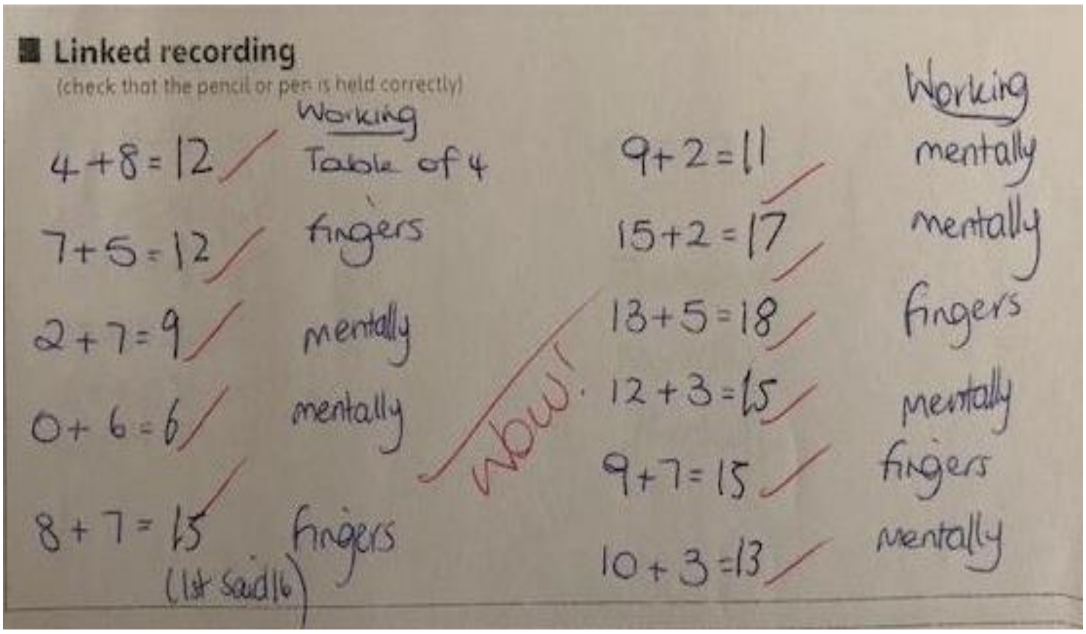

Figure 6.22: The linked recording section following the peg and hanger activity.

Apart from working out the sums, Nathan was also able to identify which strategy he had used to complete the particular sum e.g. using his fingers, working it out mentally or keeping the larger number in his mind and counting on from that numbers on his fingers. Generally, it seemed that when one of the numbers to be added was rather small (not larger than ‘3’), Nathan realised that he could work the sum out mentally without reverting to finger counting. However, when both numbers were larger than ‘3’ he went back to counting on his fingers or to keeping the larger number in his mind and counting on from that number. His strategy selection indicated that Nathan had improved in this area as he was seeing beyond finger counting, now identifying that some operations could easily be worked out mentally. This improvement possibly derived from the way the number bonds were represented through the activity with the pegs. In this exercise, the manipulatives (i.e. the hanger and pegs) were thus used as the concrete tool that would later lead to the more abstract understanding – also indicating that this tool was fundamental in the scaffolding process.

Another similar situation, in which the use of manipulatives as psychological tools seemed to support the internalization process, is the following. In the 15th session with Thomaz, the floor tiles were used as a large number line (as shown in Figure 6.23).

Figure 6.23: Large floor number line as the one used during the session.

I asked him to count backwards from one number to another by jumping backwards on the different numbers as he counted backwards. The large number line and the process of jumping were psychological tools that seemed to have facilitated his internalisation of counting backwards as during subsequent sessions it seemed that he had finally established a mental representation of the number line and could count backwards successfully by imagining that same number line he had been jumping on. He said this himself during the 15th session and in later sessions when he repeatedly closed his eyes to count backwards and said he was doing so as he could see the number line in his mind. Thomaz had never done this before these activities.

Using manipulatives seemed to be an effective tool. They did serve the purpose suggested by Vygotsky in that they appeared to help the learners to extend their use of memory and to create mental representations that would facilitate internalisation.

6.12.2.2 Visual Aids

Visual aids were another form of psychological tools used. Visual aids supported the learners to build a cognitive representation of the numbers and the number system by using the 'artificial' stimulus mentioned by Vygotsky (1978). A variety of visual aids were made use of throughout the sessions. These included: number lines, number chart, number names on cards, flashcards with important mathematics vocabulary e.g. 'count on' (seen in Figure 6.24) and cards with dots for subitizing and developing number sense.

Figure 6.24: Flashcards with the mathematics terms 'count on'.

These tools supported the children in creating a visual representation before moving on to symbolic representation that we normally tackled in the 'linked recording' section. When analyzing the data, it was evident that these visual aids supported the process of internalization. Evidence of this were situations in which the learners could use a visual aid (which they were previously unable to use) without assistance to complete a task. Some of these situations will be presented hereunder.

In the 10th session with Nathan, I used the number line as a tool to help him work out the operation '18 subtract 5'. The following conversation took place:

EZ	How can I use the number line to work out 18 minus 5?
	[Nathan could not remember how to use the number line to work it out since this had been used at school in earlier years, so I modelled how he should work the sum out using the number line to remind him of this.]
EZ	What is 14 minus 5?
Nathan	9!
EZ	How did you work it out?
Nathan	I used the number line!
	[Nathan showed me how he started from 14 and counted back 5 using the number line.

(Nathan, Session 10)

This showed that Nathan had now internalized how to use the number line as a representation to subtract the two numbers and chose to use it as a tool without my prompting also indicating internalization as pointed out in my initial indicators of internalization. As highlighted in Section 3.4 "it is up to the teacher to help students make the connection between the models and the mathematics" (Ennis & Witeck, 2007). He did not seem to know that the number line could be used for subtraction in the way he used it in this circumstance. This may have resulted from the possibly that he had used other representations at school, such as a number grid, instead of a number line to subtract. Hence, it is important for the MKO to ensure that the learner knows how a model/representation can be used to work out a specific operation. The use of this visual aid seemed to have been internalized since Nathan was also able to use this tool in subsequent sessions also tackling subtraction.

The visual aids were sometimes used as tools for scaffolding. During some of the sessions I would start the session using the visual aid, such as the number line, but aim at removing this aid by the end of the session. This would ensure scaffolding. During the second session with Seb, in which we were focusing on counting backward, I told him “For the first couple of times I am going to leave the number line and then I will remove it.” In the previous session with Seb, I had used the number line, so I thought that by leaving the number line available to begin with I would also be recapturing the learning that would have taken place during the previous session. By the end of the session, I noticed that Seb was not looking at the number line anymore and I asked him about how he was counting backwards, and he said, “I think of the number – in my mind” (Seb, Session 2). I conjecture that Seb created a mental representation of the number sequence using this visual aid and hence internalized counting backwards since he did this successfully unaided.

Another example of how visual aids were used as tools for scaffolding is the following. During the 3rd session with Andrea, I gave him some flashcards to take home with different mathematical terms that refer to ‘counting forward’. The cards read ‘count on’, ‘count up’ and ‘count forward’. I asked him to practise using the terms. During the following session, we played the paper clip activity and when I asked Andrea to instruct me to count on up to 20 from the number the paper clip was on. He did this correctly and used the different terms to instruct me out of his own initiative. First, he put the paper clip on the number ‘3’ and asked me to “count up from 3 to 20” (Andrea, Session 3) and then he placed it on the number ‘12’ and asked me to “count forwards from 12 to 20”.

6.12.2.3 Digital Tools

When analyzing the data, I also focused on the role of digital tools in supporting scaffolding. As explained in Section 3.4, digital tools were used during some of the sessions. These also seem to have facilitated the internalization of the numeracy components at hand. Although Vygotsky did not, of course, make direct reference to the use of digital representations when speaking about cultural tools, the analysis of my data supported other similar studies (Borba & Bartolini Bussi, 2008 & Mariotti, 2009) that have indicated that in today’s world these digital experiences can also be used as tools having the same function as other psychological tools. As Lerman (2014) highlighted: within being neo-Vygotskian there is a struggle. This is that, “we want to remain faithful to Vygotsky’s own work and ideas as well as make them relevant for today's context" (p. 24). Their main purpose was to provide the learners with circumstances in which they could either practice a skill, for example, continuing a sequence of numbers, or recall a concept more readily, such as ordering numbers from least to greatest. Some games were also chosen because they would help with building a mental representation of the skills or concept being tackled. I decided to include digital games in the intervention programme because children generally love playing these games. Moreover, I knew that Thomaz really liked using the computer and I thought that this strategy could serve as a means of facilitating internalisation. Including these games was not part of the original model I was following as I had only earmarked psychological and technical tools, as presented by Vygotsky (1978), as cultural tools (Section 3.6; Figure 3.1).

As explained in Section 3.4, digital games have been explored by others (Borba & Bartolini Bussi, 2008; Mariotti, 2009) as 'technological tools' that contribute to semiotic mediation. Although my finding that digital tools may serve as effective means for scaffolding has been supported by other studies (Borba & Bartolini Bussi, 2008; Mariotti, 2009), a further idea explored in this study is that these digital tools may facilitate the internalisation of the numeracy components which are the foundations for mathematics learning. The analysed interactions between the MKO and the learner when digital games were being used, demonstrated that these games were important in helping the children to practice the skill at hand or support the learner to recall the concept more readily in future sessions as will be demonstrated hereunder.

In Section 2.10.2 I referred to computer-assisted intervention programmes which have been developed. To a certain extent, in such intervention, technology takes the place of the MKO in supporting the learner. Nonetheless, through my observations in my own intervention sessions, I could conjecture that for intervention to be more effective, the MKO needs to maintain her/his crucial role, whilst digital programmes can act as a 'cultural tool' which the MKO can make use of to ensure that internalization is successful. The main reason why I conjecture this is that when I made use of a digital game, the children still needed a re-explanation of what they had to do in the task since the digital instructions given were not enough for them to understand how the task had to be completed. Moreover, in some situations, although the learner was 'cheating' or not completing the task correctly, the computer did not detect this, but it was the MKO who had to go back to supporting the child and ensuring that the task was done well.

An example of this is what took place during the 2nd session with Seb, mentioned in Section 6.9.3 to illustrate how contingency management was used in this situation. Seb was asked to play a game on the computer to practise counting back. In the game he was given several numbers arrange from largest to smallest by counting backwards. However, after the first two tries, I realised that he was 'cheating' because rather than counting backwards to order the numbers, he was counting forwards. So, I drew his attention to this and ensured that he completed the task by counting backwards as otherwise the task would not have served its purpose. Hence, in this situation, I believe that the effectiveness of the digital tool used was only maximized thanks to the role of the MKO. Another instance was when during another session with Nathan, Session 10, Nathan had to complete an online game called 'Minus Mission'. In the game, several subtraction sums came down on the screen and Nathan had to shoot the number sentence that would result in the given answer. Although the digital game served as a tool to support the internalization process since, by the end of the session, Nathan was able to complete the subtraction sums in the linked recording section alone, the MKO still had a crucial role. Primarily, after the instructions given by the digital game itself, the MKO had to re-explain the instructions and modeled playing the game once since the Nathan had not understood the instructions given by the digital game itself. Moreover, throughout the game, I encouraged him to keep trying and praised him for his effort. I also recaptured and summarized his results to help him and pointed out any miscues he had made.

In all other sessions, in which online games were used as a digital tool, it was evident that the MKO still served an important role and therefore I feel that although computer-assisted intervention can be helpful, internalisation may be more effective when these digital interventions serve as tools for the MKO to facilitate internalization rather than replace the MKO.

6.12.2.4 Tools that serve as Technical and Psychological

Vygotsky suggests that some tools may be viewed as both technical and psychological (Bartolini Bussi et al., 2012). This was discussed in Section 3.4 in which I pointed out that Vygotsky (1981) explicitly mentions that *language*, *various systems for counting*, *diagrams* and *mechanical drawings*, amongst others, are examples of such tools. When designing the programme I ensured that different technical and psychological tools were used. However, I had not thought about tools that may act as both. When analyzing the data, I noticed that some of the tools used in the intervention programme acted as both technical and psychological tools, for example dice and cubes.

As explained in Section 6.12.1, dice and cubes were used as technical tools. These two tools were used as a means for number generation or for providing a group of objects which the learner could estimate the number of. Hence, in these situations both tools were replaceable by any other tool that would serve the same purpose since they were not used because of a specific property which would help alter the children's mental representations. However, in some other situations, these same tools had a different function. For example, at times dice were used intentionally because of their capacity to serve as a visual aid to support the development of number sense and subitizing through the internalization of the number patterns represented on them. Hence, here dice served the function of a psychological tool. Similarly, cubes were sometimes used as mere objects to be counted but on other occasions they served the purpose of helping the children to internalize the number bonds, for example by representing two different numbers which added together would make ten. In the latter situation, the cubes served the purpose of providing the tactile and visual tools that would support the development of mental representations that would help the children to recall the numbers at a later stage, hence extending their capacity of memory (as explained by Vygotsky, 1978).

As illustrated through the observations presented here, tools may serve either a technical or a psychological purpose, however, some of them have the potential to act as both. This indicates that when the MKO selects which tools are most appropriate to support the internalization process, s/he should bear in mind that some tools may have more than one function. Thus, a technical tool may have a greater impact on facilitating internalization when its potential of being a psychological tool, if available, is also made use of intentionally.

6.12.2.5 Updated tools

After analyzing the data, I felt that the list of cultural tools that I had originally identified and outlined in Figure 6.20, could be extended and further developed. It was evident that throughout the intervention sessions, digital games served as a psychological tool too, which allowed the MKO to facilitate internalization. Moreover, following the analysis phase, I could also conjecture that some of the cultural tools that I made use of during the intervention programme had been used as both a psychological and a technical tool indicating that the same object may serve two distinct purposes – both very useful to helping the MKO in facilitating internalization. Hence, in Figure 6.25, I present an updated list of tools including the identification of the new tools identified (in green) and the awareness of how the same tool (referring to the tools in the middle) could be labeled as both technical and psychological depending on how it was used.

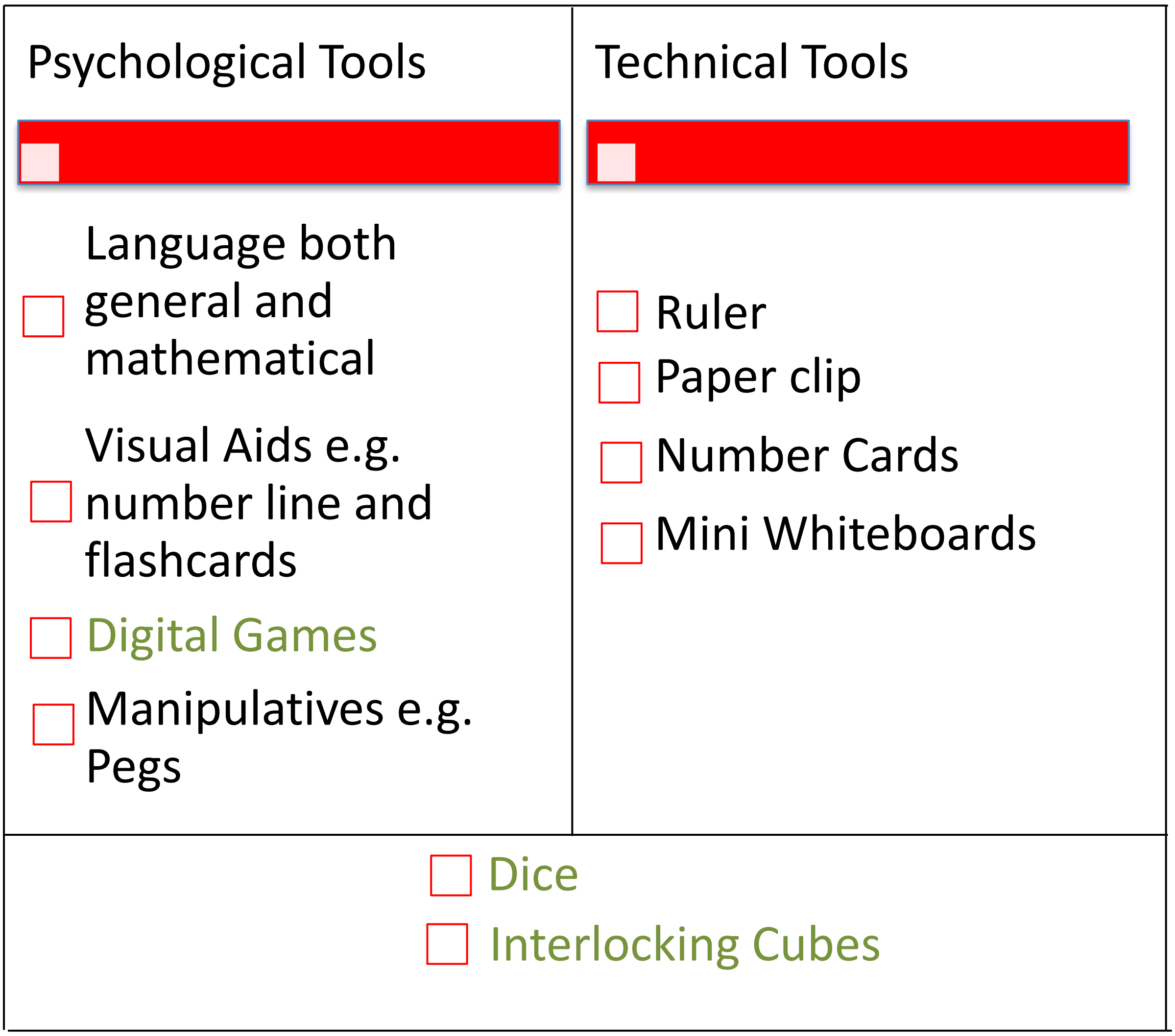

Figure 6.25: An updated list of cultural tools that helped the MKO to support internalization – the new items have been added in blue.

Without the use of cultural tools, the MKO would not have been able to provide the appropriate social interaction situations that would allow the learners to internalize the numeracy components. The data collected confirms that cultural tools are indispensable for the 'transformative collaborative practice' (Vianna & Stetsenko, 2006) needed for internalization to take place. Hence, I believe that all three forms of strategies: the MKO-driven, Learner-driven strategies and Tools-assisted strategies, are indispensable factors to the success of the transformative collaborative practice implied by Vygotsky which not only leads to internalisation but also show that it has taken place. In Chaper 7 I will discuss how these three strategies come together in what I view to be symbiotic relationship. However, prior to doing so I feel that it would be beneficial to illustrate the applicability of what has been discussed throughout Chapter 6 by providing an examplication of the cultural influences and strategies specific to one of the case studies.

6.13 An Exemplication of the Strategies through one Case Study: Thomaz

The baggage the other learners brought along, not only influenced my personalisation of the intervention programme but also their gains from the programme. Hence, since I am working within an interpretivist paradigm, I feel it would be helpful to provide an exemplication of the strategies presented throughout Chapter 6 in relation to one of the pupils studied. What I wish to show here is an exemplification of what the gains from the inidividualised programme were for one learner as a model which can be applied to the other participants. Moreover, as I describe how the learner's individual profile influenced the impact the intervention had, I will seek to show how the information I had gathered about the learner allowed me to personalise the intervention programme to ensure that effective teaching and learning took place. I have chosen to focus on Thomaz in this section because Thomaz's profile is possibly the most complex profile when compared to that of the other learners. Thomaz was identified as having both MLD and RD and also as having high levels of anxiety. Moreover, Thomaz was also screened by the Dyscalculia Screener (DS) (Butterworth, 2003) as having dyscalculia. Thomaz's difficulties in the numeracy components were severe. As a result, I only managed to work on two of the components with him because his grasp of the concepts was slow when compared to the other participants of the intervention programme. As shown in Table 6.3 the components intervened upon were: Counting Verbally (consisting of Counting Verbally, Counting On and Counting Back concepts) and Counting Objects (consisting of Order Irrelevance principle) (see Table 4.2 for further explanation of components). The number of sessions carried out for each one was: four (Counting Verbally), four (Counting On), nine (Counting Back) and three (Order Irrelevance). Examples that will be given in this account will thus be in relation to one of these components.

Much data was collected about Thomaz prior to planning his intervention programme. The data emerged through the interviews with his mother and teacher, the classroom observations as well as the various tests that he was asked to complete. This triangulation of research methods shed light on several important aspects of Thomaz's mathematical learning profile such as his learning style, level of mathematics anxiety and his domain-general abilities. Moreover, the cultural influences pertaining to his previous childhood experiences related to mathematics, both at school and at home, were revealed. Undoubtedly all these aspects, which form part of the cognitive and affective baggage that the learner brought to the intervention programme, had an impact on the outcomes of the programme itself and its benefits for Thomaz.

Cultural influences include aspects related to Thomaz's race, education, socioeconomics and gender. Thomaz was Maltese and was being brought up in a middle-class family. He attended an independent, fee paying school before joining our school. Both his parents were employed. This was important information because, as discussed in Section 2.8.1, socio-economic background can impinge on mathematics achievement. As part of Thomaz's mathematical learning profile it was also crucial to look into the cultural influences that Thomaz brought with him to the learning situations pertaining specifically to his childhood experiences of mathematics both at school and outside of school. During the interview with his mother, she revealed that Thomaz may not have had a solid foundation to the subject of mathematics. She believed that this may have resulted from several factors including lack of multisensory learning experiences in the subject or teaching styles that did not match his mathematics learning style. However, one cannot exclude that his inability to grasp the basic mathematical skills and concepts needed to engage in age-expected mathematics tasks may have derived from his cognitive learning profile. When assessed for the non-verbal reasoning component of the IQ Test, Thomaz scored lowest when compared to the other participants. Moreover, his score, that of the 27^{th} percentile, indicated that although he had an average IQ, his ability in this area was rather low. The weakness of this domain-general ability may have hindered him from achieving age-appropriate milestones in mathematics. It may have also been a main contributor to his low scores in the standardized numeracy scores obtained in both the BNST and Chinn's (2012) assessment. His literacy scores were also the lowest when compared to those of the other participants, hence his RD were most accentuated.

Another aspect of Thomaz's learner profile was a high level of mathematics anxiety. Thomaz's mother said, "he does not enjoy mathematics and is scared of the subject". This was quantified when the anxiety test was administered, since his score was the highest one obtained (55) and was almost 2 Standard Deviations (SD) higher than the average score for boys of his age. Thus Thomaz' mathematics anxiety could not be underestimated and the programme had to focus on this area too. A last aspect worth mentioning is Thomaz's learning style. When I administed the learning styles questionnaire (Chinn, 2012) to Thomaz, he demonstrated having most of the characteristics of an 'inchworm' learner (see Section 2.7.1, Figure 2.8). This meant that Thomaz avoided verifying whether an answer was correct, preferred to follow procedures, and therefore to learn facts by rote. Thomaz was the learner who started from the lowest levels of mathematics attainment. Although he found the programme beneficial, both because he mastered most of the numeracy components tackled and because his self-esteem in the subject seemed to increase (as seen in Section 6.11.1), his scores on the numeracy standardized tests made least improvement. This may suggest that his profile continued to impinge on his mathematical development. In addition, his learner profile may have also had an impact on the

Learner-driven strategies which Thomaz would engage in. These will be discussed in Section 6.13.2.

In this exemplication of strategies particularly focused on Thomaz's intervention sessions, I will tackle all the MKO-driven, Learner-driven and Tools-assisted strategies that I have discussed in Sections 6.9 – 6.12 in the same order. In the following section I will thus begin by giving examples of the MKO-driven strategies that have been previously presented (Section 6.9 and Section 6.10) and how Thomaz found these to be effective scaffolds for the internalisation of the numeracy components. I will then carry out a similar exercise for the Learner-driven strategies and the Tools-assisted strategies discussed in Section 6.11 and Section 6.12 respectively. The aim of this will be to illustrate the applicability of each strategy which has been discussed specifically to Thomaz's internalisation of the numeracy components, to show how the strategies also served in personalising the intervention programme and to illustrate how these strategies increased his self-confidence mathematics.

6.13.1 MKO-driven Strategies: Thomaz

All the MKO-driven strategies presented in Sections 6.9 and 6.10 were used with Thomaz during different sessions. In this section I will provide examples of each one. Modelling was the first MKO-driven strategy presented in Section 6.9. This strategy was used during every session with Thomaz. Primarily, since Thomaz loved playing computer games I used these often with him. Before letting Thomaz play a game, I modelled it for him by playing it myself. Secondly this strategy was used before allowing Thomaz to complete other tasks on his own, to ensure he had understood the task at hand. For example, during Session 18, which targeted the component of 'Order Irrelevance', Thomaz had to count a given set of objects. He then had to say how many there were if he were to start counting from a different object. I modelled this task first before asking Thomaz to do it himself. This was important to set clear goals for each task and to ensure that Thomaz had a good understanding of what was being expected of him.

Feedback was provided throughout all sessions. Both verbal and written feedback was given. This allowed Thomaz to appreciate whether he had been completing the task correctly and to reflect upon any miscues he had made. One way in which written feedback was given was after Thomaz had completed the 'Linked Recording' section at the end of each session. An example of this sort of feedback is demonstrated through Figure 6.26 taken from Thomaz's eighth session.

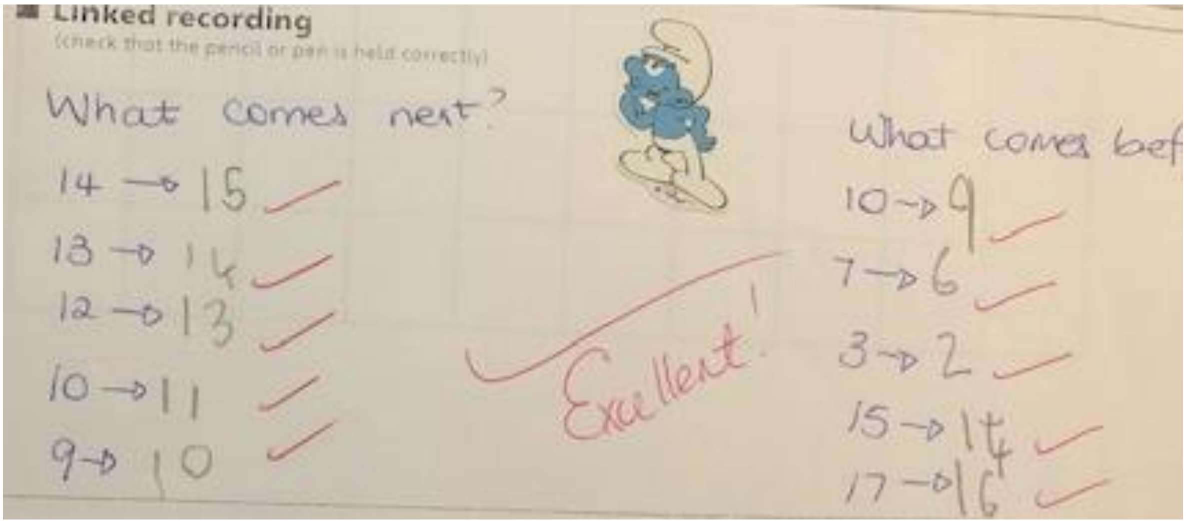

Figure 6.26 ‘Linked Recording’ Section taken from Thomaz’s eighth session.

Sometimes, I also allowed him to attach the sticker himself as he loved doing this. During this same session, written feedback was also given through the ‘Comments’ section. This can be viewed in Figure 6.27.

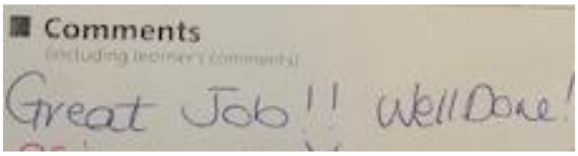

Figure 6.27 ‘Comments’ section taken from Thomaz’s eighth session.

This type of feedback had a positive impact on Thomaz’s affective domain since after completing the Linked Recording section, he would ask me **‘How did I fare?’**, eagerly waiting for a positive reply. Moreover, his beaming smile when I wrote a positive comment was very evident. Since children with MLD, like Thomaz, usually have low self-confidence in mathematics (Stevens et al., 2004) such positive feedback is crucial.

Since Thomaz would always ask me to play a computer game, I used this as one of the ways of using the strategy of Contingency Management. On various occasions I promised Thomaz he could play a computer game related to the numeracy component being tackled if he focused during the main activity of the session and completed the ‘Linked Recording’ section correctly. Another way in which Contingency Management was used particularly with Thomaz was when, on some occasions, he was not focused or would start to talk about things that were not related to the session *per se*. In these situations, I used Contingency Management to re-direct him by shifting the conversation back to the activity and sometimes also asking him to repeat the main activity maintaining his focus.

Instructing was also used in all of Thomaz's sessions. Since Thomaz had RD, it was important for me to provide oral instructions regarding what was expected in a given task. It was also very often used as a strategy to tell Thomaz how to complete a given computer game. For example, during Session 5, Thomaz had to complete an online game in which he had to count verbally to order the numbers on the caterpillar. Figure 6.28 shows the task which was to be completed.

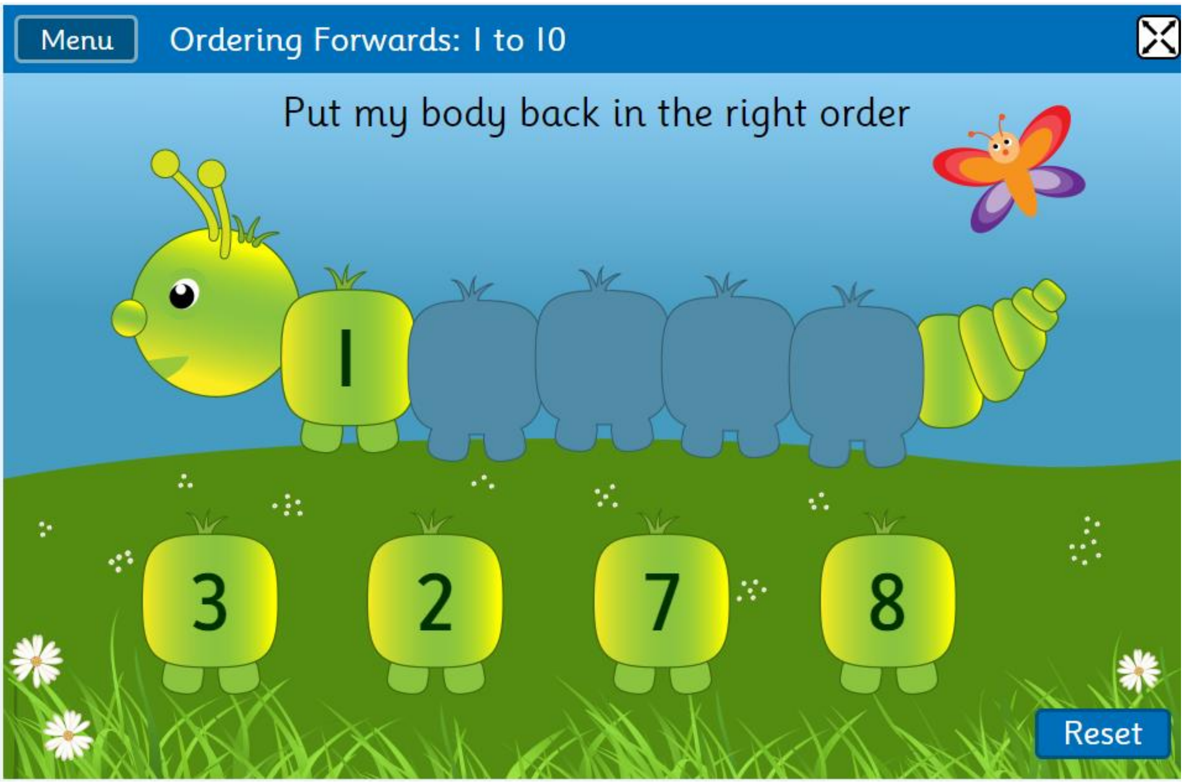

Figure 6.28 shows online task that Thomaz was asked to complete during Session 5. Taken from http://www.topmarks.co.uk/ordering-and-sequencing/caterpillar-ordering.

Although this task had written instructions at the top, I felt that it was important for me to read the instructions, and to add to them, so that Thomaz had a clear understanding of what he had to do. In this case, the use of instructions, evidently shows the importance of the MKO when digital tools are used (see Section 6.12.2.3).

Questioning was an indispensable MKO-driven strategy which was used throughout the sessions with Thomaz. As suggested by the Catch Up Numeracy® programme, the questions asked were related to:

- Prediction e.g. 'What do you think the answer will be?';
- Process e.g. 'How are you going to work it out?' and
- Reflection e.g. 'What will you do next time?'

A situation that exemplifies how questioning supported the internalisation of the numeracy components is one taken from Thomaz's sixth session. Session 6 focused on the 'Counting Verbally: Counting On' component. During this session, questioning not only allowed me to understand that Thomaz had difficulties with subitizing (as also mentioned by his mother during the interview) but allowed me to support Thomaz in thinking about the process he was engaging in. During the session, Thomaz had to throw two dice and count on from one number to another. However, Thomaz hesitated to say the numbers on the dice, prompting me to note that he was not using an efficient strategy to count the dots. The following conversation took place:

EZ	Look at the dice Thomaz. What number is on each dice?
Thomaz	[Thomaz was very hesitant] 3 and … 6
EZ	How come it took you such a long time Thom?
Thomaz	**Don't know**…
EZ	How did you count the dots?
Thomaz	One by one
EZ	Ok, now look at the pattern of the dots and try to remember the pattern. Can you notice anything that will help you to remember them?
Thomaz	Yes…in six there are two of the three…
EZ	Exactly. That is right. What about 2 and 4? [Showed Thomaz the dice showing 2 and that showing the number 4]
Thomaz	This as well…there are two of the twos in four…
EZ	Correct. So now try not to count these dots but try to remember what the pattern is showing.

Through the questions used in this conversation, I found out more about the process Thomaz was using to read the number shown on each die. Moreover, I also supported him in looking at the number patterns and internalising these since in subsequent similar activities Thomaz seemed much faster at completing these activities. As outlined at the start of Chapter 6, some of the strategies highlighted in the various sections of this Chapter overlap and are closely intertwined. In this example, taken from Session 6 with Thomaz, one can also note that Cognitive Structuring was taking place. As will be explained in Section 6.13.3, dice with dots were particularly used with Thomaz because of his poor number sense. Thus, giving him strategies of identifying number patterns without having to count in ones was very important. The questions asked during this activity, and the constant reminder to look at the patterns and their meaning, rather than count the dots on the dice in ones, ensured that I, as the MKO, provided the explanation of this newly taught concept that could support the internalization process by providing the right cognitive structure for the learner to be able to internalize the new learning.

Task Structuring took place on a few occasions with Thomaz. Since I knew the participants very well, the tasks presented were usually geared towards their actual zone of development, hence Task Structuring was generally not needed. A situation of when Task Structuring was needed with Thomaz was that which took place in Session 14. During this session we were focusing on 'Counting Back'. Thomaz had to throw three dice, add the numbers on the dice and count back from that number to 0 since the number range being worked with was 0 -18. I was very pleased to see that during this session Thomaz was no longer counting the dots on the dice but that he was able to recognise the pattern and say the number without hesitation. However, Thomaz was finding adding the three numbers together difficult. Hence, since the main focus on the session was not addition but counting back, after giving him three chances, I decided to use the number cards instead and asked him to pick a number from 0 to 18 and to count back from that number to '0'. This allowed Thomaz to maintain his focus on internalising counting back rather than adding three numbers which was something we had not yet covered, and which was evidently as yet beyond his ZPD.

Thomaz enjoyed Role Inversion. An example of how this strategy was used with Thomaz is when, in Session 16 and Session 17, he had to choose where the paper clip landed, and I had to count from that number back to '0' (see also Section 6.13.2). This strategy is what I chose to name as both a Learner-driven strategy as well as an MKO-driven one. Although it is evident that the strategy is being used and led by the Thomaz himself as he took up the role of the MKO, my role was also important. I led the strategy by purposely making mistakes in order to find out whether Thomaz would notice and whether he had internalised the numeracy component and was able to correct me. Hence, as already argued in Section 6.11.3, Role Inversion had a dual role and positively contributed to the internalisation process in both ways.

Recapturing was a strategy that Thomaz seemed to find very effective. When I did not ask him about what we had done during the previous session, he would tell me himself. During the pre-assessment I had carried out before the intervention programme, Thomaz had demonstrated that he was unaware of the 'order irrelevance' principle when counting. Hence, during Session 18, I set out to intervene upon this numeracy component. Thomaz seemed to grasp this very quickly and in fact he was very confident at saying how many cubes there where in a set when he started counting from different coloured cubes. This confidence is evident in the comment he wrote in the comment box during this session as shown in Figure 6.29: "I no😊" [I know].

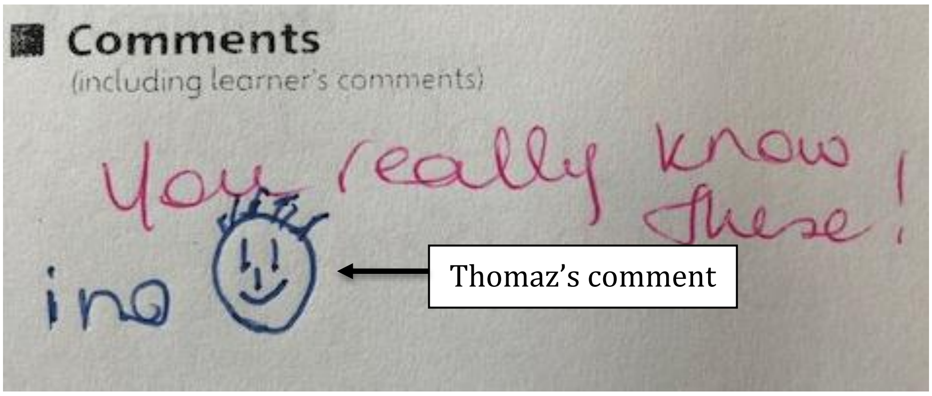

Figure 6.29 Thomaz's comment during Session 18.

Following the activities done in Session 18, Thomaz found Recapturing easy during Session 19. As soon as I asked him what we were doing during the previous session he was quick to reply *'start counting from any cube, the number remains the same'*. Apart from helping Thomaz remember what we had covered from one session to another, Recapturing was also important to engage him in the new learning and to relate this to what he already knew.

Another important MKO-driven strategy that I often used with Thomaz was Prompting. This was used on several occasions to help him to re-focus, or to think again, about an answer he would have given hastily without thinking about whether his answer could possibly be correct. An example of when Prompting was used with Thomaz is that which took place in Session 4. During this session we were focusing on the 'Counting Verbally: Counting On' component. Thomaz did not count correctly from 0 to 20 the first time I asked him to. He went through the number sequence very fast, missing out on some numbers and saying others in the wrong order. Hence, I asked him to repeat the sequence and prompted him to say the numbers slowly, also prompting him when he was hesitant or said the wrong number. This seemed to support him in saying the number sequence correctly the third time round. One of the noticeable mistakes Thomaz was making as he counted from 0 to 20 was that he confused '12' with '20'. This happened during three of the sessions, Session 15, Session 16 and Session 17. In Session 18 Thomaz seemed to have finally internalised these two numbers and their position on the number line since he did not repeat the same miscue. Prompting was not the only MKO-driven strategy used to support the internalisation process of this point. Another strategy used was Miscue Analysis. Every time Thomaz made the mistake I would explain the difference in value between '12' and '20' using cubes to show that '12' is made up of one ten and two ones, whilst '20' is made up of two tens. Figure 6.30 illustrates how this was done.

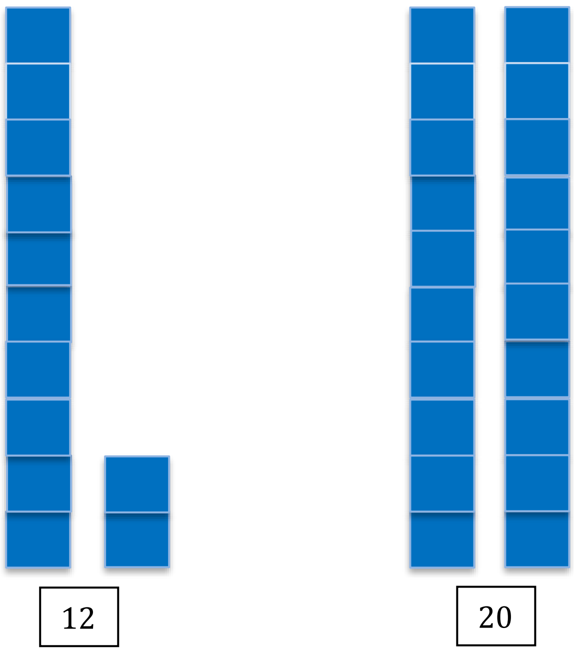

Figure 6.30 '12' and '20' represented using cubes.

Moreover, I showed Thomaz where '12' and '20' where on the number line. Miscue Analysis seemed to have a positive impact on Thomaz's internalisation process. When I asked him to recall the miscues he was making during previous sessions, he was able to do so. However, because he would give replies quickly, he sometimes made the same miscues. In the case of confusing number '12' and '20', this was evident because he kept making the same mistake until after Session 17.

The last MKO-driven strategy that I will focus upon in relation to Thomaz was that of Putting the Learner at Ease. Thomaz felt very confident during the sessions and would talk to me about various things that were not related to the session itself. On one occasion, for example, during Session 19, he started to talk about his favourite game on his iPad and explaining to me that he prefers playing it on his Ipad rather than on his father's mobile because his dad's mobile was 'small' (meaning it had a small screen). Various other occasions of Thomaz talking to me about things which were not related to the session are evident in the audio recordings. Moreover, the comments that Thomaz wrote in the comment boxes, some of which have presented here, are evidence of the confidence with which Thomaz left every intervention session. The programme undoubtedly had a great impact on Thomaz's self-confidence in the numeracy components tackled by the programme.

6.13.2 Learner-driven Strategies: Thomaz

In this section I will seek to demonstrate how the Learner-driven strategies highlighted in Section 6.11 were effective scaffolds for supporting Thomaz's internalisation of the numeracy components. Some of the examples which will follow have already been presented in the previous sections, however others are new and will serve to provide further insight into Thomaz's internalisation process.

The 'self-evaluation' strategy discussed in Section 6.11.1, seemed to be beneficial to Thomaz. As he thought of his learning process and his improvement in the specific numeracy component being focused upon, his self-confidence in mathematics was given a boost. As Silver (1985) suggests, children with MLD usually have low self-confidence in mathematics. This was true for Thomaz as confirmed by his mother during her interview. The positive comments that Thomaz passed during the sessions like *'Is the session already over?' 'Can I try this again?'* showed that his attitude towards mathematics had improved. Moreover, the comments he wrote in the self-evaluation box were clear evidence that Thomaz was enjoying mathematics during his mathematics sessions with me and that these enabled him to have a new perspective of the subject. For example, during his 4th session (presented in Section 6.11.1) Thomaz wrote 'I love it!' in the self-evaluation section. This reflected an improvement on the affective level.

When working on the Counting Objects component with Thomaz I focused on the Order Irrelevance principle. When doing the pre-assessment, as well as during the previous session, Thomaz was asked to count a number of objects and then to say how many there would be if he had to start counting from another of the objects. Thomaz kept re-counting the objects and sometimes also miscounted. He did not realise that the number of objects remained the same. Encouraging Thomaz to analyse this miscue was helpful for Thomaz to internalise the numeracy component at hand. During the 19th session, for example, he remembered that the number of objects in a set does not change, no matter from where you start counting the objects and he recalled his previous session in which he mistakenly thought that "one of them [referring to the cubes] goes away". This illustrates that he had internalised the principle of 'order irrelevance'. The strategy of reflecting upon the miscue he had done during previous similar tasks possibly helped him to remember that the number remains the same no matter from where you start counting. This is very evident in the audio recordings for both Session 19 and Session 20, in which he repeats the words "the same" with great conviction, when I ask him the number of objects there would be had I to start counting from a different coloured cube. Miscue Analysis, as also suggested in Section 6.11.2, was thus an effective scaffold for Thomaz to master the Order Irrelevance principle.

Thomaz was a bubbly child who worked with enthusiasm through the different mathematics tasks which I would have prepared for him. He especially enjoyed helping me set up activities. During the 16th session, Thomaz was doing the slippery paper clip activity (Catch Up®, 2009). In this activity, the paper clip lands on a number given on a number line and the learner has to count back from the number to 0. Since in previous sessions I had asked Thomaz to take up my role, he asked me to do so once again. Thomaz chose the number the paper clip landed on and I had to count back. Sometimes I purposely miscounted and waited for his response. During this session, he sometimes did not realise that I had not counted correctly. Due to this, I had the chance to re-explain and rehearse counting back with him whilst focusing on the numbers he confused most. Hence, role inversion was effective in getting Thomaz to reflect about the miscues I had made and revise the number sequence accordingly. This seemed to have a positive impact since when we did this activity in the 17th session, which was the last session during which we were focusing on counting back, Thomaz always realized what was wrong, thus showing me that he had internalised counting back by this last session.

After demonstrating how the Learner-driven strategies discussed more generically earlier were effective with Thomaz, I will now move on to explaining how the Tools-assisted strategies had a similar impact.

6.13.3 Tools-assisted Strategies: Thomaz

Throughout the intervention sessions with Thomaz both psychological tools and technical tools were used. These seemed to act as effective scaffolds to support the internalisation of the numeracy components being tackled.

The technical tools used were ones that are generally used in mathematics lessons, such as a ruler. As outlined in Section 6.12.1, technical tools did not directly impinge on the internalization process because they were not being used as a mathematical representation in themselves. However, they had an indirect role in ensuring that the activities planned and administered were successfully completed to facilitate the internalization process. Unlike the chosen psychological tools, the technical tools used were not used in a specific individualized way but according to the situation's need (for example, a ruler was needed if a straight line had to be drawn). Hence, I feel that giving an exemplification of how the technical tools were used is not necessary here. Since technical tools have been discussed at length in Section 6.12.1, including with reference to relevant literature, I will here restrict myself to the psychological tools that were of specific support to Thomaz.

As already explained in Section 6.12.2, semiotic mediation and the use of language as a psychological tool was used throughout all the sessions. In that section, I demonstrate how semiotic mediation was fundamental in providing the rich interactions which were necessary for internalisation to take place for all the participants. Moroever, I specifically give an example of a learning situation with Thomaz during his 11th intervention session. In the example given I showed how semiotic mediation was essential in identifying that Thomaz still had a weakness with counting backwards. However, it is interesting to note that other psychological tools were chosen according to the learner's needs and interests. As already highlighted, Thomaz loved using the computer and completing digital games. Thus, I used digital games with him more than with the other participants. These served as a great motivator for Thomaz and one of the main reasons why he enjoyed his sessions so much. Another psychological tool which I specifically used with Thomaz were dice. As revealed in the interview with his mother, Thomaz still had difficulties with number sense and understanding number magnitude. Hence, I hoped that using dice would support him in internalising their number patterns and in developing a mental representation of these same patterns. Thomaz had also not yet internalised the number sequence. I found it helpful to use number lines and number grids extensively with him. I also created a large number line on the floor so that he could jump from one number to another as this could facilitate the internalisation of the number sequence. Specific activities that I carried out involving this large number line have been presented in Section 12.2.1. By the end of the intervention programme, Thomaz had internalised the number sequence; could count forward and backwards to/from 20 and choose the greater number from two given numbers, indicating that these different psychological tools contributed to the impact of the intervention programme on the learner's cognitive development.

Following this account of how each of the MKO-driven, Learner-driven and Tools-assisted strategies supported Thomaz's internalisation of the numeracy components, I will now return to the general discussion. In the next Chapter I will demonstrate how these three strategies came together to ensure that effective teaching and learning took place throughout the programme. A further discussion of how this occurred specifically in relation to Thomaz will be given in Section 7.3.

Chapter 7
Discussion and Conclusion

Chapter 7: Discussion and Conclusion

7.1 Looking at the Holistic Picture: bringing the findings of the study together

This thesis had a main research question:

i. Is an intervention programme carried out with children having MLD only and both MLD and RD beneficial to these learners? Which characteristics of the intervention programme were effective with each group of learners?

It also had four other subsidiary questions, namely:

ii. What are the teachers' and parents' perspectives on how the children's MLD affects their daily lives at school and at home?

iii. Do children with solely MLD and those with both MLD and RD have similar mathematical profiles? Are both groups of children strong/weak in the same areas?

iv. Do the children assessed with MLD or both MLD and RD and also with a profile of Dyscalculia as identified by the Screener (Butterworth, 2003) have difficulties in all numeracy components or in just a few?

v. Is mathematics anxiety one of the difficulties experienced by the learners?

The analysis of data that has been presented in this thesis has shed light on different strategies that seem to have been effective in supporting the learners with both MLD and MLD and RD to internalize the numeracy components. However, it is now important to bring all the pieces of the 'puzzle' together to articulate the answers to my original research questions.

7.2 Answering the Subsidiary Questions

I will begin by addressing the subsidiary questions that have already been discussed to outline an answer to each one. The first subsidiary question has been addressed through the first sections in Chapter 6 in which a detailed profile of each learner was presented together with the findings from the data collected through the interviews and the observations carried out in class. Keeping in mind the child's profile of strengths and needs ensured that the intervention programme was indeed personalised and as effective as possible. This also permitted the learner to remain at the centre of learning to ensure that the study maintained the principles of the interpretivist paradigm that underpinned it.

To answer the second and third subsidiary questions, I looked at the pre-intervention scores in the formative assessment carried out to assess the ten numeracy components. These were presented in Table 6.7. Table 6.7 showed the numeracy components each learner had difficulties with and whether the participants had solely MLD or both MLD and RD. Table 6.7 showed that no two learners had the same areas of difficulties in the ten numeracy components supporting arguments put forward by other research. Primarily, this indicates that there is nothing such as 'mathematics ability' but that is it more correct to speak of 'mathematical abilities' (Dowker, 2005a). The learners' different profiles indicate that mathematics is componential and that one may struggle in one component but not in another. Furthermore, this finding indicates that learners with MLD and both MLD and RD are a heterogeneous group of learners and that the profile of difficulties each individual learner presents is unique. This supports all previous studies carried out (Bartelet et al., 2014; Dowker, 2005, Geary, 2010; Kaufmann & Nuerk, 2005), and which have been discussed in the literature review of this thesis. Some similarities were highlighted between the areas of difficulties; however, it was concluded that the profile of difficulties was not determined by whether the learner had only MLD or both MLD and RD (see Section 6.7). This finding indicated the importance of assessing learners individually and the need to create an individualised profile of mathematics/numeracy strengths and needs for intervention to be effective. It also suggested that one-to-one intervention may be more effective than small group intervention especially if the group of learners have a different profile of needs.

This difference between the learners' profiles makes it difficult to determine which specific strategies were more effective with both groups of learners. The findings from the data have illustrated that both groups of learners did internalise most of the numeracy components intervened upon. However, when analysing the pre- and post- test scores, I could deduce that the progress made by the different participants seemed to depend more on domain-general factors such as working memory and non-verbal reasoning ability. This corroborates findings from other studies that highlight the impact of these abilities on mathematical achievement (Geary et al., 2007; Karagiannakis & Cooreman, 2015; Lefevre et al., 2005). Thomaz (RD and MLD group), who had the lowest score in the Aston Auditory Sequential memory test (Table 5.4) and the lowest percentile score in the non-verbal reasoning ability test (47^{th}), registered least gain from the programme. These results were similar to Ethan's (also RD and MLD group), who also had low scores in these two domain-general tests. On the contrary, Seb who was also part of the MLD and RD group, achieved the highest scores on the domain-general tests and seems to have made most progress out of the six participants. This suggests that the effectiveness of different strategies may depend on each individual child's domain-general profile. Whether the child belongs to the group of learners with solely MLD or that of both MLD and RD did not seem to make a difference. Here I conclude that it seems more important to collect data about the children's mathematical and general profiles, as was done in my study, to ensure that intervention is tailor made to each learner's profile.

The third subsidiary question could also be addressed through the data in Table 6.7. The child who had been assessed by the Dyscalculia Screener (Butterworth, 2003) as actually having dyscalculia was Thomaz. Thomaz's mathematical learning profile illustrated that he did not have difficulties with all the numeracy components indicating that learners with dyscalculia also have strengths, which can be tapped upon to make intervention more beneficial. However, it was interesting to note that Thomaz did present the greatest number of areas of difficulty – a total of '18' when the average areas of difficulties were '12.83'. This indicates that learners with dyscalculia have a wider spread of difficulties than learners with only MLD or MLD and RD, hence leading to the more severe nature of their difficulties. Albeit this finding for this one learner, it would be important to study a larger sample of learners with dyscalculia to understand whether this reality is the same for other dyscalculic learners too.

The last subsidiary question to be addressed is that of mathematics anxiety. The scores obtained in Chinn's (2012) mathematics anxiety questionnaire were presented in Table 6.8 (Section 6.7). The scores of the six pupils were above the 'average' range determined by the norms for the questionnaire. The average score given by Chinn (2012) for male 9-year-olds in the UK is that of '36.7', but all the pupils scored higher than this. Moreover, three of the learners scored almost two Standard Deviations (SD) higher than average on the test and one of them scored one SD higher than average. Only two pupils had anxiety levels that were within average range. These scores indicate that learners with MLD tend to have higher than average levels of mathematics anxiety. This finding is in line with other literature which suggests that learners with MLD need support in enjoying mathematics tasks more (Silver, 1985) and in raising their self-confidence in mathematics (Stevens et al., 2004). Despite this finding, a pattern of results for the scores obtained was not noted as the total number of areas of difficulty did not tally with the result obtained in the questionnaire. Thus, the anxiety scores obtained by the learners did not seem to correlate to the severity or nature of their difficulties. As explained in Section 6.7, for example although Thomaz was struggling in the highest number of numeracy components, his anxiety was the lowest. On the contrary, although Mike had the lowest number of numeracy components needing intervention, his anxiety score was the highest. Moreover, Seb, struggling in 11 numeracy components (placing him in the middle), scored '40' on the anxiety test component. This was the second lower score again indicating no evident correlation. The scores obtained by the participants also illustrated that none of them had very high anxiety levels as all the scores were below the parameter (a score of 59) for this as indicated by Chinn's (2012) norms for the questionnaire.

After answering each of the subsidiary research questions, I will now move on to answering the main research question and to bringing together all the data collected, and the findings put forward.

7.3 Findings for the Main Question

The main question I set out to answer through this thesis was:

> Is an intervention programme carried out with children having MLD only and with children having both MLD and RD beneficial to these learners? Which characteristics of the intervention programme are effective with each group of learners?

Through my analysis of data, I showed how all the learners had found the programme beneficial both on a cognitive and an affective level. Primarily, I discussed how the learners registered progress in the scores obtained in the standardised tests as well as the numeracy components intervened upon. Secondly, I showed how the programme was also beneficial in supporting the children's internalisation of the numeracy components being tackled and in increasing the children's self-confidence in the subject. These observations indicate that learners with MLD can achieve in mathematics when the specific gaps in their learning of mathematics are addressed and when the affective domain is taken into account. This is in line with other research that has investigated MLD (Kaufmann et al., 2003). The six learners made a significant gain in their number ages and proved to have internalized the numeracy components addressed when they were assessed after the intervention programme. The fact that the post-assessment, for most components, took place at least six months after the intervention on the specific component also shows that the internalisation was longer term rather than momentary. Hence, a discussion of what 'worked' with these children could take place focusing on the interactive strategies that were used as part of the programme.

What I will now seek to provide is a view of what I believe has led to this success. As a teacher-researcher working within an interpretivist paradigm, I was interested in understanding the characteristics for effective intervention for MLD. When planning the intervention programme I used Vygotsky's theories to shape the strategies used throughout the sessions. I also used themes taken from his principles to analyse the data obtained through the audio recordings. However as illustrated in several sections of Chapter 6 some of Vygotsky's theories have been re-shaped to apply specifically to the internalisation of the numeracy components. New themes emerged, and further effective strategies seemed to surface.

In my conceptual framework (see Section 3.6) I explained what I have considered to be evidence of internalisation. Three types of strategies for supporting the internalization process were identified: MKO-driven strategies, Learner-driven Strategies and Tools-assisted strategies. Vygotsky's theories were applied throughout when designing the intervention programme and when looking for predetermined themes. Hence, for each one of these three types of strategies, several themes were identified. Some were used as *strategic scaffolds* (Hobsbaum et al., 1996) (see Section 3.5) and were used intentionally when planning the intervention sessions. These were:

- the development of a mathematical profile for each learner so that the sessions were learner-centred.
- the seven strategies mentioned by Tharp (1993) - modelling, feedback, contingency management, instructing, questioning, cognitive structuring and task structuring;
- technical and psychological tools; and

Other themes emerged as the data was being analysed. These were:

- further MKO-driven strategies – role inversion, recapturing, prompting, putting the learner at ease and miscue analysis;
- the idea of Learner-driven strategies which included: self-evaluation, miscue analysis led by learners' reflection and taking the role of the teacher; and
- digital tools as psychological tools and tools that serve as both technical and psychological.

Each group of strategies has been discussed at length throughout my Analysis Chapter (Chapter 6). What I now propose is that these three strategies seem to come together in a symbiotic relationship to ensure that the learner is supported in the passage from the actual zone of development to the potential zone of development in the ZPD. I have tried to find the way in which all these strategies tie together and how they merge to provide effective intervention for children having only MLD and both MLD and RD. I not only looked at each of the three forms of strategies individually but also looked into how the three forms of strategies are related and connected. This is a development of Rogoff's (1995) 'participatory appropriation' (explained in Section 3.3) in which she extended Vygotsky's theories by looking at how the learners' active participation in the task itself leads to development. The more holistic pedagogical model developed through this study provides further answers as to what it is that shifts the social plane into the individual one (Vygotsky's main concern (Wertsch, 1985; see Section 3.3)) in relation to the internalisation of the numeracy components. This is what I will explain in the following paragraph.

My data analysis prompted me to determine that a learner's mathematical profile (including domain-general ability such as working memory, previous experiences in mathematics and strengths and weaknesses in this area of learning) is a determing factor in the success of the learner. The previous experiences gained by the learner, that are part of this profile, are unchangeable. However, this mathematical profile, including domain-general ability, leads to some of the Learner-driven strategies and dispositions that can support the intervention's success. This idea has been discussed at length through the exemplification of how Thomaz's profile impinged on the dispositions he brought to the learning situations and how it influenced the Learner-driven strategies used (see Section 6.13 and Section 6.13.1). The relationship between the Learner-driven strategies and the MKO-driven strategies is manifested through social interaction and dialogue that is rich enough to provide the opportunity for internalisation to take place. Both Lerner-driven strategies and MKO-driven ones are closely related to the Tools-assisted strategies. The Learner-driven strategies are connected to the Tools-assisted strategies through an engagement in multisensory experiences that enhance internalisation. Whereas the MKO-driven strategies are connected to Tools-assisted ones because the MKO makes use of tools to facilitate the internalisation process. Tools-assisted strategies also allow the MKO to use scaffolds that are specifically relevant to mathematics; especially when mathematical tools are used such as the number line. Hence, the internalisation of the numeracy components, which are the basis for all mathematics learning, seems to be the result of:

- Rich social interaction and dialogue;
- Cultural tools which are used by the MKO to guide the learner within the ZPD; and
- An engagement in Multisensory experiences.

To bring together all these findings, I thus suggest the pedagogical model for teaching - learning presented in Figure 7.1 which I propose to be beneficial when it underpins effective intervention programmes.

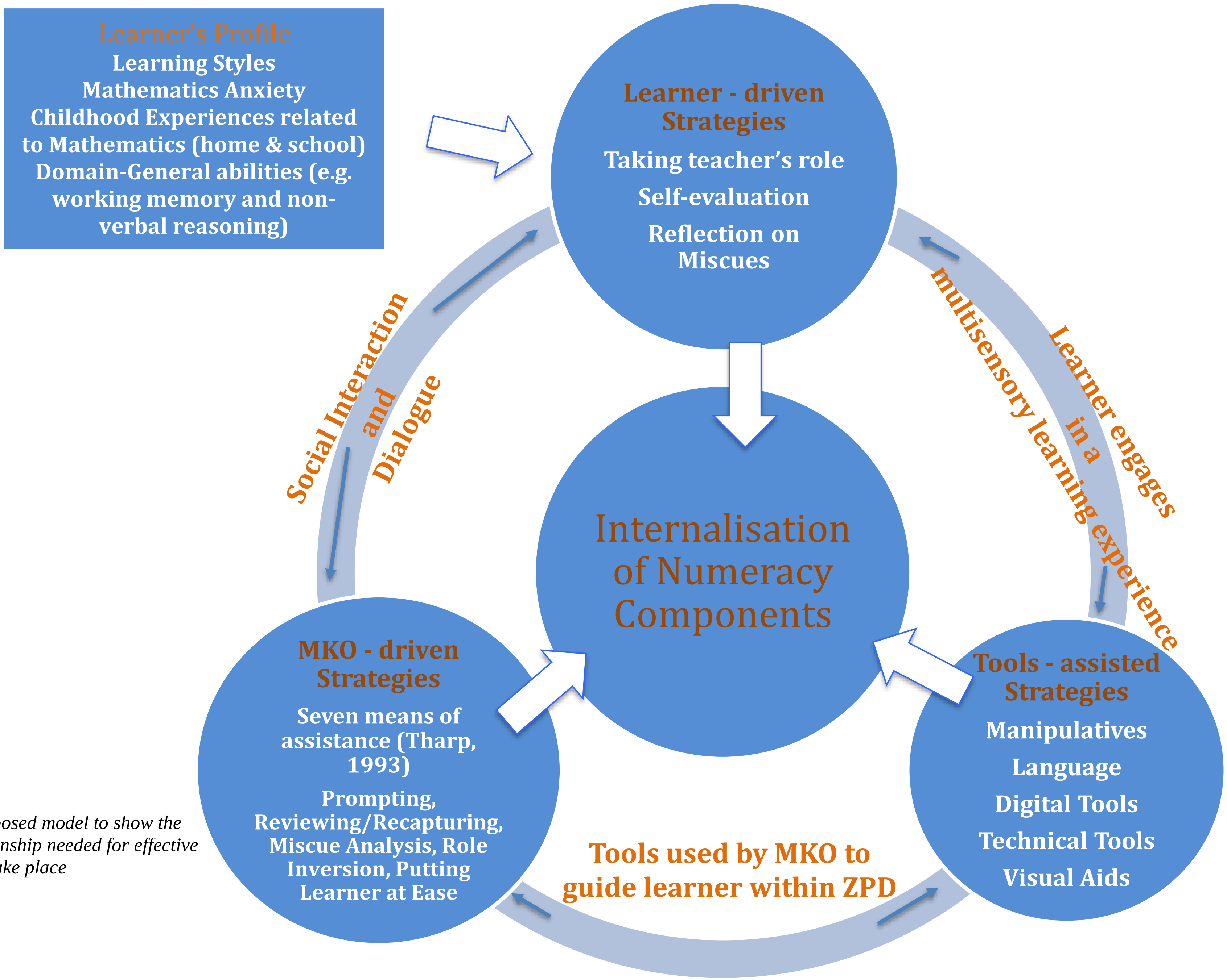

Figure 7.1: Proposed model to show the symbiotic relationship needed for effective intervention to take place

The pedagogical model presented in Figure 6.22 captures all the findings of this study that allowed me to answer the main research question regarding what it is that works with children having MLD or both MLD and RD. It is, in my view, what the transformative collaborative practice between the cultural tools, cultural influences (presented through the mathematical profile) and other significant others (as proposed by Vygotsky in Vianna and Stetsenko, 2006) that is specific to the internalisation of the numeracy components and hence to the basis of mathematics learning. The learner is central in the process in line with Vygotsky's theory. Hence, intervention should start with a deep understanding of the learner's profile regarding mathematical experiences, abilities and dispositions. Although previous experiences are not changeable, these experiences together with the learner's abilities and dispositions can be used when developing and designing individualized intervention to support internalisation.

The three fundamental learning opportunities which support internalisation mentioned earlier can only be brought about if the MKO makes effective use of all the MKO-driven strategies identified: the seven means of assistance identified by Tharp (1993) based on Vygotsky's theories and the additional five strategies identified through the themes that have emerged in this study. Moreover, the learner should be encouraged to make use of the Learner-driven strategies that have been identified to engage in the learning process more effectively and be empowered to take hold of future learning. Finally, the cultural tools, which have been extended and applied to mathematics education, should play an important role in the provision of any intervention. Technical tools were essential for the carrying out of different tasks. In this intervention programme, these included: the ruler, mini whiteboard, number cards and paper clip. Psychological tools were crucial too as they provided the necessary mental representations for the learners to internalize the numeracy components. The psychological tools used in this study were: language, visual aids, manipulatives and digital tools. It should also be noted that some tools may act as technical and psychological tool and may thus have a greater impact on facilitating the internalisation process.

Section 7.3.1 Exemplification of Model's Application to one of Thomaz's sessions

In Sections 6.13, I provide a detailed analysis of how the three different types of strategies were used as part of Thomaz's intervention programme to facilitate his internalisation of the numeracy components. Following the argument put forward in Section 7.3 about the relationship between these three types of strategies, I will now seek to illustrate how this symbiotic relationship was realised in one of Thomaz's session. This will be done to illustrate the applicability and validity of the model.

For the purpose of this task I have chosen to focus on Session 14 which tackled the numeracy component of 'Counting Back' within a number range of '0 – 18'. Nonetheless I wish to highlight that the analytical process can be replicated for other sessions. I choose to focus on this session because several of the identified strategies were used during this session. The plan which I had prepared for Session 14 can be seen in Table 7.1.

Table 7.1: Plan for Session 14 with Thomaz

<table>
<tr><td>Numeracy Component</td><td>Counting Verbally: Counting Back</td></tr>
<tr><td>Review</td><td>Ask Thomaz to recall what has been done during the previous session. Revise counting back by giving the numbers from 0 to 15 to Thomaz and asking him to place them in the correct sequence starting from the number 15. After this, ask him to build towers with the cubes to represent the numbers.</td></tr>
<tr><td>Main Activity</td><td>Ask Thomaz to throw three dice, estimate the number of dots altogether and to count from the total number on the dice back to 0 first using a number line and later with no help. Model the task for him. Repeat the activity, this time throwing the dice yourself and counting back to '0' from the number you get. Make mistakes at times.
Following this, complete the online game:
http://pbskids.org/curiousgeorge/busyday/rocket/

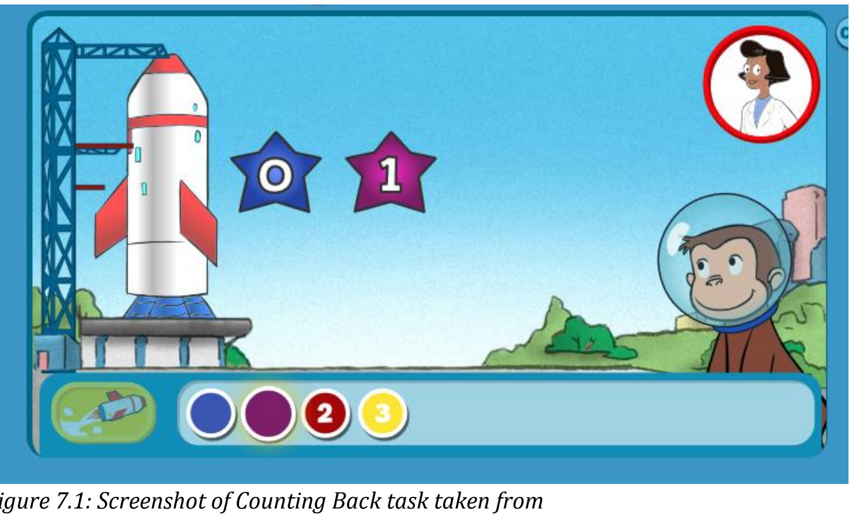

Figure 7.1: Screenshot of Counting Back task taken from
http://pbskids.org/curiousgeorge/busyday/rocket/

Repeat the instructions given by the online game if Thomaz does not understand them. Model the task if needed.</td></tr>
<tr><td>Linked Recording</td><td>To throw dice once again and then write the sequence from that number back to 0.</td></tr>
<tr><td>Questions</td><td>Which number comes before x? How many dots on the dice altogether? How are you counting the dots? What are the other terms for *counting back*?</td></tr>
</table>

In Session 14 all the MKO-driven, Learner-driven and Tools-assisted strategies were used. Each one is listed in Table 7.2.

Table 7.2 The different forms of strategies used during Session 14 with Thomaz

MKO-driven Strategies	Learner-driven Strategies	Tools-assisted Strategies
Modelling The Main Activity had to be modelled and the online game was to be modelled if Thomaz did not understand the instructions. **Feedback** Oral feedback was given as Thomaz completed the different activities as part of the 'Main Activity' section of the session. Written feedback was given on the 'Linked Recording' task. **Contingency Management** This was used when praising Thomaz if he counted back correctly from the chosen number. **Instructing** Instructions were given on how to complete each given task as well as to re-explain the online game. **Questioning** Questions were asked about the process of counting the dots on the dice and the process of adding the three numbers. Questions were also asked about prediction when Thomaz was asked to estimate the number of dots on the dice. **Cognitive Structuring** Dice were used since Thomaz had a poor number sense. The dot patterns on the dice would help him to develop mental representations of these patterns. **Task Structuring** Task Structuring took place when Thomaz found it difficult to add the three numbers on the dice and therefore after three chances, I decided to continue using number cards as per the review activity to maintain the focus on counting back. **Role Inversion** MKO to make mistakes which Thomaz had to correct when counting back from the chosen number. **Recapturing** Used at the start of the session when asking Thomaz to recap the previous session. **Prompting** Prompting was used throughout the session especially when Thomaz could not complete a given task, particularly when he was hesitant as he counted back from the chosen number. **Putting the Learner at Ease** When asked to complete the sequence in the Linked Recording section, Thomaz said 'Yayy! This is exactly like Minecraft!' showing his ease with the task. **Miscue Analysis** When Thomaz said or wrote the wrong number as part of a sequence, the MKO had to draw his attention to this and support him in analysing his mistakes.	**Self-Evaluation** Thomaz completed the Comments section saying, 'Great Work' and drawing a smiley face since he had done well during the session. **Miscue Analysis** The learner had to count back from the number '12' to '0' as part of the Linked Recording activity. He wrote '12, 13, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0'. Through questioning Thomaz was encouraged to think through the sequence again and realized his mistake. This seemed to help him internalise the number sequence since he was able to complete the number sequence correctly during the following session. **Taking the Teacher's Role** Thomaz was asked to take my role during the Main Activity in which he had to ask me to take a number and count back from that number to '0' correcting me when I made a mistake.	**Technical Tools** Dice were used for their capacity to generate a number to count back from. **Psychological Tools** ***Language*** Language was used throughout the session to ask questions, give instructions, prompt the learner, provide feedback and to model the tasks. It played a crucial part in the interaction with the child. ***Manipulatives*** In this session towers of interlocking cubes were used to support the internalisation of the number sequence. Dice were also used as psychological tools to support the development of number sense. ***Visual Aids*** The number cards and the number line were the visual aids used in this session to facilitate the internalisation of this numeracy component. ***Digital Tools*** The online game used. **Tools serving as both Technical and Psychological Tools** Dice were used to generate a number but also to support the development of Thomaz's number sense.

As outlined by the proposed pedagogical model (see Figure 7.1) the Learner-driven strategies relate to the Tools-assisted ones when multisensory experiences are provided. In Session 14 this relationship can be seen in the ‘Review’ part of the session in which Thomaz was asked to put the sequence of the numbers in order using the number cards and to build corresponding towers with interlocking cubes to represent each number. The aim of this multisensory task was to support the learner to internalise the number sequence using concrete objects as psychological tools. As discussed in Section 6.12.2.1 these tools facilitated the development of mental representations which are much needed for the understanding and use of the number sequence. This was evident because in subsequent sessions Thomaz was much more confident with counting backwards and eventually showed to have internalised the number sequence. The way in which both these strategies came together through the multisensory experience were an important factor which contributed to Thomaz’s success. Learner-driven strategies and Tools-assisted ones also came together in the ‘Main Activity’ in which the number line was used by the learner as a pictorial representation of the number sequence. This was used as a strategic scaffold which could facilitate the child’s internalisation of the numeracy component and allow him to move towards completing similar tasks without this aid in the future. Much like the towers of interlocking cubes, the number line was used to facilitate the development of a mental representation. This seemed to have successfully taken place since, by the end of the sessions focusing on this component, Thomaz no longer needed any aids to be able to count backwards within the number range ‘0 – 20’.

The pedagogical model (Figure 7.1) demonstrates that the Learner-driven strategies and the MKO-driven ones come together when rich social interaction and dialogue are provided. In Session 14 with Thomaz, several opportunities for rich social interaction and dialogue were provided through the various tasks and activities carried out. Primarily the questions posed, the feedback given, and the instructions provided by the MKO supported the learner to make use of Learner-driven strategies such as Miscue Analysis. Moreover, the activity in which I myself had to count back from a chosen number to ‘0’ and Thomaz had to correct me when I made a mistake, allowed important dialogue to unfold through which Thomaz and I could identify any difficulties which still needed to be targeted. Such activities also seemed to serve as an important means of reflection on the learner’s part which possibly supported the internalisation of the numeracy component focused upon.

Finally, the pedagogical model (Figure 7.1) also suggests that the MKO-driven strategies are linked to Tools-assisted ones since the strategies used by the MKO make use of tools, both technical and psychological, in order to facilitate the internalisation process and to guide the learner within their zone of proximal development. If we take, for example, the activity with the dice used in Session 14, had I not used the dice intentionally as a strategic scaffold to develop Thomaz's number sense, the other MKO-driven strategies that were used would possibly not have been effective enough to support this development. Hence, Tools-assisted strategies are an asset to the MKO and should be used with intention since it is only in this way that the maximum benefit of any learning situation can be accessed. Similarly, I used the digital game to provide an engaging activity which the learner would surely enjoy. The effectiveness of the use of the other MKO-driven strategies, such as questioning, instructing and feedback was maximized through this Tools-assisted strategy.

The various ways through which the Learner's profile impinges on the learning situation and the outcome of the intervention programme have been outlined in Section 6.13 and Section 6.13.1 in particular relation to Thomaz's profile. As a development of this, I have now provided the detailed exemplification of how the proposed pedagogical model (Figure 7.1) was realised in Session 14 carried out with Thomaz. The findings of this critical interpretation indicate that intervention for MLD should:

- Ensure that rich social dialogue and interactions between the Learner and the MKO are provided by intertweaving the use of MKO-driven strategies to the use of Learner-driven ones;
- Make use of Tools-assisted strategies to maximise the benefits of learning situations. The MKO-driven strategies are dependent on Tools-assisted ones for the successful inernalisation of the numeracy components;
- Provide as many opportunities for multisensory experiences as possible through which Learner-driven strategies can interweave with Tools-driven ones to support the learner in developing mental representations of the numeracy concepts and skills which are much needed for the more complex learning of mathematics.

7.4 Contributions to and Implications for Research and Practice

This study has contributed to knowledge currently available about MLD both vis-à-vis theory and practice. The findings of this study have contributed to the understanding of the nature and degree of mathematical difficulties of different learners and of those belonging to two main groups of children - those with only MLD and both MLD and RD. This has been an area limitedly studied (Cirino et al., 2007; Jordan & Hanich, 2000; Jordan & Montani, 1997) (Section 2.12 and Section 2.12.1). Furthermore, very few studies, except for Tressoldi, Rosati, and Lucangeli (2007), have taken multiple-case intervention studies as I have and looked at their specific profiles of difficulties. Hence a significant contribution is also the qualitative analysis of the learners' mathematical profiles as presented in this study, and the finding that the learners' profiles were not dependent on whether they were part of the MLD only or MLD and RD group but rather on individual abilities. Moreover, although in recent times different interventions have been developed for supporting learners with MLD, "it is still true to say that most educational interventions thus far have targeted just one component, most commonly factual knowledge trained by drilling" (Koponen et al., 2018, p.3). Hence, the contribution of this study to research is an important one as it looks at all the ten numeracy components and the various strategies that may lead to their internalisation. In addition, many of the studies available in the field of MLD are of a quantitative nature (Petrill & Plomin, 2007; Vukovic, 2012) and explore whether the children find various intervention programmes beneficial (Fischer et al., 2011; Holmes & Dowker, 2013; Wright, Martland, Stafford & Stanger, 2002). They do not usually provide an insight into what contributed to making the programme beneficial as I have attempted to do through this thesis. Conducting these six case studies, working with the children over a long period of time, and analyzing the data as an interpretivist researcher who looked at pre-determined themes, as well as any emerging themes, allowed me to gain a deep insight into what it is that actually supported children having mathematics learning difficulties. Only a limited number of studies (Dowker, 2004, 2009; Emerson, 2015; Kaufmann et al., 2003) have looked at what 'works' with children having MLD.

Although Vygotsky's theories have been applied to mathematics education in previous studies (Abtahi, 2014; Abtahi, 2017; Mariotti, 2009), none, to my knowledge, have applied Vygotsky's theories to understanding which strategies are effective in supporting learners who have difficulties with mathematics learning. This study has suggested that Vygotsky's theories concerning the notions of 'internalisation', 'cultural tools', the 'Zone of Proximal Development (ZPD)' and the role of the 'More Knowledgeable Other (MKO)' may be beneficial to understanding MLD and to providing effective intervention for MLD. I thus propose that the pedagogical model presented in Section 7.3 (Figure 7.1), which is underpinned by these theories, may be used to study further the implications of Vygotsky's theories to supporting learners with MLD in achieving further in the subject. It has also shown how Rogoff's (1995) idea of 'participatory appropriation' is one of the many facets that shift the social plane to the individual one (Wertsch, 1985). Hence, the pedagogical model presented here, outlines a more holistic view of the facets which lead to this shift and how they are connected in the case of supporting learners with MLD. This undoubtedly contributes to answering Vygotksy's main concern about how learning, and thus development, takes place with a focus on the numeracy components which are much needed for the learning of more complex mathematics.

The implications of this study for practitioners are numerous. Most importantly, it shows that children with MLD can make improve in mathematics if given the right intervention. This accentuates other similar findings (Holmes & Dowker, 2013; Kaufman et al., 2003). It implies that most learners can succeed at mathematics and that it is important that individual intervention is provided, as early as possible, to support these learners in achieving their full potential in the subject. The qualitative nature of this study allowed me to explore, at great depth, what is effective with learners struggling with MLD. The close relationship that developed between me, as the MKO, and the learner, served as a 'magnifying glass' for looking at daily practices that teachers may already use in class and explore how these may be used together for greater effectiveness. Hence, the pedagogical model that I have developed based on the findings of this thesis, is also an original contribution to a better understanding of the transformative collaborative practice (Vianna & Stetsenko, 2006) that needs to take place for the internalisation of the numeracy components. The various additional strategies identified for all three types of strategies are also another main contributor towards practical strategies needed during intervention for MLD.

Another contribution to the field of practice is the finding that no two learners with MLD had the exact same profile of difficulties. This implies that, any intervention that is to be done with children struggling with mathematics, is to start with a thorough assessment. It is only through such as assessment that the teacher would be able to identify the areas of difficulties that need to be addressed. A deep understanding of the learner's mathematical profile, including knowledge and dispositions will contribute to the effectiveness of any intervention for MLD. Findings that show that domain-general abilities such as working memory and non-verbal reasoning indicate that these are important factors which determine mathematical achievement. Hence, it may also be appropriate to follow the suggestion of Karagiannakis and Cooreman (2015) who state that, "MLD students show greatest progress when provided with direct, explicit, and multisensory instructions adapts to their individual learning profile, strengths and weaknesses" (p.223).

Another important implication of this study for practice is that the type of intervention given to learners with MLD needs to address their gaps in numeracy first. The way in which this is done is fundamental. This study showed that intervention that is well-planned and individually-designed, tends to be more effective than one which makes use of random strategies. Moreover, it is fundamental that the MKO makes use of cultural tools, specific to mathematics learning, to provide multisensory experiences to the learners. The MKO also needs to ensure that rich social interaction and dialogue is provided through various learning opportunities in which the learners can express themselves mathematically and think out aloud about the processes undertaken for solving a task and their miscues. The learner should be encouraged to develop strategies, such as those of self-evaluation and miscue analysis for internalisation to be more effective. These are necessary to empower the learner to tackle future difficulties he may encounter when learning mathematics.

A final observation which is worth noting here is that one test was more sensitive to the progress registered through the intervention programme than the other test. The six participants demonstrated to have gained much more from the intervention programme through the BNST (Gillham & Hesse, 2001) rather than from Chinn's (2012) timed test. This may imply that unless given additional time on assessments, learners with MLD would not be able to show what they know in relation to the topic being assessed. Hence, if the aim of our assessment is to monitor progress, it may be wise to remove the timed factor to gauge their progress more accurately.

7.5 Limitations and Recommendations for Further Research

The findings of this thesis have contributed to our knowledge of MLD and our understanding of children who have MLD and both MLD and RD. Nonetheless, case studies do not allow for generalization. It would be interesting to see whether the findings for this group of male learners would be similar to those from a group of female learners. For example, research about mathematics anxiety has indicated that females show higher mathematics anxiety levels than males (Devine et al., 2012). In this study, I did not note an obvious connection between anxiety levels and the children's profile of difficulties, however this may not be the case for girls. Moreover, since a few difficulties were seen to be common to all the learners in this study, it would be insightful to see whether these areas of difficulties are also common to female learners, and, of course, other groups of children (for example older and younger children).

It would also be interesting to explore whether the benefits of the programme are retained following a longer period of time. The fact that re-assessment took place exactly at the end of the programme, may mean that the children's newly acquired mathematical knowledge, skills and competencies were still 'fresh', facilitating their recall of them.

I acknowledge that my background and understanding of MLD has undoubtedly influenced my interpretation of the data. The fact that I analysed the data using pre-determined themes ensured that focus was maintained on Vygotskian principles. Other similar qualitative studies, with a different focus, may have interpreted the data differently. The beauty of conducting case studies is that they allow for an in-depth understanding of the learning processes taking place. I am aware that one own's interpretation in such situations is inevitable, hence my choice for an interpretivist underpinning. Bearing this in mind, it is important that the model I have developed is applied to other learning situations. Its effectiveness could also be tested when learners are not in a one-to-one learning situation but rather in small groups. It may also be interesting to see whether this model may be applied to the general mathematics teaching and learning that takes place in primary classrooms.

7.6 A Concluding Note

This study has shed light on new areas of MLD that, to my knowledge, have not yet been explored. Within its limitations, it has made some original contributions to epistemology and has, in my view, provided a better understanding of what facilitates the internalisation of the numeracy components that are the basis of mathematics learning. Effective intervention for learners struggling with mathematics is a field of research that is still heavily under-researched but that is very important for teacher-researchers, like myself, whose ultimate goal is to ensure that each learner is given the right learning opportunities to succeed at mathematics. My personal engagement and interest in the learners themselves have been evident throughout this thesis, relating well to my choice of framing this study within an interpretivist paradigm. I hope that the findings highlighted here will provide new insights on how Vygotsky's theories may help to develop the right practice to facilitate for the internalisation of mathematics learning. Furthermore, I hope that the model that has emerged from my analysis of the data, will prove to be a valid tool for teachers to use when designing and developing intervention for mathematics difficulties. Finally, I hope that the ultimate, but most important, beneficiaries of my study, will be children struggling with mathematics learning, who may in the future be given the most beneficial form of interventions to enable them to internalize the numeracy components, that are important to mathematics learning.

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Appendices

Appendix A: Tasks used by Bartelet et al. (2014, pgs. 660-662)

2.3. *Cognitive processing tasks*

All the cognitive processing tasks had good reliabilities, ranging from .88 to .97. For the timed tasks acc-RT scores (# of items correct/s) were calculated to control for speed-accuracy tradeoffs (Salthouse & Hedden, 2002). The total number of correct items was divided by the total response time in seconds. The outcome for the short-term working memory tasks, the estimation task and the number line 0–1000 tasks (see detailed descriptions below) were accuracy (% correct) scores, mean deviation scores and absolute error, respectively, because no reaction time data was available for these tasks. To calculate mean deviation scores the numerosity of an item was subtracted from the participant's verbal estimate of the numerosity, followed by the division of the numerosity of that same item (Actual Numerosity–Estimated Numerosity/Actual Numerosity).

2.3.1. *Dot comparison*

In this task, consisting of three practice and 64 test items, two arrays of dots were simultaneously displayed on a computer screen in random order. Children were instructed to push the button corresponding to the larger numerosity as fast as possible. Numerosities ranged from 1 to 9 (level 1) and 10–39 (level 2) with a distance ratio of .25–.33, .50, .66 or .75 (smaller/larger number). Each ratio was presented 16 times. Displays were controlled for total area filled and total perimeter. For half of the stimuli the total area of the dot arrays is the same, which causes the total perimeter to be greater on the side with more dots. For the other half of the stimuli, the total perimeter of the dots arrays is the same, which causes the total area to be greater on the side with less dots. The side of the larger array of dots was counterbalanced to ensure that for each ratio the larger numerosity was equally often on the right and left hand side.

2.3.2. *Arabic numeral comparison*

Children simultaneously saw two Arabic numerals differing by one of four distance ratios (.25–.33; .50; .66; and .75). After four practice items children responded to 64 test trials containing either one digit numbers (level 1) or two digit numbers (level 2). Also in this task each ratio was presented 16 times and the side of the larger number was counterbalanced for each ratio. Children were required to push the button corresponding to the larger number.

2.3.3. *Verbal–Arabic matching*

During the verbal–Arabic matching task 4 practice items were administered, followed by two levels of 32 items. Depending on the level children heard a one or two digit number word through a headphone, immediately followed by a visual display of an Arabic number on a computer screen. If the numbers matched, children had to push the green button. Otherwise they were supposed to push the red button. Positive and negative responses were equally distributed and randomly ordered across levels.

2.3.4. *Dot enumeration*

Counting was operationalized as a dot enumeration task composed of 6 practice and 45 test items. Each stimulus comprised a black square filled with 1–9 white dots of varying sizes in a non-linear order. Though presented in a random order, no displays containing the same numerosity were presented in immediate succession. Half of the trials were area-matched and half of the trials were perimeter-matched to control for the influence of perimeter and cumulative surface area. One-third of the stimuli displayed quantities falling within the subitizing range (1, 2, or 3) and two-third of the items presented magnitudes falling within the counting range (4, 5, 6, 7, 8, or 9). Children were asked to state their answer aloud, while simultaneously pushing the green button.

2.3.5. *Matching objects*

In the matching objects task children were required to decide which of two quadrilateral planes shown in the bottom of a display contained the same amount of objects as a quadrilateral plane presented in the top. Object quantity ranged from 1 to 6. Children indicated their choice by pushing the corresponding button. After 6 practice items three levels of 15 trials were administered. All planes were controlled for luminance and object size was varied to minimize perceptual cues. Degree of difficulty was increased across levels by enhancing the homogeneity of objects within and between planes. For this purpose different objects were used, specifically four animals (bunny, horse, dog and cat) and four fruits (banana, apple, pear and pineapple). In level one all objects incorporated within an item belonged to the same category. They varied between planes in level two. Finally, in level three objects varied within and between planes.

2.3.6. *Estimation*

Children quickly saw an array of white dots followed by a mask after 750 ms to minimize enumeration or arithmetic-based strategies. They were asked to estimate how many dots were displayed in six practice and 67 test items. If children were hesitant to do so they were encouraged to guess and reminded by the test administrator that counting was not allowed. Displays contained 1, 2, 3, 5, 7, 11 or 16 dots arranged in a random spatial arrangement to avoid processing biases due to the recognition of familiar patterns. As in the dot comparison task, all items were controlled for total area filled and total perimeter. Each numerosity occurred multiple times. Three short breaks were included at fixed time points to avoid fatigue.

2.3.7. *Number line 0–1000*

In the number line task children saw a number line with a starting point marked by the digit zero and an endpoint labeled by the digit 1000. The target numbers were written above the line in the center of the screen. Before proceeding to the 26 test trials, understanding of the task was checked through the administration of 3 practice items. However, children received no feedback regarding the accuracy of placement to avoid potential calibration.

2.3.8. *Baseline response time*

This measure was composed of 20 test trials in which children saw a row of four empty squares. When an animation figure appeared they had to push the button corresponding to its location as fast as possible.

2.3.9. *Verbal short-term working memory*

Children heard a number of pseudowords through their headphones, while facing away from the computer screen. At the end of each item children heard a specific tone indicating that the pseudoword string ended. Subsequently they were asked to repeat the pseudowords of a given item in the same order. After two practice items, the strings length increased across the 13 test trials from 2 to 6 pseudowords.

2.3.10. *Spatial short-term working memory*

In the spatial short-term working memory task a row of four empty squares was displayed on the computer screen. In alternating order a square lighted up red. Children were asked to memorize in which sequence squares turned red. Each item was followed by a mask, which disappeared after children indicated the sequence they memorized by pushing the corresponding buttons. A total of four practice and 13 test items were administered. The number of items in the sequence increased across trials.

2.3.11. *Nonverbal IQ*

The Colored Progressive Matrices is a normed untimed visuo-spatial reasoning test for children in the age range of 5–11 (Raven, Court, & Raven, 1995). Children saw a colored pattern and were asked to select the missing piece out of 6 choices.

2.4. *Arithmetic fluency tasks*

2.4.1. *MLD selection task*

The selection criterion was based on the total score of the Dutch standardized TempoTest Automatiseren (TTA) (De Vos, 2010). This paper-and-pencil measure is a timed arithmetic fluency task standardized for grade 1 to grade 6. It consists of four subtests: addition, subtraction, multiplication and division. Per subtest children had 2 min to mentally compute as many operations as possible. If children had a composite score below the 16th percentile, they were classified as MLD.

2.4.2. *Addition fluency task*

Children viewed a typed sum along the top of a computer screen. Simultaneously two answers were displayed below the operation. The instruction was to select the correct answer by pushing the corresponding button. Next to three practice stimuli, two levels with 20 items each were included. In the first level the maximum sum was 10, whereas the sum of the operations in level two exceeded 10.

2.4.3. *Subtraction fluency task*

In this task a subtraction operation was presented to children. At the bottom of the computer screen two answers were displayed. Children were asked to push the button corresponding to the correct answer. Also this task was composed of three practice items and two levels of 20 trials each. The minuend was 10 or lower in level one and higher than 10 in level two.

2.4.4. *Multiplication fluency task*

At the top of a computer screen a multiplication equation is displayed. Below this operation two answers are presented and children were required to push the button corresponding to the correct answer. The measure consisted of three practice stimuli and 40 test trials.

2.5. *Descriptive statistics of the MLD and normal sample*

In Table 1 mean age, mean standard score on a nonverbal IQ task, mean percentile score on a mathematics achievement, verbal and spatial short-term working memory test and raw scores on the remaining cognitive measures are reported separately for the MLD and normal sample. MLD children were significantly older than normal achieving children. On all cognitive tasks the typical achieving group significantly outperformed children with MLD.

Tasks used by Bartelet et al. (2014, pgs. 660-662)

Appendix B: Identification Checklist Henderson et al. (2003, p. 84 - 85).

Identification Checklist

NAME: ____________________ DOB: _____________ DATE: __________________

Personal Issues

Has very high levels of fear and anxiety when it comes to Maths	
Lacks confidence- even when they produce the correct answer	
Worries about working more slowly and inaccurately	
Will often adopt avoidance strategies	
Often develops 'learned helplessness' strategies	
Often presents messy work	
Gets mixed up with left and right	
Dislikes whole group interactive sessions	
Has difficulties with everyday skills requiring Maths, e.g. Money, time, planning	

Numbers and the Number System

Has difficulty linking mathematical words to numerals	
Has difficulty transferring from the concrete to abstract thinking	
Has difficulty understanding mathematical concepts	
Has difficulty copying numbers correctly	
Has difficulty with place value	
Reverses or inverts numbers	
Finds remembering Maths rules and formulae difficult	
Finds sequencing the order and the value of numbers difficult	
Can have difficulty up and down the number line	
Is inconsistent from day to day in what they know and can do	
May have difficulties reading the analogue clock, and understanding and relating to the passage of time	

Calculations

Has difficulty aligning numbers in the correct columns	
Is confused about which basic symbols to use	
Is not confident and avoids estimating and checking answers	
Has difficulty learning times tables and often are never able to achieve automatic recall of table facts	
Has difficulty dealing with money and time	
Has difficulty using a calculator	

Solving Problems

Has problems choosing the correct strategies to solve word problems	
Has sound technical reading skills but fails to understand the mathematical language	
Forgets the beginning before getting to the end when reading a longer word problem	
Is unable to see vertical tables within a word question	
is distracted by additional visual images on busy pages	
Has weak estimation skills and cannot make sound judgements about their answers to calculations	
Forgets how to round up and down with regard to place value	
Fails to remember the sequence of calculations needed to solve a multi-step word problem	
May look back to check that they are using the same procedure for the same type of question but inadvertently copy the answer from the previous sum	

Measures

Has difficulty linking the Maths terms to their abbreviations, e.g. Centimetre to cm	
Has difficulty remembering to work in the same unit of measure within a question e.g. mixes together centimetres and metres	
Gets confused when asked to select appropriate measures to suit different tasks	
Forgets the formula	
Has trouble recognising symbols and abbreviations, e.g. Cm^2 and cm^3	
Often forgets what the abbreviations mean within the formula	
Has difficulty reading equipment scales accurately	
Has difficulty with the mathematical language of money which will influence their understanding of questions	
Forgets the properties of shapes and are unable to connect the formula to the correct shape or process	

Handling Data

Slower working, anxious when working with charts and graphs	
Has difficulty making connections between the pictorial representation for a numerical value	
Confuses the vertical axis and horizontal axis in graphs and co-ordinates	
Fails to recognise the zero in a graph which results in inaccurate answers	
Has difficulty interpreting data patterns, graphs and charts	
Has difficulty handling specific similar vocabulary e.g. Median, mode, mean, range	

Checklist produced by Henderson et al. (2003, p. 84 - 85).

Appendix C: Letter to the Secretariat for Catholic Education

Name and Surname: Esmeralda Zerafa
Address: Lloret de Mar, Triq il-Gross, Marsascala MSK 2250
Tel: 27313186/99854860
E-mail: esm4ever@hotmail.com; ecas0001@um.edu.mt
Course being followed: M.Phil. Leading to a Ph.D. (Part-time) – 1st Year
Principal Supervisor: Dr. M. T. Farrugia (University of Malta)
Co-supervisors: Dr. V. Martinelli (University of Malta)
Dr. A. Dowker (St. Hilda's College, Oxford University)
Title of Thesis: Helping Children with Mathematics Learning Difficulties: An Intervention Programme carried out with Children with Mathematics Learning Difficulties only, and children with both Mathematics Learning Difficulties and Reading Difficulties

Further Information about Thesis

I am currently a complementary teacher at ------------------. After finishing a Masters degree in Dyscalculia and Mathematics Learning Difficulties (MLD), I am currently pursuing a Ph.D. in the same area extending my research to also include Reading Difficulties (RD). The inital part of the study will involve standardizing three standardised tests currently used in the UK in Maltese classrooms. Two of these tests are related to numeracy skills and are the Basic Number Screening Test (Hodder Education, 2001) and the Progress in Mathematics 9 (Clausen-May et al., 2009) and another is related to reading ability, the Single Word Reading Test (6-16) (Foster, 2007). After standardizing these tests in Maltese Church Schools for boys I shall use them to identify the learners for my main study. Maltese Church Schools for boys have been chosen to participate because this population is closest to the sample I shall need for the main study. The main part of the research project is of a qualitative nature. I have proposed that I carry out a numeracy intervention programme, currently being used in the UK, with six children in Grade 5 (9-10years old) over the next scholastic year 2013/2014. It may also be repeated the following scholastic year 2014/2015 if six students are not found from one cohort. The six children will be made up of three children identified as having MLD only and three children having MLD and RD. Due to the nature of my current work i.e. that of a complementary teacher, I would like to seek consent to carry out these sessions with children already entrusted to my care subject that they are identified, through the standardised tests, as having the difficulties mentioned. The programme will be made up of fun, interactive and multi-sensory activities to shed greater light on which strategies help children with MLD to gain the mathematics skills they should have acquired at a much younger age. The purpose of the study is to also identify whether the same intervention strategies are effective with both children with MLD only and MLD and RD. Sessions might be tape recorded for the sole purpose of listening to the sessions again whilst analysing the data. If granted this consent, I shall naturally also ask for written consent from the respective Heads of schools (my Head of School is already aware of the research) as well as the children and the parents involved in the main part of the study. All data gathered will be treated with strict confidentiality and will be strictly used for the purpose of this research project.

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Foster, H. (2007). *Single Word Reading Test 6-16.* London: GL Assessment.

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Appendix D: Letter from Secretariat for Catholic Education granting permission

MALTESE EPISCOPAL CONFERENCE
Secretariat for Catholic Education

The Head
All Church Schools (Boys)

30th January, 2013

Ms Esmeralda Cassar, currently reading a Ph.D at The University of Malta, under the supervision of Dr M.T. Farrugia and Dr A. Dowker from St Hilda's College, University of Oxford requests permission to conduct tests with children with Mathematics Learning difficulties and also with children who have Reading Difficulties in year 5.

The Secretariat for Catholic Education finds no objection for Ms Esmeralda Cassar to carry out the stated exercise subject to adhering to the policies and directives of the school concerned.

ChM

Rev Dr. Charles Mallia
Delegate for Catholic Education

Appendix E: Letter from Head of School granting access

19th December 2012

Ms Esmeralda Zerafa
Lloret de Mar, Triq il-Gross
Marsaskala

Dear Ms Zerafa,

Following your request, permission is being granted to you to conduct research at [redacted] This refers to your research project, entitled: 'Helping Children with Mathematics Learning Difficulties: An Intervention Programme carried out with Children with Mathematics Learning Difficulties only, and children with both Mathematics Learning Difficulties and Reading Difficulties'

I understand that

- Your research will involve assessing the students in Year 5 during the scholastic year 2013/2014, thereby identifying particular children with Maths Learning Difficulties, and both Maths Learning Difficulties and Reading Difficulties, and carrying out an intervention programme currently being used in the UK for numeracy difficulties;
- The programme will take up more or less 15 minutes twice weekly;
- You will be asking parents for their consent.

Wishing you success with your research, for your own, and the students' benefit.

Regards

Appendix F: Checklist for Dyspraxia

Checklist for How to Recognise a Child with DCD/Dyspraxia in the Classroom ….

Gross motor difficulties

- Falls, trips and bumps
- Seems awkward when walking up and down stairs, may do one at a time
- Seems awkward when running, may have their arms out to the side or seem unbalanced.
- May struggle in PE with catching and throwing, balance and negotiating apparatus as well as following actions.
- Doesn't seem to be aware of their own body's boundaries seeming to have poor spatial skills
- Fidgets in their chair or when on the carpet or tends to slump over their desks or need to lean against a table or others when on the carpet.
- May struggle to ride a trike or a bike in the playground.
- May struggle in swimming lessons to coordinate their bodies or may lack strength in the water struggling to stay afloat.

Fine motor difficulties

- Appears to struggle using their two hands together when using scissors
- Drawing skills are below what you would expect for their level of learning
- Pre/writing skills are behind what you could expect given their verbal skills and their reading ability.
- Tends to avoid construction tasks or appears to find these tricky
- Has delayed self care skills.

Sensory difficulties

- Seems to have difficulties regulating own activity levels
- Seems to have difficulty regulating their own emotions
- Appears to be unable to filter out extraneous noise or visual stimulus
- Can be overly sensitive to loud noises

Cognitive difficulties

- Poor attention span
- Difficulties with short term memory
- Difficulties sequencing tasks
- Seems to have poor problem solving abilities

Taken from National Health Service (NHS) (n.d.)

- Finds it hard to listen, think and do all at the same time for example creative writing in literacy.
- May talk themselves through tasks to help them organise their thoughts and their actions.

Organisation difficulties

- Difficulties transferring skills
- Difficulties organising own belongings, tray, school bag, equipment for lessons.
- May not have a tidy appearance, leaves clothes twisted, inside out, back to front, shoes on wrong feet.
- Loses or forgets things.
- Gets confused or muddled in their thoughts, their speech or actions.
- Struggles to follow a timetable & navigate around the school.
- Doesn't tend to like change in routine at the last minute.
- Most significant difficulties are with new/novel tasks which are not routine or well rehearsed.
- Tends to look to others for prompts as to what to do.

Social and Emotional difficulties

- Seems to lack confidence
- Often seems to be the quiet child at the back of the class or the other extreme the 'class clown'.
- Can get easily frustrated, angry or anxious
- Often prefers to play with younger children or prefers adult company
- Doesn't make friends easily, limited play during break times.
- Often needs to take control of social situations or will prefer to be on their own.

Appendix G: Sample of Chinn's (2012) Tests

The 60 second test for addition

Name ____________________ Date ________ Age _____ (y) ________ (m)

2 + 1 = ____	2 + 2 = ____	1 + 3 = ____
5 + 2 = ____	3 + 3 = ____	2 + 4 = ____
3 + 5 = ____	5 + 5 = ____	6 + 5 = ____
6 + 4 = ____	4 + 4 = ____	5 + 4 = ____
2 + 8 = ____	9 + 1 = ____	4 + 6 = ____
3 + 7 = ____	6 + 3 = ____	6 + 6 = ____
6 + 7 = ____	2 + 7 = ____	3 + 6 = ____
5 + 7 = ____	8 + 4 = ____	4 + 9 = ____
9 + 5 = ____	7 + 6 = ____	8 + 9 = ____
9 + 8 = ____	9 + 6 = ____	8 + 8 = ____
5 + 8 = ____	8 + 7 = ____	9 + 9 = ____
7 + 8 = ____	8 + 9 = ____	7 + 7 = ____

Score ________ Average score for age group______

Mathematics test 15 minutes

Name ________________ Male/Female ________ Date ________ Age ________(Y)

---

1. 2 + 5= ____

2. 7 + 8 = _____

3. 19 − 4 = ____

4. 5 − 4 + 3 = _____

---

5. 34 = 4 + ___

6. 400 + 600 = ____

7. 100 − 58 = _____

8. 16 − 8 = _____

9.
$$\begin{array}{r} 36 \\ +\underline{54} \\ \underline{\quad} \end{array}$$

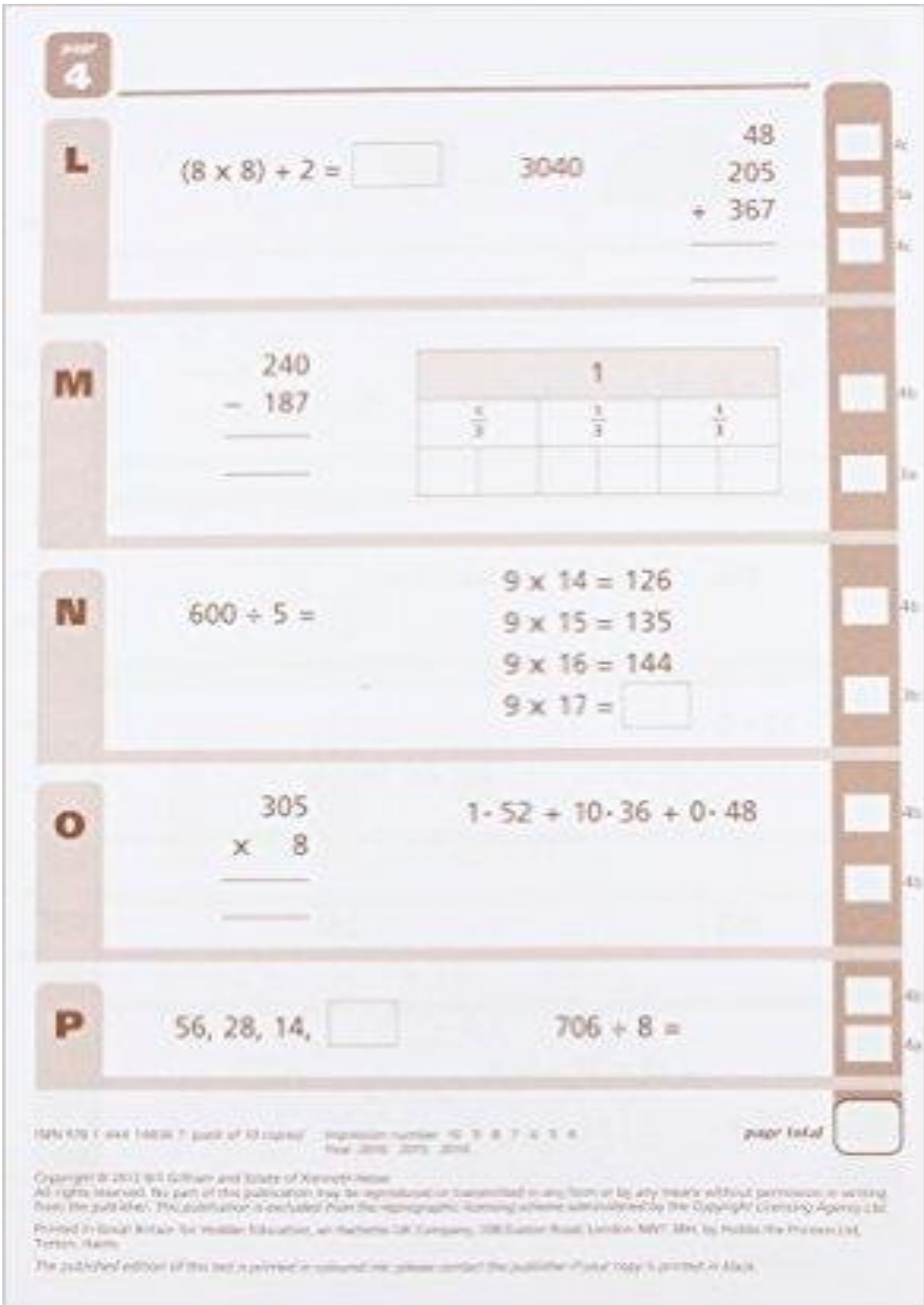
10.
$$\begin{array}{r} 827 \\ -\underline{705} \\ \underline{\quad} \end{array}$$

11. 9 = __ − 4

12.
$$\begin{array}{r} 33 \\ -\underline{16} \\ \underline{\quad} \end{array}$$

Taken from Chinn (2012).

Appendix H: Sample page from the BNST (Gillham & Hesse, 2001)


4

L

(8 x 8) + 2 =

3040

48
205
+ 367

M

240
− 187

1
1/3 1/3 1/3

N

600 ÷ 5 =

9 x 14 = 126
9 x 15 = 135
9 x 16 = 144
9 x 17 =

O

305
x 8

1·52 + 10·36 + 0·48

P

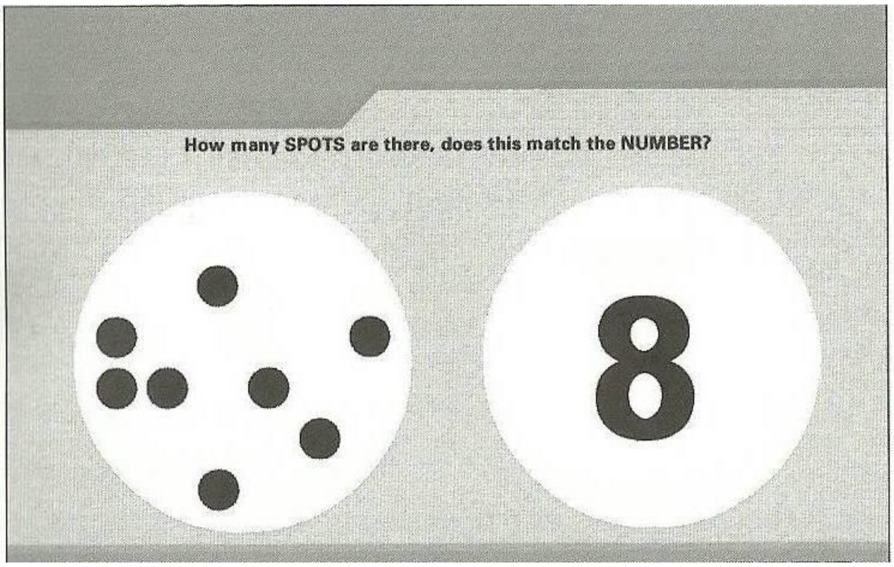
56, 28, 14,

706 ÷ 8 =

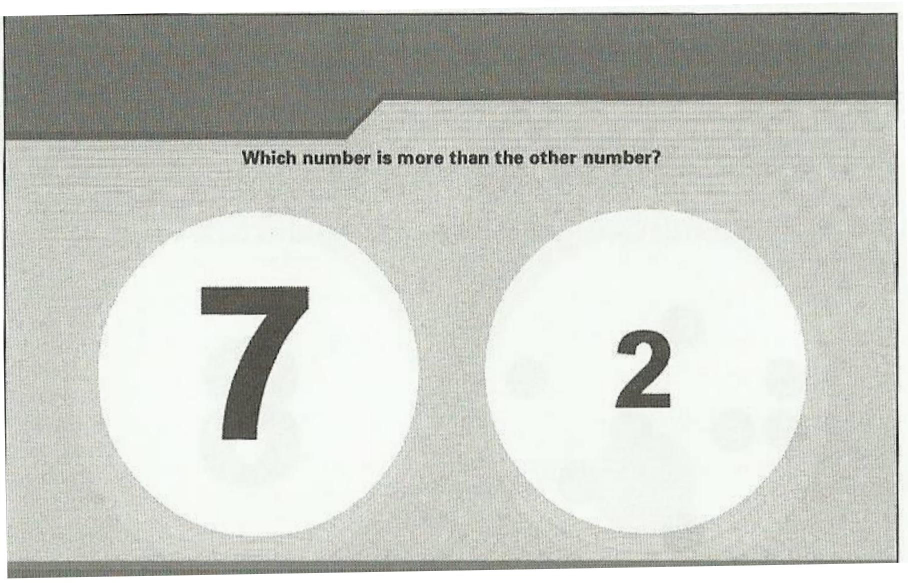
page total

Taken from Basic Number Screening Test (Gillham & Hesse, 2001).

Appendix I: Screenshots from Dyscalculia Screener (Butterworth, 2003)



Dot Enumeration



Numerical Stroop

Taken from Butterworth (2003).

Appendix J: Interview Questions for Parent/s

Questions for Parent/s

- Has your child always had difficulties with mathematics?
- When did you first notice that your son was struggling with mathematics?
- Did your son engage in mathematics activities when they were young – like counting pegs, nursery rhymes with numbers etc. Do you remember any other mathematics activities he used to do?
- Does he find particular difficulties in other subjects?
- How does he feel during exams especially when preparing for a mathematics exam?
- Which mathematical topics does he feel most comfortable with?
- Does he find difficulties with mathematical concepts such as money and time?
- Does your child do his mathematics homework alone at home?
- If not, what do you observe him doing whilst completing given tasks?
- What kind of help do you usually give him when he is doing his homework?
- Do you think that your child needs to be given extra help in particular mathematics topics? If yes, which are these topics?
- Is your son currently getting other help in mathematics other than that being received at school?
- Are you concerned about your son's knowledge of mathematics? Why?

Appendix K: Interview Questions for Class Teachers

Questions for Class Teachers

- How does X perform in Mathematics – class participation, school work, homework, exams, test etc.?
- Are there particular topics in which he performs well?
- Are there particular topics in which he needs support?
- Can you recall any situations which showed you that X is struggling with mathematics?
- What kind of help is X receiving in the classroom?
- Can he normally finish off the school work given for mathematics on his own? If not, what kind of help does he normally ask for?
- Does X participate – ask and interact with yourself and his peers - during mathematics lessons?
- Does X normally participate by listening to the explanation?
- Does he normally ask for help or is reluctant to do so?
- Does he participate more/less than in other sessions?
- Does X have difficulties with other subjects?
- Would you say that X suffers from mathematics anxiety?
- If yes, which mathematics tasks make him most anxious?
- Have you ever observed the child using mathematics language?
- Which language have you observed the learner making use of?
- How are X's parents supporting him with his difficulties in mathematics?

Appendix L: Guidelines for Classroom Observations

Points for Observing the Children in Class

Student

1. Does child ask questions and interact with the teacher and students during the lesson?
2. Does the child listen attentively to the lesson?
3. What kind of behaviour does the child exhibit during the mathematics lesson?
4. Does the child try to answer any of the teacher's questions?
5. If yes, what kind of answers does the child provide? If the answer is definitely incorrect, how do the other children react? Is the answer given related to the question? Why did the child come up with that answer?
6. Does the learner reach the objective of the lesson? Does he internalise the learning – can he use the new strategy or lesson learnt to solve the mathematical tasks he has been given? Can he re-explain the lesson back to you?
7. Does the learner display any kind of mathematics anxiety during the lesson?
8. Does the learner exhibit any internalisation of the mathematics language used either by using it himself or explaining to you its meaning or how it can be used in context?
9. Has the learner understood what has to be done for homework? Can he work out two examples from the homework tasks given and explain to you what he has to do?

Teacher

10. How does the teacher interact with the child?
11. Does the mathematics lesson cater for the child's mathematics learning style as per Chinn's inchworm and grasshopper style?
12. What kind of help does the teacher give the child during the school work?
13. What kind of mathematics language is used during the lesson?
14. How are activities scaffolded and do they seem to engage the particular learner? Engage is to be taken to mean to help the student visualise the concept, carry out any investigation given, listen attentively, interact with peers in case of group work and use the strategies being introduced within a context.

Appendix M: Consent Form for Parents

Dear Parent/s,
I am Esmeralda Zerafa, currently reading for a Ph.D at the University of Malta about Mathematics Learning Difficulties. My research project includes assessing boys in Church schools in mathematics and reading so as to be able to standardise tests which have currently only been standardised in the UK. The short assessment in mathematics will be done as a whole class and will be similar to any other mathematics assessment which the children are normally given at school. I will then ask the children to read a few words on an individual basis. This research will help me identify the norms at which children are performing in mathematics and reading and therefore will help me to identify children who are performing below these norms later on. If you do not wish your child to participate in this study, please send the form below to the school by not later than Friday 22nd February.
I thank you in advance for your support.

Esmeralda Zerafa
B.Ed. (Hons.) M.Ed. (Melit.)

- -

I, parent of ____________________ in Grade 5 ___________ do not wish my child to participate in this study.

____
Signature

Appendix N: Information Sheet for Parents

Ms. Esmeralda Zerafa
School's Name and Address
20th December, 2012

E-mail address: esmeralda.zerafa@gmail.com

Dear Parent/Guardian,
I am currently reading for a Ph.D. at the University of Malta. My study will deal with identifying Mathematics Learning Difficulties and finding strategies to help children overcome them. I will first assess the children through tests to identify children who are having difficulties with mathematics. The assessment will include a number of standardised exercises. These will be conducted during school hours but arrangements will be made so that the children do not miss important lessons. Afterwards, I would like to conduct a teaching programme to help the children found to have difficulties. Once the assessment takes place, I will inform you of the results and let you know if I think that your son should follow the programme which I shall be offering (free of charge) in order to revise and improve his mathematics. The programme will be conducted as part of the child's daily mathematics lesson. During the lessons I will be taking short notes and tape recording the sessions to be able to listen to them at a later stage and take note of anything important for my study.
Kindly note that all assessments will be carried out by myself and all information will be kept in strict confidence. Any data that will be included in my study will not reveal the names of the participants, nor the schools. Fictitious names will be used to ensure anonymity.
Please fill in the attached form stating whether you would like your child to participate in this assessment and, should there be the need, for your child to participate in a programme to help him with his mathematics concepts and skills. I thank you in advance for your support. If you have any queries, please do not hesitate to contact me through a note, an e-mail or a phone call at school.
Yours truly,

Ms Esmeralda Zerafa B.Ed. (Hons.), M.Ed. (Melit.)
Complementary Teacher

Ms. Esmeralda Zerafa
Isem u indirizz tal-iskola
20 ta' Diċembru, 2012

Indirizz elettroniku: esmeralda.zerafa@gmail.com

Għeżież ġenitur/gwardjan,
Jien qiegħda nagħmel Dottorat (*Ph. D.*) dwar diffikultajiet speċifiċi fil-matematika. L-istudju se jgħinni biex possibilment nidentifika din id-diffikultà f'numru ta' tfal u nsib strateġiji biex ngħinhom jegħlbu d-diffikultajiet tagħhom. L-ewwel se nitlob lit-tfal jagħmlu numru ta' eżerċizzji tal-matematika. Dan l-assessjar isir fil-ħinijiet tal-iskola imma se jsir arranġament sabiex it-tfal ma jitilfux lezzjonijiet importanti. Wara dan il-pass jien nixtieq nagħmel numru ta' lezzjonijiet mat-tfal sabiex intejjeb il-ħiliet tat-tfal fil-kunċetti bażiċi tal-matematika.
Għaldaqstant, wara l-assessjar jien se ninfurmakom dwar ir-riżultat ta' wliedkom u ngħidilkom jekk it-tifel tagħkom għandux jieħu sehem fil-lezzjonijiet li nkun se noffri (mingħajr ebda ħlas) bħala parti mil-lezzjonijiet tal-matematika tagħhom ta' kuljum. Matul dawn il-lezzjonijiet jien se nieħu xi noti qosra u se nirrekordja dak li nkunu qegħdin ngħidu sabiex inkun nista' nirreferi għal dan aktar tard f'każ li jkolli bżonn nikteb xi ħaġa importanti għall-istudju tiegħi.
Nixtieq nenfasizza li l-assessjar isir minni biss u li l-informazzjoni <u>kollha</u> li tinġabar se tinżamm b'mod kunfidenzjali. Filwaqt li l-informazzjoni kollha miġbura se tinżamm anonima, partijiet minnha se tidher fl-istudju tiegħi. Iżda, l-isem tat-tifel ma jintużax. Jekk jogħġbok qed nitolbok timla l-formola li tinsab mehmuża fejn tgħid jekk tixtieqx li t-tifel tiegħek jieħu sehem f'dan l-istudju, kif ukoll fin-numru ta' lezzjonijiet wara l-assessjar jekk insib li għandu bżonn isaħħaħ ċertu ħiliet tal-matematika. Nirringrazzjak bil-quddiem tal-għajnuna tiegħek. Jekk ikollok xi mistoqsijiet ikuntatjawni permezz ta' nota li tibgħatu mat-tifel tagħkom, e-mail jew telefonata l-iskola.

Dejjem tagħkom,

Ms. Esmeralda Zerafa B.Ed. (Hons.), M.Ed. (Melit.)
L-Għalliema tal-*Complementary*

Appendix O: Consent Form for Parents of Main Participants

Consent Form

Tick as appropriate:

I understand that:

i. this study will entail following some lessons in mathematics which will be spread upon one scholastic year;

ii. my child has the right to refuse from participating in this study but that he will still be given the help needed to overcome his difficulties, if any;

iii. my child has the right to stop participating in this study at any stage;

iv. I will be given feedback with regard to the child's progress;

v. all information will be kept in strict confidence; and

vi. if any information related to my son's progress is used in the thesis write-up, his name will not be used in order to ensure anonymity.

and therefore **<u>would</u>** like my child (Name) ________________ of Grade ___________ to participate in this study. ☐

but I **<u>would not</u>** like my child (Name) ______________ of Grade _________to participate in this study. ☐

____

Parent's/Guardian's Signature

Formola ta' Kunsens

Immarka kif meħtieġ:

Jien nifhem li

i. dan l-istudju se jitlob li t-tifel jieħu sehem f'xi lezzjonijiet tal-matematika li se jkunu mifruxa fuq perjodu ta' sena skolastika;
ii. t-tifel tiegħi għandu d-dritt li ma jieħux sehem f'dan l-istudju iżda xorta jingħata l-għajnuna meħtieġa skont il-bżonn;
iii. it-tifel għandu d-dritt jieqaf jieħu sehem f'dan l-istudju meta jrid;
iv. se ningħata informazzjoni dwar kif ikun sejjer it-tifel;
v. l-informazzjoni kollha li se tingħabar se tinżamm kunfidenzjali; u
vi. jekk ikun hemm il-ħtieġa li tintuża' xi informazzjoni dwar il-progress tat-tifel tiegħi, l-isem tat-tifel mhux se jidher imkien minħabba fini ta' anonimità.

u għalhekk **nixtieq** li t-tifel tiegħi (Isem) ________________ ta' Grade ________ jieħu sehem f'dan l-istudju. ☐

iżda **ma nixtieqx** li t-tifel tiegħi (Isem) ________________ ta' Grade ________ jieħu sehem f'dan l-istudju. ☐

____

Firma tal-Ġenitur jew Gwardjan

Appendix P: Information Sheet for Pupils (Phase 2)

<u>Information Sheet</u>

Dear Pupil,

I shall be carrying out a study about difficulties which children may have in mathematics. I would like to invite you to take part in this study. First I will ask you to work out a number of mathematical tasks. Then, if I find that you are having difficulties with some of the topics, I will organise a number of enjoyable activities which will help you to understand better those topics. Throughout the sessions we will have together, I will be taking short notes and tape recording our sessions so that I can then listen to the tapes again and take note of any important details that I need to include in my research.

You have a right not to participate in this study or to withdraw from the study at any point in time. You will still be entitled to your complementary sessions, which will continue to help you to overcome your difficulties.

If you would like to participate in this study please sign the consent form that you will be given.

Thank you for your help,

Ms Esmeralda Zerafa
Complementary Teacher

<u>Karta b'Informazzjoni Meħtieġa</u>

Għażiż Student,

Jien se nagħmel riċerka dwar id-diffikultajiet li tfal jistgħu jkollhom fil-matematika. Jiena nixtieq nistiednek biex tieħu sehem f'dan l-istudju. L-ewwel se nitolbok taħdem numru ta' eżerċizzji tal-matematika. Imbagħad jekk minn dawn l-eżerċizzji nara li għandek bżonn għajnuna f'ċerta topiks, jien se norganizza numru ta' attivitajiet divertenti sabiex ngħinek tifhem aktar dawk it-topiks li int ma tantx tkun fhimt. Matul dawn il-lezzjonijiet, jien se nkun qiegħda nżomm noti qosra u se nirrekordja b'*tape recorder* l-attivitajiet sabiex inkun nista' nerġa' nisma' t-*tapes* u nikteb dettalji importanti għar-riċerka tiegħi.

Nixtieq nagħmilha ċara li għandek id-dritt li ma tipparteċipax f'dan l-istudju u li tkun tista' tieqaf tipparteċipa meta tkun tixtieq. Jekk tagħżel li ma tieħux sehem xorta se jkollok il-lezzjonijiet tal-*complementary* tas-soltu biex jgħinuk tiżviluppa l-ħiliet li hemm bżonn.

Jekk tixtieq taċċetta l-istedina tiegħi biex tieħu sehem f'dan l-istudju jekk jogħġbok iffirma fuq il-formola li biha inti tista' tagħti l-kunsens tiegħek biex tieħu sehem.

Grazzi tal-għajnuna tiegħek,

Ms Esmeralda Zerafa
L-Għalliema tal-*Complementary*

Appendix Q: Consent Form for Pupils (Phase 2)

<u>Consent Form for Participating Children</u>

Name of the person doing the research: Ms. Esmeralda Zerafa
Address: School's address
Contact No.: School's contact number

The purpose of the study is to target children's difficulties in mathematics.

I shall be collecting my information by:
a. asking you to work out some tasks as part of a test;
b. asking you to participate in a number of enjoyable activities, which will be tape recorded, and which I am going to organise to help you understand better the topics you find difficulties with (if there would be the need).

I shall be using the information to create activities which will allow you to understand mathematical topics better and I will be writing the results in my thesis which I need to hand in as part of my Ph.D. studies.

I **guarantee** that:
i. Your real name will not be used in the study.
ii. You will remain free to stop taking part in the study at any time and for whatever reason. In the case you decide to stop taking part, all the information gathered will be destroyed.
iii. I will not try to deceive you in any way whilst collecting the information.

I agree to the conditions.
Name of Pupil: ______________________
Signature: ___________________ Date: _________
I agree to the conditions.
Researcher: ________________________ Date: _________

<u>Formula għall-Kunsens tat-Tfal</u>

Isem min se jagħmel l-istudju: Ms. Esmeralda Zerafa
Indirizz: Indirizz tal-iskola
Contact No.: Numru tal-iskola

L-istudju se jsir sabiex niskopri d-diffikultajiet li l-istudenti qegħdin isibu fil-matematika u niprova ngħinhom jegħlbuhom.

Jien se niġbor l-informazzjoni billi:

a. nitolbok taħdem xi eżerċizzji tal-matematika li huma parti minn *test;*
b. nitolbok tieħu sehem f'numru ta' attivitajiet, li se jiġu rrekordjati, u li se nkun qiegħda norganizza sabiex ngħinek telgħeb diffikultajiet f'topiks partikolari (jekk dan ikun meħtieġ).

L-informazzjoni miġbura se tintuża biex noħloq attivitajiet li jgħinuk tifhem aktar it-topiks partikolari. L-informazzjoni se tintuża ukoll għat-teżi li għandi nikteb bħala parti mid-Dottorat (*Ph.D.*).

Garanzija:
Jien se nimxi ma' dawn ir-regolamenti:

i. M'iniex se nuża l-isem veru tiegħek fl-istudju.
ii. Inti fil-libertà li tieqaf mir-riċerka x'ħin trid u għal kull raġuni. F'każ li tieqaf, kull oġġett rekordjat u informazzjoni miġbura tinqered mill-ewwel.
iii. Bl-ebda mod m'int ser tiġi mqarraq fil-proċess tal-ġbir ta' informazzjoni.
iv.

Naqbel ma' dawn il-kundizzjonijiet imsemmija hawn fuq.

Isem l-istudent: ______________________
Firma: ___________________ Data: _________
Naqbel ma' dawn il-kundizzjonijiet imsemmija hawn fuq.
Firma tar-Riċerkatur: _______________________ Data: _________

Appendix R: Translation of BNST (Gillham & Hesse, 2001) instructions to Maltese

Basic Number Screening Test – Traduzzjoni tal-Istruzzjonijiet għall-Malti

Se nagħmlu *test* tan-numri mingħajr ma nużaw il-*calculators.*

Kollha se nkunu qegħdin nagħmlu l-istess parti fl-istess ħin imma nixtieqkom taħdmu waħedkom mingħajr ma tgħinu lil ħaddieħor u mingħajr ma tistaqsu lil ħaddieħor għall-għajnuna. Nixtieq inkun naf x'tafu tagħmlu mingħajr għajnuna.

Waqt li tkunu qegħdin tagħmlu t-*test* jista' jkun li tiktbu tweġiba li tkunu tixtiequ tibdlu għax taħsbu li din hi ħażina. Jekk tibdlu t-tweġiba tagħkom araw li din il-bidla tkun ċara. *(Uri fuq il-wajtbord.)*

Per eżempju, jekk tridu tibdlu numru li diġa' ktibtu, aqtgħuh b'mod ċar, hekk (29, 39, jew ½, ¼) u iktbu n-numru t-tajjeb ma' ġenbu. Tippruvawx tiktbu fuq in-numru, hekk (29).

Agħmlu l-istess jekk tiġu mitluba tagħmlu linja li tgħaddi fuq numru u tiddeċiedu li tkunu għamiltu l-linja fuq numru ħażin (7, 9); aqtgħu n-numru l-ħażin b'mod ċar.

Jekk ma taqtgħux n-numru l-ħażin b'mod ċar jien mhux se nkun naf liema tweġiba għandi nikkoreġi - għalhekk aqtgħu n-numru l-ħażin bil-mod kif urejtkom.

Meta nitlobkom sabiex timlew xi ħaġa bil-kulur jew bil-lapes, għamluha hekk (///////) b'linji dritti, għax hekk tħaffu l-aktar.

Issa se nqassam il-karti. Tiktbu xejn fuqhom sakemm ngħidilkom jien.
(Qassam il-karti, bl-ewwel paġna fuq.)

Tistgħu taraw fejn tgħid '*NAME*' u '*CLASS*'? *(Waqfa)* Iktbu isimkom u l-klassi. (*Waqfa)*
Issa fejn hemm miktub '*SCHOOL'* iktbu l-isem tal-iskola tiegħkom. *(Waqfa) (N.B. Fil-klassijiet bikrin tal-primarja għandu mnejn ikollok tikteb din l-informazzjoni int sabiex huma jkunu jistgħu jikkopjawha.)*
Isimgħu sew dak li se ngħid. Se nagħmlu partijiet żgħar mit-*test* kull darba u qabel kull parti se ngħidilkom x'għandkom tagħmlu. Tinkwetawx jekk ma tistgħux tagħmlu xi wħud minnhom – ħadd ma jista' jagħmilhom kollha.

Se nibdew billi nipprattikaw ftit fuq dawk li qegħdin taraw fi tmiem l-ewwel paġna.

Tistgħu tħarsu kollha lejn RINGIELA Y? *(Waqfa)*

Hemm għandkom kaxxa b'ħafna tuffieħ fiha. (*Waqfa)*. Pinġu linji madwarhom sabiex tqiegħdu t-tuffieħ fi **gruppi ta' tlieta.** L-ewwel grupp diġa' lest. *(Waqfa sakemm jaħdmu t-tfal)* Iktbu n-numru ta' tuffieħ li jifdal barra minn grupp fil-kaxxa li hemm fit-tarf.

ERĠA' IRREPETI L-ISTRUZZJONIJIET JEKK IKUN HEMM IL-BŻONN.

Issa morru għal RINGIELA Z. L-ewwel hemm tliet *shapes. (Waqfa)*

Pinġi bil-lapes jew kulur **l-ikbar** wieħed. *(Waqfa sabiex jaħdmu t-tfal)*

F'RINGIELA Z tistgħu taraw kaxxa oħra – kaxxa twila u dejqa – bi ftit numri fiha. *(Waqfa)* Agħmlu linja li tgħaddi minn fuq **l-iżgħar** numru. *(Waqfa sabiex jaħdmu t-tfal)*

Kien hawn xi ħadd minnkom li ma kienx ċert minn dak li kellu jagħmel?

(Ara jekk hemmx xi diffikultà, u erġa' spjega jekk ikun hemm il-bżonn)

Minn issa 'il quddiem ma nistax ngħinkom.

Aqilbu l-paġna u sibu RINGIELA A fit-tieni paġna.

Paġna 2

RINGIELA A
L-ewwel hemm numru ma' ġenb kaxxa vojta. *(Waqfa)* **Għoddu erba' aktar** ma' dak in-numru u iktbu t-tweġiba fil-kaxxa. *(Waqfa sabiex jaħdmu t-tfal)* F'RINGIELA A hemm somma. Aħdmuha u iktbu t-tweġiba fil-kaxxa l-vojta. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA B
L-ewwel għandkom tliet numri u żewġ kaxxi vojta. *(Waqfa)* Iktbu **ż-żewġ** numri **li jmiss** f'dawk il-kaxxi vojta. *(Waqfa sabiex jaħdmu t-tfal)*

Imbagħad għandkom it-tpinġija ta' ċikkulata. *(Waqfa)* Immaġinaw li ommkom tgħidilkom li tistgħu taqtaw *one quarter* jew kwart minnha biex tiekluha. Pinġu bil-lapes jew bil-kuluri ***a quarter*** jiġifieri **kwart** miċ-ċikkulata, biex turu dak li kiltu. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA C
Fl-ewwel parti għandkom somma bit-tweġiba. *(Waqfa)* Taħtha iktbu żewġ pari **differenti** ta' numri li meta tgħoddhom flimkien jagħtuk l-istess tweġiba. *(Waqfa sabiex jaħdmu t-tfal)* F'RINGIELA C hemm ukoll somma. *(Waqfa)* Ħares sewwa lejn is-sinjal li jurik dak li għandek tagħmel, aħdem is-somma u ikteb it-tweġiba. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA D
Hawn għandkom tliet *shapes*, kull wieħed b'parti minnu bil-griż. *(Waqfa)* Agħmlu *tick* kbira fuq ix-*shape* li għandu **eżatt nofsu** bil-griż. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA E
Hemm għandkom kaxxa kbira b'ħafna siġar fiha. *(Waqfa)* Pinġu linji madwarhom biex tqiegħduhom **f'gruppi ta' sebgħa** – imbagħad iktbu n-numru ta' siġar **li jifdal** barra minn grupp fil-kaxxa ż-żgħira li hemm fit-tarf. *(Waqfa sabiex jaħdmu t-tfal)*

ERĠA' IRREPETI L-ISTRUZZJONIJIET JEKK IKUN HEMM IL-BŻONN.

Morru għal paġna 3

RINGIELA F
Hemm għandkom żewġ *circles. (Waqfa)* L-ewwel waħda għandha nofsha bil-griż u ma' ġenbha hemm miktub *'a half'* jiġifieri *nofs*. *(Waqfa)* Iktbu fil-kaxxa li qiegħda ma' ġenb is-*circle* l-oħra liema parti **minn din** is-*circle* hi griża. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA G
Hawn għandkom tliet somom differenti. *(Waqfa)* Ħarsu sewwa lejn is-sinjali li jurukom dak li għandkom taħdmu u iktbu t-tweġibiet. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA H
L-ewwel għandkom numru ma' ġenb kaxxa vojta. *(Waqfa)* Fil-kaxxa iktbu n-numru li juri ***t-tens.*** *(Waqfa sabiex jaħdmu t-tfal)* Imbagħad għandkom somma. *(Waqfa)* Ħarsu sewwa lejn is-sinjal li jurikom dak li għandkom tagħmlu u iktbu t-tweġiba. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA I
L-ewwel għandkom somma. *(Waqfa))* Ħarsu sewwa lejn is-sinjal li jurikom dak li għandkom tagħmlu u iktbu t-tweġiba fil-kaxxa. *(Waqfa sabiex jaħdmu t-tfal)*
F'RINGIELA I hemm ukoll grupp ta' numri. *(Waqfa)* Dawn in-numri ntagħżlu apposta għax isegwu *pattern*. *(Waqfa)* Iktbu liema numru **jmiss** sabiex tkomplu l-*pattern*. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA J
L-ewwel għandkom numru **kbir** ma' ġenb kaxxa vojta. *(Waqfa)* Fil-kaxxa iktbu n-numru **li jiġi wara** dan in-numru – in-numru li jiġi eżatt wara. *(Waqfa sabiex jaħdmu t-tfal)*
F'RINGIELA J għandkom ukoll numru **kbir** bejn żewġ kaxxi vojta. *(Waqfa)* F'dawn il-kaxxi iktbu n-numru li jiġi **eżatt qabel** dan in-numru, imbagħad dak li jiġi **eżatt wara**. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA K
L-ewwel għandkom kif numru hu magħmul minn *ten* u xi *units*. *(Waqfa)* Araw sewwa kif dan isir, imbagħad bl-istess mod, uru minn xiex hu magħmul in-numru ta' taħt . *(Waqfa sabiex jaħdmu t-tfal)*

Aqilbu għal paġna 4

RINGIELA L
L-ewwel hemm somma. *(Waqfa)* Aħdmuha u iktbu t-tweġiba. *(Waqfa sabiex jaħdmu t-tfal)*
Imbagħad għandkom numru b'erba' numri. *(Waqfa)* Agħmlu linja kbira li tgħaddi minn fuq in-numru li jirrappreżenta l-***hundreds***. *(Waqfa sabiex jaħdmu t-tfal)* F'RINGIELA L hemm ukoll somma oħra. *(Waqfa)* Aħdmuha u iktbu t-tweġiba. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA M
L-ewwel hemm somma. *(Waqfa)* Aħdmuha u iktbu t-tweġiba. *(Waqfa sabiex jaħdmu t-tfal)*
F'RINGIELA M għandkom tpinġija li turi kif **one whole** tista' titqassam f'ħafna partijiet indaqs. *(Waqfa)* L-ewwel għandkom it-tpinġija sħiħa u mbagħad tistgħu tarawha f'partijiet iżgħar indaqs: għandkom il-*fractions* miktubin fiha. *(Waqfa)* Imbagħad din it-tpinġija reġgħet ġiet maqsuma f'partijiet iżgħar li huma indaqs . *(Waqfa)* Iktbu ***fractions*** biex turu kull parti x'qiegħda tirrappreżenta. *(Waqfa sabiex jaħdmu t-tfal)*

ERĠA' IRREPETI L-ISTRUZZJONIJIET JEKK IKUN HEMM IL-BŻONN.

RINGIELA N
L-ewwel għandkom somma. *(Waqfa)* Araw sewwa x'tip ta' somma hi u mbagħad aħdmuha u iktbu t-tweġiba. *(Waqfa sabiex jaħdmu t-tfal)*
Imbagħad għandkom ringiela ta' somom oħra. *(Waqfa)* In-numri jsegwu *pattern* speċjali. *(Waqfa)* Araw sewwa n-numri u mlew in-numri li huma neqsin. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA O
L-ewwel għandkom somma. *(Waqfa)* Aħdmuha. *(Waqfa sabiex jaħdmu t-tfal)*
Imbagħad għandkom somma bid-*decimals.* *(Waqfa)* Erġgħu iktbuha fl-ispazju ta' taħt u aħdmuha. *(Waqfa sabiex jaħdmu t-tfal)*

RINGIELA P

L-ewwel għandkom grupp ta' numri. *(Waqfa)* Dawn in-numri jsegwu *pattern* speċjali. *(Waqfa)* Iktbu liema hu n-numru **li jmiss** billi tkomplu l-istess *pattern.* *(Waqfa sabiex jaħdmu t-tfal)*

F'RINGIELA P ukoll, sibu t-tweġiba għas-somma. *(Waqfa sabiex jaħdmu t-tfal)*
ERĠA' IRREPETI L-ISTRUZZJONIJIET JEKK IKUN HEMM IL-BŻONN.

Issa spiċċajna t-*test*, imma jekk hawn xi ħadd li jixtieq jerġa' jiċċekkja xi ħaġa jew ikompli xi parti li kien ħalla barra, jista' jagħmel dan issa, imma jien ma nistax nerġa' naqralkom l-istruzzjonijiet.