How to Classify a Government Can a perceptron do it?

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Abstract **– The electoral cycle literature has developed in two clearly distinct phases. The first one considered the existence of non-rational (naive) voters whereas the second one considered fully rational voters. It is our view that an intermediate approach is more appropriate, i.e. one that considers learning voters, which are boundedly rational. In this sense, one may consider perceptrons as learning mechanisms used by voters to perform a classification of the incumbent in order to distinguish opportunistic (electorally motivated) from benevolent (non-electorally motivated) behaviour of the government. The paper explores precisely the problem of how to classify a government showing in which, if so, circumstances a perceptron can resolve that problem. This is done by considering a model recently considered in the literature, i.e. one allowing for output persistence, which is a feature of aggregate supply that, indeed, may turn impossible to correctly classify the government.**

Keywords - *Classification, Elections, Government, Output Persistence, Perceptrons.*

1. Introduction

An electoral cycle created by governments is a phenomenon that seems to characterise, at least in some particular occasions and/or circumstances, the democratic economies. As it is generally accepted, the short-run electorally-induced fluctuations prejudice the long-run welfare. Since the very first studies on the matter, some authors offered suggestions as to what should be done against this electorally-induced instability. For some authors, ever since the seminal paper of Nordhaus (1975), a good alternative to the obvious proposal to increase the electoral period length is to consider that voters abandon a passive and naive behaviour and, instead, are willing to learn about government's intentions.

The electoral cycle literature has developed in two clearly distinct phases. The first one, which took place in the mid-1970s, considered the existence of non-rational (naive) voters. In accordance with the rational expectations revolution, in the late 1980s the second phase of models considered fully rational voters. It is our view that an intermediate approach is more appropriate, i.e. one that considers learning voters, which are boundedly rational. In this sense, one may consider perceptrons as learning mechanisms used by voters to perform a classification of the incumbent in order to distinguish opportunistic (electorally motivated) from benevolent (non-electorally motivated) behaviour of the government. The main objective of this paper consists precisely on studying the problem of how to classify a government showing in which, if so, circumstances a perceptron, can resolve that problem. To achieve this objective we will consider a recent version of a stylised model of economic policy, i.e. a version based on an aggregate supply curve embodying output persistence. See Gärtner (1996,1997,1999,2000) and/or Caleiro (2009,2012).

The rest of the paper is structured as follows. Section 2 offers the analysis of the bounded rationality approach as a motivation for the use of perceptrons as learning devices. Section 3 then presents the characteristics of the perceptron, which will be used to perform the classification of the government task. Section 4 explores the problem of how to classify a government showing in which, if so, circumstances the perceptron can resolve that problem. Section 5 concludes.

2. The Bounded Rationality Approach

Generally speaking, learning models have been developed as a reasonable alternative to the unrealistic informational assumption of rational expectations models. Moreover, through learning models it is possible to study the dynamics of adjustment between equilibria which, in most

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rational expectations models, is ignored. Although a number of different studies modelling learning have been presented, two main classes of models can be distinguished: rational learning and boundedly rational learning models.[1] In rational learning models, it is assumed that agents know the true structural form of the model generating the economy, but not some of the parameters of that model. In boundedly rational learning models, it is assumed that agents, while learning is taking place, use a 'reasonable' rule, e.g., by considering the reduced form of the model.

Salmon (1995) is, to the best of our knowledge, one of the very few references where an innovative bounded rationality approach such as neural networks learning has been applied in a policy-making problem. We propose to use this approach within a political business cycles context (see also Caleiro, 2013). That being said, we will consider that bounded rationality voters have to classify economic policies and outcomes as coming from opportunistic or from benevolent government behaviour.

3. The Methodology of Perceptrons

Given the characteristics of perceptrons it, thus, seems appropriate to consider that the above mentioned classification task can be performed under this formulation of bounded rationality agents. Given that (artificial) neural networks are simulations of how biological neurons are supposed to work, the structure of human brains, where processing units, the so-called neurons, are connected by sinapses, is approximated by (artificial) neural networks. In our case, a single-layer linear classifier, known as perceptron, will be used to perform the classification task or, in other words, will be used to determine the vector of weights and bias specifying a line on the space output-inflation such that two sub-sets of points – obviously the opportunistic and benevolent ones – are defined.

4. The Classification of the Government

In the electoral business cycle literature, one of the most crucial conclusions is that the short-run electorally-induced fluctuations prejudice the long-run welfare. In fact, because the electoral results depend on voters' evaluation, we can consider that, if electoral business cycles do exist, it is because voters, through ignorance or for some other reason, allow them to exist. This point introduces a well-known problem of electorally-induced behaviour punishment and its related problem of monitoring. In reality, voters often cannot truly judge/classify if an observed state/policy is the result of a *self-interested/opportunistic government* or, on the contrary, results as a *social-planner/benevolent* outcome, simply because voters do not know the structure, the model or the transmission mechanism connecting policy values to state values.

Even so, voters do 'anticipate' the possible economic damage resulting from such *myopic* behaviour by governments and, especially closer to the elections, start to *classify* policies and outcomes as potentially being the result of an 'electoralist' strategy. This is done in order not to be 'fooled' by the incumbent government or simply to punish the incumbent government in case of clear *signals* of electorally-induced policies. In other words, a classification is made, so that for a sufficiently small sub-set of policies classified as 'electoralist', voters usually do not take that as a serious motive for punishment, but others, regarded as serious deviations, are punished. Note the difference between this approach and the one considered, for instance, in Minford (1995). Here, it is assumed that "*voters penalise absolutely any evidence that monetary policy has responded to anything other than news*", by 'absolutely' meaning that there is enough withdrawal of voters to ensure electoral defeat. In general, this classification task is made difficult by ignorance of the structural form of the model transforming policies in outcomes and also simply because information gathering costs money and time.

4.1. The model

Recently some authors have assumed an extended version of the standard aggregate supply curve $y_t = \bar{y} + \beta(\pi_t - \pi_t^e)$, where y_t denotes the level of output (measured in logarithms) that deviates from the natural level, $\bar{y}$, whenever the inflation rate, π_t, deviates from its expected level π_t^e, by considering

[1] Westaway (1992) prefers to distinguish closed-loop learning, where agents learn about the parameters of the decision rule, from open-loop learning, where agents form an expectation of the path for a particular variable which they sequentially update. As is pointed out, closed-loop learning will be virtually identical to the parameter updating scheme using Kalman filtering.

$$y_t = (1-\eta)\bar{y} + \eta y_{t-1} + \delta\left(\pi_t - \pi_t^e\right), \qquad (1)$$

where η measures the degree of output persistence. See Gärtner (1999) for an output persistence case and/or Jonsson (1997) for an unemployment persistence case. As acknowledged in Gärtner (1999), only at that time authors have started to pay due attention to the consequences of considering that relevant macroeconomic variables, *in reality*, show some degree of persistence over time. In fact, a casual observation on reality shows that Europe has been facing a problem in what concerns unemployment which indeed reflects persistence. Given the close connection between unemployment and output, it should be possible to 'translate' our results in terms of output to results in terms of unemployment.

When normalizing the natural level of output such that $\bar{y} = 0$ the aggregate supply curve reduces to:

$$y_t = \phi y_{t-1} + \alpha\left(\pi_t - \pi_t^e\right), \qquad (2)$$

where, following the hypothesis of adaptive expectations,

$$\pi_t^e = \gamma \pi_{t-1}, \qquad (3)$$

where $0 \le \phi \le 1$ and $0 \le \gamma \le 1$.

As said before, a most common kind of linear classifiers for classification purposes is the so-called perceptron. In order to perform the task of classifying the government, in what concerns its behaviour during the mandate, it is required the determination of the opportunistic and benevolent solutions. These solutions differ in accordance with the way time periods are discounted: whereas for society, therefore also for a benevolent government, future periods should be less important than present ones, this is not the case with an opportunistic government, as future moments, i.e. those closer to the election day, are more vital than present ones, in order to explore the decay in the memory of voters.

Having said that, concerning the government's objective function, we make the standard assumption that the incumbent faces a mandate divided into two periods, t =1,2, such that society's welfare during the mandate, i.e. the benevolent government's objective function is given by:

$$U = U_1 + \rho U_2, \qquad (4)$$

where ρ is the social rate of discount, whereas opportunistic government's objective function is :

$$V = \mu V_1 + V_2, \qquad (5)$$

where μ is the degree of memory of the electorate. In (4) and (5) we also admit that

$$U_t = V_t = -\tfrac{1}{2}\pi_t^2 + \beta y_t. \qquad (6)$$

In these circumstances it is worth immediately noticing that, in general, excepting if $\mu\rho = 1$, the policies that maximise social welfare (4) are not the ones that maximise popularity (5). As it plausible to assume that both ρ and μ do not exceed 1, it is immediately clear that only in the case of perfect memory, i.e. $\mu = 1$, and both periods being equally important for society, i.e. $\rho = 1$, an opportunistic government will behave exactly as a benevolent one. This fact allows for making it plausible to ask the question: how to classify a government?, whose answer is supposed to be given by a perceptron when separating optimal outcomes into two parts: the opportunistic and the benevolent ones. In other words, the opportunistic and benevolent solutions (policies and outcomes) will constitute the necessary inputs for the perceptron application. Given the classification task format, let us precisely define what will be called *opportunistic* or 'electoralist' *inputs*, that is policies, and *opportunistic outputs*, that is outcomes, to be compared with *benevolent inputs* and *benevolent outputs*.

Clearly, the opportunistic policy and outcomes will be, respectively, the values of inflation and output which result from the maximisation of (4) and (5) subject to (2) and (3). This immediately leads to the optimal policies:[2]

$$\pi_1^B = \alpha\beta\left(1 - \rho(\gamma - \phi)\right), \qquad (7)$$

$$\pi_2^B = \alpha\beta, \qquad (8)$$

$$\pi_1^O = \alpha\beta\left(1 - \frac{\gamma - \phi}{\mu}\right), \qquad (9)$$

$$\pi_2^O = \alpha\beta. \qquad (10)$$

Those policies lead to the optimal output levels:

[2] From this point onwards, the superscripts *B* and *O* identify an element as, respectively, concerning the benevolent and the opportunistic government.

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$$y_1^B = \phi y_0 + \alpha\left(\alpha\beta\left(1-\rho\left(\gamma-\phi\right)\right)-\gamma\pi_0\right), \qquad (11)$$

$$y_2^B = \phi\left(\phi y_0 + \alpha\left(\alpha\beta\left(1-\rho\left(\gamma-\phi\right)\right)-\gamma\pi_0\right)\right) + \alpha\left(\alpha\beta - \gamma\alpha\beta\left(1-\rho\left(\gamma-\phi\right)\right)\right), \qquad (12)$$

$$y_1^O = \phi y_0 + \alpha\left(\alpha\beta\left(1-\frac{\gamma-\phi}{\mu}\right)-\gamma\pi_0\right), \qquad (13)$$

$$y_2^O = \phi\left(\phi y_0 + \alpha\left(\alpha\beta\left(1-\frac{\gamma-\phi}{\mu}\right)-\gamma\pi_0\right)\right) + \alpha\left(\alpha\beta - \gamma\alpha\beta\left(1-\frac{\gamma-\phi}{\mu}\right)\right). \qquad (14)$$

Before proceeding with the classification task, it is relevant to note that there are, in fact, two possible patterns for the political business cycle: i) a typical one, where inflationary expansions take place immediately before the elections and ii) an atypical one, where the inflationary expansions take place immediately after the elections.[3] Given that:

$$\pi_2^B - \pi_1^B = \alpha\beta\rho\left(\gamma-\phi\right),$$

$$\pi_2^O - \pi_1^O = \alpha\beta\frac{\gamma-\phi}{\mu},$$

the typical pattern will be observed when $\gamma > \phi$ and the atypical one when $\gamma < \phi$. Plainly, when $\gamma = \phi$ there will be no cycle at all.

Given the optimal solutions, (7) to (14), it is straightforward to verify that, because

$$\pi_1^B - \pi_1^O = \alpha\beta\left(\gamma-\phi\right)\frac{1-\mu\rho}{\mu},$$

$$\pi_2^B - \pi_2^O = 0,$$

$$y_1^B - y_1^O = \alpha^2\beta\left(\gamma-\phi\right)\frac{1-\mu\rho}{\mu},$$

$$y_1^B - y_1^O = -\alpha^2\beta\left(\gamma-\phi\right)^2\frac{1-\mu\rho}{\mu},$$

the typical pattern will then be characterised by $\pi_2^B > \pi_1^B$, $\pi_2^O > \pi_1^O$, $\pi_1^B > \pi_1^O$, $\pi_2^B = \pi_2^O$ and $y_1^B > y_1^O$, $y_2^B < y_2^O$, whereas the atypical pattern will be characterised by $\pi_2^B < \pi_1^B$, $\pi_2^O < \pi_1^O$, $\pi_1^B < \pi_1^O$, $\pi_2^B = \pi_2^O$ and $y_1^B < y_1^O$, $y_2^B < y_2^O$.

Given that, in the previous mandate, no matter the kind of government, $\pi_0 = \alpha\beta$ it is possible to further simplify the optimal output levels expressions, (11) to (14), to:

[3] This means that, in general, not possible to always use the observed pre-elections expansions as empirical evidence supporting the existence of an opportunistic behaviour of the government as, in fact, even some experienced scholars incorrectly do.

$$y_1^B = \phi y_0 + \alpha^2 \beta(1 - \rho(\gamma - \phi) - \gamma), \quad (15)$$

$$y_2^B = \phi^2 y_0 + \alpha^2 \beta\left(\phi - 2\phi\rho\gamma + \rho\phi^2 - \phi\gamma + 1 - \gamma + \gamma^2 \rho\right), \quad (16)$$

$$y_1^O = \phi y_0 + \alpha^2 \beta \frac{\mu + \phi - \gamma - \gamma\mu}{\mu}, \quad (17)$$

$$y_2^O = \phi\left(\phi y_0 + \alpha^2 \beta \frac{\mu + \phi - \gamma - \gamma\mu}{\mu}\right) + \alpha\left(\alpha\beta - \gamma\alpha\beta\left(1 - \frac{\gamma - \phi}{\mu}\right)\right). \quad (18)$$

4.2. The classification task

The optimal inflation rates, (7) to (10), and output levels, (15) to (18), define the coordinates of four points in the (y, π) space. This space is to be partioned, if possible, in two sub-spaces by a linear decision boundary – in that consists the classification task – by the perceptron. See figure 1.

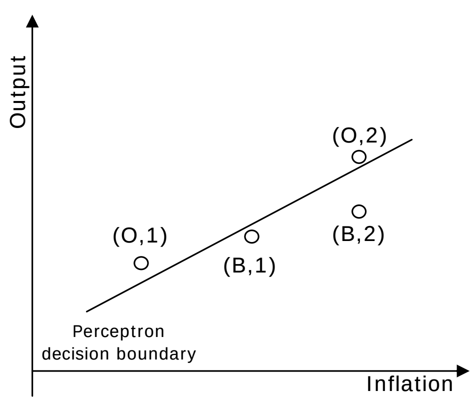

Figure 1 – The perceptron classification

Figure 1 allows visualising the opportunistic and benevolent trajectories in the inflation-output, (y,π), space, showing an example where the classification of the government is possible to be achieved by the perceptron.

There are, therefore, four points located in the (y,π) space, two of each type, *O* and *B*. This makes possible to draw two line segments connecting the two points of each kind. If these two line segments cross, it is impossible to obtain a decision boundary. This can be checked by a system of equations involving two convex combinations between these points defining the intersection between the straight line segments. They cannot be separated if the two parameters, λ_1, λ_2 in the convex combinations:

$$\lambda_1 \begin{bmatrix} y_1^B \\ \pi_1^B \end{bmatrix} + (1 - \lambda_1)\begin{bmatrix} y_2^B \\ \pi_2^B \end{bmatrix} = \lambda_2 \begin{bmatrix} y_1^O \\ \pi_1^O \end{bmatrix} + (1 - \lambda_2)\begin{bmatrix} y_2^O \\ \pi_2^O \end{bmatrix}, \quad (19)$$

are both between 0 and 1.

Given the optimal inflation rates, (7) to (10), and output levels, (15) to (18), the solutions for λ_1, λ_2 in (19) are:

$$\lambda_1 = \frac{\alpha^2 \beta}{\phi\mu} \frac{(\phi - \gamma)^2}{(1 - \phi)y_0 + \alpha^2 \beta(\gamma - 1)}, \quad (20)$$

$$\lambda_2 = \frac{\alpha^2 \beta\rho}{\phi} \frac{(\phi - \gamma)^2}{(1 - \phi)y_0 + \alpha^2 \beta(\gamma - 1)}.[4] \quad (21)$$

Plainly, in general, the possibility to classify the government depends upon the initial level of output, y_0.[5] Figure 2 thus represents those two solutions (20) and (21) as a function of y_0.

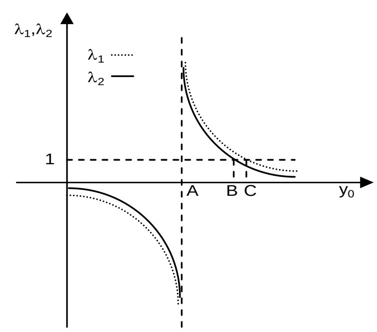

Figure 2 – The influence of initial output level

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[4] Note that $\lambda_1 - \lambda_2 = \frac{1 - \rho\mu}{\mu} \frac{\alpha^2 \beta}{\phi} \frac{(\gamma - \phi)^2}{(1 - \phi)y_0 + \alpha^2 \beta(\gamma - 1)}$.

[5] When $\phi = \gamma$, both λ_1, λ_2 are equal to zero, meaning that both types of governments behave the same.

In order to have $\lambda_1 = 1$ in (20), – point C in figure 2 – the initial level of output must be:

$$y_0 = \alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1-\gamma)\mu}{\phi(1-\phi)\mu}, \qquad (22)$$

whereas, in order to have $\lambda_2 = 1$ in (21), – point B in figure 2 – the initial level of output must be:

$$y_0 = \alpha^2 \beta \frac{\rho(\phi - \gamma)^2 + \phi(1-\gamma)}{\phi(1-\phi)}. \qquad (23)$$

As y_0 given by (22) is higher than y_0 given by (23),[6] this means that for

$$y_0 > \alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1-\gamma)\mu}{\phi(1-\phi)\mu}, \qquad (24)$$

$\lambda_1 < 1$ and, therefore, also that $\lambda_2 < 1$. Moreover,

$$y_0 > \alpha^2 \beta \frac{1-\gamma}{1-\phi} \qquad (25)$$

guarantees that both λ_1, λ_2 are positive. See point A in figure 2. After noticing that y_0 given by (22) is higher than y_0 given by (25),[7] it is possible to consider an initial condition

$$y_0 > \alpha^2 \beta \frac{(\phi - \gamma)^2 + \phi(1-\gamma)\mu}{\phi(1-\phi)\mu}, \qquad (26)$$

such that it is impossible to associate *all* the observed behaviours to the correct type of government. In all the other cases, the classification task can be resolved by the perceptron.

Notwithstanding that conditionally, there is a fundamental exception. When output does not show any persistence over time, *i.e.* $\phi = 0$, which is, indeed, the most considered case in the literature, it is possible to show that a straight line with intercept between $\alpha^2\beta\gamma(\gamma\rho - 2)$ and $\alpha^2\beta\gamma\frac{\gamma - 2\mu}{\mu}$ and slope equal to $\alpha(\gamma+1)$ will always divide the space in a correct way, this being eventually the result of the perceptron classification.[8]

Plainly, in practical terms, given that a learning process takes place, from the training of the perceptron does not usually result a straight line with the above mentioned characteristics. Most importantly, given that the two straight lines connecting the two pairs of points in the output-inflation space are parallel, this guarantees that the space is linearly separable. Figure 3 shows this situation.

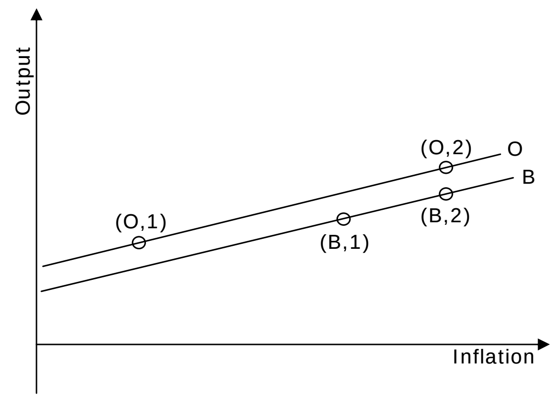

Figure 3 – A particular(ly interesting) case

5. Concluding Remarks

The paper explores a crucial aspect in the issues of political business cycles by considering the effects of boundedly rational voters, a fact that has been largely ignored by the literature (Caleiro, 2013). The classification task performed by that kind of voters is done by the use a perceptron in a model allowing for output persistence. It is shown that when output does not persist the classification task can always be resolved. Conversely, the resolution of the classification task, when output persists over time, depends crucially on the initial conditions.

As a direction for future improvements we would like to explore the possible dynamics of convergence for output in order to check, in the long-run, the real importance of the initial level of output. As, indeed, the steady state cycle, for each kind of government are characterised by a level of output below the one identified by (26), hypothetically the resolution of the classification task may become more probable over time.

[6] Note that $\alpha^2\beta\frac{(\phi-\gamma)^2+\phi(1-\gamma)\mu}{\phi(1-\phi)\mu} - \alpha^2\beta\frac{\rho(\phi-\gamma)^2+\phi(1-\gamma)}{\phi(1-\phi)} = \alpha^2\beta(\phi-\gamma)^2\frac{1-\mu\rho}{\phi\mu(1-\phi)} > 0.$

[7] Note that $\alpha^2\beta\frac{(\phi-\gamma)^2+\phi(1-\gamma)\mu}{\phi(1-\phi)\mu} - \alpha^2\beta\frac{1-\gamma}{1-\phi} = \alpha^2\beta\frac{(\phi-\gamma)^2}{\phi\mu(1-\phi)} > 0.$

[8] The mathematical details are available upon request.

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