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## Accuracy of Intermittent Demand Forecasting Systems in the Enterprise

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**Abstract:**

**Purpose:** The main purpose of this article is to find the best forecasting method for intermittent demand times series, from the company's point of view.

**Design/Methodology/Approach:** Intermittent demand forecasting systems were constructed based on the Croston's, SBA, TSB, SES and MA methods. A real database from the warehouse center, containing over sixteen thousand items, was used. Accuracy measures were also discussed. Forecasting methods were compared for all products and for separate demand categories (intermittent, lumpy, erratic, smooth).

**Findings:** It was determined that the TSB method outperforms other methods for all products. The worst procedures were found to be Croston's and SBA, which performed even worse than SES or MA. The same conclusions were true for intermittent and lumpy categories. In case of erratic and smooth items different results were obtained. It was determined that the SBA method performed best, while the TSB method yielded the poorest results.

**Practical Implications:** The main conclusion is that to judge accuracy of forecasting systems first the proper forecast error measures should be chosen. Based on obtained results, TSB method seems to be the best for intermittent demand times series and this method is recommended for enterprises dealing with intermittent demand.

**Originality/value:** Since such error measures as MASE or scaled MAE favored an underestimated (or even zero) forecast, in the article a new error metric is proposed, which was named scaled Compound Error (sCE). It is a scaled error, and it considers forecast biasedness.

**Keywords:** Intermittent demand forecasting, accuracy measures, scaled compound error, Croston's method, SBA method, TSB method, exponential smoothing.

**JEL classification:** C53, E27, L81.

**Paper Type:** Research study.

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## 1. Introduction

In enterprises, one usually deals with a substantial number of items, thus forecasting systems are required. Forecasts need to be computed simultaneously for multiple products. This affects the methodology that might be applied. Forecasting methods should be robust to any outliers, elastic and preferably uncomplicated, if various situations are to be considered. Robustness is easier to obtain with simpler methods.

In this article it is claimed that to set up a forecasting system, forecast accuracy for multiple methods should be verified based on real data. Simulation studies are important from a theoretical point of view, but it is rather impossible to mimic all possible forecasting situations. It is also hard to verify theoretical assumptions, for example regarding theoretical distributions, if only few non-zero sales are available and times series are short. In this research five forecasting systems, based on such methods as Croston's, SBA, TSB, SES and MA, were constructed. Real data for about sixteen thousand products were used. Forecast accuracy was verified for all products and for product categories (lumpy, intermittent, smooth, and erratic).

Another problem that was considered involved a proper forecasting accuracy measure. Statistical errors that are usually applied (for example MASE) favor underestimated, sometimes even zero forecasts. On the other hand, inventory-based measures often require a lot of specific information about service or stock levels, which may be unavailable. Therefore, a new forecasting measure is proposed in this article. One which avoids, at least to some extent, the above-mentioned drawbacks.

There are two general aims of this article. One is to propose the best forecasting method for the analyzed enterprise, in which most of the items are intermittent. But to evaluate the methods, an appropriate forecasting accuracy measure must be employed. An appropriate measure is understood as a measure that considers also forecast bias. Therefore, the main issues are related to choosing the most satisfactory forecasting method and an appropriate forecasting accuracy measure for the intermittent demand forecasting system in the analyzed enterprise. As was already mentioned, a new forecasting measure, named scaled Compound Error (sCE), is proposed. The analyses presented in the article might be useful to enterprises dealing with intermittent demand forecasting and stock management.

## 2. Literature Review

Most of the literature on intermittent demand forecasting is focused on Croston's method and its variants (Croston, 1972), (Syntetos, 2001), (Willemain *et al.*, 1994). Croston's method is based on exponential smoothing that is applied separately to demand size and demand intervals. These two counterparts are then divided to obtain an estimate of demand per period. There are some assumptions related to Croston's method, such as geometrically distributed demand intervals, normal distribution of demand size or independence of these two counterparts. A detailed

discussion of assumptions and stochastic models underlying Croston's method are presented in (Shenstone and Hyndman, 2005).

It was shown that Croston's method is biased (Rao, 1973), (Syntetos and Boylan, 2001), (Boylan and Syntetos, 2007). To remove this bias, some modifications of Croston's method were proposed, for example SBA (Syntetos–Boylan Approximation) (Syntetos and Boylan, 2005). The drawback of this method is that it could lead to underestimated forecasts in the case of fast-moving items, especially if they are sold in all periods. However, Syntetos (2001) proposed a solution to this problem by introducing a new estimator, which is unbiased also for non-intermittent demand.

Both SBA and Croston's methods are based on demand intervals that could be updated only in periods with non-zero sales. If there are many obsoletes, it could result in overestimated forecasts. In TSB method sales probability is estimated instead of demand intervals (Teunter, Syntetos and Babai, 2011). Sales probability might always be updated, even in periods with no demand, hence forecasts for obsoletes are decreasing. Apart from the TSB method, there are also other proposals dealing with obsolescence. For example, hyperbolic exponential smoothing is proposed in (Prestwich *et al.*, 2014), where forecasts decay hyperbolically, while in the TSB method the decay is exponential.

Originally, in Croston's method there is one smoothing constant, the same one for the demand size and demand intervals. In this research, in the case of Croston's, the SBA and TSB methods, two smoothing constants were used, separately for the demand size and demand intervals (or for sales probability in TSB method). To sum up, many assumptions of Croston's method were questioned. Apart from that, in the case of time series with high proportion of zeros, where there are often only few positive sales, it is sometimes impossible to verify any theoretical assumptions with regard to, for example, sales theoretical distributions. Therefore, while constructing a forecasting system in an enterprise, it is important to analyze forecast accuracy for real data sets, which was also emphasized in (Dosz  n, 2019).

From a theoretical point of view, in the case of intermittent demand, Croston's, SBA or TSB methods should be superior to much simpler methods such as SES (Simple Exponential Smoothing) or MA (Moving Average). Certain studies confirm this claim (Teunter and Duncan, 2009). However, empirical research has also proven that SES or MA result in better forecasts, e.g. (Syntetos, 2001), (Dosz  n, 2019). Therefore, in this study the SES and MA methods were applied as well.

There are many other methodological proposals of dealing with intermittent demand forecasting. Model-based methods are sometimes employed, such as DARMA (Discrete Auto-Regressive Moving Average) or INARMA (INteger-valued Auto-Regressive Moving Average) (Engelmeyer, 2016). There are also proposals based on bootstrapping (Willemain, Smart and Schwarz, 2004), (Snyder, 2002), (Syntetos,

Babai and Gardner, 2015), (Teunter and Duncan, 2009). Sometimes such methods as stochastic simulation (Shukur, Doszyń and Dmytrów, 2017), count data sales distributions (Kolassa, 2016; Snyder, Ord, and Beaumont, 2012) or temporal aggregation (Nikolopoulos *et al.*, 2011) are also applied. Those kinds of methods are not considered in this article.

A high proportion of zeros makes the measurement of forecasts accuracy difficult. Numerous problems related to forecast accuracy of that kind of time series are discussed e.g., in (Goodwin and Lawton, 1999; Hyndman and Koehler, 2006; Hyndman, 2006). Scale-dependent metrics, such as Mean Squared Error (MSE), Mean Absolute Error (MAE) or Geometric MAE (GMAE) are rather useless, because forecasts for multiple data series must be compared. Moreover, percentage-error metrics, such as Mean Absolute Percentage Error (MAPE) or symmetric MAPE (sMAPE), are impossible to obtain because of zeros.

It is claimed that relative or percentage errors must be used if forecasts for numerous products are compared. Errors like MAE are sometimes related to mean error (Kolassa and Schutz, 2007). Also mean-based error measures are proposed, in which forecast are related to in-sample means instead of empirical values in the forecast horizon (Prestwich *et al.*, 2014). However, in-sample means for slow-moving items are often close to zero, which might inflate these errors and affect their distributions.

In this research Mean Absolute Scaled Error (MASE) was applied as well. This error measure is recommended for intermittent demand data (Hyndman and Koehler, 2006). It is scale-independent and could be computed in almost all cases, also for times series with many zeros. There are also various other proposals. To limit the impact of outliers, Relative Geometric Root Mean Squared Error (RGRMSE) is sometimes recommended (Syntetos and Boylan, 2005). Also, non-parametric errors, such as Percentage Best (PBt) or Percentage Better (PB) forecasts are applied (Kolassa and Schutz, 2007; Syntetos, 2001). It could be claimed that the type of a forecast error measure often determines which method is chosen (Engelmeyer, 2016). Methods which perform most favourably in terms of such error measures as MASE or MAE may lead to the weakest inventory performance.

Engelmeyer (2016) emphasised that forecasts should be evaluated with regard to inventory optimization, but not statistical error measures. However, this conclusion might be the result of the properties of the error measures applied. In this article it is stated that a forecast error ought to consider biasedness. As biased, error measure favouring underestimated (or overestimated) forecasts is understood. Measures like MASE or sMAE (scaled MAE) favour methods yielding the most underestimated (the lowest) forecasts. Therefore, a new error measure, avoiding this disadvantage, is proposed in the next paragraph. Linking forecast error measures to inventory performance is an interesting idea. Among other metrics, service or stock level measures are usually employed (Engelmeyer, 2016; Wallström and Segerstedt,

2010). The application of service and stock holding levels are discussed in (Teunter and Duncan, 2009). Wallström and Segerstedt (2010) new metrics, such as Periods in Stock (PIS), Number of Shortages (NOS) or Cumulated Forecast Error (CFE), are proposed. It is also emphasized that complementary error measures ought to be applied rather than just a single measure.

It is not easy to indicate which method is superior in forecasting intermittent demand. For instance, in Syntetos and Boylan (2005) the Croston's, SBA, SES and MA methods were applied to real data sets. Such errors as ME, sME (scaled ME), RGRMSE, PB and PBt indicated that the SBA method is preferable. In Teunter, Syntetos and Babai (2011) the TSB, SBA, Croston's and SES methods were compared in a simulation experiment. The conclusion was that regarding ME and MSE errors, the TSB method surpasses all others. In Prestwich *et al.* (2014a) five methods were applied CR, SBA, SY, TSB, HES. SY is the unbiased estimator of the SBA method for non-intermittent demand proposed in Syntetos (2001). The methods were verified with respect to MASE, sMAE and U2. U2 is a well-known Theil's measure. The TSB and HES methods were pointed as being the most satisfactory.

Quite similar methods were also employed in Doszyń (2019). These involved CR, SBA, TSB, SES, MA, SESAP. The last of those methods (SESAP) is dedicated to seasonal intermittent items and could be described as SES for the same subperiods (months, for example). MASE, sME and sRMSE (scaled Root Mean Squared Error) were used in comparisons. It was generally concluded that TSB should be preferred.

Summarizing, numerous studies favor the TSB method, however the results are at times inconclusive. Therefore, empirical verification should precede the choice of the method applied in a real forecasting system. In the case of forecasting error measures, metrics taking biasedness into account ought to be applied. Moreover, inventory-based measures might be useful if enough information is available to apply them.

### 3. Research Methodology

Forecasting systems for the following methods were constructed:

- a. Croston's method (CR),
- b. Syntetos – Boylan Approximation, SBA (Syntetos and Boylan, 2005),
- c. TSB (Teunter, Syntetos and Babai, 2011),
- d. simple exponential smoothing (SES),
- e. moving average (MA),
- f. zero forecasts (ZF).

The first three methods are dedicated directly to intermittent demand forecasting. The last three of them have been applied mostly as a benchmark. In the following Table 1 symbols are presented.

**Table 1.** Symbol descriptions

Symbol	Description
$x_t$	demand (both zero and non-zero) in period $t$
$x_t^+$	demand size (non-zero sale)
$\hat{x}_t$	smoothed demand (in-sample values)
$\hat{x}_t^+$	smoothed demand size
$\tau_t$	demand interval
$\hat{\tau}_t$	smoothed demand interval
$p_t$	sales occurrence indicator (zero-one variable)
$\hat{p}_t$	smoothed sales probability
$q_t$	number of periods since the last non-zero sale
$\alpha, \beta \in (0,1)$	smoothing factors
$k$	smoothing range length (MA method)
$n$	number of (in-sample) periods
$h$	number of ex post forecasts (forecast horizon)
$\hat{x}_{pt}$	ex post forecast for period $t$ ( $t = n + 1, \dots, n + h$ )
$\bar{x}_{1-n}$	in-sample mean

**Source:** Own elaboration.

In Croston's method the demand size and demand intervals are updated only in periods with non-zero sale. Thus, if  $x_t > 0$ , then:

$$\begin{aligned} \hat{x}_t^+ &= \hat{x}_{t-1}^+ + \alpha(x_t^+ - \hat{x}_{t-1}^+) \\ \hat{\tau}_t &= \hat{\tau}_{t-1} + \beta(q_t - \hat{\tau}_{t-1}) \end{aligned} \quad (1)$$

On the other hand, if  $x_t = 0$ , then  $\hat{x}_t^+ = \hat{x}_{t-1}^+$  and  $\hat{\tau}_t = \hat{\tau}_{t-1}$ . Therefore, smoothed values are not updated if there is no sale. In that case only the number of periods since the last non-zero sale is increased by one:  $q_t = q_{t-1} + 1$ . Smoothed demand constitutes a relation of these two counterparts:  $\hat{x}_t = \hat{x}_t^+ / \hat{\tau}_t$ , so the smoothed demand size is divided by the smoothed demand interval.

As was demonstrated by (Syntetos and Boylan 2005) Croston's method is biased, because  $E(x_t^+ / \tau_t) \neq E(x_t^+) / E(\tau_t)$ . The SBA estimator was proposed to avoid such biasedness. SBA is exactly the same as Croston's method, but smoothed demand is adjusted by the factor that is supposed to limit biasedness, hence  $\hat{x}_t = (1 - \beta/2) \hat{x}_t^+ / \hat{\tau}_t$ , where  $\beta$  is the smoothing constant used to adjust demand intervals. In the TSB method, instead of demand intervals, sales probability is used, which is updated also in periods with no demand.

If  $x_t > 0$ , then:

$$\hat{x}_t^+ = \hat{x}_{t-1}^+ + \alpha(x_t^+ - \hat{x}_{t-1}^+) \quad (3)$$

$$\hat{p}_t = \hat{p}_{t-1} + \beta(1 - \hat{p}_{t-1}) \quad (4)$$

In turn, if  $x_t = 0$ , then:

$$\hat{x}_t^+ = \hat{x}_{t-1}^+ \quad (5)$$

$$\hat{p}_t = \hat{p}_{t-1} + \beta(0 - \hat{p}_{t-1}) \quad (6)$$

Smoothed demand is a product of adjusted demand size and sales probability:  $\hat{x}_t = \hat{x}_t^+ \hat{p}_t$ . With respect to sales level, TSB is the same as CR and SBA. However, sales probability is updated in TSB in each period. This solution is better on account of obsolesces.

SES and MA were used as benchmarks:

$$\text{SES: } \hat{x}_t = \hat{x}_{t-1} + \alpha(x_t - \hat{x}_{t-1}) \quad (7)$$

$$\text{MA: } \hat{x}_t = \frac{1}{k} \sum_{t-k}^{t-1} x_t \quad (8)$$

In case of zero forecasts it was assumed that forecasts for each item are always equal to zero. Overall, a single forecast error in period  $t$  is equal to:  $e_t = x_t - \hat{x}_{pt}$ . The following four ex post error metrics were used to verify forecast accuracy:

➤ scaled Mean Error (sME):

$$sME = \frac{\frac{1}{h} \sum_{t=n+1}^{n+h} e_t}{\bar{x}_{1-n}} = \frac{\bar{e}_t}{\bar{x}_{1-n}} \quad (9)$$

➤ scaled Mean Absolute Error (sMAE):

$$sMAE = \frac{\frac{1}{h} \sum_{t=n+1}^{n+h} |e_t|}{\bar{x}_{1-n}} = \frac{\overline{|e_t|}}{\bar{x}_{1-n}} \quad (10)$$

➤ Mean Absolute Scaled Error (MASE):

$$MASE = \frac{\frac{1}{h} \sum_{t=n+1}^{n+h} |e_t|}{\frac{1}{n-1} \sum_{t=2}^n |x_t - x_{t-1}|} = \frac{\overline{|e_t|}}{\overline{|\Delta x_t|}} \quad (11)$$

where  $\overline{|\Delta x_t|}$  is an average absolute change in a sample,

➤ scaled Compound Error (sCE):

$$sCE = \frac{|\bar{e}_t| + \overline{|e_t|}}{\bar{x}_{1-n}} = |sME| + sMAE \quad (12)$$

All the above measures are scale-independent and can always be calculated for intermittent demand time series (if in the past there was at least one positive sale, so

$\bar{x}_{1-n} > 0$ ). The disadvantage of scaled errors is that for highly intermittent series denominator ( $\bar{x}_{1-n}$ ) might be close to zero, what inflates such measures. However, this does not affect the conclusions if forecasting methods are compared.

Scaled Mean Error (sME) provides information on forecast biasedness, which is important with respect to both stock level and customer service level. If forecasts are overestimated ( $sME < 0$ ) stock level would be inflated, and storage costs might be too high. On the other hand, if  $sME > 0$ , then forecasts are underestimated, thereby decreasing customer service level, and reducing profits.

In general, low values of scaled Mean Absolute Error (sMAE) are preferable. In MASE forecasts are compared with naïve forecasts in a sample period, so if  $MASE < 1$ , then forecasts are better than a naïve alternative. In the case of intermittent data, these measures might prefer methods that generate underestimated forecasts, often even zero forecasts. If there is no sale in many forecasted periods, the method offering lower forecasts (or even zeros) would feature lower sMAE or MASE. However, lower forecasts would be biased (underestimated).

For that reason, a new forecasting measure is proposed in this article, which is called scaled Compound Error (sCE). The measure is also scale independent. It could be expressed as a sum of an absolute value of scaled Mean Error and scaled Mean Absolute Error, thus it considers biasedness. What is the logic behind the scaled Compound Error? For the methods that overestimate demand indications of sCE are similar as in the case of sMAE or MASE, but for the methods that underestimate forecasts the conclusions are different. MASE or sMAE, contrary to sCE, prefer underestimated forecasts, because intermittent data involves a multitude of zeros.

Let us assume that there are three types of forecasts: underestimated, unbiased, and overestimated (Table 2). There are ex post forecasts for five weeks, for these three methods. The empirical values in the examined periods equal, respectively: 0, 0, 5, 0, 0. Five pieces of an item were sold only in the third week, so the product might be treated as intermittent. Forecasts for a singular item are analyzed, hence errors are not scaled, but this does not affect the conclusions.

In this example Compound Error (CE) is computed as  $CE = |\bar{e}_t| + \overline{|e_t|}$ . With respect to biasedness the second method (UF) is superior. However, for this method a mean absolute error is equal to  $\overline{|e_t|} = 8/5$  and it is higher than in the case of zero forecasts (ZF), for which  $\overline{|e_t|} = 1$ . Therefore, such metrics as MAE or MASE point to zero forecasts as being the most satisfactory. However, zero forecasts are biased, they are underestimated.

On the other hand, compound error (CE) gives the lowest result when the second method is applied, for which forecasts are unbiased and it seems that this method should be indicated as the preferred option. According to CE, zero forecasts (ZF)



and overestimated forecasts (OF) perform far worse. To sum up, CE is recommended as it considers biasedness, what is important especially with respect to customer service level.

**Table 2.** Forecasts examples for an item with sales equal to 0, 0, 5, 0, 0 ( $CE$  – compound error)

Type of forecasts	Forecasts	$e_t$	$ e_t $	$\bar{e}_t$	$ \bar{e}_t $	$CE$
Underestimated forecasts (zero forecasts – ZF)	0 0 0 0 0	0 0 5 0 0	0 0 5 0 0	1 1	2	
Unbiased forecasts (UF)	1 1 1 1 1	-1 -1 4 -1 -1	1 1 4 1 1	0 8/5	8/5	
Overestimated forecasts (OF)	2 2 2 2 2	-2 -2 3 -2 -2	2 2 3 2 2	-1 11/5	16/5	

**Source:** Own elaboration.

#### 4. Empirical Results

The analyzed data set consisted of weekly sales time series from a distribution and warehouse center located near Szczecin. Szczecin is a medium-sized city in the North-Western Poland. The range of observations encompassed 210 weeks, yet many products have a shorter sales history (they were introduced later). In this study demand and sales are treated synonymously because customer service level is close to one.

The data base contained 16399 items, but the analysis concerned only those products that featured more than 10 observations (offered for at least 11 weeks) and at least two positive sales. For each product forecasts for the last 5 weeks were computed, thus it was assumed that a minimal number of observations should be at least twice as high (so higher than 10). To compute the demand intervals in the Croston's type methods, two (or more) positive demands are required. That is why, only the times series with more than one positive sale were considered. The final number of items examined was equal to 13783. The data for the last 5 weeks for each item were used to obtain forecast errors, hence all computations were made for 205 weeks (or for a shorter period if time series were shorter). As a rule, in the case of all methods the last smoothed values (smoothed values for the week 205) were taken as ex post forecasts.

In the literature, e.g. in Syntetos (2001), and Wallström and Segerstedt (2010), products are usually classified into four groups erratic, lumpy, smooth or intermittent. The analyzed data set was also classified into those groups, according to sales frequency ( $p$ ) and squared coefficient of variation ( $CV^2$ ). Sales frequency for a given product was understood as a share of weeks with positive sales. In the Croston's type methods, instead of sales frequency, demand intervals are considered. Sales frequency is better because it could be updated even in periods

with no demand. Squared coefficient of variation was computed for positive demand as a quotient of variance and squared mean:  $CV^2 = Var(x_t^+)/(\overline{x_t^+})^2$ .

In the classification scheme cut-off values were set at  $p = 0.75$  and  $CV^2 = 0.5$ . Similar values could be found e.g. in (Wallström and Segerstedt, 2010), but instead of sales frequency ( $p$ ), an average demand interval equal to 4/3 was proposed. The inverse of this value gives sales frequency equal to 0.75.

A more detailed discussion concerning demand classification was presented in (Syntetos, 2001), where it was stated that cut-off values should not be set arbitrarily, but they ought to be computed on the basis of the forecasting methods performance. However, cut-off values obtained in this way were close to  $p = 0.75$  and  $CV^2 = 0.5$ . There are also other classification schemes, for example, a multi-criteria inventory classification was proposed in (Engelmeyer, 2016). The classification results are presented in Table 3:

**Table 3.** Products classification with average number of weeks ( $\bar{n}$ ) and average sales frequency ( $\bar{p}$ ) for each group

$CV^2 \backslash p$	$p \leq 0.75$	$p > 0.75$
$CV^2 \leq 0.5$	Intermittent $n = 9143; \bar{n} = 154; \bar{p} = 0.11$	Smooth $n = 139; \bar{n} = 190; \bar{p} = 0.93$
$CV^2 > 0.5$	Lumpy $n = 3932; \bar{n} = 175; \bar{p} = 0.30$	Erratic $n = 569; \bar{n} = 188; \bar{p} = 0.87$

*Source:* Own computations.

Most of the items (95%) were classified as intermittent (9143) or lumpy (3932). There were 569 erratic and only 139 smooth items, so in the examined company slow-moving items dominated. As expected, average sales frequency was incredibly low for intermittent ( $\bar{p} = 0.11$ ) and lumpy items ( $\bar{p} = 0.30$ ). It was quite different for smooth or erratic demand, where it was close to 0.90. An average number of observations in each group ranged between 154 and 190 weeks, thus time series were long, but lower than 205 (observation range).

Forecast errors were computed for all 13783 products and for all groups. For the CR, SBA and TSB methods there were two smoothing constants,  $\alpha$  and  $\beta$ , equal to, respectively, 0.05, 0.10 and 0.15, providing nine combinations for each method. In the literature smoothing constant values in the range 0.05–0.20 are usually suggested, e.g. (Syntetos, 2001). Here, the upper limit was set at 0.15 because weekly data were considered. Lower values are suggested for more frequent data. Moreover, a higher number of combinations would needlessly limit clarity. At times it is suggested that smoothing constants should be optimized, see (Kourentzes, 2014). However, for many products, a majority of which have only a few positive sales, this could lead to very fragile results.

In the SES method  $\alpha$  was also set at 0.05, 0.10 and 0.15. In the MA method smoothing range length ( $k$ ) was equal to 39, 19, 12. This is the result of the formula  $k \approx \frac{2}{\alpha} - 1$  proposed in (Syntetos, 2001). This renders the results of the MA method, at least to some extent, comparable. Forecasting errors for all products are presented in the following Table 4:

**Table 4.** Average forecasting errors for all 13783 products

Method	$\alpha$	$\beta$	sME	sMAE	MASE	sCE
CR	0.050	0.050	−0.860	2.357	<i>1.659</i>	<i>3.216</i>
		0.100	−0.711	2.218	1.570	2.930
		0.150	−0.657	2.164	1.533	2.821
	0.100	0.050	−0.846	2.341	1.637	3.187
		0.100	−0.701	2.206	1.550	2.907
		0.150	−0.648	2.153	1.516	2.802
	0.150	0.050	−0.838	2.332	1.622	3.170
		0.100	−0.695	2.199	1.537	2.894
		0.150	−0.644	2.147	1.504	2.791
SBA	0.050	0.050	−0.815	2.319	1.631	3.134
		0.100	−0.630	2.150	1.518	2.781
		0.150	−0.540	2.066	1.459	2.606
	0.100	0.050	−0.802	2.304	1.609	3.106
		0.100	−0.620	2.138	1.499	2.759
		0.150	−0.532	2.056	1.442	2.588
	0.150	0.050	−0.795	2.295	1.595	3.090
		0.100	−0.615	2.131	1.487	2.746
		0.150	−0.528	2.050	1.432	2.578
TSB	0.050	0.050	−0.030	1.517	0.966	1.547
		0.100	0.050	1.421	0.900	1.471
		0.150	0.042	1.407	0.891	1.449
	0.100	0.050	−0.037	1.517	0.965	1.554
		0.100	0.042	1.423	0.899	1.465
		0.150	0.032	1.410	0.891	1.442
	0.150	0.050	−0.042	1.518	0.964	1.560
		0.100	0.035	1.425	0.900	1.460
		0.150	0.024	1.412	0.892	<b>1.437</b>
MA	0.050		0.115	1.423	0.892	1.538
	0.100		0.125	1.402	0.877	1.527
	0.150		0.017	1.475	0.924	1.492
SES	0.050		−0.038	1.539	0.975	1.577
	0.100		0.046	1.453	0.914	1.499
	0.150		0.037	1.454	0.915	1.491
ZF			0.908	<b>0.908</b>	<b>0.587</b>	1.815
min			−0.860	0.908	0.587	1.437
max			0.908	2.357	1.659	3.216

**Note:** The best values are presented in bold print and the poorest values are in italics (for the last three error measures)

**Source:** Own computations.

Generally, with respect to sCE, the TSB method was found to be superior. The worst forecasting results involved CR and SBA, which performed much worse than SES or MA. What may come as a surprise, with respect to average sMAE and MASE, zero forecasts occurred to be superior. They led to much lower errors than other methods. Thus, scaled compound error (sCE) provided more acceptable conclusions. It yields the lowest value for TSB ( $\alpha = \beta = 0.15$ ). Both sCE and other considered measures provided the poorest results for the CR method ( $\alpha = \beta = 0.05$ ). Therefore, with respect to the highest errors, the indications of sCE are consistent with sMAE and MASE.

Based on sME it could be noticed that biasedness of CR and SBA was the highest for all smoothing constant values. These methods generated highly overestimated forecasts. For instance, for CR ( $\alpha = \beta = 0.05$ ) an average sME was equal to  $-0.86$ , so forecasts were much higher than empirical values. The bias of SBA was similar, but slightly lower. On the other hand, the TSB, SES and MA methods were only slightly biased. As expected, ZF resulted in the most underestimated forecasts.

What might also be surprising, the CR and SBA methods yielded less satisfactory results in terms of biasedness in comparison to SES and MA. It was because the CR and SBA methods require that the size and demand intervals be adjusted only if the demand was positive, which rarely occurred because of intermittence. It might be claimed that Croston's type methods were not able to handle obsoletes or, generally, highly intermittent items. In SES and MA adjusted values were decreasing also in weeks with no demand, which resulted in lower forecasts errors. It was also valid for TSB.

Apart from that, the reduction of smoothed values was faster for higher smoothing constants. This was the reason why, overall, forecasts errors were lower for higher smoothing factors, especially for  $\beta$ , which were used to adjust demand intervals (CR, SBA) or sales probability (TSB).

Moreover, due to smoothing constant values, a certain relation was observed. Forecast errors were lower for higher smoothing constants. In the case of the CR, SBA and especially for the TSB methods the impact of  $\beta$  was much stronger than the impact of the same values of  $\alpha$ . Particularly in the case of TSB, for higher values of  $\beta$ , adjusted values reached zero faster, which reduced such errors as sMAE and MASE. With respect to sMAE and MASE, the CR and SBA methods were performing the most poorly. TSB, SES or MA yielded far better results. As was mentioned above, sMAE and MASE approached their lowest values for ZF.

Therefore, those types of measures are insufficient in the context of intermittent demand and that is why the sCE metric is proposed. To sum up, according to sCE, TSB was found to be the best. The poorest forecasts were obtained with the use of CR, SBA and ZF. Once more, we can notice that MA and SES were performing

better than CR or SBA, which was surprising from a theoretical point of view. The general explanation is that CR and SBA could not cope with obsolete items (or with temporal obsolescence). In the CR and SBA, the adjusted values were updated only in the weeks featuring positive demand, which led to an overestimation. In TSB, SES and MA smoothed values were updated even if the sales stood at zero. The examined error measures were presented for each demand category (intermittent and lumpy; smooth and erratic) in the following two Tables 5 and 6:

**Table 5.** Average forecasting errors for 9143 intermittent and 3932 lumpy products

Category			Intermittent products				Lumpy products			
Method	$\alpha$	$\beta$	sME	sMAE	MASE	sCE	sME	sMAE	MASE	sCE
CR	0.050	0.050	-1.105	2.698	1.883	3.803	-0.420	1.846	1.284	2.266
		0.100	-0.915	2.520	1.768	3.435	-0.342	1.774	1.236	2.117
		0.150	-0.840	2.448	1.721	3.287	-0.326	1.752	1.220	2.078
	0.100	0.050	-1.095	2.687	1.867	3.782	-0.398	1.819	1.243	2.216
		0.100	-0.907	2.512	1.755	3.419	-0.325	1.752	1.200	2.077
		0.150	-0.833	2.440	1.709	3.274	-0.312	1.733	1.186	2.045
	0.150	0.050	-1.089	2.681	1.857	3.770	-0.383	1.802	1.216	2.185
		0.100	-0.903	2.507	1.746	3.410	-0.313	1.737	1.175	2.050
		0.150	-0.830	2.437	1.701	3.267	-0.303	1.720	1.164	2.023
SBA	0.050	0.050	-1.055	2.653	1.849	3.708	-0.386	1.821	1.266	2.207
		0.100	-0.824	2.439	1.707	3.263	-0.278	1.728	1.202	2.006
		0.150	-0.709	2.331	1.632	3.040	-0.231	1.685	1.170	1.915
	0.100	0.050	-1.045	2.642	1.834	3.687	-0.364	1.795	1.226	2.159
		0.100	-0.817	2.430	1.694	3.247	-0.261	1.707	1.167	1.968
		0.150	-0.703	2.324	1.621	3.027	-0.218	1.667	1.138	1.885
	0.150	0.050	-1.040	2.636	1.824	3.675	-0.350	1.778	1.200	2.128
		0.100	-0.813	2.426	1.686	3.239	-0.250	1.693	1.144	1.943
		0.150	-0.700	2.320	1.614	3.021	-0.209	1.655	1.117	1.864
TSB	0.050	0.050	-0.029	1.616	0.999	1.645	-0.011	1.418	0.912	1.429
		0.100	0.096	1.482	0.906	1.578	-0.019	1.395	0.894	<b>1.414</b>
		0.150	0.097	1.461	0.893	1.558	-0.051	1.395	0.894	1.447
	0.100	0.050	-0.029	1.614	0.996	1.644	-0.030	1.426	0.914	1.456
		0.100	0.094	1.481	0.904	1.576	-0.042	1.404	0.898	1.446
		0.150	0.095	1.461	0.891	1.555	-0.075	1.405	0.898	1.481
	0.150	0.050	-0.030	1.613	0.994	1.644	-0.041	1.430	0.914	1.471
		0.100	0.092	1.482	0.903	1.574	-0.056	1.409	0.900	1.465
		0.150	0.092	1.461	0.891	<b>1.553</b>	-0.091	1.411	0.901	1.503
MA	0.050		0.162	1.473	0.889	1.635	0.049	1.419	0.904	1.468
	0.100		0.182	1.448	0.874	1.630	0.031	1.410	0.894	1.440
	0.150		0.084	1.525	0.921	1.609	-0.105	1.480	0.940	1.585
SES	0.050		-0.035	1.637	1.007	1.672	-0.028	1.447	0.924	1.475
	0.100		0.096	1.513	0.919	1.608	-0.039	1.437	0.915	1.476
	0.150		0.098	1.506	0.915	1.604	-0.075	1.453	0.926	1.528
ZF			0.898	<b>0.898</b>	<b>0.549</b>	1.797	0.947	<b>0.947</b>	<b>0.621</b>	1.893
min			-1.105	0.898	0.549	1.553	-0.420	0.947	0.621	1.414
max			0.898	2.698	1.883	3.803	0.947	1.846	1.284	2.266

**Note:** The best values are presented in bold print and the poorest values are in italics (for the last three error measures)

**Source:** Own computations.

Overall, in the case of intermittent and lumpy products, which accounted for 95% of all products, the conclusions are the same as before. Due to sCE, the most satisfactory forecasts were obtained for TSB and the poorest ones – for CR and

SBA. SES and MA provided better results than CR and SBA did. Intermittent items constituted 66.3% of all products. In terms of biasedness, the poorest results were the forecasts obtained for CR and SBA. These methods generated substantially overestimated forecasts. Forecasts for TSB, SES and MA featured only a slight bias. With respect to sMAE and MASE, the ZF method produced the most satisfactory forecast results. These error values were also low for TSB, SES and MA. Both sMAE and MASE were the highest for CR and SBA.

In the intermittent demand category, according to sCE, forecasts were the most favorable for TSB ( $\alpha = \beta = 0.15$ ). Other smoothing constants were also satisfactory in the case of TSB results. The highest values of sCE were related to CR, SBA and ZF. Contrary to the theoretical assumptions, such methods as SES and MA performed better than CR or SBA did. For intermittent items there was also a regularity of errors being smaller for higher values of  $\alpha$  and, especially, for  $\beta$ . For instance, for TSB with  $\alpha = 0.15$  and  $\beta$  equal to 0.05, 0.10, 0.15, an average scaled compound error (sCE) was decreasing and it was equal to 1.644, 1.574, 1.553. The changes are rather significant.

On the other hand, sCE for TSB with  $\beta = 0.10$  and  $\alpha = 0.05, 0.10, 0.15$  was equal to 1.578, 1.576, 1.574 and it was only slightly decreasing. Similar conclusions are true also for other methods and smoothing constants as well as for other error measures (sMAE and MASE). This relation proves that in the examined enterprise higher smoothing constants values occurred to be preferable. The company's offer features mostly slow-moving items, with numerous zero sales periods. High smoothing constants rendered the forecasts to quickly approach zero and becoming closer to empirical (usually zero) sales. Although, such a kind of obsolescence is typically temporary, therefore smoothing constants should not be exceedingly high.

Share of lumpy products was high and equal to 28.5%. Overall, with only a few exceptions, average forecast errors for lumpy products were lower than for intermittent items. It is especially true for CR and SBA, for which sMAE and MASE are much lower. In the case of other methods, the improvement was less evident. For lumpy products, on account of greater sales variability, higher errors were expected. Higher sales frequencies might be a possible explanation of better results for lumpy products. In the case of lumpy items average sales frequency was equal to  $\bar{p} = 0.30$ , while for intermittent items it was  $\bar{p} = 0.11$ . In the case of less intermittent items (items with higher sales frequency), lower forecasts errors are expected. On average, lumpy products in this enterprise were less intermittent.

For lumpy items forecast were also the most biased (overestimated) when applying the CR and SBA methods, yet the bias was lower than for intermittent category. The TSB and SES methods led to only a slight overestimation. In the case of MA, forecasts were weakly underestimated for  $\alpha = 0.05$  or  $\alpha = 0.10$  and overestimated for  $\alpha = 0.15$ . As before, sMAE and MASE were the lowest for ZF and the highest

for CR and SBA. With respect to these measures, the TSB method resulted in quite satisfactory forecasts. SES and MA were performing only slightly worse than TSB. In terms of sCE the forecasts obtained by means of TSB ( $\alpha = 0.05, \beta = 0.10$ ) demonstrated the best results. SES and MA outperformed ZF, CR or SBA. For lumpy items, an increase of smoothing constants did not diminish forecast errors, contrary to intermittent products. Average errors for smooth and erratic products are presented in the following Table 6:

**Table 6.** Average forecasting errors for 139 smooth and 569 erratic products

Category			Smooth products				Erratic products			
Method	$\alpha$	$\beta$	sME	sMAE	MASE	sCE	sME	sMAE	MASE	sCE
CR	0.050	0.050	-0.164	0.649	0.865	0.812	-0.118	0.818	0.848	0.936
		0.100	-0.172	0.650	0.867	0.823	-0.122	0.819	0.850	0.941
		0.150	-0.178	0.652	0.869	0.830	-0.126	0.820	0.851	0.947
	0.100	0.050	-0.167	0.648	0.862	0.815	-0.108	0.810	0.841	0.918
		0.100	-0.176	0.649	0.865	0.825	-0.112	0.811	0.842	0.923
		0.150	-0.182	0.651	0.867	0.833	-0.117	0.813	0.844	0.929
	0.150	0.050	-0.173	0.652	0.869	0.825	-0.111	0.812	0.843	0.923
		0.100	-0.182	0.654	0.871	0.836	-0.114	0.813	0.844	0.927
		0.150	-0.188	0.656	0.874	0.845	-0.119	0.814	0.846	0.934
SBA	0.050	0.050	-0.141	0.640	0.852	0.781	-0.095	0.808	0.839	0.903
		0.100	-0.126	0.633	0.842	0.759	-0.074	0.800	0.830	0.875
		0.150	-0.108	0.626	0.832	<b>0.734</b>	-0.055	0.793	0.823	0.848
	0.100	0.050	-0.144	0.639	0.850	0.783	-0.085	0.801	0.831	0.886
		0.100	-0.129	0.632	0.840	0.761	-0.065	0.793	0.823	0.858
		0.150	-0.111	<b>0.625</b>	<b>0.831</b>	0.737	-0.046	<b>0.786</b>	<b>0.816</b>	<b>0.832</b>
	0.150	0.050	-0.150	0.643	0.856	0.794	-0.088	0.803	0.833	0.890
		0.100	-0.135	0.637	0.847	0.772	-0.068	0.795	0.825	0.862
		0.150	-0.118	0.629	0.837	0.747	-0.049	0.787	0.817	0.836
TSB	0.050	0.050	-0.188	0.651	0.869	0.839	-0.142	0.815	0.846	0.957
		0.100	-0.195	0.653	0.872	0.848	-0.146	0.815	0.846	0.961
		0.150	-0.200	0.656	0.874	0.856	-0.151	0.815	0.846	0.966
	0.100	0.050	-0.209	0.654	0.871	0.863	-0.156	0.809	0.840	0.964
		0.100	-0.217	0.656	0.874	0.873	-0.161	0.809	0.841	0.969
		0.150	-0.222	0.658	0.877	0.880	-0.166	0.809	0.841	0.976
	0.150	0.050	-0.233	0.660	0.882	0.893	-0.181	0.813	0.845	0.994
		0.100	-0.241	0.663	0.885	0.904	-0.187	0.813	0.846	1.000
		0.150	-0.247	0.665	0.888	0.911	-0.193	0.814	0.847	1.007
MA	0.050		-0.165	0.651	0.868	0.816	-0.120	0.833	0.868	0.953
	0.100		-0.144	0.638	0.847	0.782	-0.068	0.798	0.826	0.866
	0.150		-0.230	0.672	0.897	0.902	-0.154	0.831	0.864	0.986
SES	0.050		-0.167	0.648	0.863	0.816	-0.113	0.816	0.846	0.929
	0.100		-0.178	0.649	0.864	0.827	-0.111	0.810	0.842	0.921
	0.150		-0.188	0.654	0.872	0.842	-0.119	0.815	0.847	0.934
ZF			0.756	0.756	0.993	1.512	0.823	0.823	0.870	1.647
min			-0.247	0.625	0.831	0.734	-0.193	0.786	0.816	0.832
max			0.756	0.756	0.993	1.512	0.823	0.833	0.870	1.647

**Note:** The best values are presented in bold print and the poorest values are in italics (for the last three error measures)

**Source:** Own computations.

Differently than before, in the case of smooth and erratic items, the best forecasts were obtained by means of the SBA and CR methods in terms of sCE. TSB performed more poorly than SES and MA did.



Only 139 smooth items were recorded. In this product category forecasts errors were much lower than for intermittent or lumpy category. As indicated before, high sales frequencies improved the forecasting results. Furthermore, differences between methods became less apparent. Except ZF, all the methods resulted in overestimated forecasts for all the analyzed cases. Obviously, ZF were underestimated. Differently from previous cases, biasedness reached its greatest values for TSB method. The least biased methods proved to be SBA and CR. This finding is contrary to the theory and it might come as a surprise because SBA and CR are dedicated to intermittent or lumpy products. The methods recommended for fast-moving items (SES, MA) yield a higher bias than SBA or CR.

Due to sMAE and MASE, the forecasts obtained by means of SBA ( $\alpha = 0.10, \beta = 0.15$ ) were the best performing. In turn, ZF occurred to be the worst performing method. According to sCE, SBA ( $\alpha = 0.05, \beta = 0.15$ ) was also performing highly favorably. All error measures indicated the ZF and TSB methods as the least satisfactory options. CR, SES and MA performed very similarly and slightly better than TSB. The number of erratic items also remained very low. Only 569 products fell into this category. The relations between forecasting error measures were similar as in the case of smooth items.

Overall, for erratic items all methods, except ZF, led to overestimated forecasts. The values of an average sME were negative. The least bias was attributed to the SBA method. The most overestimated results were the forecasts obtained through the TSB and MA methods. Due to biasedness, CR and SES were yielding very similar results.

According to sMAE and MASE, SBA ( $\alpha = 0.05, \beta = 0.15$ ) was providing the most satisfactory effects. Overall, the SBA method was also the best performing in the erratic category. The poorest forecasts due to sMAE were obtained for MA with  $\alpha = 0.05$ , but an average sMAE for ZF was close to that value. According to MASE, ZF produced the worst results. The indications of sCE are similar. The best forecasts were achieved for SBA ( $\alpha = 0.10, \beta = 0.15$ ), while the worst ones were for ZF and TSB. Such methods as CR, SES and MA operated very similarly. The changes of smoothing constants were not clearly related to the examined errors. In the case of smooth or erratic items, forecast errors like sMAE and MASE worked properly and led to similar conclusions as sCE.

## 5. Summary and Concluding Comments

The main aim of the article was to find the best forecasting method for the analyzed company. After a preliminary classification with respect to weekly sales frequency and the coefficient of variation for a demand size, it was established that 95% of the company's products belong to the intermittent or lumpy category. Therefore, in the considered enterprise intermittence is a crucial forecasting issue. Three forecasting



methods dedicated to intermittent demand were verified (Croston's, SBA, TSB). The methods such as SES, MA and zero forecasts served as benchmarks.

Zero forecasts were employed to demonstrate that in the case of intermittent or lumpy items popular forecasting error measures are insufficient for selecting the best forecasting method. Forecast accuracy measures like sMAE or MASE led to the conclusion that zero forecasts are best, which is unacceptable due to consumer service level. Those kinds of errors favor lower forecasts because an analysis of intermittent or lumpy demand involves many zeros. Still, the methods with lower forecast levels are biased as well.

In this article it was emphasized that forecasting errors considering biasedness should be considered. Therefore, a new forecasting error measure was proposed, which was named scaled Compound Error (sCE). It considers not only forecasts precision, but also biasedness, which is important especially with respect to consumer service level. It appeared that sCE, like sMAE or MASE, indicated the same forecasting methods as worst. However, sMAE or MASE, contrary to sCE, favored zero forecasts.

Forecasts accuracy was verified for all products and for specified groups of items. In the case of intermittent and lumpy items, the TSB method was proven to be the recommended. What is interesting, such methods as Croston's and SBA led to highly overestimated forecasts and performed worse than SES or MA. These conclusions are contradictory to what is found in theory. The Croston's or the SBA methods are dedicated to intermittent demand and are expected to be superior to SES or MA. Therefore, empirical verification is crucial if a forecasting system in the enterprise is developed.

A question might be posed, why the Croston's type methods perform more poorly than expected? The general explanation is that in these methods smoothed values are not updated for periods with no demand. Sales levels and demand intervals are not decreased in the periods with no demand, which renders forecasts to be overestimated. In the TSB method, if sales are equal to zero, the forecast level is decreasing. It also holds true for SES or MA. Therefore, the Croston's and the SBA methods could not handle obsolesces.

The conclusions are quite different for smooth and erratic items. In these groups the SBA method worked best, and the TSB method proved to perform most poorly. SBA was also the most unbiased method. It was also surprising, because, from the theoretical perspective, forecasts for fast-moving items computed by SBA should be underestimated by the factor  $\beta/2$ .

In summary, in the considered enterprise the forecasting system should be based on the scaled Compound Error (sCE) and the TSB method with smoothing constant values within the range of 0.10–0.15. In the future research the presented analysis

will be repeated for different data sets, to verify the obtained results.

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