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The Analysis of Some Trading Strategy on the Stock Market with the Liquidity Shortage

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Abstract:

Purpose: *The aim of this article is to determine and analyze the optimal trading strategy of purchasing a given number of the stock shares with the constraint of the constant trade velocity when the trading is executed and with the possibility of the liquidity shortage of the market.*
Design/Methodology/Approach: *The market stock price is modelled as a stochastic process with the trend which is characterized with use of functions desribed by two parameters and this seems to induce the flexibility availaible with respect to fitting the model parameters to the stock price evolution forecasts. The considered optimization problem is posed with the use of the theory of probability. The mathematical analysis is applied to obtain the optimal trading strategy. The exemplary results of the numerical computations, with the use of the formulas shown in the article, are included.*
Findings: *The optimal strategy of purchasing the stock shares in the considered set of trading strategies is determined and the significance of the trend in the stock market stock price is shown.*
Practical Implications: *The shortage of the liquidity in the stock market implies the transaction costs which are induced by the market impact. The profitability of the transaction may be also affected by the trend in the stock price. The optimization of trading on the stock market may imply the increase of the profitability of investing on the stock market.*
Originality/Value: *The results shown in the article are original and can be applied by the stock market participants who implement the trading strategies with the constant speed of the trade execution, to increase the expected profit of the investment in the stock shares.*

Keywords: *Liquidity shortage, market impact, transaction cost, stock price.*

JEL classification: *C6, G11.*

Paper Type: *Research paper.*

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1. Introduction

Understanding and measuring transaction costs is important in theory and practice of finance. Transaction costs are recognized as a substantial determinant of the financial investment performance. Moreover, the level of transaction costs can be a characteristic of the stock market liquidity. Transaction costs may significantly lower the return attained by application of the financial investment strategy. The major sources of transaction costs which may influence the profitability of financial investments are commissions (and similar payments), bid-ask spreads and the market impact (Elton *et al.*, 1999; Sharpe *et al.*, 1999). Broker commissions are explicit costs of trading. They are usually easy to evaluate before the start of the trade and therefore they do not generate the financial risk. In this article, the commissions and similar explicit transaction costs paid by the investor are not considered in determining the transaction cost of the purchase of the shares of the stock.

The market impact (also called the price impact) in the stock market, is the change in the stock price generated by the transaction. The level of the price impact is one of the measures of the liquidity shortage of the market. The market impact occurrence is disadvantageous for the initiator of the trade, the price rises when the trade initiator is a buyer and the price drops when the market participant initiating the trade is a seller. Thus, the price impact is one of the sources of the transaction costs and may significantly worsen the performance of the investment strategy. Market impact on the financial market may by moving the price unfavorably for the trade initiator turn the investment strategy into a financial failure. Some examples showing how important is market impact are: the fiasco of Metallgesellschaft in 1993 and the cancelling of the portfolio of Jérôme Kerviel by Societé Générale in 2008 (Schied and Slynko, 2011). The transaction cost induced by the market impact is a nondecreasing function of the volume of the trade. The dependence of the price impact on the trade volume was considered for example in (Bouchaud, 2009) and (Kociński, 2015). A significant factor influencing the market impact can be a speed of the trade.

The dependence of the market impact on the trade velocity was studied for example in (Kociński, 2018). The problem of the accurate assessment of the price impact of the planned transaction is important theoretically and practically. It seems that taking market impact into account in the analysis of the investment strategies may be profitable. The possibility of generation of the pre-trade estimate of the transaction cost as the function of the size of the planned trade can be an advantage of the computer program analyzing the financial data to support the optimization of the investment strategy of the participant of the stock market. It seems that the market impact assessment software can be an important tool for the investors on the stock markets (Gatheral, 2010). In practice of the financial management, it is important to verify whether the coefficients and even the functional form of the market impact formula reflect the current stock market data. The empirical analysis of the price impact is described for example in (Zarinelli *et al.*, 2014).

Another important source of the transaction costs is the bid-ask spread. The bid-ask spread can be described as the difference between the ask price and the ask price of the stock share. It seems that the bid-ask spread can be considered as one of the measures of the stock market liquidity (Anand and Karagozoglu, 2006). The problem of estimation of the bid-ask spread is considered for example in (Corwin and Schultz, 2012).

In the model considered in the article the transaction cost is implied by the bid-ask spread and the payment arising from the market impact. The price impact cost per one unit of the stock is modelled as the product of the non-negative constant β and the velocity of the trade execution. Thus, if $\beta > 0$, then reducing the speed of the execution of trading, decreases the cost which is a consequence of the market impact.

However, the result of the trading strategy may be significantly influenced by the trend in the stock price. This trend may be the result of the aggregate trading activity on the stock market in case of the appearing of the information about the stock investment profitability. For example, the positive trend in the stock price may be the effect of the announcement that the profitability of the company that emitted the stock shares is higher than expected by the participants of the stock market. The negative trend in the evolution of the price of the stock may be for example the effect of the statement that the stock share dividend will be lower than forecasted by the investors on the stock market. The trend in the article is described by the nondecreasing function describing the dependence of the expected stock price on time and this function can be fitted to the expectation regarding the stock price process. The optimization of the execution of the trade in the market model with the price impact was considered for example in (Almgren and Chriss, 2000; Kociński, 2018).

The purpose of the article is to determine the trading strategy which minimizes the expected amount of money spent for purchasing X stock shares, where X is the non-negative parameter, with the constraint that the speed of the implementation of the strategy of purchasing the shares of the stock is constant. Moreover, the purpose of this article is to determine the expected value and the standard deviation of the amount of money spent by implementing the considered optimal trading strategy. In the article there also numerical computations where the importance of the shape of trend in the stock price is shown.

2. The Spread and the Market Impact as the Sources of the Transaction Cost

One of the important characteristics of a market is the type of its execution system. In this respect there are three major types of markets, quote driven markets, order driven markets, and brokered markets. The spread and the market impact are important characteristics of trading on the stock market and in case of the order driven market the transaction cost induced by the spread and the market impact can be assessed by the order book. the order book is an important element of the order driven market. Table 1 shows the exemplary first five rows of the order book for the stock from the order driven stock market at some point in time during the trading session.

Table 1. *The first five rows of the order book for the shares of the exemplary stock at some moment during the trading day.*

Offers to buy		Offers to sell	
Size	**Price**	**Price**	**Size**
1500	5.4	5.5	2400
1200	5.38	5.55	2500
2000	5.36	5.56	2500
2500	5.32	5.58	4200
3000	5.31	5.61	4500

Source: *Own elaboration.*

The second row of Table 1 shows the number of the stock shares to sell at the bid price (1500), the bid price (5.4), the ask price (5.5) and the number of the stock shares to buy at the ask price (2400). The market price of the considered stock price is computed as the arithmetic average of the bid and ask price end equals 5.45. The bid-ask spread in this case equals 0.1.

Consider an investor on the stock market who at time 0 places market order to buy X stock shares. If the stock market was frictionless then, the amount of money spent by this investor for the stock purchase would be 5.45 per share of the stock. Denote by x_i the number of the stock shares purchased for the price S_i while executing the considered market order and let n symbolize the number of the values of the stock price occurring while implementing this market order. The transaction cost for the considered purchase of X shares of the stock is given by the formula:

$$\sum_{i=1}^{n} x_i (S_i - S_0),$$

where S_0 the stock market price at time 0 and the transaction cost per share of the stock share is given for the considered transaction as follows:

$$\frac{\sum_{i=1}^{n} x_i (S_i - S_0)}{X}.$$

The transaction cost induced by the market impact per share of the stock is given by the following formula:

$$\frac{\sum_{i=1}^{n} x_i (S_i - S_0^a)}{X}.$$

where S_0^a is the ask price at time 0.

The calculation of the transaction costs per share of the stock and the component of the transaction cost induced by the price impact per share of the stock for the transactions of purchase of the stock in case of the order book described in Table 1 is shown in Table 2.

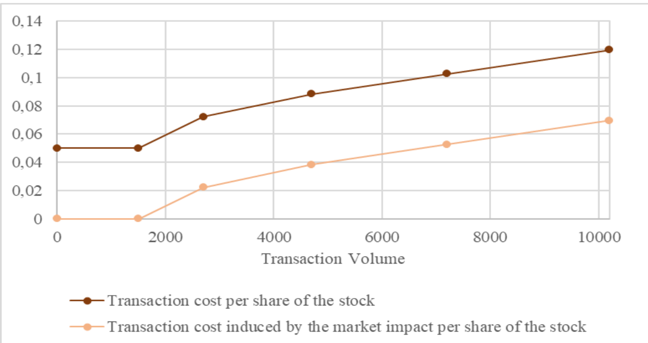
Table 2. *The transaction cost per one stock share for the exemplary stock.*

Trading size	Transaction cost per share of the stock	Transaction cost induced by the market impact per share of the stock
1500	0.05	0.00
2700	0.07	0.02
4700	0.09	0.04
7200	0.10	0.05
10200	0.12	0.07

Source: *Own elaboration and Table 1.*

In the considered example the transaction cost per unit of the stock increases with the increase of the transaction volume. The graph of the approximations of the transaction costs per share of the stock and the transaction costs induced by the market impact per share of the stock for the transactions of purchase of the stock in case of the order book described in Table 1 is shown in Figure 1.

Figure 1. *The approximated transaction cost per share of the stock for the exemplary stock.*



Source: *Own elaboration and Table 2.*

3. The Model of the Stock Market

Let S_t symbolize the market price of a share of the stock at time t. The process of the stock market price is given as follows:

$$S_t = S_0\left(1 + at^b + \sigma W_t\right) \text{ for } 0 \le t \le T, \tag{1}$$

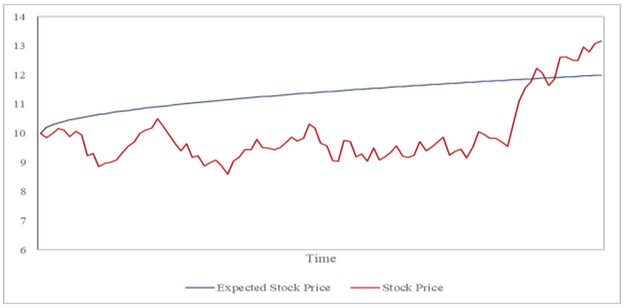
where $a > 0$, $b \ge 0$, $\sigma \ge 0$ and W_t is the value of the Wiener process at time t. Theoretically modelling the stochastic dynamics of the stock price by (1) implies the possibility of the negative stock price. However, it seems that in the case when the time horizon of the trading strategy is one day and the parameter σ is fitted to the stock price data from the stock exchange, it is likely that the probability of the negative stock price during the trade execution can be considered as sufficiently small in practical application of the considered model.

By the definition of the Wiener process it follows that the expected value of W_t is 0 for each $0 \leq t \leq T$ that is:

$$E(W_t) = 0 \quad \text{for } 0 \leq t \leq T , \tag{2}$$

The exemplary picture of the trajectory of the stock price with the expected stock price as the function of time is shown in Figure 2.

Figure 2. *The exemplary graphic of the trajectory of the stock price and the expected stock price.*



Source: *Own elaboration.*

The parameters a and b can be determined be means of the expectation of the stock price evolution in the time interval $(0, T]$. If there are given two expected stock prices $E(S_{t_1})$ and $E(S_{t_2})$ at the moments t_1 and t_2, respectively such that $0 < t_1 \leq T$ and $0 < t_2 \leq T$ then the pair (a, b) is given as the solution of the following system of equations:

$$\begin{cases} E(S_{t_1}) = S_0(1 + at_1^b) \\ E(S_{t_2}) = S_0(1 + at_2^b) \end{cases} \tag{3}$$

By (3) the following formula for b is obtained:

$$b = \frac{\ln\left(\frac{E(S_{t_2})}{S_0} - 1\right) - \ln\left(\frac{E(S_{t_1})}{S_0} - 1\right)}{\ln(t_2) - \ln(t_1)}$$

For a given value of the parameter b the value of the coefficient a can be calculated as follows:

$$a=\left(\frac{E(S_t)}{S_0}-1\right)\frac{1}{t^b}\ \text{for}\ 0\le t\le T$$

Notice that the expected stock prices $E(S_{t_1})$ and $E(S_{t_2})$ can be determined by the values of the expected returns on the stock over the time intervals $[0,t_1]$ and $[0,t_2]$. Consider the example such that $T=1$ and it is expected the expected stock price over the half of the period $[0,T]$will increase by 20% and at the moment T will be 30% percent higher than the initial price S_0. Then the parameter a equals 0.3 and the value of the parameter b can be evaluated as 0.58. The given value of the parameter a is equal to the expected value of the stock price at time 1. For the given value of a the parameter b characterizes the shape of line of the stock price trend and this shape may be important on optimization of trading on the stock market.

Consider the participant of the stock market who intends to buy X stock shares in the interval $(0,T)$ and wants to minimize the amount of money spent for the stock purchase. It is assumed that the velocity of the market participant's purchase execution is constant in the time interval where it is positive. The assumption of the constant speed of the purchase simplifies the considered minimization problem. Moreover, it seems that the effect of the constraint of the constant velocity of the trade, on the market where the realized trading velocity can be significantly different from speed of the trade planned by the market participant, can be moderate in practice.

It is easily seen that in the considered model the optimal execution of the stock buying starts at time 0. Thus, the considered minimization can be implemented over the set of strategies with the trade execution starting at time 0. Denote by v_φ the velocity of trade execution for the strategy φ with trading starting now 0. Let t_φ be the moment defined as follows:

$$t_\varphi=\frac{X}{v_\varphi} \tag{4}$$

It is easily seen that t_φ is the measure of the duration of the execution of the strategy φ. Denote by Φ the set of the purchase strategies with the trading starting at time 0 and the constant execution speed in the time interval where it is positive. Denote by S_t^φ the trade price at time t when applying the strategy φ. It is assumed that the trade price S_t^φ for the strategy φ is given as follows:

$$S_t^\varphi=S_t+S_0\left(\alpha+\beta v_\varphi\right) \tag{5}$$

where α and β are the nonnegative constants.

If the transaction volume is X then, the transaction costs induced by the market impact equals $\beta S_0 X v_\varphi$. Notice that for a fixed value t_φ the market impact cost is a linear function of the volume of the transaction.

Let $C(\varphi)$ denote the amount of money spent by applying the strategy φ. The number of the stock shares purchased by executing the strategy φ from the set Φ at an infinitesimal time interval dt from the time interval $(0, t_\varphi)$ equals $v_\varphi dt$. Thus, the following formula holds:

$$C(\varphi) = v_\varphi \int_0^{t_\varphi} S_t^\varphi dt \tag{6}$$

By (6) the expected value of $C(\varphi)$ is given by as follows:

$$E(C(\varphi)) = v_\varphi \int_0^{t_\varphi} E(S_t^\varphi) dt \tag{7}$$

By (1), (2) and (6) the standard deviation of $C(\varphi)$ is given by the following formula:

$$SD(C(\varphi)) = \sigma v_\varphi \sqrt{E\left(\left(\int_0^{t_\varphi} W_t^2 dt\right)^2\right)} \tag{8}$$

By (4) and (8) it follows that

$$SD(C(\varphi)) = \sigma X \sqrt{\frac{1}{3} t_\varphi} \tag{9}$$

4. The Optimization of the Purchase of the Stock Shares

Consider the strategy φ from the set Φ. By (1), (2) and (5) it follows that

$$v_\varphi \int_0^{t_\varphi} E(S_t^\varphi) dt = S_0 v_\varphi \left(\int_0^{t_\varphi} (1 + a t^b) dt + (\alpha + \beta v_\varphi) t_\varphi \right) \tag{10}$$

Thus, by (4) and (10) it is obtained that

$$E(C(\varphi)) = S_0 X \left(1 + \frac{a}{b+1} t_\varphi^b + \alpha + \beta \frac{X}{t_\varphi} \right) \tag{11}$$

Let the function ψ of the variable t be defined as follows:

$$\psi(t) = S_0 \left(X + \frac{a}{b+1} t^b X + \left(\alpha + \beta \frac{X}{t} \right) X \right) \tag{12}$$

Notice that

$$\psi(t_\varphi) = E(C(\varphi)). \tag{13}$$

Denote the derivative of ψ with respect to t by $\psi^{'}$. The function $\psi^{'}$ is given as follows:

$$\psi^{'}(t) = S_0 X\left(\frac{ab}{b+1}t^{b-1} - \beta\frac{X}{t^2}\right).$$

Denote by t^* the minimum of the function ψ over the interval $[0,T]$.

By the following properties of $\psi^{'}$:

$$\psi^{'}(t) < 0 \text{ for } t \in \left(0, \left(\frac{\beta(b+1)X}{ab}\right)^{\frac{1}{b+1}}\right)$$

$$\psi^{'}(t) = 0 \text{ for } t = \left(\frac{\beta(b+1)X}{ab}\right)^{\frac{1}{b+1}}$$

and

$$\psi^{'}(t) > 0 \text{ for } t > \left(\frac{\beta(b+1)X}{ab}\right)^{\frac{1}{b+1}}$$

the value of t^* is determined by the following equality:

$$t^* = \min\left(T, \left(\frac{\beta(b+1)X}{ab}\right)^{\frac{1}{b+1}}\right) \tag{14}$$

Denote by φ the strategy minimizing $E(C(\varphi))$ over the strategies from the set Φ. By (13) and (14) it follows that:

$$t_{\varphi^*} = \min\left(T, \left(\frac{\beta(b+1)X}{ab}\right)^{\frac{1}{b+1}}\right) \tag{15}$$

5. The Numerical Example

In this section, in the numerical computations, it is assumed that, $\alpha = 0.01$, $\beta = 10^{-5}$ $a = 0.3$, $S_0 = 1$, $\sigma = 0.2$ and $T = 1$. Thus, in the numerical example, the market model is considered with the transaction cost implied from the bid-ask spread and resulting from trading one unit of the stock equal to 0.01 of the initial market price of the stock share, the market impact cost resulting from trading the stock volume equal to 10000 equals 0.1 of the initial market price of the stock share, the average increase in the market price of the share of the stock over the time interval $[0,T]$ equals $0.3T^b S_0$, the standard deviation of the market price of the stock over the time interval

$[0,T]$ is equal $0.2S_0\sqrt{T}$ and the parameter T equals 1 (for example one day). The considered choice of values of the parameters of the model in the numerical example is one of the reasonable choices to the exemplary calculations. In Table 3 there are computed by the formula (15) the values of the duration of the strategy from the set Φ which minimizes the value of $E(C(\varphi))$.

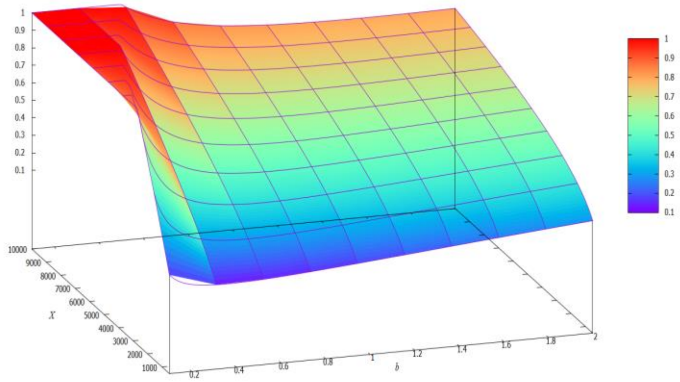
Table 3. *The values of the duration t_{φ^*} of the trading strategy φ^* which minimizes the value of $E(C(\varphi))$ over the set Φ.*

		b										
		0.2	**0.4**	**0.6**	**0.7**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
X	**500**	0.15	0.13	0.14	0.15	0.16	0.18	0.20	0.23	0.25	0.27	0.29
	1000	0.26	0.22	0.22	0.23	0.24	0.26	0.28	0.30	0.33	0.35	0.37
	1500	0.37	0.29	0.28	0.29	0.30	0.32	0.34	0.36	0.38	0.40	0.42
	2000	0.47	0.35	0.34	0.34	0.35	0.37	0.38	0.41	0.43	0.45	0.46
	2500	0.56	0.41	0.39	0.39	0.39	0.41	0.43	0.44	0.46	0.48	0.50
	3000	0.65	0.47	0.44	0.43	0.44	0.45	0.46	0.48	0.50	0.51	0.53
	3500	0.74	0.53	0.48	0.48	0.48	0.48	0.50	0.51	0.53	0.54	0.56
	4000	0.83	0.58	0.52	0.52	0.51	0.52	0.53	0.54	0.56	0.57	0.58
	4500	0.92	0.63	0.56	0.55	0.55	0.55	0.56	0.57	0.58	0.59	0.61
	5000	1.00	0.68	0.60	0.59	0.58	0.58	0.58	0.59	0.61	0.62	0.63
	5500	1.00	0.73	0.64	0.62	0.61	0.61	0.61	0.62	0.63	0.64	0.65
	6000	1.00	0.78	0.68	0.65	0.64	0.63	0.63	0.64	0.65	0.66	0.67
	6500	1.00	0.82	0.71	0.69	0.67	0.66	0.66	0.66	0.67	0.68	0.69
	7000	1.00	0.87	0.74	0.72	0.70	0.68	0.68	0.68	0.69	0.70	0.70
	7500	1.00	0.91	0.78	0.75	0.73	0.71	0.70	0.70	0.71	0.71	0.72
	8000	1.00	0.95	0.81	0.77	0.75	0.73	0.72	0.72	0.72	0.73	0.74
	8500	1.00	0.99	0.84	0.80	0.78	0.75	0.74	0.74	0.74	0.75	0.75
	9000	1.00	1.00	0.87	0.83	0.80	0.77	0.76	0.76	0.76	0.76	0.77
	9500	1.00	1.00	0.90	0.86	0.83	0.80	0.78	0.78	0.77	0.78	0.78
	10000	1.00	1.00	0.93	0.88	0.85	0.82	0.80	0.79	0.79	0.79	0.79

Source: *Own elaboration and Table 1.*

It is easily seen that in Table 3, the value of t_{φ^*} for a fixed value of b increase with the increase of the parameter X. However, the monotoicity of t_{φ^*} as the function of b for a fixed value of X depends on the volume of the transaction. In Figure 3 it is shown how the value of t_{φ^*} depends on b and X. Notice that in spite of the positive trend in the stock price, the duration period of the trade execution of the strategy φ^* may, for sufficiently large size of the transaction on the stock market, be considered as relatively long. Table 4 contains the results of computing by the formula (11) the value of the expected amount of money spent by applying the strategy φ^*.

Figure 3. *The values of* t_{φ^*} *depending on* b *and* X .



Source: *Own elaboration.*

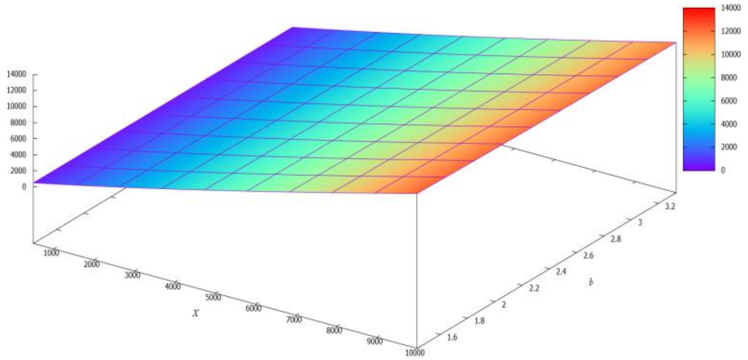
Table 4. *The values of* $E(C(\varphi^*))$

		b						
		0.25	**0.5**	**0.75**	**1**	**1.25**	**1.5**	**1.75**
X	**500**	596.25	560.26	542.30	532.39	526.38	522.47	519.77
	1000	1219.65	1149.25	1110.41	1087.46	1072.85	1062.96	1055.93
	1500	1856.04	1754.10	1694.20	1657.30	1633.10	1616.32	1604.17
	2000	2501.64	2370.88	2290.28	2239.09	2204.75	2180.55	2162.77
	2500	3154.53	2997.47	2896.76	2831.19	2786.42	2754.44	2730.70
	3000	3813.50	3632.49	3512.36	3432.49	3377.14	3337.15	3307.20
	3500	4477.70	4274.97	4136.19	4042.20	3976.21	3928.07	3891.73
	4000	5146.53	4924.17	4767.54	4659.68	4583.07	4526.69	4483.85
	4500	5819.52	5579.52	5405.86	5284.43	5197.27	5132.62	5083.20
	5000	6496.29	6240.55	6050.70	5916.03	5818.43	5745.52	5689.47
	5500	7176.53	6906.88	6701.66	6554.12	6446.23	6365.10	6302.40
	6000	7860.00	7578.18	7358.43	7198.42	7080.41	6991.11	6921.76
	6500	8547.50	8254.17	8020.72	7848.65	7720.71	7623.33	7547.36
	7000	9240.00	8934.60	8688.29	8504.57	8366.92	8261.56	8179.02
	7500	9937.50	9619.26	9360.92	9165.99	9018.85	8905.63	8816.56
	8000	10640.00	10307.96	10038.40	9832.71	9676.33	9555.37	9459.85
	8500	11347.50	11000.53	10720.58	10504.57	10339.20	10210.65	10108.76
	9000	12060.00	11696.82	11407.27	11181.41	11007.32	10871.34	10763.16
	9500	12777.50	12396.69	12098.35	11863.09	11680.55	11537.30	11422.94
	10000	13500.00	13100.00	12793.67	12549.49	12358.77	12208.42	12087.99

Source: *Own elaboration.*

It is easily seen that in Table 4, the value of $EC(\varphi^*)$ for a fixed value of X decrease with the increase of the parameter b. Consequently, the increase in the value of the parameter b may be advantageous for the stock market participant with respect to the expected amount of money spent for purchase the stock shares. The graph of $EC(\varphi^*)$ as the function of X and b is shown in Figure 4.

Figure 4. *The values of $EC(\varphi^*)$ depending on b and X*



Source: *Own elaboration.*

Table 5 contains the results of computing by the formula (9) the value of the standard deviation of the amount of money spent by applying the strategy φ^*.

Table 5. *The values of $SD(C(\varphi^*))$.*

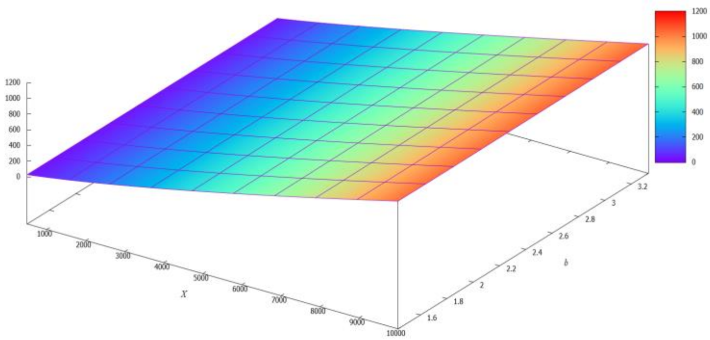
		b						
		0.25	**0.5**	**0.75**	**1**	**1.25**	**1.5**	**1.75**
X	**500**	21.37	21.27	22.83	24.67	26.49	28.20	29.77
	1000	56.39	53.60	55.66	58.67	61.79	64.78	67.54
	1500	99.48	92.03	93.75	97.40	101.43	105.37	109.07
	2000	148.82	135.05	135.71	139.55	144.17	148.82	153.23
	2500	203.39	181.85	180.80	184.45	189.37	194.51	199.47
	3000	262.53	231.90	228.57	231.66	236.64	242.08	247.44
	3500	325.76	284.81	278.67	280.89	285.71	291.27	296.88
	4000	392.73	340.32	330.86	331.91	336.36	341.89	347.63
	4500	463.13	398.19	384.96	384.56	388.43	393.79	399.55
	5000	536.74	458.24	440.81	438.69	441.82	446.87	452.53
	5500	613.36	520.34	498.27	494.20	496.40	501.01	506.48
	6000	692.82	584.35	557.25	550.98	552.10	556.16	561.34

	6500	750.56	650.16	617.65	608.96	608.85	612.22	617.03
	7000	808.29	717.68	679.40	668.07	666.57	669.16	673.51
	7500	866.03	786.84	742.42	728.24	725.21	726.92	730.73
	8000	923.76	857.54	806.65	789.42	784.74	785.46	788.64
	8500	981.50	929.74	872.04	851.57	845.09	844.73	847.22
	9000	1039.23	1003.37	938.54	914.64	906.24	904.70	906.43
	9500	1096.97	1078.37	1006.11	978.59	968.15	965.35	966.24
	10000	1154.70	1154.70	1074.70	1043.39	1030.79	1026.63	1026.62

Source: *Own elaboration.*

From Table 5 it can be noticed that the monotonicity of $SD(C(\varphi^*))$ as the function of the parameter b for a fixed value of the transaction volume X depends on the transaction volume and it is easily seen that in Table 5 standard deviation of the amount of money spent by applying the strategy φ^* for a fixed value of the parameter b increase with the increase of the transaction volume X. In Figure 5 it is shown how the value of $SD(C(\varphi^*))$ depends on X and b.

Figure 5. *The values of* $SD(C(\varphi^*))$ *depending on* b *and* X



Source: *Own elaboration.*

6. Conclusions

In the article the model with the liquidity shortage and the stock price trend which is characterized with the use of functions with two parameters is considered. In a framework of this model, the strategy of buying the shares stock minimizing the expected amount of money spent for the stock purchase is determined over the set buying strategies with the constant execution velocity in the time interval where it is positive.

It is shown that the considered optimal strategy φ^* depends on expected value of the market stock price a at the moment which is the time horizon T for the purchase of X stock shares, the shape of the considered trend, the transaction volume, and the value of T From the numerical computations included in the article it can be concluded that the volume of the transaction and the shape of the trend in the stock price may significantly influence the profitability and the risk of investing in the stock market.

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