

**Vol. 12, No. 1, 2018 Special Issue
Mathematics Education in Malta**

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Special Issue*Mathematics Education in Malta***Editorial****Michael A. Buhagiar**

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Broadly speaking, mathematics education aims “to study the factors affecting the teaching and learning of mathematics and to develop programmes to improve the teaching of mathematics” (Godino, Batanero, & Font, 2007, p. 127). As a relatively recent scientific discipline, mathematics education lacks a consolidated and dominant research paradigm. As a result, as Sriraman and English (2010) note, there have been frequent shifts in the dominant paradigm. They point out the progressive shifts from behaviourism, through to stage and level theories, to various forms of constructivism, to situated and distributed cognitions, and more recently, to complexity theories and neuroscience. Sriraman and English explain that for the first couple of decades of its life, mathematics education as a discipline drew heavily on theories and methodologies from psychology. However, by the end of the 1980s, as researchers began to focus on the social dimension of learning, theories that view mathematics as a social product began to be used, and thus, socio-cultural theories became more dominant. Sriraman and English argue that one plausible explanation for these shifts is the diverging epistemological perspectives about what constitutes mathematical knowledge; another possible explanation proposed by Sriraman and English is that mathematics education is heavily influenced by unpredictable cultural and political forces.

This drawing on a variety of theoretical approaches contributes to the progress and the richness of the discipline (Godino et al., 2007). In our view, the collection of five research papers and a commentary presented in this special issue of *Malta Review of Educational Research* (MRER) gives testimony of this diversity. For sure, the reader gets some indication of the varied research interests and research methodologies that are currently being explored in the field of mathematics education in Malta. However, although this issue aims primarily to generate ideas and discussions among readers with special interest in mathematics education, we are very much aware that MRER has a much wider audience, both locally and internationally. In view of this, we decided to favour topics that have the potential for the widest possible educational appeal. As will become clearer once we refer briefly to each of the six contributions, the non-mathematics education reader has the opportunity to come across a number of diverse issues that, albeit embedded within a myriad of mathematics education environments, can still resonate with his or her research interests.

The first two papers adopt a qualitative case study approach to focus on the professional journeys undertaken by teachers. Calleja investigates the learning journeys of secondary school teachers of mathematics as they engaged in a one-year long continuing professional development programme. He reports on teachers' community of practice experiences as they participated in a purposely-designed programme that aims to support teachers in learning to teach mathematics through inquiry. Calleja explored the teachers' motivations for joining this professional development programme and their learning experiences through in-depth interviews and a focus group. A key discussion in Calleja's paper focuses on the teachers' views about programme effectiveness. Their ongoing interactions generated in-depth practice-based understandings that call for a rethinking in the way professional development is offered to teachers in Malta. On the other hand, Buhagiar uses the notion of change to frame his analysis of one teacher's professional journey over the years from a traditional to an inquiry-based approach. Adopting a narrative research approach, Buhagiar shows that while teacher change can happen, it may be neither linear nor enduring. Furthermore, it appears to be facilitated by certain factors, including a teacher's motivation, available opportunities, and the presence of a professional learning community. In particular, Buhagiar highlights the inspirational role of a school-based educator who is willing and able to support professional learning among colleagues.

The third paper of this special issue also focuses on the role of the teacher. However, in this case, the teacher is the author himself. Borg reflects on his own experience teaching a group of six low-performing secondary school students. Borg uses radical constructivism as a guiding theory, developing a framework which he refers to as 'Mathematics-Negotiation-Learner' (M-N-L).

This framework takes into consideration the mathematics a teacher intends to teach, the classroom interaction with students, and the learners' own constructions of mathematics. In his paper, Borg shows how this framework enabled him to analyse his own teaching strategies. In Farrugia's paper, the focus now shifts to classroom interaction between a primary school class teacher and her pupils. Assuming a Vygotskian perspective, Farrugia considers the teacher to be a more knowledgeable adult, scaffolding her pupils' learning of topic related mathematical words. Farrugia's main focus of attention is the use of both Maltese and English in the interaction; she notes the apparently beneficial use of translanguaging that includes the translation of the topic-related Maltese words with which the pupils were already familiar. The final paper by Zerafa shifts the focus on the learners, in particular learners with mathematics learning difficulties. Zerafa's paper stands out from the other papers in that the author uses statistical methods to standardise mathematics tests - previously standardised in the UK - for Malta. Norms were found by administering the tests to a sample population of 10 year-old boys. The norms were then used to examine the test scores obtained by a cohort of pupils in the school where Zerafa taught. The aim of this procedure was to identify six pupils with mathematics learning difficulties with whom an intervention was to be carried out. Hence, this paper describes the process of sample selection.

This special issue ends with a personal commentary by Mamo who, apart from dedicating his professional life to the teaching of mathematics, also served for a number of years as mathematics Head of Department within the state secondary school sector. Reflecting on his headship experiences, Mamo explains his key contributions to mathematics education in Malta, and shares his perceived successes and disappointments. He also highlights what he considers to be three important elements that have guided the manner in which he had sought to carry out his role of Head of Department, namely, the love of the subject, collegiality, and professional integrity.

This special issue of MRER presents a collaborative effort, which to the best of our knowledge is the first of its kind in Malta, among individuals who are actively involved in mathematics education. A unifying factor among all the six authors is that they have all done, albeit over different periods of time, and following different types of courses, their initial teacher education at the University of Malta. As one would expect, all six individuals have experienced different career pathways after joining the profession. Notwithstanding this, all of them have sought to keep abreast with research related to the teaching and learning of mathematics. Indeed, this group of people have a passion for mathematics, enjoy teaching mathematics and learning more about how it can be taught more effectively, and have always sought to share their knowledge and expertise with pre-service and inservice teachers of mathematics. In recognition of this, all six authors are among the

regular contributors to the mathematics education programmes organised by the Faculty of Education, at the University of Malta. Over the years, Faculty has offered these programmes at both undergraduate and master's levels. More recently, moreover, mathematics education has joined the ever growing list of subject area specialisations being researched at doctoral level within our Faculty.

On a final note, we would like to dedicate this special issue of MRER to all those who have contributed in the past, in some way or another, to the advancement of mathematics education in Malta. One can locate these individuals among those who have worked in schools, who have developed education policies, who have offered support to teachers and schools, and who have guided teachers' formation and learning throughout the various stages of their professional lives. Malta owes a lot to these people, as they are the ones who have laid the solid foundations on which we continue to build today. The contributions in this special issue give testimony, in fact, to the varied and lively field that mathematics education is becoming as of late in Malta. All of this would probably not have been possible had we not benefited from the foresight, dedication and commitment of those who entered and worked the field before us. Some of them are certainly rather well-known, while others are practically invisible to the general public. Still, for us, they are all worthy of our deepest and sincerest gratitude.

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Teacher Participation in Continuing Professional Development: Motivating Factors and Programme Effectiveness

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Abstract: Teachers choose to take up professional development courses for different reasons. This paper reports on the motivations of a small group of Maltese secondary school teachers of mathematics in joining a continuing professional development (CPD) programme aiming to support them in Learning to Teach Mathematics through Inquiry (LTMI). During mathematical inquiry, students assume a central active role – wrestling with ideas, asking questions, exploring and explaining meanings – supported by the teacher as a facilitator. This paper also explores teachers’ understandings and their reported experiences of programme effectiveness. A qualitative design using thematic analysis was used to investigate views, experiences and accounts of LTMI features that teachers believed to be effective for their professional learning. The data reported here was taken from a focus group held with teachers at the end of the CPD programme, and three interviews held with the same teachers before, during and after their participation in CPD. Findings reveal intrinsic factors motivating teacher participation, namely: (1) teachers’ will to develop knowledge about teaching; (2) their beliefs about the benefits of inquiry; and (3) their need to change classroom practice. The key aspects that teachers voiced as effective throughout their CPD experience were learning by being part of a community, active learning and immersion in practice-based understandings.

Keywords: Continuing professional development; teacher motivations; inquiry-based learning; community of practice; programme effectiveness

Introduction

Teachers are usually unclear about the meaning of inquiry and how this may be translated into classroom practices (Chin & Lin, 2013; Ireland, Watters, Brownlee, & Lupton, 2012; Towers, 2010). Moreover, contextual constraints

and system restrictions are challenges that teachers often report in implementing inquiry in their teaching of mathematics (Anderson, 1996; Engeln, Mikelskis-Seifert, & Euler, 2014). One way of addressing these challenges is to offer CPD opportunities for teachers that provide pedagogical training and support in using inquiry practices (Bruder & Prescott, 2013).

Professional development is an on-going and long-term process (Loucks-Horsley et al., 2010) providing teachers with collaborative opportunities to design, implement, share, discuss and reflect (Guskey, 2002; Putman & Borko, 2000) to bring about the desired changes in classroom practice. A CPD programme was designed with this end in mind – to provide a blended approach of *off-the-job* summer workshops and *on-the-job* meetings for secondary school teachers of mathematics to immerse themselves in and learn about inquiry. By drawing on seven case studies, this paper reports on two areas: teachers’ motivations to participate and their understanding of ‘effective’ CPD. Specifically, the research questions were:

1. *What motivates teachers to learn to teach mathematics through inquiry?*
2. *What features of CPD were considered effective by the teachers?*

In the next sections, I provide literature related to CPD, design, implementation and effectiveness. This is followed by presenting the current situation in Malta with regard to teacher professional development with a focus on teacher participation in an international European Union project in promoting inquiry. Literature on inquiry-based learning (IBL) and its importance for Malta are reviewed before moving onto teacher motivations and views about what makes effective CPD. Next, I outline the study and the CPD programme design. The study methodology is then presented, followed by data analysis and key findings emerging from the qualitative data shared by seven participants. Finally, this paper outlines conclusions and implications for designing and conducting effective and replicable CPD programmes.

Continuing Professional Development

Literature on CPD reveals that there are many and varying definitions. For the purpose of this paper, CPD is taken to encapsulate the personal and the professional learning of the teacher (Earley & Bubb, 2004), that is, “those processes and activities designed to enhance the professional knowledge, skills, and attitudes of educators so that they might in turn, improve the learning of students” (Guskey, 2000, p. 16).

There seems to be two distinctive approaches to providing CPD – the ‘traditional training model’ and the ‘sociocultural model’ (Mansour,

Albalawi, & Macleod, 2014). While the ‘traditional model’ views learning as the acquisition of skills that teachers may take from a course and apply into their classrooms, the ‘sociocultural model’ values knowledge, teaching and learning as being socially created and culturally enacted. From the vast literature of studies on mathematics teachers’ CPD, it is clear that there has been a shift towards programmes that model inquiry-based pedagogies (e.g.: Back, Hirst, De Geest, Joubert, & Sutherland, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001; Loucks-Horsley et al., 2010) and *authentic activities*, that is, CPD activities that are similar to what teachers could be doing in their classrooms (see Brown, Collins, & Duguid, 1989).

There is widespread consensus about what constitutes effective professional development (Guskey, 2000; Loucks-Horsley et al., 2010; Putman & Borko, 2000). According to Loucks-Horsley et al. (2010, p. 5):

It is directly aligned with student learning needs; is intensive, ongoing, and connected to practice; focuses on the teaching and learning of specific academic content; is connected to other school initiatives; provides time and opportunities for teachers to collaborate and build strong working relationships; and is continually monitored and evaluated.

Despite all that we know about what renders CPD effective, the challenges and barriers towards successful implementation are still to be addressed (Bubb & Earley, 2013; Guskey, 2002; Putman & Borko, 2000) – both abroad and particularly in the local context. Locally, it seems that CPD providers (institutions and schools) still conceive teacher professional development as an isolated venture of off-site workshop training disconnected from practice, rather than an ongoing collaborative on-site experience of practice-oriented development and learning.

Researchers interested in teacher professional communities have drawn on the community of practice (CoP) perspective (see Jaworski, 2006b; Lave & Wenger, 1991; Wenger, 1998) – also referred to as learning communities (Attard, 2012) or professional learning communities (DuFour, 2004; Watson, 2014) – to explain the social processes shaping teacher learning. Communities of practice are dynamic learning communities in which participants engage “in actions whose meaning they negotiate with one another” (Wenger, 1998, p. 73). The identities of participants thus become shaped as they engage with others. This situated perspective of learning has been applied widely to teachers’ learning to teach (Coburn & Stein, 2006; Jaworski, 2006b; Lave & Wenger, 1991). In this study, *learning to teach mathematics through inquiry* (LTMI) was designed to bring in “a critically questioning attitude towards practice and knowledge-in-practice that allows critical reflection on the

practice of teaching” (Jaworski, 2007, p. 1693), thus engaging teachers in sharing and negotiating inquiry-based classroom practices.

Teacher Motivations to Participate in CPD

Literature shows that reasons why teachers participate in professional learning include the development of knowledge about teaching (Anderson, 2008), their ‘will to learn’ (Van Eekelen, Vermunt, & Boshuizen, 2006) and career-related purposes (Ng, 2010). Motivational theories tend to rely on two contrasting views related to human nature: (1) humans are hesitant and require some external stimulus to venture on things; or (2) motivation is internally stimulated (see Bassett-Jones & Lloyd, 2005). There may be extrinsic as well as intrinsic factors that explain what motivates people in their workplace and, in particular, what motivates teachers to pursue professional learning. As to extrinsic factors, the school or institution may ‘dictate’ or advocate teachers to engage in CPD. This is usually the case with compulsory courses locally initiated by the Ministry for Education and Employment (MEDE) to address curricular needs or when government-initiated reforms are being introduced. On the other hand, job satisfaction and the need for recognition may be considered as factors that intrinsically motivate teachers to engage in CPD. In a study with teachers in Ireland, McMillan et al. (2014) found that motivational factors fell under three categories: personal, school-related and system-wide. Teachers’ personal choice for engaging in CPD included personal interest, career advancement and a perceived need to improve their classroom practice. School-related factors were also viewed by teachers as beneficial and motivating. Following participation in CPD, teachers in the study reported by McMillan et al. (2014) were encouraged by their school to provide feedback to colleagues, hence supporting their professional learning community at school. Finally, the main system-wide motivator identified by these teachers was the mandatory nature of courses held during school hours. This scenario is very similar to the local context as generally teachers have little choice but to take the course offered to them. This constraint tends to limit teachers’ motivation to develop professionally.

Teachers’ Views of Effective CPD

There are two main approaches to understanding what ‘effective’ means in relation to CPD initiatives. For example, Joubert and Sutherland (2008) list characteristics that include encouraging purposeful networking; being grounded in classroom practice; and, supporting reflection and inquiry by teachers. On the other hand, Guskey (2000) evaluates CPD in terms of outcomes using a five-level model:

1. Participants' reactions.
2. Participants' learning.
3. Organisational support and change.
4. Changes in classroom practice.
5. Student learning.

This model offers a helpful way of looking into the outcomes of CPD at different levels. The *Researching Effective CPD in Mathematics Education* (RECME) project investigated 30 initiatives representing different models of CPD for teachers of mathematics in England, and outlined a number of factors related to effective CPD as reported by teachers (see Back et al., 2009). When Joubert et al. (2008) analysed these factors within the levels outlined by Guskey (2000), they found that participants valued the knowledge and understanding of practice demonstrated by CPD leaders and the practical advice provided during sessions because this was directly applicable to their classroom. Teachers also appreciated CPD that was stimulating, enjoyable and intellectually challenging. Long-term and reflective engagement with CPD, opportunities for networking with colleagues, an expectation to try out new ideas and report back their experiences, and opportunities for discussion were all mentioned as factors that contributed to their active involvement in CPD. In addition, teachers also reported that CPD gave them confidence, and increased passion and energy to try out new things. However, Guskey's (2000) fifth level outcome was missing as teachers did not report improved student learning as evidence of the effectiveness of their CPD. They did, however, report on improved students' attitudes towards their engagement in learning mathematics and persevering with challenging tasks.

Professional Development in Malta

Teachers in Malta are entitled to a maximum of 30 hours of CPD each year. This training time is equivalent to approximately 7.3 days, which is below the TALIS¹ average of 15.3 days dedicated to CPD in Europe (OECD, 2009). CPD duration and format are established by MEDE through a collective agreement signed with the Malta Union of Teachers, and changes may only be possible following new negotiations. CPD opportunities for teachers usually occur at school level but they are also provided by MEDE. Teachers in the independent sector may choose to attend this training, yet training is usually organised in-house (Attard Tonna & Calleja, 2010). Until 2016, MEDE was the main agent for providing in-service teacher training in Malta (see Ministry of Education and Employment, 2012). Figure 1 below delineates the CPD

¹ Teaching and Learning International Survey (TALIS) examines how countries prepare teachers to face today's diverse challenges in schools. TALIS asks teachers and school leaders about their work, working conditions and learning environments covering themes such as continuing professional development.

opportunities provided to secondary school teachers at the time of the study. Besides these CPD obligations, teachers may also undertake post-graduate courses offered by the University of Malta and other institutions.

Figure 1: CPD opportunities for teachers in Malta

CPD ACTIVITY	PERIOD	DURATION
PROFESSIONAL DEVELOPMENT SESSIONS	After school hours	3 two-hour sessions (6 hours)
SCHOOL DEVELOPMENT MEETINGS	Within school hours	3 two-hour sessions (6 hours)
SCHOOL DEVELOPMENT DAY	Within school hours	1 full-day (6 hours)
INSET TRAINING	In July (end of school year) or in September (before school year) *	3 half-day sessions (12 hours)

* Scholastic year ends after the first week of July and starts in the last full-week in September.

Generally speaking, CPD activities are still informed by a ‘deficit model’ (Brown & McIntyre, 1993) with the assumption that educators have deficiencies and CPD would serve to correct these. From my professional experience, I am aware that training is usually provided by outside experts and most sessions tend to be led by PowerPoint presentations. In such cases, CPD takes a top-down approach of knowledge transfer to participants who, in turn, end up having to listen for most of the time with little or no input from their part. Besides disregarding teacher motivation and agency in learning and development, this model is found to be ineffective (Little, 1993; Loucks-Horsley et al., 2010) due to the lack of transferability of knowledge that teachers take into their classrooms.

CPD through a learning communities approach first featured with the publication of the National Minimum Curriculum document (Ministry of Education, 1999). Yet, as Bezzina (2002, p. 65) noted, “the underlying feeling one gets is that the authorities may be assuming that it can just happen”. I believe that the development of supportive structures that enhance the ongoing professional growth of teachers is still being overlooked today and, as a result, the concept of creating and sustaining learning communities is generally missing in local schools (Attard Tonna & Calleja, 2010; Bezzina, 2006).

On a more positive note, more recently, with the publication of the National Curriculum Framework (Ministry of Education and Employment, 2012) and

the setting up of the Institute for Education² in 2015, this move seems to be regaining the much needed momentum. Indeed, a number of recent initiatives have provided a more active, practice-based, collaborative and on-going approach to CPD – for example: the *Let Me Learn* programme (see Attard Tonna & Calleja, 2010), the *Pestalozzi Action Research* project (see Brown, Gauci, Pulis, Scerri & Vella, 2015), focused training for teachers teaching the core competences learning programmes in Mathematics, Maltese and English, and the *Promoting Inquiry in Mathematics and Science across Europe* (PRIMAS) project.

PRIMAS was an international project within the Seventh Framework Programme of the European Union. Run over four years (2010-2013) in twelve European countries including Malta, the project worked at promoting IBL in mathematics and science. This project, the first of its kind for Maltese teachers, provided a range of CPD materials and ongoing support through school-based communities led by multipliers. Multipliers, who were either practising teachers or teacher educators, led CPD with small groups of teachers (Maaß & Artigue, 2013). In the case of mathematics, five multipliers (a teacher, three heads of department³ and an education officer), including myself as a head of department, were involved in creating such teacher learning communities in five state secondary schools. Notwithstanding the challenges to implement IBL lessons, Maltese teachers showed a positive orientation towards IBL and reported significantly greater use of IBL in their daily practice (see Engeln, 2013).

Inquiry-based Learning in Mathematics and in CPD

Inquiry is a multifaceted activity (Maaß & Artigue, 2013). More specifically, there seem to be common notions associated with inquiry pedagogies, namely, that they are learner-centred, investigative, problem-oriented, collaborative and question-driven (Goodchild, Fuglestad, & Jaworski, 2013; Jaworski, 2006b; Swan, 2006). In mathematics, IBL is seen to engage learners in thinking, starting off as a mediating tool through the use of tasks and over time shifting to become more “as a way of being” (Jaworski, 2006a, p. 204). The mathematical tasks that teachers use need to provide an “achievable challenge” (see Willis, 2010) requiring students to exert mental effort, but they also need to encourage creativity, decision-making and exploration.

Inquiry is becoming more relevant in the Maltese educational system and mathematics education (Ministry of Education, 2012). In addressing this,

² Established to provide high quality education through continuing professional development courses to educators at all levels.

³ Work together with school management teams to ensure high standards in teaching and learning practices.

teachers become key in developing learners' competences and in enabling them to nurture an inquiry stance to learning. This implies that teachers need to develop skills and dispositions to support learners in becoming critical thinkers as well as responsible and active citizens. Teachers may achieve this by undertaking student-centred approaches, and research shows that IBL is an effective way to support building such competences (see Towers, 2010). Through IBL, learning opportunities are aimed towards preparing learners who can create, innovate, collaborate, be critical, explore, communicate and make thoughtful decisions, hence developing key competences and skills crucial to their lives beyond school.

For teachers, using IBL requires what Greeno (2006, p. 543) calls "knowing a conceptual domain", that is, "knowing what resources are available in the domain, knowing where to find them, knowing how to use them, and anticipating the results of using them in different circumstances". Knowing a conceptual domain like IBL implies not just knowing what it means but also how it can be used with learners in different contexts. CPD is hence fundamental in offering teachers with context-related learning opportunities. By immersing participants in the process of inquiry, CPD may provide teachers with modelling experiences of inquiry teaching (Farmer, Gerretson, & Lassak, 2003). A key component of CPD is the role that professional learning tasks play in creating "opportunities for teachers to ponder pedagogical problems and their potential solutions through processes of reflection, knowledge sharing, and knowledge building" (Silver, Clark, & Ghouseini, 2007, p. 262). For learning to occur, CPD is designed to offer teachers opportunities for ongoing collaborative negotiations about the use of IBL in different classroom contexts.

The Study

LTMI is a CPD programme designed as a set of experiences offering teachers opportunities, over one scholastic year, to experience, integrate, reflect upon and develop their inquiry teaching practices. At the time of the study (academic year 2015-16), I was a teacher of mathematics and a head of department in a state secondary school. LTMI, offered to secondary school teachers of mathematics as a voluntary course, was designed as an intervention programme. It was driven and inspired by previous experiences working with teachers and particularly by my passion for designing and leading teacher professional learning. For example, I engaged with teachers at my school in various collaborative projects, such as PRIMAS and the use of formative assessment task. I also regularly contributed to professional development sessions in schools and during mathematics INSET. However, my role in LTMI was related exclusively to design, while teachers and teacher educators with experience in inquiry practices facilitated the sessions with

teachers. My role during these sessions was that of a non-participant observer – collecting feedback to improve the programme in the piloting phase and gathering field notes and other data to study teacher learning during the main phase. For the piloting phase, held during the scholastic year 2014-2015, five teachers took the programme while another 12 teachers enrolled for the main study held the following year.

The CPD Programme

The CPD programme was designed to provide LTMI experiences for teachers first through summer workshops, and then by participating in follow-up meetings held during the scholastic year (see Figure 2). The four summer workshops, led by teachers with experience in inquiry teaching, focused on four IBL features: *mathematical tasks*, *collaborative learning*, *purposeful questioning*, and *student agency and responsibility*. Summer workshops followed a consistent pattern of activities – teachers first worked collaboratively to solve a mathematical task through inquiry, then discussed their experience working on the task and later watched a video from a local classroom demonstrating a teacher using the same task with students. A subsequent activity included the analysis of a published lesson video (available on YouTube) dealing with a particular IBL feature being discussed (e.g., collaborative learning). Discussions alternated between pair, small-group and whole-class. Such discussions were intended as additional opportunities for teachers to further investigate teaching approaches, clarifying concepts and to problematize issues related to teaching through IBL. At the end of each workshop, teachers were encouraged to collaboratively plan a lesson using the activities presented and the ideas generated. The CPD materials are available online and downloadable (see www.iblmaths.com).

Follow-up meetings were then intended to provide collaborative ongoing support for teachers to discuss, evaluate and develop practice-based learning. These meetings followed a structured set of activities led by a facilitator. The opening activity prompted participants to reflect on their inquiry practices. Teachers wrote reflections on sticky notes. Reflections included personal strategies for using IBL, challenging situations encountered and classroom incidents.

This was followed by reporting back and sharing of IBL lessons and tasks. Finally, participants discussed and agreed upon an agenda for the following meeting. The facilitator's role was that of a challenger and an intervener – asking questions to support, stimulate and enable participants' thinking. Over time, this scaffolding was gradually removed to allow for increased teacher autonomy in learning about IBL, but also to nurture a self-sustaining learning

community (see Calleja, 2016 for a more detailed outline of the LTMI activities).

Figure 2: The LTMI programme

SUMMER CPD WORKSHOPS		FOLLOW-UP CPD MEETINGS
<i>A focus on understanding IBL</i>		<i>Reflecting on classroom practices</i>
1 Session	3 Sessions	Ten follow-up meetings
(4 hours)	(4 hours each)	(1¼ hours each)
<i>July 2015</i>	<i>September 2015</i>	<i>October 2015 to May 2016</i>

The Participants

Seven participants (2 males and 5 females) volunteered to contribute data to my research from a total of twelve participants (5 males and 7 females) joining the LTMI programme. Although I was working with a small number of participants, this sample still included a wide range of participant characteristics (see Figure 3). My aim was to study how IBL was understood, experienced and implemented by teachers with different teaching experience, working in different schools, and teaching different year groups. With this heterogeneous sample, I sought to identify common patterns that captured core experiences of the entire group. According to Patton (2002, p. 235), with a small sample of great diversity, data analysis would “yield important shared patterns that cut across cases and derive their significance from having emerged out of heterogeneity”.

Figure 3: Information about the participants

Teacher	School	Prior Knowledge of IBL	Teaching Experience (Years)	Year Group Taught
Sarah	State	PRIMAS	16 – 20	9
Janet	State	None	11 – 15	8
Tania	State	ITE	1 – 5	10
Greta	Church	Course	16 – 20	8
Colin	Church	ITE	1 – 5	9
Chris	Church	ITE	1 – 5	7
Jackie	Independent	None	16 – 20	10

The seven teachers taught mathematics in different secondary schools. There are two types of schools in Malta: state and non-state. State schools are governed by MEDE and operate within colleges consisting of a cluster of primary and secondary schools within particular catchment areas. There are ten of these colleges in Malta each of which is led by a principal. The non-state sector is subdivided into Church and independent Private schools.

Church schools are predominantly Roman Catholic schools heavily subsidized by the government. Private schools are set up by individuals or non-profit parents' foundations that, unlike the other schools, charge tuition fees.

Ethics approval for the research was granted by all these institutions and informed consent was then obtained from all participants and heads of school prior to conducting the research. The study adhered to the ethical principles of informed consent, confidentiality, anonymity and the right to withdraw. Pseudonyms are used to identify the participants in the study (see Figure 3 for data about the participants).

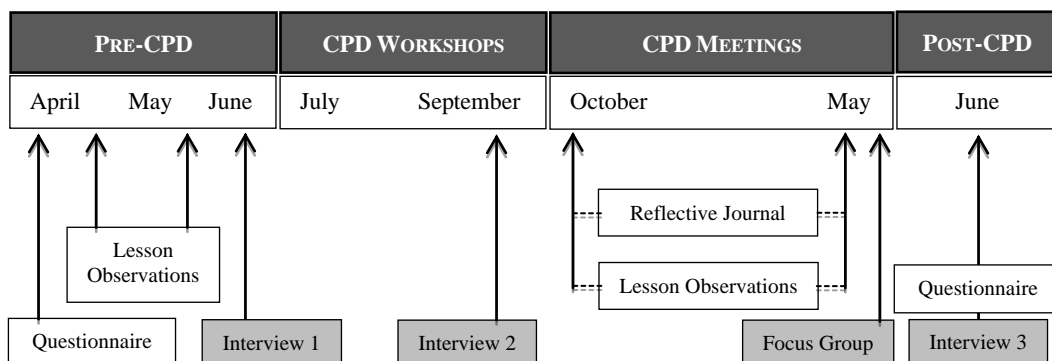
Teachers came into the CPD with different knowledge of IBL. The three younger participants had been exposed to IBL through their Initial Teacher Education (ITE) programmes. Sarah, on the other hand, had participated in the PRIMAS project and also used inquiry in her classroom. Greta had learnt about IBL in her Masters course, while Janet and Jackie were both new to IBL.

Methodology

In undertaking this research, I worked within a qualitative research paradigm with the underlying assumption that understanding of reality is embedded within a social construction (see Guba, 1990). A sound understanding of teacher learning would be gained by studying how teachers operate within the CoP created and cultivated by the CPD, and within their own work-based context.

A data-driven inductive approach (see Boyatzis, 1998) was employed to allow patterns, represented by the voices grounded in the data, to emerge from the 'realities' provided by the seven teachers. The goal was to understand multiple 'realities' across the various data sources from the teachers' perspectives, their experiences and views of effective CPD.

Figure 4: Timeline in using the data collection instruments



As this was an in-depth study focusing on a small sample of teachers, qualitative methods were chosen to collect data. Five sources were used to gather data from the participants, namely: questionnaires, lesson observations, semi-structured interviews, teacher reflective journals, and a focus group. Figure 4 shows a timeline for collecting data during the study. For the purpose of specifically answering the research questions delineated in this paper, two of these data sources were used: semi-structured interviews and a focus group.

Semi-Structured Interviews

The three semi-structured interviews (see Fontana & Frey, 2000; Kvale, 1996) conducted with the participants before, during and after teacher participation in CPD, focused on gathering data related to practices and knowledge about IBL, and their experiences engaging in CPD. The questions and situations presented touched on all aspects investigated by the research questions. While the first interview addressed aspects linked to motivations for participation, views, practices and knowledge of inquiry, the second interview investigated what participants gained from participating in the CPD workshops, and what they intended to take into their classrooms. The third, and final, interview offered teachers a retrospective, reflective analysis to describe potential challenges and learning experiences encountered in their LTMI journey to make changes towards inquiry teaching. For example, questions in the second and third interview asked participants to describe their experiences and identify LTMI activities that they found most valuable in supporting their professional development. Each interview, which took between 40 to 50 minutes, was audio recorded and later transcribed verbatim.

Focus Group

As a qualitative method for gathering data, the focus group brought together the researcher and the seven participants to discuss their CPD experiences and its effectiveness towards LTMI. Since it was difficult to get participants together at the end of the scholastic year, due to working half-days and the annual examination period, the focus group was held during our last CPD meeting. The focus group participants were led through the discussion by the researcher, acting as a moderator, using questions as probes and prompts for participants to elicit experiences, meanings and insights into effective aspects of the CPD programme. The main advantage of using the focus group was that it offered an opportunity to observe participants as they engaged in discussion about attitudes, perceptions and experiences (Krueger & Casey, 2015) related to their immersion in the CPD programme offered. The focus group took 75 minutes and was video recorded. The video recording was later transcribed for analysis.

Data Analysis

Data analysis was guided by the two research questions and conducted using a thematic approach to analysis and theory (see Braun & Clarke, 2006). Each interview and the focus group transcript were divided into chunks – usually short paragraphs of between 20 to 60 words – applying an open-ended coding technique to label comments and assign codes on the margin. Inductive coding (see Boyatzis, 1998) began with close reading of text and consideration of multiple meanings. Initial codes focused on significant statements, comments and actions that reflected teachers’ thoughts, motivations, judgements and expectations of CPD. These codes and comments were then compared and grouped to create themes. The findings reported here consider both unique cases of teachers and the shared motivations and experiences of the participants.

Findings

Teacher Motivations to Participate in the LTMI Programme

In analysing teacher motivations, I examined responses to a question from the first interview specifically asking for their motivations in participating in the LTMI programme. However, teacher motivation also emerged in teacher interactions during the focus group.

Teacher participation in the LTMI programme seemed to be driven by personal motivation factors (McMillan et al., 2014). These teachers demonstrated their personal motivation to engage in CPD, and seemed to view LTMI as a professional and personal development (see Rinaldi, 2006). The reasons that teachers in this study provided as motivations for their participation could be classified into three categories: (1) developing knowledge about teaching; (2) perceived benefits of IBL; and (3) the need to make changes to their classroom practice.

Five of the seven participants claimed that they saw LTMI as an opportunity for them to develop knowledge about teaching. One teacher, referring to a previous experience in the PRIMAS CPD commented:

When we started PRIMAS and inquiry-based learning with the multiplier, I was really interested and wanted to use more of it in my classrooms. When I got to know that this is a similar project, I could not refuse because it is something I am keen about, and in fact, I would like to do more of in my class. (Sarah)

Thus, for Sarah, LTMI offered an opportunity to continue the work she had begun during a previous CPD project. Yet, during the focus group discussion, Sarah communicated an additional motivation claiming “*I had been teaching for a long time and was fed up teaching the same way, sometimes like speaking to the*

wall... I needed something different.” It appears that her motivation was not only linked to a desire to continue to develop her knowledge about IBL but also to a change she needed in her teaching. Other participants had similar views related to knowledge development but tended to link their motivations to their perceived benefits that IBL offered.

I believe that new experiences and new learning opportunities motivated my participation. But I also see the benefits of IBL, so if I am to use inquiry in my class I feel I need to be well informed and well taught about the subject. (Colin)

For some of the teachers, LTMI represented a new learning opportunity motivated by the benefits that IBL could offer to their teaching. Another teacher went into more detail to specify why she thought IBL might help and justified why she was interested in taking up CPD.

Some students seem to struggle with learning mathematics, and you hear people saying that changing the pedagogy may help to support these students. So, if these students may get engaged in coming up with their own methods for solving a problem, then mathematics may make more sense to them and be more motivated to at least improve their achievement in mathematics and learn things that they may find useful in life. (Jackie)

For Jackie, the benefits of IBL seemed to arise from the strong position people take when they talk about it. By the word ‘people’, Jackie was possibly referring to those leading CPD sessions because it is through CPD sessions that she claims to have heard about IBL. Jackie also referred to IBL as promoting a change in pedagogy. For Jackie, it seemed that changing to a more active learning approach may better address the needs of students who are struggling with learning mathematics, and this was her main reason for joining this CPD. This leads to a third aspect emerging from teachers’ responses, also linked to an earlier reason given by Sarah – a need for change. Indeed, four of the seven teachers interviewed considered IBL and LTMI as an opportunity for them to shift their pedagogical practices. Together with Jackie, this is how the others saw LTMI:

I think we have so much content to cover. At times, I feel I want to do things differently but I am restricted by the system... this frustrates me a bit. I know there are other possibilities for delivering mathematical content. So, the fact that now I am engaged in a project that may offer a possibility for me to try new things... that encouraged me to take part. (Chris)

I believe students will find mathematics more fun learning through inquiry. I would hence like to vary the kind of lessons I provide my students with and thus making them more interesting for the students. (Greta)

I want to change the way my students learn mathematics – from copying down notes to being more active in participating and constructing knowledge, and the latter is something I am really fond of. (Janet)

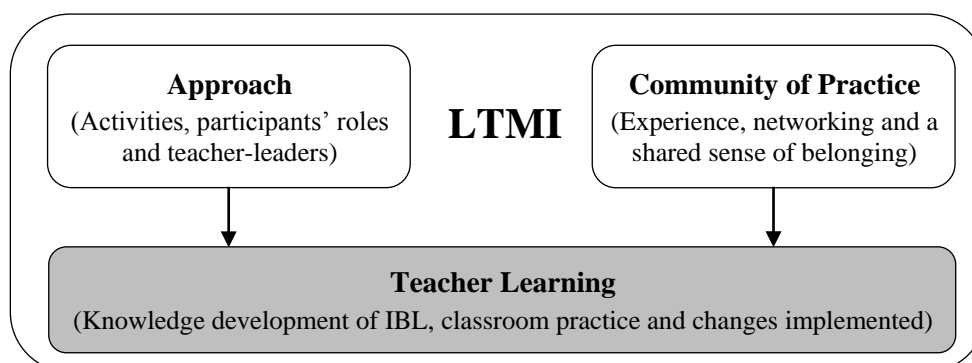
For some, LTMI represented an opening and a possibility to use other pedagogies. Like Jackie, Greta and Janet were motivated for change by the fact that they intended to support student learning (see Hunzicker, 2011; Loucks-Horsley et al., 2010), and hence improve the “service that they provide to (their) clients” (Hoyle & John, 1995, p. 17). While Jackie sought to help those falling behind, Greta believed that her students would enjoy learning mathematics through IBL. On the other hand, through IBL, Janet intended to shift the role that her students adopted in class – from passive to active learners. Hence, with the exception of Chris, whose participation in LTMI was motivated by a wish to challenge the system, it seems that for the other teachers, motivations were more directed to addressing student learning by the pedagogical shift that LTMI was offering through IBL.

Teachers’ Views of Effective Features of LTMI

In analysing features that teachers viewed as effective within LTMI, I examined responses to questions from the second and third interview, together with the focus group discussion. Teacher responses amounted to 54 separate chunks – 42 emerging from the interviews and 12 from the focus group.

Teachers perceived LTMI as effective in relation to two features: (1) the approach to CPD; and (2) the CoP experience (see Figure 5). The CPD approach adopted by LTMI related to the type of tasks and activities that teachers participated in, the active role undertaken and, those leading and facilitating CPD. The CoP experience related to their lived experience, the shared sense of belonging to a community and the networking generated. It seems that for the teachers these two features of CPD impacted positively on their learning in developing knowledge of IBL, their practice and the changes they implemented. However, the approach to running LTMI as summer workshops appeared to offer a challenge to the learning experience of two teachers who considered the approach to be particularly intensive.

Figure 5: Perceived effectiveness of LTMI on learning about IBL



The CPD Approach

Teachers valued the CPD approach adopted, and mentioned various features related to this, namely: the activities, their active role within CPD activities, the shift in role from teachers to students, the modelling of IBL practices and the fact that CPD was led by teachers. The following are excerpts taken from the interviews during which participants spoke about each of these aspects.

Activities: *The session during which we saw one of us teaching a class helped a lot because you learn from the mistakes of others, but also from the positive things. You get ideas and it was surprising to see what a teacher is able to come up with. (Tania)*

Active roles: *I liked taking the role of the student because I could get a first-hand experience and see how it (IBL) works. (Colin)*

Modelling: *I really like the fact that the sessions and the tasks we were presented with mirrored what we can do with our students. I got the message that the session structure provided a model for us of what an IBL lesson should look like. (Greta)*

CPD leaders: *Since the speakers leading the sessions were teachers themselves, I could easily relate it to what they were saying. They spoke about their personal experiences and challenges using IBL, and that helped a lot. (Janet)*

In addition, during the focus group Chris spoke at length on an additional feature that he felt contributed to the effectiveness of his CPD experience.

I think one of the things that helped make this course such a success was that this was not something imposed, but we all seem to be doing this for our own learning and also because we believe in this (IBL). It was not because somebody was checking on me that I did this, but because I wanted to grow as a teacher and for my students to learn... and I think that gives you more motivation, the fact that it is something desirable for us to do...we came to these meetings not because someone forced us to but because we wanted to. I think that made all the difference. This course was something I wished to do...and if there was a time when I could not do as much, there was nothing wrong with that...so I was free...free from the control we spoke about earlier.

Chris experienced a sense of self-directedness but also self-regulation within CPD. Chris engaged in CPD because he chose to and *wanted to grow as a teacher*. He did *something desirable*, and self-regulatory learning enabled Chris to define his own goals for learning (see Butler & Winne, 1995). At the same time, CPD allowed him to be *free from the control* of a system that, he felt, did not trust teachers as professionals. Tania echoed a similar view during the final interview claiming that *“during the follow-up meetings, the topics for discussion were not chosen for us but we were free to decide what was important for us to discuss and learn”*.

In all seven cases, teachers linked the approach to CPD adopted as effective towards their knowledge development about IBL or to improved classroom practices. Teachers generally spoke about LTMI effectiveness in terms of increased confidence and motivation in using IBL, benefits in the changes observed when students undertook more active roles, and new ways of learning that they had provided students with.

The CoP Experience

A second important aspect highlighted by teachers related to the CoP created and cultivated within CPD. Teachers attributed CPD effectiveness to the experience, networking and a shared sense of belonging that they experienced through CoP. For teachers, the community represented an opportunity to meet new people, working with teachers from different contexts that brought diverse perspectives, engaging in reflective practice and finding support from the community members.

Experience: *I could see the perspectives of other teachers from outside the small school environment that I work in, and which sometimes makes me feel enclosed within my own self. (Jackie)*

Shared

Concerns: *Besides the well-prepared content that we gained a lot from, but to feel that you are in the same boat as the others... that helps. It was one of the positive aspects of the PD. (Chris)*

Reflective

Practice: *I understood how important it is for a teacher to reflect on practice. During the meetings, we had opportunities to reflect on what we did in our classrooms. This course provided the space so that participants could reflect. (Greta)*

Collaboration: *I saw it really useful when we planned lessons together because you get the ideas of all members in the group, that provides opportunities to share views because you would not have necessarily thought about these or used such ideas in your practices. I felt that this was always a learning experience because you start considering things that you would perhaps not have thought of before. (Sarah)*

Support: *I was motivated by the fact that I could keep contact with people who value IBL, mainly because I have no-one to work with at my school. The fact that I have people whom to turn to when I have a difficulty, that is of support to me. (Janet)*

Confidence: *Overall I became more confident. Now I know that it is ok when something does not go as planned. During the PD, we saw what*

worked and what did not, so that also gave me the much-needed confidence when implementing something so new. That, I believe, also gave me the courage to try. (Tania)

Teachers mentioned aspects related to their sharing of experiences, discussing and addressing common challenges and finding support from colleagues who were *in the same boat*. This brought about a common sense of identity and belonging that supported their professional learning about their IBL practices (Potari, Sakonidis, Chatzigoula, & Manaridis, 2010). Most teachers valued the sharing of concerns and collaboration. Community served as a *support* group because teachers also shared ideas and engaged in collaborative reflections. In the following excerpt taken from the focus group, teachers discuss how the collaborative aspect cultivated within CPD supported their engagement and learning of IBL.

Chris: *I feel this was a course that didn't just speak about theory... on the contrary, we got our hands dirty, we found challenges and difficulties in using IBL, we understood that the challenges we encountered were common to all...and the most precious aspect was that we shared the positives and the negatives.*

Sarah: *Yes, and we also had the opportunity to demonstrate our work.*

Colin: *I had a vague idea about IBL and only knew the method I was trying to implement, the one method that I thought made sense. When I came here I learned about other ideas from my colleagues, I tried them out and saw the results, then came back and picked other ideas always improving on my previous knowledge of inquiry.*

Janet: *Initially I thought I just had to solve one problem (teaching through IBL) yet I found out that this led to other minor challenges that became evident during the follow-up meetings, because we were reflecting more deeply.*

Similar to the findings of the English project RECME (see Back et al., 2009), teachers reported that they valued practical experiences but, more importantly, opportunities to share and demonstrate their work. For most of the teachers, LTMI was effective because of the co-learning opportunities generated by the CoP – learning with and from others, as highlighted by Colin. For others like Janet, the CoP experience helped to uncover and address her ongoing challenges that emerged as a result of her reflective practice. It seems that the CoP contributed significantly towards the effectiveness of LTMI because teachers learnt from getting to know about the teaching methods of colleagues working in different contexts (see Butler et al., 2004; Putman & Borko, 2000). This appears to have provided them with confidence to persist and support in not giving up. Indeed, teachers described both cognitive and behavioural changes – in their knowledge of IBL and how they implemented it in their class.

However, one aspect of LTMI appeared to be puzzling for two of the participants. Janet and Greta encountered dilemmas during their CPD journey. Janet, for example, was overwhelmed by the content discussed in the summer workshops. During the focus group discussion, she claimed *“Following the summer workshops, I felt lost and couldn't make sense of what an IBL task could be like”*. The three-day CPD workshops, held in September, seemed to be too intensive for Janet. She struggled in coming to terms with understanding IBL and, hence, in identifying and choosing tasks for inquiry. The ‘block’ 12-hour sessions, held just before the scholastic year, appeared to disorient Janet in terms of translating knowledge into practice. Greta who, unlike Janet, had gained prior knowledge of IBL from her Masters course, shared a similar view. She also struggled with understanding what IBL involved and how she could successfully enact it in class. In the interview following her participation in the summer workshops, Greta spoke about this dilemma saying *“I am still unsure whether a task promotes and supports IBL”*. Moreover, during the follow-up meetings, she also repeatedly asked the facilitator to explicitly tell her whether a particular task or lesson she was doing could be classified as IBL.

Transferring knowledge from the workshop activities into the classroom is not straightforward (see Stein, Smith, & Silver, 1999). For Greta and Janet, in particular, acquiring a new body of knowledge and skills and developing new habits of practice seemed complex, and the summer workshop structure did not seem to facilitate this. Introducing teachers to new pedagogies (in this case, IBL) and building capacity to understand and enact them requires careful and well-thought designs to professional development programmes, structures and strategies. Using the three-day block INSET days available in July and September was the approach adopted in the LTMI programme. While it offered teachers the ‘whole package’ of content and the opportunity to learn about IBL before starting the scholastic year, it appeared to deny a more gradual introduction of new material over a longer period of time – at least for these two teachers.

Results and Conclusions

This paper dealt with teachers’ motivation to undertake a CPD programme. In addition, it has taken the teachers’ voice in developing an understanding of what ‘effective’ CPD means for these teachers themselves in terms of their experiences, and reactions to the LTMI activities, their learning, and changes in classroom practices. Guskey (2000) argues that factors that motivate teachers in their practice need attention in bringing about changes in classroom practices through professional development. In other words, professional developers need to consider teachers’ motivation, reasons and

needs for participation, and address these within their designs.

In Malta, participation in CPD may occasionally turn out to be an individual teacher's decision to attend a voluntary course. This decision rests on a number of motivations. Teachers in this study chose to participate in LTMI for three main reasons: (1) to develop their knowledge of IBL; (2) their beliefs about the benefits of teaching through IBL; and (3) the need to change their practices. Such findings are also reported in other studies (Anderson, 2008; Back et al., 2009). These motivations are rather personal and intrinsically motivated. The teachers participating in this study, sought to improve their knowledge and change their practices not because it was imposed on them, but as a result of their preconceptions of what IBL could provide in terms of knowledge about teaching mathematics.

The CPD programme design recognized the situative perspective (Lave & Wenger, 1991) in the process of LTMI. Lave and Wenger (1991, p. 98) view a CoP as "a set of relationships among persons, activity, and world over time". Two aspects of this theory emerge as relevant to CPD: (1) teachers' learning is enhanced by their participation in a CoP where they are supported by other members of that community; and (2) teacher implementation of reform is supported by practice-based experiences over a period of time. These two aspects resonate well with the findings arising from this study. Teachers' views of what makes LTMI 'effective' can be categorised under two aspects: the CPD approach and the CoP experience. Data indicated that teachers valued participation within a CPD approach that promoted an immersion in active learning that was hands-on, self-directed, self-regulated and involved a reflective engagement for learning. This was indeed sustained by a community that provided teachers with a shared sense of belonging to a CoP that was supportive.

The data also suggests that when CPD is designed with an approach that values and respects teachers' knowledge, when teachers are active learners and free to use the knowledge gained at their own pace (O'Sullivan & Deglau, 2006), their experience is likely to be a positive one. Teachers also indicated the modelling of IBL as effective towards their understanding and learning of teaching through inquiry. Farmer et al. (2003) argued for modelling learner-centred CPD materials as this enabled the teachers in their study to embrace new ways of teaching and integrate them into their professional practices. Four common recommendations for effective CPD include a focus on mathematical content, the use of activities that actively engage teachers in learning, planning for sustained time to learn, and developing a CoP (Garet et al., 2001; Guskey, 2000). These features of CPD are aligned to the responses teachers in this study gave related to their experiences contributing to making LTMI effective. Data suggested that teachers view LTMI as effective when it has collective participation of teachers from different schools. An added

source of learning for teachers was their reflective practice especially when carried out with others and over a prolonged period of time. Teacher empowerment is seen to stem from such prolonged engagement in social interaction. As the cases of Sarah and Chris show, teachers started to build confidence and see themselves as making personal contributions to knowledge development. Yet, particularly in the initial phase of LTMI, Greta and Janet struggled with their development of IBL. The summer workshop structure adopted appeared intensive for them and did not facilitate the gradual introduction of new material.

This paper contributes to knowledge by focusing on the LTMI learning experiences of teachers. CPD is usually designed to stimulate change from old ways of working to new and unfamiliar practices. Just like students, teachers need to be supported to learn new knowledge (Mansour et al., 2014). The evidence-based argument in this paper, stemming from the theoretical position it has taken, is that teacher learning is best 'enabled' through long term, ongoing, practice-based, self-regulated and CoP oriented CPD, in which reflective practice and networking are at the heart of the programme. Yet, addressing this implies rethinking the way CPD is planned. Bubb and Earley (2013) argue that we need not necessarily find more time but instead make better use of the time available for professional development. While using time effectively to address issues that matter for teachers is important, the argument I make here changes the focus towards *making time for collaboration*.

For the seven Maltese teachers in the study, their ongoing interaction with other colleagues to discuss their work and that of their students was key to them in developing and sustaining deeper practice-based understandings of IBL. Making time for collaboration, an important characteristic of high-quality CPD (O'Sullivan & Deglau, 2006), implies rethinking structures that provide teachers with on-the-job opportunities to meet, share, discuss and learn from one another. In Malta, but also in other countries like England and Norway (see Bubb & Earley, 2013) teachers have specific time allotted for their professional development – during and after school hours, and during training days when schools are closed for students (INSET days). However, these statutory training periods appear to offer limited opportunities for teachers to meet on a regular basis. Professional development time needs to be embedded within teachers' practice on a weekly basis – it needs to address a cultural shift in teacher learning that involves careful design, support structure and time (Stein et al., 1999). Making time for collaboration entails empowering teachers to take personal initiative in identifying needs and working with others to address these. But, more importantly, making time for collaboration requires a supportive climate (Fullan, 1993) where environments and support structures assist and motivate teachers to learn at their own pace in unhurried and non-threatening ways.

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The Mathematics Teacher who became a Promoter of Inquiry-Based Learning: A Story of Teacher Change

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Abstract: This paper presents the story of John, a mathematics teacher, who embraced 'change' at a rather advanced stage of his teaching career. As part of this development, he managed to transform his largely traditional practices to practices that advance inquiry-based learning, a pedagogical approach that is aligned to the reform visions for mathematics teaching and learning. Moreover, John is now also committed to promote this 'new' approach among other mathematics teachers. Drawing on narrative research, his case was studied to shed insights on what facilitates or hinders teacher learning and change. The narrative was co-constructed between John and the author in the form of a 'conversation' that originated from a number of Messenger chats on Facebook. The thematic analysis of the data revealed four distinct phases, so far, in John's journey towards becoming a teacher. The journey through these phases is of particular interest to anyone concerned about the impact that different teacher education initiatives have on teacher learning and change. Overall, John's story suggests that teacher change, while possibly not linear and enduring, can happen and appears to be facilitated by certain factors. These include willingness and capacity on teacher's part to change, the availability of opportunity to change, the development of a professional learning community, and the presence of someone at school who is capable and willing to lead and support teacher learning among colleagues.

Keywords: teacher change; teacher learning; teacher education; narrative research

Introduction

The focus of this paper is teacher change. More precisely, it is about change that affects what Clarke and Hollingsworth (2002) refer to as the 'personal

domain' of teachers, leading in the process to fundamental shifts in their beliefs and practices. While acknowledging that teachers learn a lot by teaching (Richardson & Placier, 2001; cited in Steinberg, Empson, & Carpenter, 2004) and as such do not necessarily require specific programmes or structures to learn (see Postholm, 2012; Attard Tonna & Shanks, 2017), change as understood here is much deeper than the 'growth' that is normally associated with established teachers (Clarke & Hollingsworth, 2002). Consequently, given that this paper is concerned with this form of transformational change in teachers, the word 'change' is used throughout to signal what Golding (2017) terms 'deep change', not the growth that derives, almost naturally, from teachers' extended experiences in schools.

In particular, this paper explores the story of a mathematics teacher – whom I am calling John – from his undistinguished beginnings, some twenty-five years ago, as a temporary contract teacher in a secondary school without any kind of teacher education to become one of the more prominent promoters of inquiry-based learning (IBL) in Malta. The change, which occurred during the latter stretch of this journey, saw John renouncing his long-standing “narrow views of mathematics and mathematics pedagogy that include conceptions of mathematics as a closed set of procedures, teaching as telling, and learning as the accumulation of information” (Lloyd & Frykholm, 2000, p. 576). He began embracing instead constructivist learning theories that encourage learners to be active constructors of their own understandings by engaging in activities that include exploring, justifying, proving, critiquing, and generalizing the ideas, representations, and procedures of their solution strategies (see Simon & Schifter, 1991). Such activities reveal an understanding of teaching as a dynamic process of inquiry into student reasoning, which is in direct contrast to the traditional notion of equating teaching to a process of transmitting a set of procedures (Zech, Gause-Vega, Bray, Secules, & Goldman, 2000). The learning benefits linked to the adoption of IBL in class appear to be significant. Hattie (2009) concludes from his analysis of the literature that these include “transferable critical thinking skills as well as significant domain benefits, improved achievement, and improved attitude towards the subject” (p. 210).

In this paper, I am primarily interested in gaining insights into what led John to change his beliefs and practices at a rather mature phase of his teaching career, why this change did not materialise before, and the prospects that his propensity for change has become sustainable and self-generative. His

trajectory towards change – which continues to run across a variety of teacher education experiences and transforming educational scenarios – can contribute to a better understanding of how teachers learn. The dynamics that drive this form of professional learning comprise changes in both the cognition and the instructional practices of teachers (see Levin & Nevo, 2009). Teacher change thus involves changing the person, and this implies in turn changing the life of that person (Hargreaves, 1997). The fact that attempts to impose change on teachers have been notoriously unsuccessful (Sikes, 2002) makes it even more crucial that one tries to understand what drives change in teachers. This understanding could then be the basis on which the development of ‘great professional development’ actually leads to ‘great pedagogy’ (see Stoll, Harris, & Handscomb, 2012).

John’s story can be very helpful in this respect. Without claiming representativeness or replicability, I am convinced that his story – which represents a single case study of teacher change – has the potential to offer a rich and holistic account that can provide important insights about the phenomenon (see Merriam, 1998). As such, it is worth divulging, analysing and reflecting upon. His story is narrated here, with accompanying reflections and commentary, along a number of sections. First, the reader is provided with information on John’s professional development and his teaching and other professional experiences over the years. The literature is then revisited to shed light on the complexities that characterise teacher change. The next section provides the background to the methodology and methods used in this research. The research findings come next, providing details about the four distinct phases that were identified in John’s professional career, so far. The insights and implications of these findings for teacher education and teacher learning are discussed in the subsequent section. The final section makes the case for reflection on John’s story and how this can inspire change in people.

John’s Professional Development and Career Pathways

John, who is in his early 40s, has been teaching mathematics at secondary level (ages 11 to 16) for almost twenty years. His decision to become a teacher can be described as ‘vocational’ (see Osborn & Broadfoot, 1993) since he had always desired to follow a teaching career. At age 18, he failed to obtain one of the entry qualifications to join the four-year Bachelor of Education (Honours) degree programme at the University of Malta which was, at that

time, one of the routes to become a warranted teacher in Malta. So John applied and was accepted to become a secondary school mathematics teacher on a temporary contract. Although it was particularly challenging for him to teach without any initial teacher education (ITE) in what was considered to be a 'difficult school', his resolve to become a teacher actually strengthened during this first year. Consequently, having attained his missing qualification, he enrolled the following year in the B.Ed. (Hons.) course with, as was customary in those days, two specialisations. His specialisations were primary education, in which students are prepared to become primary school teachers, and mathematics education, in which students are prepared to teach mathematics in secondary schools. Midway through the course, when asked to decide between the primary track and the secondary track, John chose to focus on becoming a secondary school mathematics teacher. This secondary track specialisation seeks to develop 'professional knowledge', 'professional judgement' and 'subject knowledge' (see Leask, 2009) in students by presenting them with undergraduate mathematics content courses, courses in educational theory and foundation disciplines, and general pedagogy and subject methodology courses. Moreover, in line with the curriculum of the B.Ed. (Hons.) programme, students following this track have a number of field experiences, the most notable being the two six-week block teaching practices in schools, one during the third year and the other during the fourth year of studies.

As a graduate teacher, John once again spent his first year of teaching in a state secondary school perceived by many as being 'difficult'. This school catered for students following vocational education. The following year, he was posted to another state secondary school, in which he has remained ever since. At the time of his arrival, this school aimed primarily to educate students who are more academically inclined and consequently more likely to continue with post-compulsory studies along the academic route. Over the years, however, as a result of policy developments in the local education system, the school had to discard its selective student intake policy to embrace comprehensive education policies that are based on the premise that, for both social and pedagogical reasons, it is wrong to select and segregate students (see Edwards, Whitty, & Power, 2002). Throughout his long teaching career at this school, John has predominantly taught students in their first year of secondary education. So far, only occasionally has he taught second and third year classes, and never classes in the final two years of secondary schooling. He pointed out, however, that it is the school administration, at

times in consultation with the heads of department, which decides the class allocations. In recent years, moreover, John has been on a reduced teaching load in view of his other responsibilities and duties at school.

John regularly attends the continuing professional development (CPD) sessions mandated by the sectorial agreement between the Government and the Malta Union of Teachers (see Ministry of Education, Youth and Employment, 2007). This agreement stipulates that teachers in Malta have to attend a three-day session each year, for a total of twelve hours of CPD. Secondary school teachers are normally grouped for these sessions by their subject area. These CPD out-of-school sessions usually adopt a traditional training-focussed perspective that, contrary to what happens when the perspective is learner-focused, does not present professional learning within the specific social contexts of teachers' practice (Attard Tonna & Shanks, 2017). The sectorial agreement specifies further that once every term teachers are to attend professional development sessions, each lasting two hours, organised by their school. These additional six hours of CPD, which are held after school hours, offer greater opportunities for situated professional learning as the senior management team (SMT) can link sessions to the implementation of the school's action plan and teachers can propose themes that arise from their professional needs and concerns. Apart from these mandatory professional development sessions, John also participates in other occasional CPD activities organised by the mathematics education officers (EOs) within the Directorate for Learning and Assessment Programmes. In recent years, moreover, he has begun to lead CPD sessions for mathematics teachers, both within and outside his school, that promote IBL pedagogy in mathematics classes.

The Complexity of Teacher Change

Internationally, the traditional approach to teacher learning as part of becoming a teacher tends to follow this route: First, prospective teachers are expected to enrol in an initial teacher education programme and then, once they join the profession, it is often mandatory that they attend, from time to time, some form of formal activities or events that take place either inside or outside schools (Attard Tonna & Shanks, 2017). The hierarchical nature of this approach positions teacher learning at the receiving end of expert power that exists and operates outside teachers (Barab, MaKinster, Moore, & Cunningham, 2001). Moreover, not only is the journey towards becoming a

teacher depicted as a simple and linear operation, but teacher learning is presented as a largely decontextualised activity (Attard Tonna & Shanks, 2017) in which individuals, even after they gain teaching experience, are viewed almost as if they are objects waiting to be 'in-serviced' (Wideen, 2002). The dynamics of this approach effectively ignore current conceptions of teaching and learning, such as constructivism, and do not reflect the situatedness of knowledge (Korthagen, Loughran, & Russell, 2006). As a result, the professional development of preservice and inservice teachers contrasts sharply with the very same approaches to teaching and learning that their professional education is trying to inculcate in them (Korthagen et al., 2006).

As a way out of this conundrum, Korthagen (2017) suggests that the professional development of teachers needs to be modelled on the robust body of available knowledge about how teaching can have a more positive impact on student learning. To achieve this, for a start, the journey towards becoming a teacher should be recognised for what it is. It is both complex and idiosyncratic (Flores, 2011), and this needs to be reflected in the preparation of preservice and inservice teachers. In order to break the dominant circle of traditionally trained teachers who teach in a traditional manner (Stofflett & Stoddart, 1994), one therefore has to develop professional development programmes and structures that make it possible for prospective and inservice teachers to alter pre-existing personal beliefs and images of what constitutes teaching and being a teacher (see Flores, 2011). The ultimate aim should be to change what happens inside classrooms because, as Wiliam (2010) points out, it is not enough to change what teachers know and believe unless they also change their practices. In all this, however, attention should be given to teachers' great 'sense of practicality' that determines actions according to their perceptions of what works and does not work within a specific context (Hargreaves, 1994a). So ingrained is this sense that teachers invariably resist change initiatives, even when legally imposed, which direct them towards practices of which they are not convinced (see Sikes, 2002; Hattie, 2009).

The way forward thus appears to rely on a process of dialogue, negotiation and accommodation, not imposition (Durrant & Holden, 2006). Indeed, the traditional notion, now discredited, of viewing teacher education as a process of transferring knowledge

to teachers (see Korthagen et al., 2006) has been overtaken by calls for teachers to become “active agents of their own professional growth” (Schleicher, 2012, p. 73). This shifting of responsibility on teachers necessitates that preservice and inservice teachers are exposed to ongoing opportunities to engage in professional learning that builds on the understanding of learning to teach as a life-long endeavour situated in practice (see Meissel, Parr, & Timperley, 2016). The understanding here is that ‘teachers become learners’ (Easton, 2008; Hattie, 2009) who operate along a ‘learning-to-teach continuum’ (see Feiman-Nemser, 2001; Anderson, 2004). This repositioning would facilitate, in turn, the reconciliation of the divide between theory and practice in the professional development of teachers (see, for instance, Anderson & Freebody, 2012; Korthagen, 2017). This would allow teachers to “translate new views and theories about learning into actual teaching practices in the schools” (Lunenburg, Korthagen, & Swennen, 2007, p. 586). Should this happen, teachers would be far less likely to remain sceptical about the day-to-day relevance of their professional education (see Korthagen et al., 2006; Anderson & Freebody, 2012) and to resist change (see Sikes, 2002; Anderson, 2004).

The success of this reform relies, however, on giving proper attention to how teachers learn (Steinberg et al., 2004) and, consequently, what it takes to enable and support teacher change. The topic of teacher learning – which had remained under-researched for quite a long time (Clarke & Hollingsworth, 2002) – is now attracting considerable interest from the research community (see, for instance, Borko, 2004; Easton, 2008; Hattie, 2009; Anderson & Freebody, 2012; Postholm, 2012; Stoll et al., 2012; Attard Tonna & Shanks, 2017; Korthagen, 2017). Different people tend to emphasise different aspects of teacher learning, but there is general agreement on how teachers learn most effectively. For instance, noting that the core purpose of professional learning should be to improve student achievement and outcomes, Stoll et al. (2012) conclude from their review of the literature that

...effective professional learning is school focused, school based and school led, whilst also drawing in external expertise where appropriate. Great professional development incorporates into this mix professional learning experiences that are sustained and intensive, rather than brief and sporadic, and that are undertaken collaboratively. (p. 8)

Admittedly, their conclusion is based on evidence linked to the continuing professional development of teachers. Still, the knowledge that teacher education is now viewed as a continuum, spanning across ITE and CPD,

demands that teacher learning during ITE should not only lay the foundations for future learning, but that there should also be continuity and alignment between one phase and the other. Teacher development is conceptualised in fact as a 'system' in Australia, Canada, Finland and Singapore, all of which are at the forefront of teacher education (Darling-Hammond, 2017). In each of these countries, "these systems include multiple, coherent and complementary components associated with recruiting, developing, and retaining talented individuals to support the overall goal of ensuring that each school is populated by effective teachers" (p. 294). In line with this notion of 'system', a number of characteristics of successful ITE programmes identified by Darling-Hammond (2006) mirror the spirit of what constitutes effective teacher learning as part of CPD. These include coherent learning experiences, extended and connected field experiences, links between theory and practice, and strong school-university partnerships.

As a result, looking at the wider international picture, one gets the feeling that, as advocated by Korthagen et al. (2006), we might be witnessing the development of an overarching pedagogy of teacher education. This 'new' pedagogy - in direct contrast to the traditional theory-into-practice approach to teacher education (see Korthagen et al., 2006) - places schools and practice firmly at the centre of teacher learning. Moreover, the expectation now is that "through collaborative enquiry teachers become generators of professional knowledge, agents of change and critical friends for each other" (Zellermayer & Tabak, 2006, p. 34). This approach - which recognises and relies on the professional experience, judgement and expertise of practitioners (Sikes, 2002) - requires teachers to resist the 'balkanised' culture of their work, which often sees them retiring into the isolation of their own classroom practices and keeping professional contacts with colleagues to a bare minimum (see Hargreaves, 1994b). By moving away from a life in schools partitioned from other adults, teachers open themselves to a myriad of learning experiences - such as group reflection and discussions, workshops and seminars, mentoring and coaching, and lesson study - that will help them improve their professional knowledge and develop new instructional practices (see Meissel et al., 2016). Although professional learning can also happen in the context of the individual teacher (Borko, 2004), the social dimension of learning, which requires teachers to operate in dynamic interaction with each other, needs to be recognised as an essential feature of teacher education. For it lessens the likelihood that teachers adopt what Bissessar (2014) terms an 'egg crate' model of instruction, which is both self-

contained and self-referencing, and consequently counterproductive to change.

A lot, therefore, seems to depend on the formation and nurturing of some form of professional cooperation among teachers who are not necessarily from the same school. These 'professional learning communities' – which are also referred to by a number of other names (see Willemse, Boei, & Pillen, 2016) – give teachers the opportunity to work informally with colleagues who share the same passions and concerns, who are facing the same type of problems, and who are equally interested to deepen their knowledge and expertise (Kosnik, Menna, Dharamshi, Miyata, Cleovoulou & Beck, 2015; cited in Willemse et al., 2016). The characteristics of such communities include shared values and vision, shared responsibility, reflective professional inquiry, collaboration and the promotion of both group and individual learning (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006). In this supportive environment teachers learn by re-examining what they do and how they might do it differently – a process that leads to the evolution or moulding of new practices from existing classroom practices (Harrison, Hofstein, Eylon, & Simon, 2008). Thus, the development of what Hargreaves (2000, p. 165) defines as 'professional cultures of collaboration' has the potential to address the theory-practice divide that has long been a perennial problem of preservice and inservice teacher education (see Korthagen, 2017).

Choosing a Research Methodology and Implementing the Study

The methodology used in this study echoes the strong personal and professional relationship that I had established with John when we were both involved in the EU-funded project entitled *Promoting Inquiry in Mathematics and Science Education across Europe* (PRIMAS) that sought to promote IBL in twelve European countries (see <http://www.primas-project.eu/>). In PRIMAS, which was implemented over a three-year period (2010-2013), I was part of the University of Malta team leading the project in Malta and John was one of the mathematics teachers who had agreed to explore the implementation of IBL pedagogy in his mathematics classes. His participation involved working collaboratively with a school-based group of mathematics teachers that met regularly throughout the project, under the guidance and support of their head of department (HoD), to discuss, plan and evaluate mathematics lessons that foreground inquiry-based teaching and learning strategies. John's HoD, who I am calling Paul, was one of the project's so

called ‘multipliers’ who were responsible for leading school-based CPD sessions that promoted IBL to groups of participating teachers. Although Paul attended the regular meetings held between the University team and the multipliers, which served to deepen our understanding of IBL and to discuss how best to proceed with the implementation of PRIMAS in schools, his interest in and knowledge of IBL pedagogy well preceded his participation in the project. By sheer coincidence, however, the onset of PRIMAS fitted with Paul’s plans, as a recently appointed HoD posted in a new school, to stir his colleagues away from what he considered as essentially traditional approaches to the teaching and learning of mathematics. Notwithstanding these plans, the teachers’ participation in PRIMAS remained strictly voluntary. In fact, some teachers decided not to participate in spite of being offered a reduction of two lessons per week, for the duration of the project, to compensate for the extra PRIMAS school meetings and corresponding work.

When asked by the PRIMAS international partners to produce case studies that focus on teachers participating in the project, I opted for John after I had attended a couple of project CPD sessions led by Paul. I was struck by John’s apparent willingness to change in spite of patently fearing the potential consequences of the change he sought. This ambivalence intrigued me. Thus, in agreement with John and Paul, and after making all the necessary access agreements with the school’s SMT, I observed John teach on a couple of occasions and conducted short interviews with him both prior and after the observations. The resulting case study depicted a teacher who was starting to enjoy a new way of teaching, someone who was on the verge of embracing a new teacher identity in spite of his lingering concerns related to the contextual practicalities of introducing IBL in mathematics classes (see Buhagiar, 2013). My contact with John, both personal and professional, continued to flourish after PRIMAS.

Indeed, after PRIMAS, I had numerous occasions to witness how John was growing in his knowledge of IBL and in his commitment to promote this pedagogical approach. I noted this whenever I was invited to observe him teach and each time I heard him speak about his teaching with both practising and prospective mathematics teachers. But it was during a particular CPD session that John was conducting for a small group of inservice mathematics teachers that I fully realised the extent of his professional transformation. Constantly referring to his own classroom practices, he spoke competently, confidently and enthusiastically about IBL. Moreover, he kept reassuring the

teachers present about the same concerns that I had first seen him, years back, express during the PRIMAS CPD sessions led by Paul, his HoD. Once again, John intrigued me. This time, however, I was eager to gain insights into how a teacher can pass from one understanding of teaching to another at a rather mature phase in one's career and in the process become a promoter of this new understanding. All I had to do was ask John. He immediately accepted to share his 'story' with me in the knowledge that, although I intended to publish the research findings, his identity would be protected and that no harm would come his way (see Burgess, 1989). Our comfortable and non-judgemental research relationship, as had happened before during PRIMAS, was built on mutual respect, trust and care. In line with our agreement to engage in genuine collaboration leading to the co-construction of 'his story' (see Squire, Andrews, & Tamboukou, 2013), this paper is being published after John read it and gave his consent.

My desire to explore in depth the particularity and uniqueness of John's story channelled me towards the adoption of a single case study that uses qualitative methods within an interpretive paradigm (see Simons, 2009). Moreover, the inherent potential of the case study approach for story-telling (Simons, 2009) suggested a methodology that draws on narrative research that, as Gudmundsdottir (2001) points out, involves the analysis of collected narratives, or stories, to study how individuals experience their world. Given that people do not experience 'things' in isolation, it is important that these narratives capture both the individual and the context (Moen, 2006). Aiming for this kind of overarching data, I decided to co-construct a narrative with John through online conversations using Messenger, the instant messaging service of Facebook. I saw in Facebook, which is fast becoming one of the preferred tools for professional collaboration among teachers (Bissessar, 2014), the possibility to engage in the dialogical construction of a story (see Bakhtin, 1981; cited in Squire et al., 2013) with someone I know well. Apart from the convenience of chatting at mutually convenient times from the comfort of our homes, it was always someone I can relate to and understand at the other end of my computer. John and I, however, met once at his school before we began to interact on Messenger in order to discuss the content of our 'conversations' and the logistics involved. In total, we amassed eight hours of chatting spread across six sessions over a five-week period. This online activity produced a nine thousand word narrative crafted from a carefully edited version, negotiated with John, based on the original messages shared on Messenger. A thematic analysis of this data (see Boyatzis, 1998)

identified four key phases, so far, in John's professional journey that has seen him evolve from a largely traditional teacher to become a promoter of IBL pedagogy.

The Four Phases of John's Story

Research on the work and lives of teachers suggests that they pass through different phases throughout their careers (Leitch, 2010). In this study, for instance, I noted how John's approach to professional learning changed from his initial identification with what Hargreaves (2000) terms as 'the pre-professional age' to an eventual understanding that is based on what Hargreaves (2000) terms as 'the age of the collegial professional'. Basically, John moved from a model that is characterised by practical apprenticeship and improvement through individual trial-and-error, to a model that sees teachers increasingly turning to each other for professional learning, a sense of direction and mutual support (see Hargreaves, 2000). This significant development occurred over four distinct phases, to which I now turn my attention.

Phase One: Tranquillity and Passivity

This phase in John's teaching career spanned roughly across fifteen years: from the year he spent as a teacher on a temporary contract before enrolling in the B.Ed. (Hons.) course right until he came in contact, through a colleague at school, with what was then for him a 'new' approach to the teaching and learning of mathematics. Asked to describe his pedagogical approach before and after he attended the ITE programme, John practically depicted an unchanged pedagogical scenario (see Table 1). It was as if his ITE experience had had no real impact on his teaching practices. This 'teaching as transmission' approach (see Zech et al., 2000), which continued to dominate his teaching right through this phase, was embedded within what Romberg and Kaput (1999) identify as the traditional three-segment lesson that exposes students to a cycle of exposition, practice and consolidation. Consequently, John's teaching style at this stage did not conform to constructivist learning theories which build on the notion that "learners actively construct their own understandings rather than passively absorb or copy the understanding of others" (Simon & Schifter, 1991, p. 310).

Before ITE	After ITE
<p><i>I used to teach in a very traditional manner by writing on the board many examples and the students would copy ... then they would do classroom and after that I'd give them homework.</i></p>	<p><i>My teaching was traditional. I used to teach by first presenting students with examples, after that students would work on their own, and then I would give them homework.</i></p>

Table 1: John's approach to the teaching of mathematics before and after ITE

Moreover, John's descriptions in Table 1 strongly suggest that his participation in ITE and CPD sessions during this period practically had no, or very little, effect on his pedagogy. Agreeing with this assessment, he even alleged at one point that the B.Ed. (Hons.) course had not exposed him to pedagogies other than what he now considers as traditional pedagogy. This adds weight to Kagan's (1992; cited in Flores, 2011) claim that ITE at times reinforces rather than challenges the prior beliefs of prospective teachers. Using hindsight, he conceded however that his lack of pedagogical change might have also resulted from an inability to enact in practice his intellectual understanding of theory, which according to Darling-Hammond and Snyder (2000) is a major problem in teaching and teacher education. On a more positive note, he stated that his teacher education during this phase of his career, especially throughout preservice training, had familiarised him with a variety of teaching resources and technologies that had rendered his teaching somewhat less traditional.

After the B.Ed. course my teaching remained traditional, but maybe less than before. During the B.Ed. course, and also in some CPD courses I attended, I found it helpful to learn about the use of different resources and technologies ... while before it was just talk-and-chalk, now I began to use handouts and so on. But this did not change the essence of my teaching. Even during the B.Ed. Teaching Practice, my teaching was traditional. And this situation did not change for many years after I started teaching ... the centre of my teaching was the teacher, not the students!!

Reflecting on this phase of his teaching career, John said that it was only in recent years that he began to comprehend that the introduction of new resources and technologies does not necessarily lead to improved pedagogy (see Tamim, Bernard, Borokhovski, Abrami, & Schmid, 2011). The realization that such 'tools' can only be effective as far as they allow teachers and

students to reach the desired instructional outcomes (see Tamim et al., 2011) led him to acknowledge that, in spite of his innate inclination to seek change and improvement, his approach to teaching and professional development had remained unchanged for a very long time. Indeed, during that period he retained his view of teaching as a technically simple activity and considered professional development as something that teachers acquire as they ‘experiment’ on their own inside their classes (Hargreaves, 2000). At that time, this situation represented ‘normality’ for him, something that is part and parcel of teachers’ professional lives.

Quite frankly, I used to find it easy teaching in a traditional manner ... it's always the same routine and doesn't require much effort from the teacher. And there was no one at school to lead us, to inspire us at that time ... I guess the system was like that then, cause no one ever tried to make me do things differently. Another thing ... we all used to work on our own. The maths teachers only met occasionally, say, to be informed about something, to hand in the schemes of work, to decide who will be doing the exam papers and things like that. We never met to plan lessons together, to discuss difficulties ... there was no collaboration then!

Research in Malta (see, for example, Bezzina, 2002; Attard & Armour, 2005; Buhagiar & Murphy, 2008; Brown, Gauci, Pulis, Scerri, & Vella, 2015; Attard Tonna & Shanks, 2017) repeatedly suggests that the professional isolation among teachers portrayed by John is the norm. He referred in fact to the prevalence of this situation in local schools to explain why, at that time, he used to accept it and saw no need to change it. It was only later – during phase two of his story – that he began to realise how teacher isolation, which in reality is an international phenomenon, stifles teachers’ professional development and consequently affects negatively the quality of teaching (see Biddle, Good, & Goodson, 1997; Hattie, 2009; Saha & Dworkin, 2009).

Phase Two: Enthusiasm and Turmoil

By and large, phase two spread over a four-year period, in the middle of which John and his colleagues at school were involved in the PRIMAS project. Although John was reportedly comfortable with his professional life throughout most of phase one, he claimed that towards the end of that phase he had become increasingly dissatisfied with his traditional teaching routine. Consequently, believing in his ‘talent’ to engage in more intricate forms of teaching than the transmission method, he yearned for change. John stressed, however, that the real ‘spark’ for change was the arrival in school of Paul, the new HoD.

First of all, I was bored teaching practically in the same way ... I love change. I'm always doing that something extra to avoid the vicious circle of monotony ... I also think that I have the talent to teach beyond the comfortable cycle, surely for the teacher, that relies on drilling and memory recall. But then I do not think that a teacher can change on his own ... that's certain!! So the arrival of Paul was for me a turning point, a spark ... he rekindled in me the flame that was dying out because I had fallen into the trap of traditional teaching!

John was speaking here about what led to his initial steps towards change. From the extract above, it is clear that at the time of our 'conversations' he could distinguish between 'change' and the 'growth' that teachers normally acquire over the years through their teaching experiences (see Golding, 2017). This understanding began to develop during phase two of his career: Indeed, the introduction of new resources and technologies in class, which was considered as a sign of change during phase one, was re-dimensioned to a sign of growth from phase two onwards.

Change, as understood in this paper, appears to have been motivated by three main factors. First, John's professional boredom towards the end of phase one arose, at least partially, from his self-declared love for change. Although, up to that point, this love reportedly led to growth rather than change, he remained a teacher with a 'willingness to change' that, as Hattie (2009) notes, suggests a disposition to seek better alternatives even at the cost of discontinuing the use of familiar practices. The second factor has to do with the perceived complexity of different teaching approaches. The manner in which teachers teach has not changed much over the past two centuries, and the transmission model continues to dominate (Hattie, 2009). Understandably, the long-standing tradition and technical simplicity of this teaching approach (see Hargreaves, 2000), in addition to the fact that it does lead to some form of learning (e.g., facts and skills in mathematics), make it attractive for teachers to adopt. Indeed, teachers are known to 'wash out' the pedagogies encountered during ITE and adjust to traditional ways of teaching once they join the profession (Korthagen et al., 2006). Although John did not personally experience this adjustment, he was aware from the beginning of phase two that it would be more complex to work within non-traditional models of teaching. He confided that had it not been for his belief in his 'ability to change', it would have been much harder for him to venture beyond his transmission comfort zone. Put differently, demonstrating a good measure of self-efficacy (see Bandura, 1997), he decided at that stage that he

has what it takes to meet the higher pedagogical demands of implementing IBL in class.

While the first two factors – which echo Spillane’s (1999) reference to teachers’ will and capacity to reconstruct their mathematics practices – are linked to John, the third factor is extraneous to him. Indeed, it is linked to Paul’s arrival in school. John repeatedly emphasised throughout our ‘conversations’ that his change was primarily the result of meeting Paul and working alongside him for a number of years. The advent of the new HoD was conceived by John as his ‘opportunity to change’. It was as if the encounter with Paul had created a working space for John in which he – very much in line with Vygotsky’s (1978) notion of the zone of proximal development (ZPD) – could now develop, in collaboration with other colleagues at school, under the guidance of a more capable teacher in ways that he could not do before on his own. Participation in this space was voluntary. Paul had created a parallel, two-tiered system in which, while all mathematics teachers attended the ‘normal’ departmental meetings, only volunteers, like John, attended the extra sessions that were linked specifically to PRIMAS.

ITE	CPD Courses	Paul and Colleagues
<ul style="list-style-type: none"> • mandatory participation; • ‘transfer’ of theory; • theory at university and practice in school; • student teachers expected to bridge on their own the gap between theory and practice; • teaching in isolation; • lack of support in school. 	<ul style="list-style-type: none"> • mandatory participation; • held outside school; • one-off and short duration; • delivered by experts; • passive participation; • issues identified by others and not necessarily relevant to own experiences; • lack of support in school. 	<ul style="list-style-type: none"> • voluntary participation; • held inside school; • ongoing and sustained; • collaborative approach; • active participation; • issues identified together; • cycles of planning, implementing, observing and evaluating lessons together; • ongoing support in school.

Table 2: Key characteristics of John’s different learning experiences

The information provided in Table 2 is based on John’s descriptions of his different professional learning experiences during phase one and phase two.

As evident from this table, his collaboration with Paul and other colleagues at school contrasts sharply with his ITE and CPD experiences, both of which presented him with traditional approaches to teacher education that, as Barab et al. (2001) point out, are based on expert power and are hierarchical in nature. Moreover, the embedded theory-into-practice perspective of these approaches, in which learning is perceived as a decontextualised activity, is now being increasingly challenged in view of the limitations and inadequacies (Korthagen et al., 2006) referred to earlier on in the paper. On the other hand, John's experience in school with Paul and other colleagues mirrors many of the characteristics of effective CPD programmes (see, for instance, Darling-Hammond & McLaughlin, 1995; Anderson, 2004; Harrison et al., 2008; Stoll et al., 2012). A key feature of their approach was that their quest for change did not focus on practices in a vacuum: They acted instead as a group of individuals working collegially on their practices within the specific context of their school (Postholm, 2012). Within this voluntary group, contrary to when all the mathematics teachers met as a department, Paul did not assume the role of HoD. He acted here as a leader of teacher learning (see Postholm, 2012), while remaining himself a learner among learners. John reacted very positively to this bottom-up approach to teacher learning (see Korthagen 2017), which was a new experience for him, and his enthusiasm for teaching and learning was rekindled.

Paul simply inspired me, all of us I guess ... to give you an idea of how we worked together I'll tell you about PRIMAS. We were a group of about 5 or 6 teachers who used to meet twice a week to plan a lesson. And Paul was like our manager, someone to lead us but one of us just the same! We had marvellous teamwork, all of us supporting each other ... just imagine, a group of teachers would observe a lesson and we would discuss it afterwards. Before, I would have been petrified to let anyone in my class for fear that he'll either criticise me or 'steal' my lesson ... Still, in truth, during that period I remained sceptical about the introduction of IBL, as I was afraid that I'd not finish the syllabus ... and what about students' preparation for exams? At the same time we were getting this fantastic response from students ... Quite frankly, though, it was my faith in Paul and his constant support that kept me going in spite of my anxieties and fears!

During phase two, however, John remained tormented by a fundamental professional dilemma (for a detailed account, see Buhagiar, 2013). For while he felt touched, excited, part of a team and a much more competent teacher as a result of that experience, he was not so sure that he should actually practise what he was starting to perceive as 'good teaching', certainly not for most of the time. And this was out of fear that such pedagogy would backfire on students in an educational environment that in reality values other forms of

teaching (see Korthagen, 2004). It took a good measure of resilience on his part – considered by Golding (2017) as one of the necessary conditions for teacher change – and sustained collegial support, especially from Paul, to keep moving towards new ways of viewing teaching and learning while working in a system that insists on content coverage and thrives on examination success (see, for instance, Grima & Chetcuti, 2003; Buhagiar, 2004). This permitted him to move into phase three, which he readily acknowledged as the most gratifying period of his teaching career so far.

Phase Three: Conviction and Action

There was no defining moment when phase two stopped and phase three began. Phase three, however, came to an abrupt stop after practically three years when Paul left school to take up a new position. During this relatively short period John changed from a novice and hesitant practitioner of IBL to become not only a convinced and skilful practitioner, but also a promoter of this approach. Maass, Swan and Aldorf (2017) appear to have sensed his potential when they classified him among the mathematics teachers who had developed a rather complex view of IBL during PRIMAS in spite of having very little prior experience of IBL. His transformation, though, began in the months following PRIMAS. At that point, John faced an important decision: Should he put IBL behind him and continue teaching as before, with possibly some adjustments, or should he continue learning about, and working on, the implementation of IBL? Besides the enthusiasm and the intense professional learning that he had experienced during PRIMAS, his decision to continue was based on the realization that he could work the ‘new IBL ideas’ into his existing practices in a way that is both effective and acceptable within his school context (see Harrison et al., 2008).

My IBL lessons present students with activities that can take more than one lesson ... I present students with a situation or problem that they try to solve on their own, in groups ... I go round simply to observe their thinking and work, and only offer ‘help’ through questions. IBL puts students at the centre of learning and my role is to facilitate that learning. But time is the problem with IBL ... With experience I’ve learnt however to strike a balance between using IBL and more traditional teaching that exposes students to exam-like questions and techniques that they will need in examinations. But even here, although I still make use of practice and drilling, my approach has changed because in the non-IBL lessons I insert elements of IBL like open questions, group work, presentations, class discussions and so on.

Although John’s ‘solution’, which continues to this day, involves a mix of two types of lessons, in reality there is no mental separation between his so-called

IBL lessons and the rest of his lessons (see Maass, Swan, & Aldorf, 2017). As, indeed, each type offers what can be seen as a different interpretation of how IBL can be integrated within mainstream mathematics lessons. John claimed that had he had no concerns about his operating context, mostly in relation to the examination system, he undoubtedly would have chosen to teach mathematics exclusively through what he calls 'full-blown IBL lessons'. Instead, displaying a 'sense of practicality' (see Hargreaves, 1994a), he went for a mixed teaching approach that relies on judicious use of various characteristics of IBL without jeopardising student achievement in examinations. This harmonization of his teaching efforts (see Sedova, 2017), which arguably helped him survive the 'risky business' of introducing new practices in school (Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003; cited in Harrison et al., 2008), also permitted him to further his professional learning under Paul's guidance. John's other challenge, as he transitioned from phase two to phase three, was how to continue working with Paul, and possibly other colleagues, on IBL. A new way had to be found outside PRIMAS, a project that had offered participants a number of concessions, including a reduction in their teaching load and fixed weekly meeting slots.

After PRIMAS, although we still met as a department to discuss stuff like exams and syllabi, we no longer worked on IBL as a team. I think there were different reasons for this ... some teachers could have been put off by the amount of work involved, others were perhaps never convinced about IBL, while others left school. But I wanted to continue working on IBL even if it was going to be just Paul and me ... one of the PRIMAS teachers did join us however! After PRIMAS I worked even more closely with Paul and, apart from developing many IBL lessons, we created this big bond between us ... we spent so much time together and he was a great mentor! Just to give you an idea of how we worked ... Paul would often observe my lessons, and we even filmed lessons, so that we could afterwards discuss what worked and what worked less ... Never before had I learned so much about teaching than in these last few years!

The fact that some teachers decided after PRIMAS to stop collaborating on IBL suggests that, as Cuban (1984; cited by Hattie, 2009) claims, teachers may show signs of pedagogical change for a while when they are involved in some reform initiative of which they are not convinced. But this 'change' remains surface deep and classroom practices go back to normal as soon as the push favouring that particular reform begins to recede. On the other hand, John's disposition to change and the importance that he assigned to furthering his learning under Paul's tutelage resulted in much greater determination and involvement on his part after PRIMAS (Attard Tonna & Shanks, 2017). In itself, this development indicates how crucial it is that teachers become

committed to their professional learning. For it seems that once teachers become convinced of something, they would somehow manage to find the time and the means for it, even in the absence of enticing and accommodating concessions.

Developing a professional relationship with Paul that John likened to mentoring, they now became increasingly closer, even on a personal level. In what could almost be described as a one-to-one approach to teacher learning, John had a supported, sustained, ongoing and intensive professional learning experience that was grounded in reflection and experimentation (see Darling-Hammond & Mclaughlin, 1995). This experience continued to build on the professional learning that had started during PRIMAS, albeit in a much more intensive manner. In particular, John engaged with Paul and another teacher in what Harrison et al. (2008) refer to as an evidence-based approach to collaborative inquiry. Embedded within the developmentally effective action research cycles of lesson planning, observation, assessment and reflection (Stoll et al., 2012), this approach helped them gain insights into their practices and goals, leading in the process to the creation of shared professional knowledge (Harrison et al., 2008). At this point, John started gaining the reputation of IBL 'champion teacher', basically someone who has demonstrated a degree of professional development in spite of working in a context that is not particularly conducive to it (Rebolledo, Smith, & Bullock, 2016). This was also when he began accepting invitations, received mostly through Paul, to share his experiences and expertise with both preservice and inservice teachers. This development effectively led John to become a promoter of IBL among different audiences of prospective and practising mathematics teachers. His engagement in this 'multiplicity of social spaces' offered him in turn further opportunities for professional learning, further opportunities to deepen his change (Hodgen & Johnson, 2004).

It started when Paul asked me to help him promote IBL among teachers ... he wanted a normal teacher like me to be a testimonial during meetings that IBL really works. One thing then led to another ... I've presented in many teacher meetings, including formal CPD sessions, and I've often had student teachers observe me teach IBL lessons. Once I even presented with Paul and another colleague in a national teacher conference ... I also began promoting IBL with colleagues in school, most of whom came after PRIMAS and are still young and inexperienced ... Having the chance to convince other teachers to use IBL are unique experiences of which I'm proud. For me, promoting IBL is an opportunity to continue learning, an opportunity to do something good, an opportunity to push an idea in which I believe so much!

During phase three, John reached a state of professional fulfilment like never before. Most importantly for him, he was teaching in a way that largely mirrored his beliefs, at least as far as the school context would permit. By assisting Paul, moreover, he sensed he was serving his mission to disseminate a pedagogy in which he truly believes. His professional reputation was also growing in the meantime. Indeed, his ‘by teacher for teachers’ (see Smith, Bullock, Rebolledo, & Robles López, 2016) approach as he participated in numerous teacher learning activities was gaining him recognition and respect, well beyond his school, as a skilled practitioner and a promoter of IBL. On a personal level, he also felt privileged to be working side-by-side with Paul, someone he greatly admires and is devoted to. But this most rewarding professional period for John was dealt an unexpected blow when Paul left school to assume other responsibilities. This departure led to phase four of John’s story.

Phase Four: Affliction and Hope

Phase four has been going on for slightly more than a year now. In this relatively short period John has experienced what he considers to be his gloomiest moments as a teacher. Not only is he still ‘mourning’ the loss of his mentor and friend, but he is greatly concerned that life in the school’s mathematics department could now return to the teacher isolation practices that preceded Paul’s arrival.

I’m still in shock! I felt so down when he left ... I was truly devastated! I really miss him as there was this great bond between us! I continue to feel this big void in my life at school because we used to do so many things together. Just imagine what other things we could have done had he not left. Now I’m afraid that we’ll fall back to the apathy we had before Paul came ... this thing scares me and really saddens me! I don’t want to go back to how things were before Paul!

John had begun to realise in phase two, and even more so during phase three, that when teachers collaborate together within a supportive learning community they have the opportunity to grow professionally (see, for instance, Stoll et al., 2006). For him, becoming a skilled IBL practitioner and a promoter of this pedagogy were a direct consequence of shedding his prior isolationist experiences that reflect the ‘Just leave me alone to teach my way’ mantra that, according to Hattie (2009), is common among teachers. Having grown increasingly weary during phase one of this traditional way of being a teacher and noting the multiple benefits of professional collaboration, he does not want to revert to a way of operating that has serious consequences for

teacher learning and, as a result, for student learning (see, for instance, Saha & Dworkin, 2009). Consequently, noting what he considers as disquieting changes in himself and in his colleagues, John is trying to keep Paul's spirit alive within the department, but seems to lack the conviction that he will succeed.

This year, since Paul left, we have practically stopped doing what we were doing before ... I'm afraid we're heading back to everyone on his own! I'm trying to keep things going, but I'm not the HoD nor do I want to be at this stage. Today, for instance, I took it on myself to organise a meeting ... mind you, only for teachers who are interested ... to discuss how to continue developing IBL at school. But it's hard to get things done with no HoD ... we've all taken a big slumber! We all say we miss Paul ... but I notice that without him apathy is starting to creep in. I'm even neglecting the maths room ... Our departmental meetings nowadays are like noticeboards ... serve only to inform who is expected to do what and when!

The culture of teacher collaboration in the mathematics department had started with Paul's arrival some eight years back. While, admittedly, not all the teachers at any one time were part of this culture and many teachers have left school and others replaced them over the years, there has always been a group of teachers who voluntarily collaborated with colleagues on a number of projects, not just IBL, under Paul's lead. John, however, was the only teacher who had been and remained with Paul on this collaborative experience from the very beginning. Still, given the extensive time and effort dedicated to developing a culture of collaboration within the department, it is rather surprising that signs of diminishing team spirit and dynamics began to appear almost immediately after Paul's departure. John reported, for instance, that although some teachers continued to collaborate on co-teaching, which was one of the projects initiated by Paul, by time this is becoming something that pairs of teachers do on their own steam with hardly any reference to other colleagues. Even John, who is trying to somehow hold back what he perceives as an encroaching individualistic tide within the department, admitted that he is neglecting the mathematics room that he and Paul had built from scratch and which had been the symbol of teacher collaboration in school. This spacious multi-purpose room serves to hold discussion and planning meetings, to conduct 'experimental' lessons that are observed, filmed and analysed, and also to store teaching resources. One might argue that this 'neglect' epitomises the fragility of teacher collaboration when this activity leads to practices that challenge the dominant structures and values of school (see Harrison et al., 2008), especially when there is no one capable and willing to lead teachers along this path. Not seeing himself as

someone who can shoulder this responsibility, John appears to be playing for time by proposing interim measures that would hopefully 'keep things going' until some more viable solutions are found.

I desperately need to find an ally at school if I'm to continue growing as a teacher ... I'm still in contact with Paul and I'm hoping that he'll keep coming to school ... that would give me motivation and drive! Mind you, Paul and I are planning to do something that would involve the maths teachers at school ... we're after volunteers, but I cannot give details at this stage! Another possibility for me is that a new HoD comes who has a passion for work and also believes in IBL. If I'll find someone like Paul I'll give my 200% ... but I still think that there will never be anyone like Paul!

John's desire to find an 'ally' who would support his continued professional growth suggests that while he had experienced notable change, his change has still not reached the stage that Franke, Carpenter, Levi and Fennema (2001; cited in Steinberg et al., 2004) consider as 'sustainable and self-generative'. So much so that John – who remains committed to change – is now looking for possible solutions in which he is willing to be a protagonist, but are led by others. He is working in fact, and there appear to be good prospects, to realise a project that would see him and his colleagues collaborating closely once again with Paul. On a longer term basis, he is hoping that the new HoD would be someone capable and willing to carry on where Paul left off. Notwithstanding these plans and hopes, John remains nostalgic about what has been and what could have been had Paul not left. Consequently, convinced as he is that the journey ahead is not smooth and that things might never be the same again, one might argue that John demonstrates at best what Grace (1994) terms 'complex hope'. That is, true to his resilient spirit and authentic commitment to change, he continues to seek learning opportunities with a degree of optimism in spite of recognising the complexity of what lies ahead.

Teacher Education and Teacher Learning: Insights and Implications

All the findings in this paper are based on a single case study. However, John's story has the potential to shed important insights on teacher education and teacher learning. Consequently, assuming Bassey's (2001) notion of 'fuzzy prediction', I offer here a number of insights embedded in qualified and contextualised statements that, once their implications are explored, can serve as guide to professional action.

- John experienced change, as different from growth, at a rather mature stage of his professional life. This suggests that it is never too late for a teacher to revisit and change his or her beliefs and practices. One could therefore argue that teacher education stands to benefit should it move beyond the usual ITE and CPD provisions to create additional spaces, inside and outside schools, which have the potential to ignite and advance professional learning. These spaces would serve as 'zones of enactment' in which teachers' will, capacity and prior experiences interact with reform initiatives and learning opportunities (Spillane, 1999).
- Although all the mathematics teachers in school were offered concessions to participate in PRIMAS, not everyone accepted. Again, while one might safely assume that all the participants grew professionally from that experience, it appears from John's story that only he, and possibly another teacher who continued to work with John and Paul after PRIMAS, actually changed. Apart from the opportunity to change, John attributed his professional development to his willingness and capacity to change. This suggests that while opportunity to change is possibly essential, it may not be sufficient. Consequently, one could argue that change is more likely to happen should teacher education programmes make a greater effort to instil a sense of change in prospective and practising teachers, and also provide them with the necessary pedagogical skills to handle the more complex demands of teaching. If this is to succeed, however, teachers need to operate in a school culture that is conducive to change (Anderson, 2004).
- John's beliefs and practices remained unchanged when his teacher education was based on the traditional theory-into-practice model (see Korthagen et al., 2006). On the other hand, once he experienced, through PRIMAS, a way of professional learning that brought theory and practice closely together, he entered into change mode and went on to become a promoter of change. This suggests that teacher education programmes, at all phases of teachers' professional lives, are more likely to have an impact on teacher learning should they present learning as situated, with theory and practice constantly feeding into and developing each other. This is more likely to happen when the location of theory and the location of practice are conceptualised as complementary to each other (Anderson & Freebody, 2012) or, as happened in John's case, that they actually occur under the same roof.

- John changed within a teacher community where he was encouraged to act as a learner in a welcoming, yet professionally challenging, environment that offered direction and support. In fact, he claimed that he could not have done it on his own. His experience adds testimony to reports claiming that teachers benefit when exposed to professional learning within a community, which might even include members from different schools. One might consequently suggest that, in order to enhance teacher learning, teacher education programmes for preservice and inservice teachers should consider organising their learning around communities, both within and outside schools. This would require the development of professional learning community structures (see Golding, 2017) by the host institutions, be they schools or providers of teacher education, that facilitate professional encounters through the provision of meeting slots in their schedules and adequate resources (Attard Tonna & Shanks, 2017).
- Paul played a crucial role in John's development and change. Indeed, not only did Paul offer John and other colleagues the opportunity to change, but he was also their leader of teacher learning (see Postholm, 2012). In that role, Paul inspired change, offered direction and support, and acted as their critical friend while being one of them. It was a professional relationship built on friendship and trust, not hierarchy. Noting the transformation in John as a result of this relationship, one might argue that teachers are likely to benefit should they be attached to such leaders throughout the various phases of their career. This can be realised as part of mentoring schemes that accompany teachers throughout their professional journeys. In this way, teachers would have the opportunity to engage in a continuous process of collaboration that can lead to a better understanding of teaching and learning (Wang & Odell, 2002).
- John's change trajectory has been neither easy nor linear. Most notably, this journey has included dealing with serious doubts as to whether IBL can be used successfully within a traditional education system and acute feelings of abandonment and loss following Paul's departure from school. But thanks to his resilient nature, John eventually managed to harmonise his practices while remaining sensitive to the 'requirements' of the traditional context, and to find new ways of collaborating with Paul after they had stopped teaching in the same school. John's story thus suggests that teacher change can be a rather complex and unnerving affair. One might therefore suggest that teachers seeking change should be made aware of the possibly bumpy ride ahead to the extent that they, as Sedova (2017) warns, might even experience periods of regression.

Such a forewarning might help teachers to maintain faith in their personal transformation.

- The change in John originated from Paul and remained dependent on Paul's presence in John's school life. Such was this reliance that John – who had embraced change and became a promoter of that change – lost his motivation and sense of purpose once Paul left school. This reaction by John suggests that the progression of change is more likely to be disrupted when it is 'person driven' than when it is 'school or team driven'. For it may be that when persons depart, they could leave a debilitating void behind them unless those they have led have also been prepared to progress on their own or the school in which the change is happening already has adequate structures to continue encouraging and supporting that change. It therefore appears necessary that schools become places of teacher learning (see Korthagen et al., 2006) that not only embrace individual or team initiatives, but also readily extend their structures and resources to such initiatives so that these may eventually become part of a whole-school approach to teacher professional learning.

Inviting Reflection, Inspiring Change

John's story reveals that a teacher can change along the lines of the current "global education policy attempt to move school mathematics learning towards deep conceptual understanding, rigorous reasoning, and genuine problem solving, in response to the perceived needs of 21st-century society" (Golding, 2017, p. 502). In so doing, John has succeeded where many other teachers, even from among those who claim to favour such reforms, have failed (see Golding, 2017). The possibility of pedagogical improvements in mathematics is particularly welcome because, as Esmonde (2009) points out, it is considered by societies worldwide to be an important school subject in view of its gatekeeping role to a variety of education and career opportunities. One can therefore argue that even with high status school subjects, such as mathematics, the possibility exists for professional learning initiatives that encourage, develop and sustain change in which teachers believe and are comfortable with. But John's story also signals caution, as there is evidence to suggest that change can be ephemeral unless teachers continue to find a supporting and nurturing environment. In fact, it is requiring a lot of determination on John's part to continue with his change journey following his recent setbacks at school. Still, the uneven path that has delineated his professional transformation probably carries the additional

appeal of authenticity. For the ups and downs of his journey present a narrative of a 'normal' teacher that people can relate to, reflect on and gain valuable insights from. As such, his story has the potential to inspire a 'sense of' and a 'desire for' change in a variety of interested professionals.

Reflecting my awareness that teacher learning, and consequently teacher change, is situated in given contexts and cultures that cut across space and time (Attard Tonna & Shanks, 2017), I would not encourage other teachers to look at John as a 'model' to be emulated. Instead, my aspiration is that he inspires them, as he has inspired me, to believe in and open up to the possibility of change. Moreover, in the knowledge that ITE needs to be considered as the first step in a process of ongoing professional learning (Stephens & Crawley, 1994; Bezzina, 2002; Anderson, 2004), I would suggest further that other professionals – such as heads of department, education officers, school administrators, teacher educators, and policy makers – stand to benefit from becoming aware of and reflecting on John's story. One hopes that the insights gained by these professionals could then contribute towards the development of an overarching teacher education system in which, as Bezzina (1999) suggests, teachers' professional development is addressed strategically, not haphazardly as often happens. This would enhance the quality of teachers' professional development and consequently the quality of teachers and their teaching (Walter, Wilkinson, & Yarrow, 1996). Should this happen, the students would be the ultimate beneficiaries because the improvement of their educational experience depends to a large extent on the development of teachers (Meissel et al., 2016).

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Constructivist Teaching: Mythical or Plausible?

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Abstract: Irrespective of the branch of constructivism they advocate, many constructivists argue that constructivism is a theory of learning, not of teaching, and therefore one cannot speak of such a thing as ‘constructivist teaching’ (CT). Others equate CT with a student-centred teaching methodology such as teaching for inquiry-based learning. From a radical constructivist perspective, I argue that both of these views are only partially true. The former seems to disregard the fact that teaching and learning are so interlinked that it may be virtually impossible for a teacher who strongly believes in the constructivist notion of learning not to reflect some of that belief in her/his teaching approach. The latter does not seem to acknowledge that even the most traditional and teacher-directed teaching may bring about learning, and that if learning occurs, it happens through the active construction of knowledge in the minds of the learners. Drawing on a local case study of a group of six low-performing Year 7 students (i.e., 11-year-olds) to whom I taught mathematics, I show that CT is a possibility in any classroom where the teacher is sensitive to the constructivist notion of learning. The framework I used to investigate the data was the Mathematics-Negotiation-Learner (M-N-L) framework. I devised this framework to help me to define CT and analyse the extent to which I maintain it in my lessons

Keywords: Radical Constructivism; Mathematics education; Constructivist teaching; M-N-L framework

Introduction

Radical Constructivism (RC) is built on two sets of principles about knowledge and cognition which its founder, Ernst von Glasersfeld (1990)

claims to have surmised from Piaget's theory of genetic epistemology¹ (e.g., Piaget, 1985). These two sets of RC principles are that:

- 1a. Knowledge is not passively received either through the senses or by way of communication;
- b. Knowledge is actively built up by the cognizing subject.
- 2a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
- b. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality.

(Glaserfeld, 1990, p. 22)

Principles 1a and 1b are shared by all branches of constructivism. It is Principles 2a and 2b that distinguish RC from other strands of constructivism. Glaserfeld claims that "those who merely speak of the construction of knowledge, but do not explicitly give up the notion that our conceptual constructions can or should in some way represent an independent, 'objective' reality, are still caught up in the traditional theory of knowledge" (Glaserfeld, 1991, p. 16). Riegler (2001) labels this latter type of constructivism *trivial*.

Like all mathematics teachers who draw their epistemological beliefs from RC theory, I need to keep in mind these two sets of principles during my teaching. Like all constructivists, I maintain that knowledge is not 'passed on' by the teacher or 'acquired' by the learner. My standpoint is that knowledge is constructed by the learner and that this development is facilitated by environments conducive to this knowledge construction, or what Steinbring (1998, p. 158) refers to as "learning offers." Being a *radical* constructivist, means that my understanding of 'knowledge' is not a mental representation

¹ Piaget (1985) views intellectual growth as a process of adaptation to the experiential world. This happens through a process of *assimilations* and *accommodations* of perceived information to existing mental *schemas*. When humans use an established mental schema to deal with a new perception this is called assimilation. When existing schemas do not work and need to be adapted to deal with new phenomena, humans undergo a mental process called accommodation. When humans use assimilation to deal with their experiences, Piaget says that *equilibrium* has occurred. When existing mental schemas are not viable for new experiences, a mental *perturbation* occurs, creating a state of *disequilibrium* which humans feel the need to settle. The settlement of this perturbation is called *equilibration*. This occurs by modifying the existing schema to deal with the new experience through the process of accommodation, where a state of equilibrium is regained.

of an objective reality but a viable interpretation of a person's experiential reality. This implies that the mathematics I intend to teach is *my own* construction and interpretation. It also implies that whatever mathematics is developed by the students is *their own* subjective interpretation of the mathematical realities that I coordinate and facilitate in the classroom.

One of the main research questions in a case study I carried out with a group of Year 7 students was to analyse how these RC perspectives were reflected in my teaching approach. The outcome was the development of a framework which helped me analyse my constructivist teaching.

Constructivist Teaching

The argument that constructivism is a theory of knowledge construction and not of teaching has led constructivist researchers to disagree on the legitimacy of a label such as 'constructivist teaching' (CT). Usually, such a discord originates from what different people mean by the term. Engström (2014) objects to the term CT on the grounds that it is usually equated with progressive modes of teaching. Simon (1994) says that CT is a myth because constructivism is a theory of learning and, irrespective of the teaching method being used, learners will learn by constructing concepts for themselves. Simon (1995) argues that sympathising with a constructivist notion of how one learns does not translate into a set notion of how to teach. I agree with both Simon (1995) and Engström (2014) that no particular teaching method or tools can, by themselves, constitute CT.

On the other hand, I do make a case that the term CT is legitimate if it is attributed instead to a constructivist teacher's *sensitivity* towards individual students' subjective and active constructions of knowledge. Being an avid promoter of CT, Steffe repeatedly stresses the importance of teachers' *learning* about the mathematical realities of their students (for example, Steffe, 1991; Steffe & Wiegel, 1992). In the context of mathematics education, Steffe (1991) argues that RC teachers must view themselves as persons in pursuit of knowledge about bridging the mathematics *of* students (MoS), i.e., students' constructions of mathematical concepts) and the mathematics *for* students (MfS), i.e., teachers' mathematical ideas intended to be taught to a particular student or group of students.

RC teachers are concerned with building hypothetical models of students' cognitive structures (Glaserfeld & Steffe, 1991). Based on this concern, Simon (1995) presents a practical working model of how a mathematics teacher can adopt a constructivist perspective whilst teaching. Simon (1995) explains how mathematics teaching develops from what he calls a *hypothetical learning trajectory* (HLT). This is the way teachers make hypothetical predictions of the path by which learning might proceed. Simon (1995) explains that HLT consists of the teacher's:

- i. learning goal which defines the direction of the lesson,
- ii. plan of activities aimed to achieve the learning goal, and
- iii. hypothesis of the learning process, i.e., the predictions of how students' thinking and understanding will evolve in the lesson.

These actions are 'hypothetical' because the actual learning trajectory is not knowable in advance. Glaserfeld (1994) argues that to be able to orient students' mental processes the teacher needs to have at least a hypothetical model of how the mind of a typical student operates at the outset of the lesson. I regard the use of the word 'hypothetical' (Glaserfeld & Steffe, 1991; Glaserfeld, 1994; Simon, 1995) as an acknowledgement of the fact that what learning outcomes the teacher may have in mind before the lesson starts may be changed in the course of the lesson. Such changes occur according to what the teacher learns from the students. Steffe (1991) argues that RC teachers should reflect and act upon models they build of their students' mathematical knowledge. Both Simon (1995) and Steffe (1991) suggest that constructivist mathematics teachers should help their students create connections between their mathematics and the mathematics the teacher intends to teach them. This has much in common with the *Constructivist Learning Design* proposed by Gagnon and Collay (2006).

Simon (1995) and Steffe (1991) have captured the attributes that are usually associated with fostering a mathematics teaching environment that is sensitive to constructivist notions of learning, namely to:

- i. encourage students to come to an answer in diverse ways and possibly obtain multiple correct responses,
- ii. appreciate and promote students' interventions in the lesson and invite them to articulate their understandings of the problem at hand,

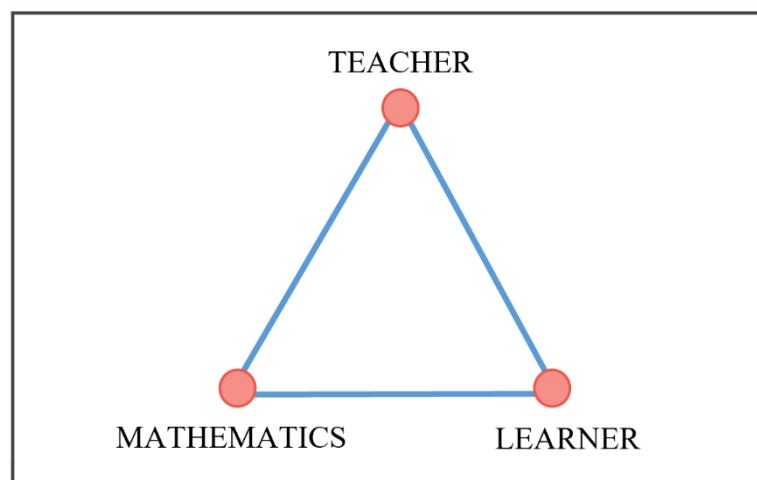
- iii. allow students to describe their strategies and engage students in debates which help them refine and adjust their strategies and understandings, and
- iv. learn about students' conceptual constructions and about students' own mathematical understandings through reflection on classroom experiences.

It seems, therefore, that there exists an approach, an attitude, and a standpoint in mathematics teaching which may be described as CT. This approach occurs when constructivist teachers, in their diverse preferred styles of teaching, make possible a two-way-traffic type of communication in their lessons, where both teacher and student are learners and both teacher and student are teachers (Freire, 1998). The relationship between the mathematical content, the learner, and the teacher is created by the need of learners to construct mathematical ideas and by the need of the teacher to learn about and orient students' mathematical understandings.

Mathematics, the Learner, and the Teacher

The dynamics between mathematical content, learners, and the teacher (including teaching approaches), most commonly referred to as the didactic triangle (Figure 1), has been in the limelight of French educational research since Brousseau (1997) put forward his theory of *les situations didactiques*. The latter are the didactical situations formed by this interlinked triplet within the classroom ethos.

Figure 1: Brousseau's didactic triangle



This rather simplistic diagram highlights the relationships between the three factors that establish the situation of a mathematics classroom: the teacher, the student, and the mathematics being taught and learnt. Schoenfeld (2012) identifies seven questions regarding one or more nodes of the didactic triangle and the relationships between them:

- 1) What is mathematics, and what version of it is the focus of classroom activities?
- 2) Who is the teacher, what does he or she bring to the classroom?
- 3) Who is the learner, what does he or she bring to the classroom?
- 4) What is the teacher's understanding (in a broad sense) of mathematics?
- 5) What is the learner's emerging understanding of mathematics?
- 6) What is the relationship between learner and teacher?
- 7) How does the teacher mediate between the learner and mathematics, shaping the learner's developing understanding of mathematics?

(Schoenfeld, 2012, p. 587)

Question 7, which is most pertinent to the subject of this paper, deals with the way the three entities relate simultaneously to each other. This question could not be tackled without considering the three triangular nodes separately (questions 1-3) and the three triangular sides, each of which connects two entities of the didactic triplet (questions 4-6). The didactic triangle even allows researchers to isolate one of the nodes of the triangle in order to elicit and expand its meaning and clarify its links with other nodes. For example, Jaworski (2012) focuses on the teacher node and identifies three interlinked activities that constructivist mathematics teachers carry out in their lessons. She calls these the *teaching triad*.

Management of Learning. This consists of the teacher's administration of the classroom activities, the students' participation in those activities, and the overall interactions fostered during the lesson. It also involves the teacher's institutional obligations and standards, assessment practices, and, most importantly, the interpretation of mathematical content.

Sensitivity to Students. This is the teacher's effort to become aware of her/his students' knowledge and thinking styles and tendencies. Such sensitivity makes students feel respected, included and cared for.

Mathematical Challenge. This is the way the teacher presents the mathematical problem to the students in a way that interests them, motivates them to learn, and promotes participation and cognitive engagement.

Jaworski's (2012) triad has much in common with ideas discussed earlier. In particular, the teacher's sensitivity to students is stressed by Steffe and Wiegel (1992) in their appeal to constructivist teachers and curriculum reformers to view mathematics knowledge as a human creation. The presentation of the 'mathematical challenge' is necessarily derived from the teacher's epistemological standpoint about the mathematical concepts she/he intends to communicate with the students. The RC teacher interprets and represents mathematical concepts as "more or less reliable ways of dealing with experiences, the only reality we know" (Glaserfeld, 1995, p. 117).

The experiences of the teacher and the students are derived from an environment which goes beyond the classroom. Chevallard (1982) introduces the notion that a didactical situation does not operate in a vacuum but is embedded within, and affected by, external social and institutional forces. The latter include government educational directives, inspecting and testing regimes and parental and community pressures. The RC teacher may well reject the idea of an *a priori* curriculum but, as Chevallard (1988) observes, the very *intention* to teach is not so much a decision of the individual teacher as it is of the society in which that teacher operates. It is society which decides what part of mathematics can be regarded as *teachable* knowledge. Chevallard (1988) argues that knowledge is inherently a tool to be put to use rather than concepts to teach and learn. He claims that it is thus an artificial enterprise to 'teach' a body of knowledge. In fact, curriculum planners need to find ways how to transform 'knowledge' from a tool to be put to use to something to be taught and learnt. He calls this the "*didactic transposition* of knowledge" (Chevallard, 1988, p. 6, original emphasis).

Once mathematical content is transformed by curriculum designers from a viable tool to a set of teachable concepts, it is the constructivist teachers' duty to "to recontextualize and repersonalize the knowledge taught to fit the student's situation" (Kang & Kilpatrick, 1992, p. 5). The RC teacher observes and reflects on the uniqueness of learners' experiential worlds and tries to find connections between the mathematical content included in the syllabus

and the learners' interpretations of that content with respect to their individual experiences.

Negotiating a Link between Teachers' and Learners' Mathematics

Literature about CT, or at least about teaching from a constructivist perspective, tends to focus mostly, if not only, on the learner. In his review of research related to CT, Gash (2014) states that the emphasis is "on the child's learning rather than just focusing on what the teacher thought was important to teach" (Gash, 2014, p. 304). I agree with Gash's argument only because his inclusion of the word 'just' implies that for a constructivist teacher *both* the child and the curriculum need to be kept in mind, for both of them constitute the didactical situation (Brousseau, 1997) which puts the teacher in the classroom in the first place.

It was Dewey who was probably the first to think of the educative process as the *interaction* between these two factors. In *The Child and the Curriculum*, Dewey (1902, p. 2) points out that teaching is influenced by two forces: "an immature, undeveloped being; and certain social aims, meanings, values incarnate in the mature experiences of the adult. The educative process is the due interaction of these forces."

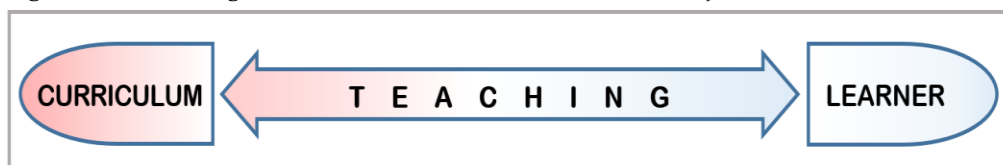
Although Dewey promotes the kind of education which allows learners to have control over their learning, he maintains that the teacher should focus on both the learner and the content to be taught. On the one hand, Dewey argues that it is unacceptable for a teacher to focus only on the content and forget about the needs of the learner. The teacher needs to draw attention to the *viability* of the subject content in the students' experiential worlds, something which today may be identified with RC. On the other hand, Dewey (1902) claims that if teachers focus only on the learners they will easily lose sight of what knowledge they have been entrusted to teach. Hence, the teacher needs to strike a balance between providing opportunities for learners to acquaint themselves with the topics in the curriculum and being sensitive to learners' individual interests and experiences. Dewey compares the learner and the learnt with two points and the teaching process with the interconnecting line drawn between those two points:

The child and the curriculum are simply two limits which define a single process. Just as two points define a straight line, so the present standpoint of the child and the facts and truths of studies define instruction.

(Dewey, 1902, p.16)

Figure 2 illustrates my understanding of Dewey's (1902) analogy that links the subject matter, the learner, and the teaching process. Dewey stresses that any teaching programme needs to be defined by the needs of the learner and the subject matter intended to be taught. The teacher's task is therefore to plan and proceed in assisting learners along their journey from their current situation to the state of developing knowledge about the subject matter.

Figure 2: Teaching seen as the line drawn between subject matter and learner



Dewey (1902) regards teaching as the negotiation process aimed at bringing together these two forces both of which demand the teacher's attention. In doing so, he acknowledges teachers' dual accountability to curricular and learners' requirements. Dewey's (1902) Curriculum-Teaching-Learner construct enriches constructivist frameworks such as those of Steffe (1991) and Simon (1995) because it takes into consideration the parameters within which school teachers operate, including, most importantly, the didactic contract between the teacher and the students (Brousseau, 1997). The constructivist frameworks proposed by Simon (1995), Steffe (1991), and Dewey (1902) were instrumental in my investigation of CT and the subsequent development of an analytic framework to investigate CT from a RC perspective.

Context and Methodology

The protagonists of my case study were six low-performing Year 7 students to whom I taught mathematics during the scholastic year 2014-15. Their pseudonyms were Dwayne, Dan, Jordan, Joseph, Omar, and Tony. The school had a policy of retaining mixed-ability classes for all subjects except for Mathematics, English, and Maltese. In these core subjects, students were divided according to their performance in the previous scholastic year. Those starting to attend the school at Year 7 were divided in these three subjects according to their performance in a national benchmark examination which Maltese students sit for at the end of Year 6.

The grades that my participants had obtained in the Year 6 benchmark exam, before entering the school, were between 1 and 3 standard deviations below the mean of the Year 7 cohort and hence they were in the lowest of three performance sets. The part of the Year 7 curriculum which featured in my research was that of introducing formal algebra by helping students to:

- i. develop meanings for numerical and algebraic expressions,
- ii. understand the use of letters as unknowns and variables, and
- iii. extend their interpretation of the equals sign.

Qualitative data was collected by a number of methods, but the data concerned with CT was obtained by video-recording a series of twenty double lessons (80 minutes each) throughout the scholastic year. As Farrugia (2006) asserts, in Maltese mathematics classrooms, English is the language of written texts, while for spoken language, technical words are usually expressed in English. The main communication medium in the lessons was Maltese and we used English to read written problems or task instructions, and to say technical words like 'plus' and 'equals.' Sometimes we code-switched to English for short intervals. The transcripts were translated immediately to English and when English was used this was indicated in parenthesis.

Throughout the lessons, I made use of the software package Grid Algebra² (GA). GA is a computer environment which is based on the multiplication grid. A typical GA interface³ is shown in Figure 3. Only multiples of a particular number are allowed in a row. For example, in R_5C_2 (Row 5 Column 2), the number 30 is allowed because it is a multiple of 5.

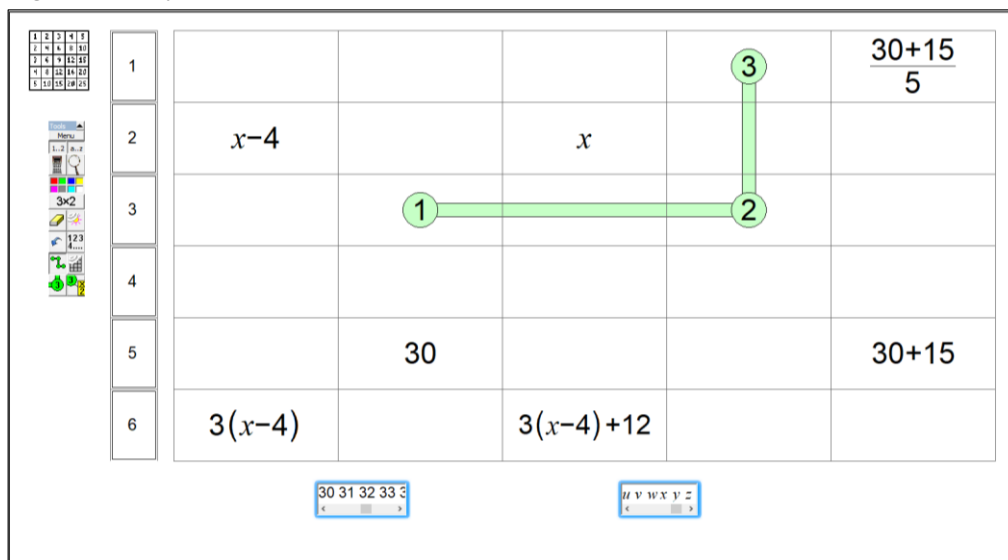
The content in one cell may be dragged into another cell and GA shows the corresponding expression. For example, dragging the 30 in R_5C_2 three cells to its right to R_5C_5 is equivalent to adding 5 three times and GA shows $30+15$. Right and left movements correspond respectively to adding and subtracting multiples of the row number. Movement from one row to another row corresponds to multiplication or division. For example, movement from R_2 to

² Developed by Dave Hewitt and distributed by Association of Teachers of Mathematics.

³ Arrows are added to show how numerical and algebraic expressions were obtained by moving the cells.

R_6 corresponds to multiplication by 3. Similarly, movement from R_5 to R_1 corresponds to division by 5 and hence, moving the expression $30+15$ from R_5C_5 to R_1C_5 results in the expression $(30+15)/5$ as shown in Figure 3.

Figure 3: A typical GA interface

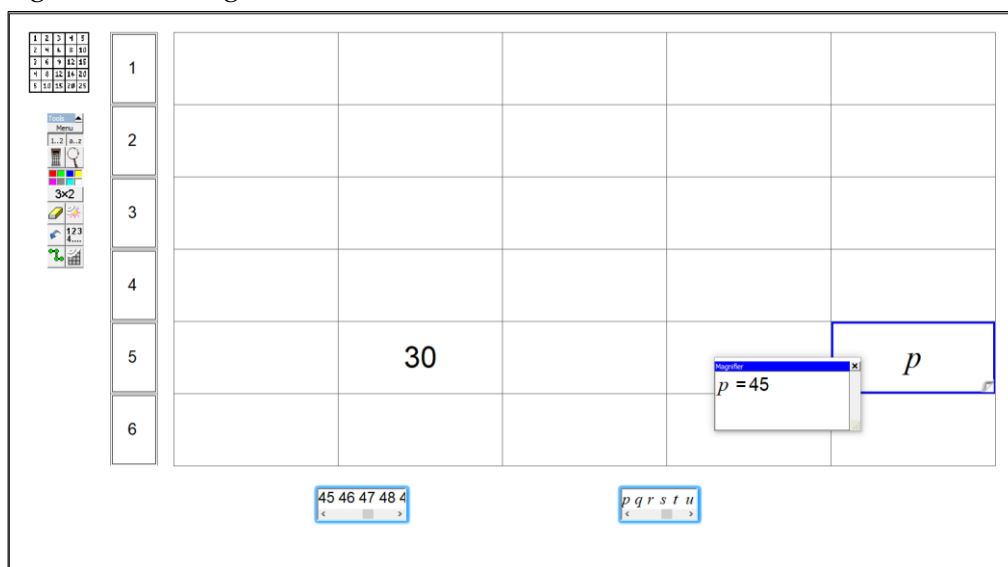


GA accepts the use of letters to represent variables or unknowns. Entering the letter x in R_2C_3 without the introduction of any other numbers in the grid, means that x represents a *variable* multiple of 2. However, if at least one number is present in the grid, that number determines the value of all the other cells in the grid. Hence, the x present in R_2C_3 in the grid shown in Figure 3, represents a *specific* multiple of 2 since there are some numbers present in the grid. Hence, it is a representation of an *unknown* (constant) rather than a variable. Evaluating neighbouring cells in Figure 3, one can see that $x=14$. The movements and respective creation of expressions described earlier may be similarly done with cells containing letters. Hence, moving x from R_2C_3 to R_2C_1 results in $x-4$, since this movement corresponds to subtracting 4. The expression $x-4$ may, in turn, be dragged onto R_6C_1 and, since jumping from R_2 to R_6 corresponds to a multiplication of 3, GA shows $3(x-4)$, and so on.

In this way, GA enables users to create and build numerical and algebraic expressions either by moving a cell and its contents from one place to another or by typing it directly with respect to its place in the multiplication grid and in relation to other expressions existing in the grid. Furthermore, it gives students the possibility to trace the movements of expressions around the grid, such as the 1-2-3 journey shown in Figure 3.

GA also allows users to input more than one expression in a single cell. Figure 4 shows a grid in which 30 is entered in R_5C_2 . As previously shown, the expression in R_5C_5 should have a value of 45. In Figure 4, GA allows users to enter a letter (say, p) inside R_5C_5 , along with the number 45. A feature in GA, called a magnifier, reveals the contents of this cell. As shown in Figure 4, the magnifier displays $p=45$ when R_5C_5 is clicked upon.

Figure 4: The magnifier feature of GA



The expression resulting in the GA magnifier was the subject of an excerpt of a lesson presented later in this paper.

The lessons were divided into two parts. The first part consisted of a class discussion about the topic at hand. The discussion was facilitated by the use of GA which was projected on the interactive whiteboard (used as a touchscreen). The second part of the lessons consisted of students working on GA tasks on their computers. While the latter was crucial in investigating students' mathematical representations and interpretations (see Borg & Hewitt, 2015), the first part was used to define and analyse CT. The framework I developed as a result of this investigation is discussed in the section that follows.

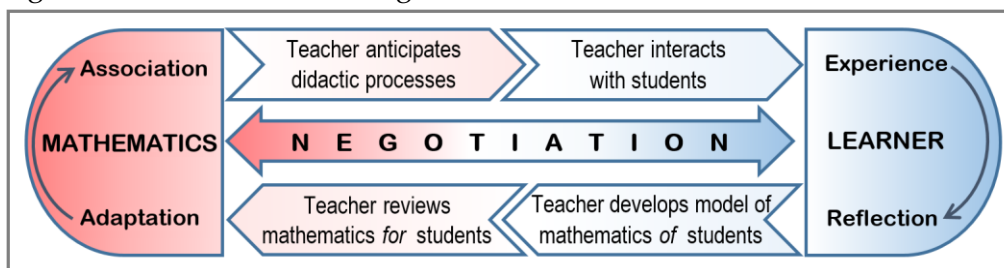
The Mathematics-Negotiation-Learner Framework

Analysing the lesson videos against the backdrop of Dewey's (1902) Curriculum-Teaching-Learner construct, I observed that I was continuously changing my purpose in the lesson due to my need to keep in mind both the mathematics I intended to teach and the mathematics being constructed by the learners. These two forces, continuously calling for my attention, necessitated negotiations from my mathematics to the learners and from the learners to my mathematics. Further analysis led to the identification of four different shifts of teaching purpose:

- i. The **M-N shift**: from my mathematics to the negotiation process. This was the moment where I changed my focus from thinking about my mathematics to making hypothetical predications about the learning process (Simon, 1995). This led to interactions aimed at providing a learning offer (Steinbring, 1998) so that students could form concepts about the mathematics I intended to teach.
- ii. The **N-L shift**: from the negotiation process to the learner. Here my focus shifted from interacting with the students to assisting students in their experience of mathematical phenomena. This involved helping students to make reflective abstractions (Piaget, 1985) of that mathematical experience.
- iii. The **L-N shift**: from the learner to the negotiation process. This refers to the moment where I learnt something about students' mathematics (Steffe, 1991) and decided to do something about it. This negotiation was not an interaction with the students but an 'internal interaction' with myself, which led to a review of the suitability of the learning offer.
- iv. The **N-M shift**: from the negotiation process to my mathematics. This was when I changed my focus from reviewing the learning offer to making associations or adaptations to my mathematics – the subset of my mental schema intended to be taught or shared with the students.

Keeping Dewey's (1902) Curriculum-Teaching-Learner construct as an overarching frame of reference, I used these shifts of focus to develop what I called the Mathematics-Negotiation-Learner (M-N-L) framework. The design and development of the M-N-L framework is discussed by Borg, Hewitt, and Jones (2016 a, b). The framework is illustrated in Figure 5.

Figure 5: The Mathematics-Negotiation-Learner framework



M-N-L builds on Dewey's (1902) Curriculum-Teaching-Learner construct by using the metaphor of two 'roads' that link (the teacher's) mathematics and the learners. These roads represent the teacher's negotiations during the lesson. The following is a description of the stages of the cycle shown in Figure 5, starting from the upper left-hand arrow that goes from mathematics to learner:

1. The Forward-negotiation Road

The forward-negotiation road is formed of the teacher's actions aimed at presenting a mathematical learning offer to the students:

- i. The teacher builds on models of the mathematics *of* the students (MoS) to *anticipate* possible didactic processes. The latter may help students to develop notions of the mathematics at hand, i.e., the mathematics *for* the students (MfS). Simon (1995) calls this a hypothetical learning trajectory since the teacher has no means of knowing in advance the actual didactic processes that may occur.
- ii. Then, the teacher *interacts* with students by making representations of MfS intended for students' constructions of MoS. The teacher makes verbal, gestural, and written representations and coordinate goal-oriented activities and discussions. 'Interaction' includes teacher exposition and teacher-coordinated activities.

2. Learner

The 'Learner' section of Figure 5 shows how this forward-negotiation road leads to students' *experience* of mathematical representations which the teacher encourages students to *reflect* upon and make abstractions. Students become learners by making abstract conceptualizations through an interplay

of experience and reflection. This is reminiscent of Kolb's (1984) experiential learning construct but with an emphasis on how the teacher reacts to students representations.

3. *Backward-negotiation Road*



- i. The Learner-to-Mathematics arrow on the right shows that the teacher builds, experiential *models* of MoS. These models are experiential because they are built entirely on the experiences of the teacher and the students. Steffe emphasises that the constructivist teacher must be a keen observer in order "to construct the mathematical knowledge of his or her students." (Steffe, personal communication, October 7, 2015). Models of MoS of individual students may serve the teacher to make inferences about the possibility of similar MoS for the rest of the class.
- ii. The arrow that follows on the left shows that the teacher uses these models of MoS to *review* MfS. This means that MoS serves as an assessment of whether the learning offer presented along the forward-negotiation road was appropriate for the students.

Each activity involved in the backward-negotiation road is a learning experience for the teacher.

4. *Mathematics*



The mathematics end of the M-N-L diagram shows that the teacher revisits her/his own mathematics, to decide whether MoS can be *associated* with it either directly or by going through some kind of *adaptation or accommodation* of her/his mental schema. The settlement of this perturbation leads to a renewed MfS and a revised anticipation of the didactic processes with which the teacher starts a new forward-negotiation road.

I consider the teacher's deliberate shifts of purpose between the four elements described above to be an indication of CT. Although some exponents of CT (e.g., Steffe et al., 1983; Steffe, 1991) tend to focus almost exclusively on the teacher's learning from and about the students (backward-negotiation road), I argue that the teacher is duty-bound *to teach* and cannot learn about students' construction of knowledge without intervening to facilitate it. Nevertheless, I argue that constructivist teachers cannot just present learning offers and, like Steinbring (1998), claim that mathematics teaching is an autonomous system.

That is, CT is dependent on students' feedback and on the actions that the teacher takes based on that feedback.

The teacher's effort to balance forward- and backward-negotiations is key to sustain regular transitions from one stage to another of the M-N-L cycle, thus maintaining the two roads which bring together mathematics and learners. CT may be analysed by studying how the teacher makes transitions between successive stages of the M-N-L cycle through shifts of teaching purpose. The extent to which the teacher manages to start, maintain, and complete M-N-L cycles may be an indication of her/his success to engage in CT. When the teacher fails to complete M-N-L cycles it may indicate a failure to engage in CT. This happens when the teacher momentarily creates roadblocks in the negotiation process which hinder the shifts of teaching purpose necessary to complete M-N-L cycles. In my study, I have identified two such roadblocks; the reader is referred to Borg et al. (2016a) for a discussion of these roadblocks. In the following section, I demonstrate how I used the M-N-L framework to analyse my CT.

Analysing CT through M-N-L Cycles

In this section, I present a continuous transcript taken from the video recording of Lesson 13. This is divided into four excerpts which I use to show how I went through two successive M-N-L cycles. The main aim of the lesson was to introduce the use of letters in the GA grid. A letter in GA could represent a specific unknown or a variable quantity.

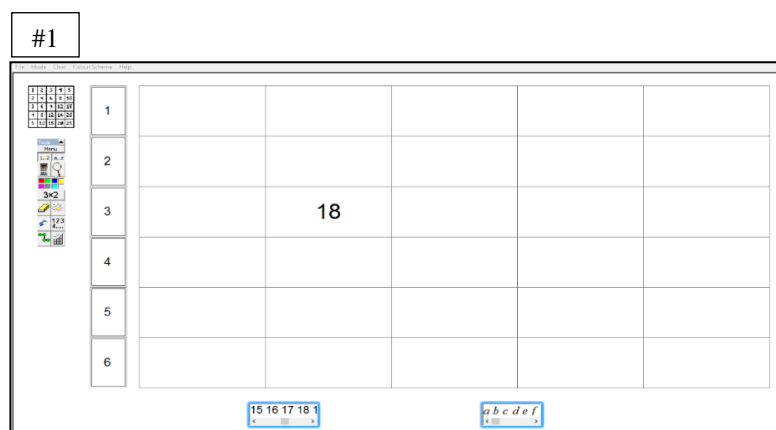
This episode occurred just 2 minutes into the lesson. As usual, the first half of the double lesson consisted of a plenary discussion. The first few minutes of class discussions consisted mainly of a teacher exposition. This was necessary since I needed to demonstrate new features of the software. Nevertheless, students' participations in such expositions were necessary since I needed students to reflect on their observations. In a typical lesson, as time went by, I usually relinquished more and more my 'control' over the discussion, where students came out to work on activities on the interactive whiteboard. This led to the second half of the double lesson where students worked in pairs on their computers. During this part of the lesson, I took on a more background, supervisory role where I assisted students only if required.

The reason for choosing this particular episode is to show that even during teacher exposition, when the teacher may be predisposed to focus more on the subject matter, CT can be achieved if the teacher is sensitive to students' knowledge constructions. This sensitivity is required for the teacher to make the necessary shifts of focus between her/his subject matter (mathematics), the negotiation process, and the learner. In this episode, a number of mathematical concepts were discussed, namely:

- i. multiples of 3,
- ii. letters standing for numbers and values of numerical expressions, and
- iii. the meaning of the equals sign.

Excerpt 1: M-N and N-L shifts (Cycle 1)

PB: ...I am going to place the number 18 here. [*Drags 18 to R_3C_2 - #1.*]



... It [the software] will let me do it.

Joseph: Because it is in the 3-times table.

...

PB: Well done! Well done! Now, if I picked a letter at random from here [*picks the letter d and drags it to R_3C_4*] and I place it over here [*Joseph raises his hand*], that *d*, first of all, what is it symbolising? [*Pointing at Joseph...*] Come, let's see.

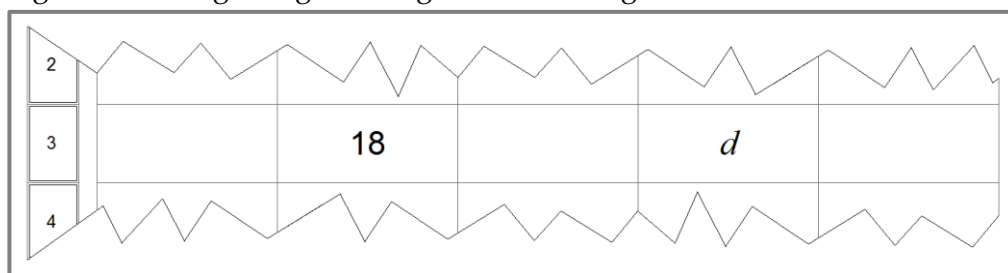
Joseph: Uh, what it is, what the answer should be. Like if you do 18 plus 3 plus 3, that is plus 6, which becomes 24, it is *d* equals 24.

This excerpt shows the beginning of an M-N-L cycle (Cycle1). At the beginning of the discussion, my initial MfS was the appreciation of the difference between variables and as unknowns. I anticipated that the students were prepared to construct notions of letters as unknowns in the GA grid by referring to neighbouring cell values. This anticipation was expressed by phrases like “I am going to...”, and “...it will let me.”

With this anticipation in mind, I changed my focus to start interacting with the students (M-N shift). This interaction was prompted by the fact that the number 18 could stay in cell R_3C_2 . I asked questions to help students reflect on why it was allowed by GA to be there. Joseph was quick to point out that this was accepted because it was a multiple of 3. This was a cue for me that I could place a letter in the grid and I inserted d in a neighbouring cell (R_3C_4) and asked the students what that letter symbolised.

Here, I shifted my focus to another teaching purpose: encouraging students to reflect on mathematical phenomena (N-L shift). This reflection encouraged Joseph to suggest a meaning for d : “like if you do 18 plus 3 plus 3”. Placing d in the neighbourhood of 18 (Figure 6) helped Joseph to interpret the symbol d , aided by the representation of its ‘container’, the cell R_3C_4 .⁴ Joseph’s interpretation of the symbol d in association with the values of the neighbouring cells is an example of Mercer’s (2000) claim that symbols (like words) gain meaning from their neighbourhood.

Figure 6: Letter gaining meaning of from its neighbourhood



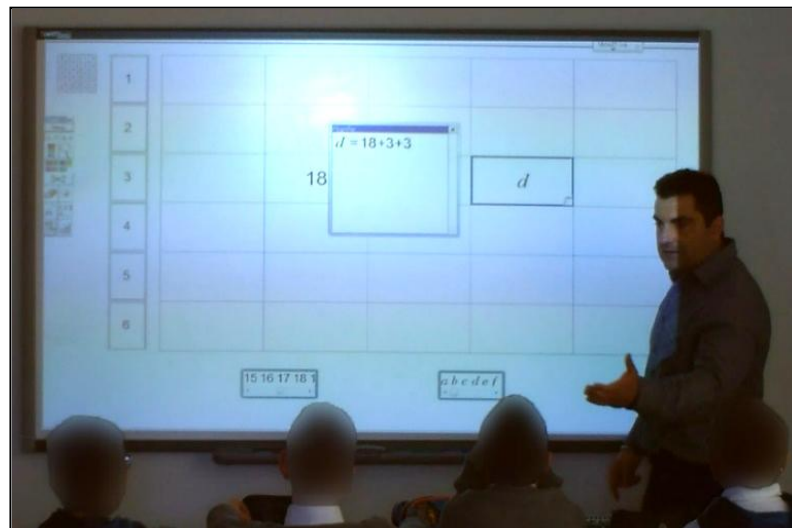
The second part of the lesson episode resumes in the following excerpt.

⁴ The interplay between conceptual interpretations and pictorial, symbolical, and kinaesthetic representations are discussed by Borg and Hewitt (2015).

Excerpt 2: L-N shift (Cycle 1)

PB: [Nodding...] All right, so what we're saying here is that d is, like, the answer of when [points to respective cells] 18 makes plus 3 plus 3. In fact, if you do like this [drags the 18 to R_3C_3 to obtain $18+3$] and like this [moves $18+3$ to R_3C_4 obtaining $18+3+3$ on the same cell as d] – all right? – we see d here and [choosing the magnifier icon] if we see ... with the magnifier here, it is telling me exactly [pointing to Joseph - #2] like you told me that [pointing to d] d [points to equals sign] is [points to respective numbers] 18 plus 3 plus 3. [Clicks on the cell to alter the expression.] If I alter here it will tell me that [points] 18 plus 3 plus 3 equals d .

#2



In this excerpt, I changed my focus from encouraging reflection to forming a model of Joseph's interpretation of the mathematics in question, i.e., his MoS (L-N shift). At first, I confirmed aloud what Joseph seemed to be thinking: "...so what we're saying here is that..." I also made cell movements corresponding to Joseph's calculation of $18+3+3$ ending on the cell containing d , and used GA's magnifier to help Joseph's classmates observe that what he seemed to be implying was that $d=18+3+3$ or that $18+3+3=d$. Building a model of Joseph's and possibly other students' MoS helped me review my original MfS, that of identifying the circumstances that made d an unknown.

Excerpt 3: N-M shift (Cycle 1) and M-N and N-L shifts (Cycle 2)

PB: But if I want, instead of doing 18 plus 3 plus 3, I can, if I want to, erase here [*erases all expressions except 18 and d*] – OK? – I can just bring up [*pointing to the number menu*] that unique number that can be here [*the cell containing d*], a single number... What is the number?

Joseph: Twenty-four.

PB: Do we agree that it is 24?

Joseph: Yes [*the others nodding*].

When I drew students' attention to the possibility of having a single number instead of $18+3+3$, Joseph proposed the number 24. At that moment, it seemed to me that Joseph, and possibly other students who were nodding to his response, were thinking of the letter d as being the answer of $18+3+3$, i.e., 24. In the above excerpt, my focus changed again from reviewing the learning offer to associating Joseph's (and possibly other students') MoS with my mathematics (N-M shift). In order to do this, I had to make adaptations of my notion of unknown as a single fixed number to accommodate Joseph's concept of unknown as 'answer'.

This shift prompted a new M-N-L cycle, with a renewed MfS: the connection between

- a letter as a single (unknown) number due to its being the value of an expression (Joseph's MoS) and
- a letter as a single fixed (unknown) number due to its neighbourhood in the GA grid (the original MfS).

I anticipated how students could make these connections as I started off a new M-N-L cycle (Cycle 2).

My purpose shifted from anticipating these connections to interacting with students to help students develop mathematical appreciations of these connections (M-N shift). I erased all the expressions, except 18 and d (Figure 6). While doing so I was hoping students would observe the link between what was in cell R_3C_4 a moment earlier ($18+3+3$) and the single number could be inserted in that cell. Previous lessons taught me that students were very competent in assigning the right numbers in GA cells, so I figured the empty

cell R_3C_4 could invoke the single number 24 in the minds of the students due to its position in relation to 18 in the 3-times table.

Excerpt 4: L-N and N-M shifts (Cycle 2)

PB: Because we're in the 3-times table and we're doing plus 3 plus 3, all right? ... I bring up the 24 ... I'll pick the 24 from here [drags 24 from the number menu to R_3C_4 containing d] ... And when I go with the magnifier there it is telling me d equals 24. ... So, d equals 24 and [clicks on the cell to alter the order] 24 equals d ...

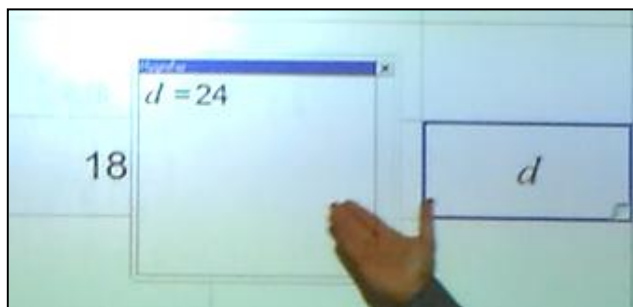
Joseph: The same.

PB: ... As such, we are not seeing an answer. When you say 'answer' it's like you have done some calculation, some plus, minus...

Joseph: 18 plus 3 plus 3.

PB: We don't have any calculation, nothing, here. So now, I cannot quite say that 'equals' is 'answer.' [Jordan shaking his head.] So what can I say that it means there [pointing to $d=24 - \#3$]?

#3



The equals?

Joseph: Equal to [in English].

Dwayne: They are the same in size.

With this in mind, I asked students what was the "unique number that can be" in R_3C_4 . Here my purpose had changed from interacting by erasing the expression $18+3+3$ to encouraging students to reflect on the single number which could be entered in that empty cell (N-L shift). It was Joseph himself who mentioned the number 24. He had already thought about it and even

mentioned it earlier (see end of Excerpt 1) where it seemed he was thinking of it as the answer to $18+3+3$.

In the above excerpt, I first wanted to orient students' thinking (Glaserfeld, 1991b) towards thinking of d as being 24 without having to think of it as the answer to a calculation. So during the experience-reflection stage, I confirmed Joseph's statement by dragging 24 into the cell containing d and proceeded to help students to observe and consider the mathematical statement $d=24$ which could be seen by clicking on the magnifier icon.

I knew that for some students, the equals sign was still just a symbol showing the answer of a computation. So, during the reflection exercise, I focused on the meaning of the equals sign in the expression $d=24$. When I asked what d 'equals' 24 meant, Joseph expressed his thinking by saying in English "equal to." The change from 'equals' to the more exact 'equal to' and his emphasis of the word 'to' gave the equality symbol a more a relational meaning. Dwayne immediately picked up on this and gave the response I was aiming for: "They are the same in size."

Dwayne and Joseph's feedback made me change my purpose from helping students to reflect on their mathematical observations to forming a model of these students' MoS (L-N shift). I confirmed Dwayne's response, and elaborated on his statement. I also said "Good", indicating a favourable review of Dwayne's statement. I was simultaneously making a favourable review of the outcome of my learning offer. In accepting that $d=24$ meant d "is the same size as" 24, Dwayne and possibly Joseph, seemed to have constructed an idea about the possibility of using the arbitrary letter d as a substitute for a constant number (unknown) irrespective of whether that number was the answer of a computation.

This led to another shift of focus: from reviewing the outcome of the learning offer to reflecting on my mathematics, i.e., my interpretation of $d=24$ (N-M shift). I knew that the neighbouring 18 meant that d *could not be anything but* 24. This concept was a subset of the original MfS. However, the original MfS included also the notion that without any other numbers in the grid, d would be a *variable* multiple of 3 and hence the statement $d=24$ would be viable if it were interpreted as in $d=..., 21, 24, 27,...$ This prompted the onset a new M-N-L cycle in which I anticipated that students could, in this way, construct the notion of d as a variable.

Table 1: Summary of two complete M-N-L cycles

Mathematics	Negotiation		Learner
My concept of 'unknown' represented by a letter in a GA environment.	I anticipate students will develop the notion of unknown when this is contrasted with a variable.	I interact by placing 18 in R_3C_2 and d in R_3C_4 . I ask students what the letter d may stand for.	Joseph says that 18 was allowed since it was a multiple of 3. Then he says that d is the answer of a computation involving 18.
The 'answer' of a calculation may also be thought of as an unknown. This holds also when the calculation is not expressed as a single number, e.g., $x=1+\sqrt{2}$	I review my original MfS and find a way how to incorporate Joseph's notion of an 'answer' within my notion of an unknown.	I create an unexpected model of Joseph's MoS concerning the letter d : a letter may stand for the 'answer' of a calculation.	
	I anticipate that students will link the notion of 'answer' and unknown if they can observe an example with the help of GA.	I use Joseph's explanation to show that d may be seen as the 'answer' of $18+3+3$. Joseph says that d could be 24.	I help students reflect on the statement $d=24$. Joseph and Dwayne elaborate on the meaning of the equals sign, viewing it as a relational symbol
I associate students' interpretations of d to my notion d as a variable.	I review the MfS. Dwayne and Joseph seem to interpret d as being <i>equal to</i> a constant.	I build a model of Joseph's and Dwayne's interpretation of the equals sign as 'same in size.'	

Table 1 above summarises how these two successive M-N-L cycles occurred by mapping each event to the respective teaching purpose. This table shows

the fast toing and froing between my mathematics and my learners' knowledge constructions as I strived for CT. The arrows indicate shifts of teacher purpose. There was an average of one M-N-L cycle per 4 minutes of plenary discussion throughout the 20 lessons.

Conclusion

The M-N-L framework gives due importance to the three constituents of Brousseau's (1997) didactic situation: the learner, the teacher, and the mathematics to be taught and learnt. Based on Dewey's (1902) idea that teaching must be defined by both curriculum and learners, the M-N-L framework places the teacher as a negotiator between mathematics and the learner. The framework suggests that the main task of the constructivist teachers is to find ways how to bridge the knowledge she/he intends to teach with the knowledge being continuously constructed by the students during the lesson.

Simon's (1995) theory of teaching mathematics from a constructivist perspective was key in the formation of what I called the forward-negotiation road. The teacher's sensitivity to students' possible constructions of knowledge enables her/him to anticipate possible didactic processes and interact with students accordingly. Based on RC, M-N-L suggests that the teacher needs to make it her/his business to know whether and how the learning offer (Steinbring, 1998) makes sense to the students.

The RC teacher gives much weight to the question of viability of mathematics as experienced by the students. In this regard, Steffe's (1991) principles of (radical) CT were crucial for the formation of M-N-L's backward-negotiation road. The teacher builds models of MoS and uses them to review MfS. The teacher synthesises students' mathematics with her/his own, sometimes requiring accommodations of her/his own mathematical schema. This puts the teacher in a better position to go back to the students with a renewed MfS and a new M-N-L cycle may commence.

The formation of the M-N-L framework, inspired chiefly by the works of Dewey (1902), Steffe (1991), Simon (1995), and Jaworski (2012), and drawing on Glaserfeld's (1990) principles of RC, showed me that the idea of CT is indeed plausible. Rather than portraying it as one set notion of how to teach, the M-N-L framework presents CT as a teaching approach resulting from the

teacher's sensitivity to RC notions of knowledge and learning. This sensitivity is the driving force behind the teacher's changes of purpose during the lesson necessary to keep both mathematics and learners in mind. The M-N-L framework proposes that:

- i. Any learning offer presented to the students is regarded by the teacher as an attempt to facilitate students' active and subjective construction of mathematics. The teacher anticipates the possible didactic situations which may lead to students' developments of mathematical ideas. The teacher thus interacts with the students in order to orient their thinking processes. In this way, the teacher helps the students to make reflective abstractions of the mathematics in question.
- ii. The RC teacher is also a learner. She/he is invested in learning about the mathematics being constructed by the students. This helps the teacher to make inferences about the success or otherwise of the current learning offer, but this exercise does not only benefit the students. When the teacher takes up the challenge of linking students' mathematics with her/his own, this enriches the teacher's own mathematical content knowledge.

The M-N-L framework is both conceptual and analytical. Besides defining CT, it also proved to be a viable tool in helping me to investigate CT in my mathematics lessons by analysing the extent to which I managed to generate and complete M-N-L cycles. It was also instrumental in identifying momentary flaws in my approach, when I created what I called 'roadblocks' (Borg et al., 2016a) that obstructed the negotiation between my mathematics and that of my students. Linking the generation and completion of M-N-L cycles with CT helped me to ascertain that these moments of failure did not render my teaching non-constructivist. Rather, such moments showed that, like anything which is not mythical, CT is not a perfect system but an endeavour of ordinary teachers who try to bring their constructivist beliefs to their daily teaching practices.

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Translanguaging with Maltese and English: The Case of *Value, Cost and Change* in a Grade 3 Classroom

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Abstract: This paper describes how a Primary school teacher in Malta used Maltese and English to teach her 8-year-old pupils meanings for the money-related English words *value, cost* and *change*. Classroom interaction data is presented to illustrate how the teacher drew on the pupils' previous knowledge of money, using related Maltese vocabulary and then introducing the English translations. My observations support international evidence of the richness of bilingual educational contexts. The translanguaging is discussed in relation to whole-class scaffolding strategies as conceptualised by Anghileri, and by Smit, van Eerde, and Bakker. I conclude that while the observed teacher appeared to be successful in her aims, her teaching style appeared to limit the potential generation of conceptual discourse on the part of the pupils. I highlight the need for research to be carried out on how scaffolding through translanguaging might pan out in learning contexts that aim to increase pupil engagement with mathematical discourse

Keywords: primary mathematics education; scaffolding; translanguaging; bilingualism in Malta

Introduction

Baker (2011) defines code-switching as a switch between languages at word or sentence level or at the level of blocks of speech. Traditionally, code-switching has tended to be viewed through a deficit perspective based on the assumption on the value of monolingualism (García, 2009). However, as explained by García, this view is being strongly challenged today, as it is being recognized that bilingualism is not a deviation from a norm but, rather, a communicative method used by many people in the world. Indeed, drawing on more than one language is commonly found in classroom

contexts where two or more languages are represented. For example, Amin (2009) reflects on the shifts from Modern Standard Arabic to English or French in the Arab region; Fu (2003) and Manyak (2001) describe respectively Chinese/English and Spanish/English switching in the U.S.; Palviainen and Mård-Miettinen (2015) discuss Finnish and Swedish in Finland; and Then and Ting (2011) write about Bahasa Malaysia and English in Malaysia. With specific reference to *mathematics* classrooms, various researchers have discussed the use of two or more languages in teaching/learning situations, including Bose and Clarkson (2016), Halai (2009), Jones (2009), Norén and Andersson (2016), and, Setati, Adler, Reed, and Bapoo (2010). In the situations discussed by these researchers, the ultimate aim of the teachers observed was effective pupil learning.

As part of his doctoral project carried out in Wales, Williams (1994) coined the term 'translanguaging'. By this he meant the planned and systematic use of two languages, for example, switching from English for reading/listening, to Welsh for speaking/writing. Subsequently, the term 'translanguaging' has gained ground in academic circles, but has taken on a wider and more flexible meaning. García and Kleyn consider 'translanguaging' to be the "deployment of a speaker's full linguistic repertoire" (2016, p.14). They highlight the relevance of translanguaging in post-colonial education contexts where the medium of instruction is often different from the language spoken by the students. In these situations, the use of the students' language(s) may be used to aid comprehension, resulting in a mix of two (or more) languages as part of the teaching/learning process.

Notably, the term 'translanguaging' is associated with a positive view of the mixing of languages. This is in contrast to the traditional association of the term 'code-switching' with negative perceptions, for example, when one views code-switching as language deficit. García and Li Wei (2014) explain that translanguaging focuses attention not on the languages *per se*, but on the *practices* of bilinguals, which may include original and complex constructions (verbal and written) on the part of the speakers. In this paper, I choose to use the term 'translanguaging' to highlight that the use of Maltese and English is an established practice in Maltese classrooms, a practice that appears to be a beneficial pedagogical tool (Camilleri, 1995). Hence, I view the practice positively.

The aim of this paper is to present data that shows how a Primary school teacher in Malta translanguaged using Maltese and English during a series of lessons on the topic 'Money'. The main focus of her lessons was to teach the children the meaning of the words *value*, *cost* and *change*. The teacher's use of the languages will be considered in terms of 'scaffolding' strategies as described by Anghileri (2006) and Smit, van Eerde, and Bakker (2013). The paper begins with an outline of the Maltese bilingual educational context.

This is followed by a description and discussion of the observed teacher's language strategies. I end the paper by recommending further related research.

Bilingual Education in Malta

Malta enjoys two official languages. One is the national language Maltese which, according to a recent self-report census, is spoken by 90% of Maltese citizens (National Statistics Office, 2014). The other official language is English, a legacy of 164 years of British colonial rule (1800 – 1964). Both languages play an important role in Maltese people's lives. Maltese enjoys respect nationally and internationally, while English is recognized as an important global language and is crucial for the local tourism industry (Camilleri Grima, 2015). Research (see, for example, Camilleri, 1995; Camilleri Grima, 2015; Gauci, 2011; Sultana, 2014) and much anecdotal evidence suggest that the vast majority of teachers use both languages to varying degrees. This is because whereas Maltese is usually the language of the teacher and students, English is considered to be the 'standard academic language' (García and Li Wei, 2014) of many school subjects, including mathematics. This situation often results in English words being embedded into stretches of Maltese speech. For example, it is customary to retain 'technical' or subject specific words in English even if Maltese equivalents exist (Camilleri, 1995)¹. This certainly applies to mathematics, for example, one might say "**Illum ser nitkellmu fuq il-quadrilaterals**" [**Today we're going to talk about** [the] quadrilaterals].

Initial teacher education has had a role to play in the development of the dominance of English in education. From 1881 up till the 1960s, that is, for a good part of the British colonial period, school administrators received their training in the U.K. (Zammit Mangion, 1992). Furthermore, between 1944 and the late 1970s, teacher training was run by British Catholic religious orders in residential programmes (Camilleri Grima, 2013). Thus, educators were traditionally enculturated into the practice of schooling through English². Certainly, one factor that prompts translanguaging in classrooms is the pervasion of English as the language of written texts. For a number of subjects, textbooks – even when produced locally – are written in English.

¹ Here I am not referring to mathematical words such as *computer* and *graph* that have been wholly assimilated into written and spoken Maltese (**kompjuter**, **graff**) and are to be found in a respected dictionary (Aquilina, 1990). Rather, I refer to words like *multiplication* and *square*, for which Maltese equivalents (**multiplikazzjoni**, **kwadru**) are found in the dictionary, and may indeed be heard in other contexts, but are not commonly used in class.

² Teacher-training moved to the University of Malta in the late 1970s.

The subjects include science and mathematics at primary level and the natural sciences, mathematics, ICT, economics, accounts, amongst others, at secondary level. Other written texts include examination papers, software, whiteboard work, worksheets and pupils' notes. As teachers and children shift between verbal interaction and written texts they tend to shift between languages, and this accounts for a good amount of the translanguaging that occurs.

Policy documents have periodically offered guidance on language use. The 1999 National Minimum Curriculum (Ministry of Education, 1999) had recommended that schools should develop their own language policies according to their own needs. However, it also recommended that mathematics, science and technology at primary level, and other subjects such as biology and economics at secondary level, be taught through English. According to the writers of this document, code-switching should only be used in cases when using English poses problems. The more recent National Curriculum Framework (Ministry for Education and Employment, 2012) recognises the need for clear direction on the language of instruction, and repeats the recommendation for school based policies. With respect to mathematics, the National Curriculum Framework is less prescriptive than the 1999 document, giving the general guideline that "mathematics concepts and language are [to be] inculcated through systematic teaching and learning activities" (p.51).

Anecdotal evidence shows that, irrespective of policy documents, the mixing of Maltese and English is an ingrained practice. Given this common language practice, I wished to study how teachers might use Maltese and English to support students' learning of mathematical ideas. In particular, I focus on the role of translanguaging. International research in mathematics education spanning over forty years suggests that for multilingual students, purposeful use of all their language resources can be beneficial; this has been pointed out by Barwell et al., (2016). In particular, García and Li Wei (2014) state that teachers can use translanguaging strategically as a scaffolding approach to ensure that emergent bilingual students engage with rigorous content, access texts and produce new language practices and new knowledge. Hence I ask the question: *Given the common use of Maltese and English in Maltese mathematics classrooms, how might a teacher's translanguaging support young students' learning of mathematical ideas?* In order to address this question I draw on the theoretical construct of scaffolding.

Scaffolding

Wood, Bruner, and Ross (1976) define 'scaffolding' as adult support which is adjusted over time until this help is removed when the learner can manage

alone. Wood et al. list six key elements that they believe scaffold learning, which include demonstration, marking critical features of the task and controlling frustration. Tharpe (1993) also suggests strategies for supporting students' learning, including feedback, questioning and cognitive structuring (e.g. explanations). The notion of scaffolding was originally conceptualised in relation with a child's Zone of Proximal Development (Vygotsky, 1978) and was related to adult-to-child interactions (Cazden, 1979; Wood et al., 1976). However, Smit et al., (2013) make a case for extending the idea to whole class contexts. They affirm that the social setting in which learning takes place fits in well with Vygotsky's work that stresses social settings, and that one can consider the ZPD of the class as a whole.

In this paper I use two interpretations of the notion of scaffolding. The first is that proposed by Anghileri (2006). Based on classroom observations, Anghileri drew up a hierarchy of scaffolding strategies used by the teachers observed. Level 1 consists of *environmental provisions* which includes the class organisation, structured tasks and artefacts. Anghileri considers this level to be the most basic one, since the provisions enable learning to take place without teacher intervention. Emotional feedback, such as drawing attention or encouragement, is also included at this level. Level 2 involves *explaining, reviewing* and *restructuring*. More specifically, this level includes teacher strategies such as modelling, prompting and probing, providing meaningful contexts, rephrasing students' talk and simplifying problems. The highest level, Level 3 involves *developing conceptual thinking*. At this level, the teacher uses strategies that encourage students to make connections, develop representational tools and to generate conceptual discourse. Here, scaffolding is more complex. As an example of scaffolding the development of representational tools, Anghileri gives the idea of using symbolic records as tools for thinking. As an example of making connections, Anghileri mentions the use of the expression 'double 6' instead of '6 add 6' in paraphrasing a pupil's suggestion. With regard to conceptual discourse, Anghileri explains that the teacher goes beyond explanations and justifications; rather, by initiating reflective shifts, what is said and done in class is rendered an explicit topic of discussion.

Another model for scaffolding is offered by Smit et al., (2013). They identify three key characteristics in the scaffolding process. One characteristic is *diagnosis*, or establishing the students' present state of knowledge. Another characteristic is *responsiveness*, which implies adapting to students' learning; Smit et al. consider this to be the heart of the scaffolding process. The third characteristic is *handing over to independence*, whereby students are able to achieve or carry out a targeted aim unaided. It is this characteristic that is the ultimate aim of the scaffolding process. Smit et al. state that successful handing over includes the fading of teacher support. Smit et al. also make a distinction between "*off-line*" and "*on-line*" application of three characteristics they identify, by which they mean the application of the strategies outside the

classroom (off-line) and as part of the classroom interaction (on-line). From their research, Smit et al. also note that in the whole-class context, the process of scaffolding is *layered, distributed* and *cumulative*. *Layering* refers to the interweaving of diagnosis, responsiveness and handover, in, and outside, classroom interaction; *distribution* refers to the fact that scaffolding occurs in a “scattered way” (p.830), that is, in various episodes over time; *accumulation* refers to the fact that students’ learning processes represent the cumulative effect of scattered diagnosis, as well as online and offline responsiveness over time.

While Anghileri’s model includes specific strategies, Smit et al.’s characteristics are more generally phrased. Since I will be referring to both models in my analysis of data, I have found it helpful to line up the two models as presented in Table 1 in order to highlight how they overlap.

Anghileri (2006)	Smit et al. (2013)	
<p><u>Level 1: Environmental provisions</u> Classroom organisation, sequencing and pacing, free play, structured tasks, self-correcting tasks, artefacts, peer collaboration, emotive feedback</p>	<p>[No parallel can be found here since Smit et al., (2013) believe that considering aspects such as classroom organisation, artefacts and so on, is stretching the notion of scaffolding too far from its original conception].</p>	
<p><u>Level 2: Explaining, reviewing & restructuring</u> Reviewing: parallel modelling, prompting & probing, interpreting students’ actions and talk Explaining & justifying, showing & telling Restructuring: providing meaningful contexts, rephrasing students’ talk, simplifying the problem, negotiating meanings</p>	<p>Diagnosis (establishing the students’ present state of knowledge) Responsiveness (adapting to students’ learning)</p>	<p>Online / Offline</p>

<p><u>Level 3: Developing conceptual thinking</u></p> <p>Making connections</p> <p>Developing representational tools</p> <p>Generating conceptual discourse</p>	<p>Handing over to independence (students carry out a targeted aim unaided)</p>	
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Table 1: Comparison of two scaffolding models (Anghileri, 2006, and Smit et al., 2013).

I now explain the context and design of my study, following which I use the afore-described models of scaffolding to interpret a primary teacher’s approach to teaching ideas related to money.

The Context and Design of the Study

The reflections offered in this paper are based on a case study of one classroom. Yin (2014) suggests that case studies are a particularly suitable research method to answer ‘how?’ type of research questions. As Stake (1995) explains, the purpose of a case study is to understand well a particular context; case studies bring to light that certain situations and learning experiences can—and do—happen or exist. My aim is to contribute to understanding the relationship between language and mathematics and hence, although conclusions from my study cannot be generalised, they can add to our understanding of this relation. The classroom observations were carried out in 2002 and served as a pilot study for a larger research project through which I was to study the use of language in elementary mathematics classrooms (see, for example, Farrugia, 2007, 2009 and 2016). The focus of the piloting was mainly the practical elements of the data collection process, but the classroom interaction itself suggested interesting points, thus prompting me to revisit it at this later date.

My choice of teacher was opportunistic (Wellington, 2000): I knew Anita (pseudonym) professionally and was aware that she was a highly motivated teacher. I approached her with a request to observe her teaching mathematics, in order to collect data about translanguaging practices. Anita taught a Grade 3 class (8-year-olds), whom I refer to as ‘pupils’ due to their young age. The home language of all the pupils was Maltese. I observed seven lessons on the topic ‘Money’, each of duration 30 – 45 minutes. I took the role of observer-as-

participant (Cohen, Manion, and Morrison, 2000) since my interest was simply to observe and reflect, and thus I did not influence the design or implementation of the lessons. The lessons were video-recorded to allow for later transcription and analysis.

Prior to the lessons, Anita informed me that the school administration recommended that English be used to teach mathematics; this was the official policy recommendation at the time (Ministry of Education, 1999). However, Anita said that she tended to use both English and Maltese. In our informal conversations prior to the series of lessons, and as the days progressed, she explained that although she tried to use English as much as possible, she felt that she needed to use Maltese to make sure that the children understood the mathematics at hand. This reasoning is similar to that of the Austrian teachers studied by Gierlinger (2015); these latter teachers taught a variety of subjects through English and for them it was the subject – rather than the language – that was priority. Anita used a whole-class teaching approach, whereby the lessons were teacher directed, and the pupils worked on the same tasks at roughly the same pace. Their desks were set in pairs in a rows-and-columns arrangement and Anita tended to stand at the front of the classroom, unless she was monitoring the children’s work during a written exercise. The textbook in use was a local publication written in English, and Anita provided the children with occasional worksheets, also written in English. The interaction observed was that described by Sinclair and Coulthard (1975) as ‘IRF’ (initiation-response-feedback), and thus the children tended to give very short, often one-word, answers to Anita’s questions or prompts. For example:

- Teacher: (Referring to a number of priced grocery items set out on the desk). What costs 12 cents? Which one costs 12 cents?
Pupil: The juice.
Teacher: The juice. Which one costs 40 cents?
Pupils: (In chorus). The soup.
Teacher: The soup, OK.

Anita assumed children’s prior knowledge of some topic related words, namely *cents*, *10c* (and similar), *amounts*, *shopping*, *money*, *coins*, *addition*, *subtraction*, *how many* and *how much*. This assumption was based on her knowledge of what had been covered in the previous grade, from her general relationship with the children as their class teacher, and also, through the periodic classroom experience of collecting money for outings and charities. Indeed, during the lessons, I noted that Anita used the afore-mentioned words without stressing them or drawing attention to them in any particular way. On the other hand, the words *value*, *cost* and *change* were assumed to be new ‘key’ English mathematical words that Anita stated she needed to focus

on. She believed that although the children were likely to be already familiar with such notions thanks to their life experiences (and therefore, with the associated Maltese terminology), she felt confident that the *English* terminology would be new to them. In order to introduce these new words, Anita made use of shopping contexts, utilising grocery items, cardboard laminated coins, role play, and handouts that depicted everyday money-related contexts. This is in line with Mercer's (2000, p.35) suggestion that "teachers can help learners make sense of technical terms by introducing them into dialogues with pupils in situations where the context helps makes meaning clear".

Using Maltese and English to Teach the Topic 'Money'

In this section I describe the approach taken by Anita to address the ideas of *value*, *cost* and *change* over the seven observed lessons. Transcripts are provided as illustrations.

Value

As Anita had anticipated, the pupils were already familiar with the coins and their values and could talk about them in Maltese. Anita started the first lesson by holding a twenty minute discussion about the coins in use, during which she asked individuals to mention amounts for which a coin existed (e.g. 2 cents) and amounts for which no coin was available (e.g. 3 cents). (NOTE: the currency in use at the time was the Maltese Lira. 1 Maltese Lira was equivalent to 100 cents). As the discussion progressed, Anita sketched existing coins on the whiteboard in the form of a circle with, for example, *1c* written within it. This conversation was held mainly in Maltese, with coin values stated in English (e.g. "ten cents"); this is a common practice locally, even outside the classroom.

Following Anita's prompts or questions, the children commented that one could buy more with a coin that had a bigger number shown on it, and that they would prefer their grandmother to give them a Maltese Lira rather than a 1 cent coin. At one point in this discussion, one of the pupils, James, stated that he would prefer one coin to another because "**tiswa ħafna**" (**it's worth a lot**) and this was promptly followed by Kenneth's suggestion "**in-numru ikbar**" (**the number is bigger**). It was at this point that the word **tiswa** was first used by a pupil named Fiona. This word is grammatically a conjugated intransitive verb and translates into '*what it's worth*'. As a conclusion to the discussion, Anita stated that she was going to ask questions, which she would write on the whiteboard. The interaction is reproduced below. In the transcription, the original speech is shown in the left-hand-side column, while I offer a translation in the right-hand-column. Any speech that was uttered in

Maltese is given in a **bold** font, and similarly for its translation. Pupils' names are pseudonyms, although in the first transcript below, pupils are numbered since they were out of camera view and hence unidentified.

Teacher:	How many Maltese coins are there? (<i>Writes this question on the whiteboard.</i>)	How many Maltese coins are there? (<i>Writes this question on the whiteboard.</i>)
Pupils:	Seven.	Seven.
Teacher:	There are 7 Maltese coins. (<i>Writes this statement on the whiteboard.</i>) Which coin has the smallest value? (<i>Writes this on the whiteboard.</i>) What am I asking? X'qed nistaqsi hawn? Which coin has the smallest value?	There are 7 Maltese coins. (<i>Writes this statement on the whiteboard.</i>) Which coin has the smallest value? (<i>Writes this on the whiteboard.</i>) What am I asking? What am I asking here? Which coin has the smallest value?
Pupil 1:	L-izghar.	The smallest.
Teacher:	Kif tidher?	As in the way it looks?
Pupils:	(<i>In chorus</i>). Le.	(<i>In chorus</i>). No.
Teacher:	OK. Mela x'inhu? Liema hi dik il-kelma li qalet Fiona, the magic word? Which coin has the smallest value? X'qed nistaqsikom?	OK. So what is it? What's that word that Fiona mentioned, the magic word? Which coin has the smallest value? What am I asking? What it's worth (its value).
Pupil 2:	Kemm tiswa.	Well done.
Teacher:	Prosit.	

The teacher went on to write the answers to this question (1 cent) on the whiteboard. She then asked and wrote '*Which coin has the largest value?*' below which she wrote the answer given by the children.

In the above stretch of interaction, translation from one language to another was used in a manner that Camilleri Grima (2013, p.563) calls 'non-explicit translation through elicitation'. This part of the lesson marked a clear shift to English as the language of the written - and more formal - text of mathematics, exemplifying one of the languages routes possible in bi/multilingual classrooms as described by Setati and Adler (2000):

Informal spoken main language → formal spoken English → formal written English

The whiteboard work also served as a bridge between the initial discussion and the written textbook exercise that was to follow immediately afterwards. In the written exercise, various sets of coins were shown and the printed text stated “Here are some sets of coins. What is the value of each set?” Here the word *value* was used in a different sense since it was used in relation to a total value of a set of coins, rather than in relation to ONE coin. The class worked out the exercise together, and Anita guided them to consider the set of coins collectively through verbal expressions that included: *altogether, add, Kemm jiswew kollha flimkien? [How much are they worth altogether?]*.

Anita used the word *value* only briefly in the following two lessons as part of a short introductory review. The word was given importance once again in the fourth lesson, now in relation to the equivalence of two sets of coins. As in the first lesson, a class discussion was used to focus on the word *value*. The following is a snippet of Anita’s questioning, where ‘they’ refers to two sets of coins and the capitalisation of the word ‘value’ indicates that Anita stressed it with her tone of voice.

“**Jiswew l-istess? [Are they worth the same?]** ...So my question in English is this: do they have the same VALUE?”

The discussion was followed by a worksheet showing sets of coins and entitled “Match the same value”. In the fifth lesson, Anita set a task on the whiteboard entitled “Draw coins to make up these values: 5c, 7c, 18c ...” In the sixth and seventh lessons the word *value* was not used.

Cost

The word *cost* was introduced in the third lesson. The word used in Maltese for *cost* is also **tiswa**. In order to introduce the word *cost* Anita now used the word **tiswa** in relation to priced objects (that is, in relation to what the object is worth rather than in relation to the value of a coin). She did this through organized role-play shopping. Groceries were placed on a table at the front of the room, with prices attached to them. The teacher called out children and instructed them to buy an item, and to give the exact amount using their set of sample coins. A pupil had picked up a packet of Chicken Soup.

- Teacher: **Kemm jiswa ċ- Chicken Soup? [How much does the Chicken Soup cost?]**
- Pupils: *(In chorus)* Forty.
- Teacher: Forty cents. How much does it COST?
- Pupils: *(Silence).*
- Teacher: *(Waving the priced packet of Soup).* How much does it cost?

Pupil 1 Forty cents.
Teacher: (*Nods*). Forty cents.

During the next purchase the word *cost* was not used, but Anita used the Maltese **tiswa** instead while showing up a newspaper:

Kemm tiswa l-gazzetta? (How much does the newspaper cost?)

In the third example, Anita switched back to English and the children pre-empted the question, seeming to anticipate what the teacher was about to request.

Teacher: (*Holding up the priced carton of juice*). How much does the juice...?
Pupils: (*Several pupils interrupt in chorus*) Twelve cents!
Teacher: First listen to the question so that you know what I am asking. How much does the juice cost?
Pupils: Twelve cents!
Teacher: **Mela [So]**, the juice costs twelve cents.

I found this stretch of interaction to be particularly interesting since, thanks to the repetitive form of the role-play structure, pupils were now able to fulfil the required interaction even before the teacher had finalised her question. Still, Anita insisted on using the word *cost* as originally intended and so she repeated - and completed - her question ("First listen to the question, so that you know what I am asking. How much does the juice cost?"). She appeared to be attempting to help the children 'glue' the new word to the related concept (Hewitt, 2001), especially since the first time Anita had used the word *cost* the pupils had remained silent. Once again, a worksheet marked a clear shift to written English. The worksheet dealt with buying fish, with the general instruction being: "*Buy some fish. How much do they cost?*" Hence the word *cost* continued to be linked with the purchasing power of money through written English.

The word *cost* was not used in the fourth and fifth lessons. In the sixth lesson, the word was used in discussion to support the meaning for the word *change*, which was the focus of attention. Hence, during this lesson the word *cost* appeared to be subordinated (Hewitt, 1996) to the new word *change*. The word *cost* was not used in the last lesson, which was also dedicated to the idea of change.

Change

The word *change* was also introduced through role play. Children were invited to approach the 'shop' with a 50c coin in order to buy something. Similarly to the introduction of the word *cost*, Anita waited for a child to use the Maltese word for change – **bqija** – and then started substituting it in the course of the interaction. For example, when Daniel was shopping, Anita used translation without a metalinguistic marker (Camilleri Grima, 2013) as indicated below.

Teacher:	X' irrid naghtik?	What should I give you?
Daniel:	Bqija.	Change.
Teacher:	Change. Very good. Change, bqija veru?	Change. Very good. Change, change right?

Once again, Anita used both languages to establish the meaning of the word, as when Derek chose to buy a carton of juice:

Teacher:	How much money does Derek have?	How much money does Derek have?
Pupils:	Twenty-five [cents].	Twenty-five [cents].
Teacher:	Can he buy it [the juice]?	Can he buy it [the juice]?
Pupils:	Yes.	Yes.
Teacher:	The juice is twelve cents. How much money does he [Derek] have left? Kemm għad fadallu? What is his CHANGE? Il-flus tiegħu qed jonqsu jew jizdiedu?	The juice is twelve cents. How much money does he [Derek] have left? How much has he got left? What is his CHANGE? Is his money decreasing or increasing?
Pupils:	Jonsqsu.	Decreasing.
Teacher:	Jonsqu. Mela rridu naghmlu...?	Decreasing. So we need to work out ...?
Pupils:	Minus.	Minus.

The role-play activity was followed by an individual worksheet related once again to buying fish at a market, this time prompting subtraction to find the change. The word *change* was not printed on the sheet.

The final lesson was dedicated to written word problems of the type: "*Anna has 42c. She spends 20c. What is her change?*" During this lesson, the words *value* and *cost* were not used, but *change* was used frequently. Anita wrote the problems on the whiteboard, each one ending in a different way: *What is*

her/his change? How much money does she/he have left? How much money has she now? For one of the problems – “John has 50c. He spends 20c” – Anita asked the pupils to finish off the story themselves in a full sentence. Different pupils offered the following endings (here reproduced as uttered by the pupils):

John has thirty cents now.

His change is thirty cents.

John has thirty cents.

The shop gave John thirty cents.

John change is thirty cents.

John has thirty cents change.

John is thirty cents.

Anita accepted these suggestions, and rephrased any that contained grammatical mistakes. For example after the suggestion “John is thirty cents”, Anita said “Yes, John HAS thirty cents”. This task was in contrast with previous ones, in the sense that here Anita gave the children the opportunity to express themselves using more language, including the key word *change*. This is in line with Lee’s (2006) suggestion that it is important for pupils to use language themselves so as to get used to the way expressions are used and to express the concepts and ideas that are encompassed by the mathematical terms.

Discussion: Scaffolding through Translanguaging

The interweaving of Maltese and English in Anita’s class supports the point that in practice it is difficult to identify boundaries between languages, a point made by Barwell et al., (2016), and one which is in line with adopting a translanguaging perspective. Hence, in Anita’s classroom, a ‘translanguaging space’ was created wherein the children’s language practices were brought together (García and Li Wei, 2014), creating what Canagarajah (2011) considers to be an integrated system. For example, the children generally used Maltese in verbal interaction, but stated numbers, prices and other topic-related words in English within the same sentence. They also referred to grocery items in English and suggested *yes* and *no* in English. The children sometimes attended to both languages simultaneously as in the case of when a written exercise was being worked out alongside a class discussion. In these situations, the written text was in English, while the verbal interaction was mixed. The children appeared to take the ‘movement’ between languages in their stride.

At Anghileri’s (2006) first level of scaffolding, one finds environmental provisions. In the observed classroom, these consisted of worksheets, sample

coins and priced items, which helped to create the context to be discussed and/or worked on. Anita first diagnosed the children's present state of knowledge 'off-line' (Smit et al., 2013, p.825) by drawing on her daily experience with the children in order to approach the lessons with certain assumptions. During the unfolding of the lessons, Anita used 'on-line' (ibid, p. 825) diagnosis as part of the process of classroom interaction. This was achieved by probing during which translanguaging played a role. One example is when Anita probed whether children were using size to determine the value of a coin. The ongoing interpretation of pupils' actions and talk, a Level 2 strategy, can also be considered to be a diagnostic strategy.

Much of the scaffolding noted was of the 'on-line' responsive type (Smit et al., 2013). I will break this down using Anghileri's (ibid) strategies at Level 2. Anita provided meaningful contexts in the form of shopping and used language with purpose in relation to this context; she frequently used explaining and reviewing. According to Anghileri (2006), probing and prompting are two strategies commonly used as part of the IRF pattern of interaction. Anita also used rephrasing, for example, when she corrected pupils' English.

One notable feature of Anita's input was the use of translation. Anita used translation to explain or to rephrase pupil talk, and even to negotiate meaning. García (2009) states that there is no simpler translanguaging than translation, and Anita used this strategy for single words or also for whole sentences or questions. Through this, Anita attempted to alter the pupils' everyday shopping experience (expressed in Maltese) into one expressed through English. I can consider this to be a scaffolding strategy in itself, one that is potentially available in a bilingual classroom. Thus in a bilingual classroom this scaffolding strategy might be added to Anghileri's model at Level 2 or to Smit et al.'s category of on-line responsive strategies. It should be noted, however, that the strategy of translating the key words *value*, *cost* and *change* was possible since the work at hand was the 'everyday' topic of Money. Thus, the pupils were already familiar with the ideas being addressed and with the Maltese vocabulary that is used to express them. Other school topics for which familiar Maltese vocabulary may be helpful are addition and subtraction, measurement and space. Possibly, for these school topics, translation might also be used as a scaffolding strategy. On the other hand, if a Maltese mathematical word and/or concept is not familiar to the pupils - as might be the case for **multiplikazzjoni** (multiplication) for young children - then translation is not helpful, since the Maltese word may be as unfamiliar as the English one. Translation is also not possible in the case of words for which no standard Maltese translation as yet exists (e.g. *square root*).

Handing over to independence (Smit et al., 2013) is a key feature of scaffolding. Anita's translanguaging from a verbal mixed code (Maltese and

English) to verbal English as part of the scaffolding processes of showing, telling and explaining clearly served as preparation for written English worksheets. The worksheets combined everyday English with ideas expressed by the new mathematical words. By the time a written exercise was set, adult support could be removed and the learners could carry out the task without assistance (Wood et al., 1976). That is, the children were able to engage with the new English mathematical words *value*, *cost* and *change* as intended by the teacher. Another strategy that aided the process of handing over was that of parallel modelling. Anita used this strategy when she expressed word problems in English herself, then asked the pupils to provide the ending to a problem. Here she offered the pupils an opportunity to 'walk alone' with the mathematics; in particular, with (English) mathematical expression. Thus I noted an element of handing over to independence with respect to Anghileri's Level 3 features *developing representational tools* (children using paper coins and symbols) and *making connections* (children linking the Maltese words to English one).

However, the teacher-directed pedagogy appeared to limit learning to aspects specifically planned by the teacher, rather than enabling an independence that provides for experimentation or innovative thinking. The fact that the tasks were structured and closed meant that the pupils themselves rarely used the new English mathematical words themselves, nor participated in lengthier discussions. Hence, the *generation of conceptual discourse*, another of Anghileri's Level Three characteristics, was very restricted. Ideally, mathematics lessons should include a stronger element of discourse in order for pupils to take ownership of ideas and to develop a sense of power in making sense of mathematics (Van de Walle, Karp, and Bay-Williams, 2013). This may be achieved through a pedagogy that requires pupils to engage in group discussion, and in lengthier discussion with the teacher, thus enriching the pupils' language input. In this scenario, the teacher's input, and hence scaffolding strategies, would be "responsive ... flexible and dynamic" (Anghileri, 2006, p.51). Possibly, a different desk arrangement would be more suitable for such activities; the traditional rows-and-columns arrangement, with all pupils facing the front of the room, was an environmental provision that was perhaps not so conducive to encouraging pupil-pupil communication; changes in seating would need to be carried out prior to any task that required discussion.

Smit et al., (2013) explain how scaffolding in whole class settings is layered, distributed and cumulative. I noted layers (diagnosis, responsiveness and handing-over) across the 7 lessons; accumulation can be considered to be the building of knowledge as the week progressed (recognition of single coins, value of sets of coins, role play using exact coins, requiring change and so on). However, systematic distribution was not so evident. Anita's use of the new words was not sustained throughout the week, so generally, the concepts

appeared to be tackled separately from each other. The only exception was when the word *cost* was used to support the learning of the word *change*. Table 2 outlines when the target words were used by the teacher over the seven lessons. In the Table, 'introduced' and 'brief mention' refer to verbal utterances; the latter indicates a quick reference by the teacher at the start of a lesson by way of linking the lesson to the previous one. By 'sustained' I mean that the word continued to be given importance in the lesson, either in the verbal interaction or through written text.

<i>Lesson</i>	<i>Key Word</i>			
	<i>Value (of a coin)</i>	<i>Value (of a set of coins)</i>	<i>Cost</i>	<i>Change</i>
1	<i>Introduced Sustained (oral/written)</i>	<i>Introduced Sustained (written)</i>		
2	<i>Brief mention</i>	<i>Brief mention</i>		
3	<i>Brief mention</i>		<i>Introduced Sustained (written)</i>	
4		<i>Sustained (written)</i>		
5		<i>Sustained (oral/written)</i>		
6			<i>Sustained (oral) Subordinated to Change</i>	<i>Introduced</i>
7				<i>Sustained (oral/written)</i>

Table 2: The use of the words *value*, *cost* and *change* over the seven lessons

As can be seen from Table 2, the word *value* in relation to one coin was not revisited from Lesson 4 onwards, and *value* was not used at all in the last two lessons. The use of *cost* was introduced, dropped, and picked up again in Lesson 6. While it is not practical to expect that every new word learnt will

continue to be used in each lesson that follows, Oxford (1990) suggests that reviewing material at spaced intervals is one strategy to help learners memorise new word meanings. According to Anghileri's hierarchy, reviewing is a Level 2 strategy. An increase in the frequency of word use would take into account learners' sensitivity to frequency of word use, a sensitivity noted by Hatch and Brown (1995) and by also myself (Farrugia, 2016).

Conclusion

The data collected as part of this study supports other international research that shows that the use of two or more languages can support teaching and learning. It was quite evident that Anita's use of translanguaging was not random or careless, but served as a valuable communicative tool (Baker, 2011). In this classroom, there was the advantage that both teacher and children shared the same two languages and general cultural background, and through my observations I concluded that all participants appeared comfortable with the language experience. I suggest that translation from Maltese to English, or vice versa, serves as a scaffolding strategy, and may be placed at Level 2 of Anghileri's (2006) strategies. Translation may be a useful strategy for some mathematics topics, in particular topics that draw on pupils' everyday experiences.

Perceiving translanguaging as a positive practice and appreciating its pedagogical benefits is something worth stressing in Malta since, locally, it is still common to find English, as a world language, being favoured amongst some educators and also policy makers. With regard to the latter, this concurs with what Barwell et al., (2016, p.21) call the "simple default position" often taken by politicians and bureaucrats. More research on translanguaging in mathematics classrooms may help to promote awareness in this regard and to highlight the benefits of what Blackledge and Creese (2010) call a flexible bilingual pedagogy.

Handing over to independence of mathematics learning was somewhat limited in the classroom observed, since the activities were very teacher directed. If pupils are to independently make connections, develop representations and generate conceptual discourse, then a more open ended, possibly investigative approach to the subject may need to be taken. Presumably, translanguaging would play a role in teacher-pupil and pupil-pupil interactions. With regard to conceptual discourse, one would need to

take into consideration the academic language of mathematics. As stated earlier in this paper, the language of written mathematics in Malta, including school and national assessments, is English. Thus, there is also the need, as explained by Setati et al., (2010), for the pupils to learn to speak and write the more formal (English) mathematical language. Even García and Li Wei (2014), who value so highly translanguaging as a tool for learning, admit the necessity of pupils engaging in certain practices required by society, such as the mastery of the dominant language practice. Such mastery can be achieved through *explicit* attention to language, something strongly recommended by mathematics educators and researchers including Murray (2004), Melanese, Chung, and Forbes (2011), and Gibbons (2015), and recently attempted by myself in another local classroom (Farrugia, 2017).

It would certainly be interesting – and important – for further research to be carried out to explore how teachers and pupils might use translanguaging in Maltese mathematics classrooms wherein pupils are given increased independence with respect to developing mathematical ideas.

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Establishing Local Norms for Two commercially available Numeracy Standardized Tests to identify Maltese Children with Mathematics Learning Difficulties

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Abstract: Mathematics Learning Difficulties (MLD) are of international and national concern. International research estimates that between four and seven percent of any population struggle with the learning of mathematics (Geary, 2004). Nonetheless, locally this field of research is still not adequately researched. Moreover, no numeracy assessment has been standardized with children in Malta. Consequently identifying children with MLD is based locally on using assessments which have been developed and standardized in other countries, in particular the U.K.. My doctorate research project aimed at finding effective strategies that help children to overcome their difficulties in Mathematics. The study was conducted with Grade 5 (9 to 10 years old) learners attending seven Catholic Church schools for boys. Six case studies were carried out with pupils attending the same school, who were selected to follow an intervention programme. The programme aimed at supporting learners with MLD to master the numeracy components that are fundamental for mathematics learning. This with the hope of finding effective strategies that would help learners struggling with mathematics to make the desired progress in the subject. This paper describes the process of sample selection. Three tests, which have been standardized in the U.K., were administered to a sample population of 352 boys out of the 704 boys attending Church schools for boys in Grade 5 and norms were established. The tests were then administered to all the boys attending Grade 5 at the school where I taught (50 pupils). The established local norms were then used to identify the boys with MLD who would participate in the intervention programme.

Keywords: Mathematics Learning Difficulties; numeracy assessment; standardization of mathematics tests; sample selection

Introduction

Despite a similar prevalence rate to that of Reading Difficulties (RD) - an estimated four to seven percent (Geary, 2004) - the field of Mathematics Learning Difficulties (MLD) is still highly unexplored and under-researched in comparison to research on RD (Moeller, Fischer, Mag, Cress, & Nuerk, 2012). This is of concern since a number of studies (Bynner & Parsons, 2005; Poustie, 2000) have suggested that MLD may have great negative implications on a learner's school life and beyond. Following a longitudinal study carried out by Bynner and Parsons (2005), the researchers reported that MLD influence adults' life chances and therefore their quality of life. Bynner and Parsons highlight that adults who lack a basic grasp of numeracy skills are less likely to find a full-time job, to have access to an employer pension, and to be home owners. Moreover, they suggest that these adults are more likely to form part of a non-working household and to develop depression due to the lack of control over their lives. Bynner and Parsons also allude that difficulties in mathematics may have even higher negative impacts than RD. Their study concluded that adults having MLD were more likely to be unemployed than other adults who exhibited RD.

Interest from the international research community has recently increased (Moeller, Fischer, Mag, Cress, & Nuerk, 2012). Many studies focus on the neurobiological causes of MLD and, therefore, on the way the brain functions, and how this differs in children having MLD. Only a small number of studies have aimed at understanding what intervention strategies work with children having MLD. Hence, a wide knowledge lacuna still remains. There is, for instance, the need to understand what kind of intervention works with children having MLD so that we can make a difference in their learning trajectory. On a positive note, however, the international research indicates that children struggling with mathematics can make great progress if given the right form of intervention (Kaufmann, Handl, & Thöny, 2003). My doctorate study thus aimed at addressing the need to add to the existing international body of knowledge within this field by exploring what works with children having MLD or both MLD and RD through six case studies.

In Malta, awareness about MLD is still limited, and most schools still do not have an intervention programme to tackle MLD. In this scenario, my doctoral research is, to the best of my knowledge, the first of its kind. It aims at identifying effective strategies that support learners in mastering numeracy skills which, are crucial for mathematics learning in general, and more

specifically with learners in Malta. Moreover, it aims at developing a better understanding of the degree and nature of the MLD presented by learners with only MLD and those with both MLD and RD (MLDRD).

One important feature of conducting case studies is the selection of the appropriate participants. Selecting the right individuals ensures that they can serve as rich cases through which a phenomenon may be explored in depth. In my doctorate study, a main hurdle in this selection process was that no numeracy test had been standardized locally. Had I not decided to start by establishing local norms, I would have had to use the standardized scores found with a different population to identify the subjects of the local case studies. This might have rendered the tests invalid and unreliable, since the scores would not have pertained to children who live in Malta and who follow a similar educational system as the participants of the intervention programme. Local norms had to be established to ensure that the right participants could be selected. This paper will explain the process through which these norms were established. It will include the methods used for data collection as well as provide a summary of the results obtained.

Defining Terms, Identifying and Assessing for MLD

Research about MLD has been given increased importance in recent years; nonetheless, studies still refer to MLD using different terms that do not always refer to the same difficulties in mathematics learning. Different studies have made use of different terms. These include *Dyscalculia* (Chinn, 2012), *Developmental Dyscalculia* (Rubinsten & Sury, 2011), *Mathematical Learning Difficulty* (Hopkins & Egeberg, 2009), *Mathematics Disability* (Geary, 1993) and *Arithmetic Learning Disability* (Koontz & Berch, 1996). These terms seem to refer to a common difficulty: a difficulty with conceptualising and applying the essential number concepts and skills that are required in order to understand and actively participate in mathematical tasks (Geary & Hoard, 2001).

In my doctorate study each term was used purposely to indicate a specific construct. The term *Mathematics Learning Difficulties* (MLD) was used to refer to all the individuals underachieving in mathematics no matter what the underlying cause may be. Dowker (2005) suggests that terms like *Difficulties with Arithmetic, Mathematics and Numeracy* have been used generically to denote all “children or adults who struggle or fail to cope with some of the

aspects of arithmetic that are necessary or desirable for educational or practical purposes” (p.11). I made use of the term *Mathematics Learning Difficulties* because poor achievement in mathematics served as the fundamental criterion for the identification of the participants in the intervention programme. Knowledge of arithmetic is made up of a wide spectrum of skills (Dowker, 2005) and difficulties in this area are complex. It is well-known that learners with MLD are a heterogeneous group of individuals who may exhibit different difficulties which may stem from a variety of biological, genetic, social, and environmental causes (Bartelet, Ansari, Vaessen, & Blomert, 2014). Since the participants of my doctorate study would form part of this heterogeneous group of learners, it was deemed necessary to use this term to refer to this construct. On the other hand, the term *Dyscalculia* was used to refer to a specific learning difficulty in mathematics, and, therefore, only one type of MLD. This is substantiated by recent studies that illustrate that individuals with dyscalculia would probably exhibit an inability to perceive and understand the numerosity (the property of magnitude) of a set of objects (Geary, Hamson, & Hoard, 2009) and a difficulty in undertaking approximate number tasks (Piazza, Pinel, Le Bihan, & Dehaene, 2007). Moreover, dyscalculia may possibly be a genetically inherited condition (Ansari & Karmiloff-Smith, 2002).

Dowker (2005) suggests that there is no such thing as *arithmetical ability* but only *arithmetical abilities*. A learner normally underachieves in numeracy due to a weakness in one or more of the fundamental numeracy components that are the foundations for mathematics learning (Chinn, 2004; Dowker, 2004). Various studies highlight the key characteristics of individuals who struggle with mathematics, all of which seem to be related to number work. These include:

- Poor number sense (Bird, 2009; Emerson & Babbie, 2010);
- A delay in understanding some of the concepts of counting (Geary et al., 2000);
- A delay in using counting techniques for addition (Jordan & Montani, 1997);
- An over reliance on finger counting strategies (Ostad & Sorenson, 2007);
- A difficulty with sequencing (Emerson & Babbie, 2010);
- A deficit in various components of working memory (Roselli, Matute, Pinto, & Ardila, 2006).

Although an individual may have a deficit in either one or more of the areas mentioned above, recent studies have suggested that the main difference between learners with MLD and those with dyscalculia is that the latter learners usually have a poor sense of number and a deficit in interpreting numerosities (Emerson & Babbie, 2010; Halberda, Mazocco, & Feigenson, 2008; Piazza et al., 2010).

The Diagnostic and Statistical Manual of Mental Disorders (DSM IV-TR) (American Psychiatric Association [APA], 2000) recommends three criteria for identifying whether a learner has Mathematics Learning Disabilities (MLD) or not. A learner may:

- i. Have lower performance in mathematics when compared to their attainment in other school subjects and general intelligence (IQ)¹;
- ii. Score two or more *standard deviations* (SD) below the norm established by any mathematics standardised test;
- iii. Not make expected improvement, even after appropriate classroom instruction.

The new Manual (APA, 2013) takes a different stance to the previous one (APA, 2000). It does not provide specific criteria for the identification of MLD but rather highlights criteria for identifying different Specific Learning Difficulties (SLD). As explained by Tannock (2012) in this new version of the DSM the definition provided is generic to SLD rather than MLD alone. This new definition is however more comprehensive as it does not focus mainly on IQ but sets new criteria for identification. These are:

- Having one of six symptoms² specified by the same manual which last at least 6 months;
- A discrepancy between actual age and achievement in the specific area;
- The learning difficulty becomes visible at the start of formal schooling;

¹ An Intelligent Quotient (IQ) is a score derived from one of many standardised tests made-up to assess intelligence. The IQ score defines one's intelligence in relation to the mean score of individuals on the same test.

² Four of these symptoms are related to literacy difficulties. The two symptoms identified in relation to number processing are: difficulty understanding number concepts, number facts and calculation and difficulty with mathematical reasoning.

- The learning difficulty is specific and not related to an intellectual disability.

I believe that having specific criteria for the identification of MLD is highly beneficial. This is because having specific criteria contributes to a stronger agreement as to which criteria are to be used to assess children having a profile of MLD. In my study, the term MLD is used to indicate learners who:

- Perform lower in mathematics when this attainment is compared to their attainment in other school subjects and general intelligence (IQ);
- Score two or more *standard deviations* (SD) below the norm established by any mathematics standardised test;
- Have difficulty understanding number concepts and number facts, struggle to perform calculations, and have problems with mathematical reasoning.

Identifying learners with MLD is not easy especially since multiple differences in definitions still exist resulting in a lack of instruments to assess for MLD. To date, I believe that the following assessment tools are currently valid tools for assessing for MLD: norm-referenced tests (also referred to as standardized tests), the use of direct observation, and the use of mathematical interviews. Using these modes of assessments ensures the proper detection of the characteristics of MLD, which I have highlighted. The use of formative assessments like the one proposed by Emerson and Babbie (2010), as well as screeners for Dyscalculia developed by Butterworth (2003), and by Trott and Beacham (2010), may also be of support in ensuring that a child is correctly assessed with a profile of MLD. In my study, The British Ability Scales (BASII) (Elliott et al., 1996) were used to test for IQ and be able to identify the first criterion. Two standardised numeracy tests (Gillham & Hesse, 2001; Chinn, 2012) were used to identify the second and third criteria. These were used in conjunction to interviews with parents and teachers as well as classroom observations to confirm specific difficulties being encountered in mathematics.

Standardized Tests

Standardized tests (STs), also referred to as Norm-referenced tests (NRTs), are the most popular means of assessing for MLD since they can show whether there is a gap between a learners' actual age and number age (age at which

the child is performing in numeracy), and, therefore, provide a clear indication of whether a learner truly has MLD. Results of STs can then be confirmed through other modes of assessment. Most STs focus on place-value, writing the numbers before and after a given number, the four operations (+, -, x, ÷), and continuing a sequence of numbers that follow a specific pattern (Butterworth, 2003; Chinn, 2012; Clausen-May et al., 2007; Emerson & Babbie, 2010; Gillham & Hesse, 2001). However, all STs will have different tasks, which are purposely graded to start from the easiest (the younger years) to more complex tasks (the older years). Every ST has a specific conversion grid that the assessor needs to use to convert the learner's raw score into a standardized score, a number age, a percentile, or a quotient. Thus, the main purpose of the ST is to assess the learner's mathematical achievement vis-à-vis their actual age and to identify a learner's number age (Shalev & Gross-Tsur, 2001). STs can be carried out on an individual basis or in groups.

Although STs have been used extensively for a variety of research projects, controversial issues still revolve around such tests (Higgins, 2009). A number of researchers (Gladwell, 2001; Phelps, 2005; Zwick, 2002) have debated the advantages and disadvantages of using standardized testing. STs are advantageous because they provide information about a learner's achievement that is more objective than that given through a teacher-created assessment. Numerous studies have indicated how teacher assessments may not be as accurate and valid as STs. Allal (2013), and, Wyatt-Smith and Klenowski (2013), indicate how teachers develop their own judgements about their pupils that then impinge on assessment. Moreover, Harlen (2004) suggests that teachers might have biases, such as those related to gender and special educational needs, which can impact assessment. The studies mentioned thus indicate that standardized testing may eliminate biases. As a result standardized tests are usually seen as more valid and reliable than teachers' observation (Marlow et al., 2014).

However, Miller (2003) has highlighted some disadvantages of standardized tests. These include that they create additional stress for teachers and learners, increase competition between students and schools, and are sometimes used to discriminate between groups of learners (Miller, 2003). Researchers (Gladwell, 2001; Phelps, 2005; Zwick, 2002) have also questioned the validity of test results resulting from these tests since they do not account for any factors that might impinge on test results such as the learner's mood

when doing the test. Notwithstanding, the arguments against standardized testing, I believe this method still remains an important and valid way of measuring a learner's achievement (Higgins, 2009), especially since they are objective. Moreover, the scores obtained from such tests allow the administrator of the test to compare the particular learner to others of his or her age. Keeping in mind that STs are not perfect, making use of a triangulation of assessment methods was a more accurate way of ensuring that the identification of the main participants was both valid and reliable. Since different STs test different spectra of mathematical content, the triangulation would allow me to confirm that the learner did have a profile of MLD and that their difficulties were not subject to the content being examined. The multiple assessments used as part of the triangulation process were: two standardized tests, summative assessments, as well as teachers' and parents' feedback about the child's achievement in mathematics. Results were also supported by other tests that would identify the characteristics of MLD. One of which was the BAS II (Elliott et al., 1996), which tested for IQ.

Cut-off Scores in STs

Every ST specifies a cut-off point, which indicates whether a learner has, in my case, MLD or not. Some also specify the degree of MLD as 'mild', 'moderate' or 'severe'. Cut-off points are test-dependent, so they have been cause for debate (Moeller et al., 2012). This has undoubtedly contributed to making it rather complex to develop a universal definition for MLD and dyscalculia. It has also augmented the difficulty of identifying the prevalence rate of MLD in the population. Different studies (Barahmand, 2008; Geary, 2010; Ramaa & Gowramaa, 2002) have used a varied range of cut-off scores for tests of similar difficulty, thus hindering researchers (Dirkset et al., 2008; Reigosa-Crespo et al., 2011) from agreeing on a universal prevalence rate. For example, a study conducted by Reigosa-Crespo et al. (2011) in Cuba made use of the 15th percentile as a cut-off point for their study. Their research indicated a prevalence rate of 3.4% for learners with MLD. On the other hand, Dirks et al. (2008), who carried out their study in the Netherlands, using a different standardized test, decided on a cut-off point of 25%, and this resulted in a prevalence rate of 10.3%.

In my doctorate study I made use of the wider construct of *MLD* as opposed to *dyscalculia*. This meant that I could include all those learners struggling with mathematics as long as they had at least an average IQ and the

characteristics highlighted earlier. The term MLD was taken to indicate all those pupils who fall below a cut-off point of approximately the 30th percentile. As a general rule, studies use this cut-off point to identify pupils who are underachieving in mathematics due to dissimilar potential causes without necessarily having a biological inherited weakness in mathematical cognition (Jordan, Kaplan, Olah, & Locuniak, 2006). This cut-off point allowed me to study a larger population of learners who are struggling with mathematics

Research Aims and Design

In my study, the use of a mixed methods approach was deemed to be very appropriate. Not having access to numeracy tests that have been standardized locally, I decided that finding local norms for the chosen standardized tests was the best way forward. This would allow me to select the participants for the intervention programme in a valid and reliable way. Following an evaluation of the processes involved in standardizing a test, I felt that it was sufficient to find norms for pupils having the same age as the eventual participants of the intervention programme (9 to 10 years old). Moreover, it was deemed suitable to find norms for pupils learning within the same educational setting as these eventual participants. Thus, I decided to standardize the test using a sample of pupils from all Church schools catering for boys.

Different STs were analysed and evaluated to find the most appropriate for the local context. Once three appropriate tests were selected these were pre-piloted with ten pupils attending the school where I taught, a Church school for boys. Following the pre-pilot study the choice of two tests out of the three piloted was concluded and a pilot study was conducted with an additional 15 pupils to ensure the suitability, reliability and validity of the tests to identify MLD. These three elements were checked for by comparing scores from the different tests as well as by carrying out observations during test administration. Teachers were also asked for their perception of the learners' abilities to compare these to the scores obtained on the tests. After analyzing the data obtained, two out of the three tests were considered suitable, reliable and valid. Following the pilot study, the standardization exercise commenced. The first step of this process was to administer the tests to a representative sample population. It was thus important to determine the sample size so as to understand how much time and resources would be

needed for the actual collection of data. As suggested by Gogtay (2010), “Sample size calculations must take into account all available data, funding, support facilities, and ethics of subjecting patients to research” (p. 517). It primarily needs to consider what type of data the research is dealing with, whether quantitative or categorical. In this case, the data was of a quantitative type. An online sample size calculator (Creative Research Systems, 2012) indicated that if scores were collected for half the population of boys in Grade 5 attending Church schools, i.e. 352 out of the 704 boys, the statistical power of the results would be sufficient in making the result valid and reliable. This sample population was calculated with a confidence level of 95% and a confidence interval of 1.2. Administering the tests to half the population took into consideration a safety margin and dropout rate.

When the sample size was determined, access was sought from the relevant entities including the Maltese Episcopal Curia and the Heads of each individual school in which the tests were to be administered. Some schools had acquired consent from parents prior to the scholastic year to carry out assessments as deemed fit but others had not. In the latter case, consent was also acquired from parents. All children were also asked for consent before the test was administered and were free to withdraw if they wanted to. A number of precautions to maintain the tests’ validity and reliability were taken whilst administering the STs. These included:

- i. Administering all the tests myself;
- ii. Introducing myself and informing the learners what the test was going to be used for;
- iii. Reassuring the learners that they needed to try their best ensuring that they did not feel stressed by the test itself.
- iv. Administering the tests to the class as a whole.

Selecting an Appropriate Standardized Test

After evaluating different tests, I decided that two tests were most appropriate: the Basic Number Screening Test (BNST) (Gillham & Hesse, 2001) and Chinn’s Number Tests (CNT) (2012). A third test, Progress in Mathematics (PIM) (Clausen-May et al., 2009), was also identified as being appropriate because it was specifically designed to use with learners in Grade 5 and had also been more recently published than the BNST. The decision of evaluating all three tests before defining the two to be used was based on two

main factors. Primarily, all three assessments were in line with our curriculum. The exercises set are tasks that focus on the same algebra and number work determined by our curriculum. Secondly, the assessments focused mostly on number operations and calculation rather than measures, data handling, shape and space – this was important since the vast majority of children with MLD display difficulties with the former areas of mathematics rather than the latter (Dowker, 2005; Emerson & Babbie, 2010).

CNT is a standardized test that can be used with both children and adults and has been standardized with different age groups in the UK. CNT does not include any written instructions but the learners merely need to work out the computations given. This feature is deemed important in the actual assessment of MLD since valid assessment should exclude other difficulties that may diagnose a learner with MLD incorrectly. This assessment has multiple parts but its main one is an assessment of the four operations (+, -, \times and \div) involving whole numbers. The pupils are first asked to work out as many addition sums as possible in one minute (maximum of 36). This exercise is repeated with subtraction sums and the pupils are given another minute to work out as many as they can out of 36 subtraction sums given. A sheet of 33 multiplication sums and another of 33 division sums follow these two exercises. The pupils are given two minutes to complete each of the multiplication and division sheets. This assessment is then followed by a 15-minute assessment which involves different types of computations including not only the four main operations with whole numbers but also simple fractions, percentages, conversions of time and length, and working with decimals. The computations vary in difficulty and are graded starting with easier tasks which become more complex in nature. All assessments were carried out with each class as a whole, reducing the variables linked to administering them on a one-to-one basis. It also made the exercise feasible, as it would have otherwise been too time consuming.

CNT has other parts of it that assess for mathematics anxiety and other basic mathematics skills. It also allows the identification of learning styles related to mathematics learning. However, these components are to be administered on a one-to-one basis and could not be standardized. As a result, due to the large numbers involved, these were not used in the process of collecting norms. These parts were however administered later to the six participants chosen for the intervention programme to gain a deeper insight into the learners' characteristics and how they learn mathematics.

The BNST was chosen because it has no written instructions; assesses a wide range of numeracy components, including the addition, subtraction, multiplication and division of whole numbers as well as fractions; and, only takes roughly 25 minutes to complete. The test was developed by Gillham and Hesse (2001) through Hodder Education and has also been standardized in the UK. The test is suitable for learners aged 7 to 12. When the test is administered the assessor has to read out the instructions for one task and the children have to complete it. Each instruction is read out twice. The instructions to the tasks are in English. Due to this, when this test was administered during the pre-pilot study, I decided to translate all the instructions to Maltese as I did not want the children's understanding of the English language to influence the score obtained. Hence, when administering the tests during the pilot study and during the actual study, I read out the instructions in both English and Maltese prior to every task. Translating the instructions involved multiple steps. These were:

- i. Translating the instructions myself;
- ii. Having the translation checked by a mathematics educator and a linguist;
- iii. Making any changes required according to the suggestions given by the reviewers of the instructions;
- iv. Having a professional translator do a back translation of the content to ensure that words and phrases were correctly interpreted. Through this process, changes to be made at word and sentence level were indicated. These changes ensured that the same meaning was attributed to the instructions in Maltese as in the English version.

PIM has different tasks for learners at different levels. For the purpose of this study I used the PIM 5, a test suitable for Grade 5 learners. All questions set are in English and it contains written questions that the learner needs to solve. The assessor may read the instructions in order to eliminate the reading variable. It takes roughly 45 minutes to complete the test. The content of the test is similar to that found in the BNST and CNT, thus focusing mostly on number work too.

As the triangulation of results would render my findings more robust, I decided that it was best to use two of the tests rather than only one to be able to compare results and ensure that the learners identified as having MLD truly did. Although the tests were not exactly the same, they tested similar

numeracy components. A pre-pilot study was carried out to determine which of the three tests identified as appropriate was most suitable for detecting MLD in local school children.

The Pre-Pilot Study

In the pre-pilot phase, ten pupils were chosen who were then in Grade 5 at the school where I taught. The Head of School granted access and consent to administer the test to the pupils was gained from both parents and pupils. Class teachers were asked to identify learners with a range of abilities for this exercise: two were low achievers; six were average achievers, and another two were high achievers in mathematics. This was crucial since I was interested in comparing how different pupils would perform in the three assessments. All ten pupils sat for the BNST. Out of the same ten pupils, five sat for CNT and the other five sat for PIM. Both subgroups were composed of one low achiever, three average achievers and one high achiever. One of the reasons why I did not give all three tests to all ten pupils was that the learners would miss too much learning time from class to complete all tests. Another was that they would have probably become too bored doing all three and the results would not have remained valid.

The data from the pre-pilot study was analysed by looking at the scores obtained by each individual pupil in the two tests that were administered to him. Using this methodology allowed me to compare the scores obtained in the two tests in order to highlight commonalities and discrepancies in performance. It was assumed that this would help me to determine the validity of a test in identifying MLD; if the scores obtained in both tests were comparable, the detection of MLD would be more accurate. In Table 1 I have illustrated the percentile scores obtained on the BNST and CNT assessment as well as the teacher's perception of the each learner's mathematical abilities. The CNT only offers percentile scores when the learner scores below the 30th percentile, if the pupil scores higher than the 30th percentile, a comment such as 'average' or 'above average' is provided to describe the pupil's achievement.

Pupil Code	BNST Percentile Score	CNT - 15-mins. Percentile Score	CNT - Addition, Subtraction, Multiplication, and Division Percentile Score	Teacher's Perception of Child's Maths Ability
PP1	5th	Below 5th	Addition: 10th Subtraction: 10th Multiplication: 5th Division: 10th	Below Average
PP2	40th	Between 10th and 5th	Addition: Average Subtraction: Average Multiplication: 10th Division: Above Average	Average
PP3	80th	25th	Addition: Average Subtraction: Average Multiplication: 10th Divi. Score: Average	Average
PP4	90th+	80th - 75th	Addition: 10th to 5th Subtraction: 25th - 10th Multiplication: 5th Division: Above Average	Above Average
PP5	90th+	50th - 40th	Addition: Above Average Subtraction: Above Average Multiplication: Average-25th Division: Above Average	Average

Table 1: A comparison of scores obtained in the BNST versus those obtained in CNT's 15-minutes assessment and that for the four operations as well as the teacher's perception of learner's mathematical abilities

The scores for the BNST and CNT were compared by looking at each of the percentile scores achieved by each individual learner on both tests. During this comparison I took note of whether the scores complemented each other. When the scores were similar it meant that both tests had placed the child within the same category of learners (i.e., 'average', 'below average', etc.). Discrepancies in scores, on the other hand, meant that the different tests were placing the same learner in two different categories. The scores obtained in both tests were also compared to the class teacher's perception of the child's mathematical abilities. A similar exercise was also carried out with the other five learners sitting for the BNST and PIM. This comparison is presented in Table 2.

Pupil Code	BNST Percentile Score	PIM Standardised Score	Comment Retrieved from PIM Manual	Teacher's Perception of Child's Maths Abilities
PP6	90th+	113	Above Average	Above Average
PP7	90th+	101	Average	Average
PP8	60th	106	Average	Average
PP9	70th	87	Below Average	Lower Average
PP10	20th	69	Very Low	Below Average

Table 2: A comparison between scores obtained in the BNST, PIM and the teacher's perception of learners' mathematical abilities

After the analysis of all the data collected from the pre-pilot study, some discrepancies in the results obtained in the different tests were evident (see, for instance, PP2, PP3, PP5 and PP9). One possible reason for the discrepancy between the scores obtained in the BNST and CNT may have been that CNT covers topics that are higher in level of difficulty since some of the computations require more complex calculation skills. Another reason may be that CNT is timed, and, therefore, the pupils' speed of working out through the tasks would have impinged on the score obtained. On the other hand, when comparing results from the BNST and PIM, discrepancies in the scores were possibly due to the learner not being able to understand the instructions since the latter involved written questions indicating what the learner had to do to complete the task. Although some discrepancies in the scores obtained by the learners in all three test were noted, some similarities in the outcomes were also evident. For example, the scores obtained agreed that PP1 and PP10 had severe MLD and that PP2 had mild-to-moderate MLD. The conclusions from the tests also supported the teachers' perception of the pupils' abilities in mathematics.

It was decided that all three tests could be considered as reliable since most of the results between the tests were coherent. However, the BNST and CNT were the assessment tools deemed most appropriate. The fact that PIM included a lot of written instructions may have resulted in an invalid picture

of the learners' abilities since a pupil may have been assessed with a profile of MLD due to the nature of his/her reading difficulties. Moreover, it took the children a long time to complete PIM: approximately between 45 minutes and an hour, which contrasted with the 20 to 25 minutes employed to complete each of the BNST and CNT. Due to this, I could observe that some pupils got bored and began to guess answers to solve the questions in the PIM. This was not observed for the other two tests, indicating that using PIM, in comparison to the BNST and CNT, might have increased the risk of obtaining invalid findings.

The Pilot Study

During the pilot study CNT and the BNST were administered in this respective order. Only the BNST was translated to Maltese because CNT has no instructions, and, therefore, no translations were needed. During this piloting phase, it was deemed necessary to administer both tests. This was done to be able to determine whether the back translation for the BNST was fine and whether there was anything else that could be done differently during the actual data collection process.

Pupil Code	BNST Score	CNT - 15-min. Assessment Scores	CNT - Addition (A), Subtraction (S), Multiplication (M) and Division (D) Scores	Teacher's Perception of Child's Maths Abilities
P1	90 th percentile	80 th percentile	A: Above Average S: Above Average M: Above Average D: Above Average	Above Average
P2	40 th percentile	20 th percentile	A: 45 th percentile S: 25 th percentile M: 35 th percentile D: Average	Low
P3	50 th percentile	12 th percentile	A: Above Average S: Above Average M: Above Average D: Above Average	Low
P4	60 th percentile	60 th percentile	A: Average S: Average M: 40 th percentile D: Above Average	Average
P5	80 th percentile	77.5 th	A: Above Average	Average

P6	90 th percentile	75 th percentile	A: Above Average S: Above Average M: Above Average D: 25 th percentile	Average
P7	80 th percentile	75 th percentile	A: Above Average S: Above Average M: Above Average D: 40 th percentile	Average
P8	60 th percentile	20 th percentile	A: 25 th percentile S: 35 th percentile M: 40 th percentile D: Average	Average
P9	40 th percentile	20 th percentile	A: 20 th percentile S: Average M: Above Average D: Above Average	Low
P10	90 th percentile	90 th percentile	A: Above Average S: Above Average M: Above Average D: Above Average	Above Average
P11	90 th percentile	80 th percentile	A: Above Average S: Above Average M: Above Average D: Above Average	Above Average
P12	80 th percentile	75 th percentile	A: Above Average S: Above Average M: Above Average D: Above Average	Above Average
P13	40 th percentile	35 th percentile	A: 25 th percentile S: 30 th percentile M: 45 th percentile D: 45 th percentile	Low
P14	60 th percentile	50 th percentile	A: 35 th percentile S: Average M: 45 th percentile D: 35 th percentile	Average
P15	70 th percentile	55 th percentile	A: 35 th percentile S: 45 th percentile M: 20 th percentile D: Above Average	Average

Table 3: Scores obtained in the pilot study by the 15 participants

Fifteen Grade 5 children (aged 9 to 10) were chosen randomly from the school where I taught. Random selection was used to ensure that the learners had different mathematical abilities. The Grade level teachers were asked to nominate four children who were low-achievers, seven average-achievers, and four high-achievers. The results obtained by these learners can be seen in Table 3.

The results obtained were generally comparable, since most of the pupils obtained similar results in both tests. This indicated that the two tests could be used in tandem so as to collect more valid and reliable data. Pupils like P1, P10, P11 and P12 clearly scored an above average score in both tests indicating that they did not have MLD. On the other hand, pupils like P2 and P13 seemed to be struggling with mathematics since all their tests scores indicate this. When pupils' scores were not coherent, such as in the case of P3, possible reasons for this were looked into. It seemed that P3 was able to successfully complete the simple addition, subtraction, multiplication and division sums but then found difficulty in both the other tests, that is, the BNST and the 15-minute test component of CNT. The latter contains more complex mathematical tasks including work with fractions and percentages. I thus checked with the class teacher to identify whether this pupil was struggling with mathematics and the teacher said he was. Anyway, since the pupil's difficulty in mathematics was detected by two of the tests, it was concluded that the test results were valid and reliable. Some other discrepancies with the scores obtained by the children were also noted. The scores obtained in the two numeracy tests by P8, P9 and P14 were slightly different since all three performed better in the BNST as opposed to CNT. Nonetheless, the discrepancy was minor and still placed the learner within the same category of achievement (i.e. whether low achieving, average or high achieving).

Following this piloting phase, I decided to change the order in which the tests were administered. Whilst carrying out the pilot study I realized that the children enjoyed doing CNT more than the BNST. This was possibly due to the fact that CNT test is timed. The children seemed to enjoy this characteristic of CNT as they took it up as a challenge to complete as many sums as possible in the time given. Thus, when administering the tests with the larger sample, the BNST was administered first. This was done with the hope of maintaining the children's motivation throughout both tests so that they would not guess answers due to boredom, and, thus, invalidate results.

The Data Collection Stage

Once access was granted by the relevant entities, appointments were set to administer the tests in the seven schools. Various validity and reliability recommendations were taken into account to maintain a high level of these in the data collection process (Cohen et al., 2007; Lincoln & Guba, 1985):

- i. All the tests were administered by myself to all learners ensuring that consistency was maintained in reading speed or fluency. In this way, additional variables that could have been problematic had the tests been administered by multiple persons were avoided;
- ii. As for the pre-pilot and the pilot study, in the actual collection of data, the children were asked to cover their work or to separate their desks so that copying was evaded;
- iii. As much as possible I sought to administer the tests to the pupils during the same period of time. In fact, all data was collected between November and December 2012, so that the pupils would have covered approximately the same topics in class, since as confirmed by the schools, all had started using the same textbook and had covered roughly the same topics. For the same reason, this period was considered suitable a year later, when assessing participants for the case studies. This same period was also then maintained the following year when assessing the participants for the case studies;
- iv. When possible, tests were also administered at the same period of time during the day, i.e. the morning. This choice was based on the fact that children are usually less restless at this time of day;
- v. All tests were administered in the children's own classroom with the presence of the teacher to ensure that the children felt secure and safe in a familiar environment.

Analysis of Data

All the tests were scored by myself. The raw scores were entered on SPSS and a z-score (standardized score) was computed for each raw score. These z-scores were then saved as variables and were later used to find the norms. The quotient was then calculated through MS Excel by using the formula 'z-score * 15 + 100'. Finally, percentiles were also smoothed. The cut-off points used were as for 99 equal groups. Since the data collected was of ordinal type, it was not assumed that the difference between one score and another

was equivalent to the interval between any other two percentiles. Therefore, for example, the difference in score between the 98th and 99th percentile could have been much larger than that between the 50th and 55th score. The tests used to analyse the data were non-parametric tests, which correlate with the type of data collected since the data had independent variables. Through this data analysis process, the median, quartiles and percentiles in multiples of 5 were worked out (5th, 10th, 15th, 20th, 25th etc.). The crucial percentile and related score needed for the identification of the main participants of the main study was the 30th percentile scores since the cut-off point I used for this research was the 30th percentile. The raw scores obtained as norms for these three percentile scores are illustrated in Table 4.

Assessment	30th Percentile Score
CNT - Addition	18 and below
CNT - Subtraction	16 and below
CNT - Multiplication	20 and below
CNT - Division	16 and below
CNT - 15-minute assessment	15 and below
Basic Number Screening Test	22 and below

Table 4: Scores extrapolated for the 30th percentile following the collection and statistical analysis of the data collected in this study

The local norms found for the specific population of Grade 5 boys attending Church schools for boys were compared to the norms achieved in the U.K. for all tests. The latter norms are ones that have been established through a sample population of the whole population of Grade 5 children in the U.K.. It was considered interesting to explore how the cohort for whom the local norms were found, actually compared to the general cohort of Grade 5 pupils in the U.K.. In Table 5, I present the local and U.K. norms for the 30th percentile.

When comparing the local established norms to the ones found in the U.K. for Grade 5 pupils, the following observations were made. Primarily, the U.K. and local scores for Chinn's (2012) assessment were very similar. In fact, the U.K. and local norms for the addition and subtraction components were identical. Moreover, the local norms for the multiplication and division components, as well as those for the 15-minute assessment, were only slightly higher than those found in the U.K.. On the other hand, an important finding was that the local norms for the BNST are higher than the U.K. norms. This

was in line with the findings from the pre-pilot and the pilot study in which some pupils did well in the BNST, but not so well in CNT. This indicates that the local population for whom the norms were found - boys attending Church schools - performed generally better in the mathematics components assessed in this test than the population with whom this test was standardized with in the U.K.. Despite this result, one must highlight that had the test been administered to a wider population in Malta, including girls and other sectors of the local education system, the findings may have been different and the difference not as accentuated.

Assessment	30th Percentile Local Score	30th Percentile UK Score
CNT - Addition	18 and below	18 and below
CNT - Subtraction	16 and below	16 and below
CNT - Multiplication	20 and below	19 and below
CNT - Division	16 and below	12 and below
CNT - 15-minute assessment	15 and below	13 and below
Basic Number Screening Test	22 and below	14 and below

Table 5: 30th percentile scores obtained by Maltese Grade 5 boys attending Church schools compared with 30th percentile scores obtained by the whole population of U.K. Grade 5 pupils

Another interesting observation was that the discrepancy between the norms achieved for the BNST and CNT test shows that, in general, the content covered in CNT, although testing similar numeracy components, was found to be more difficult than that presented in the BNST. This finding reflects the U.K. norms for both tests too, since this same discrepancy is also evident when these are compared.

Conclusions, Limitations and Recommendations for Further Research

Through this phase of my doctorate study I found norms for numeracy assessments for one group of learners - boys in Grade 5 (ages 10 to 11) Church schools. In this paper, the local norms collected were discussed and were compared to those collected in the U.K. The local norms established during this phase of my doctorate study were crucial to my intervention programme, as they allowed me to identify in a valid manner the six main participants for the qualitative part of the study. This qualitative part

consisted of six case studies. The scores obtained by the cohort at the school where I taught were compared to the established local norms. Pupils who achieved scores that were equal to or below the 30th percentile were then assessed using further tests, for example the BAS which assesses for IQ, to ensure that they had the characteristics identified in learners with MLD. These participants were also confirmed by asking for the teacher's feedback about the children's achievement in mathematics and by looking at their previous examination paper marks (those taken at the end of Grade 4). Indeed, four out of the six pupils had failed their mathematics examination. The other two had just managed to get a pass mark. Thus, having been identified with a number of criteria for MLD, these learners were asked to participate in the intervention programme.

The norm collection process was carried out for only one specific group of learners (i.e., Grade 5 boys attending Church schools for boys). Due to this, norms for other groups of learners, such as those in other levels, in other educational settings and girls, were not found. This is a limitation of this part of my study and thus, there is a need for the process of establishing norms to be replicated for different groups of learners. Educators urgently need to acquire assessment tools that accurately identify learners struggling with mathematics. This need arises from an increased awareness about MLD and the impact they might have on an individual's life. Difficulties with mathematics can persist throughout adult hood reducing life opportunities such as employment (Bynner & Parsons, 2005). Hence, these tools are essential for the early identification of mathematics learning difficulties.

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COMMENTARY

**The Past is a Foreign Country: Reflections of a Head of
Department¹**

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To Be or Not to Be, That is the Question

In October 1996 I was appointed Head of Department (Mathematics). Before that time I taught for twelve years in a primary school, then mathematics for five years at secondary level and seven years at the post-secondary level. I must say that the time spent teaching at primary level are amongst the most I cherish. At no time in my career have I felt so much in control of teaching and learning. I had my own class, I was in contact with the pupils almost all the time, I could organise activities without the constriction of time frames imposed by teaching periods that characterise post-primary education and I could organise displays of children's work on the walls of the classroom. On reflection I think my approach to teaching mathematics was oriented too much towards drill-and-practice and a transmission mode pedagogy. With hindsight I would have liked to include more constructivist pedagogies. But as a Maltese proverb states: *Wara kulhadd gharef (Everyone is wise with hindsight)*.

When I moved on to teach in a secondary school the advantages of teaching in a primary school were absent. Besides, because students were admitted after passing a competitive examination at the end of primary education, I expected that they would not only have higher competencies in mathematics but also more positive attitudes towards the subject. I was to some extent

¹ The first part of the title is borrowed from L.P. Hartley's (1986) famous novel. The exact quote is: "The past is a foreign country: they do things differently there."

disappointed in this respect and, although I was teaching mathematics at a higher level, I encountered students who struggled with mathematics and, even worse, had negative attitudes towards the subject.

Although I had always wanted to teach mathematics, as this was my subject of specialisation during the teacher training course, I really became interested in mathematics after completing a B.Sc. in mathematics and computing in 1991. I clearly remember one of our lecturers suggesting that once we complete the degree we ought to do some reading into the multi-faceted nature of mathematics. And that is what I did. After reading such gems as Davis and Hersh's *The Mathematical Experience* and Bell's *Men of Mathematics*, I was hooked on the fascinating subject that is mathematics.

The decision to apply for the post of Head of Department (HoD) was far from a straightforward one. I was happy teaching 'A' level mathematics at a local Sixth Form college. On one hand, I wished to do something different with my career that would give me the opportunity to share my love of the subject with others. On the other hand, I did not know what to expect. Which school would I be transferred to? Would I be accepted by the teachers there? In the end, I decided to apply and, as they say, the rest is history.

Unfortunately, being intrigued by a subject is one thing, sharing its beauty with others, especially if these others are students who happen to have had only negative experiences with mathematics, is another. As a mathematics teacher I have frequently, though, admittedly, not always successfully, juggled pressures to 'cover the syllabus' by activities from the compendium of mathematical activities found in Bolt's series of books (1982, 1991, 1996) or incidents from the rich history of mathematics. Now that I had been appointed HoD could I perhaps share my passion for mathematics with students and teachers alike, thus improving the negative perceptions students might have of the subject?

This short commentary consists of a number of personal reflections on my role as HoD in a secondary school. In retrospect, I have to acknowledge that, in view of my aspirations, the experience was a very positive one even though I have not always been successful in what I set out to achieve and, admittedly, some things I could have done differently.

Rules are Meant to be Broken

The official duties and responsibilities of a Head of Department are spelt out in Ministry of Education, Youth and Employment (2007). Although these stipulations look admirable on paper, in my opinion, they may be limited in raising standards in mathematics teaching and learning, especially if they are construed to regard the HoD as a mere appendage to higher grades within a bureaucratic machine. In a system in which change is managed in a top-down fashion, often with little consultation with the HoD, the latter may find him or herself compelled to implement policies with which he or she may or may not agree (Turner, 2005). For example, about two years before I retired from public service, HoDs were asked by a high-ranking official to forgo part of their duties to dedicate time to produce a series of so-called 'Reusable Learning Objects' (RLOs). These consisted of a series of IT resources that could be used by the teacher to teach mathematics. Although I did not object to being involved in the project, I felt that forgoing the duties of HoD would have a negative bearing on the teaching and learning of mathematics.

To avoid being constrained by procedures dictated from above I adopted two strategies. First of all, I always strove to be involved in all issues that concerned the teaching and learning of mathematics within my school, whether these were spelled out in the job description or not. As Pope (2010) points out, subject leaders for mathematics are responsible for the quality of the mathematics education of every person in the school, they are accountable to the school's senior leadership, they are ambassadors for their team, as well as advocates for mathematics. For example, assigning classes to different members of department was always a delicate aspect of the job that entailed constant consultation, often during summer holidays, with the teachers themselves and the head of school.

Second, during my eighteen years as HoD I have, with other colleagues from the national Directorate for Quality and Standards in Education (DQSE) and The Malta Mathematics Society, promoted a number of initiatives on a national level that seek to enhance the enjoyment of the subject and contribute to raise standards in the teaching and learning of mathematics. In spite of the fact that these endeavours have involved work that went over and above the duties stated in the job description of HoD, they have given me great satisfaction.

The Malta Mathematics Olympiad

The Malta Mathematics Olympiad dates back to the year 2000. This year was designated by the International Mathematical Union as the World Mathematical Year. The idea was that of the Malta Mathematics Society, of whom I was a member, and it aims to foster problem-solving skills, team work, and positive attitudes towards mathematics in young students in an atmosphere of healthy competition. The Malta Mathematics Olympiad is held every two years and is contested by students attending secondary schools. The event is now organised by the Mathematics Section within the Directorate for Learning and Assessment Programmes. The eighth edition of the Malta Mathematics Olympiad was held in 2017 and was contested by a record number of pupils attending schools from state, church and independent sectors. My role over the years has been to prepare questions, seek sponsors, and organise the event.

The Mathematics Venture

This activity was first organized in 2013 by the Mathematics Department of the Education Directorate in conjunction with the Department of Mathematics, Science & Technical Education, Faculty of Education, University of Malta. Its aim is to give students the opportunity to experience mathematics beyond the confines of the classroom. The venture consisted of

- a mathematics trail around Malta's ancient city, Mdina;
- a number of challenging problem solving activities; and
- a set of fun mathematical games involving collaborative work

Groups of four students visited a number of stations where they had to solve a number of problems relating to mathematics. The choice of Mdina gave participants a chance to appreciate this national gem with activities involving both mathematics and history. Besides, being a walled city with practically no traffic going through, it provided a safe environment for students to roam around.

The Mathematics Project Competition

This activity was organised in 2009 but has not been repeated since. Again the participants were students attending Forms 3 and 4 in state, church and

independent sector schools. They were required to work in pairs to produce a number of charts, a slide presentation, or models which focus on a mathematical topic. Each team could choose from a number of themes which included The Story of Number, Symmetry, Magic Squares, The Golden Ratio, Pi, Fractals, Circles, Conic Sections, Fibonacci Numbers, Prime Numbers, and so on. A short list of projects was chosen and each team had to make a presentation in front of two judges, who could probe the students' understanding of the topic chosen.

Gifted & Talented Activities

The origin of the Gifted & Talented activities was a talk to secondary school students delivered by Ian Stewart, mathematician and author of several popular books on mathematics, in November of 2006. This was followed by an activity animated by Mario Micallef, a Maltese associate professor of mathematics at Warwick University. Eventually, I was invited to take part in the project. If I remember correctly the first activity that I animated was entitled *Prime Numbers: The Atoms of Arithmetic*. These were followed by others featuring Platonic Solids, Fibonacci Numbers, Pythagoras' Theorem, Secret Codes, and Games. I must say that these activities have given me much satisfaction, and it was a joy when students approached me at the end of one of the sessions and told me how much they enjoyed it, or that they had read one of the books I had recommended during the activity.

MATHSLINE

In 1999 Peter Vassallo, then Education Officer for mathematics, and a colleague from whom I learnt considerably, proposed starting a publication dedicated to issues relating to mathematics education, especially with regard to the use of IT in mathematics education. The idea appealed to me as it provided an opportunity to put into print matters related to mathematics education. Indeed, I wrote about varied topics, but my favourite was my regular contribution on such topics as magic squares, mathematical humour, mathematics books for children, and so on. *MATHSLINE*, as the publication is called, has now been going for almost eighteen years. It includes contributions by individuals involved in mathematics education, including lecturers from the University of Malta, practising teachers, and Heads of Department. Of particular satisfaction was the twentieth issue (April 2009),

published to commemorate the 10th year of *MATHSLINE*. This issue included contributions by lecturers from the University of Malta and other individuals involved in mathematics education.

He that would Eat the Kernel must Crack the Nut

When I was appointed to the role of HoD I did not have all the competencies that I think a HoD ought to have. As a leader there are qualities that, in retrospect, I think it is important to possess. These include the ability to lead and manage people, to solve problems and make decisions, to understand the views of others, plan one's time effectively, and organise oneself well (Teacher Training Agency, 1998).

Love Thy Subject

Some attributes I did possess. For example, I knew the subject sufficiently well, and, perhaps more importantly, I was, and still am, passionate about it. Further-more, I have always sought to transmit this love of the subject to students, parents, and teachers through the way I talk, write and teach the subject. One of the most gratifying compliments I was ever paid came from one of the patrons at the pub where I often pop in for a drink ... or two. I had ordered my usual pint and bought a drink to a fellow punter with whom I had only exchanged a few words before. After a few moments of silence, he asked me whether I was involved in tennis. Clearly, I must have reminded him of someone, and I remarked that my face was common enough. "I wouldn't say common," he said. "you being a professor of mathematics." I remarked that I wasn't a professor and asked what had given him that impression. "By the way you talk about the subject. It is clear that you love the subject."

Vision

However, knowing the subject, and being enthusiastic about it, is necessary, but not sufficient. The HoD – and here I think I was initially wanting – also needs to have a vision of how the subject should be taught. What is equally important is that the HoD is able to share this vision with the School

Management Team (SMT)², and other members of his/her mathematics department. Regular subject meetings during which teachers can share good practices with their colleagues are important. One such instance occurred when my school participated in the PRIMAS (Promoting Inquiry-based Learning in Mathematics and Science) project. Participating teachers met regularly and, after some initial concerns, shared practices that promoted inquiry. I specifically remember the point in the project when I invited teachers to have one of their lessons filmed. Initially the teachers were apprehensive, but when I offered myself to be filmed and invited teachers into my class, a number of teachers not only invited me to attend lessons, but also agreed to be filmed delivering lessons.

Communication

Being a good communicator is another quality that a HoD ought to possess. As I am rather shy by nature these communicative skills did not come naturally. However, gradually I managed to overcome my timidity, and could effectively connect with teachers, parents, and members of the SMT. With experience, I learnt to address teachers as well as parents, both as a group, and individually. At times, I must admit that I was rather brusque and should have been more cautious. Telling a head of school that his ideas are nonsense is, admittedly, not a suitable manner to address anyone, let alone a member of the SMT! Admittedly, the job of a head of school is not an easy one, especially if the school happens to have a staff complement of some one hundred teachers and learning support assistants and some eight hundred students.

No Man is an Island

A delicate issue here is when members of the department, especially those who have been teaching the subject for many years, may not share the same ideas about the teaching and learning of the subject. In a system where I had no say in the choice of members of my department, I had to accept the fact that the department can function with teachers having different ideas on how the subject is taught. Indeed, the challenge for the HoD is that of “attempting

² The School Management Team includes the Head of School, Assistant Heads and Heads of Departments of other subjects.

to realise the strength bound up in diversity, whilst minimising its weaknesses.” (The Mathematical Association, 1988, p.43). For example, a pertinent issue which never fails to give rise to an animated discussion is related to the use of the calculator. Some members of staff, especially those who did not make use of this device during their compulsory schooling, insist that the use of calculators should be limited because students tend to rely excessively on the calculator, even to multiply two single-digit integers. Others, for whom technology has been part of their lives since childhood, tend to be of the opinion that technology is an important part of our lives and students should be allowed to use calculators whenever they need to.

As HoD I have always tried to establish a healthy relationship not only with the teachers within my department, but also with teachers of other subjects, other members of the SMT, personnel from the central administration (Education Officers, Assistant Directors), parents, and students. Perhaps, as a newcomer, the most difficult is to be accepted by the mathematics teachers within the school. The new HoD has to prove him/herself by demonstrating that his/her presence will make a positive difference to the teaching and learning of mathematics within the school. Indeed, I can say that many teachers, HoDs, and EOs have been a privilege to work with. However, the human relations side of the HoD goes beyond simply proving oneself to one’s colleagues. I have always felt that it is very important to remember that being a subject leader is not just about the subject but also about people. Every individual has his/her personality and a life beyond the position he/she holds that is fraught with joys and tribulations. Being aware of these makes the HoD position more difficult on one hand but more gratifying on the other.

Professional Integrity

Another quality that I have always viewed as important is to establish professional standards with respect to curriculum planning, teaching, and assessment. This I have sought to achieve by establishing high standards through example, and setting professional standards in various ways, including the writing and updating of schemes of work, assisting the Head of School in assigning teaching duties to the various members within the department, acquiring learning resources, and keeping records of student achievement. Meeting parents to discuss issues relevant to their children’s mathematics education have, in general, been very fruitful, though these had to be tackled with the utmost tact as parents tend to raise issues that might

involve individual members of the mathematics department. I remember an occasion when a parent spent a good quarter of an hour complaining that her daughter's teacher could not maintain discipline, nor explain concepts, only to realise at the end of the diatribe that the teacher was not a mathematics teacher at all!

Related to the above I have always sought, as teacher and HoD, to improve myself through reading and attending relevant courses that have enabled me to improve the teaching and learning within the department. Over the years I have also been involved in numerous professional development sessions involving teachers of mathematics. As teachers employed in state schools have to attend such courses, their perceptions of these activities are varied, although, in general, I think that they have been positive. One funny incident which I still recall was with a group of some sixteen teachers, all of whom were females except for one middle-aged gentleman. My favourite approach in these sessions was to stimulate discussion and reflection about some particular issue or other. In this particular instance, I had no problem with the ladies in the group. They became so engrossed in the subject that I could hardly venture a comment myself. The gentleman, on the contrary, refrained from taking part in the discussion, even when I asked whether he had an opinion. Finally, some half an hour from the end of the session, he raised his hand, and I, relieved, asked whether he had anything to say. "May I leave because I have to pick up my daughter from her school?"

Beyond the Department

The position of HoD entails a considerable interaction with members of the SMT and Education Officers. As pointed out by Sammons, Thomas, and Mortimore (1997), as cited in Turner (2005), the whole-school context, in conjunction with the quality of leadership afforded by the SMT may be very important in enabling the department to function effectively. I must say that throughout my eighteen years as HoD I have found that the attitude of some members of the SMT towards mathematics education left much to be desired. For example, on several occasions I clashed with Heads of School when they encouraged students and teachers to miss mathematics lessons in order to attend rehearsals for some school activity or other.

This attitude contrasts drastically with the attitude towards mathematics in other contexts. For example, in an episode reported in Stigler and Hiebert

(1999), the authors were comparing video lessons from Japan, Germany, and the USA. While watching a film of a US lesson, a voice was heard over the public address system making an announcement about transport arrangements, an occurrence that is also common in Maltese schools. The Japanese member of the team was shocked that such interruptions should take place during mathematics lessons, pointing out that these disruptions would never happen in Japan as they interrupted the flow of the lesson.

Besides duties within the school I also had commitments assisting mathematics Education Officers. These included setting of examination papers, writing of syllabi, choosing of textbooks, and perhaps, more interestingly, animating in-service courses for teachers in state and non-state schools. The extent of the success of these courses, I must say, is rather limited, due mostly to the fact that most have been obligatory, and so teachers had to attend them whether they liked it or not. The objectives of some of the courses were quite ambitious. For example, I remember the first one in which I took part, during which teachers were introduced to software such as the programming language LOGO, and the dynamic geometry software 'Cabri Geometre' and were encouraged to use them in their mathematics lessons. While I still think that there is great potential in the use of computers in mathematics lessons, for a number of reasons the impetus behind the initiative was not maintained. Probably, the main source of this failure is that Maltese teachers' beliefs about teaching and learning are still more oriented towards learning through exposition rather than learning through discovery and/or learning through exploration. However, one must also take into account the limited technical support that was, and still is, available, as well as the relative complexity of the logistics involved in organising a mathematics lesson in the computer lab. From my interactions with members of my staff as well as other mathematics teachers it is clear that both these factors considerably diminished teachers' enthusiasm and willingness to conduct lessons in the computer lab.'

Mentoring

When I had completed the second draft of this commentary I asked two of my ex-colleagues who had lately been appointed as HoDs to read it and give me feedback. From their feedback I realised that I had not explicitly mentioned a very important aspect of the duties of a HoD: that of mentoring. On reflection, and judging from the comments provided by my two ex-

colleagues, I think this apparent oversight was not due to the fact that I did not perform any mentoring, but because I may have taken this role for granted. Indeed, one cannot avoid it even if one wanted to. New members of staff regularly join the department and it is the HoD to whom they refer for guidance. However, I have not regarded the role of mentor solely as having to do with the induction of new members of staff. Indeed, without even being aware of it, mentoring has been part-and-parcel of my HoD role, from my attitude towards the subject to the manner with which I dealt with different individuals, be they members of the SMT, colleagues, parents, and pupils.

... They Do Things Differently There

In the introduction to this brief commentary I declared that on being appointed HoD I sought to share my passion for mathematics with others and to raise standards of teaching the subject. I think that there were times when my goals were achieved as well as others when they were not. However, I wish that I could have done more, especially in promoting a pedagogy that is more constructivist, one in which students are given the opportunity to work cooperatively on tasks that encourage thinking and problem solving. This does not mean that other approaches may not at times be valid.

Collegiality is another aspect of school life I think is lacking in schools and which I would have liked to foster more. Although I feel that in the schools in which I was HoD I did achieve a certain degree of success in this respect, I still think that some teachers are reluctant to share ideas, with some preferring to be isolated from their colleagues. Perhaps, this can be achieved if the HoD has some say in the selection of teachers in his/her department. This might make it possible for teachers to share a common vision about mathematics and how it is to be taught. Furthermore, teachers should be given the opportunity to observe lessons of teachers that form part of their mathematics department. I think that if the HoD manages to create an environment of trust within the department, one in which he/she observes lessons, teachers observe his/her lessons, and teachers observe other teachers at work, one can go a long way in raising standards in mathematics teaching and learning (Ofsted, 2000/01). Such cooperation can give teachers the opportunity to learn from each other, to share resources, and to prepare and evaluate lessons together.

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BOOK REVIEW

Carmel Cefai and Paul Cooper (editors): *Mental Health Promotion in Schools – Cross-Cultural Narratives and Perspectives*. Rotterdam: Sense Publishers, ISBN: 9789463510516

This book is effectively a collection of papers by some well-known authors with a largely regional focus, the Mediterranean, and Australia in particular. It is divided into three parts covering the perspectives of students, teachers, and parents on mental health issues in relation to educational settings. These three parts are preceded by an introductory section that deals with some fundamental issues in the area of social, emotional, behaviour difficulties (SEBD). The term 'mental health' covers a wide range of situations stretching from temporary circumstances knowing their origin to factors in the child's or young person's environment to psychiatric conditions that usually start manifesting themselves in adolescence. In this introductory section, Paul Cooper makes a strong case for the polymorphic nature of SEBD and the need for flexibility in addressing the issues involved. He maintains that there is a plethora of psychological families of empirically supported approaches to address the issues. He also lists a number of teacher characteristics that go a long way towards making the whole process of managing such difficulties effective. As always, when dealing with vulnerable persons, a balanced measure of scientific intervention and human sensitivity, sympathy and warmth increases the possibility of effective intervention. Education may use scientific methods but will always need to rely on the human interface that can never be substituted.

I am now selecting a paper or two from each section to afford the reader a flavour of the papers in this book. In the first section covering student perspectives, there is a reference to the issue of mental health in a Maltese context. The authors of this paper posit that 10% of the Maltese student population experiences social, emotional and/or behavioural problems and

that Maltese students rate their health and wellbeing relatively poorly, citing bullying as one of the factors underlying wellbeing difficulties. I posit that relationship issues at school are but one of the constellations of underlying factors that lead to the development of mental health issues. Other issues relate to the stability of the family unit, examination pressure, and performance worries. Another paper addresses the issue that the technological revolution within society has effectively ousted from the system a historically significant number of students who struggle to meet the so-called “normal” standards of school performance and who would have otherwise been channelled into the unskilled job market. The raising of the school leaving age on its own may not have done these students any immediate favours but the development of a vocational curriculum may have. Even so, an alternative approach focussing on multi-age inclusion and a move away from traditional curricula towards meaningful activity in a working society set-up, always guided by insightful, perceptive and responsive teachers can make for positive individual futures for these students in a relationship model of interaction and life skills development.

The second section of the book deals with teachers’ perspectives and recognition of mental health issues and how mental health can be addressed in the school environment. The various papers in this section seem to agree on one point. They agree that this needs to be interlaced in a whole school approach, namely commitment to and active participation in a shared vision of the child’s mental wellbeing along with a good support structure at school. Furthermore, parents should constitute an integral component of this tripartite setup.

Two related papers deal with the issue of staff perceptions of mental health promotion in school from an Australian and a Maltese perspective. In the Australian case study, the authors described how targeted interventions for children at risk of experiencing mental health difficulties needed support over and above what the school programme offered. Interventions delivered by other professionals were seen as key to the implementation of an integrated education-health-social welfare model. A second paper dealing with the same theme from a local perspective described Maltese school teacher’s perceptions of social and emotional learning. It shows how while some schools are receptive to the idea, there is much work that needs to be done before the concept can be developed well enough to have an impact on children’s overall mental health issues. Again, mental health must be seen against a

background of all that is taking place in a learner's life, ranging from family issues to relationships and school achievement issues. These threaten to upset the fragile stability of childhood which may be mythical more than real.

The third section focuses on stakeholders' perspectives of children's mental health issues. What stood out best in this section was a paper co-authored by Helen Askill Williams about the Kidsmatter initiative, which aims at strengthening protective factors within settings, in families, and in children, with the ultimate goal being to help families access appropriate services and counteract potential long-term problem situations. The programme serves as a focal point for parent-school collaboration and decision making in the children's best interests. A well placed final paper in this section and indeed at the end of the collection of papers refers to the worsening children's mental health situation in industrialised societies. It acknowledges that on their own, teachers are not best equipped to address children's mental health issues and that a unified effort through a partnership model such as child and adolescent mental health services (CAMHS) may be more effective.

One theme that can be drawn from these papers that is applicable to the local situation is the lack of exposure to mental health training that novice teachers report having. The responsibility for identifying these needs and managing them falls on the shoulders of more than one profession but unless these professionals talk to each other, the needs of the children will not be met as adequately as when there is synergy and unison. As professionals at the coalface, teachers must be equipped to at least recognise mental health issues when they see them and be able to involve themselves in multi- and interdisciplinary team initiatives at individual and systems level to address the needs of the children they teach.

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