Forecasting macroseismic scenarios through anisotropic attenuation: a Bayesian approach

Rotondi R.[1], Azzaro R.[2], D'Amico S.[2], Tuvè T.[2] and Zonno G.[3]

[1] C.N.R. - Istituto di Matematica Applicata e Tecnologie Informatiche,
Via Bassini 15, Milano - Italy
(e-mail: reni@mi.imati.cnr.it)

[2] Istituto Nazionale di Geofisica e Vulcanologia, sez. Catania,
P.zza Roma 2, Catania - Italy,
(e-mail: azzaro@ct.ingv.it, damico@ct.ingv.it, tuve@ct.ingv.it)

[3] Istituto Nazionale di Geofisica e Vulcanologia, sez. Milano-Pavia,
Via Bassini 15, Milano - Italy
(e-mail: zonno@mi.ingv.it)

Abstract. The seismic hazard evaluation is heavily influenced by how the attenuation of the macroseismic intensity recorded at sites distant from the epicentre of a strong earthquake is modelled. This decay is generally assumed symmetric, because due to a point source, and calculated through loglinear regression relationships. In this work we aim at two objects: quantifying, by a binomial-beta probabilistic model, the uncertainty involved in the assessment of the intensity decay, an ordinal quantity often incorrectly treated as real variable, and, given the finite dimension of the fault, modelling non-symmetric decays but exploiting information collected from previous studies on symmetric cases. To this end we transform the plane so that the ellipse having the fault length as maximum axis is changed into a circle with fixed diameter. We start from an explorative analysis of a set of macroseismic fields representative of the Italian seismicity among which we identify three different decay trends by applying a hierarchical clustering method. Then we focus on the exam of the seismogenic area of Etna volcano where some fault structures are well recognizable as well as the anisotropic trend of the attenuation. As in volcanic zones the seismic attenuation is much quicker than in other zones, we first shrink and then transform the plane so that the decay becomes again symmetric. Following the Bayesian paradigm we update the model parameters and associate the estimated values of the intensity at site with the corresponding locations in the original plane. Backward validation and comparison with the deterministic law are also presented.

Keywords. anisotropy, Bayesian inference, seismic attenuation.

Motivation

We choose to express the severity of an earthquake through its macroseismic intensity, a measure related to the damages caused on humans, animals, environment and hand-made structures. An earthquake's destructiveness depends on many factors: in addition to intensity and the local geologic conditions (stability of the ground), these factors include the distance from the epicentre, the focal depth, and the design of buildings and other structures. The set of the spatially distributed felt intensity reports constitutes the macroseismic field of an earthquake. The DBMI04 Italian database contains 58146 macroseismic observations related to 14161 sites. Being able to forecast the extention and seriousness of the damages in case a quake occurs is clearly important for interventions of mitigation of the seismic risk. The problem is how to distinguish different decay trends in the large amount of data taking into account the intrinsic uncertainty of the decay process without losing sight of the specificity of particular events. These demands match perfectly the Bayesian paradigm which updates the parameter estimates by the current data modifying their previous values, expression of all past information.

We outline the analysis performed:

a. exam of a set of macroseismic fields representative of the distribution of the seismicity of interest in order to select groups of fields homogeneous from the attenuation point of view;
b. formulation of a beta-binomial model and elicitation of the prior;
c. for each class estimation of the probability distribution of the intensity decay ΔI (or of the intensity at site I_s) conditioned on the epicentral intensity I_0 and on the epicentre-site distance;
d. shrinkage of the plane so as to reduce a linear rupture segment into a point source;
e. updating of the model parameters through the "transformed" data and assessment of so-estimated distributions to the re-transformed data;
f. comparison between circular and elliptical model on the basis of validation criteria.

The steps [a] - [c] deal with the isotropic case in which the seismic source is a point (epicentre), whereas the steps [d] - [f] extend the previous results to the anisotropic case in which we have more information on the rupture fault so that the source can be considered as a finite linear segment.

Statistical model

[a] We have considered 55 earthquakes of intensity VII-XI degree of the MCS scale occurred in the period 1560-1980. To capture the different aspects of their macroseismic fields we have characterized them through location and dispersion measures computed for each set of distances from the epicentre to those sites in which the same intensity was recorded. Then we have applied a hierarchical agglomerative method (Kaufman and Rousseeuw (1990)) to the matrix collecting this information. Three classes, $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, were identified and associated with increasing depth of the seismic source generating the earthquake. They constituted the input for the probabilistic analysis of the decay ΔI.

[b] Following Zonno et al. (2009) we assume that, conditioned on the value of I_0 and on a fixed distance from the epicentre, $\Delta I = I_0 - I_s = 0, 1, \ldots, i_0 - 1$, has a binomial distribution with parameter $(1 - p)$

$$Pr(\Delta I = i_0 - i \,|\, I_0 = i_0, p) = \binom{i_0}{i} p^i (1-p)^{i_0 - i} \quad ,$$

that is equivalent to assume that the intensity at site I_s has a binomial distribution with parameter p. Moreover, to account for the difference in ground shaking even among sites at the same epicentral distance, the parameter p, in its turn, is taken as a random variable having a beta distribution $Be(p; \alpha, \beta)$. In practice we consider J distance bins of fixed width around the epicentre and estimate different p_j, $j = 1, \ldots, J$, assuming that in all the sites within each jth distance bin ΔI has the same binomial distribution with parameter p_j.

[c] The estimation algorithm is a two-step algorithm. We apply it to each of the classes identified by clustering and for each class we proceed as follows. Given a value of I_0, in the first step we assign the hyperparameters $\alpha_{j,0}$ and $\beta_{j,0}$ of the prior distribution of p on the basis of the macroseismic fields belonging to the same class, but with a different value of I_0. In the second step, we update them on the basis of the macroseismic fields belonging to the same class and with the same value of I_0. In this way, given a value of I_0, the class and the distance bin to which the epicentre-site distance belongs, we obtain a binomial distribution with parameter p equal to its posterior mean:

$$\hat{p}_j = E\left(p_j \,|\, \mathcal{D}_j\right) = \frac{\alpha_{j,0} + \sum_{n=1}^{N_j} i_s^{(n)}}{\alpha_{j,0} + \beta_{j,0} + I_0 \cdot N_j} \quad ,$$

where $i_s^{(n)}$ is the intensity felt at the n-th site inside the j-th bin and N_j is the total number of felt intensities in that bin. The process starts with the rough estimation of the probability of null decay $Pr(\Delta I = 0 \,|\, I_0 = i_0, j) = p_j^{i_0}$ by the relative frequency of null decay $N_j(i_0)/N_j$, where $N_j(i_0)$ the number of sites where the felt intensity is equal to the epicentral intensity.

By smoothing the posterior means of p in each bin by the inverse power function $g(d) = (\gamma_1/d)^{\gamma_2}$, we can express the parameter as a function of the epicentral distance varying with continuity, and the binomial distribution in which the parameter is given by this function can be used to forecast I_s at any distance d from the epicentre. The mode of the smoothed binomial distribution is taken as estimate of the intensity I_s. Forecasting can be also given in terms of probability of exceeding a given intensity, and of the value of I_s not exceeded at least with a fixed probability value.

Transformation of the macroseismic field

[d] So far we have supposed that the isoseismal lines which bound the points of equal intensity are circular, but when the length of the rupture is such that it cannot be assimilated to a point, the isoseismal contours are elliptical and the innermost indicates the dimension and the orientation of the fault. To exploit also in these cases the information gained from the circular cases for assigning the prior distributions we transform the plane so that each point on an elliptical isoseismal line moves to a point on the circle with radius equal to the minor axis b, then we go back (c) and finally associate the estimated intensities with the original locations. The transformation consists of the following steps.

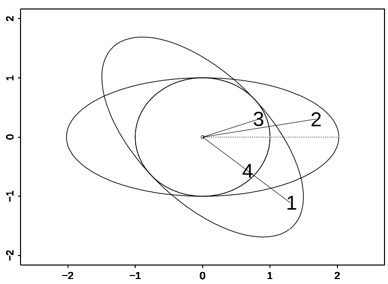

Fig. 1 - Transformation of the ellipse (2.023, 1) in the circle with radius 1, azimuth = 2.456, $\psi = -0.885$ rad.

- $\mathrm{P}_1(x_1, y_1) \longrightarrow \mathrm{P}_2(x_2, y_2)$

$$\begin{cases} x_2 = \cos(-\psi)\ x_1 - \sin(-\psi)\ y_1 \\ y_2 = \sin(-\psi)\ x_1 + \cos(-\psi)\ y_1 \end{cases}$$

being ψ the angle between the positive semi-axis x and the directrix a

- $\mathrm{P}_2(x_2, y_2) \longrightarrow \mathrm{P}_3(x_3, y_3)$

$$x_3 = x_2 \times b/a \qquad y_3 = y_2$$

- $\mathrm{P}_3(x_3, y_3) \longrightarrow \mathrm{P}_4(x_4, y_4)$

given $\theta = \arctan(y_2/x_2)$ and $\theta' = \arctan(y_3/x_3)$, we set $\phi = \theta' - \theta + |\psi|$

$$\begin{cases} x_4 = \cos(\phi)\ x_3 - \sin(\phi)\ y_3 \\ y_4 = \sin(\phi)\ x_3 + \cos(\phi)\ y_3 \end{cases}$$

Acknowledgements

This work was funded by the Italian Dipartimento della Protezione Civile in the frame of the 2007-2009 Agreement with Istituto Nazionale di Geofisica e Vulcanologia (INGV) - Project V4 FLANK Volcanology.

References

Kaufman, L. and Rousseeuw, P.J. (1990): *Finding groups in data*, Wiley, New York.

Zonno, G., Rotondi, R. and Brambilla, C. (2009): Mining macroseismic fields to estimate the probability distribution of the intensity at site. *Bulletin of the Seismological Society of America*, *99*, 5, 2876–2892.