AM 02/I.25M

L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2025 FIRST SESSION

SUBJECT:	**Applied Mathematics**
PAPER NUMBER:	I
DATE:	10th May 2025
TIME:	09:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are 10 questions in all.
Each question carries 10 marks.
The total number of marks for all the questions is 100.
Graphical calculators are **not** allowed. Scientific calculators can be used but all necessary working must be shown.
A booklet with mathematical formulae is provided.
In the paper, **i**, **j**, **k** are unit vectors along the x-, y-, z-axes of a Cartesian coordinate system.

(Take $g = 10\,\text{m s}^{-2}$.)

1. A car of mass 1800 kg accelerates uniformly from rest to $100.8\,\text{km h}^{-1}$ in a distance of 98 m. The engine is then switched off and the car comes to rest in 7 s due to a frictional force of 2000 N and a constant braking force.

   (a) Draw a diagram showing **all** the forces acting on the car during the acceleration. (2)
   (b) Find the acceleration of the car. (4)
   (c) Find the braking force of the car. (4)

   (**Total: 10 marks**)

2. Forces $\boldsymbol{F}$ and $\boldsymbol{G}$ act at points with position vectors $\boldsymbol{r}_F$ and $\boldsymbol{r}_G$ respectively, where at time t,

$$\boldsymbol{F} = \begin{pmatrix} 2 \\ 3e^t \end{pmatrix}, \quad \boldsymbol{G} = \begin{pmatrix} 2e^t \\ -2\cos t \end{pmatrix}, \quad \boldsymbol{r}_F = \begin{pmatrix} 5 \\ -\cos t \end{pmatrix}, \quad \boldsymbol{r}_G = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$$

   (a) Find the resultant force and the moment about the origin. (4)
   (b) Show that the system reduces to a single force at a fixed point P on the x-axis and find the vector equation of the line of action of the resultant at P at $t = 0$. (6)

   (**Total: 10 marks**)

3. A uniform sphere of mass m and radius a hangs by a string attached to the topmost point on the sphere. The other end of the string is attached to a vertical wall such that the sphere is in contact with the wall and the string makes an angle of 60° with the wall. The coefficient of friction between the sphere and the wall is equal to μ. Find:

   (a) the tension in the string, in terms of mg; (5)
   (b) the minimum value of μ required for the sphere to remain in equilibrium. (5)

   (**Total: 10 marks**)

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4. A parachutist jumps off a balloon at a point A which is 4 km above the ground and descends along a vertical line $ABCDE$, starting from rest. Between A and B, she accelerates uniformly at $8\,\mathrm{m\,s^{-2}}$ for t seconds until she reaches a downward velocity $v\,\mathrm{m\,s^{-1}}$. She maintains this constant velocity for 35 s until, at point C, she opens her parachute so that she decelerates uniformly for 10 s until point D, when her velocity is $4\,\mathrm{m\,s^{-1}}$ downwards. She retains this velocity until she reaches the ground E.

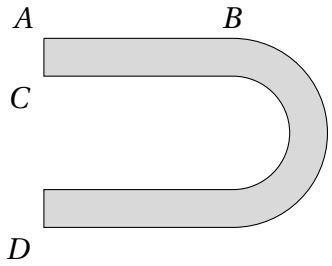
   (a) Draw a velocity-time graph for this descent. (4)

   (b) Find the distances AB, BC, and CD in terms of t. (3)

   (c) Find the velocity v, given that her stopwatch shows that the total time elapsed from A to E is 150 s. (3)

   (**Total: 10 marks**)

5. A horseshoe magnet of uniform density has shape $ABDC$ where $AB = AD = 5\,\mathrm{cm}$, $AC = 1\,\mathrm{cm}$, as shown below, and the curves are semi-circular arcs.



   The centre of mass of a uniform semi-circular disc of radius r is distant $\frac{4r}{3\pi}$ from its diameter. Ignoring the third dimension, find:

   (a) the position of the centre of mass G of the magnet, relative to the point D; (8)

   (b) the angle that AB makes with the vertical if it is suspended from point A. (2)

   (**Total: 10 marks**)

6. A particle is projected from ground level with a speed of $58\,\mathrm{m\,s^{-1}}$ at an angle with the horizontal of $\phi = \tan^{-1}\frac{20}{21}$. After travelling for 2 s the projectile hits the top of a tree.

   (a) What is the horizontal distance of the tree from the point of projection? (3)

   (b) Show that the particle hits the tree on its upward trajectory. Make sure to provide **all** the necessary calculations and explain clearly your reasoning. (4)

   (c) Calculate the height of the tree. (3)

   (**Total: 10 marks**)

7. Two smooth elastic spheres A and B of the same radius are moving on a smooth surface with speeds $6\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ in the same direction when they collide directly. The ratio of the total kinetic energy before and after the collision is $13:10$.

   Given that the mass of sphere A is m and that of sphere B is $4m$, determine the possible value of the speeds of the spheres after the collision, and find the coefficient of restitution between the spheres.

   (**Total: 10 marks**)

8. An electric car of mass 1200 kg is driven along a horizontal road at a constant speed of $54\,\mathrm{km\,h^{-1}}$. During the motion, it can be assumed that the only resistive forces acting on the car are friction and air resistance. The coefficient of friction is 0.7.

   (a) If the power output of the car at this speed is 150 kW, what is the traction force? (2)

   (b) Calculate the air resistance. (3)

   (c) Determine the work done by the car engine in moving 1.2 km. (4)

   (d) Suppose that only 75% of the electric energy is converted to work done in moving the car. How much electric energy is needed to travel 1.2 km? (1)

   (**Total: 10 marks**)

9. A light aircraft is to travel 143 km due North (**j** direction) in a straight line. It has a cruising speed of $61\,\mathrm{km\,h^{-1}}$ in still air. However, a wind is blowing with a velocity of $(11\mathbf{i}+5\mathbf{j})\,\mathrm{km\,h^{-1}}$, where **i** is in the East direction.

   (a) Find the time taken for the trip. (5)

   (b) If the airplane is to travel back under the same wind conditions, how long would it take? (5)

   (**Total: 10 marks**)

10. A car negotiates a circular bend in the road with a radius of 25 m.

    (a) If the road is flat and the coefficient of friction μ between the tyres and a dry road is 0.7, determine the maximum speed with which the car can go round the bend without slipping. (4)

    (b) When the road is wet, μ can decrease significantly. What banking angle would be required in order to make sure that the car is still able to negotiate the bend at the same maximum speed calculated in part (a)? Take the worst case scenario of $\mu = 0$. (6)

    (**Total: 10 marks**)

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MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2025 FIRST SESSION

SUBJECT:	**Applied Mathematics**
PAPER NUMBER:	II
DATE:	12th May 2025
TIME:	4:00 p.m. to 7:05 p.m.

Directions to Candidates

Answer **SEVEN** questions. There are 10 questions in all.
Each question carries 15 marks.
Graphical calculators are **not** allowed. Scientific calculators can be used but all necessary working must be shown.
A booklet with mathematical formulae is provided.
In the paper, **i**, **j**, **k** are unit vectors along the x-, y-, z-axes of a Cartesian coordinate system.

(Take $g = 10\,\text{m}\,\text{s}^{-2}$.)

1. A tower crane consists of two uniform beams AB and CD, each of equal length a and mass M. AB is vertical with B above A, while CD is horizontal on top of AB with $CB = x$ metres, where $x < a/2$.

   (a) Find the position of the centre of mass of the structure, relative to A. (6)

   (b) Determine the additional mass m, as a function of x, to be placed at point C in order that the centre of mass lies on the line AB. (6)

   (c) The crane lifts a weight $Mg/10$ from its end D. If $x = a/3$ and $m = M/2$, what horizontal force needs to be applied along DC so that there is **no** net moment on the crane about A? (3)

   (**Total: 15 marks**)

2. A uniform beam AB has length a and mass M. It is clamped horizontally at A, and a force $Mg/2$ acts vertically upwards at the point C where $BC = a/4$.

   (a) If the beam is in equilibrium, find the reaction and moment at A. (3)

   (b) Find expressions for the shearing force and bending moment at a point on the beam distant x from A. (6)

   (c) Sketch a diagram of the shearing force and bending moment along the beam. (4)

   (d) Find the minimum bending moment and the points on the beam where this occurs. (2)

   (**Total: 15 marks**)

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3. A "centrifuge ride" consists of a cylindrical drum of radius 3 m with a vertical axis. Participants enter the ride and stand up leaning against the vertical walls of the drum. The drum starts to spin rapidly, pushing the riders against the padded walls. When the drum reaches a sufficient speed, the floor of the drum is lowered, leaving riders suspended against the walls with their feet dangling in the air.

   (a) If the coefficient of friction between the participants and the wall is 0.7, find the minimum angular speed of the drum necessary before the floor can be lowered safely and explain why it is safe for both children and adults. (4)

   (b) Calculate the linear velocity experienced by a rider at this angular speed. (2)

   (c) At what angular speed does a rider experience a reaction force equal to twice his normal weight? Does this happen before or after the floor is lowered? (4)

   (d) A child releases a uniform spherical marble on the wall while the drum is spinning, which then rolls down the wall without slipping. Determine its acceleration downwards. (5)

   (**Total: 15 marks**)

4. A builder, Stan, needs to lower 90 kg of bricks from the roof of a house, which is at a height of 10 m above ground. He sets up a small light pulley at this height, hoists a barrel to the top and ties the other end of the rope at the bottom of the building. The mass of the builder is 70 kg and that of the barrel is 10 kg. He calls his companion, Gerard, on the roof to fill the barrel with the bricks.

   (a) Stan unties the rope but holds on to the rope so that he is lifted into the air as the heavy barrel descends. Calculate the speed of the barrel as it brushes past the builder when he is halfway up. (5)

   (b) When the builder reaches the pulley, the barrel hits the ground and all the bricks spill out. The builder is now heavier than the barrel, and starts to descend while holding on to the rope. What is the time taken until he lands on the bricks? (5)

   (c) Stan now lets go of the rope in panic. Calculate the momentum of the barrel when it hits his head. (5)

   [The height of the persons can be ignored.] (**Total: 15 marks**)

5. Amy is in a canoe at a point O on the shore of Valletta. She wants to cross to a point S in Senglea, 500 m due East of O. Take the x-axis to be along OS and the y-axis to be due North. She can paddle at 5.4 km h^{-1} and there is a sea current of 1.8 km h^{-1} due South.

   (a) Working in units of m s^{-1}, find the direction vector at which she should point the canoe in order to cross in the shortest time. (2)

   When Amy is halfway there, Bob drives a speedboat of mass 400 kg from O, starting from rest at time $t = 0$. Pointing the boat due East, the net force of the engine and sea resistance on the boat equals $100(1 - e^{-t/8})\mathbf{i}$ N, while the current presents a constant force of 100 N southwards.

   (b) Find expressions for the velocity and position vectors of Bob at time t. (8)

   (c) Calculate how far Bob is from Amy when she reaches three quarters of the way to S. (5)

   (**Total: 15 marks**)

6. Two smooth spheres A and B of equal radii and masses 3 kg and 6 kg respectively collide obliquely. Before the impact the velocity of A is $(12\mathbf{i}-5\mathbf{j})\,\mathrm{m\,s^{-1}}$ while that of B is $(-5\mathbf{i}+12\mathbf{j})\,\mathrm{m\,s^{-1}}$ with the unit vector $\mathbf{i}$ being along the line joining the centres of mass at the moment of impact. The coefficient of restitution between the spheres is $\frac{7}{17}$.

   (a) Determine the velocity of the spheres after the impact. (10)

   (b) Calculate the change in total kinetic energy upon impact. (5)

(**Total: 15 marks**)

7. The displacement x with time t of a particle from a fixed point O is given by the equation

$$x(t)=\sin(4t)+\sqrt{3}\cos(4t).$$

   (a) Show that $\ddot{x}=-\omega^2 x$ for some constant ω. (3)

   (b) Deduce the periodic time. (2)

   (c) Show that the motion is equivalent to

$$x(t)=2\cos(4t-\tfrac{\pi}{6}).$$ (5)

   (d) Sketch a graph of $x(t)$ against t over a whole cycle starting from $t=0$. Make sure to indicate on the sketch the positions where $x(t)=0$. (5)

(**Total: 15 marks**)

8. A hollow uniform sphere of mass 5 kg rolls without slipping down a rough inclined plane along the line of steepest descent. The plane makes an angle $\theta=\tan^{-1}\frac{7}{24}$ to the horizontal.

   (a) If the sphere starts from rest, determine its speed after moving a distance of $\frac{25}{21}$ m. (7)

   (b) What is its linear acceleration at this point? (2)

   (c) Calculate the minimum value of the coefficient of friction between the sphere and the plane so that **no** slipping occurs during the motion. (6)

(**Total: 15 marks**)

9. A uniform cube of mass 0.5 kg and side length 6 cm, and having centre O, pivots around a vertical edge AB. When the cube is motionless, an instantaneous impulse J is applied along $\overrightarrow{MO}$, where M is the centre of a face adjacent to AB. As a result, the cube starts rotating with an angular velocity of 4 rad s^{-1}.

   (a) Show that the moment of inertia of a uniform solid cube of mass M and side length d about an axis parallel to an edge, passing through the centre O, is $\frac{1}{6}Md^2$. The moment of inertia of a square lamina about the perpendicular axis through its centre can be assumed to be $\frac{1}{6}M_0b^2$, where M_0 and b are its mass and side length, respectively. (5)

   (b) Determine an expression for the moment of inertia of the cube about AB. (4)

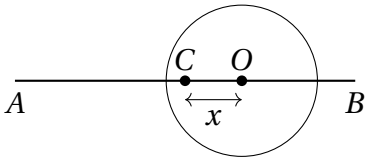
   (c) Calculate the value of the impulse J. (3)

   (d) A constant friction at the pivot brings the block to rest after 20 turns. What is the angular deceleration of the block? (3)

(**Total: 15 marks**)

Please turn the page.

10. A uniform sphere of mass $15m$ and radius $l/3$ is smoothly pivoted about its centre O. The sphere has a small hole along an axis passing through its centre and perpendicular to the pivot axis. A uniform rod AB of length $2l$ and mass m is placed and fixed inside the hole such that its centre of mass C is at a distance x from O. The effect of the small hole on the moment of inertia of the sphere can be ignored.



(a) Determine the moment of inertia of the combined system about an axis through O perpendicular to the rod. (4)

(b) Express the distance of the centre of mass G of the combined system from O in terms of x. (2)

(c) Show that the periodic time for small oscillations is

$$T = 2\pi\sqrt{\frac{l^2 + x^2}{g\,x}} \qquad (2)$$

(d) Using the facts that $\frac{\mathrm{d}}{\mathrm{d}x}T^2 = 2T\frac{\mathrm{d}T}{\mathrm{d}x}$ and $T > 0$, or otherwise, determine the values of x for which the periodic time has a stationary point. What is the value of T at these points? (7)

(**Total: 15 marks**)