AM 02/I.26S

L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2026 SECOND SESSION

SUBJECT:	**Applied Mathematics**
PAPER NUMBER:	I
DATE:	31st August 2026
TIME:	09:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are 10 questions in all.
Each question carries 10 marks.
The total number of marks for all the questions is 100.
Graphical calculators are **not** allowed. Scientific calculators can be used but all necessary working must be shown.
A booklet with mathematical formulae is provided.
In the paper, **i**, **j**, **k** are unit vectors along the x-, y-, z-axes of a Cartesian coordinate system.

(Take $g = 10\,\text{m}\,\text{s}^{-2}$.)

1. Three light rigid rods form a smoothly jointed triangular framework DEF. The lengths of the rods are $DE = 6\,\text{m}$, $DF = 10\,\text{m}$, and DE is perpendicular to EF. Particles of weights 80 N and 40 N are attached at E and F respectively. The framework is freely suspended from the joint D and hangs in equilibrium in a vertical plane.

   (a) Find the point on the line EF at which the resultant of the two weights acts. (3)

   (b) Show that the rod DE makes an angle of θ with the vertical, where $\tan\theta = \frac{4}{9}$. (2)

   (c) Determine the forces in **each** of the rods DE and EF, stating clearly which rods are in tension and which are in compression. (5)

   (**Total: 10 marks**)

2. A particle of mass 3 kg lies at rest at the point with coordinates $(6,0)$ in the x–y plane, where distances are measured in metres. For $0 \leqslant t \leqslant 4\,\text{s}$, a constant force $\boldsymbol{F}$ of magnitude 6 N and direction $4\mathbf{i}+3\mathbf{j}$ acts on the particle. At $t = 4\,\text{s}$ the force $\boldsymbol{F}$ is removed and replaced by a second constant force $\boldsymbol{Q}$, which acts for $4\,\text{s} \leqslant t \leqslant 9\,\text{s}$. The force $\boldsymbol{Q}$ is chosen so that at $t = 9\,\text{s}$ the particle is moving with velocity $2\mathbf{i}\,\text{m}\,\text{s}^{-1}$ and is located at the point $(0,3)$. Find, in **i**,**j** notation where appropriate:

   (a) the velocity and displacement from the origin of the particle at $t = 4\,\text{s}$; (4)

   (b) the constant force $\boldsymbol{Q}$; (4)

   (c) the change in kinetic energy of the particle between $t = 4\,\text{s}$ and $t = 9\,\text{s}$. (2)

   (**Total: 10 marks**)

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3. A car travels along a straight road between points A, B, C, D and E. Between A and B the car travels at a constant speed of $90\,\text{km}\,\text{h}^{-1}$. From B to C it decelerates uniformly for 40 s, reaching a speed of $27\,\text{km}\,\text{h}^{-1}$ at C. The car then maintains this speed while travelling along CD, which has length 400 m. At D the car accelerates uniformly over a distance of 800 m, reaching the point E in one minute.

   (a) Working in SI units, sketch a velocity–time graph. (3)

   Use the graph to find:

   (b) the acceleration of the car between D and E; (2)

   (c) the total distance travelled from B to E; (2)

   (d) the extra time taken to travel from B to E compared with travelling the whole distance BE at a constant speed of $90\,\text{km}\,\text{h}^{-1}$. (3)

(**Total: 10 marks**)

4. Two points P and Q lie on horizontal ground and are 80 m apart. At time $t = 0$, a particle X is projected from P with speed $25\,\text{m}\,\text{s}^{-1}$ at an angle of 30° above the horizontal, towards Q. At the same instant, a particle Y is projected from Q towards P with speed $20\,\text{m}\,\text{s}^{-1}$ at an angle of 60° above the horizontal. Assume that air resistance is negligible. Find:

   (a) the horizontal distance between the two particles in terms of t; (4)

   (b) the time at which the particles are vertically aligned; (2)

   (c) the speed of **each** particle at this instant. (4)

(**Total: 10 marks**)

5. A particle P of weight W is at rest in limiting equilibrium on the inner surface of a fixed rough hollow hemisphere of internal radius a and centre O. The coefficient of friction between the particle and the sphere is μ. Let θ be the angle between the downward vertical through O and the radius OP.

   (a) By resolving forces along the radius and along the tangent to the sphere at P in the vertical plane containing OP, show that $\mu = \tan\theta$. (4)

   (b) Find the least possible value of μ for which the particle could remain at rest at a point with $\theta = 60°$. (2)

   (c) If the frictional force equals half the weight of the particle, in magnitude, find the value of μ. (4)

(**Total: 10 marks**)

6. Two smooth elastic spheres P and Q move along the same straight line and collide directly. The mass of Q is eight times that of P. Initially, Q is at rest and P moves with speed $10\,\text{m}\,\text{s}^{-1}$. After the collision P rebounds and loses 64% of its kinetic energy. Determine:

   (a) the speeds of P and Q after the collision; (5)

   (b) the coefficient of restitution between the spheres; (2)

   (c) the percentage loss in the total kinetic energy of the whole system. (3)

(**Total: 10 marks**)

7. A human cannonball performer is projected from a cannon at a point O and lands on a safety net at point N. The point N is 42 m horizontally from O and 2 m vertically above O. The performer may be modelled as a particle and air resistance can be neglected.

   (a) Show that if (x, y) are the coordinates of a point projectile P, θ is the angle that the initial velocity of P makes with the horizontal, and u is the initial speed of P, then the following equation holds throughout the motion:

   $$y = x\tan\theta - \frac{g\,x^2}{2u^2\cos^2\theta}. \tag{7}$$

   (b) If $\theta = 45°$, determine the speed of the performer at O. (3)

(**Total: 10 marks**)

8. Two rockets of negligible mass are attached to a snow sledge of mass 7 kg, one on either side. A person of mass 63 kg is seated on the sledge. The rockets are lit and accelerate the sledge and person from rest to a speed of 20 m s^{-1} in 14 s on a smooth horizontal terrain covered with snow. Air resistance is to be ignored and the sledge and person are to be treated as a point. Find:

   (a) the total work done by the rockets during the first 14 s of motion; (3)

   (b) the average power output of the rockets over this interval. (2)

   After 14 s the rockets burn out and the sledge, with the person on it, moves up a smooth snow covered ramp inclined at an angle of θ to the horizontal, where $\tan\theta = \frac{20}{21}$. Calculate:

   (c) the maximum height above the horizontal terrain attained by the person and sledge; (2)

   (d) the distance travelled by the sledge along the ramp. (3)

(**Total: 10 marks**)

9. As observed from a point S on the shore, a daredevil D travels in a speedboat with constant speed from a point A to a point B in 10 min, where A is 5 km due East of S and B is 5 km due North of S. Beyond B, the speedboat is at risk of colliding with moored boats. Accordingly, at the instant the speedboat reaches B, a lifeguard on a jet-ski departs from a point C, which is 3 km due north of S, in order to intercept the speedboat. The jet-ski moves at a constant speed of 50 km h^{-1}. Let **i** and **j** be unit vectors in the East and North directions respectively. Find:

   (a) the velocity of the speedboat in terms of **i** and **j**, in units of km h^{-1}; (4)

   (b) the velocity that the lifeguard must maintain to intercept the speedboat, assuming the speedboat continues moving beyond B with the same velocity; (4)

   (c) the time taken in minutes for the lifeguard to intercept the speedboat. (2)

(**Total: 10 marks**)

Please turn the page.

10. A toy drone of mass 0.75 kg flies in a horizontal circle of radius 3 m at constant speed. Given that the drone completes 5 revolutions in 3π s, find:

    (a) the magnitude of the centripetal force acting on the drone; (4)

    (b) the magnitude of the resultant force exerted on the drone by its rotors; (4)

    (c) the angle that the resultant force makes with the vertical. (2)

(**Total: 10 marks**)

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MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2026 SECOND SESSION

SUBJECT:	**Applied Mathematics**
PAPER NUMBER:	II
DATE:	1st September 2026
TIME:	9:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **SEVEN** questions. There are 10 questions in all.
Each question carries 15 marks.
Graphical calculators are **not** allowed. Scientific calculators can be used but all necessary working must be shown.
A booklet with mathematical formulae is provided.
In the paper, **i**, **j**, **k** are unit vectors along the x-, y-, z-axes of a Cartesian coordinate system.

(Take $g = 10\,\mathrm{m\,s^{-2}}$.)

1. A horizontal test track is used to trial two automated delivery pods, X and Y, moving along parallel straight guide rails.

   Pod X is released from rest at the starting position A and accelerates uniformly at $4.2\,\mathrm{m\,s^{-2}}$ for 5 s, after which it continues at a constant speed.

   Pod Y is released from rest 3 s after X from the same starting position A on the parallel rail and accelerates uniformly at $5.6\,\mathrm{m\,s^{-2}}$ for 5 s, reaching a higher cruising speed than X, after which it also moves at constant speed.

   The finish marker D is 360 m from A. When X reaches D, both pods immediately begin to decelerate uniformly at $15\,\mathrm{m\,s^{-2}}$ until they come to rest.

   (a) Construct the velocity–time graphs for X and Y on the same axes. (5)
   (b) Show that Y reaches the finish line D before X. (5)
   (c) Find the distance from A at which Y overtakes X. (2)
   (d) Determine the distance between the final resting positions of X and Y. (3)

   (**Total: 15 marks**)

2. Two model aircraft A and B are tracked in a 3D Cartesian coordinate system with origin O. The position vectors relative to O at time $t = 0$ are

$$\boldsymbol{r}_A = (3,-7,-3)\,\mathrm{km}, \qquad \boldsymbol{r}_B = (-1,1,s)\,\mathrm{km}.$$

   The aircraft fly with constant velocity vectors

$$\boldsymbol{v}_A = (1,1,1)\,\mathrm{km\,h^{-1}}, \qquad \boldsymbol{v}_B = (2,-1,5)\,\mathrm{km\,h^{-1}}.$$

   (a) If the aircraft collide at some time $t > 0$, find the value of s and determine the coordinates of the collision point. (7)
   (b) If instead $s = 1$, find the shortest distance between the aircraft for $t \geqslant 0$ during their subsequent motion. Give your answer correct to 3 significant places. (8)

   (**Total: 15 marks**)

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3. A particle is projected from a point A, which is at a height h above horizontal ground, with speed U at an angle of elevation α to the horizontal. The particle moves freely under gravity with constant acceleration g acting vertically downward, and air resistance is neglected.

   (a) Taking A as the origin and the horizontal ground as the line $y=-h$, show that if the particle hits the ground at a point P which is at a horizontal distance a from the foot of the vertical through A, then
   $$ga^2\tan^2\alpha - 2aU^2\tan\alpha + (ga^2 - 2hU^2) = 0. \tag{5}$$

   (b) Deduce that the particle cannot strike the point P if
   $$U^4 < g^2a^2 - 2ghU^2. \tag{5}$$

   (c) Show that if $U^2 = g(\sqrt{a^2+h^2}-h)$, there is exactly one possible angle of projection, and that the time taken for the particle to reach P is
   $$\sqrt{\frac{U^2}{g^2} + \frac{a^2}{U^2}}. \tag{5}$$

(Total: 15 marks)

4. Three variable forces act in the xy-plane on a rigid plate at positions relative to an origin O given respectively by
   $$\boldsymbol{r}_1 = (0,0)\,\text{m}, \qquad \boldsymbol{r}_2 = (2,-1)\,\text{m}, \qquad \boldsymbol{r}_3 = (1,1)\,\text{m}.$$
   At time t (in seconds), the forces are
   $$\boldsymbol{F}_1 = (\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j})\,\text{N}, \qquad \boldsymbol{F}_2 = (2\cos t\,\mathbf{i} - \sin t\,\mathbf{j})\,\text{N}, \qquad \boldsymbol{F}_3 = (-\sin t\,\mathbf{i} + \cos t\,\mathbf{j})\,\text{N}.$$

   (a) Find the resultant force $\boldsymbol{R}(t)$ acting at O and the resultant moment $M(t)$ about O. (7)

   (b) Show that there exists a point P on the x-axis such that, for every t, the system is equivalent to a single force $\boldsymbol{R}(t)$ acting at P. Find the position vector of P. (5)

   (c) A fourth force $\boldsymbol{F}_4(t)$ is applied at the point with position vector $\boldsymbol{r}_4 = x\,\mathbf{i}$. Find $\boldsymbol{F}_4(t)$ and $\boldsymbol{r}_4$ such that, for all t, the four-force system is in equilibrium. (3)

(Total: 15 marks)

5. A particle of mass m is moving vertically downward in a long column of oil, and experiences a resistive force proportional to the particle's velocity, opposite to the particle's motion:
   $$\boldsymbol{R} = kv\,\mathbf{j},$$
   where $k>0$ is a constant, v is the particle's speed, and $\mathbf{j}$ is a unit vector directed upward. The particle reaches a terminal speed V. The initial speed of the particle is $3V$ downward.

   (a) Using Newton's Second Law, obtain a differential equation for the motion of the particle for $t \geqslant 0$. (2)

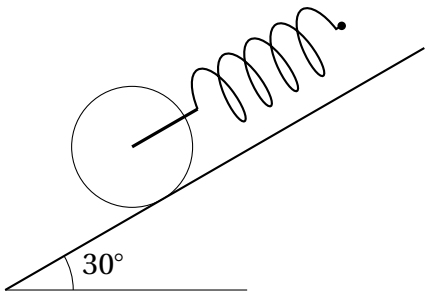
   (b) Express k in terms of m, g and V. (4)

   (c) By separating variables and integrating once with respect to t, show that
   $$v = V(1 + 2e^{-gt/V}). \tag{9}$$

(Total: 15 marks)

6. A uniform solid cylinder of mass 5 kg and radius 10 cm rests on a rough plane inclined at 30° to the horizontal. A light spring of natural length 50 cm and modulus of elasticity 100 N is attached

to the cylinder's axle so that the line of action of the spring is parallel to the plane and the cylinder is free to rotate about its axis of symmetry. The other end of the spring is fixed to a point higher up the plane. The cylinder is released from rest and rolls down the plane, extending the spring until an equilibrium extension x_0 is reached. Assume that the cylinder rolls without slipping.



(a) Determine the value of x_0. (3)

(b) Using conservation of energy, or otherwise, show that if the cylinder is displaced a further distance of 2.5 cm down the plane from the equilibrium position and released from rest, then it oscillates according to the equation:

$$5 = 3\dot{\theta}^2 + 80\theta^2$$

where θ denotes the angle of rotation of the cylinder as measured from the equilibrium position and a dot denotes differentiation with respect to time. (8)

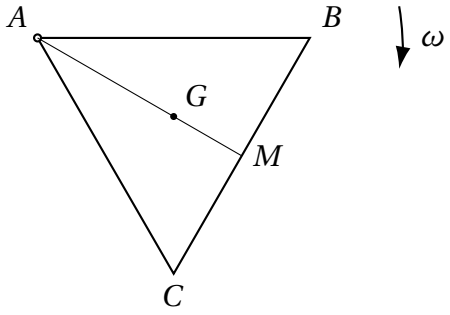
(c) The equation in part (b) can be differentiated to obtain

$$3\ddot{\theta} = -80\theta.$$

Solve for θ. (4)

(**Total: 15 marks**)

7. Three uniform rods, each of length $2a$ and mass $3m$, are joined together to form a rigid equilateral triangular frame ABC where A, B, and C are the vertices. The frame is smoothly pivoted about A so that it can rotate freely in a vertical plane.



(a) Show that the moment of inertia of the frame about the pivot point is $18ma^2$. (4)

(b) Let M be the midpoint of BC. Show that the centre of mass of the frame lies on AM, at a distance of $\frac{2}{3}\sqrt{3}a$ from A. (6)

(c) Initially, the frame is aligned so that AB is horizontal with C below AB. It is released with an angular velocity ω such that B initially moves downwards. Determine the minimum value of ω so that B reaches a point vertically above A. (5)

(**Total: 15 marks**)

8. A smooth elastic sphere P of mass 3 kg is moving with a velocity of $(4u\mathbf{i}+10u\mathbf{j})\,\mathrm{m\,s^{-1}}$ and collides obliquely with another smooth elastic sphere Q having mass 4 kg and moving with a velocity of $-3u\mathbf{i}-7u\mathbf{j}\,\mathrm{m\,s^{-1}}$ before impact, where u is a positive constant. At the moment of collision, $\mathbf{i}$ is a unit vector directed along the line joining the centres of the spheres and $\mathbf{j}$ is a unit vector perpendicular to this line. The coefficient of restitution between the spheres is equal to e. The loss in total kinetic energy due to the impact is $21u^2\,\mathrm{J}$.

   (a) Find the components of the velocities of P and Q along the line joining their centres after the collision, expressing your answers in terms of u and e. (7)

   (b) Hence determine the value of e. (5)

   (c) Calculate the angle that the velocity of P makes with the positive $\mathbf{i}$-direction after the collision. Illustrate your answer with a diagram. (3)

(Total: 15 marks)

9. A uniform square shaped lamina of area mass density σ has side length $\sqrt{\frac{10}{3}}a$. A square hole of side length x is cut out from the lamina so that the centres of the lamina and the hole coincide and their edges are parallel. The perforated lamina is smoothly pivoted at a point P, where P is the midpoint of one side of the square hole. The axis is perpendicular to the plane of the lamina and the lamina is free to move in a vertical plane.

   (a) Show that the moment of inertia of the perforated lamina about its pivot point is

$$I_{\text{Lamina}} = \frac{50a^4\sigma}{27} + \frac{5}{6}a^2\sigma x^2 - \frac{5\sigma x^4}{12}.$$

   Note that the moment of inertia of a square lamina of mass M and side length L about an axis through its centre and perpendicular to its plane is $\frac{1}{6}ML^2$. (5)

   (b) Determine the possible nonzero value(s) of x for which I_{Lamina} has stationary point(s) in terms of a. Clearly justify your working. (5)

   (c) Obtain an expression for the period of small oscillations of the perforated lamina about its *stable* equilibrium position, for the value(s) of x found in part (b). (5)

(Total: 15 marks)

10. A uniform thin rod XY of length 16 cm and mass 0.6 kg lies on a smooth horizontal surface. It is smoothly pivoted at its centre about a fixed axis perpendicular to the plane of the surface. A small block of mass 0.25 kg is projected towards the rod with speed $10\,\mathrm{m\,s^{-1}}$ so that it strikes the rod at a point 4 cm from the centre. At the instant of collision, the block's velocity is perpendicular to the rod. After the impact, the block keeps moving in the same direction but with half its initial speed.

   (a) Show, from first principles, that the moment of inertia of a uniform rod of mass m and length l about an axis through its centre and perpendicular to its length is $\frac{1}{12}ml^2$. (5)

   (b) Determine the impulse exerted by the block on the rod. (2)

   (c) Calculate the angular velocity of the rod immediately after the collision. (4)

   (d) Find the percentage change in the total kinetic energy as a result of the collision. (4)

(Total: 15 marks)