AM 27/I.25M

L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2025 FIRST SESSION

SUBJECT:	**Pure Mathematics**
PAPER NUMBER:	I
DATE:	3rd May 2025
TIME:	09:00 to 12:05

Directions to Candidates

Answer **ALL TEN** questions.
Each question carries 10 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

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1. Solve the differential equation

$$\frac{dy}{dx} = \frac{5x^2y^2 + y^2}{x^2y^5 + 4x^2},$$

given that $y = 2$ when $x = 1$. **[10 marks]**

2. (a) Let $y = \dfrac{1}{\sqrt{x^2+1}}$.

(i) Find $\dfrac{dy}{dx}$ and deduce that $\dfrac{d^2y}{dx^2} = \dfrac{2x^2-1}{(x^2+1)^{5/2}}$.

(ii) Hence show that

$$y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 + y^6 = 0.$$

[3, 2 marks]

(b) A curve has equation $y^3 + 2xy^2 + 5x^2y - x^3 = 7$. Find the equation of the line that is normal to the curve at the point $(1, 1)$. **[5 marks]**

3. Consider the function $f(x) = \dfrac{8x+5}{2x+3}$ defined for $x \in \mathbb{R}$, $x > -\dfrac{3}{2}$.

(a) Show that $f(x) = A + \dfrac{B}{2x+3}$, where A and B are constants to be found. **[3 marks]**

(b) Determine the inverse function $f^{-1}(x)$, giving its domain and range. **[5 marks]**

(c) Solve the equation $f^{-1}(x) = f(-5/4)$. **[2 marks]**

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4. (a) Use partial fractions to find the integral

$$\int \frac{2}{u^2-1}\,\mathrm{d}u.$$

[3 marks]

(b) Use (a) and the substitution $u = e^x$ to find the integral

$$\int \frac{2}{e^x - e^{-x}}\,\mathrm{d}x.$$

[3 marks]

(c) Use (a) and a suitable substitution to find the integral

$$\int \frac{1}{\cos x}\,\mathrm{d}x.$$

[4 marks]

5. The points A and B have position vectors $9\mathbf{i}+\alpha\mathbf{j}+2\mathbf{k}$ and $\beta\mathbf{i}+7\mathbf{j}-4\mathbf{k}$, respectively. The line ℓ_1 has vector equation

$$\mathbf{r} = -2\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}-\mathbf{k}).$$

(a) Let M be the midpoint of the line segment AB. Find α and β given that ℓ_1 passes through the point M. **[4 marks]**

(b) Find the length of the line segment AB. **[2 marks]**

(c) The line ℓ_2 has vector equation

$$\mathbf{r} = \mathbf{i}-5\mathbf{j}+\gamma\mathbf{k}+\mu(\mathbf{i}+\delta\mathbf{j}+4\mathbf{k}).$$

Find γ and δ given that ℓ_2 passes through M and is perpendicular to AB. **[4 marks]**

6. (a) Express $5\sin x + 12\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$, x and α are in radians, and $0 < \alpha < \pi/2$. Give the value of α correct to 2 decimal places. **[3 marks]**

(b) Hence, or otherwise, find the maximum and minimum values of $f(x)$ where

$$f(x) = \frac{30}{5\sin x + 12\cos x + 17}.$$

State also the values of x, in the range $0 < x < 2\pi$, at which they occur. **[4 marks]**

(c) Find $f'(x)$ and solve for x, in the range $0 < x < 2\pi$, the equation $f'(x) = 0$. **[3 marks]**

7. (a) Fifteen identical balls and two identical cubes are placed next to each other in a line. How many different arrangements of these objects are there? **[5 marks]**

   (b) Hence, or otherwise, determine how many different ordered triples (a, b, c) are there such that $a+b+c=15$, where a, b and c are non-negative integers.
   [Note that, for example, the triples $(0,0,15)$, $(0,15,0)$ and $(15,0,0)$ are all different valid triples.] **[5 marks]**

8. (a) If α and β are the roots of the equation $3x^2+2x+4=0$, find the equation whose roots are $1/\alpha$ and $1/\beta$. **[3 marks]**

   (b) Using the Factor Theorem or otherwise, find the factors of

   $$f(x)=2x^3-9x^2+7x+6.$$

   Hence, resolve $1/f(x)$ into partial fractions. **[3, 4 marks]**

9. In this question let $i=\sqrt{-1}$.

   (a) Let $z=a+bi$, where a and b are real numbers. If $z/z^*=c+di$, where c and d are real numbers, prove that $c^2+d^2=1$. **[4 marks]**

   (b) It is given that $3-i$ is a root of the quadratic equation

   $$z^2-(a+bi)z+4(1+3i)=0,$$

   where a and b are real numbers. Find

   (i) the values of a and b,

   (ii) the other root of the quadratic equation, given that it is of the form ki, where k is a real number.

   [3, 3 marks]

10. The line ℓ_1 is given by the equation $4y=3x-12$ and the line ℓ_2 is given by the equation $12y=6-5x$.

    (a) Sketch the two lines on the same set of axes. **[2 marks]**

    (b) Find the coordinates of the point of intersection of the two lines. **[3 marks]**

    (c) Determine the equations of the two straight lines resulting from the locus of the point which is equidistant from the lines ℓ_1 and ℓ_2. **[5 marks]**

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MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2025 FIRST SESSION

SUBJECT:	**Pure Mathematics**
PAPER NUMBER:	II
DATE:	5th May 2025
TIME:	09:00 to 12:05

Directions to Candidates

Answer **SEVEN** questions from **TEN**. Each question carries 15 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

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1. (a) Find the particular solution to the differential equation

$$(x^2+1)\frac{dy}{dx}+3xy=6x,$$

given that $y=4$ when $x=0$. **[7 marks]**

(b) Solve the differential equation

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=10\sin 2x+15\cos 2x,$$

given that $y=1$ and $\frac{dy}{dx}=2$ when $x=0$. Give your answer in the form $y=f(x)$.
[8 marks]

2. (a) Let $f(x)=\sqrt{1+x+x^2}$, where $0<x<1$.

(i) Show that $f''(x)=\dfrac{3}{4(1+x+x^2)^{\frac{3}{2}}}$.

(ii) Hence, or otherwise, obtain the first four non-zero terms in the Maclaurin series for $f(x)$.

(iii) By taking $x=\frac{1}{10}$, use the expansion to find an approximate value of $\sqrt{111}$, giving your answer correct to 2 decimal places.

[4, 3, 2 marks]

(b) The curve $\mathscr{C}_1$ has Cartesian equation given by $x^4+2x^2y^2-2xy+y^4=0$.

(i) Write the equation of $\mathscr{C}_1$ in polar form, giving your answer in the form $r^2=f(\theta)$.

(ii) The curve $\mathscr{C}_2$ has equation $r=\sin 2\theta$. Determine the polar coordinates of the points of intersection of $\mathscr{C}_1$ and $\mathscr{C}_2$ where $0\le\theta\le\frac{\pi}{2}$.

[3, 3 marks]

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3. The functions F and G are defined by $F(x)=\frac{1}{2}(e^x+e^{-x})$ and $G(x)=\frac{1}{2}(e^x-e^{-x})$.

(a) (i) Show that $\frac{\mathrm{d}F}{\mathrm{d}x}=G(x)$ and $\frac{\mathrm{d}G}{\mathrm{d}x}=F(x)$.

(ii) Show that $(G(x))^2+1=(F(x))^2$.

[2, 3 marks]

(b) Find the arc length of the curve of the function $F(x)$ between $x=0$ and $x=\ln 3$. **[4 marks]**

(c) Let $I_n=\int (G(x))^n\,\mathrm{d}x$.

(i) Use the results of part (a) to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[F(x)(G(x))^{n-1}\right]=n(G(x))^n+(n-1)(G(x))^{n-2}.$$

(ii) Hence, or otherwise, show that for $n\geq 2$,

$$nI_n=F(x)(G(x))^{n-1}-(n-1)I_{n-2}.$$

[3, 3 marks]

4. A kiosk offers 15 different types of non-alcoholic beverages. Six customers each order a drink.

(a) How many orders are possible? **[3 marks]**

(b) How many orders are possible in which at least two customers get the same drink? **[5 marks]**

(c) The kiosk offers meal deals, where one can choose any one of the 15 different beverages and any one of 10 different sandwiches. How many possible combination of meal deals are there? **[2 marks]**

(d) The beverages and the sandwiches have different prices. There are 6 beverages costing €1, 5 costing €1.50 and 4 costing €2, whereas there are 6 sandwiches costing €2, 3 costing €3 and 1 costing €4. If a meal deal is chosen at random, what is the probability that it does **not** cost more than €5? **[5 marks]**

5. The line ℓ_1 is given by the vector equation

$$\mathbf{r}=3\mathbf{i}+5\mathbf{j}-4\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+\mathbf{k})$$

and the point A has position vector $\mathbf{i}+3\mathbf{j}-4\mathbf{k}$.

(a) Find the equation of the plane Π_1 that contains both ℓ_1 and A. **[5 marks]**

(b) Find the equation of the plane Π_2 that contains ℓ_1 and is perpendicular to Π_1. **[4 marks]**

(c) Find the equation of the plane Π_3 that passes through A and is perpendicular to ℓ_1. Find the point of intersection of the three planes. **[3, 3 marks]**

6. The complex numbers α and β are given by

$$\frac{\alpha+4}{\alpha}=2-i \quad \text{and} \quad \beta=-3+i\sqrt{3}.$$

(a) Show that $\alpha=2+2i$. Find $|\alpha|$, $|\beta|$, $\arg\alpha$ and $\arg\beta$. **[1, 2 marks]**

(b) Find the modulus and argument of $\alpha\beta$. Illustrate the complex numbers α, β and $\alpha\beta$ on an Argand diagram. **[1, 3 marks]**

(c) Describe the locus of the points on the Argand diagram representing complex numbers z for which $|z-\alpha|=2|z-\beta|$. **[8 marks]**

7. The function f is given by $f(x)=\dfrac{3x^2+4x-15}{6x^2-7x-3}$.

(a) Find where the curve $y=f(x)$ cuts the coordinate axes. **[2 marks]**

(b) Find the stationary points of the curve $y=f(x)$. **[4 marks]**

(c) Determine the equations of the horizontal and vertical asymptotes of the curve $y=f(x)$. **[2 marks]**

(d) Find where the curve cuts the line representing the horizontal asymptote. **[2 marks]**

(e) Sketch the curve of $y=f(x)$ and deduce the range of the function f. **[5 marks]**

8. (Note that angles should be taken in radians in this question.)

(a) Show that the equation $5\cos x - e^x + e^{-x} = 3$ has a solution between 0 and 1. Use the Newton-Raphson method to find an approximate value of this solution, taking 0.5 as a first approximation. Do **TWO** iterations and give your working to four decimal places. **[7 marks]**

(b) (i) Evaluate the integral $\displaystyle\int_0^{0.8} 2\cos\left(e^{-x^2}\right)\mathrm{d}x$ by Simpson's Rule with an interval width of $h=0.2$. Give your answer to four decimal places.

(ii) By using the first two terms of the series expansion of $\cos t$ and taking $t=1-x^2$ (as an approximation to the expression e^{-x^2}), show that $2\cos\left(e^{-x^2}\right)$ may be approximated by the expression $1+2x^2-x^4$.
Use this expression to find another estimate of the integral evaluated in part (i). **[4, 4 marks]**

9. (a) Prove by the principle of mathematical induction that $3n^5+7n$ is divisible by 5 for all $n\geq 1$. **[6 marks]**

(b) (i) Express $\dfrac{n+3}{(n-1)n(n+1)}$ in partial fractions, and deduce by the method of differences that

$$\frac{5}{1\times2\times3}+\frac{6}{2\times3\times4}+\frac{7}{3\times4\times5}+\cdots+\frac{n+3}{(n-1)n(n+1)}=\frac{3}{2}-\frac{n+2}{n(n+1)}.$$

(ii) For the series given in (i), find the sum of the first n terms and the limit of this sum as $n\to\infty$. **[6, 3 marks]**

10. Let

$$\mathbf{A}_k = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & k \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad \text{and} \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

(a) Show that the equation $\mathbf{A}_k\mathbf{x} = \mathbf{b}$ has a unique solution for $\mathbf{x}$ exactly when $k \neq 3$.

[4 marks]

(b) Solve for $\mathbf{x}$ the equation $\mathbf{A}_1\mathbf{x} = \mathbf{b}$, giving your answer in terms of b_1, b_2 and b_3.

[4 marks]

(c) Show that if the equation $\mathbf{A}_3\mathbf{x} = \mathbf{b}$ is consistent then $b_3 = b_1 + b_2$.

[4 marks]

(d) Solve for $\mathbf{x}$ the equation $\mathbf{A}_3\mathbf{x} = \mathbf{b}$ when $b_1 = -b_2 = 1$ and $b_3 = 0$.

[3 marks]