AM 27/I.26M

L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

**ADVANCED MATRICULATION LEVEL**
**2026 FIRST SESSION**

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|---|---|
| SUBJECT: | **Pure Mathematics** |
| PAPER NUMBER: | I |
| DATE: | 2nd May 2026 |
| TIME: | 09:00 to 12:05 |

**Directions to Candidates**

Answer **ALL TEN** questions.
Each question carries 10 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

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1. Solve the differential equation

$$\frac{dy}{dx} = 3xe^{3x+y},$$

given that $y = 0$ when $x = 0$. Give your answer in the form $e^y = f(x)$. **[10 marks]**

2. (a) Let $y = x^2 \sin 2x$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, and deduce that

$$x\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4xy = -6x\sin 2x.$$

**[5 marks]**

(b) A curve has equation $y = x^2 \sin 2x + x$. Find the equation of the tangent line to the curve at the point where $x = \pi$, and find its $x$-intercept. **[5 marks]**

3. Consider the two functions

$$f(x) = e^{2x} - 4, \quad \text{defined for } x \in \mathbb{R}, \text{ and}$$
$$g(x) = \frac{1}{x-11}, \quad \text{defined for } x \in \mathbb{R}, x \neq 11.$$

(a) State the range of $f(x)$ and $g(x)$. **[2 marks]**
(b) Determine the inverse function $f^{-1}(x)$, giving its domain. **[4 marks]**
(c) Solve the equation $(g \circ f)(x) = 1$. **[4 marks]**

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4. (a) Use partial fractions to find the integral

$$\int \frac{2}{(3x-1)(3x+1)}\,\mathrm{d}x\,.$$

**[4 marks]**

(b) Use the substitution $u=\sqrt{x}$ to find the integral

$$\int \frac{9}{\sqrt{x}(9x-1)}\,\mathrm{d}x\,.$$

**[6 marks]**

5. The points $A$ and $B$ have position vectors $5\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $-\mathbf{i}-\mathbf{k}$, respectively. The line $\ell_1$ has vector equation

$$\mathbf{r}=-2\mathbf{i}+\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}-\mathbf{k}).$$

(a) Find the vector equation of the line $\ell_2$ passing through $A$ and $B$. **[3 marks]**

(b) Show that the lines $\ell_1$ and $\ell_2$ intersect. Find the point of intersection and the angle between the two lines. **[4, 3 marks]**

6. (a) The first term of an arithmetic sequence and the first term of a geometric sequence are both equal to 4. The common difference of the arithmetic sequence is 3, and the common ratio of the geometric sequence is $r>0$. Given that the sum of the first $n$ terms of the geometric sequence is twice the sum of the first $n$ terms of the arithmetic sequence, show that

$$1+r+r^2+\cdots+r^{n-1}=\frac{n(5+3n)}{4}.$$

Find $r$ when $n=3$. **[4, 2 marks]**

(b) Find the value of $k$ such that the coefficient of $x^3$ in $(1+kx)^8$ is nonzero and equal to the coefficient of $x^2$. **[4 marks]**

7. (a) Solve for $x$ the equation

$$\ln(x+1)+\ln(2x-1)=\ln(3x).$$

**[4 marks]**

(b) Let

$$g(x)=x^3-3x^2+ax+b\,,$$

where $a$ and $b$ are real numbers. Given that $x-1$ is a factor of both $g$ and its derivative $g'$, find $a$ and $b$, and deduce that $x=1$ is a triple root of the equation $g(x)=0$. Sketch the graph of $y=g(x)$.

**[3, 1, 2 marks]**

8. (a) A University Mathematics Department is organizing an inter-faculty discussion on Mathematical Physics. A committee must be formed from a pool of available staff consisting of 8 mathematicians and 6 physicists. The committee must consist of 5 members in total. Calculate the number of ways this committee can be formed if it must contain:
   (i) exactly 3 mathematicians and 2 physicists;
   (ii) at least 4 mathematicians.

   **[2, 2 marks]**

   (b) Once the committee made up of 3 mathematicians and 2 physicists is chosen, they sit in a single row of 5 chairs on a stage for a panel discussion.
   (i) How many different seating arrangements are possible if the 3 mathematicians must sit together in a block?
   (ii) How many different seating arrangements are possible if the 2 physicists refuse to sit next to each other?

   **[3, 3 marks]**

9. (a) Let $i := \sqrt{-1}$. Find the value of the real numbers $a$ and $b$ given that

$$\frac{-15+10i}{2+i} = a + ib\,.$$

   If $a+ib$ is a root of the equation

$$x^2 + \alpha x + \beta = 0\,,$$

   where $\alpha$ and $\beta$ are real numbers, find the values of $\alpha$ and $\beta$. **[2, 2 marks]**

   (b) Given that $\cos\theta = \frac{3}{5}$ and that $\frac{3\pi}{2} \le \theta \le 2\pi$, show that $\sin 2\theta = -\frac{24}{25}$ and evaluate $\sin 3\theta$.

   **[3, 3 marks]**

10. Let $T_1$ be a magnification (enlargement in both the $x$ and $y$ directions) from the origin by a scale factor of 3. Let $T_2$ be a rotation through 45° anticlockwise about the origin.
   (a) Write down the matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ associated with $T_1$ and $T_2$, respectively. **[4 marks]**
   (b) Find the matrix $\mathbf{M}$ for the compound transformation where $T_1$ is followed by $T_2$.

   **[2 marks]**

   (c) A point $P'$ has coordinates $(0, 6\sqrt{2})$ after undergoing the compound transformation represented by $\mathbf{M}$. Use the inverse matrix $\mathbf{M}^{-1}$ to find the original coordinates of the point $P$. **[4 marks]**

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**ADVANCED MATRICULATION LEVEL**
**2026 FIRST SESSION**

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|---|---|
| SUBJECT: | **Pure Mathematics** |
| PAPER NUMBER: | II |
| DATE: | 4th May 2026 |
| TIME: | 09:00 to 12:05 |

**Directions to Candidates**

Answer **SEVEN** questions from **TEN**. Each question carries 15 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

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1. (a) Find the particular solution to the differential equation

$$\sin x \frac{dy}{dx} = \sin x \sin 2x + y \cos x,$$

given that $y = 3/2$ when $x = \pi/6$. Find the exact value of $y$ at $x = \pi/4$. **[7 marks]**

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 32x^2,$$

given that $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$. Give your answer in the form $y = f(x)$. **[8 marks]**

2. Consider the function $f(x)$ defined by: $f(x) = (1-x)e^{2x}$.

(a) Find the first, second, and third derivatives of $f(x)$. **[3 marks]**

(b) Obtain the Maclaurin series for $f(x)$ up to and including the term in $x^3$. **[3 marks]**

(c) Using mathematical induction, show that the $n^{\text{th}}$ derivative of $f(x)$ is given by

$$f^{(n)}(x) = 2^{n-1}e^{2x}(2-2x-n).$$

**[6 marks]**

(d) Write down the general term of the Maclaurin series for $f(x)$ in terms of $x^n$. **[3 marks]**

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3. (a) (i) Show that $\displaystyle\int_0^{\pi/2} \sin^n x \cos^3 x \, \mathrm{d}x = \frac{1}{n+1} - \frac{1}{n+3}$ for $n \geq 0$.
[Hint: Write $\cos^3 x$ as $\cos^2 x \cos x$ and use the identity $\sin^2 x + \cos^2 x = 1$.]

(ii) The region bounded by the curve $y = \sin^{3/2} x \cos^{3/2} x$ and the $x$-axis between $x = 0$ and $x = \pi/2$ is rotated through $2\pi$ radians about the $x$-axis. Find the volume of the solid that is generated by this rotation. **[5, 3 marks]**

(b) A function $y$ is defined by $y = 9x^{7/3} + \dfrac{1}{28x^{1/3}}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, and show that $1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \left(21x^{4/3} + \dfrac{1}{84x^{4/3}}\right)^2$.

(ii) The part of the curve of this function $y$ between $x = a$ and $x = b$ is rotated by $2\pi$ radians about the $x$-axis (where $a, b > 0$). Show that the area $A$ of the surface of revolution so formed is given by

$$A = \pi \left[ 81x^{14/3} + \frac{6x^2}{7} - \frac{1}{784x^{2/3}} \right]_{x=a}^{x=b}.$$

**[3, 4 marks]**

4. A research team in a department is developing a new periodic signal for a secure communication protocol. The signal consists of a sequence of 8 high-frequency pulses: 3 identical "alpha" pulses ($\alpha$); 3 identical "beta" pulses ($\beta$); and 2 identical "gamma" pulses ($\gamma$).

(a) These 8 pulses are arranged in a circular loop around a rotating sensor.

(i) Find the number of distinct circular arrangements of these 8 pulses.

(ii) If the circular arrangement is restricted such that the two $\gamma$ pulses must be positioned directly opposite each other (separated by exactly 3 other pulses on either side), how many distinct circular arrangements are possible?

**[2, 2 marks]**

(b) From the set of 8 pulses (three $\alpha$, three $\beta$ and two $\gamma$), a random sample of 4 pulses is selected for interference testing.

(i) Find the total number of ways of selecting 4 pulses at random from the original 8.

(ii) Explain why it is impossible that this sample contains no $\alpha$ pulse **AND** no $\gamma$ pulse.

(iii) Find the probability that this sample contains no $\alpha$ pulse.

(iv) Find the probability that this sample contains at least one $\alpha$ pulse and at least one $\gamma$ pulse.

**[2, 2, 2, 5 marks]**

5. The line $\ell_1$ is given by the vector equation

$$\mathbf{r} = \mathbf{i} - \mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

and the point $A$ has position vector $3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$.

(a) Find the equation of the line $\ell_2$ that passes through $A$, intersects $\ell_1$ and is perpendicular to it. Find the point of intersection $B$ of the two lines. **[5 marks]**

(b) Find the equation of the plane $\Pi$ that contains both $\ell_1$ and $\ell_2$. **[5 marks]**

(c) The point $C$ has position vector $\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$. Show that $C$ does not lie on the plane $\Pi$. Find the equation of the line $\ell_3$ that passes through $C$ and is perpendicular to $\Pi$. Find the point $D$ where $\ell_3$ intersects $\Pi$. **[5 marks]**

6. Let $P(x, y)$ be the point on the Argand diagram representing the complex number $z = x + iy$ and satisfying the equation

$$\mathrm{Re}\left(\frac{z+ia}{z+b}\right) \geq k,$$

where $a$, $b$ and $k$ are real numbers.

(a) Show that the locus of $P(x, y)$ can be described by the equation

$$(k-1)x^2+(k-1)y^2+(2k-1)bx-ay+kb^2 \leq 0.$$

**[5 marks]**

(b) Show that when $k \neq 1$ the locus of $P(x, y)$ is a disc. Find the radius and the coordinates of the centre of this disc in terms of $a$, $b$ and $k$.

**[5 marks]**

(c) Find the maximum value of $|z|$ when $\mathrm{Re}\left(\dfrac{z+i}{z-1}\right) \geq 2$.

**[3 marks]**

(d) Describe the locus of $P(x, y)$ when $k = 1$ and $a > 0$. **[2 marks]**

7. The function $f$ is given by $f(x) = \dfrac{2x^2+9x-18}{x^2-18x+81}$.

(a) Express $f(x)$ in the form $f(x) = a + \dfrac{45(x-b)}{(x-c)^2}$, where $a, b$ and $c$ are positive integers to be found. Hence, or otherwise, determine the equations of the horizontal and vertical asymptotes of the curve $y = f(x)$. **[4 marks]**

(b) Find where the curve $y = f(x)$ cuts the coordinate axes. **[2 marks]**

(c) Find the stationary points of the curve $y = f(x)$. **[4 marks]**

(d) Find where the curve cuts the line representing the horizontal asymptote. **[1 marks]**

(e) Sketch the curve of $y = f(x)$ and deduce the range of the function $f$. **[4 marks]**

8. (a) Show that the equation $5e^{0.9x} - x^2 = 10$ has a solution between 0 and 1. Use the Newton-Raphson method to find an approximate value of this solution, taking 1 as a first approximation. Do **TWO** iterations and give your working to four decimal places.

**[6 marks]**

(b) (i) Evaluate the integral $\displaystyle\int_{0.6}^{1} \left(\frac{6e^{-x}}{x^2} + x\right) \mathrm{d}x$ by Simpson's Rule with an interval width of $h = 0.1$. Give your answer to two decimal places.

(ii) By using the series expansion of $e^{-x}$ up to and including the term in $x^4$, show that $\dfrac{6e^{-x}}{x^2} + x$ may be approximated by the expression $3 + \dfrac{6}{x^2} - \dfrac{6}{x} + \dfrac{x^2}{4}$.
Use this expression to find another estimate of the integral evaluated in part (i).

**[4, 5 marks]**

9. (a) Prove by the principle of mathematical induction that for every positive integer $n$

$$\sum_{r=1}^{n} r \times r! = (n+1)! - 1 .$$

**[7 marks]**

(b) Express $\dfrac{1}{(r+1)(r+2)(r+3)}$ in partial fractions, and deduce by the method of differences that

$$\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)},$$

where $a$, $b$ and $c$ $(a < b)$ are integers to be found. **[2, 6 marks]**

10. Let

$$\mathbf{A} = \begin{pmatrix} -8 & -5 & -5 \\ 5 & 2 & 5 \\ 5 & 5 & 2 \end{pmatrix}.$$

(a) Solve the equation

$$(\mathbf{A} - 2\mathbf{I})\mathbf{x} = \mathbf{0},$$

where $\mathbf{I}$ is the $3 \times 3$ identity matrix.

**[5 marks]**

(b) Show that $(\mathbf{A} - 2\mathbf{I})(\mathbf{A} + 3\mathbf{I}) = \mathbf{0}$. Hence, or otherwise, find $\mathbf{A}^{-1}$.

**[2, 2 marks]**

(c) Find the equation of the plane $\Pi$ whose image under the transformation $\mathbf{A}$ is the plane with Cartesian equation $x + y - z = 6$.

**[6 marks]**