AM 27/I.25S

L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2025 SECOND SESSION

SUBJECT:	**Pure Mathematics**
PAPER NUMBER:	I
DATE:	29th August 2025
TIME:	09:00 to 12:05

Directions to Candidates

Answer **ALL TEN** questions.
Each question carries 10 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

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1. Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\ln x}{x y^2},$$

given that $y = 1$ when $x = e$. Give your final answer in the form $y = f(x)$.
[Hint: Use a suitable substitution to solve the integral w.r.t. x.] **[10 marks]**

2. (a) Let $y = \sqrt{x^2 - 1}$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$, and show that

$$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1.$$

[5 marks]

(b) A curve has equation $2y^3 + 3xy^2 - x^2y + 5x^3 = 9$. Find the equation of the line that is tangent to the curve at the point $(-1, 3)$. **[5 marks]**

3. The point A has position vector $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(a) Write down the equation of the line ℓ_1 that passes through A and has direction vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. **[1 marks]**

(b) Find the position vectors of the two points B and C on the line ℓ_1 that are at a distance 6 from A. **[4 marks]**

(c) Let D be the point with position vector $5\mathbf{i} + \mathbf{j} - \mathbf{k}$. Find the angle $\angle BDC$ and the area of the triangle BDC. **[5 marks]**

[The formula for the area of the triangle is $\frac{1}{2}ab\sin\theta$, where a and b are the lengths of two sides of the triangle, and θ is the angle between these two sides.]

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4. Two linear transformations T_1 and T_2 are respectively represented by the matrices **A** and **B** given hereunder.

$$\mathbf{A} = \frac{1}{2}\begin{pmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

(a) Describe the two transformations geometrically. **[4 marks]**

(b) Find the matrix **C** representing the compound transformation obtained by applying T_1 followed by T_2. **[3 marks]**

(c) Hence, deduce the matrix **D** representing the linear transformation T_3 which should follow T_1 and T_2 (in that order) to give the identity transformation. **[3 marks]**

5. (a) Use integration by parts to find the integral

$$\int \cos(\ln x)\,\mathrm{d}x\,.$$

[7 marks]

(b) Use a suitable substitution to evaluate the integral

$$\int_0^1 2x\cos(x^2)\,\mathrm{d}x\,.$$

[3 marks]

6. (a) Find the positive integers that simultaneously satisfy the inequalities

$$0 < \frac{5}{2x-3} \leq \frac{3}{8} \quad \text{and} \quad 3x - 35 < 0\,.$$

[3 marks]

(b) Find strictly positive real numbers x and y that simultaneously satisfy

$$8^x = 2^{x+y} \quad \text{and} \quad \log_x y = 3\,.$$

[3 marks]

(c) Find the values of $r \in \mathbb{R}$ such that $r(x+2)^2 = (x-1)(x-2)$ for only one value of x. **[4 marks]**

7. (a) A student reading a 426-page book finds that she reads faster as she gets into the subject. She reads 19 pages on the first day, and her rate of reading then goes up by 3 pages each day. How long does she take to finish the book? **[4 marks]**

(b) A post is being driven into the ground by a mechanical hammer. The distance it is driven by the first blow is 8 cm. Subsequently, the distance it is driven by each blow is 9/10 of the distance it was driven by the previous blow.

(i) The post is to be driven a total distance of at least 70 cm into the ground. Find the smallest number of blows needed.

(ii) Explain why the post can never be driven a total distance of more than 80 cm into the ground. **[5, 1 marks]**

8. A number of red bags and blue bags are distributed among six people with the condition that no one gets two or more bags of the same colour. This means that any person can receive either a red bag only, or a blue bag only, or both a red bag and a blue bag, or no bags at all.
   (a) In how many different ways can the distribution happen? **[4 marks]**
   (b) It is now given that at least one of the people got a red bag, at least one got a blue bag, and at least one got both a red and a blue bag (possibly the same person). In how many ways could the distribution have happened? **[6 marks]**

   [One may assume that there are sufficient bags for any possible distribution.]

9. Consider the two functions

$$f(x) = 3 + \sqrt{x-2}, \quad \text{defined for } x \in \mathbb{R}, x > 2, \text{ and}$$

$$g(x) = \frac{15}{x-3}, \quad \text{defined for } x \in \mathbb{R}, x \neq 3.$$

   (a) Determine the inverse function $f^{-1}(x)$, giving its domain and range. **[4 marks]**
   (b) Determine the composite function $(g \circ f)(x)$ giving its domain. **[3 marks]**
   (c) Solve the equation $(g \circ f)(x) = f(89/4)$. **[3 marks]**

10. (a) Find the equation of the straight line joining the points $A(0, 3/2)$ and $B(3, 0)$. **[3 marks]**
    (b) Express $\sin\theta + 2\cos\theta$ in the form $r\sin(\theta + \alpha)$, where r is a positive number, θ and α are in degrees, and α is an acute angle. **[3 marks]**
    (c) The point Q lies on the line segment AB. Let O denote the origin $(0,0)$ and let θ be the angle $\angle AOQ$, measured in degrees. Given that $|OQ| = 2$, show that $2\sin\theta + 4\cos\theta = 3$. Calculate the value of θ which satisfies this equation. **[4 marks]**

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EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2025 SECOND SESSION

SUBJECT:	**Pure Mathematics**
PAPER NUMBER:	II
DATE:	1st September 2025
TIME:	09:00 to 12:05

Directions to Candidates

Answer **SEVEN** questions from **TEN**. Each question carries 15 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

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1. (a) (i) Use the substitution $u = \sin x$ to find the integral $\displaystyle\int \frac{\cos x}{\sin^2 x}\,\mathrm{d}x$.

(ii) Hence, find the particular solution to the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\cos x}{\sin x}y = \sin 2x\,,$$

given that $y = 2$ when $x = \frac{\pi}{2}$. Give your answer in the form $y = f(x)$. **[2, 6 marks]**

(b) Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 8\frac{\mathrm{d}y}{\mathrm{d}x} + 16y = 24e^{4x}\,,$$

given that $y = -1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -4$ when $x = 0$. Write your final answer in the form $y = (Ax^2 + B)e^{Cx}$, where A, B and C are integers that need to be found.
[When finding the particular integral, use px^2e^{4x} as a trial solution.]

[7 marks]

2. (Note that angles should be taken in radians throughout this question.)

(a) Show that the equation $8\sin x + 2x^2 - x^4 = 3$ has a solution between 0 and 1. Use the Newton-Raphson method to find an approximate value of this solution, taking 0 as a first approximation. Do **TWO** iterations and give your working to four decimal places.

[7 marks]

(b) (i) Evaluate $\displaystyle\int_{0.2}^{1} \sin 3x\left(e^x + e^{-x}\right)\mathrm{d}x$ by Simpson's Rule with an interval width of $h = 0.2$. Give your answer to four decimal places.

(ii) Given that

$$\int e^{ax}\sin 3x\;\mathrm{d}x = \frac{1}{a^2+9}(a\sin 3x - 3\cos 3x)e^{ax}$$

for any real number a, evaluate the integral in part (i) to four decimal places.

[5, 3 marks]

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3. (a) Show that

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

is the matrix corresponding to the linear transformation T that you get on $\mathbb{R}^3$ if you first reflect in the xy-plane, then rotate about the z-axis by 60^o (counter clockwise when watching in the direction of the z-axis), and finally reflect in the xz-plane.

[8 marks]

(b) Show that the plane $\sqrt{3}x - y = 0$ is invariant under the transformation T.

[4 marks]

(c) Find the vector equation of the image of the y-axis under the transformation T.

[3 marks]

4. (a) (i) Show that $\tan^2 x + 1 = \sec^2 x$.

(ii) Let $I_n = \int \tan^n x \, \mathrm{d}x$. Show that,

$$I_n = \frac{1}{n-1}\tan^{n-1} x - I_{n-2}\,.$$

[Hint: Write $\tan^n x$ as $\tan^{n-2} x \tan^2 x$.]

(iii) The region bounded by the curve $y = \tan^3 x$ and the x-axis between $x = 0$ and $x = \pi/4$ is rotated through 2π radians about the x-axis. Find the volume of the solid that is generated by this rotation. **[1, 5, 3 marks]**

(b) A function y is defined by $y = \dfrac{x^5}{15} + \dfrac{1}{4x^3}$.

(i) Evaluate $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, and show that $1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \left(\dfrac{x^4}{3} + \dfrac{3}{4x^4}\right)^2$.

(ii) The part of the curve of this function y between $x = 1$ and $x = 2$ is rotated by 2π radians about the x-axis. Find the area of the surface of revolution so formed.

[3, 3 marks]

5. Let $i = \sqrt{-1}$.

(a) (i) Find the least value of $|z|$ if $|z - 3 + i| = |z + 2 + 5i|$.

(ii) Find the greatest value of $\arg z$ if $|z - 5| = 3$.

[4, 3 marks]

(b) Using Euler's identity $e^{i\theta} = \cos\theta + i\sin\theta$, prove that if $z = \cos\theta + i\sin\theta$ then

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad \text{and} \qquad z^n - \frac{1}{z^n} = 2i\sin n\theta\,,$$

for any integer n. Hence show that

$$\cos^4\theta + \sin^4\theta = \frac{1}{4}(\cos 4\theta + 3).$$

[4, 4 marks]

6. (a) A survey was carried out with one thousand people on their use of three social media platforms, namely Whatsapp, Facebook and Instagram. The following is a list of obtained results:
   - 800 use Whatsapp;
   - 525 use Facebook;
   - 375 use Instagram;
   - 550 use exactly two types of social media platforms;
   - 50 use none.

   How many of those surveyed use all the three types of social media platforms? **[5 marks]**

   (b) A binary string of length n is generated by placing a total of n zeros and ones next to each other such that the string cannot start nor end with a zero and it cannot contain two zeros next to each other.
   (i) Find the possible number of such binary strings of length n that can be formed for each $n \in \{1,2,3,4,5\}$.
   (ii) Deduce the possible number of such binary strings of length $n = 10$.

   [5, 5 marks]

7. (a) Let the curve $\mathscr{C}$ be given by the polar equation $r = \sqrt{2\theta \sin(2\theta)}$ for $0 \leq \theta \leq \pi/2$.
   (i) By taking values of θ at intervals of $\pi/12$ radians, sketch the curve $\mathscr{C}$.
   (ii) Find the area bounded by the curve $\mathscr{C}$.

   [5, 4 marks]

   (b) Derive the Maclaurin series of $f(x)$, where $f(x) = x(\sin x + \cos x)$, giving the first three non-zero terms. **[6 marks]**

8. (a) Express $\dfrac{8}{(2n-1)(2n+1)(2n+3)}$ in partial fractions, and show by the method of differences that

$$\frac{1}{1\times 3\times 5}+\frac{1}{3\times 5\times 7}+\frac{1}{5\times 7\times 9}+\cdots+\frac{1}{(2n-1)(2n+1)(2n+3)}=\frac{1}{12}-\frac{1}{4(2n+1)(2n+3)}.$$

   [7 marks]

   (b) If $\mathbf{M} = \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}$, use the principle of mathematical induction to prove that

$$\mathbf{M}^n = \begin{pmatrix} -3n+1 & 9n \\ -n & 3n+1 \end{pmatrix},$$

   for any positive integer n.

   [8 marks]

9. The function f is given by $f(x) = \dfrac{4x^2-2x+15}{9x-5}$.
   (a) Find where the curve $y = f(x)$ cuts the coordinate axes. **[2 marks]**
   (b) Find the stationary points of the curve $y = f(x)$ and determine their nature. **[4 marks]**
   (c) Determine the equations of any asymptotes of the curve $y = f(x)$. **[5 marks]**
   (d) Sketch the curve of $y = f(x)$. **[4 marks]**

10. The planes Π_1 and Π_2 have equations $2x+3y-z=22$ and $x+2y-z=12$, respectively.

    (a) Find the equation of the line ℓ_1 where the planes intersect. **[4 marks]**

    (b) Show that ℓ_1 also lies in the plane Π_3 with equation $3x-y+4z=22$. **[4 marks]**

    (c) The point A has position vector $5\mathbf{i}+4\mathbf{j}+\mathbf{k}$. Show that A lies on the plane Π_2, and write down the equation of the line ℓ_2 passing through A and perpendicular to Π_1. **[3 marks]**

    (d) Find the position vector of the point B where ℓ_2 intersects Π_3. Show that the distance from A to B is $7\sqrt{14}$. **[4 marks]**