AM 27/I.26S

L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2026 SECOND SESSION

SUBJECT:	**Pure Mathematics**
PAPER NUMBER:	I
DATE:	31st August 2026
TIME:	09:00 to 12:05

Directions to Candidates

Answer **ALL TEN** questions.
Each question carries 10 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

---

1. (a) Express $\dfrac{5}{(2x-1)(x-3)}$ into partial fractions. **[3 marks]**

   (b) Solve the differential equation

$$5\frac{\mathrm{d}y}{\mathrm{d}x} = 2y^2 - 7y + 3,$$

   given that $y = -2$ when $x = 0$. Give your final answer in the form $y = f(x)$. **[7 marks]**

2. (a) Let $y = e^{\tan x}$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$, and show that

$$\cos^2 x \frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 2y\tan x = 0.$$

   [5 marks]

   (b) A curve has equation $y^3 + 2xy^2 - 3x^2y + x^3 = 5$. Find the equation of the line that is normal to the curve at the point $(1,-1)$. **[5 marks]**

3. The points A, B and C have position vectors $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $4\mathbf{i} - \mathbf{j} + \mathbf{k}$, and $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ respectively.

   (a) Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$. **[3 marks]**

   (b) Find the vector equation of the line ℓ that passes through A and C. **[2 marks]**

   (c) Find the position vector of the point D on the line ℓ such that the line through B and D is perpendicular to ℓ. Hence find the perpendicular distance of B from ℓ. **[5 marks]**

©The MATSEC Examinations Board reserves all rights on the examination questions in all examination papers set by the said Board.

4. (a) Consider the quadratic equation

$$a x^2 + b x + c = 0, \qquad (\star)$$

where a, b and c are real numbers, and $a \neq 0$.

(i) Use the method of completing the square to show that for equation $(\star)$ to have real roots, one must have $b^2 - 4ac \geq 0$.

(ii) Deduce that if a, b and c are all greater than zero, and $b^2 - 4ac \geq 0$, then equation $(\star)$ has roots that are less than zero.

[4, 3 marks]

(b) Show that the following quadratic equation in x

$$(e^u - 5)x^2 + 5e^u x + (4e^u + 1) = 0,$$

has real and distinct roots for every real value u. **[3 marks]**

5. (a) Use the substitution $u = 1 - x$ to find the integral

$$\int x\sqrt{1-x}\,\mathrm{d}x.$$

[5 marks]

(b) Use integration by parts to find the integral

$$\int -e^{-x}(1-x)^2\,\mathrm{d}x.$$

[5 marks]

6. (a) Prove that

$$\cos 3x = 4\cos^3 x - 3\cos x.$$

[3 marks]

(b) Let θ satisfy the equation

$$(4\cos^2\theta - 3)(4\cos^2 3\theta - 3) = 1. \qquad (\sharp)$$

(i) Show that $\cos\theta \neq 0$ and $\cos 3\theta \neq 0$.

(ii) Find the solution of equation $(\sharp)$, given that $0 \leq \theta \leq \pi$.

[3, 4 marks]

7. (a) If $x - 2$ is a factor of

$$f(x) = x^3 + a x^2 - 4x + 4,$$

find the value of a. Hence, express $f(x)$ as a product of three linear factors and resolve

$$\frac{1}{f(x)}$$

into partial fractions. **[5 marks]**

(b) Expand

$$\ln\left(\frac{1+x}{1-x}\right)$$

as a series of ascending powers of x up to and including the term in x^5. Hence, or otherwise, show that

$$\ln 3 \approx \frac{263}{240}.$$

[5 marks]

8. Consider the circle $\mathscr{C}_1$ given by the equation

$$x^2+y^2-6x+4y-12=0.$$

(a) Write down the parametric coordinates of any point on $\mathscr{C}_1$ in terms of an angle θ. **[3 marks]**

(b) Find the equations of the tangents to $\mathscr{C}_1$ that are parallel to the line $3x-4y+5=0$. **[4 marks]**

(c) A second circle $\mathscr{C}_2$ has equation $(x-10)^2+(y-5)^2=r_2^2$. Find the value of r_2 such that $\mathscr{C}_1$ and $\mathscr{C}_2$ touch each other externally. **[3 marks]**

9. Consider the two functions

$$f(x)=\frac{2x+3}{2x-3}, \quad \text{defined for } x\in\mathbb{R}, x\neq 3/2, \text{ and}$$
$$g(x)=x^2+2, \quad \text{defined for } x\in\mathbb{R}.$$

(a) State the range of $f(x)$ and $g(x)$. **[2 marks]**

(b) Determine the composite function $(f\circ g)(x)$ as a simplified algebraic fraction, giving its domain. **[4 marks]**

(c) Determine the inverse function $f^{-1}(x)$ as a simplified algebraic fraction, giving its domain. **[4 marks]**

10. A linear transformation M is defined by the matrix

$$\mathbf{M}=\begin{pmatrix} k & 2 \\ 2 & k \end{pmatrix},$$

where k is a real constant. Let ℓ be the line having equation $y=mx$, where m is also a real constant.

(a) Given that the transformation M maps points on ℓ to other points also on ℓ, find the possible value/s of the gradient m. **[5 marks]**

(b) Take $k=0$. Describe geometrically the transformation represented by **M** and hence, write **M** as the product of two matrices (each representing a linear transformation not equal to the identity). **[3, 2 marks]**

L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

ADVANCED MATRICULATION LEVEL
2026 SECOND SESSION

SUBJECT:	**Pure Mathematics**
PAPER NUMBER:	II
DATE:	1st September 2026
TIME:	09:00 to 12:05

Directions to Candidates

Answer **SEVEN** questions from **TEN**. Each question carries 15 marks.
Graphical calculators are **not** allowed however scientific calculators can be used but all necessary working must be shown.

---

1. (a) Show that the particular solution to the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=9x(x^3+1)^{1/2},$$

given that $y=27/2$ when $x=2$, is

$$y=\frac{a}{x^2}\left(x^3+1\right)^{3/2},$$

where a is a positive integer to be found.
[One may need to use the substitution $u=x^3+1$ when solving the integral w.r.t. x.]

[7 marks]

(b) Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+5y=34\cos 2x,$$

given that $y=18$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=0$. Give your answer in the form $y=f(x)$.

[8 marks]

2. (Note that angles should be taken in radians throughout this question.)
(a) Show that the equation $\cos 4x=x^3-1$ has a solution between 0 and 1. Use the Newton-Raphson method to find an approximate value of this solution, taking 0.5 as a first approximation. Do **TWO** iterations and give your working to four decimal places.

[8 marks]

(b) Evaluate $\displaystyle\int_1^2 \sin(x)\ln(1+x^4)\,\mathrm{d}x$ by Simpson's Rule with an interval width of $h=0.25$. Give your answer to two decimal places.

[7 marks]

©The MATSEC Examinations Board reserves all rights on the examination questions in all examination papers set by the said Board.

3. (a) Given that

$$\mathbf{A}=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 2 & 3 & 5\end{pmatrix},$$

find $\mathbf{A}^{-1}$. Hence, solve the equation $\mathbf{Ax}=\mathbf{y}$ when $\mathbf{y}=\begin{pmatrix}2\\ -1\\ 0\end{pmatrix}$.

[5, 2 marks]

(b) Let $\mathbf{T}=\begin{pmatrix}4 & 3\\ 2 & 5\end{pmatrix}$ and $P(a,ka)$ be a point on the line $y=kx$.

(i) $P'(x',y')$ is the image of $P(a,ka)$ under the transformation $\mathbf{T}$. Find the coordinates of P' in terms of a and k.

(ii) $\mathbf{T}$ maps the line $y=kx$ onto another straight line. Using your answer to part (i), find the equation of this straight line.

(iii) Find the Cartesian equation of the straight lines, passing through the origin, that $\mathbf{T}$ maps onto themselves. **[2, 2, 4 marks]**

4. (a) (i) Let $I_n=\displaystyle\int_0^2 x^n e^{-2x}\,\mathrm{d}x$ for $n\geq 0$. Evaluate I_0, and show that, for $n\geq 1$,

$$I_n=-\frac{2^{n-1}}{e^4}+\frac{n}{2}I_{n-1}.$$

(ii) The region bounded by the curve $y=xe^{-x}$ and the x-axis between $x=0$ and $x=2$ is rotated through 2π radians about the x-axis. Find the volume of the solid that is generated by this rotation. **[4, 5 marks]**

(b) A function y is defined by $y=-\ln(\cos x)$ for $-\pi/2<x<\pi/2$. Find $1+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$, and hence evaluate the arc length of the curve of this function between $x=\pi/6$ and $x=\pi/3$.

[3, 3 marks]

5. Let $i=\sqrt{-1}$.

(a) Find the least value of $|z|$ if $|z-(1+i)|=|z-(5-i)|$.

[4 marks]

(b) Find the largest value of $\arg z$ if $\sqrt{3}\ |z-(2+i)|=\sqrt{5}$.

[4 marks]

(c) Using Euler's identity $e^{i\theta}=\cos\theta+i\sin\theta$, prove that if $z=\cos\theta+i\sin\theta$ then

$$z^n+\frac{1}{z^n}=2\cos n\theta \qquad \text{and} \qquad z^n-\frac{1}{z^n}=2i\sin n\theta,$$

for any integer n. Show that

$$\cos 4\theta=\cos^4\theta+\sin^4\theta-6\cos^2\theta\sin^2\theta.$$

Deduce that

$$\cos^4\theta+\sin^4\theta=\frac{1}{4}(\cos 4\theta+3),$$

and use this identity to find the general solution of the following equation

$$2\cos^4\theta-3\cos^2 2\theta+2\sin^4\theta+3\sin^2 2\theta=0.$$

[2, 2, 1, 2 marks]

6. A group of 100 mathematics students were surveyed regarding their use of three digital tools: Mathematical Software (S), a Graphing Calculator (C), and a Digital Tablet (T). The following data was collected:
   - The total number of students using S is 45, using C is 50, and using T is 30.
   - Exactly 20 students use exactly two of the three tools.
   - The number of students who use only S **AND** C is twice the number of those who use all three tools.
   - The number of students who use only C **AND** T is the same as the number of those who use only S **AND** T.
   - The number of students who use none of the tools is equal to the number of students who use all three tools.

   (a) Let x represent the number of students who use all three tools, and y represent the number of students who use only C **AND** T. Formulate a system of equations to find the values of x and y. **[6 marks]**

   (b) Complete a Venn diagram showing the number of students in each of the eight regions. **[3 marks]**

   (c) A student is chosen at random. Find the probability that the student uses:
   (i) at most one tool;
   (ii) a Graphing Calculator, given that at least two tools are used.
   [2, 4 marks]

7. (a) Show that $(r+1)^2-(r-1)^2=4r$, and hence show by the method of differences that

$$\sum_{r=1}^{n} 4r = a(n^2+n),$$

   where a is an integer to be found. **[7 marks]**

   (b) Use the principle of mathematical induction to prove that

$$\sum_{r=1}^{n} r\times 2^{-r} = 2-(n+2)2^{-n}$$

   for every integer $n \geq 1$.
   [8 marks]

8. The function f is given by $f(x)=\dfrac{x^2-2x-8}{x-6}$, for $x\in\mathbb{R}$, $x\neq 6$.

   (a) Express $f(x)$ in the form $f(x)=(x+a)+\dfrac{b}{(x-6)}$, where a and b are positive integers to be found. Hence, or otherwise, determine the equations of the two asymptotes of the curve $y=f(x)$. **[4 marks]**

   (b) Find the coordinates of the point where the two asymptotes intersect. **[1 marks]**

   (c) Find where the curve $y=f(x)$ cuts the coordinate axes. **[2 marks]**

   (d) Find the stationary points of the curve $y=f(x)$. **[4 marks]**

   (e) Sketch the curve of $y=f(x)$ and deduce the range of the function f. **[4 marks]**

9. Let $f(\theta) = \left(\theta \sin(\frac{\theta}{2})\right)^2$. Let the curve $\mathscr{C}_1$ be given by the polar equation $r = f(\theta)$ for $0 \leq \theta \leq 2\pi$.
   (a) By taking values of θ at intervals of $\frac{\pi}{6}$ radians, sketch the curve $\mathscr{C}_1$. **[4 marks]**
   (b) On the same diagram, sketch the curve of $\mathscr{C}_2$ given by $r = \frac{1}{2}\theta^2$ for $0 \leq \theta \leq 2\pi$. **[3 marks]**
   (c) Find the polar coordinates of the points of intersection between $\mathscr{C}_1$ and $\mathscr{C}_2$. **[4 marks]**
   (d) Find the area enclosed by the curve $\mathscr{C}_2$ between $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$. **[4 marks]**

10. The points A, B and C have position vectors $4\mathbf{i}-3\mathbf{j}-5\mathbf{k}$, $\mathbf{j}-\mathbf{k}$ and $6\mathbf{i}-2\mathbf{j}+2\mathbf{k}$, respectively.
   (a) Find the equation of the plane Π_1 passing through A, B and C. **[5 marks]**
   (b) Find the equation of the plane Π_2 that is perpendicular to the plane Π_1, and passes through C and the midpoint of A and B. **[5 marks]**
   (c) The point D has position vector $\alpha\mathbf{i}+\beta\mathbf{j}-2\mathbf{k}$. Find α and β given that D lies on the plane Π_2 and is equidistant from A and C.
   [Hint: Note that distance is never negative and one can use $|\overrightarrow{AD}|^2 = |\overrightarrow{CD}|^2$.] **[5 marks]**