L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2025 FIRST SESSION

SUBJECT:	**Pure Mathematics**
DATE:	3rd May 2025
TIME:	9:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are **10** questions in all.
The total number of marks for all the questions in the paper is 100.
Graphical calculators are **not** allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.

1. Let $f(x) = 6x^3 + 5x^2 - 2x - 1$.
   (a) Use the Remainder Theorem to show that $x+1$ is a factor of $f(x)$. Factorise $f(x)$ completely.

   [5 marks]

   (b) Express into partial fractions $\dfrac{3+15x-12x^2}{f(x)}$ and show that

   $$\int \frac{3+15x-12x^2}{f(x)}\,\mathrm{d}x = \ln\left(\frac{|6x^2-x-1|}{(x+1)^4}\right) + k,$$

   where k is a constant.

   [5 marks]

   [**Total: 10 marks**]

2. It is thought that the relationship between two variables x and y is $y = ka^x$, where a and k are constants. An experiment is performed and the following pairs of data values (x, y) were obtained.

x	1	2	3	4	5
y	5.9	12	26	49	96

   (a) Plot an appropriate graph to verify that the relation $y = ka^x$ is valid.

   [4 marks]

   (b) Use the graph to find estimates for k and a.

   [4 marks]

   [**Total: 8 marks**]

3. (a) Solve the equation $x^2+(1-\sqrt{3})x-\sqrt{3}/2=0$ by completing the square. Leave your answer in surd form.

[5 marks]

(b) Hence, or otherwise, solve

$$\tan^2 2\theta + \tan 2\theta = \frac{\sqrt{3}}{2}\left(1+2\tan 2\theta\right)$$

for θ between $-90°$ and $90°$.

[5 marks]

[**Total: 10 marks**]

4. (a) Find the coordinates of the point on the curve

$$y=\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 \qquad (x>0)$$

where the tangent is parallel to the line $4y-3x+\sqrt{2}=0$.

[5 marks]

(b) A small square is drawn inside a larger square. The sides of the smaller square are increasing at the rate of 2 cm/s whereas the sides of the larger square are decreasing at the rate of 1 cm/s. Find the rate of change of the area enclosed by the two squares when the squares have sides of length 5 cm and 12 cm, respectively.

[5 marks]

[**Total: 10 marks**]

5. The 5^{th} term of an arithmetic progression is -5, whilst the 10^{th} term is -15.

(a) Find the value of the first term and the common difference of this progression.

[4 marks]

(b) Starting from the first term, how many terms need to be added for their sum to be -320 ?

[3 marks]

(c) The first two terms of the arithmetic progression are also the first two terms of a geometric progression. Explain why the sum to infinity of this geometric progression exists and find the sum to infinity.

[3 marks]

[**Total: 10 marks**]

6. (a) Find the coordinates of the stationary points of the curve $y = \dfrac{x^2+9}{2x}$ and determine their nature.

[5 marks]

(b) Solve the differential equation

$$x(y^2-1)\frac{\mathrm{d}y}{\mathrm{d}x} = y(y^2-3),$$

given that $y = 2$ when $x = 1$.

[7 marks]

[**Total: 12 marks**]

7. (a) A chord subtends an angle of $\pi/6$ radians at the centre of a circle. The minor segment formed between this chord and the corresponding arc has an area of $(12\pi - 36)$ cm^2. What is the radius of the circle?

[5 marks]

(b) The line ℓ has equation $y = 2x + 7$ and the curve $\mathscr{C}$ has equation $y = 2x^2 + 4x + 3$.

(i) Find the x-coordinates of the points of intersection of ℓ and $\mathscr{C}$.

[2 marks]

(ii) Evaluate the area enclosed between ℓ and $\mathscr{C}$.

[5 marks]

[**Total: 12 marks**]

8. Ms Brenda's students, 10 boys and 6 girls, are to be seated in the front row of a cinema theatre to watch a film. The row consists of 16 seats.

(a) In how many ways can the 16 students be assigned their seats?

[2 marks]

(b) What is the probability that the 6 girls sit next to each other?

[3 marks]

Ms Brenda chooses 4 students at random to purchase popcorn during intermission.

(c) In how many ways can she choose these 4 students?

[2 marks]

(d) What is the probability that she chooses 4 girls?

[3 marks]

[**Total: 10 marks**]

9. A triangle has vertices at $A(1,6)$, $B(4,2)$ and $C(5,9)$.
   (a) Find the gradients of the sides AB and AC, and hence, or otherwise, prove that the triangle is right-angled at A.

   [3 marks]

   (b) Prove that the triangle is isosceles.

   [2 marks]

   (c) Find the equation of the line ℓ containing the points A and C, giving your answer in the form $ax + by + c = 0$.

   [3 marks]

   (d) Find the distance of the point $D(2,10)$ from the line ℓ.

   [2 marks]

   [**Total: 10 marks**]

10. (a) A matrix is given by $M = \begin{pmatrix} 2 & 10 \\ 1 & a \end{pmatrix}$.
   (i) Find the value of the constant a for which this matrix does **not** have an inverse.
   (ii) Find the inverse of M in the case when $a = 4$, giving your answer in the form $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$, where e, f and h are integers and g is a rational number.

   [4 marks]

   (b) The coefficient of x^2 in the expansion of $(1-2x)^n$ is 84. Find n, given that n is a positive integer.

   [4 marks]

   [**Total: 8 marks**]